# TicTacToe - Force Draw

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In the game of tic tac toe both players have a safe strategy to force a draw. This simple – but sometimes overlooked – fact can be proved by the following diagrams.

Notation:

- Player X versus player 0, X begins.
- O means: if 0 makes another choice, X can win by the next move. Analogousely for X. A red arrow has the same meaning: if the player makes another choice, the opponent can win by the next move.
- A green arrow means: the player can choose among several moves but they should use this move to achieve a draw – or in some cases even a win.

There are 3 choices to begin the game:
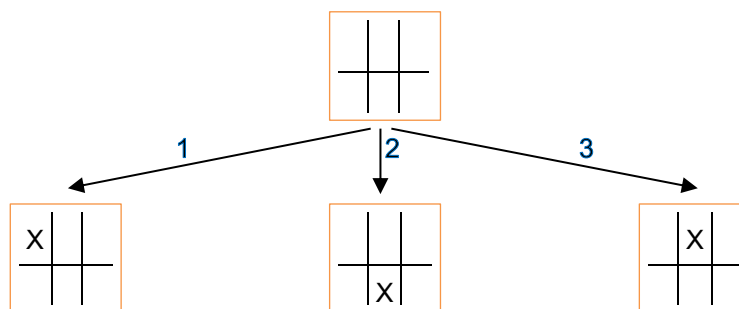
All other choices are equivalent to one of these.

Opening move 1: Player 0 has the choices 1..5, all others are equivalent.
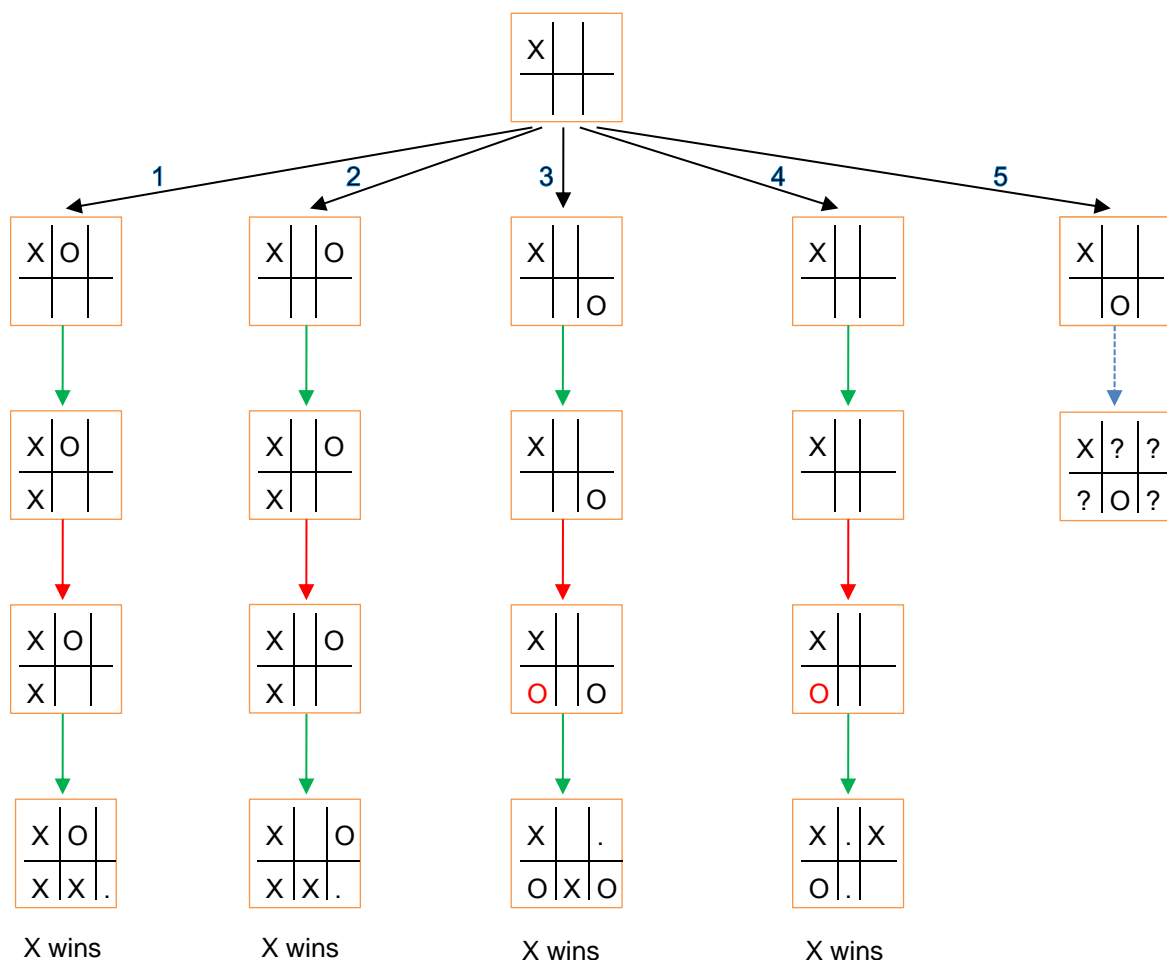
If player 0 makes one of the choices 1..4, player X has a safe strategy to win: they must just follow the green arrows, player 0 then has to follow the red arrows and will loose.

