

Teste 1 - 2021

13

1ª ocorrência de n° ímpar e retorna o índice

```

1 int firstOdd (int v[], int N) {
    // pre: N >= 0
    int i = 0;
    while (i < N && v[i] % 2 == 0) i++;
    if (i == N) return -1;
    else return i;
}

```

	0	1	2	3	N=4
v	10	4	7	5	
i=0	i=1	i=2	i=3	i=4	
			ímpar		
			ímpar		

// pos: (i == -1 || forall {0 ≤ k < N} v[k] % 2 == 0) || (0 ≤ i < N && forall {0 ≤ k < i} v[k] % 2 == 0 && v[i] % 2 == 1) ✓

return i;

i	i	v[i]
0	0	10
1	0	4
2	2	7

1.

Prova de que a cada i k < i, as i-1 k < i-1 v[k] % 2 == 0

2.

Invariante: 0 ≤ i < N && forall {0 ≤ k < i} v[k] % 2 == 0 ✓

Postul I: (i == -1 || forall {0 ≤ k < i} v[k] % 2 == 0) || (forall {0 ≤ k < i} v[k] % 2 == 0 && v[i] % 2 == 1) && 0 ≤ i < N

→ Prova Indut

{N >= 0} i = 0 {I}

N >= 0 ⇒ (i == -1 || forall {0 ≤ k < 0} v[k] % 2 == 0) || (forall {0 ≤ k < 0} v[k] % 2 == 0 && v[0] % 2 == 1) && 0 ≤ 0 < N

→ Prover Init ✓

$$\{N \geq 0\} \quad i = 0; \quad \{I\}$$

$$N \geq 0 \Rightarrow 0 \leq 0 < N \text{ and } \text{full} - \{0 \leq k < 0\} \vee [k] \% 2 == 0$$

→ Prover Util ✓

$$0 \leq i < N \text{ and } \text{full} - \{0 \leq k < i\} \vee [k] \% 2 == 0$$

$$\Rightarrow (\lambda == -1 \text{ and } \text{full} - \{0 \leq k < N\} \vee [k] \% 2 == 0) \vee$$

$$(i > N \vee [i] \% 2 == 1)$$

①

②

$$(0 \leq \lambda < N \text{ and } \text{full} - \{0 \leq k < \lambda\} \vee [k] \% 2 == 0$$

$$\text{and } [k] \% 2 == 0$$

$$\text{and } [i] \% 2 == 1)$$

$$\text{Case ① } i > N \text{ and } 0 \leq i < N \Rightarrow 0 \leq i < N \leq i$$

$$\text{F} \Rightarrow ? = \text{①} \checkmark$$

$$0 \leq i < N \text{ and}$$

$$\text{Case ② } \text{full} - \{0 \leq k < i\} \vee [k] \% 2 == 0 \text{ and } [i] \% 2 == 1$$

$$0 \leq \lambda < N \text{ and}$$

$$\text{full} - \{0 \leq k < \lambda\} \vee [k] \% 2 == 0 \text{ and } [i] \% 2 == 1$$

→ Prover Pos

$$\{0 \leq i < N \text{ and } \text{full} - \{0 \leq k < i\} \vee [k] \% 2 == 0$$

$$\text{①} \Rightarrow ? = \text{①} \checkmark$$

$$\text{②} \Rightarrow ? = \text{①} \checkmark$$

$$\{0 \leq i < N \text{ and } \text{full} - \{0 \leq k < i\} \vee [k] \% 2 == 0$$

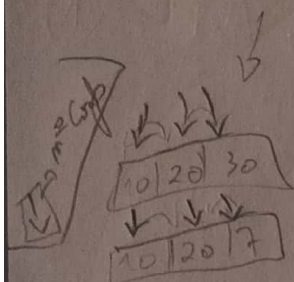
$$\text{①} \Rightarrow \{0 \leq i < N \text{ and } \text{full} - \{0 \leq k < i\} \vee [k] \% 2 == 0$$

$$\{I \wedge i = N\} \quad \lambda = -1 \quad \{I\}$$

$$\{I \wedge i \neq N\} \quad \lambda = i \quad \{I\}$$

$$\{I \wedge i\} \quad \text{if } (i == N) \lambda = -1; \text{ else } \lambda = i; \quad \{I\}$$

m^2 Comparação de $\sqrt{(\hat{n})} \% 2 = 50$



$$\overline{T}_{\text{add}}^{\text{PC}}(N) = \sum_{i=0}^{N-1} 1 = N - 1 = 0 + 1 = \underline{N} = \underline{\Theta(N)}$$

$$\overline{f_{add}}^{MC}(N) = \underline{1} = \underline{\Theta(1)} //$$

4. Caso Médio - para m² acessos ao arroyo

long Sur par ce inf
dem prob = $\frac{1}{2}$

$$\textcircled{+} \quad \overline{1(N)} = \sum_{i=1}^N \underbrace{\left(\frac{1}{2}\right) \times \left(1 - \frac{1}{2}\right)^{i-1}}_{\text{prob.}} \cdot \underbrace{i}_{n = \text{conjunctions}} + \underbrace{\left(\frac{1}{2}\right)^N \cdot N}_{\checkmark}$$

