Exercise 5.

assume $x_{acc} = 0$ or $\ddot{x} = 0$

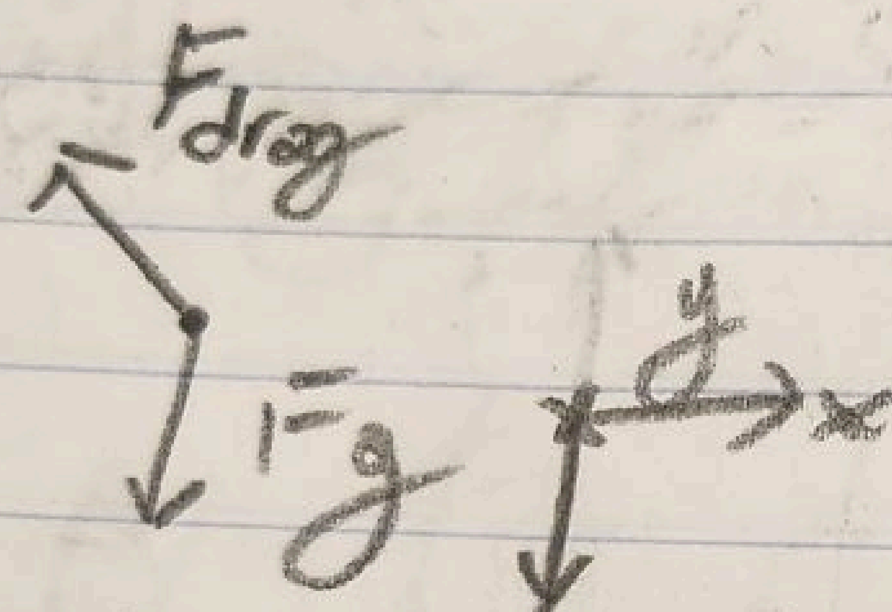
$\vec{r}(t)$ $\vec{r}(t_0) = h\,\vec{e}_y$,

$\vec{v}(t_0) = v_{x0}\vec{e}_x + v_{y0}\vec{e}_y$

Take $\vec{F}_{grav} = -mg\,\vec{e}_y$

$\vec{F}_d = -D v \vec{v}$

5a.)



$\vec{F}_\Sigma = \vec{F}_g - \vec{F}_{drag}$

$\vec{a}_\Sigma = \vec{a}_g - \frac{Dv\vec{v}}{m}$

$\ddot{x} = - \frac{D|\vec{v}|\,v_x}{m}$

$\ddot{y} = g - \frac{D|\vec{v}|\,v_y}{m}$

~~5b.)~~

~~... direction.~~

Ex. 5 cont.

5b).

IVP?

$\uparrow y$ $\quad V_x = 0 \quad V(0) = 0$

$\ddot{x} = 0$

$$\ddot{y} = -g + \frac{D}{m}\dot{y}^2$$

$$\Rightarrow \dot{V}_y = -g + \frac{D}{m} v_y^2$$

$$\Rightarrow \frac{dv_y}{dt} = -g + \frac{D}{m} v_y^2 \qquad \cancel{u = -g + \tfrac{D}{m} v_y} \quad \cancel{du = \tfrac{D}{m} dv_y}$$

$$\cancel{\Rightarrow \left(-g + \frac{D}{m} v_y\right)^{-1} dv = dt \Rightarrow \int \frac{dv}{-g + \frac{D}{m} v_y} = t + c}$$

$$\Rightarrow \int \frac{1}{u} \cdot \frac{m}{D}\, du = t + c$$

$$\cancel{\Rightarrow \frac{m}{D} \ln|u| = t + c \Rightarrow e^{\frac{m}{D}} u = e^{t+c} = ce^t}$$

$$\cancel{\Rightarrow u = ce^t \Rightarrow -g + \frac{D}{m} v_y = ce^t \Rightarrow v_y = ce^t + g}$$

$$\Rightarrow \int \frac{dv}{-g + \frac{D}{m} v_y^2} = t + c \qquad u = \sqrt{\frac{D}{mg}}\, v_y \quad \sqrt{\frac{mg}{D}}\, du = dv_y$$

$$\Rightarrow -\int \frac{dv}{g - \frac{D}{m} v_y^2} = -\frac{1}{g} \int \frac{dv}{1 - \left(\sqrt{\frac{D}{mg}}\, v_y\right)^2} = t + c$$