The question is, what happens, when player 0 chooses 5? The following diagram shows, that in this case player 0 can force a draw.

1-5

1   2   3   4
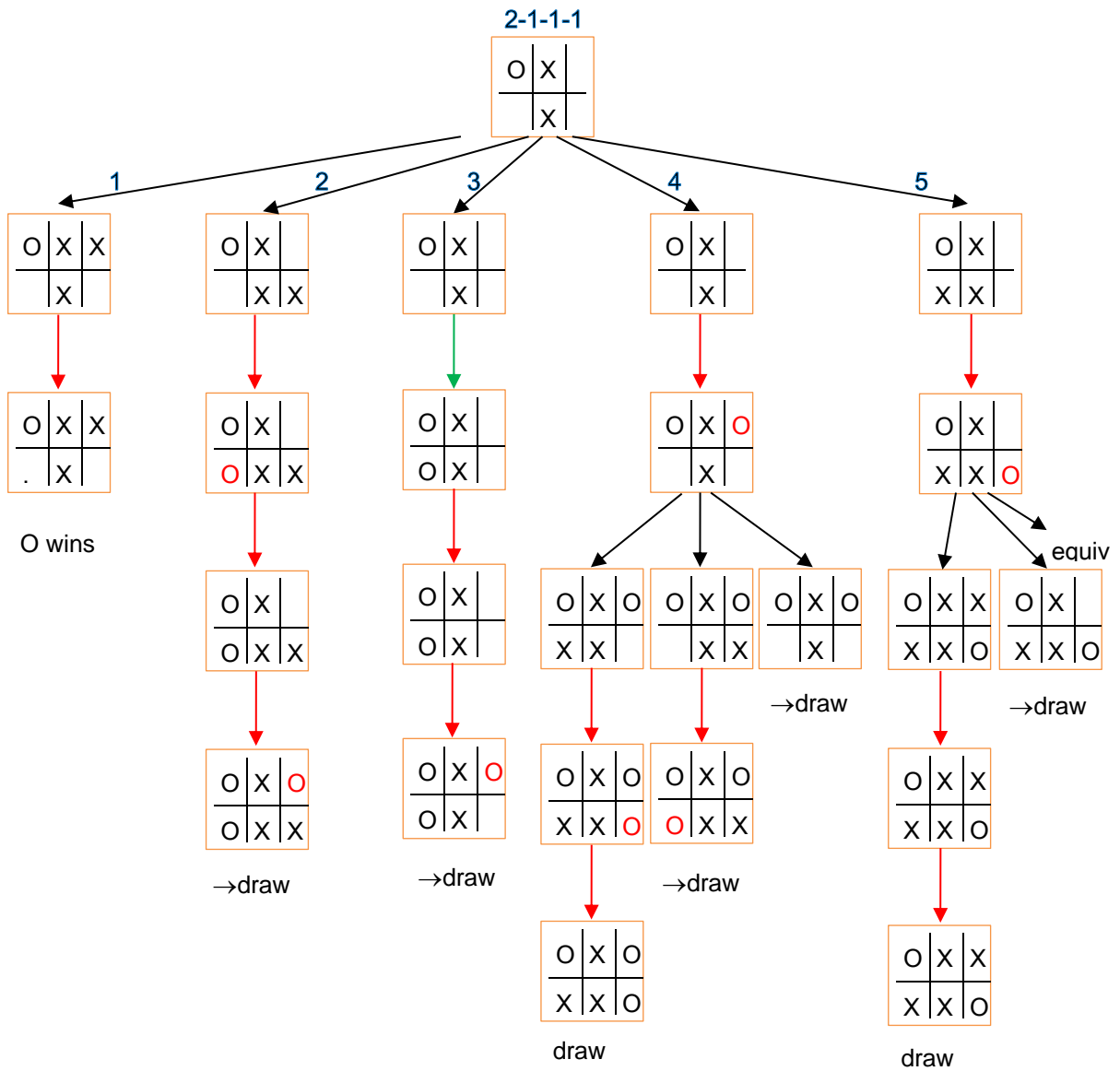
→draw

→draw

→draw

→draw

draw   draw
