PWM-Based Digital-to-Analog Conversion for Microcontrollers

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Abstract

Nowadays microcontrollers are extensively used in the development of electronic instrumentation systems. Most of these microcontrollers incorporate Pulse-Width-Modulation (PWM) modules, which can be used for implementing inexpensive Digital-to-Analog (D/A) converters, requiring only a few external components (typically a resistor, a capacitor and eventually a low-cost opamp). Despite having performance limitations, this kind of D/A converters are suitable for many applications.

This document presents the operating principles and methodologies for designing PWM-based DACs.

1 Introduction

Embedded systems, such as the ones used on electronic instrumentation, often require the generation of analog outputs. Although integrated DACs with different performance levels are available on the market, in many cases applications require a relatively low resolution, which can be attained at expenses of PWM modules, now standard on most microcontrollers, and a few inexpensive external components, resulting in simpler and less expensive circuits. The reminder of this document introduces the operating principles behind the generation of PWM-based DACS, identifies the sources of distortion and presents analytic methods to compute upper bounds for these, thus permitting the design of PWM-based DACs with specific requirements.

2 Operation Principles

PWM is a well-known technique, used to transmit signals and to control the power supplied to electrical loads, such as motors, heating devices, illumination systems (e.g. LEDs) and many others. A PWM signal is a periodic digital waveform with a variable duty cycle, defined as the ratio between the "on" and the "off" time of the signal. By varying the duty-cycle it is possible to vary

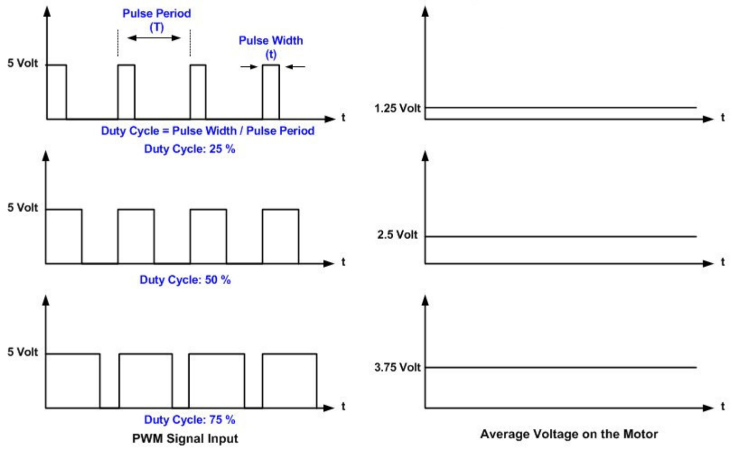

Figure 1: PWM signal average

the amount of energy delivered to the system, and thus varying the speed of motors, the intensity of LEDs, etc., as exemplified in Figure 1.

The operation principle of PWM-based DACs is quite simple, consisting in producing a PWM signal with a duty-cycle proportional to the desired analog signal, and use a low-pass filter to remove the high-frequency components, as shown in Figure 2.

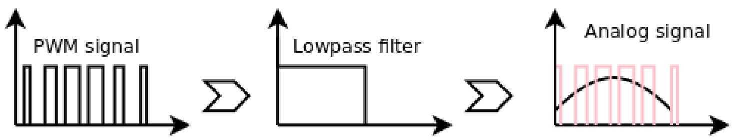

Figure 2: DAC-based PWM blocks

The performance limitations of PWM-based DACs are essentially due to tow factors:

- The limited resolution of the PWM modules;
- The non-ideality of low-pass filters, which don't block completely the unwanted frequencies.

In the following sections these non-idealities will be analyzed in detail, to allow the system designer to understand the limitations and tradeoffs on the design of this kind of ADCs, identify and quantify the contribution of each one of the distortion sources and adopt adequate design methodologies.

3 Fourier Analysis of PWM signals

As discussed on the previous section, on PWM-based DACs the DC component is the only one that contains useful information, while all the other ones are distortion and should be removed. However, practical low-pass filters have a limited attenuation, so high-frequency components shall appear on the output signal. This is called harmonic distortion, and the first step to be able to evaluate its magnitude on the output signal is to determine the spectral components of PWM signals.

