Free surface flow

Numerical methods part 2

Shaoxu Zheng (1064735), Water Science & Engineering (HI), IHE-Delft, Institute for Water Education, Delft, the Netherlands.

– v. good report though you use too little words
– you started well to analyse discretised terms then you stopped but overall good
– you show underst of what you do
– suggestions:
– make bigger graphs!

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Report mark: 8.7

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Introduction

Nice you tried both

Through coding, we obtain 12 tests for Free Surface Flow. Each set of tests applies two methods (matrix inversion and double-sweep algorithm) to calculate. Normally, the outcome is basically the same. Only when the model is unstable, double-sweep algorithm can get partial process data. Therefore, when the program reports an error, the figures used are all double-sweep algorithm.

All the test can be divided into 4 types according to the motion's state of the water flow, namely static test, steady-state test, transient test and seiche test. Below I will give an explanation for each test result and explain its cause separately. Some of them cannot calculate the result, I will comment the reasons in either numerical or physical.

For the parameters part, it is because the parameters are not selected properly. For the physical part, it is because the simulation data cannot be realized in practice.

Variable

The following table 1 is the meaning of the variables in the test data.

Table 1. Key variables

Variable	Meaning	Unit
h	Depth	m
Q	Discharge	m^3/s
L	Channel length	m
b	Constant channel width	m
S0	Bed slope	-
C	Chezy resistance coefficient	-

t	Computational time step	s
T	Total time	s
x	Cell size	m
ϴ	Time weighting parameter	-
Ψ	Space weighting parameter	-
β	Coefficient	-
Itera	Max No. of iteration	-
Err	Convergence criteria	-
g	Gravity	m/s^2

Note: ϴ, Ψ, β are three weighting parameters to execute a weighted fit. In the modelling theory, the less precise variable is given less weight. However, the more precise variable is given more weight. It reflects our lack of sureness about what we obtain when running the numerical model.

these weights have for diff. scope

In this model building method, the scheme becomes explicit as ϴ = 0. The scheme is fully implicit as ϴ = 1. Last, the scheme is fully unstable as ϴ < 0.5.Coefficitent ϴ can own any value in the scale [0, 1], so you can define any formula. In some cases, the model will report an error due to improper parameter selection, so we should be careful when selecting it.

it is for making the scheme explicit or implicit.

Case 1. Static test

Data designing idea

The flow in initially at rest and must remain at rest for a long period of simulation can be named static flow. In the designing dataset, the initial and boundary conditions are set to Discharge-free inflow, and the water level is tested for certain disturbances.

I set up three groups for comparison. The difference between the groups is the weight of ϴ. Since the parameter that affects the final state to form a stationary state is time weighting parameter.

The following table 2 is the dataset for static test.

Table 2. Variable and value

Variable	Value		
L	800		
b	70		
S0	0		
C	1000000		
t	10		
T	20000		
x	30		
ϴ	0.1	0.5	0.7
Ψ	0.5		
β	1		
Itera	10		
Err	0.01		
g	9.8		

Table 3. Initial and boundary conditions

Condition	Value	
Initial condition	h	Q
	0.5	0
Upstream boundary	h	
	2	
Downstream boundary	h	
	2	

Result and analyse

No need for such part, but ok appreciated

Case 1.1 ϴ = 0.1

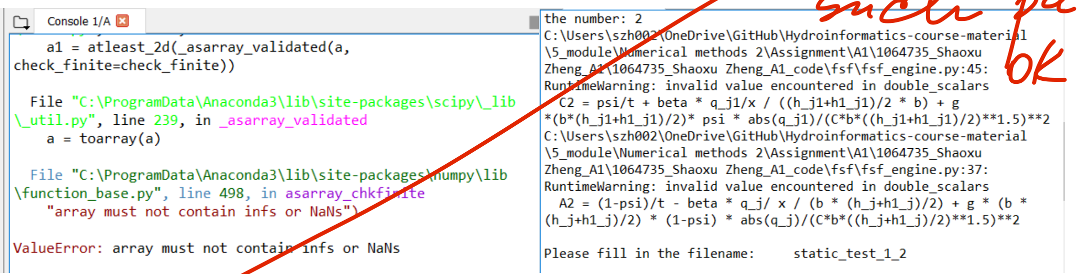

Figure 1. Error interface

When I apply matrix inversion calculation, the program reports that array contain Nans. When I run the double-sweep algorithm, the model warns that invalid value encountered in double_scalars. Since we apply ϴ = 0.1, the model is fully unstable, which causes h (depth) is less than 0, making it impossible to calculate the following term (1)

$$\frac{gA_j{}^{n+1/2}\Psi|Q_{j+1}{}^{n}|}{(Cb(h_{j+1}{}^{n+1/2})^{3/2})^2} \tag{1}$$

