Math MOE Homework 3

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1. Find the probability that you get a reward (total roll(s) ≤ 7)

$A = \{$ first roll is 1, 2, 3 $\}$ $B = \{$ first roll is 4, 5, 6 $\}$

$R = \{$ get a reward, (total ≤ 7) $\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{3}{6} = \frac{1}{2}$

Using total probability theorem: $P(R) = P(A)P(R|A) + P(B)P(R|B)$

$P(R|B) = 1$ (for all options when first roll is 4,5,6 you get reward)

$$P(R|A) = \frac{6}{6}\left(\frac{1}{3}\right) + \frac{5}{6}\left(\frac{1}{3}\right) + \frac{4}{6}\left(\frac{1}{3}\right) = \frac{15}{18} = \frac{5}{6}$$

$$P(R) = \frac{1}{2}\left(\frac{5}{6}\right) + \frac{1}{2}(1) = \frac{5}{12} + \frac{1}{2} = \frac{5}{12} + \frac{6}{12} = \frac{11}{12} = 0.9167$$

$P(R) = 0.9167$

$P(R) = 91.67\%$

2. What is the probability that the new customer is in the reckless category?

- 80% of the population is regular, 0.01 probability accident
- 20% of the population is reckless, 0.05 probability accident

$R = \{$ person is regular $\}$

$B = \{$ person is reckless $\}$

$A = \{$ person has an accident $\}$

$$P(B|A) = \frac{P(B)P(A|B)}{P(B)P(A|B) + P(R)P(A|R)} = \frac{(0.05)(0.2)}{(0.05)(0.2) + (0.01)(0.8)}$$

$$= \frac{0.01}{0.018} = 0.556$$

$P(B|A) = 0.556$

$P(B|A) = 55.6\%$

3. Three coins in a blind box.

- 1 has both heads
- 1 has both tails
- 1 has one side heads, one tails

1) What is probability that we picked normal coin?

$R = \{\text{result is heads}\}$

$A = \{\text{pick the fair coin}\}$

$B = \{\text{pick the both heads coin}\}$

$C = \{\text{pick the both tails coin}\}$

$$P(A|R) = \frac{P(A)\cdot P(R|A)}{P(A)\cdot P(R|A) + P(B)\cdot P(R|B) + P(C)\cdot P(R|C)} = \frac{\frac{1}{3}\left(\frac{1}{2}\right)}{\frac{1}{3}\left(\frac{1}{2}\right)+\frac{1}{3}(1)+\frac{1}{3}(0)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

P(picked fair coin | result is head) = 1/3 = 33.3%

What is the probability that we picked the one w/ both heads?

$$P(B|R) = \frac{P(B)\cdot P(R|B)}{P(R)} = \frac{\frac{1}{3}(1)}{3/6} = \frac{2}{3}$$

P(picked both heads coin | result is head) = $\frac{2}{3}$ = 66.7%

2) Toss twice, result is both heads, picked a fair coin?

$$P(\text{fair coin} \mid \text{get 2 heads}) = \frac{\frac{1}{3}\left(\frac{1}{4}\right)}{\frac{1}{3}\left(\frac{1}{4}\right)+\frac{1}{3}(1)+\frac{1}{3}(0)} = \frac{1/12}{5/12} = 0.2$$

P(fair coin | 2 heads) = 20%

Toss twice, result is both heads, picked both heads coin

$$P(\text{both heads coin} \mid \text{get 2 heads}) = \frac{\frac{1}{3}(1)}{\frac{1}{3}\left(\frac{1}{4}\right)+\frac{1}{3}(1)+\frac{1}{3}(0)} = \frac{\frac{1}{3}}{5/12} = 0.8$$

P(both heads coin | 2 heads) = 80%

3) Toss it n times, results are all heads. How large must n be in order to be 99% sure we have the coin w/ both heads?

$$P(B|R^n) = \frac{P(R^n|B)\,P(B)}{P(R^n)} = \frac{1\left(\frac{1}{3}\right)}{1\left(\frac{1}{3}\right) + \frac{1}{3}(0) + \frac{1}{3}\left(\frac{1}{2}\right)^n} = \frac{\frac{1}{3}}{1+\frac{1}{2}^n}$$

$$0.99 \leq \frac{1}{1+\left(\frac{1}{2}\right)^n} \rightarrow 1+\left(\frac{1}{2}\right)^n \leq \frac{1}{0.99} \longrightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{0.99} - 1$$

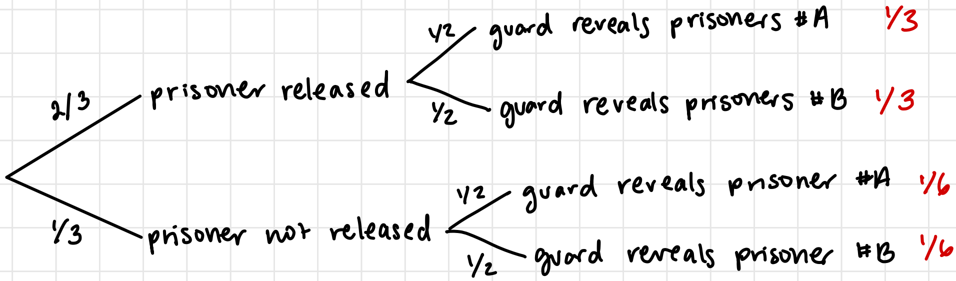
$$\left(\frac{1}{2}\right)^n \leq 0.0101$$

$$n \geq \log_2\left(\frac{1}{0.0101}\right)$$

$$n = 7$$

4 A prisoner's dilemma

Initial probability: $P(\text{released}) = 2/3$



$P(\text{prisoner is released}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

conditional prob. after guard response

Regardless of the guards response, the probability of the prisoner being released is 2/3. This is due to the fact that there are two prisoners being released so the response the guard provides will not give any additional information about the prisoner's release status. We can see this as the conditional probability is 2/3 which is the same as the original probability.