CIT-237

More about Binary Trees

December 11, 2019

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Reminder / Announcement

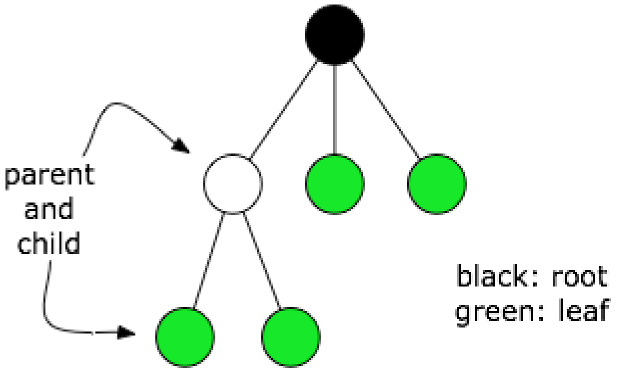
- The EXTENDED Due date for Project 3 was Monday, December 9.
- If you are still having difficulty completing Project 3, ask for help during the Lab portion of today's class.
- Our last day of class is Monday, December 16.
  - Obviously, that is the last day to demonstrate the solutions to Lab Exercises.
  - We will NOT have a Final Exam.

Tree Data Structures

- In a previous class, we introduced the concept of a Tree data structure.

tree (data structure): A data structure accessed beginning at the *root* node. Each *node* is either a *leaf* or an *internal node*. An internal node has one or more *child* nodes and is called the *parent* of its child nodes. All children of the same node are *siblings*. Contrary to a physical tree, the root is usually depicted at the top of the structure, and the leaves are depicted at the bottom.

(Source: https://xlinux.nist.gov/dads/HTML/tree.html)



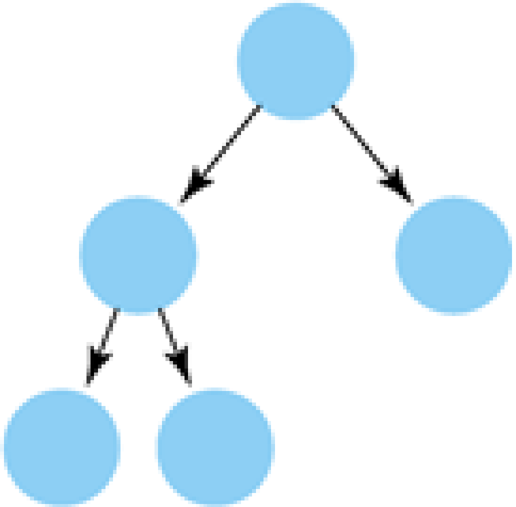
Binary Tree

- If each node in a tree is assumed to have no more that two child nodes, it is called a *binary tree*:

binary tree (data structure): A *tree* with at most two *children* for each *node*.

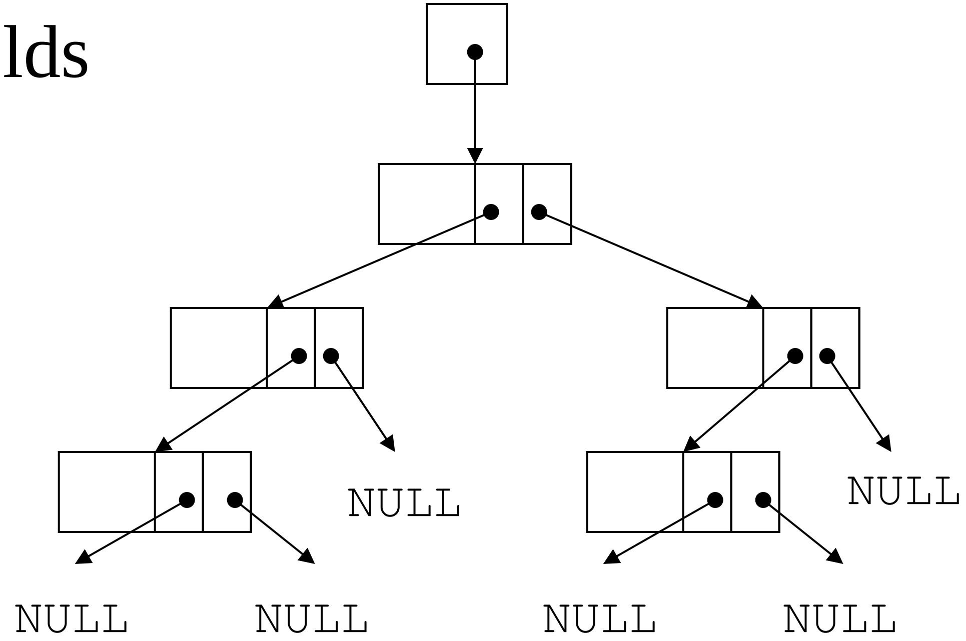
Formal Definition: A binary tree either is empty (no nodes), or has a *root* node, a left binary tree, and a right binary tree.

