Evolution of Cooperation in Networked populations

Network Science

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1 Introduction

We were proposed to deepen and apply our knowledge about complex networks to model real world dynamics. The theme that we chose was evolution of cooperation in networked populations, in the context of the Prisoner's Dilemma. Given that we come from computer science and in our first project we had implemented our own network models in Python (*Erdős–Rényi* and *Barbási–Albert*) and some of the principal metrics discussed in the course, we decided to reuse it and extend it. We are then studying lattices, complete graphs, regular random graphs and *Barabási–Albert* to see how heterogeneity affects the evolution of cooperation. Finally, we decided to do this report in LaTeX to practice for upcoming works.

2 Prisoner's Dilemma and Cooperation

Prisoners Dilemma is one game analyzed in evolutionary game theory. These games represent the interaction between two individuals in which each of the two have the option to either cooperate or defect. The interaction score is described recurring to a payoff matrix with specific parameters. In evolutionary game matrix, the payoff matrix parameters are the following: When one defects and the other cooperates, the defector scores T (corresponding to the whole benefit) and the cooperator scores S (corresponding to the whole cost). If both cooperate, they both get the benefit and split the cost, resulting in a reward R for both. If they both defect, none pays the cost but none receives the benefit, being both punished with a value P.

In the dilemma studied, $P = 0$ and $R = 1$, so mutual cooperation is favored when compared to mutual defection. With $T>1$ and$S<0$, greed and fear foster defection when interacting with other agents. This means that the optimal short-term individual option is to defect if the other individual is a defector, so that it does not get exploited ($P>S$) , and to become a defector if the other cooperates, since exploiting the cooperator is better than cooperating as well ($T>R$). Considering all this, cooperation never prevails in this dilemma seen that the gain of being a possible defector is higher being a cooperator as shown by the following, $T>R>P>S$.

An example of a real life situation represented by this dilemma can be when dealing with doping in sports. Both athletes/teams know that if they both do not use doping, neither get boosted nor damaged, leading to a fair trial. If only one of them uses it, despite the damage caused, he will gain an advantage when facing the other. Finally, when both use it they both get damaged and their advantage is cancelled out, leading again to a fair trial. In a nutshell, the ideal scenario would be that both refuse to doping, but the greed to win, and the fear of losing causes them to follow the individual short-term best option that is to use doping even though, they both know that they may get disqualified or damaged, or get the trial in the same conditions as well. So, when populations are assumed infinite and every individual has the same links, non-cooperative behaviour prevails.

When introducing the notion of networked populations, and that the agent plays the game with its neighbours there are new factors to take into account. Each node plays with their neighbours in a round, scoring a payoff. The score of the node being a cooperator equals to number of cooperators that are connected to it times R plus the number of defectors that are connected to it times S (

$$Nd * S + Nc * R \tag{1}$$

) . Fitness of the node being a defector equals the number of cooperators that are connected to it times S plus the number of defectors that are connected to it times P (

$$Nd * P + Nc * T \tag{2}$$

). If the payoff of a node is large when compared to its neighbours, the node has a high fitness, increasing the chance of replicating his strategy on his neighbours. Since the fitness of a node depends on the number of nodes linked to it and of their own strategy, there are some cases in which cooperation prevails over defection. A goal of our project is to understand when.

Considering all this, on homogeneous networks and splitting the population in 50% defectors and 50% cooperators, each node should start with the same number of cooperator and defectors as neighbours. Then the fit of every defectors is larger than of cooperators (as $T>R$) so for each generation it is expected that the number of neighbour nodes cooperators to decrease reaching the *tragedy of the commons*.

On the other hand, in heterogeneous networks, there are some cases in which cooperation can thrive. Remembering the equation when the fit of a cooperator is better than of the defector:

$$Nd * S + Nc * R > Nd * P + Nc * T \tag{3}$$

it can be seen that the number of the neighbours and their respective strategy has also a role and so it can be more important to this inequality than the relative ordering of the payoffs, even when greed and fear exist. Both defectors and cooperators benefit from occupying in highly connected positions, as they increase their chance to interact with more cooperators.

3 Variation of Temptation payoffs

The following graphs/images represent the role of variation of greed in this dilemma. The higher the greed (T), most likely it is for the node to defect if the other cooperates.

In this simulations we test how temptation affects the evolution of cooperation and we fix the value of $R = 1$, and the values of $P = S = 0$. The following plots show how cooperation evolved for each generation in the simulation and each line represents a different simulation.

