$0 \le t \le 1$

1a) $a_1$ $(0,0) \to (1,0)$ $\quad r(t) = (1-t)\langle 0,0\rangle + t\langle 1,0\rangle$

$\vec{F}(r(t)) = \langle x^2, 2xy\rangle = \langle t^2, 0\rangle$ $\qquad = \langle t, 0\rangle$

$r'(t) = \langle 1, 0\rangle$

$$\int_{a_1} \vec{F}\cdot r'(t)\,dt = \int_0^1 \langle t^2, 0\rangle \cdot \langle 1, 0\rangle\,dt = \int_0^1 t^2\,dt = \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3}$$

$0 \le t \le 1$

$a_2$ $(1,0) \to (1,1)$ $\quad r(t) = (1-t)\langle 1,0\rangle + t\langle 1,1\rangle = \langle 1, t\rangle$

$r'(t) = \langle 0, 1\rangle$

$\vec{F}(r(t)) = \langle 1, 2t\rangle$

$$\int_0^1 \vec{F}\cdot r'(t)\,dt = \int_0^1 \langle 1, 2t\rangle \cdot \langle 0, 1\rangle\,dt = \int_0^1 2t\,dt = \left[t^2\right]_0^1 = 1$$

$$\int_a \vec{F}\cdot r'(t)\,dt = \int_{a_1} \vec{F}\cdot r'(t)\,dt + \int_{a_2} \vec{F}\cdot r'(t)\,dt = \frac{1}{3} + 1 = \frac{4}{3}$$

1b) $b$ $(0,0) \to (1,1)$ $\quad r(t) = \langle t, t^2\rangle \quad 0 \le t \le 1$

$r'(t) = \langle 1, 2t\rangle \qquad \vec{F}(r(t)) = \langle t^2, 2t(t^2)\rangle = \langle t^2, 2t^3\rangle$

$$\int_0^1 \langle t^2, 2t^3\rangle \cdot \langle 1, 2t\rangle\,dt = \int_0^1 (t^2 + 4t^4)\,dt = \left[\frac{t^3}{3} + \frac{4t^5}{5}\right]_0^1$$

$$= \frac{1}{3} + \frac{4}{5} = \frac{5 + 3(4)}{15} = \frac{17}{15}$$

1c) $c$ $(0,0) \to (1,1)$ $\quad r(t) = \langle t^3, t^2\rangle \quad 0 \le t \le 1$

$r'(t) = \langle 3t^2, 2t\rangle \qquad \vec{F}(r(t)) = \langle t^6, 2t^3 t^2\rangle = \langle t^6, 2t^5\rangle$

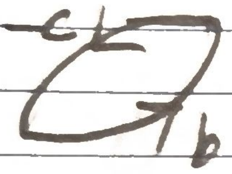
$$\int_0^1 \langle t^6, 2t^5\rangle \cdot \langle 3t^2, 2t\rangle\,dt = \int_0^1 (3t^8 + 4t^6)\,dt$$

$$= \left[\frac{3t^9}{9} + \frac{4t^7}{7}\right]_0^1 = \frac{1}{3} + \frac{4}{7} = \frac{7 + 4(3)}{21} = \frac{19}{21}$$

1a) The force is not conservative. The work done from (0,0) to (1,1) is dependent on path

$$\int_{path\ b} \vec{F}\cdot d\vec{r} \neq \int_{path\ a} \vec{F}\cdot d\vec{r} \neq \int_{path\ c} \vec{F}\cdot d\vec{r}$$

Let's reverse the path of c so that we get a closed loop with path b.




$$\int_{-c} \vec{F}\cdot d\vec{r} = -\frac{19}{21}$$

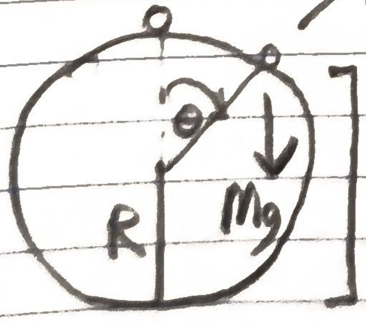
Then the line integral around the loop is:

$$\oint \vec{F}\cdot d\vec{r} = \int_{b} F\cdot d\vec{r} + \int_{-c} F\cdot d\vec{r} = \frac{17}{15} - \frac{19}{21} = \frac{8}{35} \neq 0$$

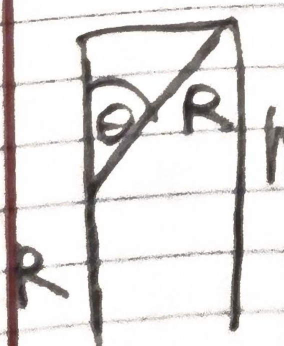
which is not zero.

Therefore the force cannot be conservative because the work done around a closed loop does not equal zero.

2a)




- The sphere of radius R is at rest and not rolling
- No air resistance acts on the puck when it slides down the sphere
- No friction acts on the puck
- The puck is a point particle




$h(\theta) = R + R\cos\theta = R(1+\cos\theta)$

at angle θ the normal force N acts perpendicular to the surface of the sphere outward

The centripetal force acts towards the center of the sphere.

The maximum possible angle is $\frac{\pi}{2}$, $90°$.
This is because there is no radial component of gravity keeping the puck on the surface of the sphere.

toward center of sphere

At $90°$, the sphere isn't underneath the puck to support it so the puck will be in a freefall.

2b) $\Delta E = \Delta K + \Delta U_{grav} = 0$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i = 0$$

Take the zero of $U_{grav}$ to be the ground $h=0$. Let $v_i = 0$, $h_i = 2R$ to show the initial state of the puck.

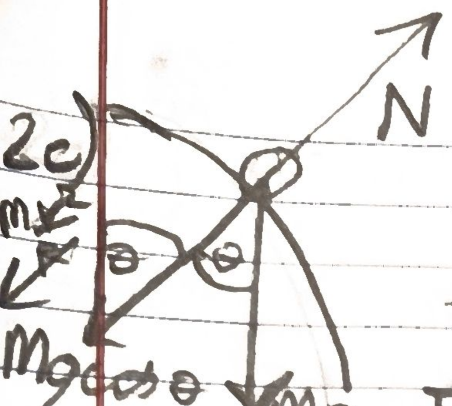
$$\frac{1}{2}mv^2 + mgh(\theta) - mg(2R) = \frac{1}{2}mv^2 + mg(R + R\cos\theta - 2R)$$

$$= \frac{1}{2}mv^2 + mgR(\cos\theta - 1) = 0$$

$$v^2 = 2gR(1-\cos\theta)$$

$$v = \sqrt{2gR(1-\cos\theta)}$$

2c)




The centripetal force is the net force acting radially towards the center of the sphere on the puck.

$$F_r = ma_r = \frac{mv^2}{R} = mg\cos\theta - N$$

$$N = mg\cos\theta - \frac{mv^2}{R} = mg\cos\theta - \frac{m}{R}(2gR(1-\cos\theta))$$

$$= mg\cos\theta - 2mg + 2mg\cos\theta$$

$$= 3mg\cos\theta - 2mg$$

The puck leaves the sphere when the normal force N is zero.

2d) $3mg\cos\theta - 2mg = 0$

$\cos\theta = \frac{2}{3}$ $\theta = \cos^{-1}\left(\frac{2}{3}\right) = 0.841\text{ rad}, 48.2°$

$h = R\left(1 + \frac{2}{3}\right) = \frac{5R}{3}$

The puck leaves the sphere at an angle of 48.2° and a height of $\frac{5R}{3}$.