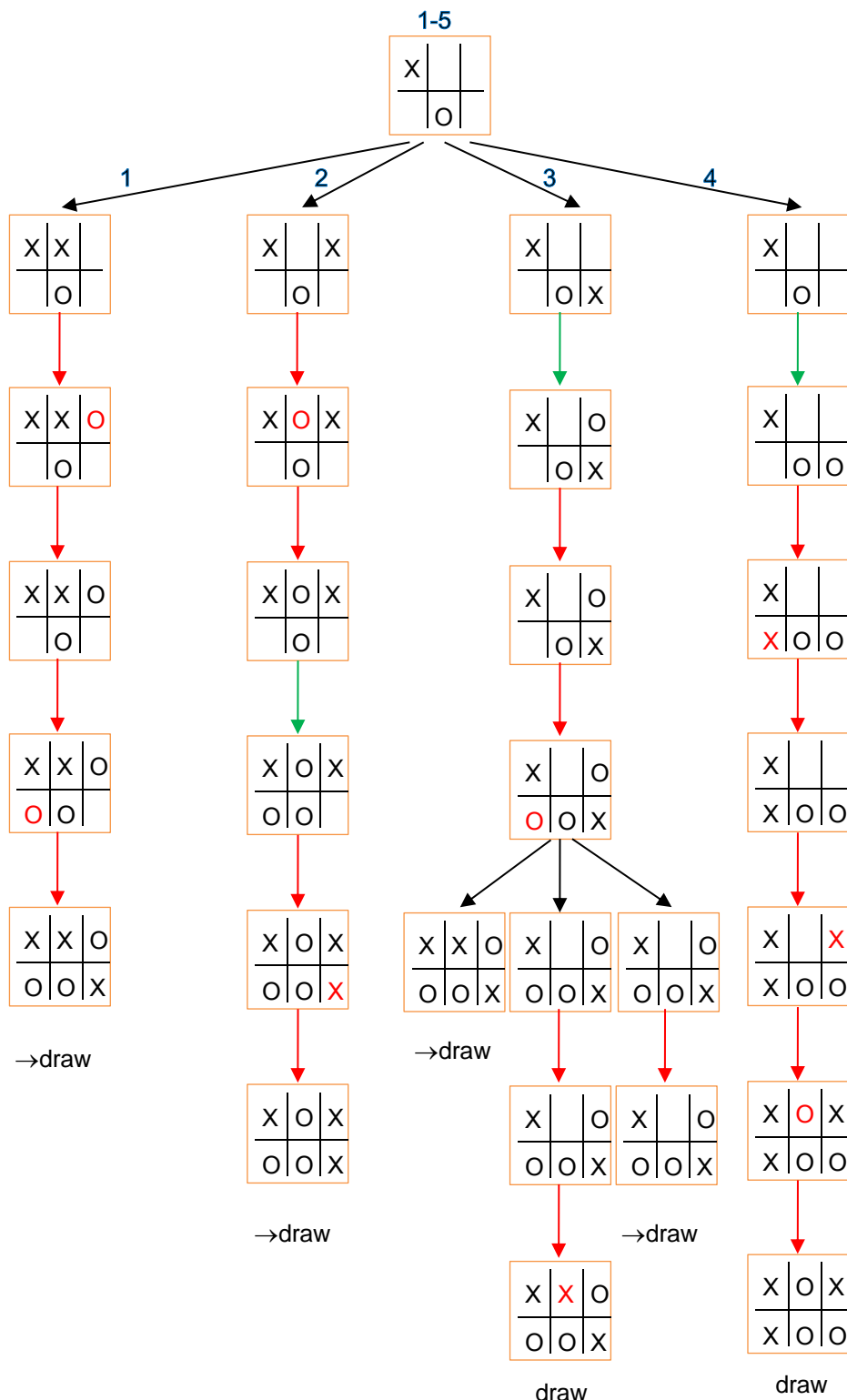
Player X can choose among 4 moves, all others being equivalent. If X chooses 1, a draw results inevitably. For the cases 2,3,4 player 0 has a choice, indicated by a green arrow, that will lead to a draw.

