Lab Assignment: Multiple Comparisons/Post Hoc Procedures

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Mar. 2, 2017

Create a Word document from this R Markdown file for the following exercises. Submit the R markdown file and the knitted Word document via D2L Dropbox.

## Preliminaries

Cars were selected at random from among 1993 passenger car models that were listed in both the Consumer Reports issue and the PACE Buying Guide. Pickup trucks and Sport/Utility vehicles were eliminated due to incomplete information in the Consumer Reports source. Duplicate models (e.g., Dodge Shadow and Plymouth Sundance) were listed at most once. Use the data set Cars93 to do the following. (Type ?Cars93 to learn more about the data.)

For the first couple of exercises we are going to use the Cars93 data set from the MASS package. We'll delete the data having to do with vans so that we are only dealing with cars. The code to load and prepare the data is here:

if (!require(MASS)){  
 install.packages('MASS')  
 library(MASS)  
}  
data(Cars93)  
Cars93 <- Cars93[Cars93$Type != 'Van',]  
Cars93$Type <- factor(Cars93$Type) # recasting Type forces the factor levels to reset  
# shorten level labels to make them fit on boxplots  
# Cm = Compact  
# Lg = Large  
# Md = Midsize  
# Sm = Small  
# Sp = Sporty  
Cars93$Type <- factor(Cars93$Type,labels=c('Cm','Lg','Md','Sm','Sp'))

Here is another trick which will simply your analysis a bit. You can attach a data frame so that it's simple to refer to the variables.

attach(Cars93)

## The following objects are masked from Cars93 (pos = 6):  
##   
## AirBags, Cylinders, DriveTrain, EngineSize,  
## Fuel.tank.capacity, Horsepower, Length, Luggage.room, Make,  
## Man.trans.avail, Manufacturer, Max.Price, Min.Price, Model,  
## MPG.city, MPG.highway, Origin, Passengers, Price,  
## Rear.seat.room, Rev.per.mile, RPM, Turn.circle, Type, Weight,  
## Wheelbase, Width

## The following objects are masked from Cars93 (pos = 7):  
##   
## AirBags, Cylinders, DriveTrain, EngineSize,  
## Fuel.tank.capacity, Horsepower, Length, Luggage.room, Make,  
## Man.trans.avail, Manufacturer, Max.Price, Min.Price, Model,  
## MPG.city, MPG.highway, Origin, Passengers, Price,  
## Rear.seat.room, Rev.per.mile, RPM, Turn.circle, Type, Weight,  
## Wheelbase, Width

summary(Price) # Price is one of the variables in the Cars93 data frame, after attaching we don't have to refer to the data frame. Don't forget to detach(Cars93) after you're done.

## Min. 1st Qu. Median Mean 3rd Qu. Max.   
## 7.40 11.75 16.40 19.55 24.75 61.90

## Exercise 1

We are going to look for differences in population mean engine revolutions per minute at maximum horsepower (RPM) of the different types of cars (Type). Assume that the RPM distributions are normal and have equal variances for the different types of cars. To use onewayComp() you'll have to load the DS705data package.

### Part 1a

Use a one step procedure to find a family of 95% simultaneous confidence intervals for the difference in population means.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 1a -|-|-|-|-|-|-|-|-|-|-|-

source('onewayComp.R')  
posthoc <- onewayComp(Rev.per.mile~Type, data=Cars93)  
posthoc$comp[,2:3]

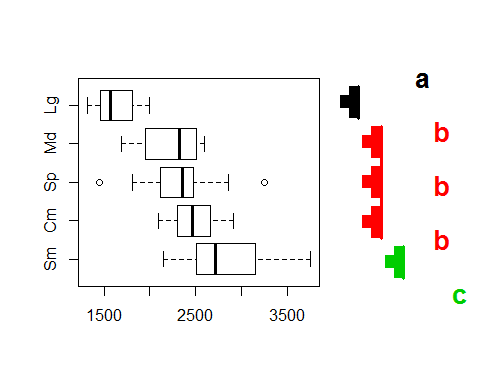
## lwr upr  
## Lg-Cm -1235.75207 -461.86157  
## Md-Cm -578.85832 70.33559  
## Sm-Cm 13.92104 669.59086  
## Sp-Cm -522.88281 200.20424  
## Md-Lg 229.72997 959.36094  
## Sm-Lg 822.86333 1558.26221  
## Sp-Lg 289.42121 1085.51386  
## Sm-Md 294.62150 897.41313  
## Sp-Md -244.83143 430.67558  
## Sp-Sm -843.96175 -162.22873

### Part 1b

Use the multcompView package to produce a boxplot and letters/T display illustrating the differences in population means. (Review slides 17-19 in Multiple Comparisons Part 2 presentation.)

