Exercise

07

TUM Department of Informatics

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Informatics 3 - Professorship of Data Mining and Analytics

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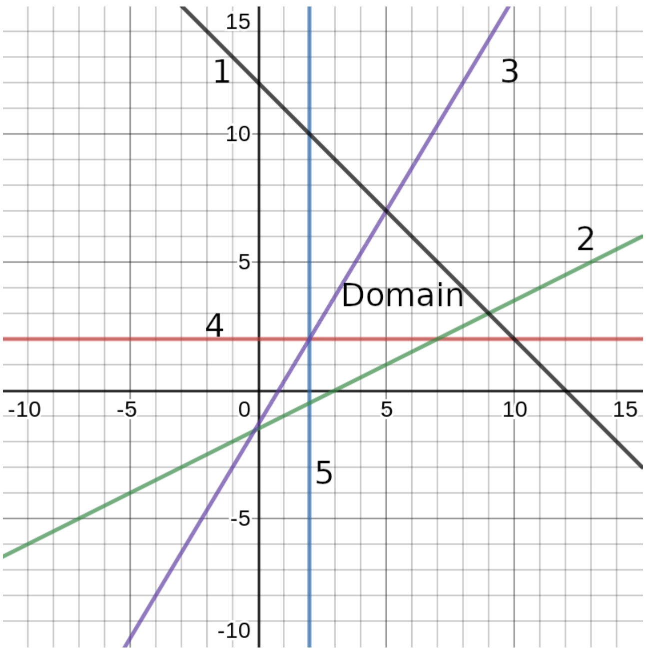
Constrained Optimization

Problem 1:

Constraints:

$$\theta_1 + \theta_2 \leq 12 \implies \theta_2 \leq 12 - \theta_1 \tag{1}$$
$$-\theta_1 + 2\theta_2 \geq -3 \implies \theta_2 \geq \frac{\theta_1 - 3}{2} \tag{2}$$
$$-5\theta_1 + 3\theta_2 \leq -4 \implies \theta_2 \leq \frac{1}{3}(5\theta_1 - 4) \tag{3}$$
$$\theta_2 \geq 2 \tag{4}$$
$$\theta_1 \geq 2 \tag{5}$$

Plot: Axis along x-dimension: θ_1, Axis along y-dimension: θ_2, simply plot the functions (1) up to (5):



$f(\theta) = 2\theta_1 - 3\theta_2$ Minimizer and maximizer both need to be a corner vertex of the domain.

Simple testing against $(2, 2)$, $(7, 2)$, $(9, 3)$ and $(5, 7)$ shows the solution:

Minimizer $\theta_{min} = (5, 7) f(\theta_{min}) = -11$

Maximizer $\theta_{max} = (9, 3) f(\theta_{max}) = 9$

Problem 2:

a)

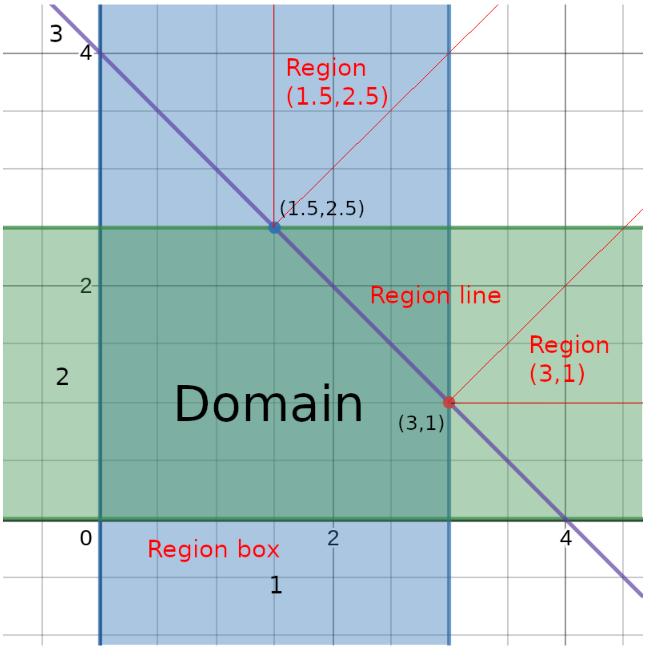
Constraints:

$$\theta_1 + \theta_2 \leq 4 \quad (1)$$

$$0 \leq \theta_1 \leq 3 \quad (2)$$

$$0 \leq \theta_2 \leq 2.5 \quad (3)$$

Plot: Axis along x-dimension: θ_1, Axis along y-dimension: θ_2, simply plot the functions (1) up to (3):



Domain corner vertices: $(0, 0)$, $(3, 0)$, $(3, 1)$, $(1.5, 2.5)$, $(0, 2.5)$

Domain regions: Domain (inside), box, line, $(3, 1)$, $(1.5, 2.5)$

$\pi_{\mathcal{X}}(p) = p$ if $p \in$ Domain (inside)

$\pi_{\mathcal{X}}(p) = (3, 1)$ if $p \in$ Region (3,1)

$\pi_{\mathcal{X}}(p) = (1.5, 2.5)$ if $p \in$ Region (1.5,2.5)

$\pi_{\mathcal{X}}(p) = \pi_{line}$ if $p \in$ Region line

$\pi_{\mathcal{X}}(p) = \pi_{box}$ if $p \in$ Region box

$$
\begin{aligned}
p \in \text{ Domain (inside)} &= \{p | p \in \mathcal{X}\} \\
p \in \text{ Region (3,1) } &= \{p | p_2 \geq 1 \land -p_1 + p_2 \leq -2\} \\
p \in \text{ Region (1.5,2.5)} &= \{p | p_1 \geq 1.5 \land -p_1 + p_2 \geq 1\} \\
p \in \text{ Region line} &= \{p | p_1 + p_2 > 4 \land -2 < -p_1 + p_2 < 1\} \\
p \in \text{ Region box} &= \{p | p \notin \mathcal{X} \land (p_1 < 1.5 \lor p_2 < 1)\}
\end{aligned}
$$

Let a be $(4, 0)$ and let b be $(0, 4)$:

$$
\begin{aligned}
\pi_{line}(p) &= a + \frac{(p-a)^T(b-a)}{||b-a||_2^2}(b-a) \\
&= \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{(p_1 - 4, p_2)\begin{pmatrix} -4 \\ 4 \end{pmatrix}}{32} \begin{pmatrix} -4 \\ 4 \end{pmatrix} \\
&= \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{16 - 4p_1 + 4p_2}{32} \begin{pmatrix} -4 \\ 4 \end{pmatrix} \\
&= \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \left(2 - \tfrac{1}{2}p_1 + \tfrac{1}{2}p_2\right) \begin{pmatrix} -1 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 2 + \frac{1}{2}p_1 - \frac{1}{2}p_2 \\ 2 - \frac{1}{2}p_1 + \frac{1}{2}p_2 \end{pmatrix}
\end{aligned}
$$

Formula taken from the lecture:

$$
\pi_{box}(p) = \begin{pmatrix} max(0, min(3, p_1)) \\ max(0, min(2.5, p_2)) \end{pmatrix}
$$

Problem 3:

Problem 4:

Appendix

We confirm that the submitted solution is original work and was written by us without further assistance. Appropriate credit has been given where reference has been made to the work of others.

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