Mathematica Demonstrations in an introduction to Quantum Field Theory and Particle Physics

A dynamic and educational notebook for QFT

By Bruno Gehlen F . da Silva

Chapter #1: Introduction and Theory

Historical background

The 20th century had its beginning marked by the great discovery of quantum physics by Max Planck and his studies of the quantization of matter and the Black Body, as we could now describe various oscillations of different modes trapped in a cavity (explains the Ultraviolet Catastrophe). Albert Einstein interpreted this quantization as massless particles called photons and used the new discoveries to find his Coefficients, in addition to the famous Photoelectric Effect. With this new formalism we can use the Annihilation and Creation operators on energy (momentum) modes and even find the Transition Rate between these states.

In the same century, we also had enormous advances in the area of relativity: we do not have an absolute frame of reference, light travels at a constant speed, and depending on the observer's velocity, they may measure different physical quantities. We found Lorentz invariant observables (symmetrical by the entire Lorentz Group), since it was already known that rotations left norms unchanged, and now we had the Lorentz Boosts. We also discovered that these boosts reduce to Galileo's at low energies (still validating traditional classical physics), and now we had objects like fields or operators invariant under all these transformations, whether continuous or discrete.

Nowadays,

Quick Formalism

Taking the classical wave solution

Talking properly about our theory, we must recalling some basic aspects of Quantum Field Theory. After the so-called “second quantization”, where we establish local (equal-time) commutation relations between the our creation/annihilation operators and normalization of states, also having in mind the action of those operators on a one-particle state:

$$[a_k, a_p^\dagger] = (2\pi)^3 \delta^3(\vec{p} - \vec{k}) \qquad \langle 0|0\rangle = 1 \qquad a_p^\dagger|0\rangle = \frac{1}{\sqrt{2\omega_p}}|\vec{p}\rangle$$

and define that ***n*** particles can simultaneously occupy a state of momentum ***p***, resulting from the direct sum of the Hilbert spaces of ***i*** particles, also called Fock space, and we can have now Quantum Scalar Fields:

$$F_\nu(H) = \bigoplus_{n=0}^{\infty} S_\nu H^{\otimes n} \qquad \phi_0(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left(a_p e^{-ipx} + a_p^\dagger e^{ipx}\right),$$

and so they can be seen as

Chapter #2: Computing

First Computations

Now,