

3a) $v(t) = v_{\text{ter}}(1 - e^{-t/\tau})$ $v_{\text{ter}} = \frac{mg}{b}$ $\tau = \frac{m}{b}$

Let $\frac{t}{\tau} \ll 1$ then we can perform a Taylor expansion of $e^{-t/\tau}$ centered at $t=0$

$$e^{-t/\tau} = e^{-\frac{bt}{m}} = 1 - \frac{bt}{m} + \frac{1}{2} \left(\frac{-bt}{m} \right)^2 - \frac{1}{6} \left(\frac{-bt}{m} \right)^3 + \dots$$

Neglect the terms in quadratic order and higher order terms,

$$e^{-\frac{bt}{m}} \approx 1 - \frac{bt}{m} \text{ for } \frac{t}{\tau} \ll 1$$

Then,

$$v(t) = \frac{mg}{b} \left(1 - \left(1 - \frac{bt}{m} \right) \right) = \frac{mg}{b} \left(\frac{bt}{m} \right) = gt$$

Therefore, $v(t) = gt$ for $\frac{t}{\tau} \ll 1$ which is very close to the time when the object is dropped from rest.

3b) $\alpha(t^2)$ is all higher order terms including t^2 in the expansion. It is significant when $\frac{t}{\tau}$ is larger.

$$v(t) = gt + \frac{mg}{b} \left(-\frac{1}{2} \frac{b^2 t^2}{m^2} \right) + \dots$$

$$= gt - \frac{bg}{2m} t^2 + \dots$$

$-\frac{bg}{2m} t^2$ is the next term in the expansion

3c) For motion close to when $v_0 = 0$, $\frac{t}{\tau} \ll 1$. Then expand $e^{-t/\tau}$

$$y(t) = \frac{mg}{b} t + \left(0 - \frac{mg}{b} \right) \frac{m}{b} \left(1 - \left(1 - \frac{bt}{m} + \frac{b^2 t^2}{2m^2} + o(t^3) \right) \right)$$

$$= \frac{mg}{b} t - \frac{m^2 g}{b^2} \left(\frac{bt}{m} - \frac{b^2 t^2}{2m^2} - o(t^3) \right)$$

Neglect $o(t^3)$ For small t

$$y(t) = \frac{mg}{b}t - \frac{mg}{b}t + \frac{mg}{b^2} \left(\frac{b^2 t^2}{2m} \right)$$

$$= \frac{1}{2}gt^2 \text{ for small } t \left(\frac{t}{\tau} \ll 1 \right)$$

$\frac{t}{\tau}$ is the small parameter in the expansions for all cases.

$$4a) m \frac{dv}{dt} = -cv^{3/2} \quad -\frac{m}{c} \frac{1}{v^{3/2}} dv = dt$$

$$-\frac{m}{c} \int_{v_0}^v v^{-3/2} dv = -\frac{m}{c} \left[\frac{v^{-1-3/2}}{-3/2} \right]_{v_0}^v = -\frac{m}{c} \left[-2v^{-1/2} \right]_{v_0}^v$$

$$= \frac{2m}{c} \left(\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_0}} \right) \stackrel{=}{=} \int_0^t 1 dt \quad \frac{1}{\sqrt{v}} = \frac{c}{2m}t + \frac{1}{\sqrt{v_0}}$$

