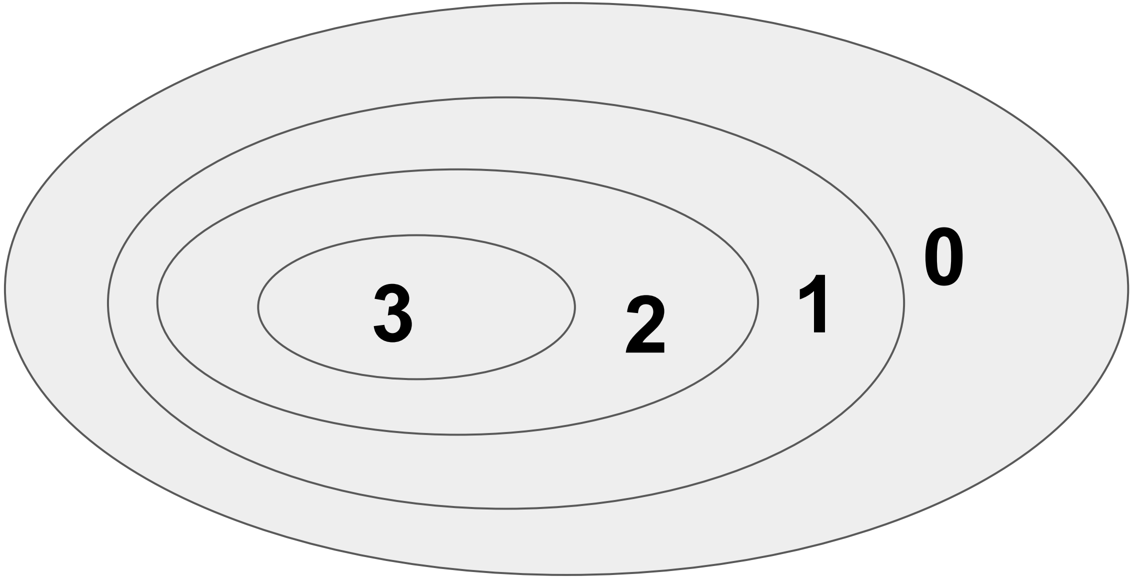
Natural Language Processing

Lecture 4
Formal grammars

Formal grammars

1. **Listing grammar**: if the language *L* consists of some chains then the language *L* can be described as a list of all chains;
2. **Generating grammar** specifies rules by which you can create any chain ("sentence") of this language;
3. **Recognizing grammar** (analytical grammar) allows to define if a word is contained in the language or not.

Generating grammars



0 - Unrestricted;
1 - Context-sensitive;
2 - Context-free;
3 - Regular

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Generating grammars

A grammar *G* is formally defined as a tuple <*N*, *Σ*, *P*, *S*>, where

- *N* is a finite set of nonterminal symbols;
- *Σ* is a finite set of terminal symbols that is disjoint from *N*;
- *P* is a finite set of production rules $\alpha \rightarrow \beta$;
- *S* is a distinguished symbol $S \in N$ that is the start symbol (sentence symbol).

Generating grammars

Alphabet is non-empty finite set, its elements are called symbols.

Chain (word, string) of symbols in alphabet Σ is a sequence of symbols from alphabet Σ. The number of symbols in chain α is called **length** and denoted as $|\alpha|$. Empty chain doesn't contain any symbols and denoted as e.

If α and β are chains then the chain $\omega = \alpha\beta$ is called **concatenation** of α and β. For any symbol a and integer k ($k \geq 0$) concatenation is denoted as a^k: $a^0 = e$, $a^1 = a$ and $a^{k+1} = a^k a$ for $k \geq 1$.

For any chains α, β and γ the chain β is **prefix** of chain $\beta\alpha$, **suffix** of chain $\alpha\beta$ and **subchain** of chain $\alpha\beta\gamma$.

Reversal of chain α (α^R) is a chain α, written in the reverse order, $e^R = e$.

Generating grammars

Example:

Binary **alphabet** is a set {0, 1}. The sequence α = 000110 is a chain with **length** 6. α is a **concatenation** of chains γ = 0001 and β = 10, i.e. $\alpha = \gamma\beta$.

Chains 0011, 000110, 1 are **subchains** of α

1^0 is **empty chain**

$\alpha^R = 011000$

$1^3 = 111$

Generating grammars

Let Σ be an alphabet. Σ^* is a set of all chains in alphabet Σ including empty chain e, Σ^+ is a set $\Sigma^* \setminus e$.