(Source: https://xlinux.nist.gov/dads/HTML/binarytree.html)



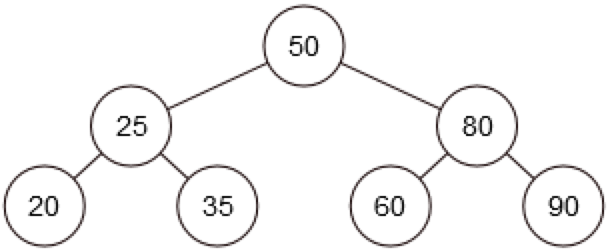
Linked-Node Binary Trees

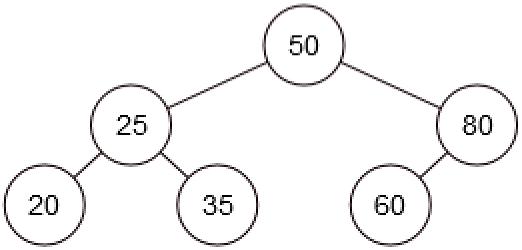
- Binary tree: a nonlinear linked structure in which each node may link to 0, 1, or 2 other nodes
- Each node contains:
  - one or more data fields
  - two pointers



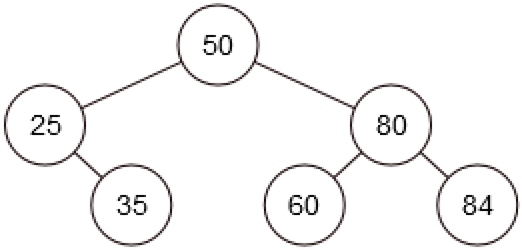
Complete Binary Tree

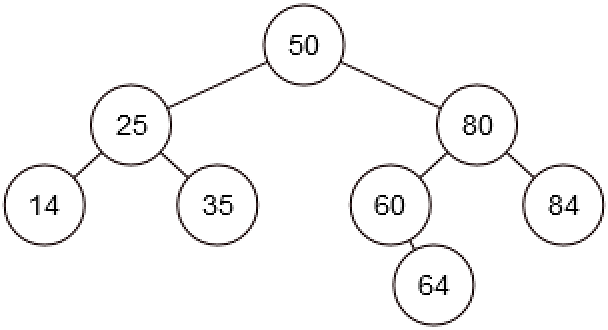
- A binary tree is *complete* if every level of the tree is full, or
- If the last level is not full, all the nodes on the last level are placed left-most.
- For example, these two example trees are *complete*:





- But these two are <u>not</u> *complete*:





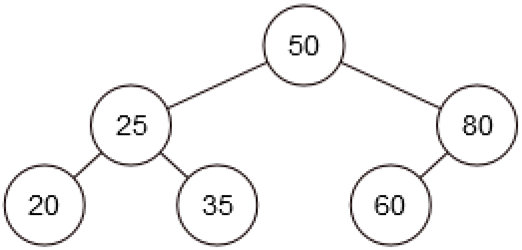
Complete Binary Tree Representation (1)

- Previously, we looked at binary trees represented by linked objects.
- An alternate representation of a <u>complete</u> binary tree, using an array, is sometimes preferable:
  - It saves space.
  - It provides an easier way for a program to find the <u>parent</u> node of any particular node.