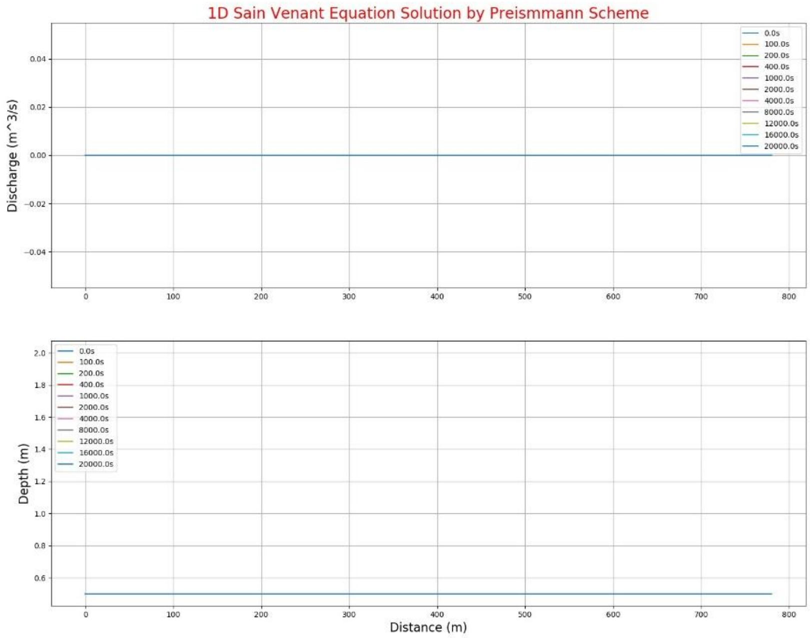

Figure 2. The result of Case 1.1

For matrix inversion scheme, this leads to appear a null value in the matrix and fail to calculate the next time's value, either Q or h. For double-sweep algorithm, because Q and h are calculated using the loop method, when this situation is encountered, but we can still run the model and end up with an empty graph. This means that h has been less than 0 from the beginning calculation.

Case 1.2 ϴ = 0.5

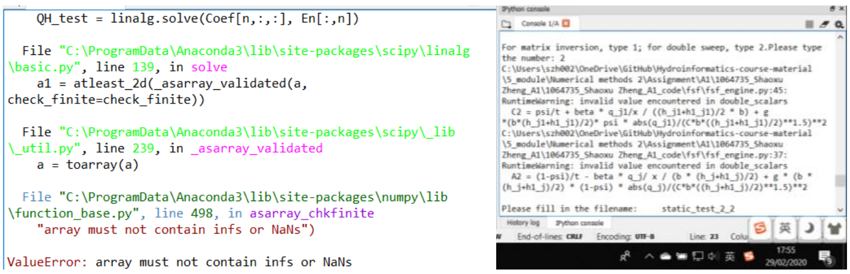

Figure 3. Error interface

Both methods report an error and the program will not run.Matrix inversion method cannot give any graph. However, double-sweep will give some part of result, the procession figure 4 is as follows.

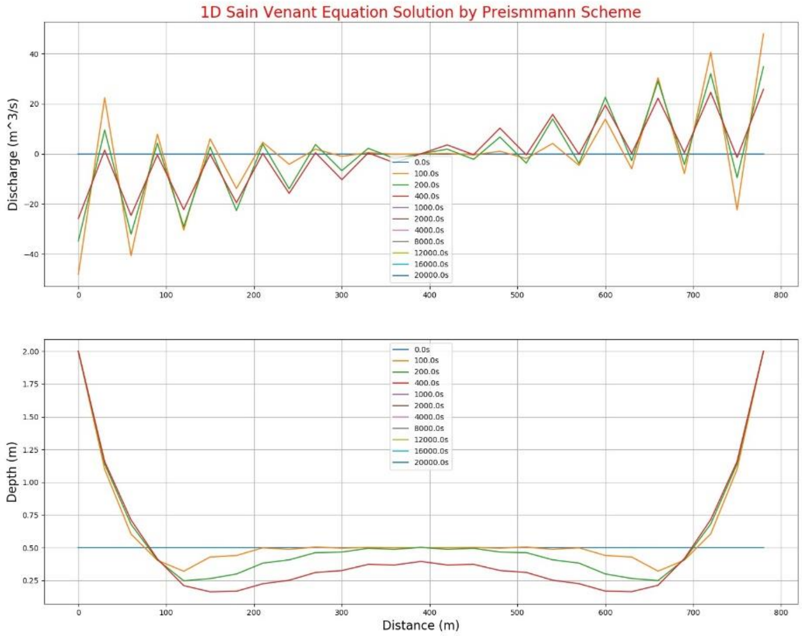

Figure 4. The result of Case 1.2

For discharge, we can see that it presents rotational symmetry phenomenon. The initial discharge is 0 m^3/s. And then, the discharge becomes unstable with jagged shock. As time increases, polylines shocks diminish and tends to 0 m^3/s. Through the depth graph, it indicates that the initial depth is 0.5 m. After that, all the water profile boundary curves intersect at 2m point. Each curve appears as an inverted saddle, and the curve shift to zero line until h is less than 0, which cannot be continually calculated.

