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Inverse Problems - Worksheet 1 Winter term 2019/2020

Consolidation assignment

Consider the following experimental setup:

Janic and Daniel step on a scale together and note their combined weight. After that, Daniel and Philipp do the same.

1. describe in detail how this constitutes a *forward* and an *inverse problem*!
2. is the inverse problem well- or ill-posed?
3. describe how the situation changes if we add another piece of data by weighing Janic and Philipp together!

Preparation assignment (finish before 22 October)

1) Complete the short survey on your expectations about the lecture "Inverse Problems" in the corresponding StudOn group

2) Peruse “`Preliminaries and Introduction`” in the lecture notes folder in StudOn. Familiarize yourself with any new concepts, definitions, and ideas. Answer the following short questions and write down their answers briefly.

   a) Are the $\|\cdot\|_{L^1}$ and $\|\cdot\|_{L^2}$ norms equivalent on the space $C([0,1])$ of continuous functions on the interval $[0,1]$? Give a proof/counterexample why this does or does not hold.

   b) Consider the setting of Theorem B.1. Give an example for an unbounded linear mapping L and show that it is indeed not continuous at 0.

c) Consider the setting of Theorem B.2. Set $X = l^2$, the set of all sequences such that $\|x\|_2 < \infty$, with $\|x\|_2^2 = \sum_{k=1}^{\infty} x_k^2$. This space is also a Hilbert space with $\langle x, y \rangle := \sum_{k=1}^{\infty} x_k \cdot y_k$.

- We define $L_1 : X \to \mathbb{R}$ by $L_1(x) = \sum_{k=1} \frac{x_k}{k}$. Prove that L_1 is linear and bounded. What is the unique $x_0 \in X$ such that $Lx = \langle x, x_0 \rangle$?

- We define $L_2(x) = \sum_{k=1} (-1)^k \cdot x_k$. Show that L_2 is unbounded. What "candidate" for x_0 could you try and why does this not work?

d) Is the identity operator $I : X \to X$ for a Banach space X a compact operator or not? Explain your answer mathematically and give special consideration for the space X.

e) Show that the (infinite) sequence $x^{(n)}$ with $x^n = (0, 0, \ldots, 0, 1, 0, \ldots)$ (with the 1 at position n) converges weakly, but not strongly in $(l^2, \|\cdot\|_2)$

Notes / Insights from class

If you have open questions from the preparation assignment, note them here and bring them to class!