exercise_12_matrix_factorization

February 2, 2020

0.1 Exporting the results to PDF

Once you complete the assignments, export the entire notebook as PDF and attach it to your homework solutions. The best way of doing that is 1. Run all the cells of the notebook. 2. Export/download the notebook as PDF (File -> Download as -> PDF via LaTeX (.pdf)). 3. Concatenate your solutions for other tasks with the output of Step 2. On linux, you can use `pdfunite`, there are similar tools for other platforms, too. You can only upload a single PDF file to Moodle.

Make sure you are using `nbconvert` version 5.5 or later by running `jupyter nbconvert --version`. Older versions clip lines that exceed page width, which makes your code harder to grade.

1 Matrix Factorization

In [1]:
```
import time
import scipy.sparse as sp
import numpy as np
from scipy.sparse.linalg import svds
from sklearn.linear_model import Ridge

import matplotlib.pyplot as plt
%matplotlib inline
```

1.1 Restaurant recommendation

The goal of this task is to recommend restaurants to users based on the rating data in the Yelp dataset. For this, we try to predict the rating a user will give to a restaurant they have not yet rated based on a latent factor model.

Specifically, the objective function (loss) we wanted to optimize is:

$$\mathcal{L} = \min_{P,Q} \sum_{(i,x)\in W} (M_{ix} - \mathbf{q}_i^T \mathbf{p}_x)^2 + \lambda \sum_x \|\mathbf{p}_x\|^2 + \lambda \sum_i \|\mathbf{q}_i\|^2$$

where W is the set of (i, x) pairs for which the rating M_{ix} given by user i to restaurant x is known. Here we have also introduced two regularization terms to help us with overfitting where λ is hyper-parameter that control the strength of the regularization.

Hint 1: Using the closed form solution for regression might lead to singular values. To avoid this issue perform the regression step with an existing package such as scikit-learn. It is advisable to use ridge regression to account for regularization.

Hint 2: If you are using the scikit-learn package remember to set `fit_intercept = False` to only learn the coefficients of the linear regression.

1.1.1 Load and Preprocess the Data (nothing to do here)

In [2]:
```
ratings = np.load("exercise_12_matrix_factorization_ratings.npy")
```

In [3]:
```
# We have triplets of (user, restaurant, rating).
ratings
```

Out[3]:
```
array([[101968,   1880,      1],
       [101968,    284,      5],
       [101968,   1378,      2],
       ...,
       [ 72452,   2100,      4],
       [ 72452,   2050,      5],
       [ 74861,   3979,      5]], dtype=int64)
```

Now we transform the data into a matrix of dimension [N, D], where N is the number of users and D is the number of restaurants in the dataset. We store the data as a sparse matrix to avoid out-of-memory issues.

In [4]:
```
n_users = np.max(ratings[:,0] + 1)
n_restaurants = np.max(ratings[:,1] + 1)
M = sp.coo_matrix((ratings[:,2], (ratings[:,0], ratings[:,1])), shape=(n_users, n_resta
M
```

Out[4]:
```
<337867x5899 sparse matrix of type '<class 'numpy.int64'>'
        with 929606 stored elements in Compressed Sparse Row format>
```

To avoid the cold start problem, in the preprocessing step, we recursively remove all users and restaurants with 10 or less ratings.

Then, we randomly select 200 data points for the validation and test sets, respectively.

After this, we subtract the mean rating for each users to account for this global effect.

Note: Some entries might become zero in this process -- but these entries are different than the 'unknown' zeros in the matrix. We store the indices for which we the rating data available in a separate variable.

In [5]:
```
def cold_start_preprocessing(matrix, min_entries):
    """
    Recursively removes rows and columns from the input matrix which have less than mi

    Parameters
    ----------
    matrix       : sp.spmatrix, shape [N, D]
                   The input matrix to be preprocessed.
    min_entries : int
                   Minimum number of nonzero elements per row and column.
```

```
    Returns
    -------
    matrix      : sp.spmatrix, shape [N', D']
                  The pre-processed matrix, where N' <= N and D' <= D

    """
    print("Shape before: {}".format(matrix.shape))

    shape = (-1, -1)
    while matrix.shape != shape:
        shape = matrix.shape
        nnz = matrix>0
        row_ixs = nnz.sum(1).A1 > min_entries
        matrix = matrix[row_ixs]
        nnz = matrix>0
        col_ixs = nnz.sum(0).A1 > min_entries
        matrix = matrix[:,col_ixs]
    print("Shape after: {}".format(matrix.shape))
    nnz = matrix>0
    assert (nnz.sum(0).A1 > min_entries).all()
    assert (nnz.sum(1).A1 > min_entries).all()
    return matrix
```

