

$$a_1 (0,0) \rightarrow (1,0) \quad r(t) = (1-t)\langle 0,0 \rangle + t\langle 1,0 \rangle = \langle t, 0 \rangle \quad 0 \leq t \leq 1$$

$$\vec{F}(r(t)) = \langle x^2, 2xy \rangle = \langle t^2, 0 \rangle$$

$$\int_{a_1} \vec{F} \cdot r'(t) dt = \int_0^1 \langle t^2, 0 \rangle \cdot \langle 1, 0 \rangle dt = \int_0^1 t^2 dt = \left[ \frac{t^3}{3} \right]_0^1 = \frac{1}{3}$$

$$a_2 (1,0) \rightarrow (1,1) \quad r(t) = (1-t)\langle 1,0 \rangle + t\langle 1,1 \rangle = \langle 1, t \rangle \quad 0 \leq t \leq 1$$

$$\vec{F}(r(t)) = \langle 1, 2t \rangle \quad r'(t) = \langle 0, 1 \rangle$$

$$\int_{a_2} \vec{F} \cdot r'(t) dt = \int_0^1 \langle 1, 2t \rangle \cdot \langle 0, 1 \rangle dt = \int_0^1 2t dt = \left[ t^2 \right]_0^1 = 1$$

$$\int_a \vec{F} \cdot r'(t) dt = \int_{a_1} \vec{F} \cdot r'(t) dt + \int_{a_2} \vec{F} \cdot r'(t) dt = \frac{1}{3} + 1 = \frac{4}{3}$$

$$b) \quad b (0,0) \rightarrow (1,1) \quad r(t) = \langle t, t^2 \rangle \quad 0 \leq t \leq 1$$

$$r'(t) = \langle 1, 2t \rangle \quad \vec{F}(r(t)) = \langle t^2, 2t(t^2) \rangle = \langle t^2, 2t^3 \rangle$$

$$\int_0^1 \langle t^2, 2t^3 \rangle \cdot \langle 1, 2t \rangle dt = \int_0^1 (t^2 + 4t^4) dt = \left[ \frac{t^3}{3} + \frac{4t^5}{5} \right]_0^1$$

$$= \frac{1}{3} + \frac{4}{5} = \frac{5 + 12}{15} = \frac{17}{15}$$

$$c) \quad c (0,0) \rightarrow (1,1) \quad r(t) = \langle t^3, t^2 \rangle \quad 0 \leq t \leq 1$$

$$r'(t) = \langle 3t^2, 2t \rangle \quad \vec{F}(r(t)) = \langle t^6, 2t^3 t^2 \rangle = \langle t^6, 2t^5 \rangle$$

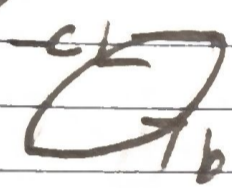
$$\int_0^1 \langle t^6, 2t^5 \rangle \cdot \langle 3t^2, 2t \rangle dt = \int_0^1 (3t^8 + 4t^6) dt$$

$$= \left[ \frac{3t^9}{9} + \frac{4t^7}{7} \right]_0^1 = \frac{1}{3} + \frac{4}{7} = \frac{7 + 12}{21} = \frac{19}{21}$$

1d) The force is not conservative. The work done from  $(0,0)$  to  $(1,1)$  is dependent on path

$$\int_{\text{path } b} \vec{F} \cdot d\vec{r} \neq \int_{\text{path } a} \vec{F} \cdot d\vec{r} \neq \int_{\text{path } c} \vec{F} \cdot d\vec{r}$$

Let's reverse the path of c, so that we get a closed loop with path b.



$$\int \vec{F} \cdot d\vec{r} = -\frac{19}{21}$$

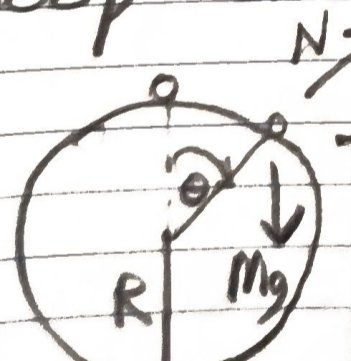
Then the line integral around the loop is:

$$\oint \vec{F} \cdot d\vec{r} = \int_b \vec{F} \cdot d\vec{r} + \int_{-c} \vec{F} \cdot d\vec{r} = \frac{17}{15} - \frac{19}{21} = \frac{8}{35} \neq 0$$

which is not zero.

Therefore the force cannot be conservative because the work done around a closed loop does not equal zero.

2a)



- The sphere of radius  $R$  is at rest and not rolling


- No air resistance acts on the puck when it slides down the sphere

- No friction acts on the puck

- The puck is a point particle

$h(\theta) = R + R \cos \theta = R(1 + \cos \theta)$

at angle  $\theta$  the normal force  $N$  acts perpendicular to the surface of the sphere outward



The centripetal force acts towards the center of the sphere.

The maximum possible angle is  $\pi/2, 90^\circ$ .

There is no other force acting on the puck towards the center other than the centripetal force.

The centripetal force is zero once the puck leaves the surface.

$$2b) \Delta E = \Delta K + \Delta U_{\text{grav}} = 0$$

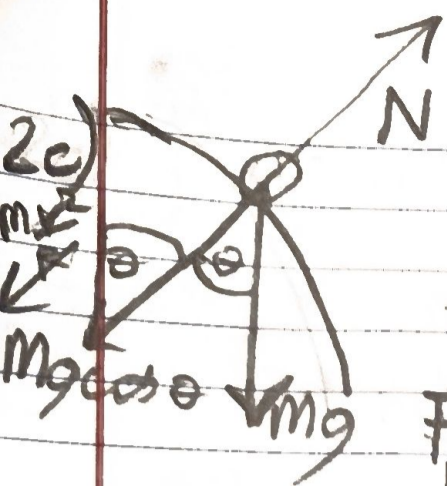
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + Mgh_f - Mgh_i = 0$$

Take the zero of  $U_{\text{grav}}$  to be the ground  $h=0$ . Let  $v_i=0$ ,  $h_i=2R$  to show the initial state of the puck,

$$\begin{aligned} \frac{1}{2}mv^2 + mgh(\theta) - mg(2R) &= \frac{1}{2}mv^2 + mg(R + R\cos\theta - 2R) \\ &= \frac{1}{2}mv^2 + mgR(\cos\theta - 1) = 0 \end{aligned}$$

$$v^2 = 2gR(1 - \cos\theta)$$

$$v = \sqrt{2gR(1 - \cos\theta)}$$



The centripetal force is the net force acting radially towards the center of the sphere on the puck.

$$F_r = m a_r = \frac{m v^2}{R} = m g \cos \theta - N$$

$$N = m g \cos \theta - \frac{m v^2}{R} = m g \cos \theta - \frac{m (2 g R (1 - \cos \theta))}{R}$$

$$= m g \cos \theta - 2 m g + 2 m g \cos \theta$$

$$= 3 m g \cos \theta - 2 m g$$

The puck leaves the sphere when the normal force  $N$  is zero.

2d)  $3 m g \cos \theta - 2 m g = 0$

$$\cos \theta = \frac{2}{3} \quad \theta = \cos^{-1}\left(\frac{2}{3}\right) = 0.841 \text{ rad}, 48.2^\circ$$

$$h = R \left(1 + \frac{2}{3}\right) = \frac{5R}{3}$$

The puck leaves the sphere at an angle of  $48.2^\circ$  and a height of  $\frac{5R}{3}$ .