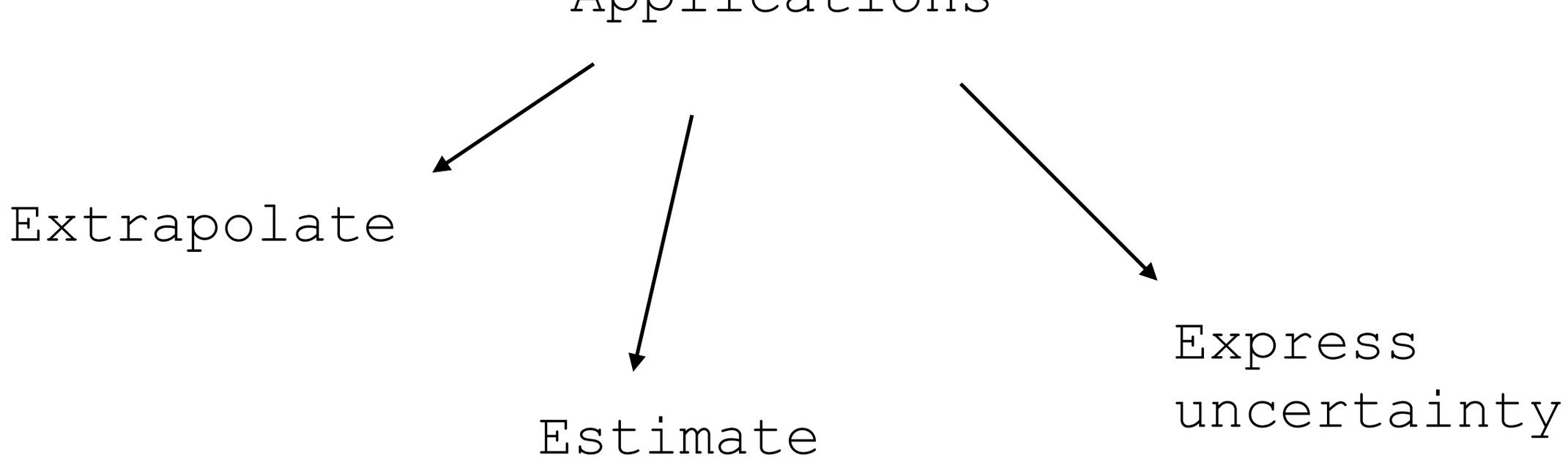
Lesson 13

Tuesday 3/12/24

Inference

Conclusions = Data + Assumptions (Manski, 2011)



Probability

frequency version

Relative fr

$$p(\text{event}) = \frac{\text{\# of times an event occurs}}{\text{\# of times the event could have occurred}}$$

the proportion or fraction of times we would expect an event to occur (expressed on a [0,1] scale.

Example: probability that someone drawn at random from the population gets arrested at least once by age 25.

Probability

Degree of belief version

p(event) = the likelihood or chance an event occurs expressed on the [0,1] scale.

Example: a jury decides that someone is liable in a civil lawsuit by a preponderance of the evidence (meaning it is more probable than not that the defendant is liable).

Bounding Rule

Probabilities must be in the [0,1] interval; a probability of zero means the event is impossible; a probability of one means the event is certain.

Example: p(age at prison release = 8) is zero; means there is no chance that someone age 8 could be released from prison.

