

- 3a)  $V(t) = V_{ter}(1 - e^{-\frac{t}{T}})$   $V_{ter} = \frac{Mg}{b}$   $T = \frac{m}{b}$
- Let  $\frac{t}{T} \ll 1$  then we can perform a Taylor expansion of  $e^{-\frac{t}{T}}$  centered at  $t=0$
- $$e^{-\frac{t}{T}} = e^{\frac{-bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{-bt}{m}\right)^2 + \frac{1}{6} \left(\frac{-bt}{m}\right)^3 + \dots$$
- Neglect the terms in quadratic order and higher order terms,
- $$e^{\frac{-bt}{m}} \approx 1 - \frac{bt}{m} \text{ for } \frac{t}{T} \ll 1$$
- Then,
- $$V(t) = \frac{Mg}{b} \left(1 - 1 + \frac{bt}{m}\right) = \frac{Mg}{b} \left(\frac{bt}{m}\right) = gt$$
- Therefore,  $V(t) = gt$  for  $\frac{t}{T} \ll 1$  which is very close to the time when the object is dropped from rest.
- 3b)  $O(t^2)$  is all higher order terms including  $t^2$  in the expansion. It is significant when  $t$  is larger.
- $$V(t) = gt + \frac{Mg}{b} \left(-\frac{1}{2} \frac{b^2 t^2}{m^2}\right) + \dots$$
- $$= gt - \frac{bgt^2}{2m} + \dots$$
- $-\frac{bgt^2}{2m}$  is the next term in the expansion
- 3c) For motion close to when  $V_0 = 0$ ,  $\frac{t}{T} \ll 1$ . Then expand  $e^{-\frac{t}{T}}$ .
- $$y(t) = \frac{Mg}{b} t + \left(0 - \frac{Mg}{b}\right) \frac{m}{b} \left(1 - \left(1 - \frac{bt}{m} + \frac{b^2 t^2}{2m^2} + O(t^3)\right)\right)$$
- $$= \frac{Mg}{b} t - \frac{M^2 g}{b^2} \left(\frac{bt}{m} - \frac{b^2 t^2}{2m^2} - O(t^3)\right)$$
- Neglect  $O(t^3)$  for small  $t$

$$y(t) = \frac{M_0}{b} t - \frac{M_0 g}{b} t + \frac{M_0^2}{b^2} \left( \frac{b^2 t^2}{2m^2} \right)$$

$$= \frac{1}{2} g t^2 \text{ for small } t \quad (\frac{t}{T} \ll 1)$$

$\frac{t}{T}$  is the small parameter in the expansions for all cases.

$$4a) M \frac{dv}{dt} = -cv^{3/2} \quad -\frac{M}{c} \frac{1}{v^{3/2}} dv = dt$$

$$-\frac{M}{c} \int_{v_0}^v v^{-3/2} dv = -\frac{M}{c} \left[ \frac{v^{-1+3/2}}{1-3/2} \right]_0^v = -\frac{M}{c} \left[ -2v^{-1/2} \right]_0^v$$

$$= \frac{2M}{c} \left( \frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_0}} \right) = t \quad \frac{1}{\sqrt{v}} = \frac{ct + \frac{1}{\sqrt{v_0}}}{2M}$$

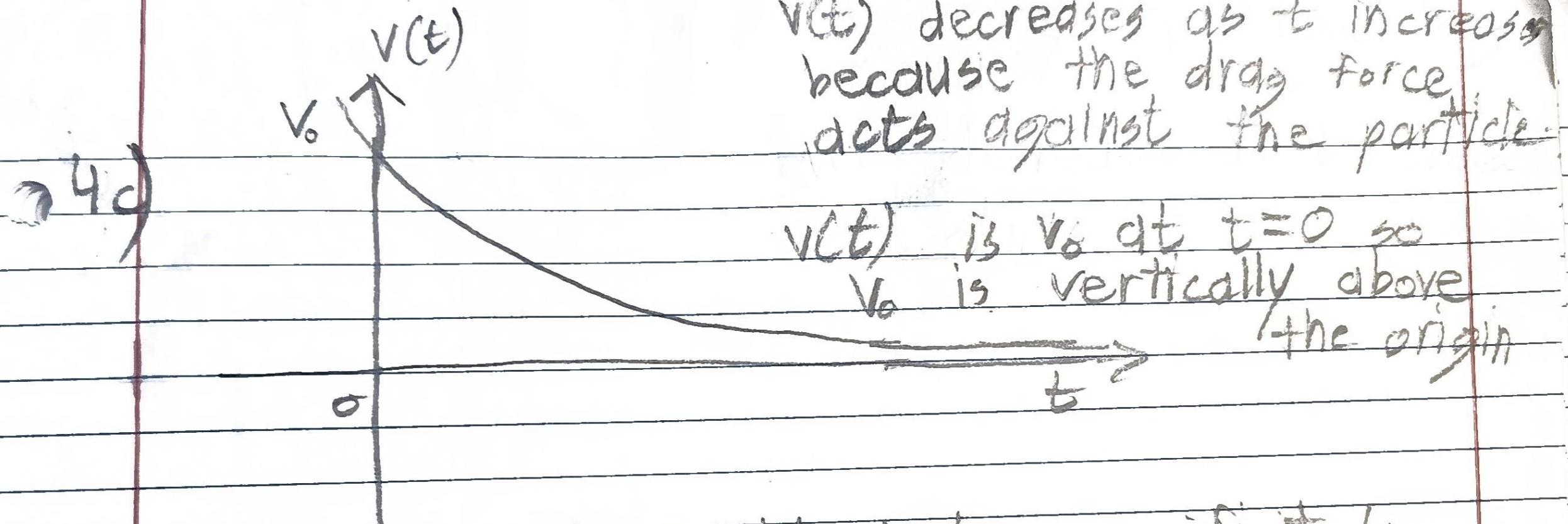
$$v(t) = \frac{1}{\left( \frac{ct}{2M} + \frac{1}{\sqrt{v_0}} \right)^2}$$

$$4b) \lim_{t \rightarrow 0} v(t) = \frac{1}{\left( 0 + \frac{1}{\sqrt{v_0}} \right)^2} = \frac{1}{\left( \frac{1}{v_0} \right)} = v_0$$

This agrees as the object must initially start with  $v_0$  as the velocity when  $t=0$ .

It also agrees since as  $t$  gets bigger then  $v$  is smaller.  $t$  is in the denominator.

The drag force is the only force acting on the particle so it should decelerate the particle.



4d) No since it would take an infinitely long time ( $t \rightarrow \infty$ ) for  $v=0$ .