Formally a set Σ^* is defined by the following rules:

1. $e \in \Sigma^*$;
2. if $\alpha \in \Sigma^*$ and $a \in \Sigma$ then $\alpha a \in \Sigma^*$;
3. $\alpha \in \Sigma^*$ iff α is a chain in alphabet Σ due to 1 and 2.

Set Σ^* is called **Kleene closure**.

For a set of symbols we get a set of chains:

$\{‘a’, ‘b’, ‘c’\}^* = \{e, ‘a’, ‘b’, ‘c’, ‘aa’, ‘ab’, ‘ac’, ‘ba’, ‘bb’, ‘bc’, ‘ca’, ‘cb’, ‘cc’, ‘aaa’...\}$.

Generating grammars

Example:

The grammar G is a tuple $<\{A, S\}, \{0, 1\}, P, S>$, where P consists of the following rules:

- $S \rightarrow 0A1$;
- $0A \rightarrow 00A1$;
- $A \rightarrow e$.

A, S - nonterminal symbols;

0, 1 - terminal symbols;

S - start symbol.

Generating grammars

Let $G = \langle N, \Sigma, P, S \rangle$ be a grammar. The relation of a **direct deducibility** $\Rightarrow_G$ on a set $(N \cup \Sigma)^*$ is defined as follows: if $\alpha\beta\gamma$ is a chain from $(N \cup \Sigma)^*$ and $\beta \rightarrow \sigma$ is a rule from P then $\alpha\beta\gamma \Rightarrow_G \alpha\sigma\gamma$.

Transitive closure of the relation of a direct deducibility is denoted as $\varphi \Rightarrow^+_G \psi$.

Reflective and transitive closure of the relation $\Rightarrow_G$ is denoted as $\varphi \Rightarrow^*_G \psi$. **Reflexivity**: $\varphi \Rightarrow^*_G \varphi$; **transitivity**: $\varphi \Rightarrow^*_G \psi$, $\psi \Rightarrow^*_G \theta$ then $\varphi \Rightarrow^*_G \theta$.

The chain α is called **deduced chain** in grammar G if $S \Rightarrow^*_G \alpha$.

The language generated by grammar G (denoted as $L(G)$) is a set of terminal deduced chains of grammar G, i.e. $L(G) = \{\alpha : \alpha \in \Sigma^*, S \Rightarrow^*_G \alpha\}$.

Regular grammar

G = <{A, S}, {0, 1}, P, S> and the deduction $S \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 0011$ by the rules.

One can show that the grammar generates the language $L(G) = \{0^n1^n : n \geq 1\}$.

Context-free grammar

$G = \langle\{E, T, F\}, \{a,+,*,(,)\}, P, E\rangle$, where P consists of the following rules:

- $E \rightarrow E + T \mid T$;
- $T \rightarrow T * F \mid F$;
- $F \rightarrow (E) \mid a$.

We can obtained the following deduction: $E \Rightarrow E{+}T \Rightarrow T{+}T \Rightarrow F{+}T \Rightarrow a{+}T \Rightarrow a{+}T{*}F \Rightarrow a{+}F{*}F \Rightarrow a{+}a{*}F \Rightarrow a{+}a{*}a$.

Context-sensitive grammar

Let grammar P is defined by the following rules:

- $S \rightarrow aSBC$;
- $S \rightarrow abC$;
- $CB \rightarrow BC$;
- $bB \rightarrow bb$;
- $bC \rightarrow bc$;
- $cC \rightarrow cc$.

Possible deduction: $S \Rightarrow aSBC \Rightarrow aabCBC \Rightarrow aabBCC \Rightarrow aabbCC \Rightarrow aabbcC \Rightarrow aabbcc$.

This grammar generates a language $L(G) = \{a^n b^n c^n : n \geq 1\}$.

Regular sets

Regular operations: union, concatenation, iteration. Regular sets are come out from elementary languages as a result of applying a finite number of regular operations.

Let Σ be a finite alphabet. **Regular set** in alphabet Σ is defined by the following recursive features:

1. $\varnothing$ is a regular set;
2. $\{e\}$ is a regular set;
3. $\{a\}$ is a regular set $\forall\ a \in \Sigma$;
4. if P, Q are regular sets then sets $P \cup Q$, PQ, P^* are also regular sets;
5. there isn't any others regular sets in alphabet Σ.

Regular sets are sets of symbols chains on a given alphabet, made in specified way (with using of regular operations).

