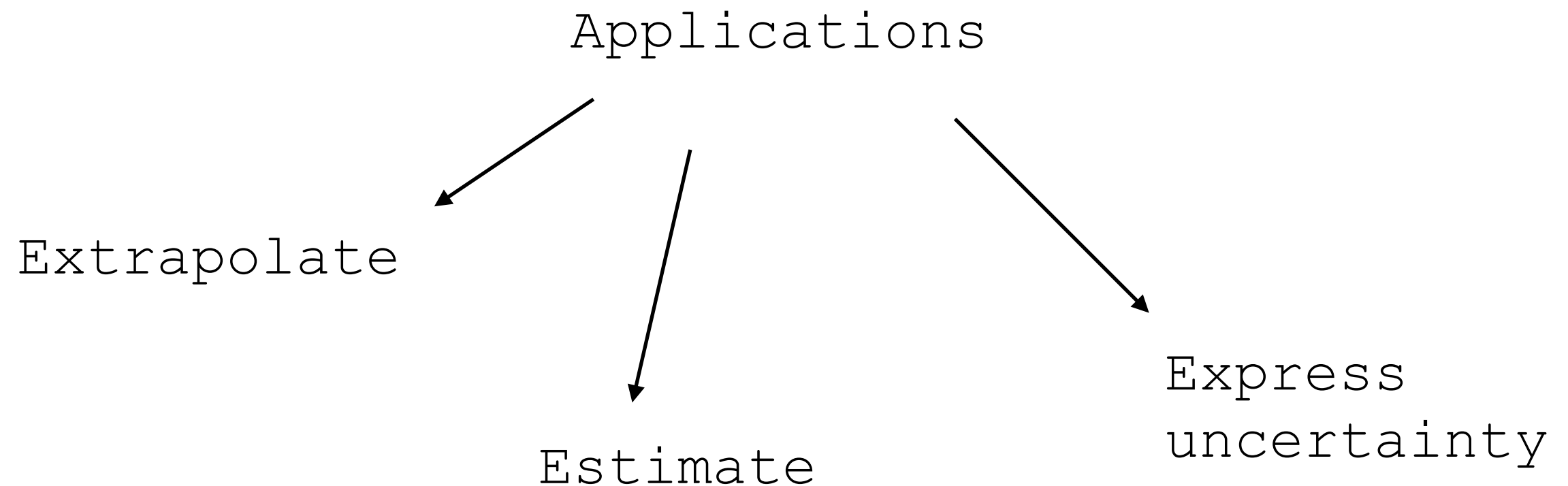


# Lesson 13

Tuesday 3/12/24

# Inference

Conclusions = Data + Assumptions (Manski, 2011)



# Probability

Relative frequency version

$$p(\text{event}) = \frac{\text{\# of times an event occurs}}{\text{\# of times the event could have occurred}}$$

the proportion or fraction of times we would expect an event to occur (expressed on a  $[0,1]$  scale.

Example: probability that someone drawn at random from the population gets arrested at least once by age 25.

# Probability

Degree of belief version

$p(\text{event}) =$  the likelihood  
or chance an  
event occurs  
expressed on the  
[0,1] scale.

Example: a jury decides that someone is liable in a civil lawsuit by a preponderance of the evidence (meaning it is more probable than not that the defendant is liable).

# Bounding Rule

Probabilities must be in the  $[0,1]$  interval; a probability of zero means the event is impossible; a probability of one means the event is certain.

Example:  $p(\text{age at prison release} = 8)$  is zero; means there is no chance that someone age 8 could be released from prison.