1.1.2 Task 1: Implement a function that subtracts the mean user rating from the sparse rating matrix

```
In [6]: def shift_user_mean(matrix):
    """
    Subtract the mean rating per user from the non-zero elements in the input matrix.

    Parameters
    ----------
    matrix : sp.spmatrix, shape [N, D]
             Input sparse matrix.
    Returns
    -------
    matrix : sp.spmatrix, shape [N, D]
             The modified input matrix.

    user_means : np.array, shape [N, 1]
                 The mean rating per user that can be used to recover the absolute rat

    """

    # TODO: Compute the modified matrix and user_means
    non_zeros = (matrix>0)
    user_means = matrix.sum(1) / non_zeros.sum(1)
    matrix -= sp.csr_matrix(user_means).multiply(non_zeros)
```

```
        assert np.all(np.isclose(matrix.mean(1), 0))
        return matrix, user_means
```

1.1.3 Split the data into a train, validation and test set (nothing to do here)

```
In [7]: def split_data(matrix, n_validation, n_test):
            """
            Extract validation and test entries from the input matrix.

            Parameters
            ----------
            matrix          : sp.spmatrix, shape [N, D]
                              The input data matrix.
            n_validation    : int
                              The number of validation entries to extract.
            n_test          : int
                              The number of test entries to extract.

            Returns
            -------
            matrix_split    : sp.spmatrix, shape [N, D]
                              A copy of the input matrix in which the validation and test entr

            val_idx         : tuple, shape [2, n_validation]
                              The indices of the validation entries.

            test_idx        : tuple, shape [2, n_test]
                              The indices of the test entries.

            val_values      : np.array, shape [n_validation, ]
                              The values of the input matrix at the validation indices.

            test_values     : np.array, shape [n_test, ]
                              The values of the input matrix at the test indices.

            """

            matrix_cp = matrix.copy()
            non_zero_idx = np.argwhere(matrix_cp)
            ixs = np.random.permutation(non_zero_idx)
            val_idx = tuple(ixs[:n_validation].T)
            test_idx = tuple(ixs[n_validation:n_validation + n_test].T)

            val_values = matrix_cp[val_idx].A1
            test_values = matrix_cp[test_idx].A1

            matrix_cp[val_idx] = matrix_cp[test_idx] = 0
```

```
            matrix_cp.eliminate_zeros()

            return matrix_cp, val_idx, test_idx, val_values, test_values
```

In [8]:
```
M = cold_start_preprocessing(M, 20)
```

```
Shape before: (337867, 5899)
Shape after: (3529, 2072)
```

In [9]:
```
n_validation = 200
n_test = 200
# Split data
M_train, val_idx, test_idx, val_values, test_values = split_data(M, n_validation, n_tes
```

In [10]:
```
# Remove user means.
nonzero_indices = np.argwhere(M_train)
M_shifted, user_means = shift_user_mean(M_train)
# Apply the same shift to the validation and test data.
val_values_shifted = val_values - user_means[np.array(val_idx).T[:,0]].A1
test_values_shifted = test_values - user_means[np.array(test_idx).T[:,0]].A1
```

1.1.4 Compute the loss function (nothing to do here)

In [11]:
```
def loss(values, ixs, Q, P, reg_lambda):
    """
    Compute the loss of the latent factor model (at indices ixs).
    Parameters
    ----------
    values : np.array, shape [n_ixs,]
        The array with the ground-truth values.
    ixs : tuple, shape [2, n_ixs]
        The indices at which we want to evaluate the loss (usually the nonzero indice
    Q : np.array, shape [N, k]
        The matrix Q of a latent factor model.
    P : np.array, shape [k, D]
        The matrix P of a latent factor model.
    reg_lambda : float
        The regularization strength

    Returns
    -------
    loss : float
           The loss of the latent factor model.

    """
    mean_sse_loss = np.sum((values - Q.dot(P)[ixs])**2)
    regularization_loss =  reg_lambda * (np.sum(np.linalg.norm(P, axis=0)**2) + np.sum

    return mean_sse_loss + regularization_loss
```

1.2 Alternating optimization

In the first step, we will approach the problem via alternating optimization, as learned in the lecture. That is, during each iteration you first update Q while having P fixed and then vice versa.