### -|-|-|-|-|-|-|-|-|-|-|- Answer 1b -|-|-|-|-|-|-|-|-|-|-|-

require(multcompView)  
padj\_extract <- function(formula, data){posthoc$comp[,'p adj']}  
multcompBoxplot(Rev.per.mile ~ Type, data=Cars93, horizontal=TRUE, compFn='padj\_extract')



### Part 1c

Summarize your findings about the differences in population mean RPM for the different types of cars. Which means are different and by how much? You should start with "We are 95% confident that ..."

### -|-|-|-|-|-|-|-|-|-|-|- Answer 1c -|-|-|-|-|-|-|-|-|-|-|-

We are 95% confident that:

The Large is 1235.75 to 461.86 less than the Compact.

The Small is 13.92 to 669.59 more than the Compact.

The Medium is 229.73 to 959.36 more than the Large.

The Small is 822.86 to 1558.26 more than the Large.

The Sporty is 289.42 to 1085.51 more than the Large.

The Small is 294.62 to 897.41 more than the Medium.

The Sporty is 843.96 to 162.23 less than Small.

### Part 1d

Use the REGWQ multi-step procedure (function regwqComp in DS705data) to test for pairwise differences in population mean RPM at (Don't forget each comparison includes an adjusted P-value and an adjusted significance level. See the presentation for more details.) How do the results compare to the one-step procedure you chose in 1b)?

### -|-|-|-|-|-|-|-|-|-|-|- Answer 1d -|-|-|-|-|-|-|-|-|-|-|-

source('regwqComp.R')  
posthoc <- regwqComp(Rev.per.mile~Type,data=Cars93)  
posthoc[,c(1,4,5,6)]

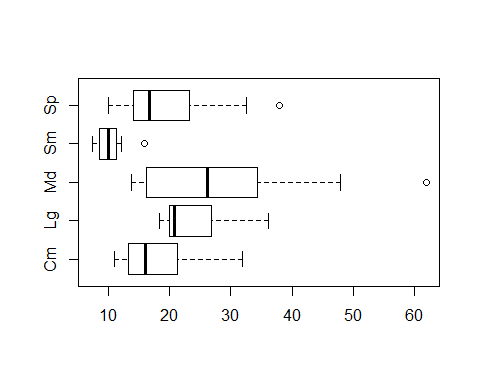
## diff p adj alpha adj rej H\_0  
## Lg-Cm -848.80682 1.985789e-07 0.05000000 1  
## Md-Cm -254.26136 7.967877e-02 0.03030722 0  
## Sm-Cm 341.75595 4.688896e-03 0.02030827 1  
## Sp-Cm -161.33929 2.165067e-01 0.02030827 0  
## Md-Lg 594.54545 1.915434e-05 0.02030827 1  
## Sm-Lg 1190.56277 0.000000e+00 0.05000000 1  
## Sp-Lg 687.46753 2.009454e-05 0.03030722 1  
## Sm-Md 596.01732 2.464740e-06 0.05000000 1  
## Sp-Md 92.92208 4.447358e-01 0.02030827 0  
## Sp-Sm -503.09524 2.704065e-04 0.03030722 1

It appears that in this case, both tests result in rejecting the null hypothesis for the same pairs at

## Exercise 2

Now we are going to analyze differences in prices for different types of cars in the Cars93 data set. The boxplot below shows that the prices are skewed and variances are different.

boxplot(Price~Type,horizontal=TRUE)