Complete Binary Tree Representation (2)

- While our conceptual picture of any binary tree is as a tree, we can choose to represent a *complete* binary tree using an array.

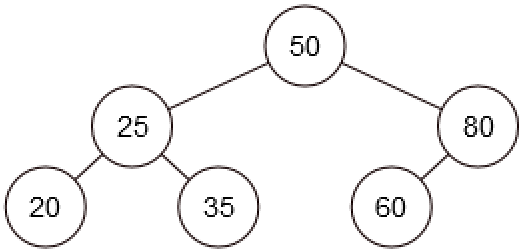
For example, the tree shown below:

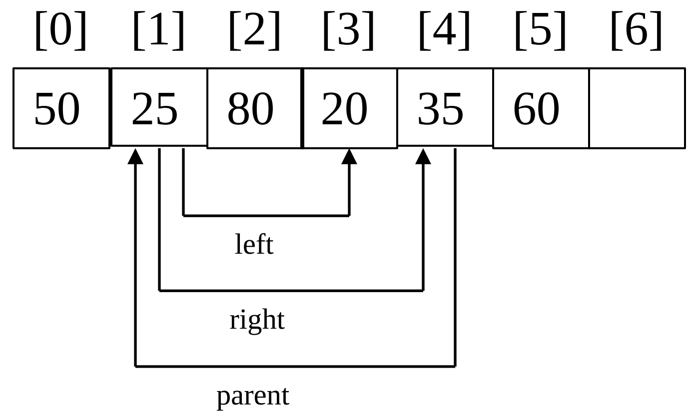


can be represented by the following array:

[0]	[1]	[2]	[3]	[4]	[5]	[6]
50	25	80	20	35	60	

Complete Binary Tree Representation (3)

