CS570 Spring 2022: Analysis of Algorithms   Exam I

	Points		Points
Problem 1	20	Problem 5	18
Problem 2	9	Problem 6	16
Problem 3	6	Problem 7	13
Problem 4	8	Problem 8	10
	Total	**100**	

Instructions:

1. This is a 2-hr exam. Open book and notes. No electronic devices or internet access.
2. If a description to an algorithm or a proof is required, please limit your description or proof to within 150 words, preferably not exceeding the space allotted for that question.
3. No space other than the pages in the exam booklet will be scanned for grading.
4. If you require an additional page for a question, you can use the extra page provided within this booklet. However please indicate clearly that you are continuing the solution on the additional page.
5. Do not detach any sheets from the booklet. Detached sheets will not be scanned.
6. If using a pencil to write the answers, make sure you apply enough pressure, so your answers are readable in the scanned copy of your exam.
7. Do not write your answers in cursive scripts.
8. This exam is printed double sided. Check and use the back of each page.

1) 20 pts

Mark the following statements as **TRUE** or **FALSE** by circling the correct answer. No need to provide any justification.

[TRUE/FALSE]

If we add *1* unit to the cost of the two lowest cost edges in the graph G, then the cost of the MST of G will increase by *2* units

[TRUE/FALSE]

Every weighted undirected graph has at least one MST

[TRUE/FALSE]

We say that an algorithm runs in $O(1)$ if it takes it constant time to run when the problem size $n=1$

[TRUE/FALSE]

A binary tree with k levels has 2^k-1 nodes.

[TRUE/FALSE]

The worst-case time complexity of merge sort is $O(n^2)$

[TRUE/FALSE]

The worst case runtime of binary search satisfies the recurrence relation $T(n) = 2T(n/2)+c$ where c is a constant

[TRUE/FALSE]

If a directed acyclic graph with 4 nodes has a unique topological ordering (*ABCD*), then it must have at least 3 edges.

[TRUE/FALSE]

Prim's algorithm cannot handle negative cost edges

[TRUE/FALSE]

A binary max heap can be converted into a min heap by reversing the order of the elements in the heap array.

[TRUE/FALSE]

For $n>3$, a directed graph with n nodes and n edges can be strongly connected.

2) 9 pts

Circle ALL correct answers (no partial credit when missing some of the correct answers). No need to provide any justification.

i- If a binomial heap contains these three trees in the root list: B0, B1, and B3, after 2 Extract_min operations it will have the following trees in the root list. (3 pts)

a) a B0 and a B3

b) a B1 and a B2

c) a B0, a B1 and a B2

d) None of the above

ii- What's the solution to the recurrence $U(n) = 2U(n/2) + n \log n + 2n$. (3 pts)

a) $U(n) = \Theta(n \log n)$

b) $U(n) = \Theta(n \log^2 n)$

c) $U(n) = \Theta(n \log \log n)$

d) None of the above

iii- Let $G(V,E)$ be a weighted undirected connected graph. Which of the following are true? Choose all that are true. (3 pts)

(a) Minimum edge in a graph is always part of a MST.

(b) Minimum edge in a cycle is always part of a MST.

(c) Maximum edge in a cycle is never part of a MST.

(d) Maximum edge in a graph is never part of a MST.

3) 6 pts

For the given recurrence equations, solve for $T(n)$ if it can be found using the Master Method. Else, indicate that the Master Method does not apply.

i) $T(n) = T(n/2) + 2^n$

ii) $T(n) = 5T(n/5) + n \log n - 1000n$

iii) $T(n) = 2T(n/2) + \log^2 n$

iv) $T(n) = 49T(n/7) - n^2 \log n^2$

4) 8 pts
Analyze the worst-case complexity of the following code snippets and provide a tight upper bound for each case. No explanations necessary. Each part has 2 pts.

