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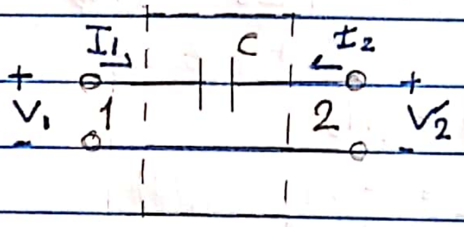
∘ Pré-Lab 02

2.



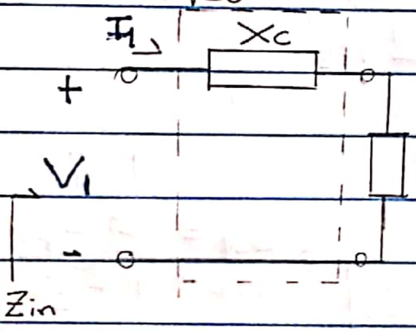
3.

$$b_1 = S_{11}\,a_1 + S_{12}\,a_2 \qquad \Rightarrow \quad S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} \;(1);\quad S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0}$$

$$b_2 = S_{21}\,a_1 + S_{22}\,a_2 \qquad\qquad S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0} \;(2);\quad S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0}$$

↳ Cálculo S_{11}:

$$a_1 = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}} \;(3);\quad b_1 = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}} \;(4)$$



$$Z_{in} = X_C + Z_0 \;(5)$$

$$V_1 = I_1 Z_{in} \Rightarrow I_1 = \frac{V_1}{Z_{in}} \;(6)$$

∘ (5) → (6):

$$I_1 = \frac{V_1}{X_C + Z_0} \;(7)$$

∘ (7) → (3):

$$a_1 = \frac{V_1 + \frac{V_1 Z_0}{X_C + Z_0}}{2\sqrt{Z_0}} \Rightarrow a_1 = \frac{V_1\left[1 + Z_0/X_C + Z_0\right]}{2\sqrt{Z_0}} \;(8)$$

∘ (7) → (4):

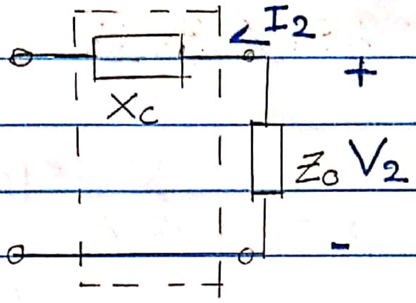
$$b_1 = \frac{V_1 - \frac{V_1 Z_0}{X_C + Z_0}}{2\sqrt{Z_0}} \Rightarrow b_1 = \frac{V_1\left[1 - Z_0/X_C + Z_0\right]}{2\sqrt{Z_0}} \;(9)$$

∘ (8) e (9) → (1):

$$S_{11} = \frac{\cancel{V_1}\left[1 - Z_0/X_C + Z_0\right]}{\cancel{2\sqrt{Z_0}}} \cdot \frac{\cancel{2\sqrt{Z_0}}}{\cancel{V_1}\left[1 + Z_0/X_C + Z_0\right]}$$

$$S_{11} = \frac{1 - Z}{1 + Z} \;(10);\quad Z = \frac{Z_0}{X_C + Z_0}$$

↳ Cálculo S_{21}:



$I_2 = -I_1$ (12)

$V_2 = \frac{V_1 Z_0}{X_c + Z_0}$ (13)

• (6) → (11):

$I_2 = \frac{-V_1}{X_c + Z_0}$ (14)

$b_2 = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$ (11)

• (13), (14) → (11)

$$b_2 = \frac{\frac{V_1 Z_0}{X_c + Z_0} + \frac{V_1 Z_0}{X_c + Z_0}}{2\sqrt{Z_0}} \rightarrow b_2 = \frac{\frac{\cancel{2} V_1 Z_0}{X_c + Z_0}}{\cancel{2}\sqrt{Z_0}} \rightarrow b_2 = \frac{V_1 Z_0 / X_c + Z_0}{\sqrt{Z_0}} \quad (15)$$

• (15), (8) → (2):

$$S_{21} = \frac{\cancel{V_1} Z_0 / X_c + Z_0}{\cancel{\sqrt{Z_0}}} \cdot \frac{2\cancel{\sqrt{Z_0}}}{\cancel{V_1}[1 + Z_0 / X_c + Z_0]} \rightarrow S_{21} = \frac{2 Z_0 / X_c + Z_0}{1 + Z_0 / X_c + Z_0}$$

$$S_{21} = \frac{2Z}{1+Z} \quad (16); \quad Z = \frac{Z_0}{X_c + Z_0}$$

↳ Por simétria e reciprocidade, temos que:

$S_{22} = S_{11}$ e $S_{12} = S_{21}$

4.

↳ Para $f = 1\,GHz$, $C = 1\,pF$ e $Z_0 = 50\,\Omega$, temos:

$$X_c = \frac{1}{j2\pi f . C} = \frac{1}{j2\pi . 1\times10^{9} . 1\times10^{-12}} \Rightarrow X_c = -j159,15$$

$$Z = \frac{Z_0}{X_c + Z_0} = \frac{50}{50 - j159{,}15} = \frac{50}{166{,}82\angle -72{,}56°}$$

$$Z = 0{,}3\angle 72{,}56°$$

↳ Temos S_{11}:

$$S_{11} = \frac{1 - (0{,}09 + j0{,}286)}{1 + (0{,}09 + j0{,}286)} = \frac{0{,}91 - j0{,}286}{1{,}09 + j0{,}286} = \frac{0{,}954\angle -17{,}45°}{1{,}127\angle 14{,}70°}$$

$$\boxed{S_{11} = 0{,}846\angle -32{,}15°} = S_{22}$$

↳ Temos S_{21}:

$$S_{21} = \frac{0{,}6\angle 72{,}56°}{1{,}127\angle 14{,}70} = \boxed{0{,}532\angle 57{,}86°} = S_{12}$$

↳ Portanto, temos:

$$S = \begin{bmatrix} 0{,}846\angle -32{,}15° & 0{,}532\angle 57{,}86° \\ 0{,}532\angle 57{,}86° & 0{,}846\angle -32{,}15° \end{bmatrix}$$

• Prova Real:

↳ No Livro "Microwave and RF Design network, Volume 3" de Michael Steer, temos para uma conexão em série:

$$S_{11} = S_{22} = \frac{\bar{Z}}{\bar{Z} + 2} ; \quad \bar{Z} = Z / Z_0$$

$$S_{21} = S_{12} = \frac{2}{\bar{Z} + 2}$$

$$\bar{Z} = \frac{-j159{,}15}{50} = \frac{159{,}15\angle -90°}{50} = 3{,}183\angle -90° = 0 - j3{,}183$$

$$S_{11} = \frac{3,183 \angle -90}{2 - j3,183} = \frac{3,183 \angle -90}{3,759 \angle -57,86} = \boxed{0,847 \angle -32,14°}$$

$$S_{21} = \frac{2}{3,759 \angle -57,86°} = \boxed{0,532 \angle 57,86°}$$

↳ Assim comprovando os valores encontrados.