3.1 Homogeneous networks

Homogeneous networks are those in which all nodes have the same degree distribution. This includes complete graphs, lattices and regular random graphs with fixed degree for every node. Thus the heterogeneity is very low. In this type of networks interactions are assumed homogeneous such that all individuals are in equivalent positions, and have the same number of interactions. It is known that cooperation is not favoured in networks as these.

The random regular graph is built by randomly connecting each node to a fixed number of k nodes.

3.1.1 Complete graph

Heterogeneity of this model equals 0. Every node has N - 1 links.

$$fitC = (Nc - 1) * R + Nd * S \tag{4}$$

$$fitD = Nc * T + (Nd - 1) * P \tag{5}$$

Since the network is complete the number of neighbours a node has that are cooperators (defectors) is the total number of cooperators (defectors).

If $R = T$, then $fitC - fitD = -R$, so even here cooperation fails in this model.

3.1.2 Lattice graph

Heterogeneity of this model equals 0. This model has 1024 nodes and every node has 4 links. The conclusion is the same as the previous since its heterogeneity value is equal to the previous, and as shown by the figure even with a low greed value ($T = 1.2$) the simulations always ends up at full defection. This conclusions follows for the subsequent model.

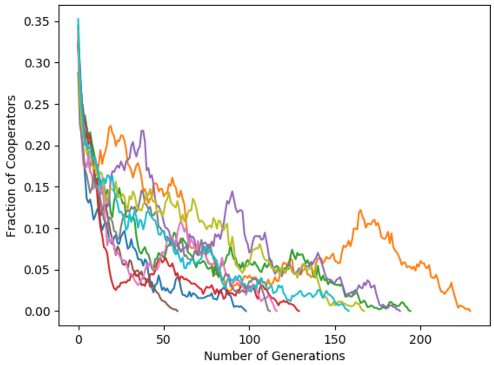


Figure 1: Lattice graph with $T = 1.2$ and $S = 0$

3.1.3 Regular random graph

This regular graph has 1000 nodes and average degree = 4.

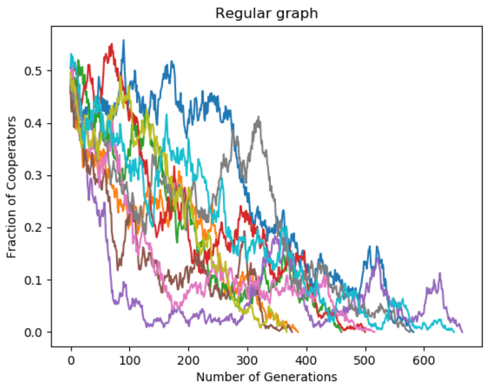


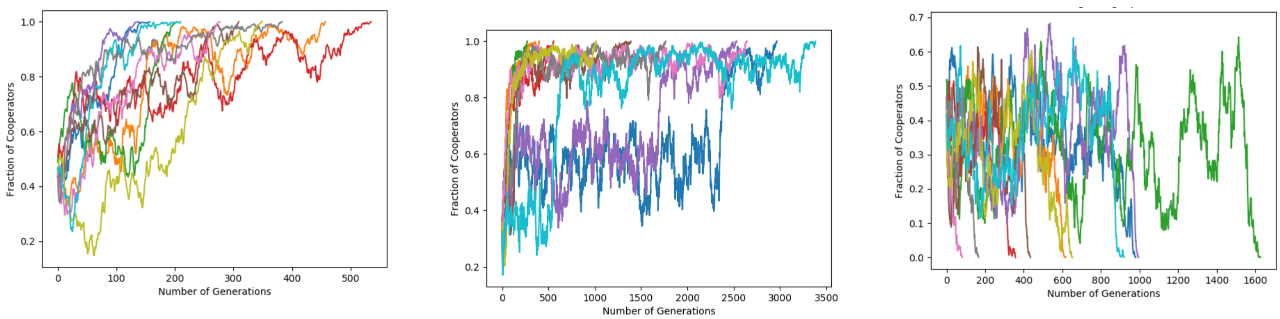
Figure 2: Regular graph with $T = 1.2$ and $S = 0$

3.2 Heterogeneous networks

Real populations are heterogeneous, where some nodes have a degree much higher than others. Thus, these networks show a much larger heterogeneity. Heterogeneity embeds the intuition that different individuals engage in different numbers of interactions with different intensities. In such conditions, cooperation is enhanced and can thrive. In the next subsection, it will be presented the results from cooperation in Barabasi-Albert model, in which nodesToAdd represents the number of links that each node gets when he is inserted in the model. Thus, the average degree of the model is 2 times nodesToAdd.

