Teste 2 - 14/15

1

a) $(a+b+c)^3 = \sum_{t_1+t_2+t_3=3} \binom{3}{t_1,t_2,t_3} a^{t_1} \cdot b^{t_2} \cdot c^{t_3}$

b) $(a+2)^3 = (a+1+1)^3 = \sum_{t_1+t_2+t_3=3} \binom{3}{t_1,t_2,t_3} a^{t_1} \cdot 1^{t_2} \cdot 1^{t_3}$

$= \sum_{t_1+t_2+t_3=3} \binom{3}{t_1,t_2,t_3} a^{t_1}$

$\alpha_0 \Rightarrow a^{t_1} = 1 \Leftrightarrow t_1 = 0$

$t_2+t_3=3$

& $t_2=3$ e $t_3=0$ $\quad \binom{3}{0,3,0} = 1$

$t_2=2$ e $t_3=1$ $\quad \binom{3}{0,2,1} = \frac{3}{2!1!} = 3$

$t_2=1$ e $t_3=2$ $\quad \binom{3}{0,1,2} = \frac{3}{1!2!} = 3$

$t_2=0$ e $t_3=3$ $\quad \binom{3}{0,0,3} = 1$

$\alpha_0 = 1+3+3+1 = 8$

$\alpha_1 \Rightarrow a^{t_1} = a^1 \Rightarrow t_1 = 1$

$t_2+t_3=2$

& $t_2=2$ e $t_3=0$ $\quad \binom{3}{1,2,0} = \frac{3}{1!2!} = 3$

& $t_2=1$ e $t_3=1$ $\quad \binom{3}{1,1,1} = 6$

& $t_2=0$ e $t_3=2$ $\quad \binom{3}{1,0,2} = 3$

$\alpha_1 = 12$

$\alpha_2 \Rightarrow a^{t_1} = a^2 \Rightarrow t_1 = 2$

$t_2+t_3=1$

$t_2=0$ $t_3=1$ $\quad \binom{3}{2,0,1} = 3$

$t_2=1$ $t_3=0$ $\quad \binom{3}{2,1,0} = 3$

$\alpha_2 = 6$

$\alpha_3 \Rightarrow a^{t_1} = a^3 \Rightarrow t_1 = 3$

$t_2+t_3=0$

$t_2=0$ $t_3=0$ $\quad \binom{3}{0,0,3} = 1$

2 $\binom{n}{0}\binom{n}{k} + \binom{n}{1}\binom{n-1}{k-1} + \dots + \binom{n}{k}\binom{n-k}{0} = \binom{n}{k}2^k$

$\binom{n}{0}\binom{n}{k} + \binom{n}{1}\binom{n-1}{k-1} + \dots + \binom{n}{k}\binom{n-k}{0} =$

$\sum_{i=0}^{k}\binom{n}{i}\cdot\binom{n-i}{k-i} = \sum_{i=0}^{k} \frac{n!}{i!(n-i)!}\cdot\frac{(n-i)!}{(n-i-(k-i))!\,(k-i)!}$

$= \sum_{i=0}^{k} \frac{n!}{i!(n-k)!(k-i)!} = \sum_{i=0}^{k}\frac{n!\,k!}{(n-k)!\,k!\,i!(k-i)!} = \frac{n}{(n-k)!\,k!}\sum_{i=0}^{k}\frac{k}{i!(k-i)!}$

$= \binom{n}{k}\cdot\sum_{i=0}^{k}\binom{k}{i} = \binom{n}{k}2^k$

$a_n - 6a_{n-1} + 9a_{n-2} = 8$

$a_n = a_n^{(1)} + a_n^{(2)}$

$a_n^{(1)} \Rightarrow a_n = a_n^{(1)} \Leftrightarrow a_n - 6a_{n-1} + 9a_{n-2} = 0$

