Discussion 10

1. Given the SAT problem from lecture for a Boolean expression in Conjunctive Normal Form with any number of clauses and any number of literals in each clause. For example,

$$(X_1 \vee \neg X_3) \wedge (X_1 \vee \neg X_2 \vee X_4 \vee X_5) \wedge \ldots$$

Prove that SAT is polynomial time reducible to the 3-SAT problem (in which each clause contains at most 3 literals.)

2. The *Set Packing* problem is as follows. We are given m sets $S_1, S_2, \ldots S_m$ and an integer k. Our goal is to select k of the m sets such that no selected pair have any elements in common. Prove that this problem is **NP**-complete.

3. The *Steiner Tree* problem is as follows. Given an undirected graph $G=(V,E)$ with nonnegative edge costs and whose vertices are partitioned into two sets, R and S, find a tree $T \subseteq G$ such that for every v in R, v is in T with total cost at most C. That is, the tree that contains every vertex in R (and possibly some in S) with a total edge cost of at most C.
Prove that this problem is **NP**-complete.