### -|-|-|-|-|-|-|-|-|-|-|- Answer 2a -|-|-|-|-|-|-|-|-|-|-|-

It should be fairly clear that the price data is not from normal distributions, at least for several of the car types, but ignore that for now and use the Games-Howell procedure with confidence level 90% to do simultaneous comparisons (if interpreting the -values use ).

posthoc <- onewayComp(Price~Type, data=Cars93, var.equal=FALSE)  
posthoc

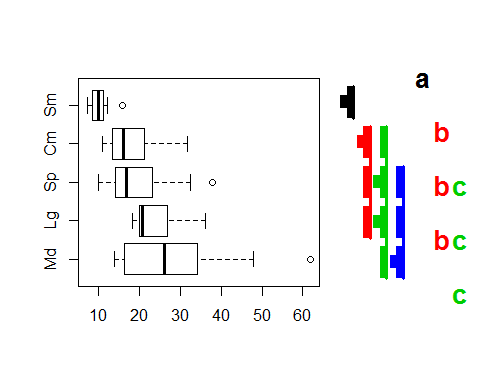
## $call  
## onewayComp(formula = Price ~ Type, data = Cars93, var.equal = FALSE)  
##   
## $comp  
## diff lwr upr t p  
## Lg-Cm 6.087500 -1.43351665 13.608517 2.3977076 2.524630e-02  
## Md-Cm 9.005682 0.06552497 17.945839 2.9017065 6.489106e-03  
## Sm-Cm -8.045833 -13.29611220 -2.795554 -4.6636919 2.240512e-04  
## Sp-Cm 1.180357 -6.76232482 9.123039 0.4357599 6.666721e-01  
## Md-Lg 2.918182 -6.45877344 12.295137 0.9010497 3.745297e-01  
## Sm-Lg -14.133333 -20.46424531 -7.802421 -7.2190114 1.705110e-05  
## Sp-Lg -4.907143 -13.36906316 3.554777 -1.7142915 9.992807e-02  
## Sm-Md -17.051515 -24.90872496 -9.194305 -6.4360269 1.739174e-06  
## Sp-Md -7.825325 -17.53995770 1.889308 -2.3196827 2.650442e-02  
## Sp-Sm 9.226190 2.45658636 15.995795 4.2447828 8.107007e-04  
## p adj rej H\_0  
## Lg-Cm 1.266833e-01 0  
## Md-Cm 3.759910e-02 1  
## Sm-Cm 1.192465e-04 1  
## Sp-Cm 9.923626e-01 0  
## Md-Lg 8.956675e-01 0  
## Sm-Lg 2.789471e-09 1  
## Sp-Lg 4.312457e-01 0  
## Sm-Md 8.678284e-08 1  
## Sp-Md 1.495367e-01 0  
## Sp-Sm 5.542557e-04 1  
##   
## $pairw  
##   
## Pairwise comparisons using t tests with unpooled SD   
##   
## data: Price and Type   
##   
## Cm Lg Md Sm   
## Lg 0.12668 - - -   
## Md 0.03760 0.89567 - -   
## Sm 0.00012 2.8e-09 8.7e-08 -   
## Sp 0.99236 0.43125 0.14954 0.00055  
##   
## P value adjustment method: one.step

### Part 2b

Use the multcompView package to produce a boxplot and letters/T display illustrating the differences in population means. We want to make the comparisons at , but the multcompBoxplot command assumes and that is difficult to change. So instead divide the adjusted p-values by 2 before calling the multcompBoxplot (something like this should work: out$comp[,'p adj'] <- out$comp[,'p adj']/2 ).

### -|-|-|-|-|-|-|-|-|-|-|- Answer 2b -|-|-|-|-|-|-|-|-|-|-|-

padj\_extract <- function(formula, data){posthoc$comp[,'p adj']/2}  
multcompBoxplot(Price ~ Type, data=Cars93, horizontal=TRUE, compFn='padj\_extract')



### Part 2c

Summarize the differences in the population mean prices for the different cars at . Since you have confidence intervals you should explain how the mean prices differ and by how much.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 2c -|-|-|-|-|-|-|-|-|-|-|-

At , we reject the null hypothesis for the Md-Cm, Sm-Cm, Sm-Lg, Sm-Md, and Sp-Sm pairs.