Therefore, Time and space weighting parameter equalling to 0.5 and 0.5 does not guarantee stability and convergence, and 0.5 is just experimental value. In some cases, it is solvable under certain conditions and no solution under certain conditions. In view of this situation, you have two choices, increase weight to 0.7, or change the initial values to about 1.5m. The first hypothesis will be discussed in the following test. The second hypothesis is achievable, and the following is the testing figure.

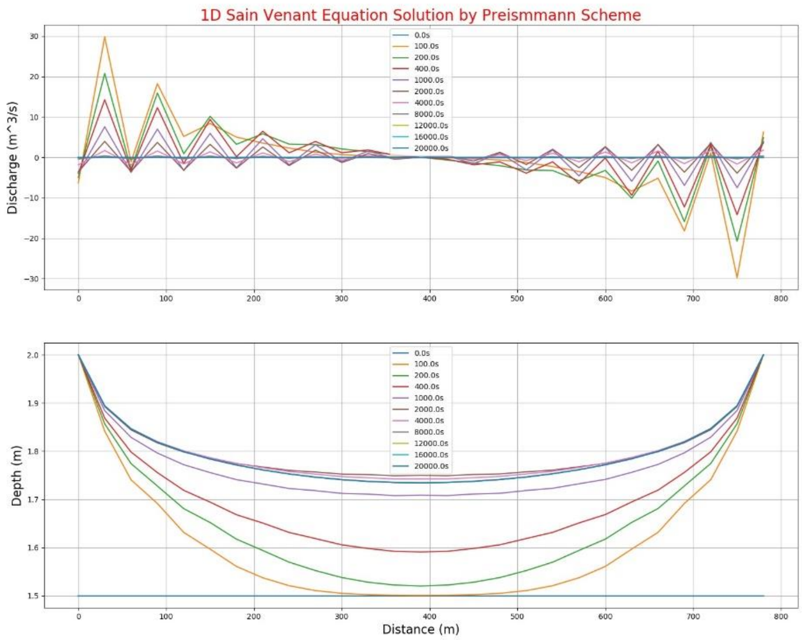

Figure 5. The result of Case 1.2(initial h = 1.5m)

Through observation, the change of discharge is the same as previous one, but finally closes at line Q=0 m^3/s. However, the depth curve is impossible to appear in reality. The water surface is sunken and has maintained this state without any outside help. This shows that the correct simulation can only be obtained by changing the initial conditions on the premise that the schemes (parameters in this case) are correctly solvable. Otherwise the simulation is useless.

Case 1.3 ϴ = 0.5

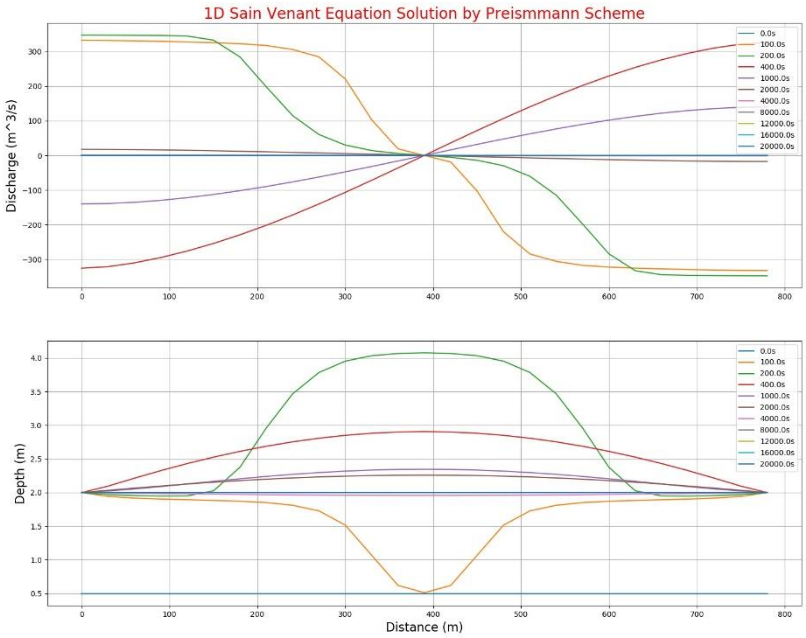

Figure 6. The result of Case 1.3

This graph shows the procession how the depth rise from 0.5m to 2m. Observing the discharge diagram, at the initial stage, the absolute value of the flow at t=100 is greater than the value of t=200s, which means that there is irregular movement. The data after t=400s shows a damped vibration, and the peak value is getting smaller and smaller. After a long period, the final curve will gradually approach the initial condition Q=0. The Q satisfies the conditions envisaged by the static test.

For depth graph, expect that the initial condition of t=0 is a line, the rest are curves intersecting at 2m at two boundaries. The peaks of these curves decrease with time and eventually the profile approximate a straight line of h=2m.This meets the requirements and assumptions of the static test. However, the curve is not symmetrical with line h=2m. The curve's peak value of t=200s exceeds 4m. Furthermore, this is already greater than the difference between the height of the final state and the initial state. If we assume the flow as a spring disturbed and starts to vibrate, that cannot be happened in the real world.