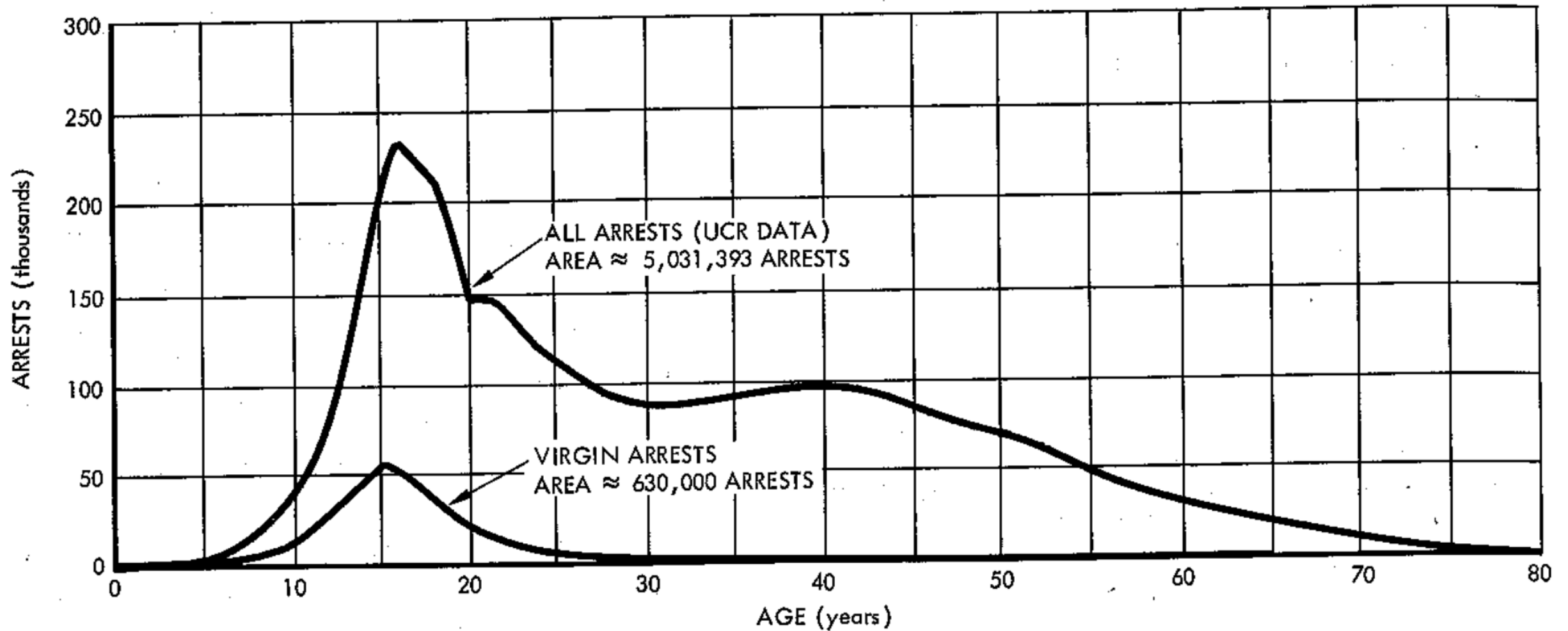
# Complements

If  $p(a)$  is the probability that event  $a$  occurs, then  $p(\text{not } a)$  is  $1 - p(a)$ . We say that  $p(\text{not } a)$  is the complement of  $p(a)$ .

Example: A sample of people were all arrested at age 16. For each of these people, the arrest can be classified as either a "first-time" arrest or a "recidivist" arrest (but not both). We can say that  $p(\text{recidivist})$  is the complement of  $p(\text{first time})$ .

# Age and Arrest (Christensen's Chart, 1967)

FIGURE J-2. 1965 ARRESTS BY AGE FOR ALL NONTRAFFIC OFFENSES



# Restricted Addition Rule

If  $a$  and  $b$  are mutually exclusive events then  $p(a) + p(b) = p(a+b)$ .  
In words,  $p(a+b)$  is the probability that  $a$  or  $b$  occurs  $\rightarrow p(a \text{ or } b)$

Example: A sample of people were all arrested at age 16. For each of these people, the arrest can be classified as a "first-time" arrest or a "recidivist" arrest. These two types of arrests are mutually exclusive and exhaustive. So,  $p(\text{first time}) + p(\text{recidivist}) = 1$ . In words, we can say that the probability that an arrest is either a first-time or a recidivist arrest is 1.



# General Addition Rule

If  $a$  and  $b$  are not mutually exclusive events then:

$$p(a+b) = p(a) + p(b) - p(a \text{ and } b)$$

Example: consider a sample of people who have been convicted of domestic violence. We follow each of these people for 3 years and document arrests for new crimes against the same victim. Here is the formula we will use:

$$p(v+pr) = p(v) + p(pr) - p(v \text{ and } pr)$$

# Numerical Example of General Addition Rule

Example: consider a sample of people who have been convicted of domestic violence. We follow each of these people for 3 years and document arrests for new crimes against the same victim. Here are our data:

	Violent= No	Violent= Yes	Total
Property=No	51	35	86
Property=Yes	36	18	54
Total	87	53	140

What is the probability that someone drawn at random was arrested for either a violent or a property offense?