We are 95% confident that:

The Medium is 0.067 to 17.95 more expensive than the Compact.

The Small is 13.30 to 2.80 less expensive than the Compact.

The Small is 20.46 to 7.80 less expensive than the Large.

The Small is 24.91 to 9.19 less expensive than the Medium.

The Sporty is 2.47 to 16.00 more expensive than the Small.

## Exercise 3.

Since the price data is likely not normally distributed, the Games-Howell procedure was not entirely appropriate. However we can use bootstrapping to estimate the P-values and confidence intervals since the theoretical sampling distribution is likely not accurate.

### Part 3a

Repeat part 2a) using bootstrapping, by setting nboot=10000, in the onewayComp() function.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 3a -|-|-|-|-|-|-|-|-|-|-|-

posthoc <- onewayComp(Price~Type, data=Cars93, nboot=10000)  
posthoc

## $call  
## onewayComp(formula = Price ~ Type, data = Cars93, nboot = 10000)  
##   
## $comp  
## diff lwr upr t p p adj rej H\_0  
## Lg-Cm 6.087500 -3.124131 15.2991315 1.9278958 0.0217 0.2711 0  
## Md-Cm 9.005682 1.278316 16.7330476 3.3999009 0.0086 0.0207 1  
## Sm-Cm -8.045833 -15.850282 -0.2413846 -3.0075298 0.0000 0.0432 1  
## Sp-Cm 1.180357 -7.426560 9.7872743 0.4000801 0.6656 0.9900 0  
## Md-Lg 2.918182 -5.766628 11.6029912 0.9802417 0.3690 0.8030 0  
## Sm-Lg -14.133333 -22.886798 -5.3798682 -4.7102692 0.0000 0.0013 1  
## Sp-Lg -4.907143 -14.383047 4.5687611 -1.5107381 0.0932 0.4803 0  
## Sm-Md -17.051515 -24.226554 -9.8764765 -6.9329786 0.0000 0.0000 1  
## Sp-Md -7.825325 -15.865896 0.2152465 -2.8392040 0.0266 0.0565 0  
## Sp-Sm 9.226190 1.111511 17.3408698 3.3168985 0.0002 0.0235 1  
##   
## $pairw  
##   
## Pairwise comparisons using t tests with pooled SD   
##   
## data: Price and Type   
##   
## Cm Lg Md Sm   
## Lg 0.2711 - - -   
## Md 0.0207 0.8030 - -   
## Sm 0.0432 0.0013 <2e-16 -   
## Sp 0.9900 0.4803 0.0565 0.0235  
##   
## P value adjustment method: one.step

### Part 3b

Repeat 2b) using the results produced by bootstrapped Games-Howell. Again use .

### -|-|-|-|-|-|-|-|-|-|-|- Answer 3b -|-|-|-|-|-|-|-|-|-|-|-

posthoc <- onewayComp(Price~Type, data=Cars93, var.equal=FALSE, nboot=10000)  
posthoc

## $call  
## onewayComp(formula = Price ~ Type, data = Cars93, var.equal = FALSE,   
## nboot = 10000)  
##   
## $comp  
## diff lwr upr t p p adj rej H\_0  
## Lg-Cm 6.087500 -3.9187075 16.093707 2.3977076 0.0261 0.2074 0  
## Md-Cm 9.005682 -3.2261037 21.237467 2.9017065 0.0065 0.1217 0  
## Sm-Cm -8.045833 -14.8451916 -1.246475 -4.6636919 0.0040 0.0278 1  
## Sp-Cm 1.180357 -9.4952591 11.855973 0.4357599 0.6678 0.9915 0  
## Md-Lg 2.918182 -9.8459219 15.682286 0.9010497 0.3829 0.8887 0  
## Sm-Lg -14.133333 -21.8493582 -6.417308 -7.2190114 0.0017 0.0025 1  
## Sp-Lg -4.907143 -16.1887451 6.374459 -1.7142915 0.1004 0.4747 0  
## Sm-Md -17.051515 -27.4932310 -6.609799 -6.4360269 0.0002 0.0052 1  
## Sp-Md -7.825325 -21.1206975 5.470048 -2.3196827 0.0294 0.2280 0  
## Sp-Sm 9.226190 0.6598824 17.792499 4.2447828 0.0103 0.0387 1  
##   
## $pairw  
##   
## Pairwise comparisons using t tests with unpooled SD   
##   
## data: Price and Type   
##   
## Cm Lg Md Sm   
## Lg 0.2074 - - -   
## Md 0.1217 0.8887 - -   
## Sm 0.0278 0.0025 0.0052 -   
## Sp 0.9915 0.4747 0.2280 0.0387  
##   
## P value adjustment method: one.step

### Part 3c

Explain these results in context as you did in 2c).