1.2.1 Task 2: Implement a function that initializes the latent factors Q and P

In [12]:
```python
def initialize_Q_P(matrix, k, init='random'):
    """
    Initialize the matrices Q and P for a latent factor model.

    Parameters
    ----------
    matrix : sp.spmatrix, shape [N, D]
             The matrix to be factorized.
    k      : int
             The number of latent dimensions.
    init   : str in ['svd', 'random'], default: 'random'
             The initialization strategy. 'svd' means that we use SVD to initialize P
             the entries in P and Q randomly in the interval [0, 1).

    Returns
    -------
    Q : np.array, shape [N, k]
        The initialized matrix Q of a latent factor model.

    P : np.array, shape [k, D]
        The initialized matrix P of a latent factor model.
    """
    np.random.seed(0)

    N, D = matrix.shape
    if init == 'svd':
        U, s, V = svds(matrix, k=k)
        S = np.diag(s)
        Q = U.dot(S)
        P = V
    else:
        Q = np.random.random((N, k))
        P = np.random.random((k, D))

    assert Q.shape == (matrix.shape[0], k)
    assert P.shape == (k, matrix.shape[1])
    return Q, P
```

1.2.2 Task 3: Implement the alternating optimization approach

In [15]:
```python
def latent_factor_alternating_optimization(M, non_zero_idx, k, val_idx, val_values,
                                           reg_lambda, max_steps=100, init='random',
```

```
                                   log_every=1, patience=5, eval_every=1):
    """
    Perform matrix factorization using alternating optimization. Training is done via
    i.e. we stop training after we observe no improvement on the validation loss for
    amount of training steps. We then return the best values for Q and P oberved duri

    Parameters
    ----------
    M                 : sp.spmatrix, shape [N, D]
                        The input matrix to be factorized.

    non_zero_idx      : np.array, shape [nnz, 2]
                        The indices of the non-zero entries of the un-shifted matrix
                        nnz refers to the number of non-zero entries. Note that this
                        from the number of non-zero entries in the input matrix M, e.
                        that all ratings by a user have the same value.

    k                 : int
                        The latent factor dimension.

    val_idx           : tuple, shape [2, n_validation]
                        Tuple of the validation set indices.
                        n_validation refers to the size of the validation set.

    val_values        : np.array, shape [n_validation, ]
                        The values in the validation set.

    reg_lambda        : float
                        The regularization strength.

    max_steps         : int, optional, default: 100
                        Maximum number of training steps. Note that we will stop earl
                        no improvement on the validation error for a specified number
                        (see "patience" for details).

    init              : str in ['random', 'svd'], default 'random'
                        The initialization strategy for P and Q. See function initial

    log_every         : int, optional, default: 1
                        Log the training status every X iterations.

    patience          : int, optional, default: 5
                        Stop training after we observe no improvement of the validati
                        iterations (see eval_every for details). After we stop traini
                        observed values for Q and P (based on the validation loss) an

    eval_every        : int, optional, default: 1
                        Evaluate the training and validation loss every X steps. If w
```

```python
                      of the validation error, we decrease our patience by 1, else 

    Returns
    -------
    best_Q            : np.array, shape [N, k]
                        Best value for Q (based on validation loss) observed during t

    best_P            : np.array, shape [k, D]
                        Best value for P (based on validation loss) observed during t

    validation_losses : list of floats
                        Validation loss for every evaluation iteration, can be used f
                        loss over time.

    train_losses      : list of floats
                        Training loss for every evaluation iteration, can be used for
                        loss over time.

    converged_after   : int
                        it - patience*eval_every, where it is the iteration in which 
                        or -1 if we hit max_steps before converging.

    """

    Q,P = initialize_Q_P(M, k, init)
    best_Q = Q
    best_P = P
    best_loss = -1
    validation_losses = []
    train_losses = []
    converged_after = -1
    train_idx = tuple(non_zero_idx.T)

    reg = Ridge(alpha=reg_lambda, fit_intercept=False)

    nnz_mask = sp.coo_matrix((np.ones(len(non_zero_idx)),
                              (non_zero_idx[:,0],non_zero_idx[:,1])),
                             shape=M.shape, dtype="uint8").tocsr()
    rows = nnz_mask.tolil().rows
    cols = nnz_mask.T.tolil().rows

    for i in range(max_steps):
        if i % eval_every == 0:
            # evaluate losses
            val_loss = loss(val_values, val_idx, Q, P, reg_lambda)
            validation_losses.append(val_loss)

            train_loss = loss(M[train_idx].A1, train_idx, Q, P, reg_lambda)
```

```
                train_losses.append(train_loss)

                if best_loss <= -1 or val_loss < best_loss:
                    best_Q = Q
                    best_P = P
                    best_loss = val_loss
                    current_patience = patience
                else:
                    current_patience -= 1

                if current_patience == 0:
                    converged_after = i - patience * eval_every
                    break

            print("Iteration ", i)

            # fix Q
            for rating_idx in range(M.shape[1]):
                nnz_idx = cols[rating_idx]
                res = reg.fit(Q[nnz_idx], np.squeeze(M[nnz_idx, rating_idx].toarray()))
                P[:, rating_idx] = res.coef_

            # fix P
            for user_idx in range(M.shape[0]):
                nnz_idx = rows[user_idx]
                res = reg.fit(P[:, nnz_idx].T, np.squeeze(M[user_idx, nnz_idx].toarray()))
                Q[user_idx, :] = res.coef_

        return best_Q, best_P, validation_losses, train_losses, converged_after
```