# Numerical Example of General Addition Rule (Continued)

	Violent=No	Violent=Yes	Total
Property=No	51	35	86
Property=Yes	36	18	54
Total	87	53	140

$$p(\text{violent}) = 53/140 = 0.379$$

$$p(\text{property}) = 54/140 = 0.386$$

$$p(v \ \& \ pr) = 18/140 = 0.129$$

$$p(v+pr) = p(v) + p(pr) - p(v \text{ and } pr)$$

$$p(v+pr) = 53/140 + 54/140 - 18/140 =$$


$$= 0.379 + 0.386 - 0.129 = 0.636$$

# Union and Intersection Notation

$$p(a+b) = p(a) + p(b) - p(a \text{ and } b)$$

=

$$p(a \text{ **U** } b) = p(a) + p(b) - p(a \cap b)$$



union  
or



intersection  
and

# Restricted Multiplication Rule

If events  $a$  and  $b$  are independent,  
then  $p(a \text{ and } b) = p(a) \times p(b)$

$a = \text{fair coin flip} \rightarrow \text{heads}$

$b = \text{fair coin flip} \rightarrow \text{tails}$

$$p(a \text{ and } b) = p(\text{HT}) = 0.5 \times 0.5 = 0.25$$

# Simulation Demonstrating Validity of Restricted Multiplication Rule

```
> a <- vector()
> b <- vector()
>
> for(i in 1:10000000) {
+   a[i] <- ifelse(runif(n=1)>0.5, "H", "T")
+   b[i] <- ifelse(runif(n=1)>0.5, "H", "T")
+ }
>
> table(a,b)
      b
a      H      T
H 2501200 2500166
T 2498130 2500504
> mean(a=="H")*mean(b=="T")
[1] 0.2501018
>
```

*Advanced - not on exam*

# Textbook, pp. 161–162

Experiment: flip a fair coin two times and count the number of heads arising from the 2 flips. The sample space for this experiment is: 0 heads, 1 head, 2 heads.

Repeat this experiment  $k$  times.

	$k = 10$	$k = 100$	$k = 1000$	$k = 10,000$	$k = 100,000$
# heads = 0	3	20	262	2472	24919
# heads = 1	5	59	504	5036	50074
# heads = 2	2	21	234	2492	25007

## Joint Probability Assuming Independence

	Weapon = No	Weapon = Yes	Total
Crime = No	335	11	346
Crime = Yes	70	16	86
Total	405	27	432

$p(\text{carry a weapon}$   
 $\text{and no criminal}$   
 $\text{involvement})$

Restricted  
Multiplication  
Rule:

$$\begin{aligned} p(w \ \& \ c) &= p(w) \times p(nc) = 27/432 \times 346/432 \\ &= 0.063 \times 0.801 = 0.050 \end{aligned}$$



# General Multiplication Rule

	Weapon = No	Weapon = Yes	Total
Crime = No	335	11	346
Crime = Yes	70	16	86
Total	405	27	432

$p(\text{carry a weapon}$   
 $\text{and no criminal}$   
 $\text{involvement})$

$$p(a \ \& \ b) = p(a) \times p(b|a)$$

$$\begin{aligned} p(w \ \& \ nc) &= p(w) \times p(nc|w) = 27/432 \times 11/27 \\ &= 0.063 \times 0.407 = 0.026 \end{aligned}$$

# Binomial Probability Distribution

$$p(y = r) = \binom{N}{r} p^r q^{(N-r)}, \text{ for } r = 0, 1, \dots, N$$

which can also be written as:

$$p(y = r) = \frac{N!}{r!(N-r)!} p^r q^{(N-r)}, \text{ for } r = 0, 1, \dots, N$$

where  $p$  is the probability of the event of interest occurring on any given experiment,  $q = 1 - p$ ,  $y$  is the observed number of events,  $r$  represents the outcomes in the sample space, and  $N$  is the number of experiments.

# Coin Flipping Experiment Revisited

Experiment: flip a fair coin  $N = 2$  times and count the number of heads arising from the 2 flips. The sample space for this experiment is:  $r=0$  heads,  $r=1$  head, and  $r=2$  heads.

$$p(y = 0) = \frac{2!}{0!(2-0)!} 0.5^r 1 - 0.5^{(2-0)} = 0.5^2 = 0.25$$

$$p(y = 1) = \frac{2!}{1!(2-1)!} 0.5^1 1 - 0.5^{(2-1)} = 2 \times 0.5 \times 0.5 = 0.5$$

$$p(y = 2) = \frac{2!}{2!(2-2)!} 0.5^2 1 - 0.5^{(2-2)} = 0.5 \times 0.5 = 0.25$$

Compare to what we got before...