Fourier theory [3] states that periodic signals can be decomposed into an infinite sum of harmonics, at integer multiple frequencies of the fundamental period. To simplify the analysis, and without loss of generality, the time origin of the PWM signal is shifted to make it representable by an even mathematical function, as shown in Figure 3.

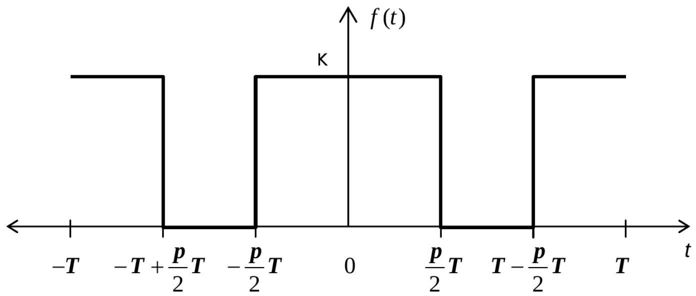

Figure 3: PWM signal representation

In Figure 3 **k** represents the signal amplitude and **T** and **p** ($0 \leq p \leq 1$) the PWM signal period and duty-cycle, respectively.

An even periodic function **f(t)** can be represented in Fourier series as follows:

$$f(t) = A_0 + \sum_{n=1}^{\infty} \left(A_n \cdot \cos\left(\frac{2 \cdot n \cdot \pi \cdot t}{T}\right) + B_n \cdot \sin\left(\frac{2 \cdot n \cdot \pi \cdot t}{T}\right)\right) \quad (1)$$

Where:

$$A_0 = \frac{1}{2T} \int_{-T}^{T} f(t)dt; A_n = \frac{1}{T} \int_{-T}^{T} f(t) \cdot \cos\left(\frac{2 \cdot n \cdot \pi \cdot t}{T}\right) dt \quad (2)$$

A_0 is the average value of $f(t)$. Referring to Figure 3, one can see that, by symmetry, it is enough to average the signal between 0 and $\frac{T}{2}$, thus:

$$A_0 = \frac{k \cdot p \cdot \frac{T}{2}}{\frac{T}{2}} = k \cdot p \quad (3)$$

The symmetry of the PWM signal can also be used to compute A_n. Equation 1 can also be computed only on the interval $[-\frac{T}{2}, \frac{T}{2}]$, thus:

$$A_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cdot\cos(\frac{2\cdot n\cdot\pi\cdot t}{T})dt \tag{4}$$

As:

$$f(t) = \begin{cases} k & : if\ 0 \leq |t| \leq p\cdot\frac{T}{2} \\ 0 & : otherwise \end{cases} \tag{5}$$

then Equation 4 becomes:

$$A_n = k\cdot\frac{2}{T}\int_{-p\cdot T/2}^{p\cdot T/2}\cos(\frac{2\cdot n\cdot\pi\cdot t}{T})dt \tag{6}$$

Integrating we obtain:

$$A_n = k\cdot\frac{2}{T}\left(\frac{1}{\frac{2\cdot n\cdot\pi}{T}}\cdot\left[\sin(\frac{2\cdot n\cdot\pi}{T}\Big/_{-p/2\cdot T}^{p/2\cdot T}\right)\right. \tag{7}$$

and finally:

$$A_n = \frac{2\cdot k}{n\cdot\pi}\sin(n\cdot\pi\cdot p) \tag{8}$$

Noting that $f(t)$ is even, and so B_n is null, we can replace equations 8 and 3 on Equation 1, obtaining:

$$f(t) = k\cdot p + \sum_{n=1}^{\infty}\left[\frac{2\cdot k}{n\cdot\pi}\sin(n\cdot\pi\cdot p)\cdot\cos\left(\frac{2\cdot n\cdot\pi\cdot t}{T}\right)\right] \tag{9}$$

A step-by-step deduction of these equations can be found in [2].

