# **Robot Programming**

## Localizing on a Distance Map

G. Grisetti

{grisetti}@diag.uniroma1.it

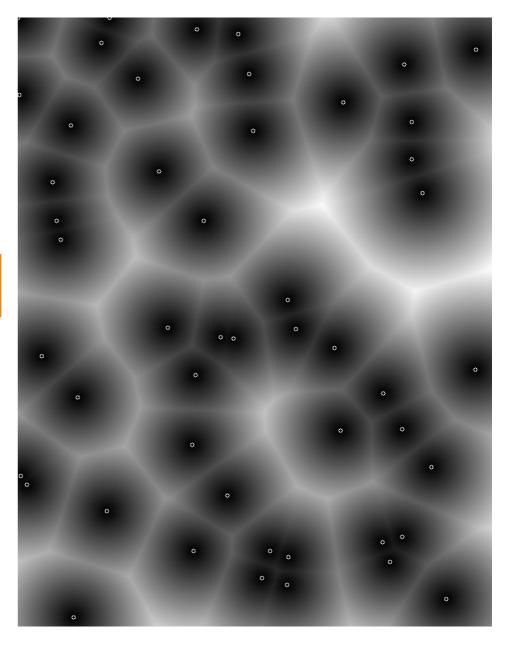
Department of Computer, Control, and Management Engineering Sapienza University of Rome

## **DMap for Localization**

Let our reference be the set of points shown to the right

Formulate localization as a minimization problem

$$\mathbf{X}^* = \operatornamewithlimits{argmin}_{\mathbf{X}} \sum_{j} \lVert d(\mathbf{X}\mathbf{z}_j) 
Vert_{\Omega}^2$$
 Robot pose



## **Dmap for Localiation**

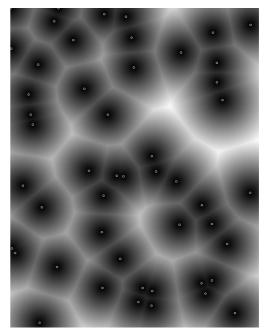
Let our reference be the set of points shown to the right

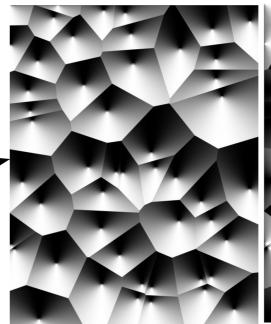
 Embed the data association in the cost function

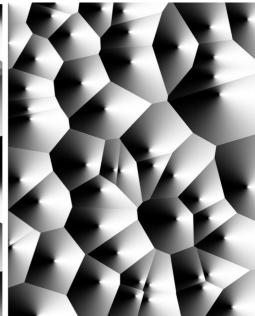
$$\mathbf{X}^* = \underset{\mathbf{X}}{\operatorname{argmin}} \sum_{j} ||d(\mathbf{X}\mathbf{z}_j)||^2$$

 The distance function is differentiable so is the cost

$$\frac{\partial d(\mathbf{X}\mathbf{z}_j)}{\partial \mathbf{X}} = \frac{\partial d(\mathbf{y})}{\partial \mathbf{y}} \Big|_{\mathbf{y} = \mathbf{X}\mathbf{z}_j} \frac{\partial \mathbf{X}\mathbf{z}_j}{\partial \mathbf{X}}$$







#### **Gauss-Newton**

Iteratively solve this problem

$$\mathbf{X}^* = \underset{\mathbf{x}}{\operatorname{argmin}} ||\mathbf{e}_i(\mathbf{X})||^2$$