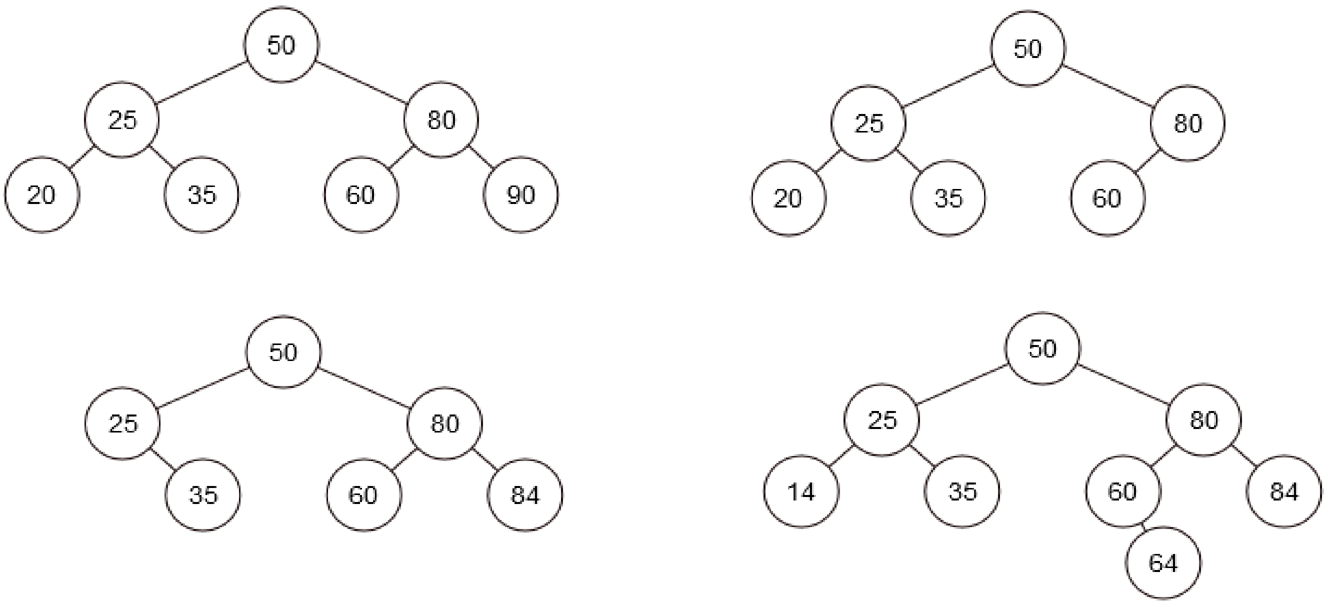




- For a node at position *i*:
  - its left child is at position *2i+1*
  - its right child is at position *2i+2*
  - its parent is at index (*i-1)/2* (Integer division – remainder is discarded.)
- For example: The node for element 25 is at position 1.
  - its left child (element 20) is at position 3 (*2*1+1*)
  - its right child (element 35) is at position 4 (*2*1+2*)
  - Element 25 is the parent of element 35: position of element 35 is 4. The parent of the element at position 4 is at position (4-1)/2 = 1.

Binary Search Trees

- We have seen a common use of a binary tree: a *binary search tree*.
  - Any node in the *left* subtree of a node contains data that is "less than" the data in the current node.
  - Any node in the *right* subtree of a node contains data that is "greater than" the data in the current node.
- All of the trees shown below are examples of binary search trees:



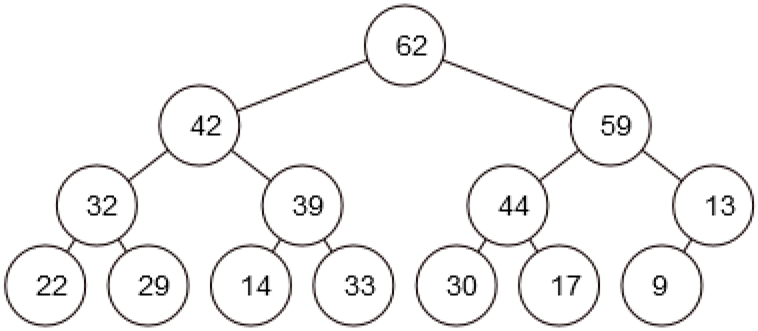
- But *not all* binary trees are binary search trees.

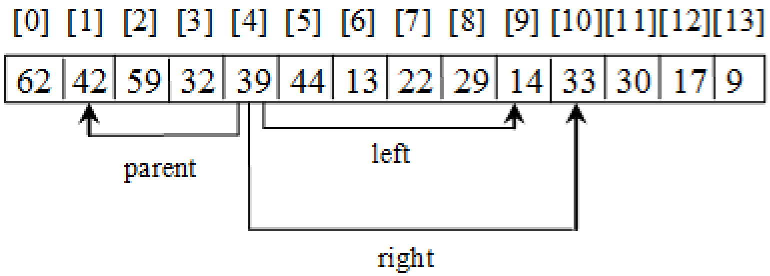
Binary Heap

- The term ***heap*** has multiple meanings:
  - In everyday life, a heap might be a disorganized jumble of items, such as a "heap" of clean, unsorted laundry.
  - We refer to the region of memory where dynamically allocated objects are created as a "heap".
  - In Data Structures, a heap is a special kind of binary tree.
- A *binary heap* is a binary tree with the following properties:
  - It is a complete binary tree.
  - If it is a "max-heap", then the data in each node is greater than or equal to the data in any of its children. If it is a "min-heap", then the data in each node is less than or equal to the data in any of its children. (The examples in this lecture are all "max-heap" examples.)

Representing a Heap

- For a node at position *i*:
  - its left child is at position *2i+1*
  - its right child is at position *2i+2*
  - its parent is at index (*i-1)/2*.
- In the example below: the node for element 39 is at position 4.
  - its left child (element 14) is at position 9 (*2*4+1*)
  - its right child (element 33) is at position 10 (*2*4+2*)
  - its parent (element 42) is at position 1 (*(4-1)/2*).
- This particular example is a ***max-heap***: the data in any particular node is greater than or equal to the data in each of the child nodes.





Modifying a Heap

- Whenever a node is added to a binary heap, or removed from a binary heap, the structure may need to be adjusted to restore the *max-heap* or *min-heap* rules:
  - Scan the heap to find any nodes that break the *max-heap* (or *min-heap*) rule.
  - Swap the two offending values so that they obey the rule.
  - Continue scanning and swapping until the entire tree obeys the heap rules.