Regular expressions

Regular expressions (RE) in alphabet *Σ* is defined recursively in the following way:

1. Ø is a RE denoting a regular set Ø;
2. *e* is a RE denoting a regular set {*e*};
3. if $a \in \Sigma$ then a is RE denoting a regular set {*a*};
4. if *p* and *q* are REs denoting regular sets *P* and *Q* respectively then: union (*p*+*q*) is RE denoting a regular set $P \cup Q$; concatenation (*pq*) is RE denoting a regular set *PQ*; iteration $(p)^*$ is RE denoting a regular set P^*;
5. there isn't any others REs.

Regular expressions

Example 1:

The set of all chains composed from 0 and 1 and ended with a chain 001 can be described with RE $(0+1)^*011$.

Example 2:

RE $(a+b)(a+b+0+1)^*$ denotes the set of all chains from $\{0, 1, a, b\}^*$ started with *a* or *b*.

Regular expressions

Check if a string is ended with ‘*.com*’ or ‘*.org*’ or ‘*.ru*’

```
public static boolean test(String testString) {
    Pattern p = Pattern.compile(".+\\.(com|org|ru)");
    Matcher m = p.matcher(testString);
    return m.matches();
}
```

Regular expressions

Check if users names is valid ("*@_BEST*" and "*miha_10*"): the first one is not valid, the second one is valid.

```
public static boolean manualCheck(String userNameString) {
    char[] symbols = userNameString.toCharArray();
    if (symbols.length < 3 || symbols.length > 15) return false;
    String validationString = "abcdefghijklmnopqrstuvwxyz0123456798_";
    for (char c : symbols) {
        if (validationString.indexOf(c)== -1) return false;
    }
    return true;
}

public static boolean checkWithRegExp(String userNameString) {
    Pattern p = Pattern.compile("^[a-z0-9_]{3, 15}$");
    Matcher m = p.matcher(userNameString);
    return m.matches();
}
```

Regular expressions

Replace all variants of “*Тайланд*” with “*Россия*”.

```
public class Rexep {
    public static final String TEXT = "Мне очень нравится Тайланд. " +
            "Таиланд - это то место, куда бы я поехал. тайланд - мечты сбываются!";
    public static void main (String[] args) {
        System.out.println(TEXT.replaceAll("[Тт]а[ий]ланд", "Россия"));
    }
}
```

Relationships between regular sets, regular grammars and finite-state machines

Ways to determine the regular languages:

- regular grammars;
- finite-state machines (FSM);
- regular sets

Regular expressions and FSMs

- For any regular language, determined by a regular expression, one may build a FSM determining the same language;
- For any regular language, determined by a FSM, one may get a regular expression determining the same language

Regular grammars and FSMs

- Based on a regular grammar one may build the equivalent FSM;
- For determined FSM one may build the equivalent regular grammar

Example:

Let a rightmost regular grammar $G = (\{A, S\}, \{a, b, c\}, P, S)$, where P consists of the following rules:

- $S \rightarrow aS$;
- $S \rightarrow bA$;
- $A \rightarrow e$;
- $A \rightarrow cA$.

This grammar describes the same language $L(G) = \{a^n bc^n : n \geq 1\}$ as the regular expression $a^* bc^*$.

Recognizing grammars

Finite-state machine (FSM) is a transducer which allows to compare an input with the corresponding output, moreover this output can depend on not only the current input but also on previous FSM performance.

Non-determined finite-state machine (NFSM) is a tuple $M = <Q, \Sigma, \sigma, q_0, F>$, where:

- Q - finite non-empty set of states;
- Σ - input alphabet (non-empty set of symbols);
- σ - mapping a set $Q*\Sigma$ on set Q, which is called **state-transition function**;
- q_0 - initial state, the element of Q;
- F - the set of final states.

Recognizing grammars

If $M = <Q, \Sigma, \sigma, q_0, F>$ is FSM then a pair $(q, \omega) \in Q * \Sigma^*$ is called the **machine M configuration**. (q_0, ω) is called **start configuration** and (q, e) where $q \in F$ is called **final configuration**.

The step of machine *M* performance is a binary relation $\vdash_M$, determined on the configurations. If $\sigma(q, a)$ contains p then $(q, a\omega) \vdash_M (p, \omega)$ for each $\omega \in \Sigma^*$.

The relation $\vdash_M^+$ is a transitive closure of the relation $\vdash_M$. The relation $\vdash_M^*$ is reflexive and transitive closure of the relation $\vdash_M$.

