

- 3a) $V(t) = V_{ter}(1 - e^{-\frac{t}{T}})$ $V_{ter} = \frac{Mg}{b}$ $T = \frac{m}{b}$
- Let $\frac{t}{T} \ll 1$ then we can perform a Taylor expansion of $e^{-\frac{t}{T}}$ centered at $t=0$
- $$e^{-\frac{t}{T}} = e^{\frac{-bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{-bt}{m}\right)^2 + \frac{1}{6} \left(\frac{-bt}{m}\right)^3 + \dots$$
- Neglect the terms in quadratic order and higher order terms,
- $$e^{\frac{-bt}{m}} \approx 1 - \frac{bt}{m} \text{ for } \frac{t}{T} \ll 1$$
- Then,
- $$V(t) = \frac{Mg}{b} \left(1 - 1 + \frac{bt}{m}\right) = \frac{Mg}{b} \left(\frac{bt}{m}\right) = gt$$
- Therefore, $V(t) = gt$ for $\frac{t}{T} \ll 1$ which is very close to the time when the object is dropped from rest.
- 3b) $O(t^2)$ is all higher order terms including t^2 in the expansion. It is significant when t is larger.
- $$V(t) = gt + \frac{Mg}{b} \left(-\frac{1}{2} \frac{b^2 t^2}{m^2}\right) + \dots$$
- $$= gt - \frac{bgt^2}{2m} + \dots$$
- $-\frac{bgt^2}{2m}$ is the next term in the expansion
- 3c) For motion close to when $V_0 = 0$, $\frac{t}{T} \ll 1$. Then expand $e^{-\frac{t}{T}}$.
- $$y(t) = \frac{Mg}{b} t + \left(0 - \frac{Mg}{b}\right) \frac{m}{b} \left(1 - \left(1 - \frac{bt}{m} + \frac{b^2 t^2}{2m^2} + O(t^3)\right)\right)$$
- $$= \frac{Mg}{b} t - \frac{M^2 g}{b^2} \left(\frac{bt}{m} - \frac{b^2 t^2}{2m^2} - O(t^3)\right)$$
- Neglect $O(t^3)$ for small t

$$y(t) = \frac{Mg}{b}t - \frac{Mg}{b}t + \frac{M^2 g}{b^2} \left(\frac{b^2 t^2}{2m^2} \right)$$

$$= \frac{1}{2}gt^2 \text{ for small } t \quad (\frac{t}{T} \ll 1)$$

$\frac{t}{T}$ is the small parameter in the expansions for all cases.

$$4a) M \frac{dv}{dt} = -cv^{3/2} \quad -\frac{M}{c} \frac{1}{v^{3/2}} dv = dt$$

$$-\frac{M}{c} \int_{v_0}^v v^{-3/2} dv = -\frac{M}{c} \left[\frac{v^{-1+3/2}}{1-3/2} \right]_0^v = -\frac{M}{c} \left[-2v^{-1/2} \right]_0^v$$

$$= \frac{2M}{c} \left(\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_0}} \right) = \int_0^t dt \frac{1}{\sqrt{v}} = \frac{c}{2m} t + \frac{1}{\sqrt{v_0}}$$

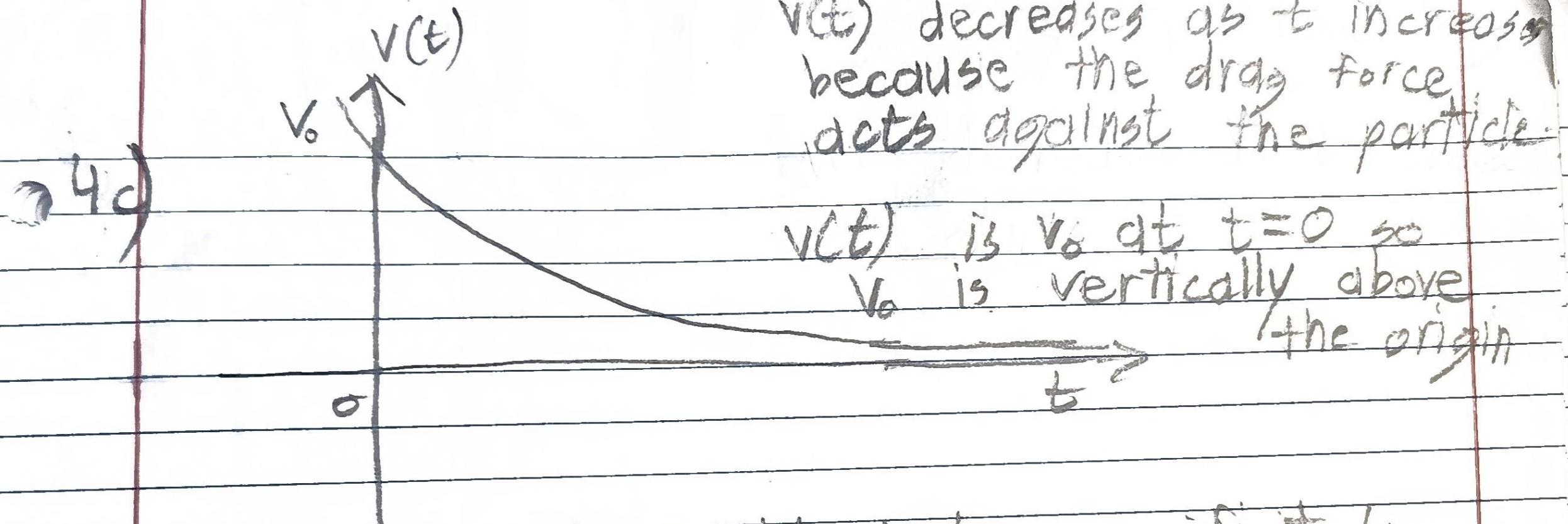
$$v(t) = \frac{1}{\left(\frac{c}{2m} t + \frac{1}{\sqrt{v_0}} \right)^2} = t$$

$$4b) \lim_{t \rightarrow 0} v(t) = \frac{1}{\left(0 + \frac{1}{\sqrt{v_0}} \right)^2} = \frac{1}{\left(\frac{1}{v_0} \right)} = v_0$$

This agrees as the object must initially start with v_0 as the velocity when $t=0$.

It also agrees since as t gets bigger then v is smaller. t is in the denominator.

The drag force is the only force acting on the particle so it should decelerate the particle.



4d) No since it would take an infinitely long time ($t \rightarrow \infty$) for $v=0$.