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A Metric Equational System for Quantum Computation

Motivation and Context

Program equivalence and underlying theories typically hinge on the idea that equivalence is binary, *i.e.* two programs are either equivalent or they are not [Win93]. Whilst in the case of classical programming this is usually fine, in other computational paradigms it is a too coarse-grained perspective [DN23a]. Specifically in quantum computation it manifests itself at multiple fronts [HHZ$^+$19, DN23a, Wat18]. For example the fact that one works only with a *finite* set of basic quantum gates, and that these are used to approximate the remaining ones, already strongly suggests the development of appropriate notions of *approximate* program equivalence [DN23a]. Also the central issue of quantum decoherence calls for such notions, for it is unreasonable to expect that our idealised quantum algorithm will run perfectly on a quantum device – what we will observe is instead a mere approximation [HHZ$^+$19, Pre18]. Yet another example is found in the day-to-day substitution of 'costly' quantum operations by less costly ones: the latter might not be equivalent to the original ones but are informally regarded as being 'almost' equivalent.

A notion of approximate equivalence for quantum programming was (briefly) studied in [DN23b]. Technically it uses linear λ-calculus as basis – *i.e.* programs are written as linear λ-terms – which has deep connections to both logic and category theory [GLR95, Ben94]. A notion of approximate equivalence is then integrated in the calculus via the so-called *diamond norm*, which induces a metric (roughly, a distance function) on the space of quantum programs (seen semantically as completely positive trace-preserving super-operators) [Wat18]. Some positive results were achieved in this setting but much remains to be done.

Goals

The notion of approximate equivalence for quantum programming explored in [DN23b] does not take important operations into account. Specifically the corresponding mathematical model does not include measurements, nor classical control flow, nor discard operations. Also the corresponding typing system is often times too strict, and cannot properly handle multiple uses of the same resource, such as sampling exactly n-times from a distribution. The overarching goal of this M.Sc. project is to tackle the aforementioned limitations.

A successful completion of this goal will provide a fully-fledged quantum programming language on which to study metric program equivalence in various scenarios. This includes not only quantum algorithmics – where for example the number of iterations in Grover's algorithm involves approximations – but also in quantum information theory, where for example quantum teleportation and the problem of the discrimination of quantum states have important rôles [NC10].

Research Plan

The first three months of this project are devoted to a background study on the topics of programming theory, λ-calculus, and (graded) typing systems that are suited to the use of a resource multiple times [Win93, DN23b, DN23a]. The next three months are allocated to extending the quantum model in [DN23b] with measurement, classical control flow, and also discard operations. The subsequent two months will be dedicated to enriching the typing system in [DN23b] so it can properly support multiple uses of the same resource. Finally the following two months will be devoted to writing the dissertation. The table below summarises the work plan just described. Throughout the whole project we will use a number of simple case-studies to illustrate and benchmark the prospective results.

Task	Oct	Nov	Dez	Jan	Fev	Mar	Apr	May	Jun	Jul
Background Study	X	X	X							
New Quantum Operations				X	X	X				
Enriched Typing System							X	X		
Dissertation									X	X

References

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