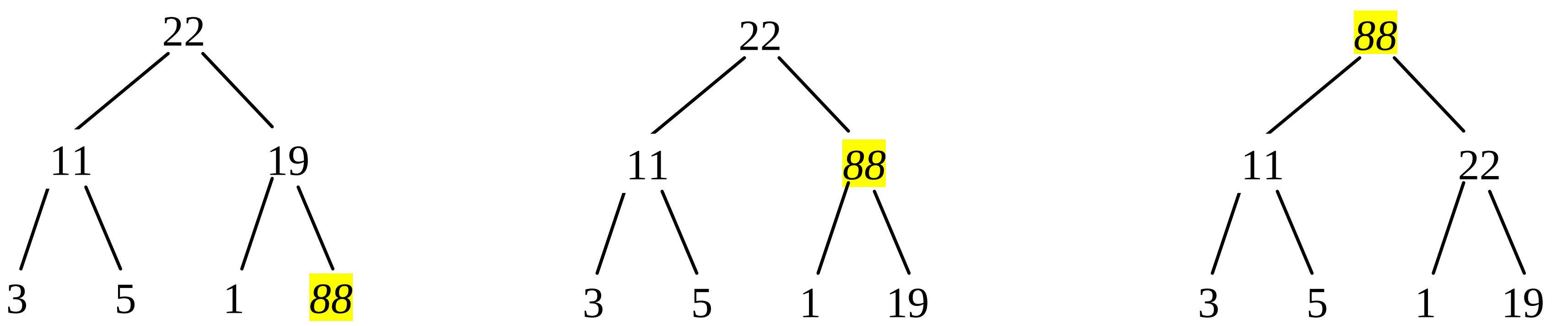
- Adding a node:
  - The new node is added at the bottom of the tree, so that the tree is still a *complete* binary tree.
  - The tree is scanned / adjusted starting at the bottom.

- Removing a node:
  - The node removed is always the **root** node.
  - The right-most node of the bottom row is temporarily moved to the root position.
  - The tree is scanned / adjusted starting at the root.

Details of the Scan / Adjust Process

Each time a node is added, the scan / adjust process moves the new value to the correct position.

In this example, the value 88 is added to an existing max-heap.

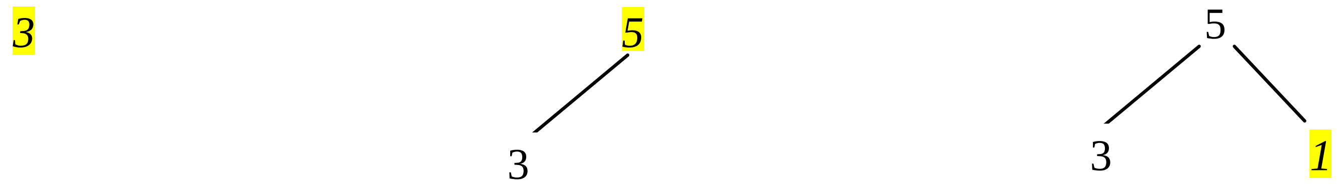


(a) Add 88 to a heap (b) After swapping 88 with 19 (b) After swapping 88 with 22

Adding elements to a Heap

This example begins with an empty *max-heap* and then the values 3, 5, 1, 19, 11, and 22 are added.

