Round Table Puzzle and the Problem Solving process

Ishaan Gupta

December 2023

1 Introduction

I've got this really old puzzle from a Stanford Math puzzle book. (The Stanford Mathematics Problem book, 1970), But it's an interesting one. No Mathematical background required for this as we will use simple ground level logic and induction to solve this one. I used one method, but I'm confident that there is a slew of much smarter and elegant methods that may even help with concepts pertaining to other stuff completely unrelated to the problem at hand.

So, a big disclaimer. I recommend everyone give this a try before checking my solution. There will be hints if you seem to need a slight push. The point of this publication is to get inputs from the readers on approaches that may be applicable. Feel free to mail in you attempts, even failed ones, along with your thought processes at :

ishaangupta875@gmail.com

2 The Puzzle

10 people sit at a round table. You have 10 dollars that have to be distributed among these 10 people, in such a way that, any particular person has exactly half of the sum of his adjacent member.

Meaning that if I point to any person on the table, call them A, the sum of the money in hand with the person to the left of A and to the right of A must equal twice the money with A himself. This condition needs to be satisfied for all the members on the table. Figure 1 shows an image of the setup, although the order of seating is irrelevant, a visualisation always helps derive patterns.

Figure 2 shows a obvious first solution, where 1 USD is distributed to everyone. The original books treats this as an obvious fact, considering this might be the first thing you even try, saying 'Are there any other solutions to this problem?'.

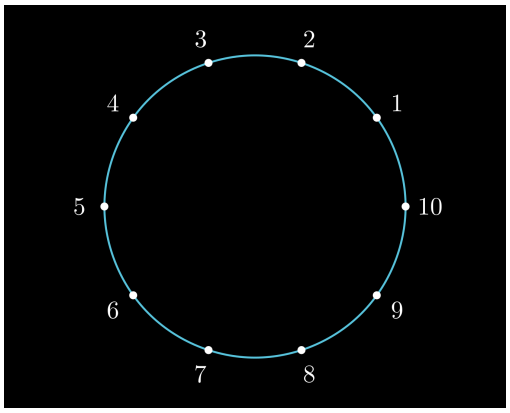

Figure 1: The setup

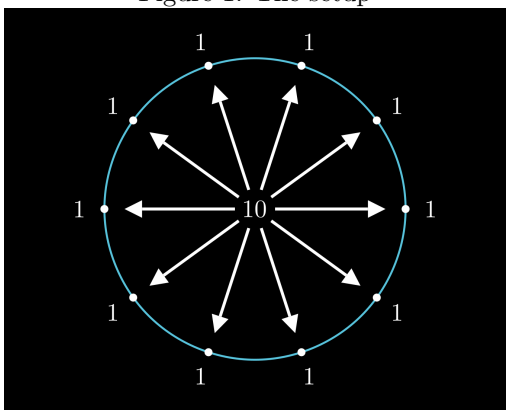

Figure 2: First solution

Th objective is to find another solution, or to prove that another solution cannot exist. Following are a few hints that might help nudge you towards my solution, or if we are lucky, maybe a solution of your own !. Do give the problem 10 Minutes at least before you look for guidance within the hints.

3 Hints

1. Number down the people, draw yourself a diagram. Try making arrows to show the direction of increasing money.

2. It always helps to somehow incorporate the '*Defining Features*' of the setup into your thinking process. In this case that might be the fact that the table is round. Think about solutions for people sitting in a straight line, and what exactly changes when the setup is changed to a round table.

3. Try simplifying the problem. In this case, it helps to think about only 5 or evm 3 people sitting at the table. Solve a similar but simpler problem and try extend that logic to the main problem at hand as well. It might be the fact that any domino effects that you set on though the table will always loop around. if A implies B which implies C, as you keep going through this process you might end up with contradictions.

4. Maybe you remember arithmetic progressions and maybe it hits you that in an arithmetic progression any given term is the mean of the term before and after. Maybe that line of thinking helps you a more 'Mathy' answer than I did. PS: The book does employ the concept of arithmetic progressions in its solution.

