Università di Pisa

The Electric Vehicle Transport Problem

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1 Introduction

A green logistic service provider needs to buy a new fleet of electric vehicles to set up a transportation network to ship a given number of parcels from some origin depots (depot 1, depot 2 and depot 3) to some destinations (destinations 4, 5, 6, 7 and 8). The manager needs to decide what kinds of vehicles to buy to create the new fleet. Specifically, it has to decide among three different types, namely scooter, electric bike, and electric motorbike. A scooter costs 1000€ and may transport at most 4 parcels, an electric bike costs 1200€ and may transport at most 6 parcels, an electric motorbike costs 1700€ and may transport at most 9 parcels. The policy of the company is to buy only vehicles of one type to create the new fleet. However, there are no restrictions on the number of vehicles of the chosen type that can be bought by the company. Unfortunately, electric vehicles have a limited autonomy and a vehicle, no matter of the type, has enough charge to travel along one single link (e.g., moving from depot 2 to destination 7 and then from destination 7 to destination 8 is not possible for any vehicle type).

The tables below report the available links for the transportation and the supply (with "+" in the table) and the demand (with "-" in the table) of parcels.

Node	Supply (+) and demand (-)
Depot 1	+38
Depot 2	+45
Depot 3	+28
Destination 4	-17
Destination 5	-16
Destination 6	-18
Destination 7	-26
Destination 8	-34

Available links
(1,4)
(1,5)
(1,3)
(2,3)
(2,7)
(3,5)
(3,6)
(3,8)
(7,8)

1. Formulate an Integer Linear Programming (ILP) model to decide what type of vehicles and how many vehicles the manager should buy to minimize the cost of purchase, by satisfying the demand of parcel transportation;
2. Implement the model via the modeling language AMPL and solve it by means of the optimization solver CPLEX;
3. How does the solution change if, for congestion reasons, at most 4 vehicles may move on each link of the network?

2 Defining the input data and decision variables

This problem can be defined as a type of Basic Network Flow Problem. Indeed, the goal is to satisfy the demand of parcel transportation, in our case by minimizing the cost of purchase of one type of vehicle k among the set K. In order to state the problem, we introduce the input data:

- $G = (N, A)$ Directed graph with N nodes and A links
- b_n = Balance of node n, b_n is $\begin{cases} < 0 & \text{if it's a supply node} \\ > 0 & \text{if it's a demand node} \\ 0 & \text{if it's a transshipment node} \end{cases}$, $\forall n \in N$
- x_{ij}^k = Parcels to be sent along (i, j) by vehicle k, $\forall (i, j) \in A$
- w_k = Capacity of k vehicle, $\forall k \in K$
- z_{ij}^k = Number of k vehicles needed to satisfy the demand along (i, j), $\forall k \in K$, $\forall (i, j) \in A$

- c_k = *Cost of k vehicle*, $\forall k \in K$
- $N = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $K = \{1, 2, 3\}$

Since not all vehicles are going to be used, indeed it is required to choose one vehicle among the set of vehicles, we added a decision variable y_k.

$$y_k = \begin{cases} 1 & \text{if } k \text{ vehicle is chosen} \\ 0 & \text{otherwise} \end{cases} \quad ,\forall k \in K$$

The graph at Figure 1 shows our logistic network.

