Practical Session 5

Optimization

1 Convexity

Problem 1: Show that affine functions of the form $\boldsymbol{w}^T \boldsymbol{x} + b$ are both convex and concave.

Problem 2: Show that a twice differentiable function $f(\boldsymbol{x})$ with a convex domain is convex if and only if its Hessian or second derivative is positive semidefinite: $\nabla^2 f(\boldsymbol{x}) \geq 0$ for all $\boldsymbol{x} \in \mathrm{dom}(f)$.

2 Logistic Regression

Problem 3: Prove that the objective function of logistic regression

$$E(\boldsymbol{w}) = -\ln p\big(\boldsymbol{y} \mid \boldsymbol{w}, \boldsymbol{X}\big) = -\sum_{i=1}^{N} \big(y_i \ln \sigma(\boldsymbol{w}^T \boldsymbol{x}_i) + (1 - y_i) \ln(1 - \sigma(\boldsymbol{w}^T \boldsymbol{x}_i))\big) \tag{1}$$

is convex. What is the benefit of having a convex function for optimization?

3 Optimization methods

Problem 4: Discuss the following topics:

- Condition number
- Consistency, convergence, stability
- Stiffness