FMS M **accepts** the chain $\omega \in \Sigma^*$ if $(q_0, \omega) \vdash_M^* (q, e)$ for some $q \in F$. **The language determined (recognized, accepted)** by FSM M (denoted as $L(M)$) is a set of input chains, accepted by FMS M:

$L(M) = \{\omega : \omega \in \Sigma^*$ and $(q_0, \omega) \vdash_M^* (q, e)$ for some $q \in F\}$.

Recognizing grammars

FMS begins to work in state q_0, reading symbols from the input chain x one by one. Read symbol transits the FSM into another state according to the state-transition function. In the end FSM is in a state q'. If this state is final then FMS accepts the chain x.

Determined finite-state machine (DFSM) is a finite-state machine $M = <Q, \Sigma, \sigma, q_0, F>$ where a set of state-transition functions $\sigma(q, a)$ contains no more than one state for any states $q \in Q$ and for input symbols $a \in \Sigma$.

Recognizing grammars

Example:

Build DFSM *M* which recognizes two nulls which are standing together.

This DFSM is a tuple $M = \langle\{p, q, r\}, \{0, 1\}, \sigma, p, \{r\}\rangle$ where σ is defined by the following table:

State	**Input**	
	0	1
p	$\{q\}$	$\{p\}$
q	$\{r\}$	$\{p\}$
r	$\{r\}$	$\{r\}$

Recognizing grammars

Example:

Build NFSM accepting chains in the alphabet {1, 2, 3} for which the last symbol is already contained in the chain. For example: 121 is accepted, 31312 is not accepted.

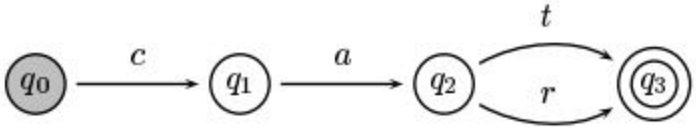
NFSM is a tuple M = <{q_0, q_1, q_2, q_3, q_f}, {1, 2, 3}, σ, q_0, {q_f}>, σ is defined by the following table:

State	**Input**		
	1	2	3
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_3\}$
q_1	$\{q_1, q_f\}$	$\{q_1\}$	$\{q_1\}$
q_2	$\{q_2\}$	$\{q_2, q_f\}$	$\{q_2\}$
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3\}$
q_f	$\varnothing$	$\varnothing$	$\varnothing$

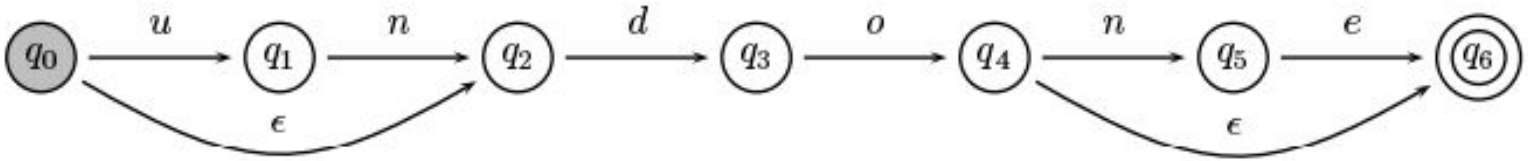
Recognizing grammars

The diagram of states (**the graph of transitions**) is annotated oriented graph, the heads correspond to the states, arcs correspond to the transitions from one state to another.

Let M = <Q, Σ, σ, q_0, F> be the FSM. If symbol $a \in \Sigma : q \in \sigma(p, a)$ exists then in the graph of transitions there is an arc (p, q).

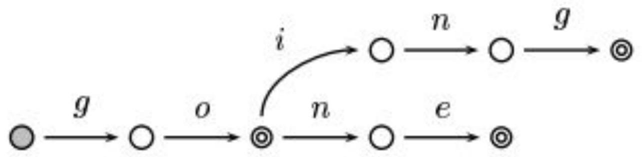


The language accepted by the following automaton is $\{$*do, undo, done, undone*$\}$:

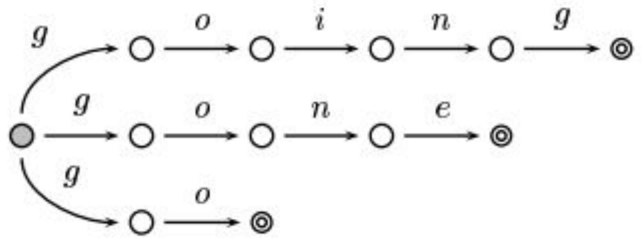


The following three finite-state automata are equivalent: they all accept the set {*go, gone, going*}.

