Università di Pisa

The Oil Company Problem

Logistics Project 2021/22

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Contents

1 Introduction

An oil company has a tanker truck that it uses to deliver fuel to customers. The tanker has five different storage compartments with capacities to hold 2,500, 2,000, 1,500, 1,800 and 2,300 gallons, respectively. The company has an order to deliver 2,700 gallons of diesel fuel, 3,500 gallons of regular unleaded gasoline, and 4,200 gallons of premium unleaded gasoline. If each storage compartment can hold only one type of fuel, how should the company load the tanker?

2 Formulating an ILP model for the oil company problem.

2.1 Defining the input data and decision variables

- I = set of orders of gallons of fuel = $\{1, 2, 3\}$
- J = set of storage compartments = $\{1, 2, 3, 4, 5\}$
- x_{ij} = amount of fuel i to be loaded into compartment j
- $y_{ij} = \begin{cases} 1 & \text{if } x_i > 0 \\ 0 & \text{otherwise} \end{cases}$

2.2 ILP Model

$$\begin{aligned}
\text{maximize} \quad & \sum_{i \in I} \sum_{j \in J} x_{ij} \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 2700, \\
& x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 3500, \\
& x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 4200, \\
& x_{i1} = 2500 y_{i1}, \\
& x_{i2} = 2000 y_{i2}, \\
& x_{i3} = 1500 y_{i3}, \\
& x_{i4} = 1800 y_{i4}, \\
& x_{i5} = 2300 y_{i5}, \\
& y_{11} + y_{21} + y_{31} \leq 1, \\
& y_{12} + y_{22} + y_{32} \leq 1, \\
& y_{13} + y_{23} + y_{33} \leq 1, \\
& y_{14} + y_{24} + y_{34} \leq 1, \\
& y_{15} + y_{25} + y_{35} \leq 1, \\
& x_{ij} \geq 0 \ \forall i \in I, \\
& y_{ij} \in \{0, 1\} \ \forall j \in J
\end{aligned}$$

3 AMPL Implementation

As input for the model, we had the corresponding demands for each type of fuel i which are:

- Diesel = 2700
- Regular gas = 3500
- Premium gas = 4200

While for the capacities of compartment j they are as follows:

- c1 = 2500
- c2 = 2000
- c3 = 1500
- c4 = 1800
- c5 = 2300

Input data

```
set fuel := diesel regular premium;
set compartment := c1 c2 c3 c4 c5;

param demand :=
diesel 2700
regular 3500
premium 4200;

param max_capacity :=
c1 2500
c2 2000
c3 1500
c4 1800
c5 2300;
```

Regarding the model, it was first necessary to set the parameters regarding the demand of each type of fuel and the capacity of each compartment. Then we defined a binary variable to indicate the assignment of each fuel to each compartment. We set the objective function, which is to maximize the amount of fuel loaded into each compartment. As constraint we specified that the demand must be satisfied, and that each storage compartment can only hold one type of fuel. Also the load of fuel in each compartment must not exceed its capacity.

Model

```
set fuel;
set compartment;

param demand {fuel};
param max_capacity {compartment};

var assigned {fuel, compartment} binary;
var assigned_amount{fuel, compartment} >=0;

maximize total_fuel_loaded: sum {i in fuel, j in
   compartment} assigned_amount[i,j] * assigned[i,j];

# The order needs to be delivered
subject to demand_constr {i in fuel}:
sum {j in compartment} assigned_amount[i,j] == demand[i];

# Each storage compartment can hold only one type of fuel
subject to compartment_constr {j in compartment}:
sum {i in fuel} assigned[i,j] <= 1;

# Amount of fuel cannot exceed the maximum capacity of
   each compartment
subject to capacity_constr {j in compartment}:
sum {i in fuel} assigned_amount[i,j] <= max_capacity[j];
```

3.1 AMPL results

Integer infeasible
0 branch-and-bound nodes
No basis

4 Model modification

As seen from the results presented on section 3.1, the solution is infeasible for the model, it cannot be solved. This is due to one of its constraints, the one regarding the demand: the total capacity is equal to 10100, while the demand is higher, 10400. This leads to the need of reformulating the model, this time allowing the model to not reach the previous demand, and instead focus on minimizing the shortages produced by this fact.

