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Inverse Problems - Worksheet 2 Winter term 2019/2020

1 Consolidation assignment

Show that the (infinite) sequence $x^{(n)}$ with $x^n = (0, 0, \ldots, 0, 1, 0, \ldots)$ (with the 1 at position n) converges neither weakly, nor strongly in $(l^1, \|\cdot\|_1)$.

2 Preparation assignment

1) **Peruse Chapter 2.1 of the textbook**. If you are stuck at some point, use the Hypothesis annotation tool on the PDF to get help. It is likely that many of your peers have the same problem. To take down private notes that will only be visible to you, check the appropriate box, but keep in mind that you can help others by sharing your insights, too.

2) We consider the interval $[0, 1]$ for the discrete convolution model (section 2.1.2 in the textbook), with discretization parameter $n = 5$. Calculate explicitly:

   a) $(x_j)_j =$

   b) equation (2.8): $\int_0^1 g(x)\,\mathrm{d}x \approx$

   c) For $\psi_0(x)$ as in (2.1): $\mathbf{p} =$

   and $\mathbf{A} =$

   d) For $\psi_0(x) = \chi_{[-a,a]}$: $\mathbf{p} =$

   and $\mathbf{A} =$

3) Run the Matlab/Octave file `Ex1_ContinuousData.m`. Which influence has changing the convolution parameter `a`?

4) Run the Matlab file `Ex2_DiscreteData.m`[1]. Describe in detail how different grids are used in order to avoid an inverse crime.

5) Run the Matlab file `Ex3_NaiveRecon.m`. To see the influence of noise, edit file `Ex2_DiscreteData.m` at the appropriate position and run `Ex3_NaiveRecon.m` again.

   Comment on the reason of the quality of each reconstruction.

6) Rerun all files after having implemented a different convolution kernel, for example the one from question 2d).

If you have open questions from the preparation assignment, note them here and bring them to class.

3 Notes / Insights from class

[1]Note: Both `Ex2_DiscreteData.m` and `Ex3_NaiveRecon.m` need a dataset created by `Ex1_ContinuousData.m`, i.e. you need to run all three files in this order or you will get an error.