Although the simulation of static in the two figures is in line with reality, there are still some places that are strange. Whether it is a strange disturbance of discharge at both boundaries in the discharge graph, or the peak value of the profile at a certain moment in the depth graph exceeds the starting and ending difference. It's all caused by abrupt changes in the starting and boundary conditions. This is also because everything in reality is continuous. This phenomenon is the flaw of the model when building. But after a period of time, these anomalies disappeared because the model automatically solved this problem. This also illustrates that the initial value affects the outcome but only causes a certain degree of disturbance at the beginning. In contrast, the boundary conditions are more important due to the influence of the entire exercise process.

By comparison, Case 3(ϴ = 0.5) is the most appropriate solution. Since its weighting parameters setting are more reasonable, it satisfies various imaginations of static test. Initial conditions are not variables that determine the results of the simulation mostly procession. Its setting is only partially abnormal at the beginning time. There only be two situations, one is that the model warns an error since h is less than 0 due to initial disturbance, another is that the model normal run in the most of the time without effect on outcomes.

Case 2. Steady test

Data designing idea

This situation is where the flow is initially flowing in a steady state and the boundary conditions remains unchanged. We can imagine that both discharge and depth remain on the certain horizontal line without any change. To verify the effect of roughness, we increase the roughness making the Chezy resistance coefficient equal to 10000. The depth's profile will slightly change every time period. This test is relatively simple and only gives a set of data for testing.

Table 3. Variable and value

Variable	Value
L	16000
b	125
S0	0
C	10000
t	100
T	80000
x	1000
Ɵ	0.5
Ψ	0.5
β	1
Itera	15
Err	0.01
g	9.8

Table 4. Initial and boundary conditions

<table>
<tr><td>Condition</td><td colspan="2">Value</td></tr>
<tr><td rowspan="2">Initial condition</td><td>h</td><td>Q</td></tr>
<tr><td>12.7</td><td>400</td></tr>
<tr><td rowspan="2">Upstream boundary</td><td colspan="2">Q</td></tr>
<tr><td colspan="2">400</td></tr>
<tr><td rowspan="2">Downstream boundary</td><td colspan="2">Q</td></tr>
<tr><td colspan="2">400</td></tr>
</table>

Result and analyse

The matrix inversion method cannot provide a graph, the same happens in the double-sweep situations. Since the second method can give a profile before reporting the error.

```
    QH_test = linalg.solve(Coef[n,:,:], En[:,n])

  File "C:\ProgramData\Anaconda3\lib\site-packages\scipy\linalg
\basic.py", line 139, in solve
    a1 = atleast_2d(_asarray_validated(a,
check_finite=check_finite))

  File "C:\ProgramData\Anaconda3\lib\site-packages\scipy\_lib
\_util.py", line 239, in _asarray_validated
    a = toarray(a)

  File "C:\ProgramData\Anaconda3\lib\site-packages\numpy\lib
\function_base.py", line 498, in asarray_chkfinite
    "array must not contain infs or NaNs")

ValueError: array must not contain infs or NaNs
```

```
Please type the test number you want. (1~12):    4

Now you should choose which method you apply, whether matrix
inversion or double sweep.

For matrix inversion, type 1; for double sweep, type 2.Please type
the number: 2
C:\Users\szh002\OneDrive\GitHub\Hydroinformatics-course-material
\5_module\Numerical methods 2\Assignment\A1\1064735_Shaoxu
Zheng_A1\1064735_Shaoxu Zheng_A1_code\fsf\fsf_engine.py:45:
RuntimeWarning: invalid value encountered in double_scalars
  C2 = psi/t + beta * q_j1/x / ((h_j1+h1_j1)/2 * b) + g
*(b*(h_j1+h1_j1)/2)* psi * abs(q_j1)/(C*b*((h_j1+h1_j1)/2)**1.5)**2

Please fill in the filename:    1
```

Figure 7. Error interface

The following are the results of the model simulation before the error is reported.