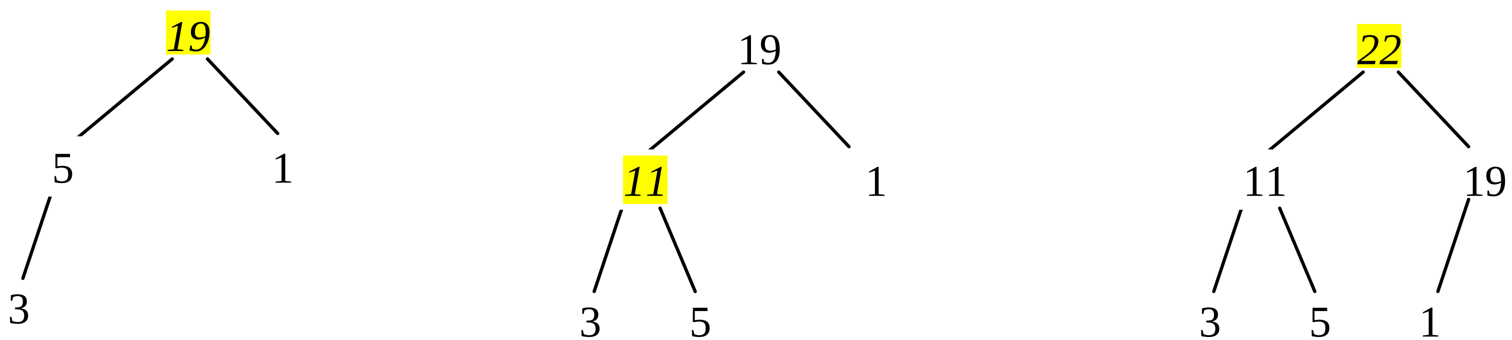
The scan / adjust process must be performed when inserting each new value.



(a) After adding 3

(b) After adding 5

(c) After adding 1



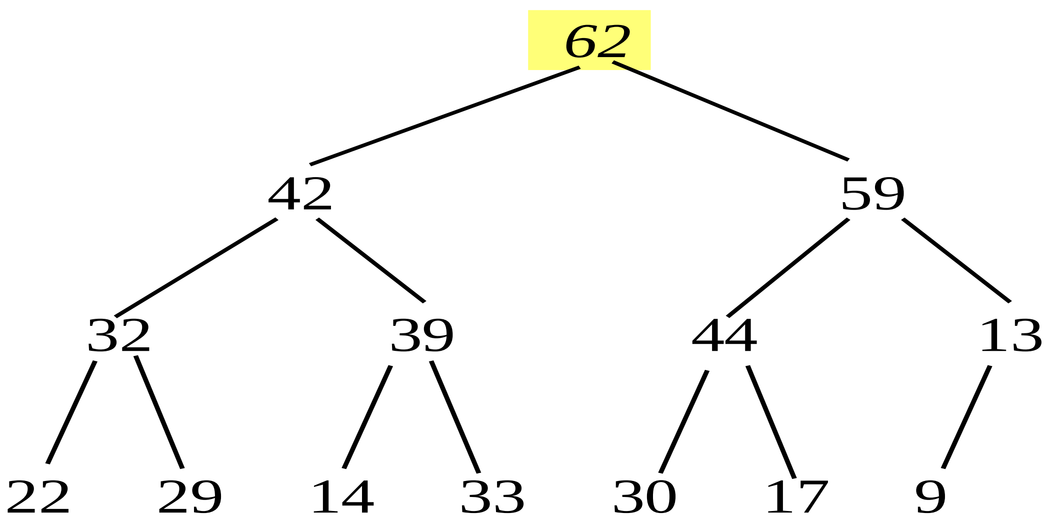
(d) After adding 19

(e) After adding 11

(f) After adding 22

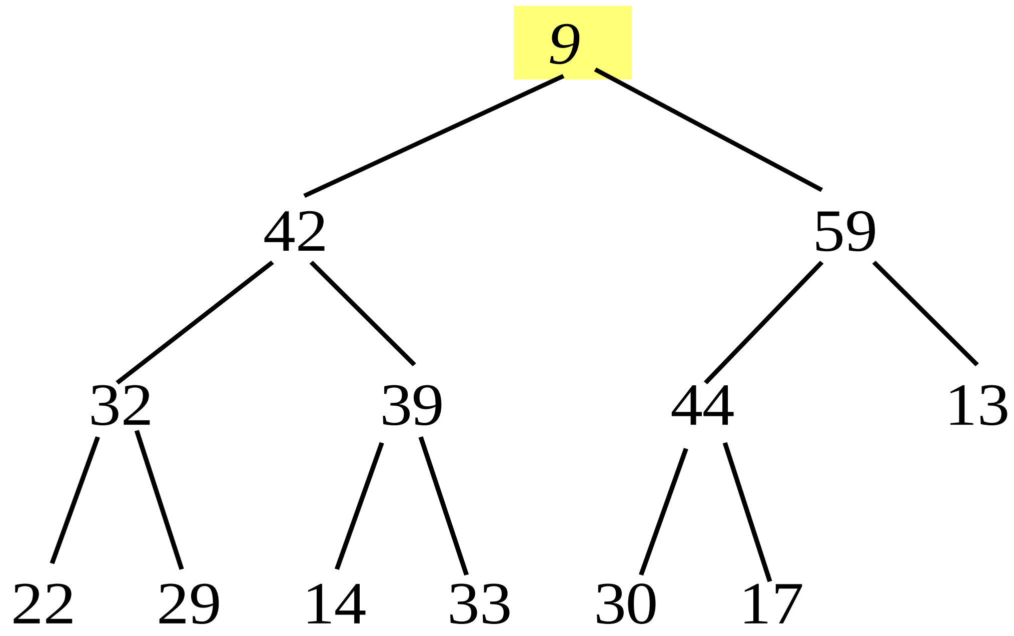
Removing the root node (1)