$$\mathbf{e} : Dom(\mathbf{X}) \to \Re^m$$

$$||\mathbf{v}|| := \mathbf{v}^T \mathbf{v}$$

Requires e(..) to be differentiable w.r.t. and euclidean perturbation around X

### Gauss-Newton(on manifold)

Clear **H** and **b** 

$$\mathbf{H} \leftarrow 0 \qquad \mathbf{b} \leftarrow 0$$

For each measurement, update h and b

$$egin{array}{lll} \mathbf{e}_i & \leftarrow & \mathbf{d}_i(\mathbf{X}^*\mathbf{z}_i) \ \mathbf{E}_i & \leftarrow & \left. rac{\partial \mathbf{d}_i(\mathbf{T}(\mathbf{\Delta}\mathbf{x})\mathbf{X}^*\mathbf{z}_i)}{\partial \mathbf{\Delta}\mathbf{x}} \right|_{\mathbf{\Delta}\mathbf{x}=\mathbf{0}} \ \mathbf{H} & \leftarrow & \mathbf{H} + \mathbf{E}_i^T\mathbf{E}_i \ \mathbf{b} & \leftarrow & \mathbf{b} + \mathbf{E}_i^T\mathbf{e}_i \end{array}$$

Update the estimate with the perturbation

$$\mathbf{\Delta x} \leftarrow \operatorname{solve}(\mathbf{H}\mathbf{\Delta x} = -\mathbf{b})$$
 $\mathbf{X}^* \leftarrow \mathbf{T}(\mathbf{\Delta x})\mathbf{X}^*$ 

### Gauss-Newton(on manifold)

Clear **H** and **b** 

$$\mathbf{H} \leftarrow 0 \qquad \mathbf{b} \leftarrow 0$$

For each measurement, update h and b

$$\mathbf{p}_{i} \leftarrow \mathbf{X}^{*}\mathbf{z}_{i}$$
 $\mathbf{e}_{i} \leftarrow \mathbf{d}(\mathbf{p}_{i})$ 
 $\mathbf{E}_{i} \leftarrow \operatorname{grad}^{T}(\mathbf{p}_{i}) \begin{pmatrix} 1 & 0 & -\mathbf{p}_{i}.y \\ 0 & 1 & \mathbf{p}_{i}.x \end{pmatrix}$ 
 $\mathbf{H} \leftarrow \mathbf{H} + \mathbf{E}_{i}^{T}\mathbf{\Omega}_{i}\mathbf{E}_{i}$ 
 $\mathbf{b} \leftarrow \mathbf{b} + \mathbf{E}_{i}^{T}\mathbf{\Omega}_{i}\mathbf{e}_{i}$ 

Update the estimate with the perturbation

$$oldsymbol{\Delta} \mathbf{x} \leftarrow \operatorname{solve}(\mathbf{H} oldsymbol{\Delta} \mathbf{x} = -\mathbf{b})$$
 $\mathbf{X}^* \leftarrow \mathbf{T}(oldsymbol{\Delta} \mathbf{x}) \mathbf{X}^*$ 

### **Implementation**

#### Ingredients

- Functions to display
- Functions to map from world to grid and viceversa
- Calculation of Gradients on dmap
- Facilities for Linear Algebra (Eigen helps)

#### **Testing**

- Spawn some points on a grid
- Compute the dmap and the gradients
- Express these points in world coordinates
- Express these points in sensor coordinates (to simulate a measurement)
- Implement GN

### **Hands On**

# **Full Fledged Implementation**

In the repo, you find a working Dmap Localization

- The algorithm has a state X (the current estimate of the robot position)
- On startup the algorithm subscribes to
  - \map topic to get the occupancy grid on which to update distance and gradients
  - the \initial\_pose topic to allow manually setting the initial guess
  - the \odom msg that expresses the position of the robot w.r.t. the odom frame (dead reckoning)
  - The \scan msg containing a laser scan

# **Full Fledged Implementation**

•Whenever it receives an odomery reading **U**\in **SE2** (relative position measured from the encoders), it updates its pose

```
U= odom_before.inverse()*odom_now
X=X*U
```

 Whenever it receives a scan, it computes the endpoints in the robot frame

```
x_i=z_i*cos(angle_i), y_i=z_i*sin(angle_i)
```

and updates the estimate based using GN registration (seen before)

# **Full Fledged Implementation**

The localizer publishes the pose of the robot w.r.t the map through a tf message, but not directly, as this would generate more than one root in the transform tree

- Map→base\_link = T\_localizer
- Odom→base\_link = T\_odom

hence it generates a transform Map→Odom s.t.

```
T_localizer=T_exported*T_odom
T_exported=T_localizer*T_odom.inverse()
```