Exam 3 Practice Questions Part II

Preamble: Consider the following contingency table examining the effect of a drug treatment program (randomly assigned to treatment = 1 and randomly assigned to control = 0) on substance use relapse rates (no relapse = 0, relapse = 1).

	$R = 0$	$R = 1$	Total
$T = 0$	322	165	487
$T = 1$	373	119	492
Total	695	284	979

1. Calculate $p(R = 1|T = 0)$.

   A. 0.537

   B. 0.662

   C. 0.339

   D. 0.497

   E. 0.242

   Solution:

$$p(R = 1|T = 0) = \frac{\text{Number of Controls Who Relapsed}}{\text{Number of Controls}} = \frac{165}{487} = 0.339$$

2. Calculate $p(R = 1|T = 1)$.

   A. 0.537

   B. 0.662

   C. 0.339

   D. 0.497

   E. 0.242

   Solution:

$$p(R = 1|T = 0) = \frac{\text{Number of Treated Who Relapsed}}{\text{Number of Treated}} = \frac{119}{492} = 0.242$$

3. Calculate the relative risk statistic so that you can comment on how many times more likely it is for a control case to relapse in comparison to a treatment case.

   A. 0.714

   B. 1.372

   C. 1.489

   D. 2.203

   E. 1.845

   Solution:

$$\text{Relative Risk} = \frac{p(R=1|T=0)}{p(R=1|T=1)} = \frac{0.339}{0.242} = 1.372$$

4. Calculate the difference between the relapse rates for the two groups.

   A. 0.097

   B. 0.124

   C. 0.308

   D. 0.031

   E. 0.242

   Solution:

$$\Delta = p(R=1|T=1) - p(R=1|T=0) = 0.242 - 0.339 = -0.097$$

   Note: the difference between the two numbers is a positive number; sometimes people want to calculate a *treatment effect* where one needs to specify which number comes first and which number comes second. Usually, in these cases, we would put $p(R=1|T=1)$ first and $p(R=1|T=0)$ second as we did above. For this example, the treatment effect would be a negative number.

5. Test the hypothesis that treatment and relapse are independent of each other using a chi-square test of independence. Use a 0.01 significance level for your test. What decision do you make?

   A. reject

   B. fail to reject

   C. not enough information to tell

   Solution:

   Critical value of chi-square test = 6.635 (w/1 degree of freedom)

$$\text{Chi Square Test} = \frac{(O_A - E_A)^2}{E_A} + \frac{(O_B - E_B)^2}{E_B} + \frac{(O_C - E_C)^2}{E_C} + \frac{(O_D - E_D)^2}{E_D}$$

Cell	O	E	$O-E$	$(O-E)^2$	$[(O-E)^2]/E$
A	322	695*487/979 = 345.725	-23.725	562.876	562.876/345.725 = 1.628
B	165	284*487/979 = 141.275	23.725	562.876	562.876/141.275 = 3.984
C	373	695*492/979 = 349.275	23.725	562.876	562.876/349.275 = 1.612
D	119	284*492/979 = 142.725	-23.725	562.876	562.876/142.725 = 3.944

So, now we are able to calculate our chi-square test statistic which is:

$$\text{Chi-Square Test Statistic} = 1.628 + 3.984 + 1.612 + 3.944 = 11.168$$

and, since $11.168 > 6.635$, we reject the hypothesis of independence and conclude that the 2 variables (treatment and relapse) are empirically related to each other.

6. Based on the information in this table, estimate Yule's Q for this table.

   A. -0.108

   B. -0.143

   C. -0.233

   D. -0.314

   E. -0.181

   Solution:

$$Q = \frac{AD - BC}{AD + BC} = \frac{322 \times 119 - 165 \times 373}{322 \times 119 + 165 \times 373} =$$

$$\frac{38318 - 61545}{38318 + 61545} = \frac{-23227}{99863} = -0.233$$

7. Reconsidering the contingency table in problem 11, estimate a 90% confidence interval for Yule's Q. Does the 90% confidence interval you estimated include the number zero?

   A. yes

   B. no

   C. not enough information to tell

   Solution:

$$Q - z\text{-multiplier} \times \sqrt{\frac{(1-Q^2)^2(\frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \frac{1}{D})}{4}}$$

   Q = -0.233
   z-multiplier for two-tailed $\alpha = 0.10 = 1.645$

$$\text{LCL} = -0.233 - 1.645 \times \sqrt{\frac{(1 - 0.233^2)^2(\frac{1}{322} + \frac{1}{165} + \frac{1}{373} + \frac{1}{119})}{4}}$$

$$\text{LCL} = -0.233 - 1.645 \times \sqrt{\frac{(1 - 0.054)^2(\frac{1}{322} + \frac{1}{165} + \frac{1}{373} + \frac{1}{119})}{4}}$$

$$\text{LCL} = -0.233 - 1.645 \times \sqrt{\frac{0.895 \times (0.003 + 0.006 + 0.003 + 0.008)}{4}}$$

$$\text{LCL} = -0.233 - 1.645 \times \sqrt{\frac{0.895 \times 0.020}{4}}$$

$$\text{LCL} = -0.233 - 1.645 \times \sqrt{\frac{0.018}{4}}$$

$$\text{LCL} = -0.233 - 1.645 \times \sqrt{0.005}$$

$$\text{LCL} = -0.233 - 1.645 \times 0.071$$

$$\text{LCL} = -0.233 - 1.645 \times 0.071 = -0.350$$

$$\text{UCL} = -0.233 + 1.645 \times 0.071 = -0.116$$

8. The historical 1-year bond revocation rate from the local drug court is 0.48 (or 48%). We draw a random sample of 150 people from the drug court docket from 1 year ago and check to see how many of the people who "bonded out" were revoked within the 1-year follow-up period. We find out that 83 of the 150 people were revoked. Test the hypothesis that the recent bond revocation proportion is equal to the historical proportion (a non-directional, two-tailed test with $\alpha = 0.05$ or $p < .05$).

   A. reject

   B. fail to reject

   C. not enough information to tell

   Solution:

   - Population $P = 0.48$
   - This is a single sample hypothesis test for a proportion (two-tailed).
   - We have a large sample ($n = 150$).
   - Look up the two-tailed z-value for $\alpha = 0.05$: $z = 1.96$.
   - $\hat{\theta} = \frac{83}{150} = 0.553$

$$z\text{-test} = \frac{\hat{\theta} - P}{\sqrt{P(1-P)/n}}$$

$$z\text{-test} = \frac{0.553 - 0.48}{\sqrt{0.553(1-0.553)/150}}$$

$$z\text{-test} = \frac{0.073}{\sqrt{0.553 \times 0.447/150}}$$

$$z\text{-test} = \frac{0.073}{\sqrt{0.002}}$$

$$z\text{-test} = \frac{0.073}{0.045} = 1.622$$

   Since $1.622 < 1.96$, we fail to reject the hypothesis that the recent bond revocation rate is equal to the long-term average.