4.1 Defining the new input data and decision variables

- I = set of orders of gallons of fuel = {1, 2, 3}
- J = set of storage compartments = {1, 2, 3, 4, 5}
- x_{ij} = amount of fuel i to be loaded into compartment j
- $y_{ij} = \begin{cases} 1 & \text{if } x_i > 0 \\ 0 & \text{otherwise} \end{cases}$
- z_i = amount by which the order of fuel i is short

4.2 Modified ILP Model

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i \in I} z_i \\
\text{subject to} \quad & x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 2700, \\
& x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 3500, \\
& x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 4200, \\
& x_{i1} = 2500 y_{i1}, \\
& x_{i2} = 2000 y_{i2}, \\
& x_{i3} = 1500 y_{i3}, \\
& x_{i4} = 1800 y_{i4}, \\
& x_{i5} = 2300 y_{i5}, \\
& y_{11} + y_{21} + y_{31} \leq 1, \\
& y_{12} + y_{22} + y_{32} \leq 1, \\
& y_{13} + y_{23} + y_{33} \leq 1, \\
& y_{14} + y_{24} + y_{34} \leq 1, \\
& y_{15} + y_{25} + y_{35} \leq 1, \\
& x_{ij} \geq 0 \ \forall i \in I, \\
& y_{ij} \in \{0, 1\} \ \forall j \in J
\end{aligned}
$$

4.3 AMPL Modification

Now that the model has changed, we just added a variable to know the amount of fuel loaded into each compartment in order to know how short we are on a certain order. Also, the demand constraint changed to allow a partial satisfaction of the demand, so a partial delivery.

Model

```
set fuel;
set compartment;

param demand {fuel};
param max_capacity {compartment};

var assigned {fuel, compartment} binary;
var assigned_amount {fuel, compartment} >=0;
var total_fuel = sum {i in fuel, j in compartment}
   assigned_amount[i,j] * assigned[i,j];

minimize shortage: sum {i in fuel} demand[i] -
   total_fuel;

# The order needs to be delivered
subject to demand_constr {i in fuel}:
sum {j in compartment} assigned_amount[i,j] <=
   demand[i];

# Each storage compartment can hold only one type of
   fuel
subject to compartment_constr {j in compartment}:
sum {i in fuel} assigned[i,j] <= 1;

# Amount of fuel cannot exceed the maximum capacity of
   each compartment
subject to capacity_constr {j in compartment}:
sum {i in fuel} assigned_amount[i,j] <= max_capacity[j];
```

4.4 AMPL results

i	**j**	**x**
Diesel	Compartment 1	2500
Diesel	Compartment 2	0
Diesel	Compartment 3	0
Diesel	Compartment 4	0
Diesel	Compartment 5	0
Regular Gas	Compartment 1	0
Regular Gas	Compartment 2	2000
Regular Gas	Compartment 3	1500
Regular Gas	Compartment 4	0
Regular Gas	Compartment 5	0
Premium Gas	Compartment 1	0
Premium Gas	Compartment 2	0
Premium Gas	Compartment 3	0
Premium Gas	Compartment 4	1800
Premium Gas	Compartment 5	2300

5 Conclusion

During the first instance, the model was infeasible due to the fact that the combined demands was of $x_{i \in I} = 10400$ while the combined capacities of $x_{j \in J}$ is only $= 10100$. This forces the model to allow certain orders to be short, thus changing the objective to minimize said shorts, instead of delivering the demand. By reformulating the model, it is shown that the optimal way to load the tanker while minimizing the shortages would be to load:

$$x_{11} + x_{22} + x_{23} + x_{34} + x_{35}$$

With quantities as follows:

$$2500_{11} + 2000_{22} + 1500_{23} + 1800_{34} + 2300_{35}$$

Therefore effectively loading every compartment to their full capacity, while maintaining the minimum amount of shortages at:

$$z_1 = 200, \; z_2 = 0, \; z_3 = 100$$

References

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