Exercise

01

TUM Department of Informatics

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Informatics 3 - Professorship of Data Mining and Analytics

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Submission date Munich, November 23, 2019

Optimization

Problem 1:

a)

$h(x) = g_2(g_1(x))$ is not convex in this context:

Since g_2 and g_1 are convex: $g_2'' \geq 0$ and $g_1'' \geq 0$ but g_2 can be decreasing so $g_2'(x)$ does not have to be positive $\forall x$.

$$[h(x)]' = g_2'(g_1(x)) * g_1'(x)$$
$$[h(x)]'' = [g_2'(g_1(x)) * g_1'(x)]'$$
$$[h(x)]'' = g_2''(g_1(x)) * (g_1'(x))^2 + g_2'(g_1(x)) * g_1''(x)$$

So $[h(x)]''$ can be ≤ 0 if $g_2'(g_1(x)) \leq 0$ ($(g_1'(x))^2$ is always ≥ 0)

b)

$h(x) = g_2(g_1(x))$ is convex in this context:

Since g_2 and g_1 are convex: $g_2'' \geq 0$ and $g_1'' \geq 0$ also g_2 is non-decreasing so $g_2'(x) \geq 0$.

$$[h(x)]' = g_2'(g_1(x)) * g_1'(x)$$
$$[h(x)]'' = [g_2'(g_1(x)) * g_1'(x)]'$$
$$[h(x)]'' = g_2''(g_1(x)) * (g_1'(x))^2 + g_2'(g_1(x)) * g_1''(x)$$

So $[h(x)]''$ can only be ≥ 0. ($(g_1'(x))^2$ is always ≥ 0)

c)

$h(x) = max(g_1(x), ..., g_n(x))$ is always convex:

$$\begin{aligned}
h(\lambda x + (1-\lambda)y) &= max(g_1(\lambda x + (1-\lambda)y), ..., g_n(\lambda x + (1-\lambda)y) \\
&\leq max(\lambda g_1(x) + (1-\lambda)g_1(y), ..., \lambda g_n(x) + (1-\lambda)g_n(y)) \\
&\leq max(\lambda g_1(x), ..., \lambda g_n(x)) + max(\lambda g_1(y), ..., \lambda g_n(y)) \\
&= \lambda h(x) + (1-\lambda)h(y)
\end{aligned}$$

Problem 2:

Problem 3:

Problem 4:

Appendix

We confirm that the submitted solution is original work and was written by us without further assistance. Appropriate credit has been given where reference has been made to the work of others.

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