Polinómio característico: $x^n - 6x^{n-1} + 9x^{n-2} = 0$

$x^{n-2}(x^2 - 6x + 9) = 0$

$(x-3)^2 = 0$

$x = 3$

3 é raiz dupla do polinómio característico

logo

$a_n^{(1)} = (C_0 + C_1 n)\,3^n$

$a_n^{(2)} = 8 \Rightarrow$ polinómio

1 não é raiz característica, logo

$a_n^{(2)} = A_0$

Para saber A_0, substitui-se a_n por $a_n^{(2)}$ na relação de recorrência

$A_0 - 6A_0 + 9A_0 = 8$

$4A_0 = 8$

$A_0 = 2$

$a_n = (C_0 + C_1 n)\,3^n + 2$

$$\begin{cases} a_0 = (c_0 + c_1\cdot 0)\cdot 3^0 + 2 \\ a_1 = (c_0 + c_1\cdot 1)\cdot 3^1 + 2 \end{cases} = \begin{cases} 4 = c_0 + 2 \\ 11 = 3(c_0+c_1)+2 \end{cases} \Leftrightarrow \begin{cases} c_0 = 2 \\ 2 + c_1 = 3 \end{cases}$$

$$\Leftrightarrow \begin{cases} — \\ c_1 = 1 \end{cases}$$

$$a_n = (2+1n)3^n + 2$$

3

$a_n = 2n\, a_{n-1}$

$$f(x) = \sum_{k=0}^{\infty} a_k \frac{x^k}{k!} = a_0 \frac{x^0}{0!} + \sum_{k=1}^{\infty} a_k \frac{x^k}{k!} = 1 + \sum_{k=1}^{\infty} (2k a_{k-1}) \frac{x^k}{k!}$$

$$= 1 + 2x \sum_{k=1}^{\infty} a_{k-1} \frac{x^{k-1}}{(k-1)!} = 1 + 2x \sum_{k=0}^{\infty} \frac{a_k x^k}{k!} = 1 + 2x f(x)$$

$$f(x) = 1 + 2x f(x)$$
$$f(x) = \frac{1}{1-2x}$$

$$f(x) = \frac{1}{1-2x} = \sum_{n=0}^{\infty} (2x)^n = \sum_{n=0}^{\infty} 2^n x^n$$

Uma vez que $\displaystyle\sum_{k=0}^{\infty} a_k \frac{x^k}{k!} = f(x) = \sum_{k=0}^{\infty} 2^k x^k$

$$\sum_{k=0}^{\infty} a_k \frac{x^k}{k!} = \sum_{k=0}^{\infty} 2^k x^k$$

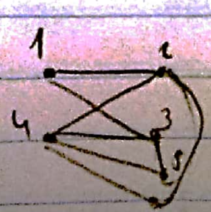
$$\frac{a_k}{k!} = 2^k \Leftrightarrow a_k = 2^k k!$$

4

$$\binom{-3/2}{3} = \frac{-\frac{3}{2}\cdot\left(-\frac{3}{2}-1\right)\cdot\left(-\frac{3}{2}-2\right)}{3!} = \frac{-\frac{3}{2}\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{3!} = \frac{-3\times5\times7}{6\times2\times2\times2} = -\frac{35}{16}$$

5

a)



Não pois tem um ciclo de comprimento ímpar: {43, 35, 54}

it	1	2	3	4	5	6	z	Temp
0	(0, —)	$(\infty, —)$	$(\infty, —)$	$(\infty, —)$	$(\infty, —)$	$(\infty, —)$	1	{2,3,4,5,6}
1		(4, 1)	(2, 1)	$(\infty, -)$	$(\infty, -)$	$(\infty, -)$	3	{2,4,5,6}
2		(4, 1)		(8, 3)	(4, 3)	(17, 3)	5	{2,4,6}
3		(4, 1)		(6, 5)		(17, 3)	2	{4,6}
4				(6, 5)		(17, 3)	4	{6}
5						(10, 4)	6	{}

Caminho mais curto:

1 - 3 - 5 - 4 - 6