Exercise

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Informatics 3 - Professorship of Data Mining and Analytics

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Linear Algebra

Problem 1: Dimensions of matrices A, B, C, D, E, F

$$A \in \mathbb{R}^{M \times N}, \qquad B \in \mathbb{R}^{1 \times M}, \qquad C \in \mathbb{R}^{N \times P} \tag{1}$$

$$D \in \mathbb{R}^{Q}, \qquad B \in \mathbb{R}^{N \times N}, \qquad C \in \mathbb{R}^{1} \tag{2}$$

Problem 2: $f(x) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j M_{ij}$ using only matrix-vector multiplications.

$$f(x) = x^T M x \tag{3}$$

Problem 3:

(a) Conditions for unique solution x for any choice of b in $Ax = b$

$$rank(A) = M, \qquad det(A) \neq 0, \qquad ker(A) = \{0\}$$

(b) Unique solution x for any choice of b in $Ax = b$ with eigenvalues of A: $\{-5, 0, 1, 1, 3\}$

$det(A) = \prod_i \lambda_i = -5 * 0 * 1 * 1 * 3 = 0 \implies$ No unique solution

Problem 4: Properties of eigenvalues of A in $BA = AB = I$

$$BA = AB = I \implies B = A^{-1} \tag{4}$$

A has to be invertable $\implies det(A) \neq 0 \implies \forall i : \lambda_i \neq 0$

Problem 5: A is PSD if and only if it has no negative eigenvalues

Definition of eigenvalue: $Ax = \lambda x$

$$PSD \Leftrightarrow x^T A x \geq 0 \tag{5}$$

$$PSD \Leftrightarrow x^T A x = x^T \lambda x = \lambda x^T x = \lambda \sum_i x_i^2 \geq 0 \tag{6}$$

$$\sum_i x_i^2 \geq_{always} 0 \implies \forall \lambda : \lambda \geq 0 \tag{7}$$

Problem 6: $B = A^T A$ is PSD for any A

$$B = A^T A \implies Bx = \lambda_B x = A^T A x = \lambda_A \lambda_A x = \lambda_A^2 x \tag{8}$$

$$\lambda_B = \lambda_A^2 \implies \lambda_B \geq_{always} 0 \tag{9}$$

B has to be PSD for any choice of A. $\square$

Calculus

Probability Theory

Appendix

We confirm that the submitted solution is original work and was written by us without further assistance. Appropriate credit has been given where reference has been made to the work of others.

Munich, October 17, 2019, Signature Marcel Bruckner

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