① $\sigma = \xi \cdot \omega_n \quad \omega_d = \omega_n \sqrt{1-\xi^2}$

$$M_p = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \qquad \ln(M_p) = \frac{-\pi \xi}{\sqrt{1-\xi^2}}$$

$$\ln^2(M_p)(1-\xi^2) = \pi^2 \xi^2$$

$$\ln^2(M_p) = (\pi^2 + \ln^2(M_p))\xi^2$$

$$\xi = \frac{\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}}$$

$$t\Big|_0^{100\%} = \frac{\pi - \arccos \xi}{\omega_n \sqrt{1-\xi^2}}$$

$$\omega_n = \frac{\pi - \arccos \xi}{\sqrt{1-\xi^2} \cdot t\Big|_0^{100\%}}$$

$\sigma = \xi \cdot \omega_n \quad \omega_d = \omega_n \cdot \sqrt{1-\xi^2}$

$P_1 = -\sigma + \omega_d j$

$P_2 = -\sigma - \omega_d j$

② Apenas no matlab

③ $$C(s) = \frac{K_p s + K_i}{s}$$

$$G(s) = \frac{1}{Ls + R}$$

$$G_f(s) = \frac{\dfrac{K_p s + K_i}{Ls^2 + Rs}}{1 + \dfrac{K_p s + K_i}{Ls^2 + Rs}} \therefore \frac{K_p s + K_i}{Ls^2 + (K_p + R)s + K_i}$$

Polos: $\Delta = (K_p + R)^2 - 4LK_i$

$$P_{1,2} = \frac{-(K_p + R) \pm \sqrt{(K_p + R)^2 - 4LK_i}}{2L}$$

$$\xi\omega_n = \frac{(K_p + R)}{2L}$$

$$\frac{\sqrt{(K_p + R)^2 - 4LK_i}}{2L} = \omega_n \sqrt{1 - \xi^2}\, j$$

$$K_p = 2\xi\omega_n L - R$$

$$\frac{(K_p + R)^2 - 4LK_i}{4L^2} = -\omega_n^2(1-\xi^2)$$

$$K_i = \frac{1}{4L}\left((K_p + R)^2 + 4L^2\omega_n^2(1-\xi^2)\right)$$

Substituindo K_p:

$$K_i = \frac{1}{4L}\left(4\xi^2\cancel{\omega_n^2}L^2 - \cancel{4\xi^2\omega_n^2L^2} + 4L^2\omega_n^2\right)$$

$$K_i = L\omega_n^2$$

4 $F(s) = \frac{1}{K_p + K_d s}$

$$C(s) = K_p + K_d s$$

$$G(s) = \frac{1}{ms^2 + bs}$$

$$G_f(s) = F(s)\left(\frac{C(s)G(s)}{1+C(s)G(s)}\right)$$

$$G_f(s) = \frac{1}{K_p + K_d s}\left(\frac{\frac{K_p + K_d s}{ms^2 + bs}}{1 + \frac{K_p + K_d s}{ms^2 + bs}}\right)$$

$$G_f(s) = \frac{1}{K_p + K_d s} \cdot \frac{K_p + K_d s}{ms^2 + (b + K_d)s + K_p}$$

$$G_f(s) = \frac{1/m}{s^2 + \frac{(b + K_d)}{m}s + \frac{K_p}{m}}$$

$$2\xi\omega_n = \frac{b + K_d}{m} \qquad K_d = 2\xi\omega_n m - b$$

$$\omega_n^2 = \frac{K_p}{m} \qquad K_p = m\omega_n^2$$