### -|-|-|-|-|-|-|-|-|-|-|- Answer 3c -|-|-|-|-|-|-|-|-|-|-|-

At , we reject the null hypothesis for the Sm-Cm, Sm-Lg, Sm-Md, and Sp-Sm pairs

We are 95% confident the Small is 14.51 to 1.58 less expensive than the Compact.

We are 95% confident the Small is 21.47 to 6.80 less expensive than the Large.

We are 95% confident the Small is 26.98 to 7.12 less expensive than the Medium.

We are 95% confident the Sporty is 1.08 to 17.37 more expensive than the Small.

## Exercise 4.

One step procedures like Tukey-Kramer and Games-Howell are conservative (lower power) so they can miss significant differences between population means. If you don't need the confidence intervals, then a multi-step procedure such as the Bonferroni-Holm step-down procedure may be used to get more power.

### Part 4a

Repeat 2a, but this time use the Bonferroni-Holm procedure at to compare the population mean prices for the different types of cars. Use bootstrapping and unequal variances.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 4a -|-|-|-|-|-|-|-|-|-|-|-

posthoc <- onewayComp(Price~Type, data=Cars93, var.equal=FALSE, nboot=10000, adjust='holm')  
posthoc

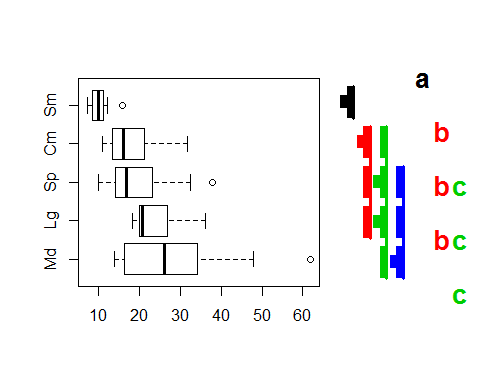
## $call  
## onewayComp(formula = Price ~ Type, data = Cars93, var.equal = FALSE,   
## nboot = 10000, adjust = "holm")  
##   
## $comp  
## diff lwr upr t p p adj rej H\_0  
## Lg-Cm 6.087500 NA NA 2.3977076 0.0277 0.1310 0  
## Md-Cm 9.005682 NA NA 2.9017065 0.0074 0.0518 0  
## Sm-Cm -8.045833 NA NA -4.6636919 0.0027 0.0216 1  
## Sp-Cm 1.180357 NA NA 0.4357599 0.6724 0.7720 0  
## Md-Lg 2.918182 NA NA 0.9010497 0.3860 0.7720 0  
## Sm-Lg -14.133333 NA NA -7.2190114 0.0021 0.0189 1  
## Sp-Lg -4.907143 NA NA -1.7142915 0.1029 0.3087 0  
## Sm-Md -17.051515 NA NA -6.4360269 0.0001 0.0010 1  
## Sp-Md -7.825325 NA NA -2.3196827 0.0262 0.1310 0  
## Sp-Sm 9.226190 NA NA 4.2447828 0.0109 0.0654 0  
##   
## $pairw  
##   
## Pairwise comparisons using t tests with unpooled SD   
##   
## data: Price and Type   
##   
## Cm Lg Md Sm   
## Lg 0.131 - - -   
## Md 0.052 0.772 - -   
## Sm 0.022 0.019 0.001 -   
## Sp 0.772 0.309 0.131 0.065  
##   
## P value adjustment method: holm

### Part 4b

Repeat 2b to produce the boxplot with T and letter displays for the output in 4a. Don't forget to manually adjust the P-values to "fool" the plot into use the significance level to produce the plot.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 4b -|-|-|-|-|-|-|-|-|-|-|-

padj\_extract <- function(formula, data){posthoc$comp[,'p adj']/2}  
multcompBoxplot(Price ~ Type, data=Cars93, horizontal=TRUE, compFn='padj\_extract')



### Part 4c

As in 2c, explain the mean price comparisons in context. Did you find any mean price differences that weren't previously revealed?