- If we remove the root node (node with hightest value), we have to put *something* in its place, so we initially choose the last node, and then reposition that node to its correct position in the tree.
- In this example, we choose to remove the **62** value:



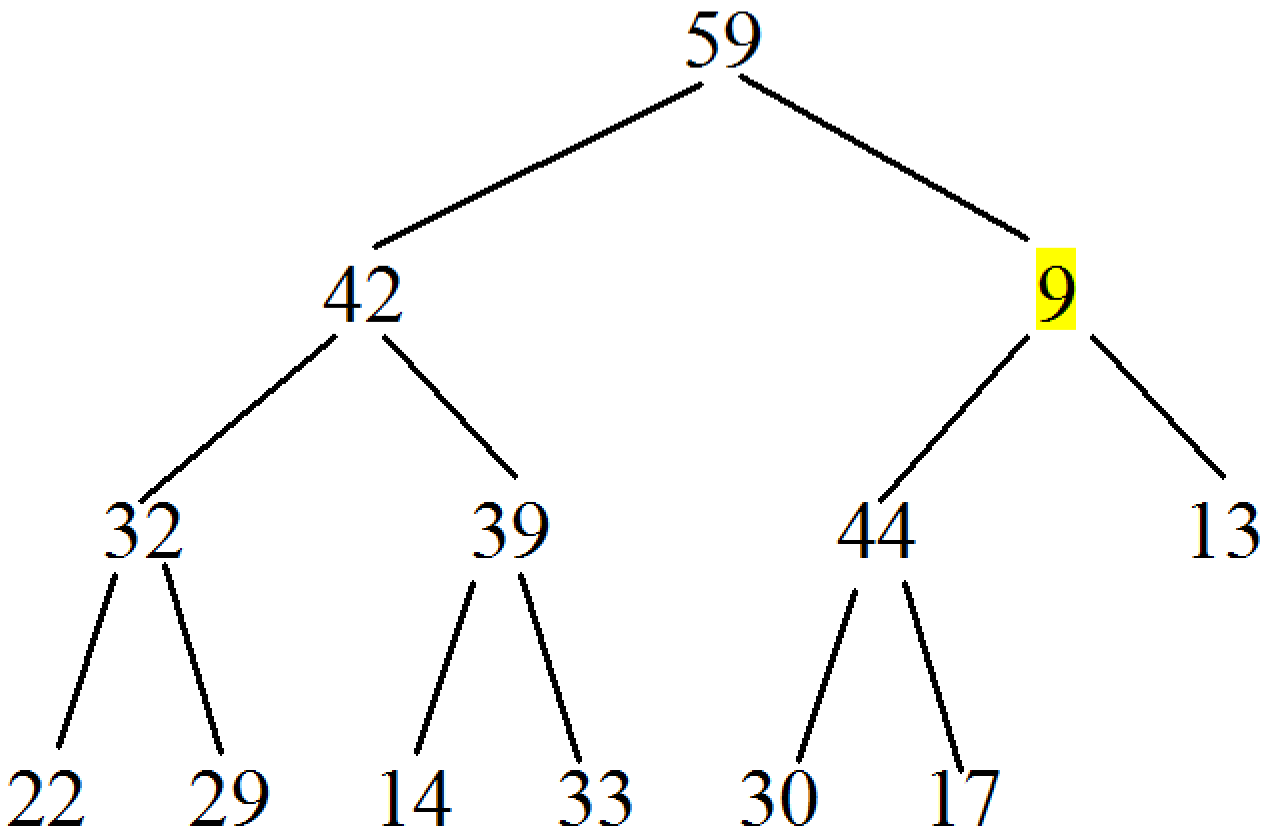
Removing the root node (2)

- So we fill in the root position with the last node of the last row: in this case, the **9** value.