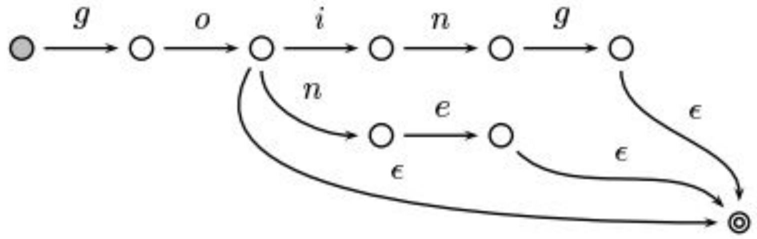
A_1



A_2



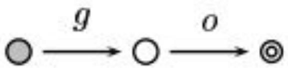
A_3



Note that A_1 is deterministic: for any state and alphabet symbol there is at most one possible transition. A_2 is not deterministic: the initial state has three outgoing arcs all labeled by g. The third automaton, A_3, has ϵ-arcs and hence is not deterministic. While A_2 might be the most readable, A_1 is the most compact as it has the fewest nodes.

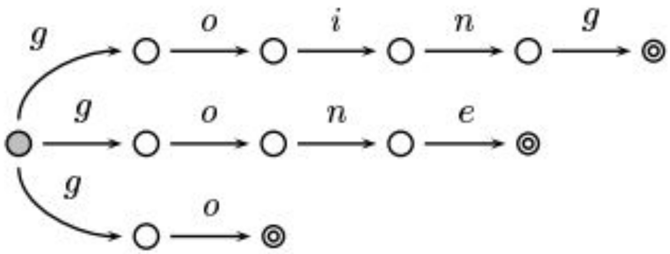
FSM in NLP: dictionaries

go :



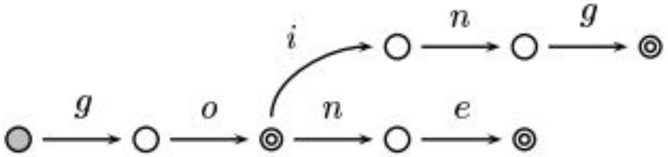
To represent more than one word, we can simply add paths to our "lexicon", one path for each additional word. Thus, after adding the words *gone* and *going*, we might have:

go, gone, going :



This automaton can then be determinized and minimized:

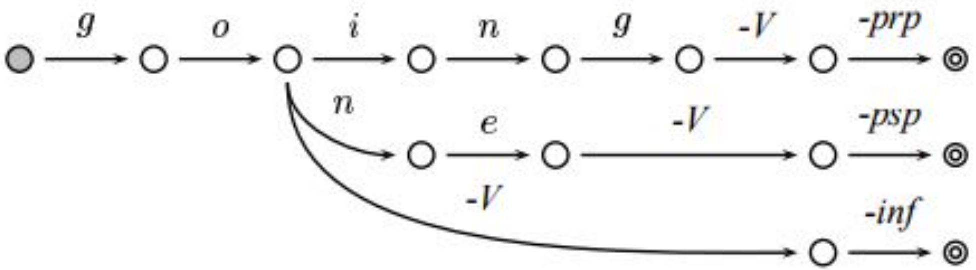
go, gone, going :



FSM in NLP: morphology

$$\Sigma = \{a, b, c, \ldots, y, z, \text{-}N, \text{-}V, \text{-}sg, \text{-}pl, \text{-}inf, \text{-}prp, \text{-}psp\}$$

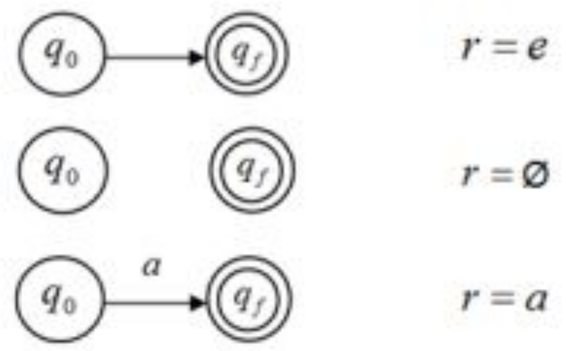
With the extended alphabet, we might construct the following automaton:



The language generated by the above automaton is no longer a set of words in English. Rather, it is a set of (simplisticly) "analyzed" strings, namely {*go-V-inf, gone-V-psp, going-V-prp*}.

