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Inverse Problems - Worksheet 8 $\qquad$ Winter term 2019/2020

1   Consolidation assignment

Let K be the Radon transformation operator.

1. Formulate the reconstruction problem (with given tomography data $f = Ku$ where K is the Radon operator and u is the unknown phantom) as an optimization problem for u with an additional Tikhonov regularization term penalizing the gradient.

2. Deduce the necessary optimality conditions for the former problem.

3. Characterize the occuring adjoint Radon operator K^* of K mathematically.

2   Preparation assignment

1) **Peruse** the pdf titled "Bayesian Inference" in StudOn.

2) Do the exercises 21.1, 21.2 and 21.3 (interspersed in the reading assignment).

3) Ponder the following ("Simpson's") paradox (the numerical values are artificially constructed). Consider a carrot cake. It consists of 67% batter and 33% frosting. The batter consists of 21% fat and the frosting consists of 75% fat.

   On the other hand, there is chocolate cake, consisting of 90% batter and 10%frosting, with the batter having a fat content of 30% fat and the frosting being 90% fat.

   This means that each component of the chocolate cake has a higher percentage of fat than the carrot cake's components. Do the math to see that in fact the carrot cake contains more overall fat than the chocolate cake. Think about this apparent contradiction and solve it using a sketch.

   This statistical paradox is quite pervasive and it's good to develop a sense for it.

3   Notes / Insights from class