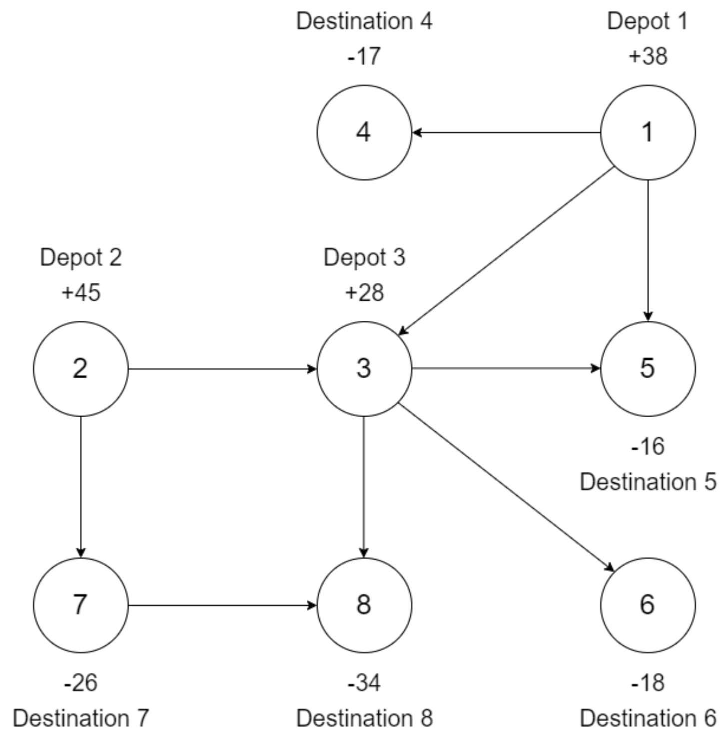

Figure 1. Network displaying nodes, their supply/demand and available links

3 Formulating a general ILP Model

1. Objective function
$$min \sum_{(i,j)\in A,\ k\in K} c_k z_{ij}^k$$

2. Flow conservation constraints
$$\sum_{(i,j)\in BS(n)} x_{ij}^k - \sum_{(j,i)\in FS(n)} x_{ji}^k = b_n y_k, \forall n \in N, \forall k \in K$$

3. Capacity constraints
$$z_{ij}^k w_k \geq x_{ij}^k, \forall (i,j) \in A, \forall k \in K$$

4. One type of vehicle policy
$$\sum_{k\in K} y_k = 1$$

5. Integrality and non-negativity constraints
$$x_{ij}^k, z_{ij}^k \ \geq 0, integer$$

6. Binary decision variable
$$y_k \in \{0, 1\}$$

The flow conservation constraint (2), assures the respect of the balance of node n, so the total supply of parcels minus the total demand of parcels of a node, delivered using k vehicle, must sum the balance of the node, given that k vehicle is used. Constraint (3) expresses that the number z of k vehicles needed for each link (i,j) times its capacity must be greater or equal than the number x of parcels sent along link (i,j). Thus satisfying the capacity constraint. Since the manager must only choose 1 type of vehicle, constraint (4) ensures that the sum of $y_k = 1$.

3.1 ILP Model for the electric vehicle problem

$$\begin{aligned}
\min\ & 1000z_{14}^1 + 1000z_{15}^1 + 1000z_{13}^1 + 1000z_{23}^1 + 1000z_{27}^1 + 1000z_{35}^1 + 1000z_{36}^1 \\
& + 1000z_{38}^1 + 1000z_{78}^1 + 1200z_{14}^2 + 1200z_{15}^2 + 1200z_{13}^2 + 1200z_{23}^2 + 1200z_{27}^2 \\
& + 1200z_{35}^2 + 1200z_{36}^2 + 1200z_{38}^2 + 1200z_{78}^2 + 1700z_{14}^3 + 1700z_{15}^3 + 1700z_{13}^3 \\
& + 1700z_{23}^3 + 1700z_{27}^3 + 1700z_{35}^3 + 1700z_{36}^3 + 1700z_{38}^3 + 1700z_{78}^3
\end{aligned}$$

Subject to

$$-x_{13} - x_{14} - x_{15} \geq -38y_1$$
$$-x_{23} - x_{27} \geq -45y_1$$
$$x_{13} + x_{23} - x_{35} - x_{36} - x_{38} \geq -28y_1$$
$$-x_{13} - x_{14} - x_{15} \geq -38y_2$$
$$-x_{23} - x_{27} \geq -45y_2$$
$$x_{13} + x_{23} - x_{35} - x_{36} - x_{38} \geq -28y_2$$
$$-x_{13} - x_{14} - x_{15} \geq -38y_3$$
$$-x_{23} - x_{27} \geq -45y_3$$
$$x_{13} + x_{23} - x_{35} - x_{36} - x_{38} \geq -28y_3$$
$$x^1_{14} = 17y_1$$
$$x^1_{15} + x^1_{35} = 16y_1$$
$$x^1_{36} = 18y_1$$
$$x^1_{27} - x^1_{78} = 26y_1$$
$$x^1_{38} + x^1_{78} = 34y_1$$
$$x^2_{14} = 17y_2$$
$$x^2_{15} + x^2_{35} = 16y_2$$
$$x^2_{36} = 18y_2$$
$$x^2_{27} - x^2_{78} = 26y_2$$
$$x^2_{38} + x^2_{78} = 34y_2$$
$$x^3_{14} = 17y_3$$
$$^3x_{15} + x^3_{35} = 16y_3$$
$$x^3_{36} = 18y_3$$
$$x^3_{27} - x^3_{78} = 26y_3$$
$$x^3_{38} + x^3_{78} = 34y_3$$
$$z^1_{14}4 \geq x^1_{14}$$
$$z^2_{14}6 \geq x^2_{14}$$
$$z^3_{14}9 \geq x^3_{14}$$
$$\vdots$$
$$y_1 + y_2 + y_3 = 1$$
$$x^1_{i4}, x^2_{14}, x^3_{14}, x^1_{15}, \ldots \geq 0, integer$$
$$z^1_{i4}, z^2_{14}, z^3_{14}, z^1_{15}, \ldots \geq 0, integer$$
$$y_1, y_2, y_3 \in \{0, 1\}$$