Thus, for the opening move 1 player 0 can force a draw. But also player X can force a draw, namely following the moves 1-5-1.

Now, let's investigate the 2nd opening move:
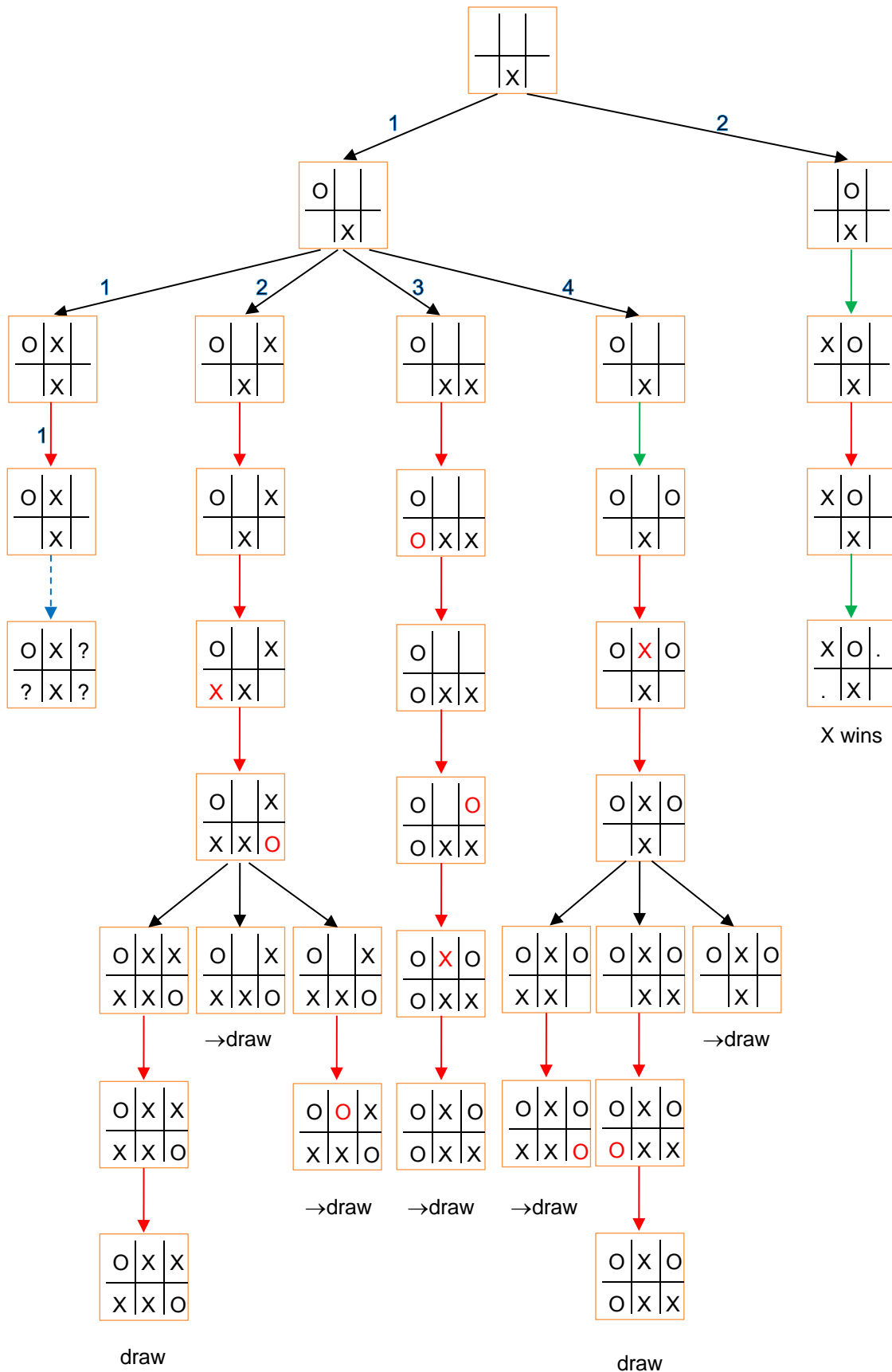
Player 0 has the choices 1 and 2, all others being equivalent. When choosing 2, player X has a safe strategy to win, following the green arrows. When choosing 1, player X has 4 choices to respond (all others being equivalent). Choices 2 an 3 lead to a draw, choice 4 lets player 0 force a draw by following the green arrow. The question is, what happens if player X chooses 1? Then player 0 has only 1 choice, and after that player X has 5 choices. This is investigated separately in the following diagram.