4 My Solution

Let me go through my thought process to get to a solution.

Consider a setup with only 3 people and only 3 USD, as discussed in Hint No.3. We number these three people, 1, 2 and 3 respectively.

Now we point to Person 1. Person 2 and 3 sit beside of 1. There is 3three possibilities to consider. 2 and 3 have the same amount of money, 2 has more than 1, 3 has more than 1.

1. Both 2 and 3 have an equal amount of money. Now for 1 to have the mean of 2 and 3, all 1,2,3 need to have the same amount of money.

   As an exercise, try extending this to the original puzzle, and see that it does indeed correspond to the given solution, where everyone gets the same amount of money, 1 USD.

2. In the second case, 2 has more money than 1. The mean of two numbers must lie between those 2 numbers. So if 2 has less money than 1, then 3 necessarily has more money than 1. Lets draw arrows in the direction of increasing money to keep track. (Figure 4.1)

   Now applying the same logic to 2, 1 has less money than 2, so 3, the other neighbour to 2, must have more money than 1 and 2. However, on applying this to 3, since 2 has less money than 3, 1 must have money money than 2. This contradicts the first result that 3 has more money than 1. So such a distribution is not possible for 3 people at a round table.

   Extend this to 10 people and see if it applies.

3. In the third case, 3 has more money that 1, meaning 2 has less money than 1. Very similar logic to point 2 applies, and we see this case is impossible as well due to contradiction with the original statement.

   Since Case 1 is already covered, and case 2 and 3 are not possible, we can say 'There is no other way of distributing 10 USD among 10 people at a round table such that every person has the MEAN money of their neighbours.'

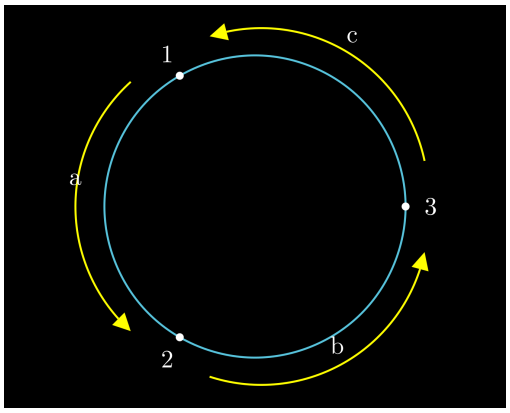

Figure 3: (a)Arrows in the direction of increasing money. If any 2 of a,b,c are true, then the third one must be false.

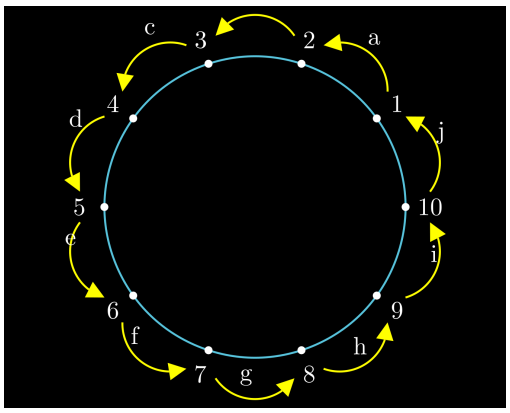

Figure 4: (b)Extending to 10 people, all of these arrows much hold except for one, which leads to proof by contradiction

5 Concluding statements

Here is the book where the puzzle was taken from, with solutions.

https://mathsmartinthomas.files.wordpress.com/2018/09/stanfordproblem.pdf

This one involves arithmetic progressions and little more math, but the idea is the same. I hope this was a worthwhile lesson on problem solving. Thank you for reading all the way through, you are now just a bit smarter and better at the *'Systematic Method of problem solving'*

6 Reader Exercise

The defining feature, the round table leads us to the conclusion. Make sure to try and think how this would work out if there was no table, and everyone was just standing in a straight line. Try and generalize the amount of money everyone has in such a setup.