4 Optimal solution obtained using AMPL and CPLEX

The optimal solution found solving the problem with AMPL and CPLEX is to have the manager buy electric bikes.
The optimal number of parcels to be sent through each link is:

$$x_{13}^{2}{}^{*} = 5, x_{14}^{2}{}^{*} = 17, x_{15}^{2}{}^{*} = 16, x_{23^{*}}^{2} = 3, x_{27}^{2}{}^{*} = 42, x_{35}^{2}{}^{*} = 0, x_{36}^{2}{}^{*} = 18, x_{38}^{2}{}^{*} = 18, x_{78}^{2}{}^{*} = 16$$

The optimal number of electric bikes to buy for each link is:

$$z_{13}^{2}{}^{*} = 1, z_{14}^{2}{}^{*} = 3, z_{15}^{2}{}^{*} = 3, z_{23}^{2}{}^{*} = 1, z_{27}^{2}{}^{*} = 7, z_{35}^{2}{}^{*} = 0, z_{36}^{2}{}^{*} = 3, z_{38}^{2}{}^{*} = 3, z_{78}^{2}{}^{*} = 3$$

The electric motorbike and the scooter are not bought by the manager.

$$z_{13}^{1}{}^{*} = z_{14}^{1}{}^{*} = z_{15}^{1}{}^{*} = z_{23^{*}}^{1} = z_{27}^{1}{}^{*} = z_{35}^{1}{}^{*} = z_{36}^{1}{}^{*} = z_{38}^{1}{}^{*} = z_{78}^{1}{}^{*} = 0$$

$$z_{13}^{3}{}^{*} = z_{14}^{3}{}^{*} = z_{15}^{3}{}^{*} = z_{23}^{3}{}^{*} = z_{27}^{3}{}^{*} = z_{35}^{3}{}^{*} = z_{36}^{3}{}^{*} = z_{38}^{3}{}^{*} = z_{78}^{3}{}^{*} = 0$$

Total cost = 28,800
Number of vehicles to buy = 24

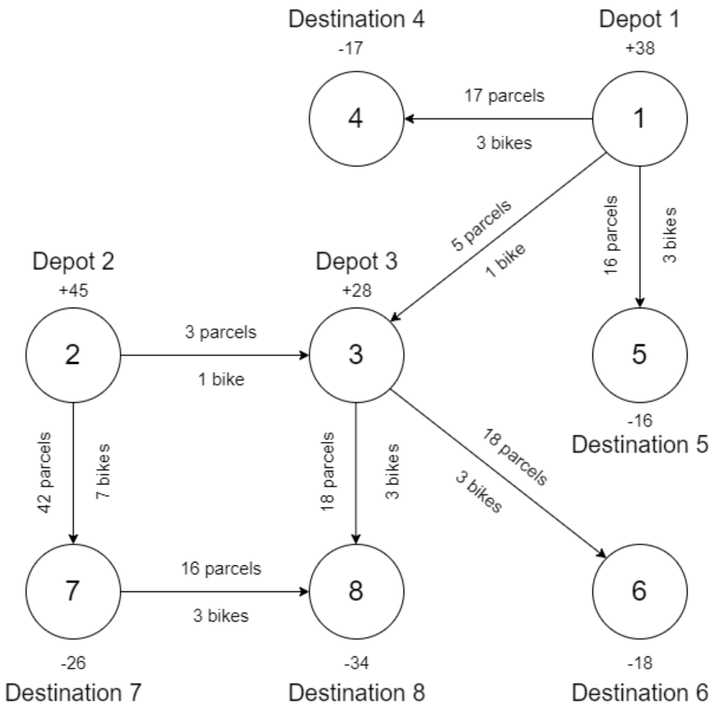

Figure 2. Network displaying optimal number of parcels and vehicles to minimize cost