1.2.3 Train the latent factor (nothing to do here)

```
In [16]: Q, P, val_loss, train_loss, converged = latent_factor_alternating_optimization(M_shif
                                                                                    k=100,
                                                                                    val_va
                                                                                    reg_la
                                                                                    max_st
```

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```

```
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```

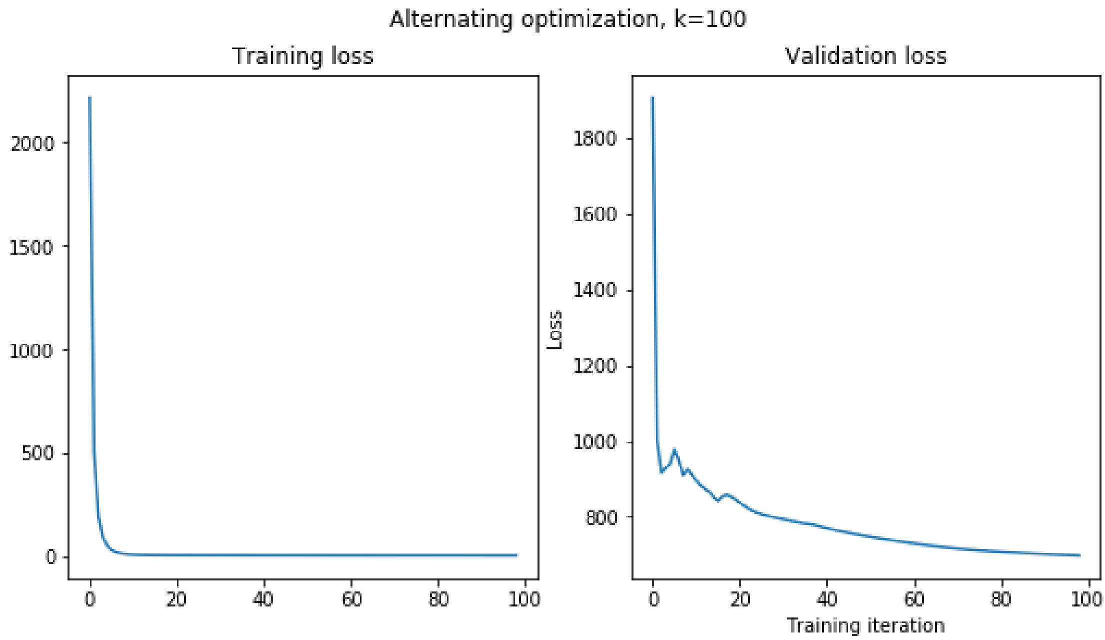
1.2.4 Plot the validation and training losses over for each iteration (nothing to do here)

```
In [17]: fig, ax = plt.subplots(1, 2, figsize=[10, 5])
         fig.suptitle("Alternating optimization, k=100")
```

```
ax[0].plot(train_loss[1::])
ax[0].set_title('Training loss')
plt.xlabel("Training iteration")
plt.ylabel("Loss")


ax[1].plot(val_loss[1::])
ax[1].set_title('Validation loss')
plt.xlabel("Training iteration")
plt.ylabel("Loss")

plt.show()
```




```
In [ ]:
```