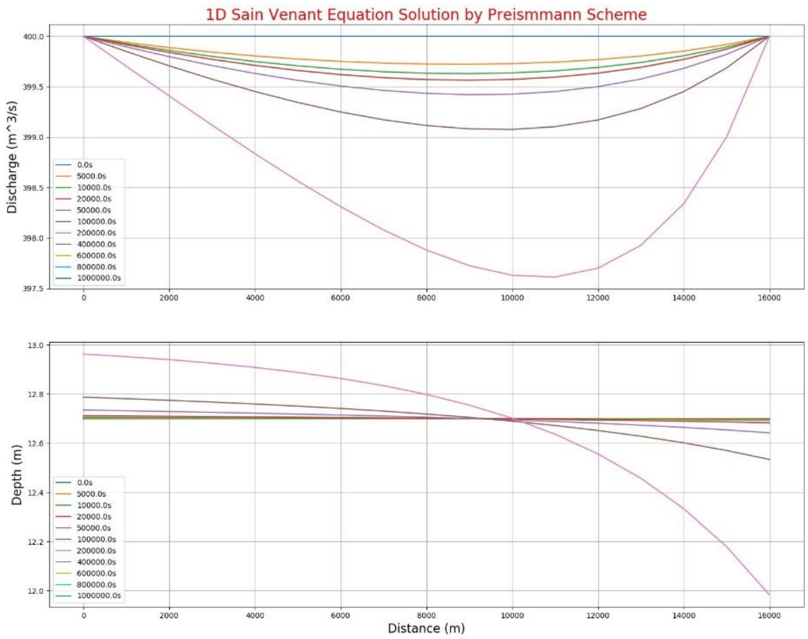

Figure 8. The result of Case 2

For the discharge figure, we can see that the discharge starts as a straight line Q=400m^3/s, and then discharge changes slightly in the beginning time. Each curve is sunken and the lowest dipped point locates in the end of the time. Over time, the lower the value of the lowest point of discharge for the entire channel, the more it moves backward (close to the downstream boundary). The simulated flow finally stopped at about 398.5m^3/s.

The curve anomaly occurs at the downstream boundary, since the discharge should always fall along the channel due to friction. However, boundary conditions are constant, so discharge will rise strangely to a fixed value -- 400m^3/s.

For the depth figure, it illustrates that the initial depth is a straight line h=12.7m. The first half of the curve increase slowly from upstream to downstream. Before the inflection point of the discharge occurs, the depth starts to decrease, and the curve drops faster when it approaches the downstream. Over time, the downstream data drops to less than 0, at which time the model starts to report error.

13.268	13.3923	13.5061	13.5909	13.6182	13.5421	13.2816	12.6679	11.2185	2.62464
13.2696	13.3943	13.5086	13.5939	13.6219	13.5467	13.2873	12.6749	11.2269	2.53893
13.2712	13.3964	13.5111	13.5971	13.6258	13.5515	13.2932	12.6823	11.236	2.44923
13.2728	13.3984	13.5137	13.6003	13.6298	13.5565	13.2994	12.6902	11.2459	2.35506
13.2745	13.4006	13.5164	13.6036	13.634	13.5617	13.306	12.6986	11.2568	2.25584
13.2762	13.4027	13.5191	13.6071	13.6383	13.5671	13.313	12.7077	11.2688	2.15085
13.278	13.405	13.5219	13.6106	13.6428	13.5728	13.3204	12.7174	11.2821	2.03918
13.2798	13.4073	13.5249	13.6143	13.6475	13.5789	13.3282	12.7281	11.297	1.91967
13.2816	13.4096	13.5279	13.6181	13.6524	13.5853	13.3367	12.7398	11.314	1.79076
13.2835	13.412	13.531	13.6221	13.6575	13.5921	13.3459	12.7527	11.3335	1.65035
13.2855	13.4146	13.5342	13.6263	13.663	13.5994	13.356	12.7673	11.3564	1.49536
13.2875	13.4172	13.5376	13.6307	13.6688	13.6073	13.3672	12.784	11.3841	1.32109
13.2895	13.4199	13.5411	13.6354	13.6751	13.6159	13.3797	12.8038	11.4192	1.11958
13.2917	13.4227	13.5448	13.6403	13.6819	13.6256	13.3943	12.828	11.467	0.875063
13.2939	13.4256	13.5487	13.6456	13.6893	13.6365	13.4118	12.8601	11.544	0.544547
nan	nan	nan	nan	nan	nan	nan	nan	nan	nan
nan	nan	nan	nan	nan	nan	nan	nan	nan	nan
nan	nan	nan	nan	nan	nan	nan	nan	nan	nan
nan	nan	nan	nan	nan	nan	nan	nan	nan	nan
nan	nan	nan	nan	nan	nan	nan	nan	nan	nan
nan	nan	nan	nan	nan	nan	nan	nan	nan	nan

Figure 9. The depth value in the variable interface

The time weighting parameters equal to 0.5 will lead to the downstream depth to be infinitely close to 0 until it fails to be calculated. So I change it to 0.8.

