3D Scanning & Motion Capture

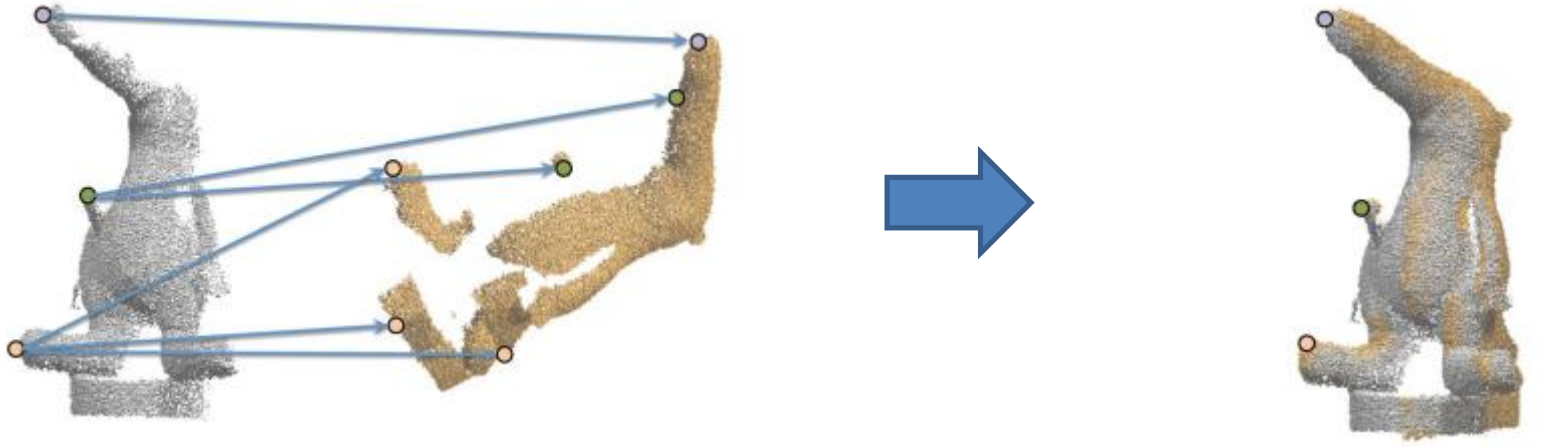
Exercise - 4

Dejan Azinović, Manuel Dahnert

Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation



Procrustes

- Problem: Align two objects using known correspondences

  →scaling, translation, rotation

  - Compute center of gravity of both objects
  - Scale one object to match the avg. distance from all vertices to the center of gravity

Procrustes

- Problem: Align two objects using known correspondences

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  - Translation is given by the vector between the center of gravity of both objects

Procrustes

- Problem: Align two objects using known correspondences

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  - Assume objects that are zero centered
    - Target object: $\{x_0, .. x_{n-1}\}$
    - Moving object: $\{\hat{x}_0 , .. \hat{x}_{n-1}\}$

$$\sum_i \|x_i - R \cdot \hat{x}_i\|_2^2 \rightarrow min$$

$$\left\|X - \hat{X}R^T\right\|_F^2 \rightarrow min$$

Procrustes

- Problem: Align two objects using known correspondences

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$$\left\| X - \hat{X}R^T \right\|_F^2 \rightarrow min$$

$\|A\|_F^2 = trace(A^T A)$

Cyclic invariance of trace:

$trace(ABC) = trace(CAB)$

$$\left\| X - \hat{X}R^T \right\|_F^2 = trace(X^T X - X^T \hat{X} R^T - \left(\hat{X}R^T\right)^T X + \left(\hat{X}R^T\right)^T (\hat{X}R^T)) \rightarrow min$$

$$trace(-X^T \hat{X} R^T - \left(\hat{X}R^T\right)^T X + \left(\hat{X}R^T\right)^T (\hat{X}R^T)) \rightarrow min$$

$$-2 \cdot trace(X^T \hat{X} R^T) \rightarrow min$$

$$trace(X^T \hat{X} R^T) \rightarrow max$$

SVD: $X^T \hat{X} = USV^T$

$$trace(USV^T R^T) \rightarrow max$$

$$trace(SV^T R^T U) \rightarrow max$$

Singular values →positive

Product of orthogonal matrices →max if equal to Identity

Procrustes

- Problem: Align two objects using known correspondences

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$$\left\| X - \hat{X} R^T \right\|_F^2 \rightarrow min$$