Recognizing grammars

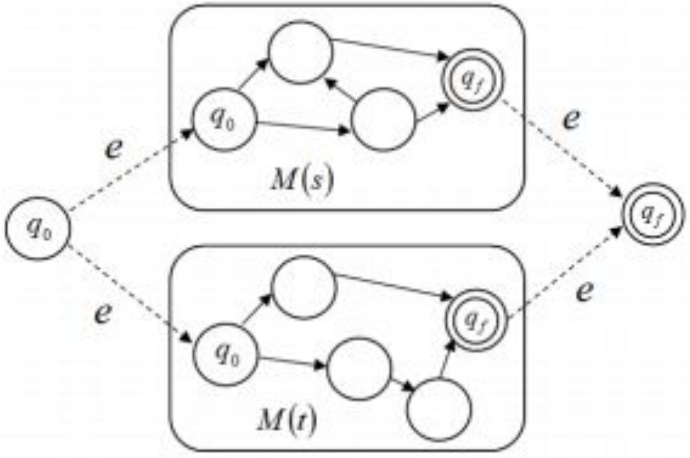
According to the definition, a set of chains in the alphabet is called **regular** iff it is one of the sets Ø, {*e*}, {*a*} ∀ *a* ∈ Σ or it can be deduced from these sets by applying the operations of joining, concatenation and iteration.



Building FSM M (s+t)

q_0 (out of the frame) is new initial state;

q_f (out of the frame) is new final state



Building FSM M (s+t)

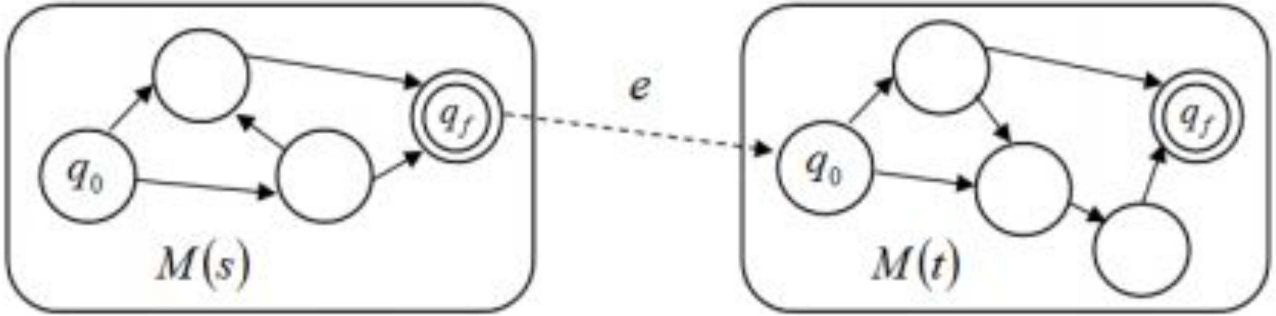
Let R_1 be the following relation, mapping some English words to their German counterparts:

$$R_1 = \{\textit{tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit, pineapple:Ananas, coconut:Koko}\}$$

Let R_2 be a similar relation: $R_2 = \{\textit{grapefruit:pampelmuse, coconut:Kokusnuß}\}$. Then

$$\begin{aligned} R_1 \cup R_2 = \{&\textit{tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit,} \\ &\textit{grapefruit:pampelmuse, pineapple:Ananas, coconut:Koko ,coconut:Kokusnuß}\} \end{aligned}$$

Building FSM M (st)



Building FSM M (st)

Let R_1 be the following relation, mapping some English words to their German counterparts:

$$R_1 = \{\textit{tomato:Tomate, cucumber:Gurke, grapefruit:Grapefruit,}$$
$$\textit{grapefruit:pampelmuse, pineapple:Ananas, coconut:Koko ,coconut:Kokusnuß}\}$$

Let R_2 be a similar relation, mapping French words to their English translations:

$$R_2 = \{\textit{tomate:tomato, ananas:pineapple, pampelmousse:grapefruit,}$$
$$\textit{concombre:cucumber, cornichon:cucumber, noix-de-coco:coconut}\}$$

Then $R_2 \circ R_1$ is a relation mapping French words to their German translations (the English translations are used to compute the mapping, but are not part of the final relation):

$$R_2 \circ R_1 = \{\textit{tomate:Tomate, ananas:Ananas, pampelmousse:Grapefruit, pampelmousse:Pampelmuse,}$$
$$\textit{concombre:Gurke, cornichon:Gurke, noix-de-coco:Koko, noix-de-coco:Kokusnuße}\}$$

Thank you for your attention!