Equation 9 shows the dependency between the harmonic's amplitude and the duty cycle (p). If $p = 0$ then $f(t) = 0$ and if $p = 1$ then $f(t) = k$, i.e., constant signals with amplitude 0 and k, respectively. If $p = 0.5$, then:

$$A_n^{p=0.5} = \frac{2\cdot k}{n\cdot\pi}\sin\left(\frac{n\cdot\pi}{2}\right) \tag{10}$$

and thus:

$$\begin{array}{lll} A_1^{p=0.5} & = \frac{2\cdot k}{pi}\cdot\sin(\frac{\pi}{2}) & \simeq 0.637\cdot k \\ A_2^{p=0.5} & = \frac{2\cdot k}{2\cdot\pi}\cdot\sin(\pi) & \simeq 0 \\ A_3^{p=0.5} & = \frac{2\cdot k}{3\cdot\pi}\cdot\sin(\frac{-\pi}{2}) & \simeq -0.212\cdot k \\ \dots & \dots & \dots \end{array} \tag{11}$$

Those are the well-known spectral component weights for square waves.

Simulating Equation 9 for $0 \leq p \leq 1$ allows to see that the maximum power associated with the PWM harmonics occurs for $p = 0.5$. Therefore, for system design purposes, we can use the amplitudes shown in Equations 10 and 11. A similar result could be reached formally, by noting that the energy of the first harmonic prevails over the other ones and that $\sin(1 \cdot \pi \cdot p)$, with $0 \leq p \leq 1$ has a maximum at $p = 0.5$.

4 Filters

Low-pass filters are studied on introductory electronics courses, so this section is mostly focused on the specific aspects relevant to PWM-based DACs.

As seen in the previous section, PWM signals contain, in addition to the DC component, unwanted spectral components at $1/T, 2/T, ...$, which should be removed by an analog low-pass filter. Obviously, the quality of the filtered signal depends heavily on the choice of the low-pass filter. Active filters (based on OPAMPS) have several advantages when compared with passive ones (based only on resistors, capacitors and inductors). One of the most important ones is that the behavior of active filters is not affected by the electrical load of the circuits in which they are inserted to, a phenomenon that usually cannot be neglected with passive filters.

However, it is necessary to recall that the main reasons to use PWM-based DACs is simplicity and economy. For active filters the gain-bandwidth product of the OPAMPS can be a limitation. In practice it is desirable to employ OPAMPS with a bandwidth at least 5 to 10 times greater than the highest expected input frequency, to prevent significant distortion. As PWM signals have important high-frequency harmonic components, the implementation of active filters may require the use of expensive OPAMPS. The additional cost of these kind of OPAMPS and the increased circuit complexity may defeat the justification for using PWM-based DACs.

For the class of systems envisaged on this paper, the input load is not usually a problem for passive filters, since the PWM signal comes from a microcontroller, and these devices have a relatively low output impedance. On the output side, it may be necessary to drive high loads. In these cases it suffices adding an OPAMP, in voltage-follower configuration. As at the output the signal is already filtered and the gain is unitary, an ordinary low cost OPMAP can be used.

The two key properties of the low-pass filter to consider are the bandwidth and the rate of frequency rolloff. The filter bandwidth is defined as the frequency at which the output magnitude of the circuit is -3 dB of the nominal pass-band magnitude value. Obviously, in PWM-based DACs the filter bandwidth limits the maximum signal frequency that the converter can handle. The stop-band rolloff rate is high-frequency slope of the signal magnitude frequency response. Both bandwidth and rolloff rate define the ripple on the output of the DAC signal due to the harmonics. Recall that the stop-band rolloff rate is -20 dB/decade for 1^{st} order analog filters and -40 dB/decade for a 2^{nd} order analog filters.

Figure 4 shows a 1^{st} order RC low-pass filter. These kind of filters are extremely simple and cheap (only one resistor and one capacitor), having as main and often important limitation a -20 dB/decade stop-band rolloff rate.

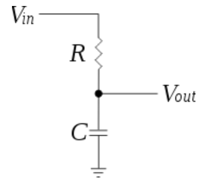

Figure 4: 1^{st} order low-pass RC filter

For this class of filters we have:

$$\begin{array}{lll} Time\ constant & \tau = R \cdot C & (s) \\ Transfer\ function & \frac{V_o(s)}{V_i(s)} = \frac{1}{\tau \cdot s + 1} & \\ Bandwidth & BW = \frac{1}{\tau} & (rad/s) \end{array} \quad (12)$$

Second-order low-pass filters offer a 40dB/decade stop-band attenuation, thus doubling the performance of 1^{st} order filters. The transfer function and bandwidth for this kind of filters are:

$$\begin{array}{ll} Transfer\ function & \frac{V_o(s)}{V_i(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2 \tau \cdot s + 1} \\ Bandwidth & BW = \omega_n \left[(1 - 2\xi) + \sqrt{4\xi^4 - 4\xi^2 + 2} \right]^{\frac{1}{2}} \end{array} \quad (13)$$

where ω_n is the undamped natural frequency (in $(rad/s)^2$) and ξ is the damping ratio (adimensional).