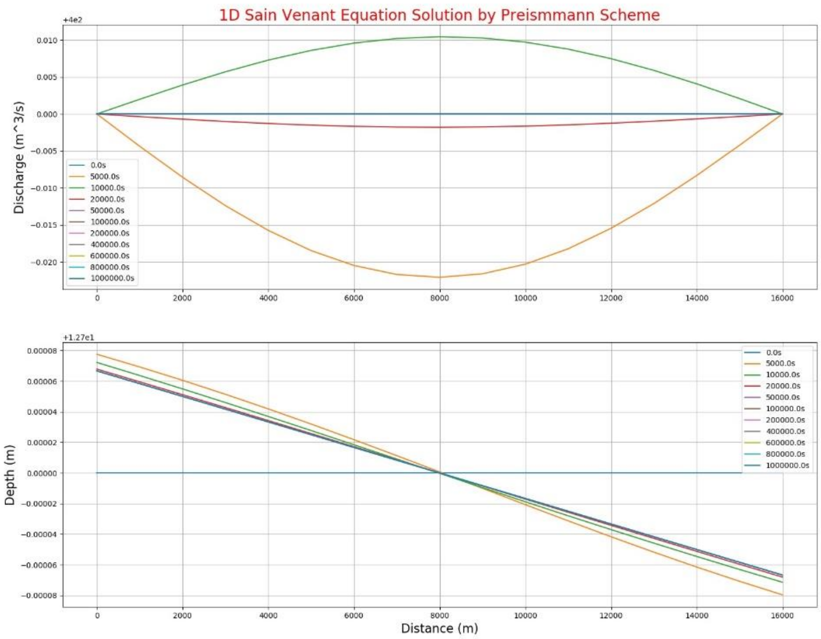

Figure 10. The result of Case 2

When I change the time weighting parameter to 0.8, the program no longer reports error. The discharge image oscillates along the line Q=400m^3/s. The depth's value in the front channel is gradually higher, but the value goes down over time, forming a dammed water. Note that all the changes in both graphs are within a small area.

Combined with relevant knowledge, the depth's profile should form a dammed water due to the friction. And the discharge will keep the same along the channel. In the end, the profile of both Q and h will be fixed after a long time due to the balance with friction.

This abnormal phenomenon occurs because both the upstream and downstream boundary conditions are set to discharge, and it cannot happen for the left and right discharges to always remain equal in the friction cases. This leads to the discharge cannot change in the boundary. This involves the designing idea of boundary conditions. In general, we should avoid to set two boundaries to be the same variable. Otherwise, this cannot be achieved in reality, and the response is an error warning in the model. ✓ ✓ good

Case 3. Transient test

Data designing idea

This situation is where the fluid initially flowing in a steady state and the boundary conditions are subjected to a sudden change. When designing the data, I no longer consider the impact of friction, so the coefficient is set to infinity (1000000). Here we still test different time weighting parameters, 0.1, 0.5, and 0.7 separately. I will test two cases, one is constant boundary conditions, and the other is periodic functions. At last, the test should carry out computations for 3-4 full cycles, when possible. 1 full cycle is a travelling back and forth of the wave.

Table 5. Variable and value

Variable	Value		
L	7200		
b	100		
S0	0		
C	1000000		
t	25		
T	20000		
x	72		
Ɵ	0.5	0.55	0.8
Ψ	0.5		
β	1		
Itera	15		
Err	0.01		
g	9.8		

Table 4. Initial and boundary conditions

Condition	Value	
Initial condition	h	Q
	13.8	625
Upstream boundary	Q	
	700	
Downstream boundary	Q	

	625

Result and analyse

Case 3.1 ϴ = 0.5

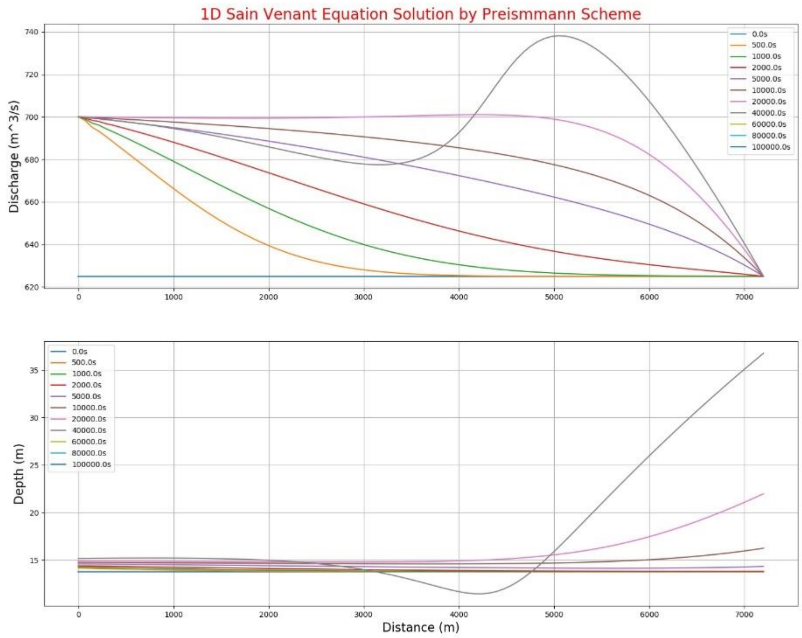

Figure 11. The result of Case 3.1

The matrix inversion cannot give the correct result because of the depth less than 0 in some point. In THE discharge diagram, the two boundaries of the curve are fixed. The curve is perpendicular to the line connecting the two points of upstream and downstream, and then propagates to the downstream direction. At t=40000s, local minimum and local maximum occur in the flow curve occurring anomalies.

For depth figure, the depth rise over time in the front of the channel. In the end of the channel, the depth sharply go up over time. At t=40000s, the curve appears dipping phenomenon. This is the last curve in the graph, so we have reason to infer that the depth will report an error since the depth is less than 0.