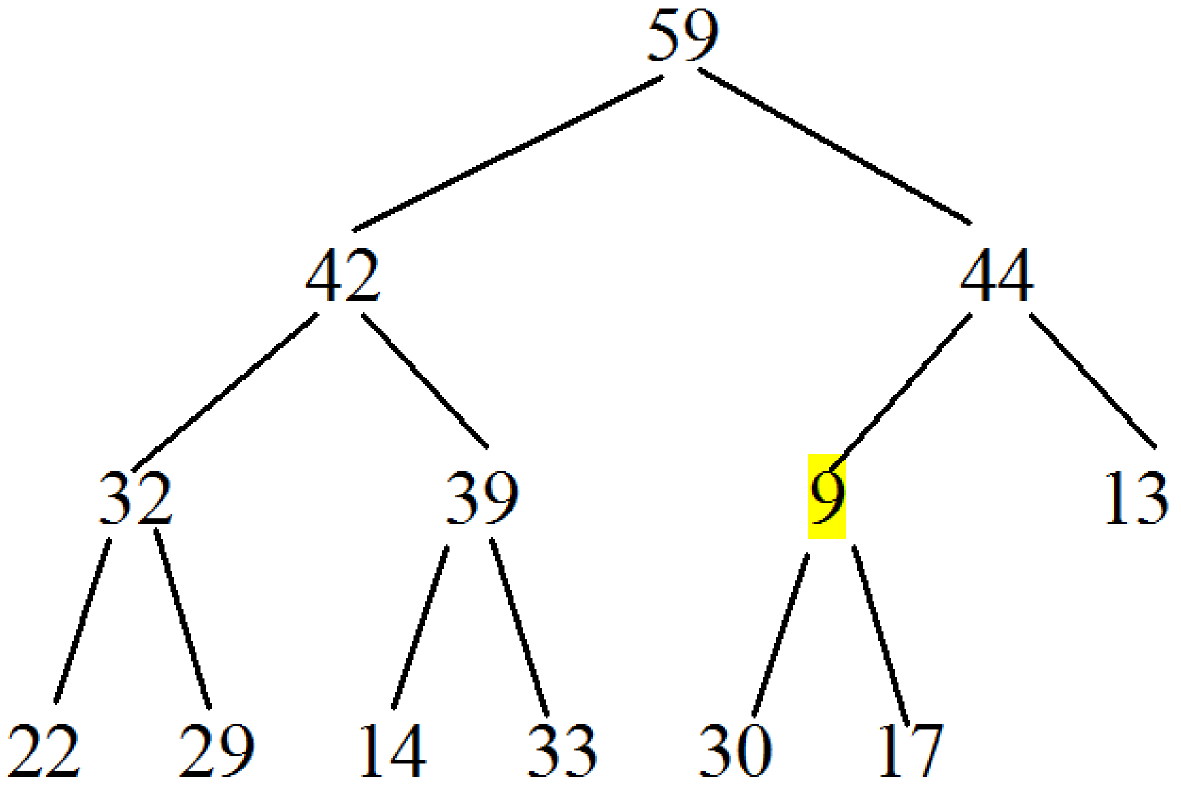
Removing the root node (3)

- Next we need to find the correct position for the **9** value:
  - swap it with the larger of its two children (in this case, **59**).



Removing the root node (4)

- The **9** value is still not in the correct position:
  - swap it with the larger of its two children (in this case, **44**).



Removing the root node (5)

- The **9** value is still not in the correct position:
  - swap it with the larger of its two children (in this case, **30**).
- Finally, the tree is again obeying the rules for a heap.