Second-order low-pass filters can be built by cascading two first order RC low-pass filters or recurring to an inductor, as shown in Figure 5.

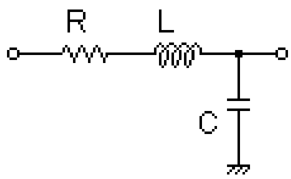

Figure 5: 2^{nd} order low-pass RLC filter

For the cascaded RC filters we have:

$$\begin{array}{l} \omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \\ \xi = \frac{R_1 C_1 + R_1 C_2 + R_2 C_2}{2\sqrt{R_1 R_2 C_1 C_2}} \end{array} \quad (14)$$

where $R_n; C_n$ refer to the resistor/capacitor pair of each one of the first order RC filters.

For RLC filters we have:

$$\begin{aligned} \omega_n &= \frac{1}{\sqrt{LC}} \\ \xi &= \frac{R}{2}\sqrt{\frac{C}{L}} \end{aligned} \tag{15}$$

The use of inductors is often avoided due to its size, cost and non-idealities. However, for PWM-based DACs, its use is viable since only one is needed and the allowed tolerance is high (exact bandwidth is not a critical issue).

Second order filters bring additional flexibility to system designer. For PWM-based DACs, the step response is an important characteristic, since it defines the time that the DAC output takes to change from one voltage level to another one. For this reason, it is desirable to use a damping ratio as low as possible. However, low damping ratios result in large overshoot and settling times. To prevent overshoot ξ must be equal to or greater than 1. On the other hand, to avoid having a resonant peak in the frequency response magnitude of the filter, ξ must be equal or bigger than 0.707. The choice of the damping ratio should be made according with the particular application requirements, but generally it is set to the interval $0.707 \leq \xi \leq 1$

5 PWM signal generation

Most modern microcontrollers incorporate hardware PWM modules that can be used to generate stable PWM signals with minimum code and processing overhead.

Figure 6 shows a simplified diagram of the PWM modules of Microchip PIC32 microcontrollers, which is representative of the PWM modules found on today's microcontrollers.

These modules are based on a timer (TMRx), two comparators and two comparison registers (PRx and OCxRS). The timer is fed by a clock, derived from the peripheral clock (PBCLK), affected by a pre-scaler (divider). At the beginning of each PWM cycle the output (OCx signal) is set to a logical "1". When the timer reaches the value present on the OCxRS register, the output signal is reset. The timer continues to be incremented and when it reaches the value stored in the PRx register, a new cycle is started.

For the realization of DACs, the two key PWM module parameters are the the clock frequency and the resolution of the timer and comparison registers.

As discussed in Sections 2, 3 and 4, PWM signals have unwanted spectral components at the fundamental frequency and harmonics. These components originate signal distortion and should be removed, or, in practice, attenuated. As the attenuation of filters increases with the frequency (Section 4), it results that higher PWM clock frequencies allow obtaining a smaller harmonic distortion and/or use simpler filters. Therefore, one of the guidelines for designing this kind of systems is using the highest possible clock frequency to drive the PWM module.

From the description of the internal operation of PWM modules, provided above, it results immediately that the resolution of the DAC is conditioned by