4.1 Solution considering the additional constraint

How does the solution change if, for congestion reasons, at most 4 vehicles may move on each link of the network?

$$z_{ij}^{k} \leq 4, \forall (i,j) \in A, \forall k \in K$$

We now add the additional constraint, which limits the number of vehicles that can flow through a link (i,j) to at most 4. In case of congestion, the optimal solution changes. Now, the vehicle the manager should buy is the motorbike, since it is the only vehicle capable of satisfying the demand while sending 4 or less vehicles at a time on a single link.

The optimal number of parcels to be sent through each link is:

$$x_{13}^{3}{}^{*} = 5, x_{14}^{3}{}^{*} = 17, x_{15}^{3}{}^{*} = 16, x_{23*}^{3} = 19, x_{27}^{3}{}^{*} = 26, x_{35}^{3}{}^{*} = 0, x_{36}^{3}{}^{*} = 18, x_{38}^{3}{}^{*} = 34, x_{78}^{3}{}^{*} = 0$$

The optimal number of electric motorbikes the manager should buy for each link is:

$$z_{13}^{3}{}^{*} = 1, z_{14}^{3}{}^{*} = 2, z_{15}^{3}{}^{*} = 2, z_{23}^{3}{}^{*} = 3, z_{27}^{3}{}^{*} = 3, z_{35}^{3}{}^{*} = 0, z_{36}^{3}{}^{*} = 2, z_{38}^{3}{}^{*} = 4, z_{78}^{3}{}^{*} = 0$$

The electric bike and the scooter are not bought by the manager.

$$z_{13}^{1}{}^{*} = z_{14}^{1}{}^{*} = z_{15}^{1}{}^{*} = z_{23*}^{1} = z_{27}^{1}{}^{*} = z_{35}^{1}{}^{*} = z_{36}^{1}{}^{*} = z_{38}^{1}{}^{*} = z_{78}^{1}{}^{*} = 0$$

$$z_{13}^{2}{}^{*} = z_{14}^{2}{}^{*} = z_{15}^{2}{}^{*} = z_{23}^{2}{}^{*} = z_{27}^{2}{}^{*} = z_{35}^{2}{}^{*} = z_{36}^{2}{}^{*} = z_{38}^{2}{}^{*} = z_{78}^{2}{}^{*} = 0$$

Total cost = 28,900
Number of vehicles to buy = 17

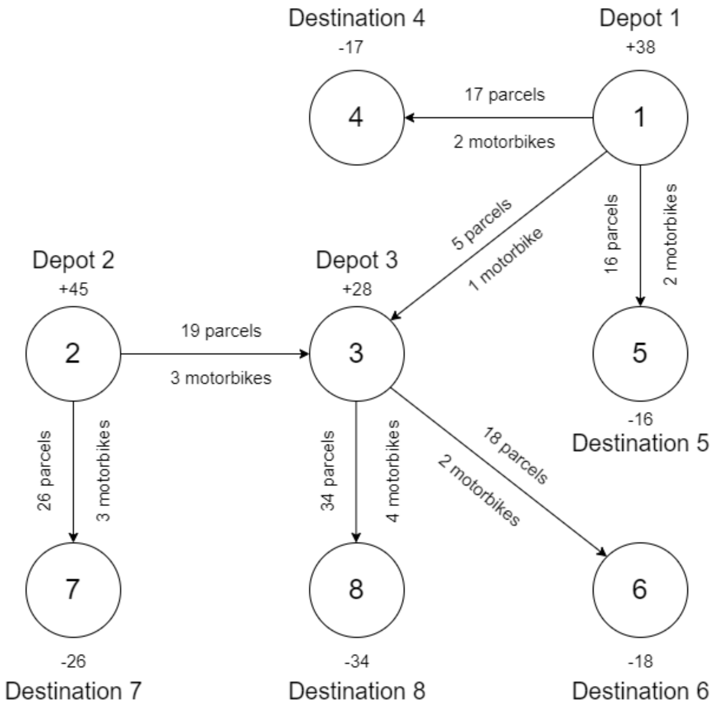

Figure 3. Network displaying optimal number of parcels and vehicles to minimize cost after additional constraint

References

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