$$\begin{cases} f(x+h) = f + hf' + \frac{h^2}{2} f'' + \frac{h^3}{3!} f''' + \frac{h^4}{4!} f'''' \\ f(x-h) = f - hf' + \frac{h^2}{2} f'' - \frac{h^3}{3!} f''' + \frac{h^4}{4!} f'''' \\ f(x+2h) = f + 2hf' + \frac{(2h)^2}{2} f'' + \frac{(2h)^3}{3} f''' + \frac{(2h)^4}{4!} f'''' \\ f(x-2h) = f - 2hf' + \frac{(2h)^2}{2} f' - \frac{(2h)^3}{3!} f''' + \frac{(2h)^4}{4!} f'''' \end{cases}$$

$$a f(x+h) + b f(x-h) + c f(x+2h) + d f(x-2h)$$

$$= \frac{f(x+2h) - 4 f(x+h) + 6 f(x) - 4 f(x-h) + f(x-2h)}{h^4}$$

0 1

$$a f + b f + c f + d f = \cancel{6 f} + a h f' - b h f' + 2 c h f' - 2 d h f'$$

2 3

$$+ a \frac{h^2}{2} f'' + b \frac{h^2}{2} f'' + c \frac{(2h)^2}{2} f'' + d \frac{(2h)^2}{2} f'' + a \frac{h^3}{3!} f''' - b \frac{h^3}{3!} f''' + c \frac{(2h)^3}{3!} f''' - d \frac{(2h)^3}{3!} f'''$$

$$+ a \frac{h^4}{4!} f'''' + b \frac{h^4}{4!} f'''' + c \frac{(2h)^4}{4!} f'''' + d \frac{(2h)^4}{4!} f''''$$

0

$$a \frac{h^4}{4!} f'''' + b \frac{h^4}{4!} f'''' + c \frac{(2h)^4}{4!} f'''' + d \frac{(2h)^4}{4!} f''''$$

$$= -0 - 1 - 2 - 3 + a f(x+h) + b f(x-h) + c f(x+2h) + d f(x-2h)$$

0 $-\frac{4!}{h^4}\left(af+bf+cf+df\right)$

1 $-\frac{4!}{h^4}\left(ahf'-bhf'+c2hf'-d2hf'\right)$

2 " $\left(a\frac{h^2}{2}f''+b\frac{h^2}{2}f''+c\frac{(2h)^2}{2}f''+d\frac{(2h)^2}{2}f''\right)$

3 " $\left(a\frac{h^3}{3!}f'''-b\frac{h^3}{3!}f'''+c\frac{(2h)^3}{3!}f'''-d\frac{(2h)^2}{3!}f'''\right)$

$$\left.\begin{aligned}4!\,a\,f(x+h)&=-4f(x+h)\\ 4!\,b\,f(x-h)&=-4f(x-h)\\ 4!\,c\,f(x+2h)&=f(x+2h)\\ 4!\,d\,f(x-2h)&=f(x-2h)\end{aligned}\right] \Rightarrow \begin{aligned}a&=-\frac{1}{3!}\\ b&=-\frac{1}{3!}\\ c&=\frac{1}{4!}\\ d&=\frac{1}{4!}\end{aligned}$$

$$\Rightarrow -4!\left(-\frac{1}{3!}f-\frac{1}{3!}f+\frac{1}{4!}f+\frac{1}{4!}f\right)=\frac{4!}{3!}f+\frac{4!}{3!}f+f+f$$

$$=4f+4f-f-f=6f \leq \Rightarrow \begin{cases}4!\,c\,f(x+2h)=-f(x+2h)\\ 4!\,d\,f(x-2h)=-f(x-2h)\end{cases} \Rightarrow \begin{cases}c=-\frac{1}{4!}\\ d=-\frac{1}{4!}\end{cases}$$

$$\Rightarrow \sum f''''=-0-1-2-3+f(x+h)+f(x-h)-f(x+2h)-f(x-2h)$$

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$$\Rightarrow -4!\left(-\frac{1}{3!}f' + \frac{1}{3!}f' - \frac{1}{4!}f' + \frac{1}{4!}f'\right) = 0f'$$

$$\Rightarrow -4!\left(-\frac{1}{3!}\frac{1}{2}f'' - \frac{1}{3!}\frac{1}{2}f'' - \frac{1}{4!}\frac{1}{2}f'' - \frac{1}{4!}\frac{1}{2}f''\right)$$

$$\Rightarrow \boxed{\frac{4}{2}f'' + \frac{4}{2}f'' + \frac{1}{2}f'' + \frac{1}{2}f''^{*} = 6f''\cdot h^2}$$

Las f''' también se anulan.

$$\frac{1}{3!}f'''' - \frac{1}{3!}f''''a - \frac{2^4}{4!} - \frac{2^4}{4!} = x$$

$$\Leftrightarrow -4f'''' - 4f'''' - 2^4 - 2^4 = 4!\,x$$

$$-f'''' - f'''' - 2^2 - 2^2 = 3!\,x$$

$$-2 - 2^2 - 2^2 = 3 \cdot 2 \cdot x \Leftrightarrow -f - 2f - 2f = 3x$$

$$\Rightarrow f = -\frac{3}{5}x.$$ Los coeficientes son:

$$a = b = \frac{3}{5 \cdot 3 \cdot 2} = \frac{1}{10}; \quad c = d = \frac{3}{4 \cdot 3 \cdot 2 \cdot 5} = \frac{1}{40}.$$

$$\boxed{\frac{6}{h^2}\left(-\frac{3}{5}\right) = -\frac{18}{5h^2}f''(x)}$$

$$\boxed{\text{Y queda: } f''''(x_j) = \frac{f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h) + f(x-2h)}{h^4} - \frac{18}{5} \cdot h^{-2} f''(x)}$$