i)
```
k = 0
for i = 1 to n
    k = k + 1
endfor
```

ii)
```
k = 1
for i = 1 to n
    j = 1
    while j <= i
        k = k + 1
        j = j * 2
    endwhile
endfor
```

iii)
```
k = 570
i = 1
while i < n
    i = i * k
endwhile
```

iv)
```
x = 0
k = 57075
for i = 1 to min of (n, k)
    for j = 1 to log n
        x = x + k
    endfor
endfor
```

5) 18 pts

During Spring break, which is m days long, USC is making its alumni park available to charitable organizations for fundraising events. However, only 1 event can be hosted on each day from day 1 to m. Each event lasts for exactly 1 day.

There are N prospective events. For each event i, there is a deadline D_i, denoting the last day on which it can be hosted, and expected funds F_i, denoting the funds it is expected to raise if it is hosted on or before its deadline. You cannot schedule an event after its deadline, i.e. event i can only be scheduled on days 1 through D_i only.

The objective is to create a schedule that will maximize the expected funds raised. Your schedule will assign n events to m days.

Note : n<=N and n<=m. (It may not be possible to schedule all events before their deadlines, in which case you will have to skip some events)

a) Consider the greedy algorithm that assigns n events to m days without any gaps and in increasing order of event deadline D_i (if by doing so an event i happens to land after its deadline Di then that event is skipped, i.e. not scheduled). Give a counterexample that shows this algorithm will not always yield an optimal solution. (5 pts)

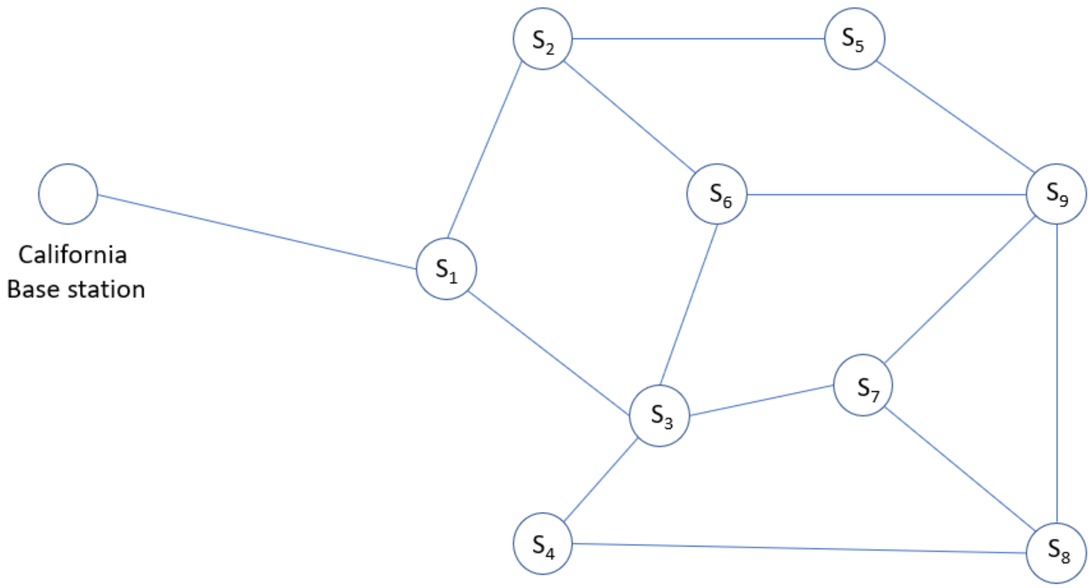
b) Consider the greedy algorithm that assigns n events to m days without any gaps and in decreasing order of Expected Funds F_i (if by doing so an event i happens to land after its deadline Di then that event is skipped, i.e. not scheduled). Give a counterexample that shows this algorithm will not always yield an optimal solution. (5 pts)

See next page for part c

c) Give an efficient greedy algorithm that will result in an optimal solution to this problem. No need to provide a proof of correctness. (8 pts)

6) 16 pts
Because of a bad update, you (the CTO of SpaceX) are in a hurry to fix the ongoing issue with the Starlink satellite. Your talented engineers have prepared the urgent fix, which needs to be sent to all the satellites as soon as possible.