2-1-1-1

O wins

→draw   →draw

→draw   →draw

draw   →draw   equiv

→draw

draw

If player X chooses 1, player 0 will win. If player X chooses 2,4 or 5, a draw will result. If player X chooses 3, then player 0 can force a draw by following the green arrow. Thus, also in case 2-1-1-1 player 0 can force a draw.

Summarizing: also for opening move 2 player 0 can force a draw.

Opening move 3 remains. Player 0 has 5 choices to respond, up to equivalences. It suffices, to show, that choice 3 allows player 0 to force a draw.

3-3

1   2   3   4

→draw   →draw   →draw   →draw
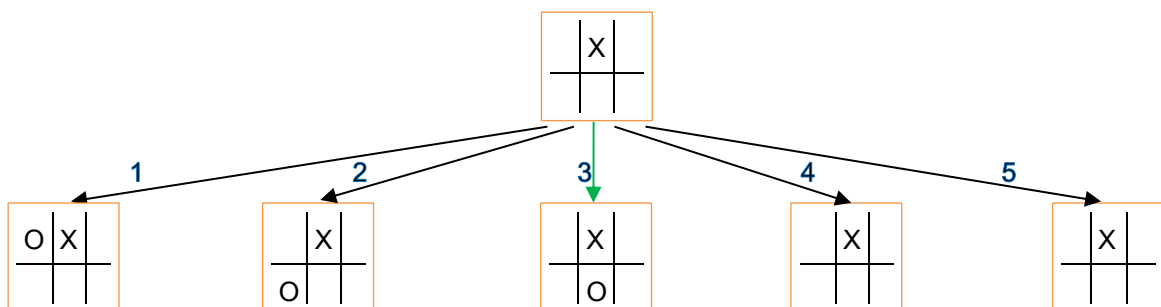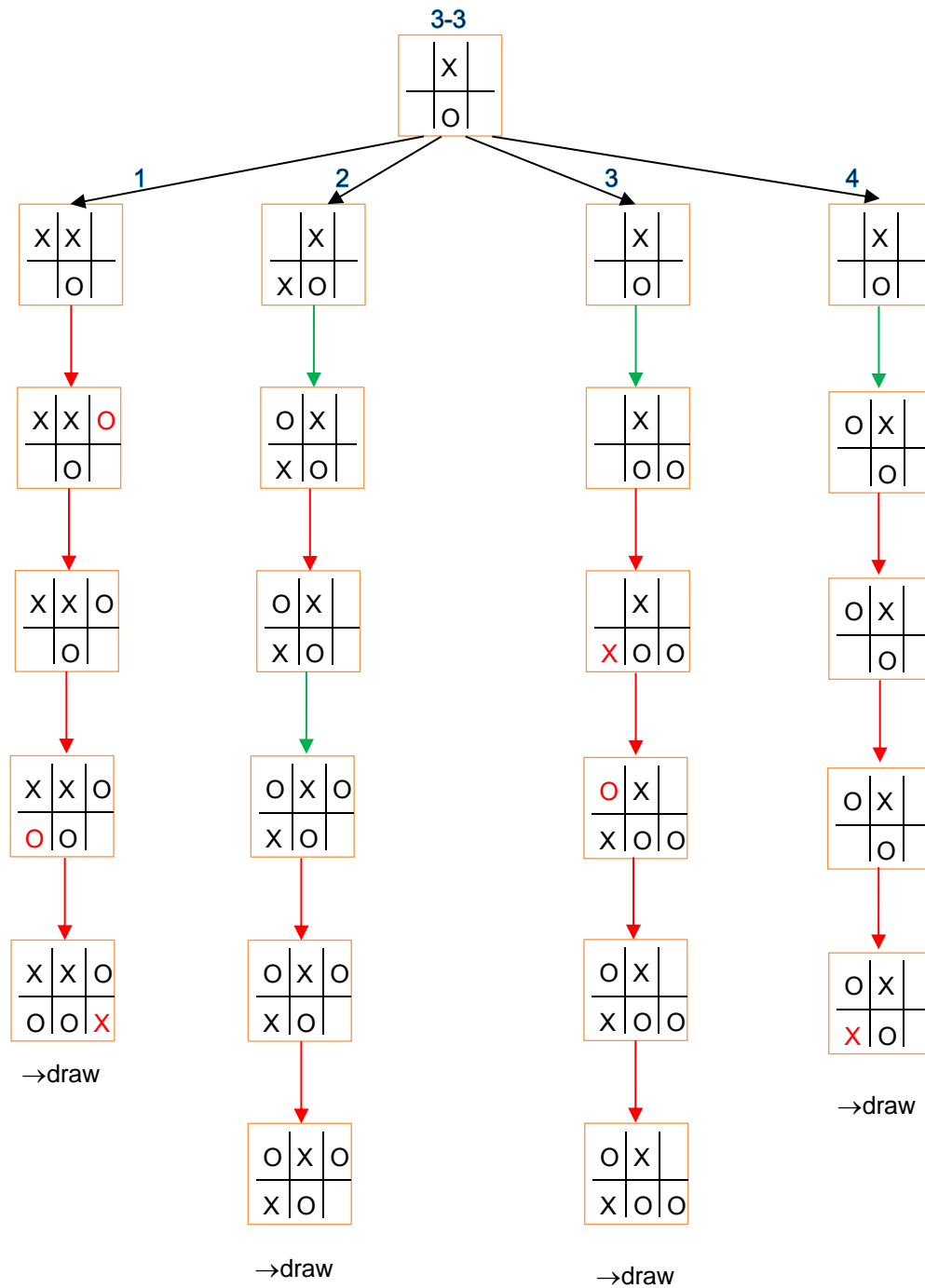
Player X has 4 choices, choice 1 will lead to a draw, choices 2 to 4 let player 0 follow the green arrows to force a draw.

Summarizing: for all 3 opening moves of player X player 0 can force a draw. And player X can force a draw by beginning with opening move 1.

Thus, both players have a safe strategy to force a draw.

q.e.d.