### -|-|-|-|-|-|-|-|-|-|-|- Answer 4c -|-|-|-|-|-|-|-|-|-|-|-

At , we reject the null hypothesis for Md-Cm, Sm-Cm, Sm-Md, and Sp-Sm pairs. I do not see any new price differences.

### Part 4d

In Exercises 2, 3, and 4 you used 3 different methods to analyze the differences in population mean prices for the different types of cars. Which analysis do you think is the most reliable? Why?

### -|-|-|-|-|-|-|-|-|-|-|- Answer 4d -|-|-|-|-|-|-|-|-|-|-|-

In my opinion, the bootstrapped version is most appropriate. This is because the Price data is most likely not normally distributed (required for Exercise 2). I also like the confidence intervals that it provides which is not the case with Exercise 4.

## Exercise 5

Build a custom contrast matrix that compares the average of average small and compact prices to the average of the other car types and also compares the mean prices of the midsize and compact cars. (You may have to use levels(Type) to see the ordering of the levels) Use the Bonferroni-Holm procedure at the 10% significance level with bootstrapping and unequal variances to make the comparisons. Summarize your results.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 5 -|-|-|-|-|-|-|-|-|-|-|-

K <- rbind('Sm-Cm to Other'=c(-1/2,1/3,1/3,-1/2,1/3),  
 'Midsize to Compact'=c(1,0,-1,0,0))  
  
posthoc <- onewayComp(Price~Type, data=Cars93, var.equal=FALSE, nboot=10000, adjust='holm', con=K)  
posthoc

## $call  
## onewayComp(formula = Price ~ Type, data = Cars93, var.equal = FALSE,   
## con = K, nboot = 10000, adjust = "holm")  
##   
## $comp  
## diff lwr upr t p p adj rej H\_0  
## Sm-Cm to Other 9.447430 NA NA 6.080276 0.0000 0.0000 1  
## Midsize to Compact -9.005682 NA NA -2.901707 0.0081 0.0081 1  
##   
## $pairw  
##   
## Pairwise comparisons using t tests with unpooled SD   
##   
## data: Price and Type   
##   
## Cm Lg Md Sm  
## Lg <2e-16 - - -   
## Md 0.0081 - - -   
## Sm - - - -   
## Sp - - - -   
##   
## P value adjustment method: holm

## Exercise 6

Since the price distributions are skewed it makes more sense to talk about median prices than mean prices.

### Part 6a

The Kruskal-Wallis and Dunn procedures aren't appropriate for comparing population median prices, why? Explain.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 6a -|-|-|-|-|-|-|-|-|-|-|-

These require distributions with the same shape, only possibly shifted from one and another.

### Part 6b

We're going to make 4 simultaneous confidence intervals for price data (compact - small, sporty-small, midsize-sporty, midsize-compact). If we want familywise confidence level 90%, what confidence level should you use for each individual comparison according to the Bonferroni correction?

### -|-|-|-|-|-|-|-|-|-|-|- Answer 6b -|-|-|-|-|-|-|-|-|-|-|-

for 90% confidence, . We have 4 comparisons, so each individual needs to be 0.025 (0.10/4). As such, the confidence level for each needs to be 97.5% (1-0.025)

### Part 6c

Use the boot package (as in the class presentation) to bootstrap the 4 confidence intervals for the specified differences of population median prices. You'll have to write the helper function and make sure you are referring to the correct columns of the Cars93 data. Don't forget to install and load the 'boot' package.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 6c -|-|-|-|-|-|-|-|-|-|-|-

require(boot)  
bootMedianDiff <- function(d, i)  
{  
 medians <- tapply(d[i,5],d[,3],median)  
 c(  
 medians[1]-medians[4],   
 medians[5]-medians[4],   
 medians[3]-medians[5],   
 medians[3]-medians[1]   
 )  
}  
boot.object <- boot(Cars93, bootMedianDiff, R=5000, strata=Cars93$Type)  
boot.ci(boot.object, conf=1 - 0.10 /4, type='bca', index=1)$bca[4:5]