3.2.1 Barabasi-Albert : Starting with a complete graph with 2 nodes and nodesToAdd = 2

This model has an heterogenity of 26.948. As seen by the following plots, with $88T = 2.1$, plot (b), cooperation still prevails, but not as easily as with $T = 1.8$, plot (a), in which all the simulations end before 1000 generations contrasting to plot (b). For plot (c), with $T = 2.5$, defection prevails, but cooperators can reach almost 70% of the network, showing that cooperation can still resist to defection for a bit of time.



(a) T = 1.8 and S = 0

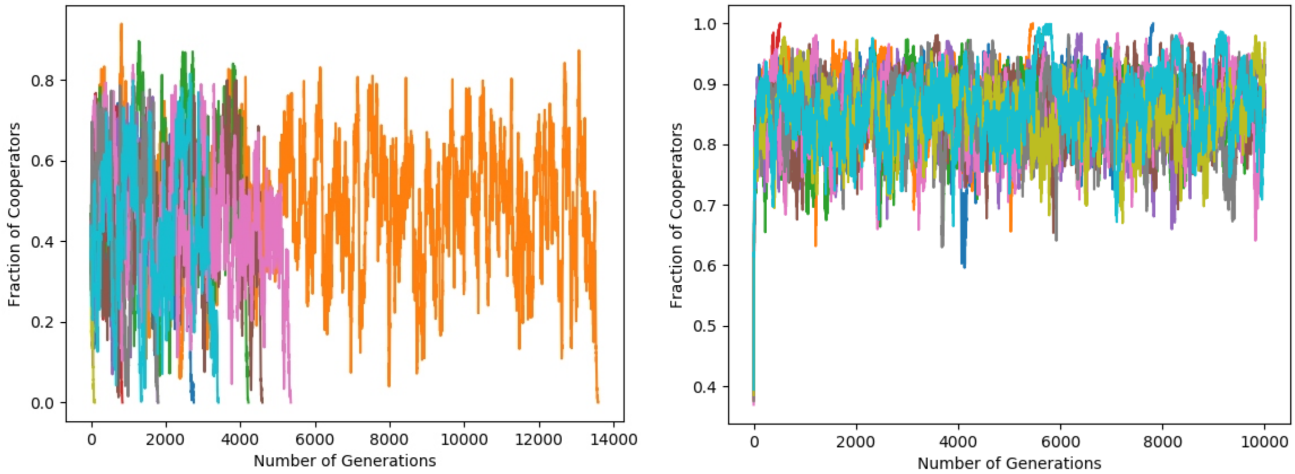
(b) T = 2.1 and S = 0

(c) T = 2.5 and S = 0

Figure 3: Comparing BA model

4 Init strategies in Heterogeneous networks

In this section it is shown the role of the hubs in the evolution of cooperation, if they can help cooperators to exist in hard conditions.



(a) Random Strategy

(b) topHubsStrategy, fraction = 20%

Figure 4: Comparing initial strategies in BA model $T = 2.2$ and $S = 0$

Please check section *Methods* for an explanation of this strategies.