$$v(t) = \frac{1}{\left(\frac{c}{2m}t + \frac{1}{\sqrt{v_0}} \right)^2}$$

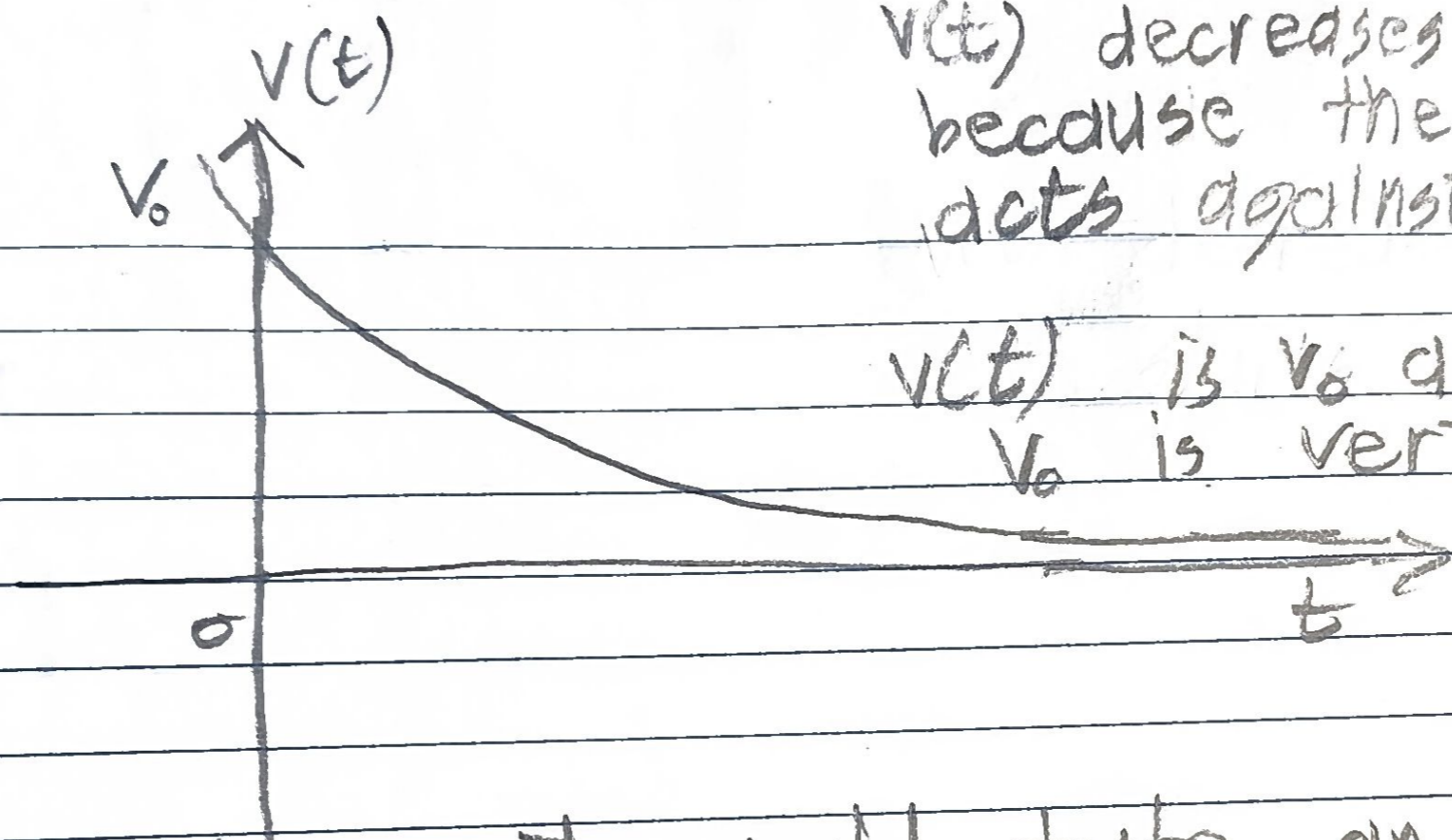
$$4b) \lim_{t \rightarrow 0} v(t) = \frac{1}{\left(0 + \frac{1}{\sqrt{v_0}} \right)^2} = \frac{1}{\left(\frac{1}{v_0} \right)} = v_0$$

This agrees as the object must initially start with v_0 as the velocity when $t=0$.

4c) It also agrees since as t gets bigger then v is smaller. t is in the denominator.

The drag force is the only force acting on the particle so it should decelerate the particle.

4c)



$v(t)$ decreases as t increases because the drag force acts against the particle.

$v(t)$ is v_0 at $t=0$ so v_0 is vertically above the origin

4d) No since it would take an infinitely long time ($t \rightarrow \infty$) for $v=0$.