The fix is first sent from the base station situated in California to satellite S_1. From there, you can send the fix from satellite S_i to a satellite S_j if there is a link (edge) between S_i and S_j. You know the time required to send the fix over this link from satellite S_i to satellite S_j is T_{ij}. We can assume that if a satellite S_i needs to send the fix to k of its neighbors the k messages can be sent out at the same time. In other words, messages from a satellite to its neighbors go out in parallel not sequentially. However, a satellite can send the fix to its neighbors only AFTER it has fully received it from a neighbor.



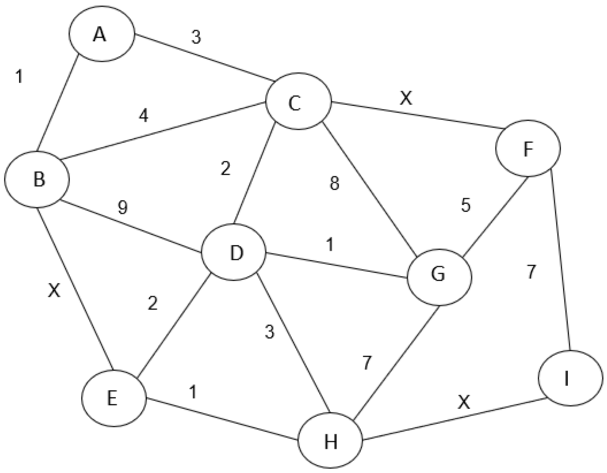
Note: The above diagram is only an example. The network may have any number of satellites in it.

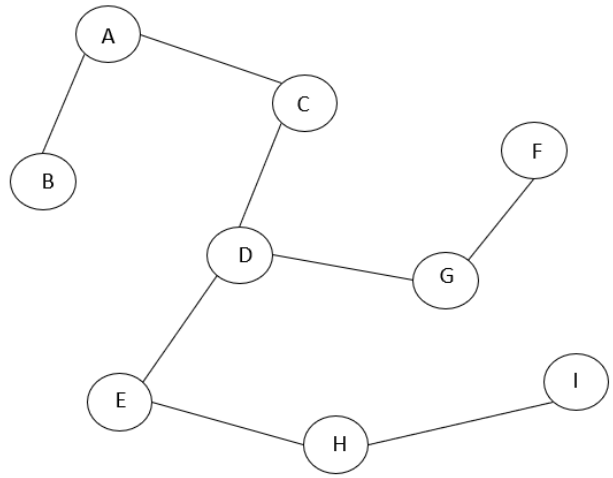
a) Provide an algorithm to determine the minimum time required to propagate the update to all satellites. In other words, we need to minimize the time between the first message leaving the base station and when the last satellite completes receiving the update (8 pts)

See next page for part b

b) Assuming that each satellite requires a single processing time D_i before broadcasting the fix to any nearby satellites that need to receive the fix, how would you modify the solution in part a to determine the minimum time required to propagate the fix to all satellites. (8 pts)

7) 13 pts
Consider the weighted undirected graph *G* on the left (see below graphs). Suppose the graph *T* on the right is the unique MST of *G*. Find the value of X, assuming X is an integer. You must provide the reasoning for your answer.





8) 10 pts

Consider the following set of preference lists for three men and women:

m_1	m_2	m_3	w_1	w_2	w_3
w_2	w_1	w_3	m_2	m_3	m_1
w_1	w_2	w_1	m_3	m_1	m_2
w_3	w_3	w_2	m_1	m_2	m_3

True or false: m_1 is a valid partner of w_1. If true, give a stable matching in which m_1 and w_1 are matched; if false, prove that m_1 is not a valid partner of w_1.

Additional Space

Additional Space

Additional Space