N Conf. S.
2 man - 1
or 2 people

Nancy
e me N-
e' you too

2. $\overline{T(N)} = \Theta(1)$ \rightarrow expected $\Theta(1)$

$$\left(\frac{1}{2}\right) \times 1 + \left(\frac{1}{4}\right) \times 2 + \frac{1}{8} \times 3 + \dots$$

no paravel aonde no inicio

1° Lanes 0.50% Cu^{+} (1/2)
do 0.5 Lanes 75% Cu^{+} (1/2 + 1/2)
" " 87.5% " (1/2 + 1/4 + 1/4)

② `int elem (int x, int u[], int N) {`

`// pre: N >= 0`

`if (N == 0) r = 0;`

`else if (x == u[0]) r = 1;`

`else r = elem (x, u+1, N-1);`

`// pos: (x == 0 && forall {0 <= k < N} v[k] != x) || (x == 1 && exists {0 <= k < N} v[k] == x)`

`return r;`

`}`

`int diferentes (int v[], int N) {`

`int r = 0;`

`if (N != 0) {`

`r = diferentes (v+1, N-1);`

`if (!elem (v[0], v+1, N-1)) r++;`

`}`

`return r;`

`}`

1.

2. diferentes m dem MC ou PC

$$T_{diff}(N) = \begin{cases} 0 & \Leftarrow N \leq 1 \\ T_{diff}(N-1) + T_{elem}(N-1) & \Leftarrow N > 1 \end{cases}$$

(x N=1 → def(0) → elem(0) return 0
N = 1 → 1ª comparação

mª comparação x == u[0]

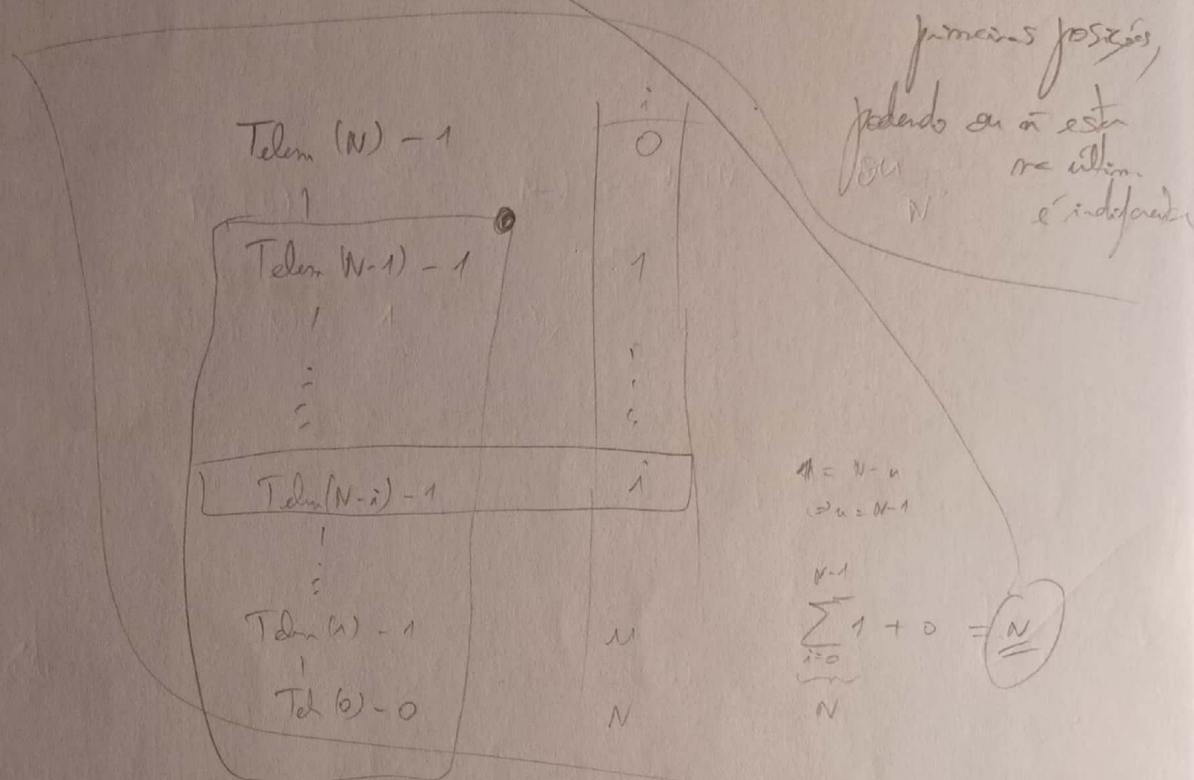
MC \rightarrow Considere-se sempre qual

$$T_{elem}(N) = 1$$

MC de \rightarrow qual x está na 1ª posição any

$$T_{elem}^{PC}(N) = 1 + T_{elem}(N-1) = N$$

PC de \rightarrow qual x n está na 5 N-1



Então, MC e PC decompõem de funções elementares,

$$T_{diff}^{MC}(N) = T_{diff}(N-1) + 1 \quad \leftarrow N < 1$$

$$T_{diff}(N) + 1 \quad \leftarrow N > 1$$

$$T_{diff}^{PC}(N) = T_{diff}(N-1) + \boxed{N-1}$$