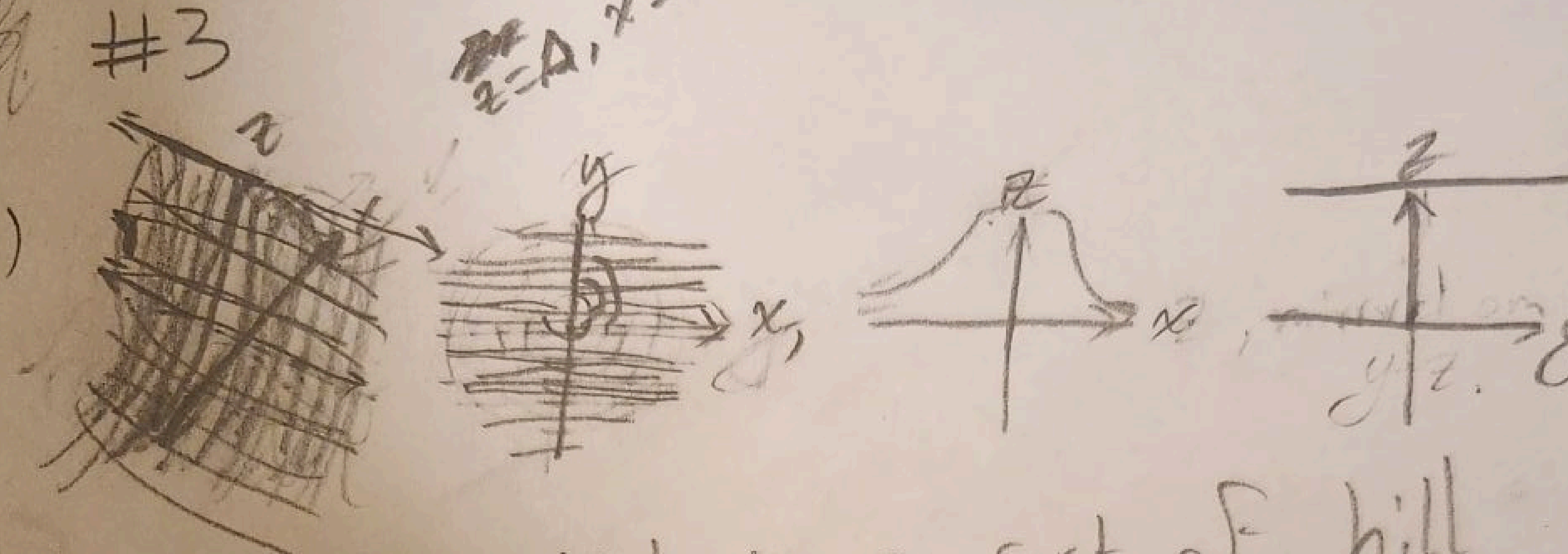
#3

3 a.)



3. b.) the potential is a sort of hill shape, with a critical point at $(0, 0, A)$. When A increases, the height of the potential hill increases, when a increases the breadth of the hill increases. If $x, z = 0$, so $y \neq 0$, $V(x,y,z) = 0$,

3 c.) Expecting a particle to fall or move, rather, away from the y-axis, towards the ~~ground~~ x-y plane.

3 d.) ~~Thee~~ There appears to be an unstable equilibrium line along y-axis.

3 e.) $\vec{F} = -\nabla V = \frac{\partial}{\partial x} A\exp\left(-\frac{x^2+z^2}{2a^2}\right) + \frac{\partial}{\partial y} 0 + \frac{\partial}{\partial z} A\exp\left(-\frac{x^2+z^2}{2a^2}\right)$

$= -\frac{A}{a^2} \left\langle x\exp\left(-\frac{x^2+z^2}{2a^2}\right), 0, z\exp\left(-\frac{x^2+z^2}{2a^2}\right)\right\rangle$

$\nabla \times \vec{F} = 0 \therefore \vec{F}$ is conservative.

ex. #4 ex. 4

$\boxed{4a.)}$ $v = \frac{\alpha}{x} = \alpha(x^{-1})$

$\frac{d}{dt}\frac{1}{x}$

$$\dot v = \frac{\dot\alpha x - \alpha \dot x}{x^2} = \frac{\dot\alpha}{x} - \frac{\alpha v}{x^2}$$

$$a = \dot v$$

$$ma = F = m\dot v = m\left(\frac{\dot\alpha}{x} - \frac{\alpha}{x^2}\cdot\frac{\alpha}{x}\right)$$

$$= m\left(\frac{\dot\alpha}{x} - \frac{\alpha^2}{x^3}\right)$$

$$F = \frac{m}{x}\left(\frac{\dot\alpha}{x} - \frac{\alpha^2}{x^2}\right) = \frac{m}{x}\left(\dot\alpha - \left(\frac{\alpha}{x}\right)^2\right).$$