## [1] 2.35000 11.14574

boot.ci(boot.object, conf=1 - 0.10 /4, type='bca', index=2)$bca[4:5]

## [1] 3.5 14.2

boot.ci(boot.object, conf=1 - 0.10 /4, type='bca', index=3)$bca[4:5]

## [1] 0.15 19.50

boot.ci(boot.object, conf=1 - 0.10 /4, type='bca', index=4)$bca[4:5]

## [1] 0.881592 19.652097

### Part 6d

Explain the results of your intervals.

### -|-|-|-|-|-|-|-|-|-|-|- Answer 6d -|-|-|-|-|-|-|-|-|-|-|-

We are 90% confident that:

Compact is 2.35 to 10.65 more expensive than Small

Sporty is 3.35 to 14.10 more expensive than Small

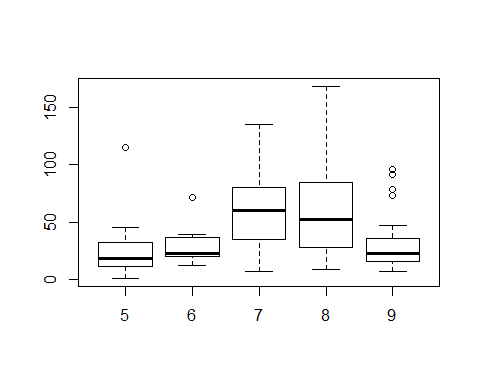
Midsize in 0.30 to 19.55 more expensive than Sporty

Midsize is 1.00 to 19.41 more expensive than Compact

## Exercise 7

The airquality data set that is built into R looks at air quaility measures in New York City, including ozone levels, for 5 months in 1973. We are going to estimate differences in population median ozone levels for the 5 months using the Dunn procedure which is a traditional follow up to the Kruskal-Wallis test. Here is a boxplot and the Kruskal-Wallis test:

detach(Cars93) # we don't need the Cars93 data now   
data(airquality)  
boxplot(Ozone~Month,data=airquality)



kruskal.test(Ozone ~ Month, data = airquality)

##   
## Kruskal-Wallis rank sum test  
##   
## data: Ozone by Month  
## Kruskal-Wallis chi-squared = 29.267, df = 4, p-value = 6.901e-06

The boxplot shows that the distributions of ozone levels are similar, if not identical, for the 5 months. The Kruskal-Wallis test, assuming that the population distributions have the same shapes, shows that there is significant evidence that the population median ozone levels are not the same for all 5 months. Use the Dunn procedure (as shown in the presentations) with Bonferroni-Holm adjusted p-values to see which months have different population median ozone levels. Use Summarize your findings. (Don't forget to install and load the correct package here.)

### -|-|-|-|-|-|-|-|-|-|-|- Answer 7 -|-|-|-|-|-|-|-|-|-|-|-

require(dunn.test)  
with(airquality, dunn.test(Ozone, Month, method='holm', alpha=0.05))

## Kruskal-Wallis rank sum test  
##   
## data: Ozone and Month  
## Kruskal-Wallis chi-squared = 29.2666, df = 4, p-value = 0  
##   
##   
## Comparison of Ozone by Month   
## (Holm)   
## Col Mean-|  
## Row Mean | 5 6 7 8  
## ---------+--------------------------------------------  
## 6 | -0.925158  
## | 0.5323  
## |  
## 7 | -4.419470 -2.244208  
## | 0.0000\* 0.0745  
## |  
## 8 | -4.132813 -2.038635 0.286657  
## | 0.0002\* 0.1037 0.7744  
## |  
## 9 | -1.321202 0.002538 3.217199 2.922827  
## | 0.3729 0.4990 0.0052\* 0.0121\*

The month pairs of 5-7, 8-5, 9-7, 9-8 show differences.