5 Conclusion/Analysis

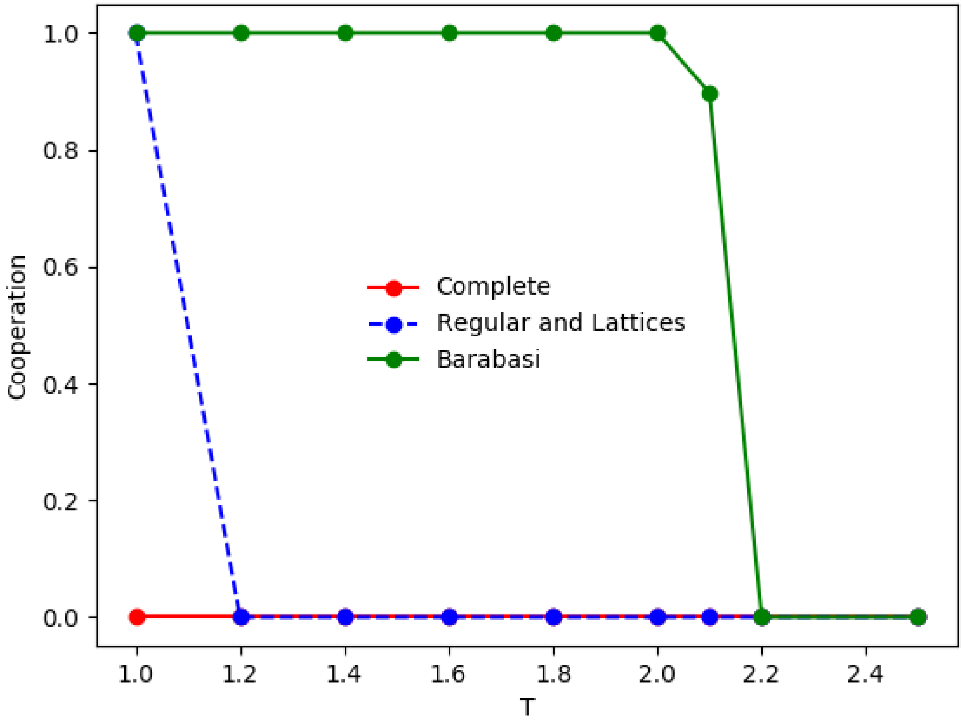


Figure 5: Evolution of cooperation associated with increase in T

From the results of the simulations it can be concluded that graphs with more heterogeneity can make cooperation thrive in conditions that fail in homogeneous networks. Also hubs play a crucial role in guaranteeing that cooperation grows. Our demonstrations show that even for a low fraction of only the larger hubs as the only initial cooperators of the network, cooperation can be possible in such conditions as in the prisoner's dilemma with a high T.

In homogeneous networks, only when $R = T$ cooperation is possible as for small increases of T cooperation is already impossible. However, in the complete graph, cooperation is not possible at all in the Prisoner's Dilemma as shown before. Lattice graphs tend to resist (a bit) longer than regular graphs. This is due to the higher clustering that is present in this graphs, which allows clusters of cooperators to form and resist invasion by defectors longer. In regular random graphs, this effect is not present; shortcuts are formed, inhibiting the formation of clusters and allowing defectors to spread more effectively.

In heterogeneous networks, the hubs remain pure cooperators until the cooperators' density vanishes; thanks to their percolation properties, the subgraph of all pure cooperators therefore remains connected, and the pure defectors start by forming various small clusters which finally merge and invade the whole network.

Simulations in *Barabasi* show that sometimes there is a quick and large shift in the evolution of cooperation. This happens when a group of nodes can change the hub's strategy and consequently the hub influences other smaller degree nodes. Also in *Barabasi*, since the model is built by the combined mechanisms of growth and preferential attachment, it will show clustering between the higher "older" degree nodes, which favours cooperation. That is why only a small fraction of these hubs as cooperators can overcome defection so well.

Summing up, heterogeneity, hubs and the possibility of cooperators to form clusters are factors that favour cooperation.

6 Methods

As mentioned, we use our own code to build the network models. We use the same code for *Barabási–Albert* model that we used in Project 1. For complete graphs we used the same code we did to *Erdős–Rényi* with probability of 1 that a node connects to each other node; lattices are built by connecting each node to two others, the right one and the bottom one, and making the extremes to connect as if it were circular to make all nodes have N-S-E-W interactions; or regular graphs we use the configuration model by assigning a degree to each node (k=4), represented as stubs or half-links, randomly select a stub pair and connect them. Then it is randomly chosen another pair from the remaining $2 * links - 2$ stubs and they are connected. This procedure is repeated until all stubs are paired up.

The evolution of cooperation is computed synchronously, in that in every generation is calculated the fitness of all the nodes; every node randomly picks a neighbour node to imitate and this replication is done with a probability given by the function

$$p = [1 + e^{-\beta(fitB - fitA)}]^{-1} \tag{6}$$

This method demands that a temporary vector of strategies of the next generation to be stored.

For homogeneous networks the initial strategy of each node can be calculated by (**randomStrategy**) sampling 50% of the population as Cooperators and those not in that list are assigned as Defectors; (**staticStrategy**) in which the nodes with ids under the 50% of the size of the graph are Cooperators and the rest are the Defectors;

For heterogeneous networks, the initial strategy can be computed using the methods mentioned before; the **selectTopHubs** strategy selects the fraction of the nodes with the highest degree making them Cooperators or Defectors (decided with parameter **nice**).

The heterogeneity of a network is calculated as $<k>^2 - <k^2>$.

References

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[2] Barrat, A., M. Barthelemy, and A. Vespignani. *Dynamical processes on complex networks.* Cambridge University Press, 2008.

[3] Prisoner's Dillema,
`https://en.wikipedia.org/wiki/Prisoner%27s_dilemma`