Case 3.2 and Case 3.3 ϴ = 0.55, 0.8

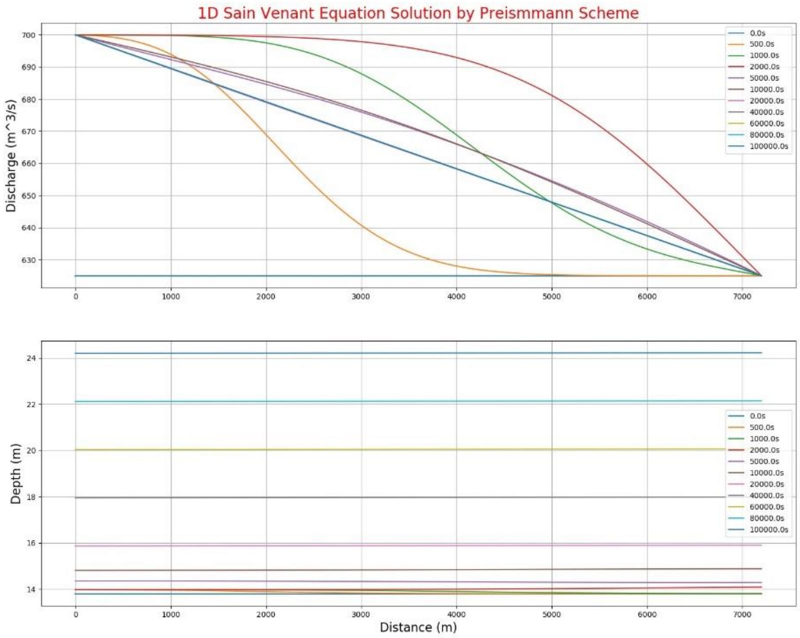

Figure 12. The result of Case 3.2

The change trend of the flow is basically consistent with the above case, but its flow began to fall and fluctuates after t =2000s. The final curve and the line connecting the two boundary points are close or even completely fitted. For depth, its level steadily go up. When t =100000s, the depth is around 24m.

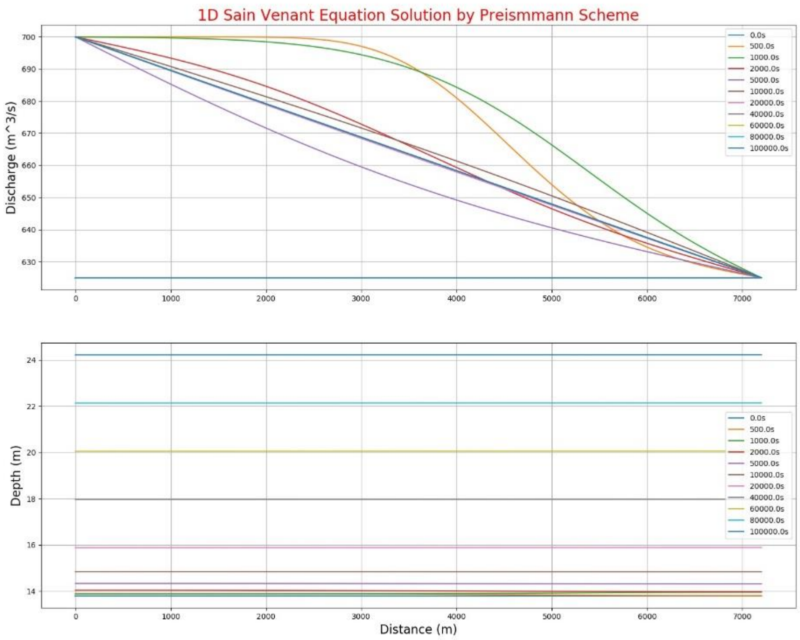

Figure 13. The result of Case 3.3

For Case 3.3, the discharge change is accelerated. Compared with the previous case, there are no other changes. The depth is almost the same as the previous case.

Overall, the discharge will eventually form a straight line connecting the two boundary conditions, and the depth will continue to rise. According to analyse, parameters will affect the trend of discharge and depth and the speed of convergence. It is more accurate when applying $\Theta = 0.8$. Here we can make inferences that the closer Θ to 1, the better the model fits the real situation.

Case 3.4 Θ = 0.5

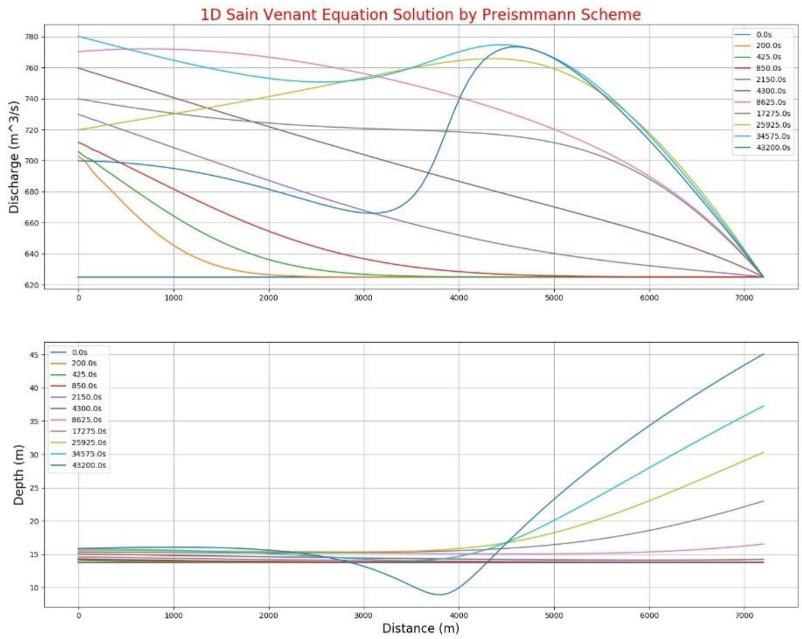

Figure 14. The result of Case 3.4

The discharge chart is generally chaotic, and the discharge will continue to decline in the early stage. But over time, the discharge profile appears the phenomenon of first descending, then rising and then falling. Next, for the depth chart, the same exception occurs for depth as the Case 3.1.