$\boxed{4b.)}$ $F(x) = -kx + \frac{kx^3}{\alpha^2}$, $k > 0$

$\dot k = 0$

$\dot\alpha = 0$

$F = \frac{d}{dx}U(x) \Rightarrow U = \int F(x)\,dx$

chose $U_0 = 0$

$$\int F(x)\,dx = \int -kx\,dx + \int \frac{kx^3}{\alpha^2}\,dx = -k\int x\,dx + \frac{k}{\alpha^2}\int x^3\,dx$$

$$= -\frac{kx^2}{2} + \frac{k}{\alpha^2}\cdot\frac{x^4}{4}$$

$$U(x) = \frac{k}{4}x^2\left(\left(\frac{x}{\alpha}\right)^2 - \frac{1}{2}\right) + U_0$$

Taking $U(0) = \frac{k\alpha^2}{4}$, $U(x) = \frac{k}{4}x^2\left(\left(\frac{x}{\alpha}\right)^2 - \frac{1}{2}\right)$.

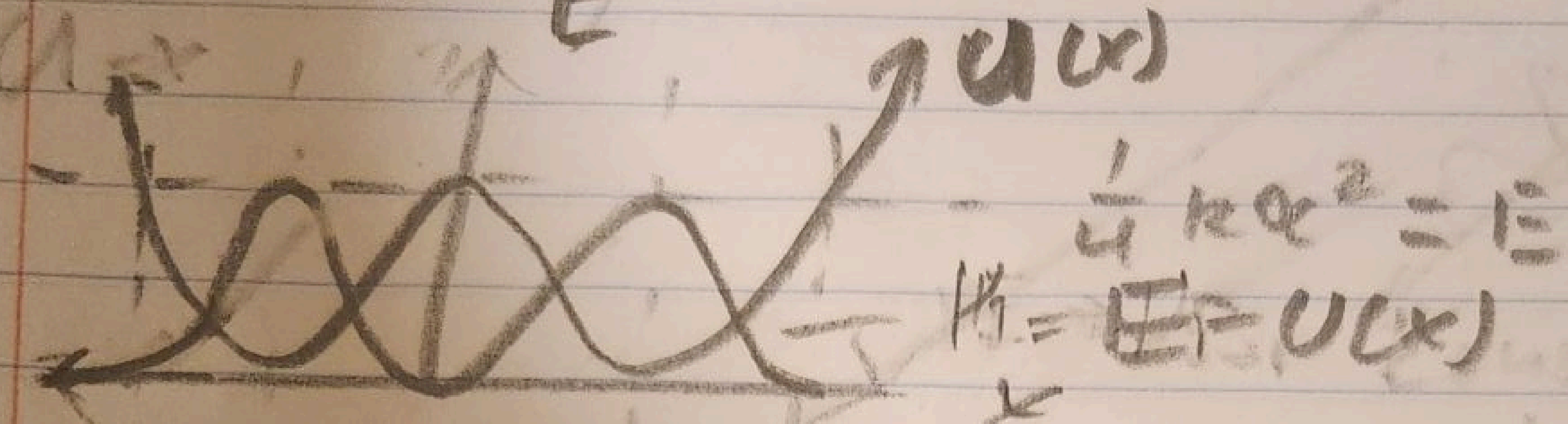
unstable equilibrium at $(0, U(0))$, stable equilibrium at $(\alpha, 0)$, $(\alpha, 0)$.

$\boxed{\text{Ex } 4c.)}$ #4

$$E = \frac{1}{4} k\alpha^2 = U + K$$

$$\max(U) = \infty,\ x \to \infty;\ \text{if } E = \frac{1}{4} k\alpha^2,$$

$$\max(U) = \underbrace{\frac{1}{4} k\alpha^2}_{E},\ \max(K) = \frac{1}{4} k\alpha^2$$



either at an unstable equilibrium, ~~then~~ $(x, U, K) = (0, E, 0)$, or oscillating around stable equilibrium, $(x, U, K) =$ $(\alpha, 0, E), (-\alpha, 0, E)$.