① a) $R\dot{Q} + \frac{Q}{C} = V \qquad Rm + \frac{1}{C} = 0$

$Q_h = b_1 e^{\frac{t}{RC}}$

$Q_p = VC$

$\tau = \frac{1}{RC} = \frac{1}{10\cdot10^{3}\cdot10^{-6}} = 100$

b) $Q = b_1 e^{-\frac{t}{RC}} + VC$

$Q(0) = 0 \longrightarrow 0 = b_1 + VC$

$b_1 = -VC$

$Q = VC\left(1 - e^{-\frac{t}{RC}}\right)$

$Q(0{,}02) = 5\cdot10^{-6}\left(1 - e^{-2}\right)$

$Q = 4{,}323\cdot10^{-6}$

② a) Considerando $L=0$:

$$\begin{cases} V = Ri + V_b \\ \tau_m - b\omega = J\dot{\omega} \end{cases}$$

$$\begin{cases} \tau_m = K_t i \\ \omega = K_\omega V_b \end{cases}$$

$P_{ele} = P_{mec}$

$\tau_m \omega = V_b i$

$\cancel{\tau_m}\cancel{\omega} = \frac{\cancel{\omega}}{K\omega} \frac{\cancel{\tau_m}}{K_t}$

$K\omega = \frac{1}{K_t}$

$$\omega = \frac{V_b}{K_t}$$

$V_b = \omega K_t$

$$\begin{cases} V = Ri + \omega K_t \rightarrow i = \frac{V - \omega K_t}{R} \\ K_t i - b\omega = J\dot{\omega} \end{cases}$$

$$J\dot{\omega} + \underbrace{\left(b + \frac{K_t^2}{R}\right)}\omega = \frac{VK_t}{R}$$

$$J m + u = 0 \qquad \text{, Const. Tempo}$$

$$m = -\frac{u}{J} \qquad \tau = \frac{J}{b + \frac{Kt^2}{R}}$$

b) $\omega_h = b_1 e^{-\frac{u}{J}t}$

$$\lim_{t \to \infty} \omega_h = 0$$

$$\omega_p = cte. \qquad 0 + \left(b + \frac{Kt^2}{R}\right)\omega_p = \frac{VKt}{R}$$

$$\omega = \omega_h + \omega_p \qquad \omega_p = \frac{VKt}{bR + Kt^2}$$

$$\lim_{t \to \infty} \omega = \lim_{t \to \infty}(\omega_h + \omega_p) = \lim_{t \to \infty} \omega_p = \omega_p$$

3) $$\begin{cases} m_1 \ddot{x}_1 - b(\dot{x}_2 - \dot{x}_1) - K(x_2 - x_1) = 0 \\ m_2 \ddot{x}_2 + b(\dot{x}_2 - \dot{x}_1) + K(x_2 - x_1) = F \end{cases}$$

$$\ddot{x}_1 = -\frac{K}{m_1}x_1 - \frac{b}{m_1}\dot{x}_1 + \frac{K}{m_1}x_2 + \frac{b}{m_1}\dot{x}_2$$

$$\ddot{x}_2 = \frac{k}{m_2}x_1 + \frac{b}{m_2}\dot{x}_1 - \frac{k}{m_2}x_2 - \frac{b}{m_2}\dot{x}_2 + F$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{b}{m_1} & \frac{k}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & \frac{b}{m_2} & -\frac{k}{m_2} & -\frac{b}{m_2} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{B} F$$

$$Y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}}_{D} + 0 \cdot F$$