  - Compute SVD of the Cross-Covariance Matrix

$$X^T \hat{X} = USV^T$$

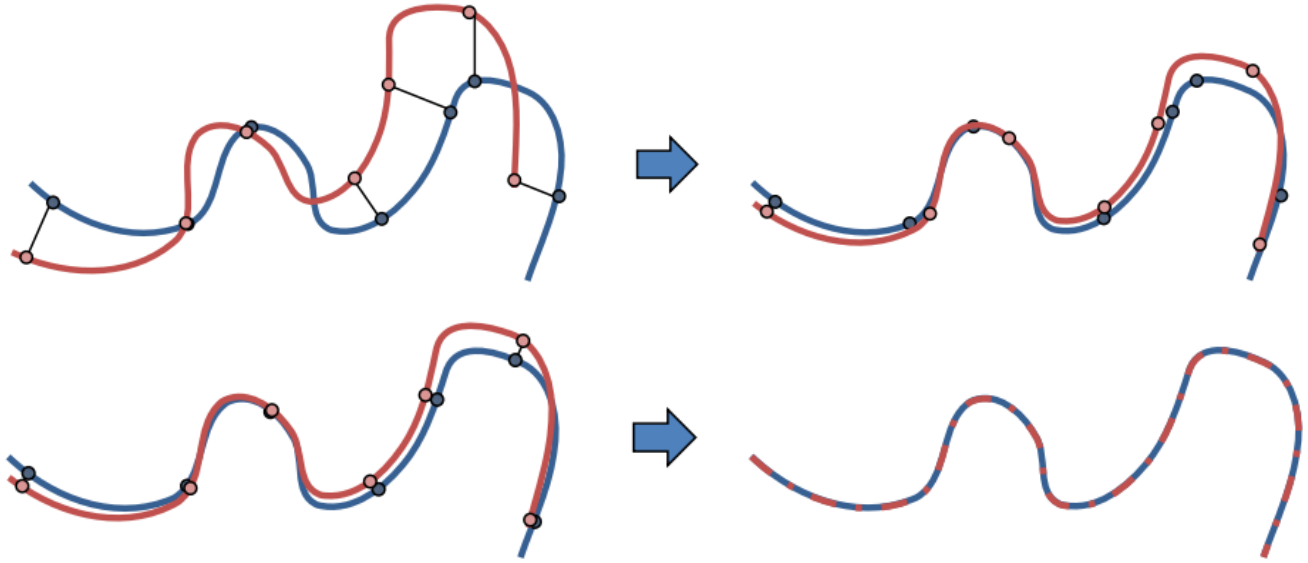
  - Compute the rotation

$$R = UV^T$$

ICP (Iterative Closest Point)

- Problem: Align two objects with unknown correspondences
  - Iterate:
    - Estimate correspondences using the current alignment and nearest neighbors
    - Use the correspondences to compute new alignment based on
      - Point-to-point distances
        - » Procrustes

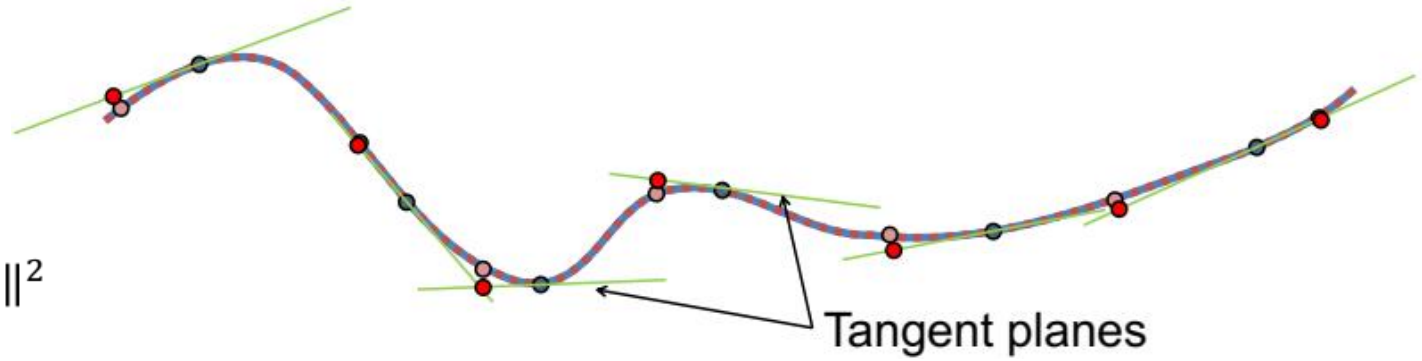
$$\min_{R,t} \sum_i \|p_i - (Rq_i + t)\|^2$$



ICP (Iterative Closest Point)

- Problem: Align two objects with unknown correspondences
  - Iterate:
    - Estimate correspondences using the current alignment and nearest neighbors
    - Use the correspondences to compute new alignment based on
      - Point-to-point distances
      - Point-to-plane distances
        - » Faster convergence
        - » Non-linear!

$$\min_{R,t} \sum_i \|(p_i - (Rq_i + t)) \cdot n_i\|^2$$



ICP (Iterative Closest Point)

- Problem: Align two objects with unknown correspondences
  - Iterate:
    - Estimate correspondences using the current alignment and nearest neighbors
    - Use the correspondences to compute new alignment based on
      - Point-to-point distances
      - Point-to-plane distances
    - Use weighting of correspondences and pruning
      - Good correspondences are close, have similar normal, …
      - Prune correspondences to border

$$\min_{R,t} \sum_i w_i \|p_i - (Rq_i + t)\|^2$$