Case 3.5 and Case 3.6 ϴ = 0.55, 0.8

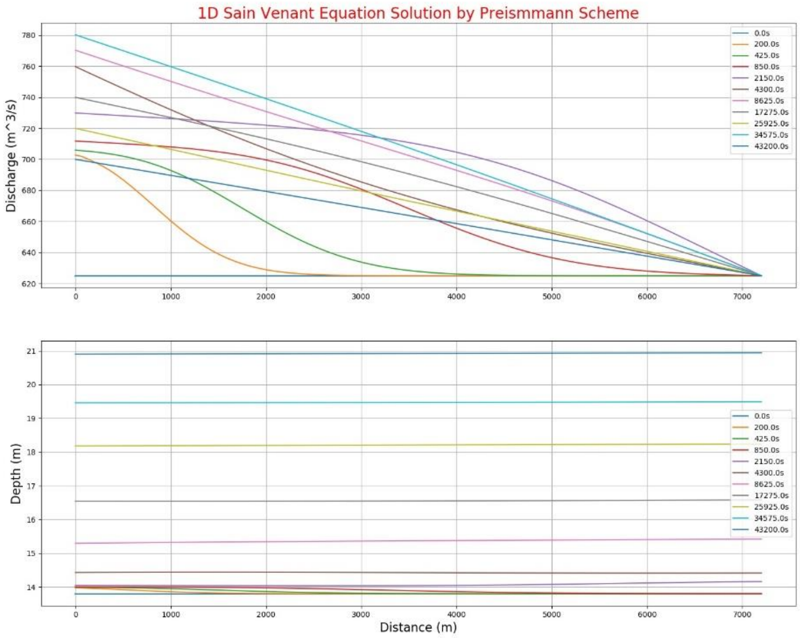

Figure 15. The result of Case 3.5

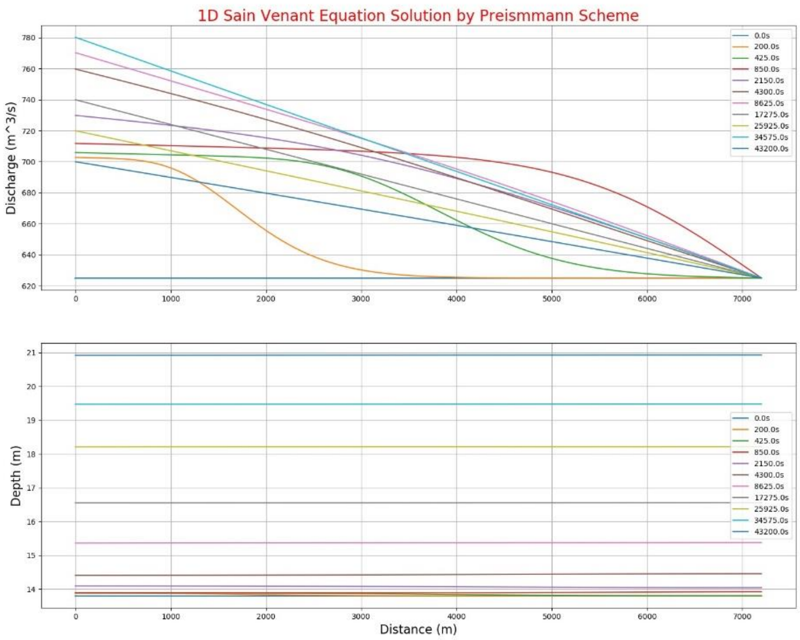

Figure 16. The result of Case 3.6

For these two schemes, the discharge profile is continuously declining while the depth level is slowly rising. However, in the case of theta=0.8, the discharge changes more rapidly, and it is easier to form a final state, straight line.

Conclusion

1. The time and space weighting parameters influence the model simulation effect. Comparison of Case 1 and Case 3, it is concluded that the Θ value closer to 1 (implicit), the more the model fits the reality. In experience, Ψ close to 0.5 is the optimal parameter of the model.
2. The initial conditions affect the previous model data, but these disturbances will be automatically handled by the model and will not interfere with the final result. Unless the initial disturbance causes depth to be less than 0, the program warns an error.
3. Boundary conditions determine the final result, so we try to avoid setting two boundary conditions to the same variable. That's why our downstream boundary in Mike 11 is set to a rating curve. This phenomenon is difficult to appear in reality.