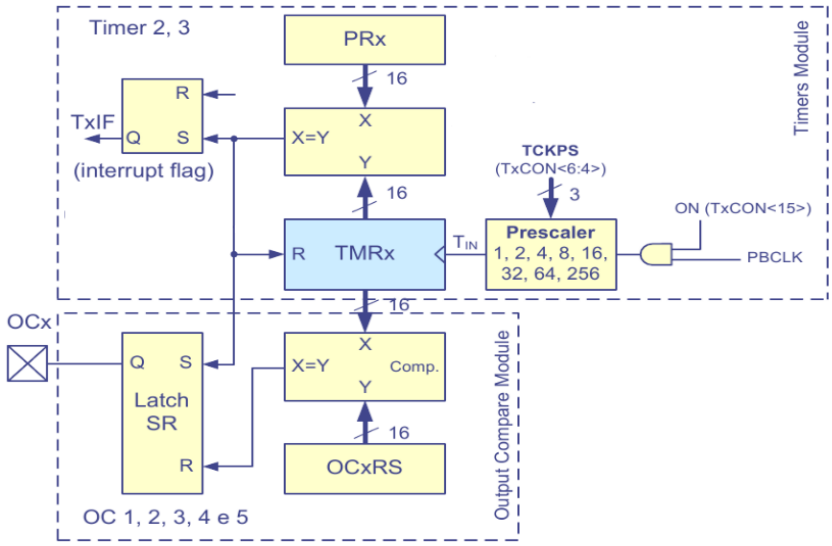

Figure 6: Microchip PC PWM module - simplified diagram

the number of bits of the timer and comparison registers. What is less obvious is that the DAC resolution also depends on the clock frequency. In fact, for a given clock value, obtaining higher PWM frequencies may imply that the timer will not reach is full-scale value, thus reducing the effective resolution.

Therefore, the selection of the PWM clock shall result in a tradeoff between the harmonic distortion and the PWM resolution.

6 Example

Lets consider now an concrete example to clarify the issues associated with the design of PWM-based DACs. Assume that it it used a 3.3V Microchip PIC32 microcontroller with the PBClock set to 40MHz. The maximum PWM clock frequency is obtained without prescaling, resulting in a timer clock $f^{clk} = 40MHz; T^{clk} = 25ns$. As PIC32 PWM modules are based on 16 bit registers, the maximum PWM period (minimum PWM frequency) that can be attained in these conditions is approximately $T^{pwm} \simeq 25ns * 2^{16} \simeq 1.64ms; f^{pwm} \simeq 610Hz$.

Consider that to reduce the harmonic distortion it is used a PWM frequency of $10KHz \quad (period 100\mu s)$. As the PIC32 is supplied with 3.3V, the PWM resolution at 610Hz is $r^{pwm}_{610Hz} = \frac{3.3V}{\frac{1.64ms}{25ns}} \simeq 0.05mV$. For the 10KHz PWM frequency, the resolution becomes $r^{pwm}_{10KHz} = \frac{3.3V}{\frac{100\mu s}{25ns}} \simeq 0.823mV$, that is, a degradation of more than one order of magnitude.

Lets now consider that this DAC should handle signals up to 10Hz. To minimize the attenuation of signals withing the pass-band, the filter bandwidth is set to 100Hz. It is also assumed, for sake of simplicity, that it is used a first order RC low-pass filter. From Equation 12 we compute $\tau = \frac{1}{2\pi\ 100} = 1.59ms$ and the attenuation at several frequencies of interest, shown on the second column of Table 6. The amplitude of the PWM spectral components at the output, shown on the 3^{rd} column of Table 6, is obtained multiplying these gains by the corresponding unfiltered signal amplitudes, given by Equation 10.

Frequency (Hz)	Gain	PWM Spec. Comp. Amplitude (mV)
10	0.9950	$\cdots$
10k	0.0100	$3.3 \cdot 0.6366 \cdot 0.0100 = 21.00$
30k	0.0033	$3.3 \cdot 0.2122 \cdot 0.0033 = 2.311$
50k	0.0020	$3.3 \cdot 0.1273 \cdot 0.0020 = 0.8402$
70k	0.0014	$3.3 \cdot 0.0909 \cdot 0.0014 = 0.4149$

Comparing the PWM resolution above computed ($r^{pwm}_{10KHz} \simeq 0.823mV$) with the amplitude of the PWM spectral components, shown in Table 6, we can conclude that the harmonic distortion clearly dominates. Thus, in this case, the DAC performance could be improved significantly improved simply by increasing the PWM frequency.

7 Conclusions

This document presents the principles of operation and design aspects of PWM-based DACs. Firstly it is presented an overview of this kind of DACs. Then, the main functional blocks are identified and their impact on the overall performance of this kind of devices is analyzed, both qualitatively an quantitatively. The main design tradeoffs, namely the ones involving bandwidth, PWM frequency, timer resolution and filter complexity are discussed in detail and illustrated with an example.

The interested reader can find additional information on application notes such as [5] and [1]. Generic information about active filters can be found in [4] while PWM filters are treated in detail in [6].

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