$$\Rightarrow -\frac{1}{g} \int \sqrt{\frac{mg}{D}} \cdot \frac{du}{1-u^2} = -\frac{1}{g}\sqrt{\frac{mg}{D}} \int \frac{du}{1-u^2} = -\sqrt{\frac{m}{gD}} \cdot \left(-\frac{1}{2} \ln\left|\frac{u-1}{u+1}\right|\right) = t + c$$

$$\Rightarrow e^{-2(t+c)\sqrt{\frac{m}{gD}}} = \frac{u-1}{u+1} = 1 + \frac{-2}{u+1}$$

$$\Rightarrow ce^{-2t} = 1 + \frac{-2}{u+1} \Rightarrow -\frac{1}{2} ce^{-2t} + \frac{1}{2} = \frac{1}{u+1} = \cancel{\frac{1}{\sqrt{\frac{D}{mg}}\, v_y + 1}} \longrightarrow$$

$$\Rightarrow ce^{-2t} + \frac{1}{2} = \frac{1}{u+1} \Rightarrow u+1 = \frac{1}{ce^{-2t}+\frac{1}{2}}$$

$$\Rightarrow u = \frac{1}{ce^{-2t}+\frac{1}{2}} + 1$$

$$\Rightarrow \sqrt{\frac{m g}{D}}\, v_y = \frac{1}{ce^{-2t}+\frac{1}{2}} + 1$$

$$\boxed{\Rightarrow v_y = \sqrt{\frac{D}{mg}}\left(\frac{1}{ce^{-2t}+\frac{1}{2}}\right) + \frac{\sqrt{D}}{\sqrt{mg}}}$$

$V(0) = 0$

$$0 = \sqrt{\frac{D}{mg}}\left(\frac{1}{ce^{-0}+\frac{1}{2}}\right) + \frac{\sqrt{D}}{\sqrt{mg}}$$

$$-\frac{\sqrt{D}}{\sqrt{mg}} = \frac{\sqrt{D}}{\sqrt{mg}}\left(\frac{1}{c+\frac{1}{2}}\right)$$

$$-1 = \frac{1}{c+\frac{1}{2}} \Rightarrow c + \frac{1}{2} = \frac{1}{-1} = -1$$

$$\therefore \quad \Rightarrow c = -\frac{1}{2}$$

$$v_y(t) = -2\left(\frac{\sqrt{D}}{\sqrt{mg}\,(e^{-2t}-1)}\right) + \sqrt{\frac{D}{g}}$$

$= v_t$

or, equivalently

$$\vec{v}(t) = v_t \tanh\left(\frac{gt}{v_t}\right)$$

✓

Ex. 5 cont.
(5c.)

$v(t) = V_t \tanh\left(\frac{gt}{V_t}\right)$ $\quad u = \frac{gt}{V_t}$, $\frac{V_t}{g} du = dt$

$\int v(t)\,dt = \int V_t \tanh\left(\frac{gt}{V_t}\right) dt$

$y(t) = \frac{V_t^2}{g} \int \tanh(u)\,du$

$\cosh(-x) = \cosh(x)$

$y(t) = \frac{V_t^2}{g} \ln(\cosh(u)) + C$

$y(t) = \frac{V_t^2}{g} \ln\left(\cosh\left(\frac{gt}{V_t}\right)\right) + C$

$y(0) = 0$

$0 = \frac{V_t^2}{g} \ln(\cosh(0)) + C$

$0 = \frac{V_t^2}{g} \ln(1) + C = 0 + C$

$\boxed{C = 0}$

$\therefore y(t) = \frac{V_t^2}{g} \ln\left(\cosh\left(\frac{gt}{V_t}\right)\right) + C$

$= \frac{m}{D} \ln\left(\cosh\left(\frac{gt}{V_t}\right)\right)$

$v_\infty = \sqrt{\frac{mg}{D}}$

Ex 5. 5c cont.

$m = 0.2\,kg$

$g = 9.81\frac{m}{s^2}$

$y(0) = 2$

$y(t) = -\frac{m}{D}\ln\left(\cosh\left(\frac{gt}{v_\infty}\right)\right) + 2$

$0 = -\frac{0.2}{0.2}\ln\left(\cosh\left(\sqrt{\frac{9.81 \cdot 0.2}{0.2}}\, t\right)\right) + 2$

$\Rightarrow 0 = -1\ln(\cosh(\sqrt{g}\, t)) + 2$

$\Rightarrow 2 = \ln(\cosh(\sqrt{g}\,t))$

$\Rightarrow e^2 = \cosh(\sqrt{g}\,t)$

$\Rightarrow \frac{\cosh^{-1}(e^2)}{\sqrt{g}} = t = 0.858\text{ s}$