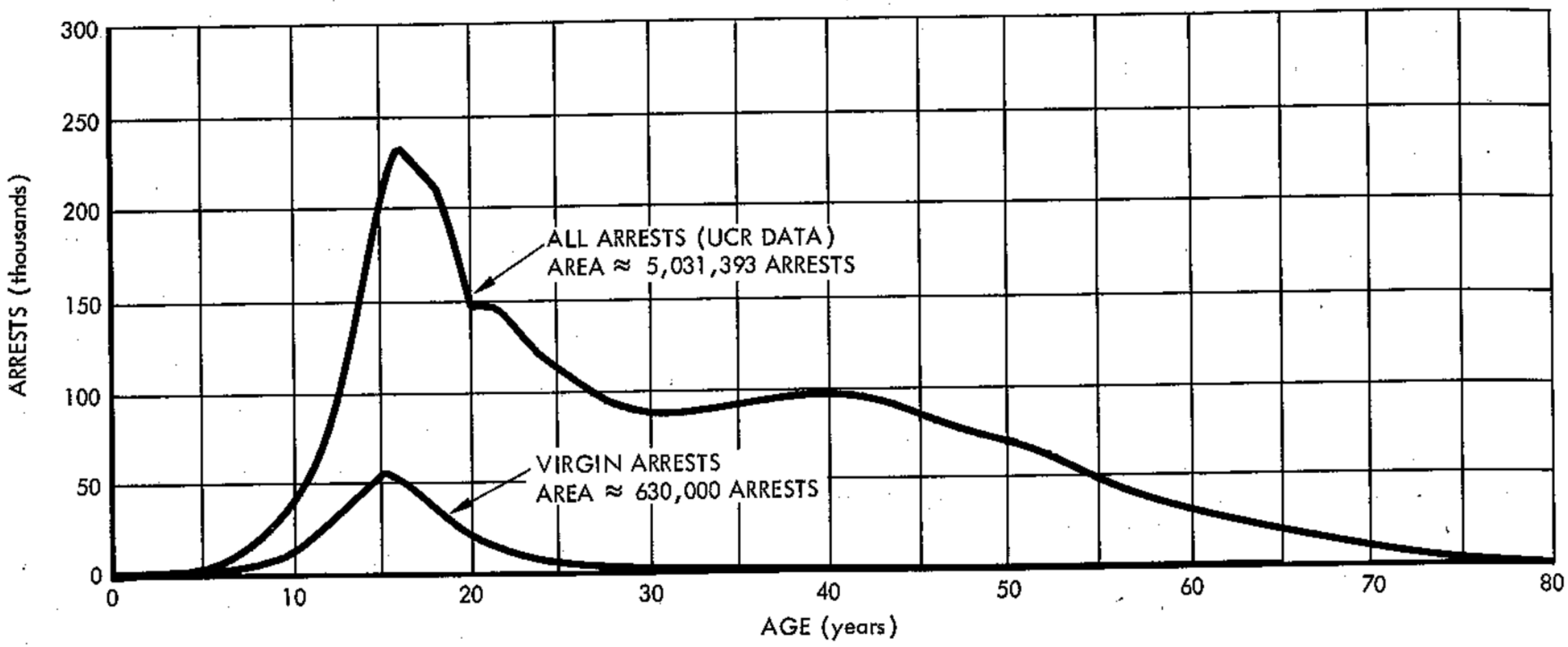
Complements

If *p*(a) is the probability that event *a* occurs, then *p*(not a) is 1 - *p*(a). We say that *p*(not a) is the complement of *p*(a).

Example: A sample of people were all arrested at age 16. For each of these people, the arrest can be classified as either a "first-time" arrest or a "recidivist" arrest (but not both). We can say that *p*(recidivist) is the complement of *p*(first time).

Age and Arrest (Christensen's Chart, 1967)

FIGURE J-2. 1965 ARRESTS BY AGE FOR ALL NONTRAFFIC OFFENSES



Restricted Addition Rule

If *a* and *b* are mutually exclusive events then *p*(a) + *p*(b) = *p*(a+b). In words, *p*(a+b) is the probability that *a* or *b* occurs → p(a or b)

Example: A sample of people were all arrested at age 16. For each of these people, the arrest can be classified as a "first-time" arrest or a "recidivist" arrest. These two types of arrests are mutually exclusive and exhaustive. So, *p*(first time) + *p*(recidivist) = 1. In words, we can say that the probability that an arrest is either a first-time or a recidivist arrest is 1.

General Addition Rule

If *a* and *b* are <u>not</u> mutually exclusive events then:

$$p(\text{a+b}) = p(\text{a}) + p(\text{b}) - p(\text{a and b})$$

<u>Example</u>: consider a sample of people who have been convicted of domestic violence. We follow each of these people for 3 years and document arrests for new crimes against the same victim. Here is the formula we will use:

$$p(\text{v+pr}) = p(\text{v}) + p(\text{pr}) - p(\text{v and pr})$$

Numerical Example of General Addition Rule

Example: consider a sample of people who have been convicted of domestic violence. We follow each of these people for 3 years and document arrests for new crimes against the same victim. Here are our data:

	Violent= No	Violent= Yes	Total
Property=No	51	35	86
Property=Yes	36	18	54
Total	87	53	140

What is the probability that someone drawn at random was arrested for either a violent or a property offense?

Numerical Example of General Addition Rule (Continued)

	Violent= No	Violent= Yes	Total
Property=No	51	35	86
Property=Yes	36	18	54
Total	87	53	140

p(violent) = 53/140 = 0.379

p(property) = 54/140 = 0.386

p(v & pr) = 18/140 = 0.129

$$p(\text{v+pr}) = p(\text{v}) + p(\text{pr}) - p(\text{v and pr})$$

$$p(\text{v+pr}) = 53/140 + 54/140 - 18/140 =$$

$$= 0.379 + 0.386 - 0.129 = 0.636$$

Union and Intersection Notation

$$p(a+b) = p(a) + p(b) - p(a \text{ and } b)$$

$$=$$

$$p(a \cup b) = p(a) + p(b) - p(a \cap b)$$

union
or

intersection
and

Restricted Multiplication Rule - Assumes Independence

	Weapon = No	Weapon = Yes	Total
Crime = No	335	11	346
Crime = Yes	70	16	86
Total	405	27	432

p(carry a weapon and no criminal involvement)

Restricted Multiplication Rule:

$$p(w \ \& \ c) = p(w) \times p(nc) = 27/432 \times 346/432$$

$$= 0.063 \times 0.801 = 0.050$$

General Multiplication Rule - Does Not Assume Independence

	Weapon = No	Weapon = Yes	Total
Crime = No	335	11	346
Crime = Yes	70	16	86
Total	405	27	432

p(carry a weapon and no criminal involvement)

$$p(a \ \& \ b) = p(a) \times p(b|a)$$

$$p(w \ \& \ nc) = p(w) \times p(nc|w) = 27/432 \times 11/27$$

$$= 0.063 \times 0.407 = 0.026$$