In[702]:=

```
SetDirectory[NotebookDirectory[]];
(*Defining where the plots must be saved and the list of these paths.*)
overleafDrafpath =
  "/Users/brunogoes/Dropbox/Aplicativos/Overleaf/00Goes-Bruno-PhDThesis/
    Figures/Chap6";
overleafFinalpath =
  "/Users/brunogoes/Dropbox/Aplicativos/Overleaf/01Goes-Bruno-PhDThesis/
    Figures/Chap6/";
overleafNotespath =
  "/Users/brunogoes/Dropbox/Aplicativos/Overleaf/SPI-Magnetic-field/Figures/
    Spontaneous-emission/";
pathsList = {overleafDrafpath, overleafFinalpath, overleafNotespath};
(*PlotStyling*)
fontsize = 24;
alphabetlabel = {"(a)", "(b)", "(c)", "(d)", "(e)", "(f)", "(g)",
   "(h)", "(i)", "(j)", "(k)", "(l)", "(m)", "(n)", "(o)", "(p)", "(q)",
   "(r)", "(s)", "(t)", "(u)", "(v)", "(w)", "(x)", "(y)", "(z)"};
ClearAll[MPLColorMap]
<< "http://pastebin.com/raw/pFsb4ZBS";
MPLColorMap["Inferno"];

Clear[savePlot];
savePlot[nameAndExtension_, plot_, path_ : pathsList] := Do[
  Export[PlotsPath <> nameAndExtension, plot], {PlotsPath, path}]
```

Chapter 6: SPI subjected to a in-plane magnetic field

Author: Bruno Ortega Goes

Parameters set definitions and loading experimental data/simulations☑

In[10]:=
```
(*All the physical parameter are real*)
$Assumptions =
  {nx^2 + ny^2 + nz^2 == 1 &&
     Ωg > 0 &&
     Ωe > 0 &&
     ΩR > 0 &&
     ΩL > 0 &&
     γ > 0 &&
     Γ > 0};
```

2) Exporting experimental data

It is necessary to change the path of the experimental data in your notebook if you want the program to run!

In[630]:=
```
PrettyTiming[data150 = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/Data/Data_150mT.csv"];
 data250 = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/Data/Data_250mT.csv"];
 data350 = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/Data/Data_350mT.csv"];
 data450 = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/Data/Data_450mT.csv"];]
dataexp1list = {data150, data250, data350, data450};
```

```
0h : 0m : 0s
```

In[632]:=
```
PrettyTiming[data150dcp = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/DataFig2c/Data_150mT_dcp.csv"];
  data250dcp = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/DataFig2c/Data_250mT_dcp.csv"];
  data350dcp = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/DataFig2c/Data_350mT_dcp.csv"];
  data450dcp = Import[
    "/Users/brunogoes/Dropbox/Research-projects/22--SPIMagneticField/
      ExperimentalData/DataFig2c/Data_450mT_dcp.csv"];]
datadcplist = {data150dcp, data250dcp, data350dcp, data450dcp};
```

```
0h : 0m : 0s
```

3) Parameters Exp.1: Probing the dynamics and coherence of a semiconductor hole spin via acoustic phonon-assisted excitation

In[635]:=
```
Clear[Experimentalγ, ExperimentalΩe, ExperimentalΩg]
τtrion = 450;(*This is constant because it
  depends solely on the cavity used in the experiment*)

Experimentalγ[τtrion_] := 1 / τtrion;(*ps^-1*)
ExperimentalΩe[B_] := (0.38 * 58 / 658) B(*ps^-1*);
ExperimentalΩg[B_] := (0.38 * 58 / 658) B(*ps^-1*);

(*The following experimental physical parameters follow Ref. https://
  arxiv.org/pdf/2207.05981.pdf*)
PhysicalParameters0  = {Ωe → ExperimentalΩe[0.0] / Experimentalγ[τtrion],
    Ωg → ExperimentalΩg[0.0] / Experimentalγ[τtrion], γ → 1.};
PhysicalParameters30  = {Ωe → ExperimentalΩe[0.03] / Experimentalγ[τtrion],
    Ωg → ExperimentalΩg[0.03] / Experimentalγ[τtrion], γ → 1.};
PhysicalParameters150  = {Ωe → ExperimentalΩe[0.15] / Experimentalγ[τtrion],
    Ωg → ExperimentalΩg[0.15] / Experimentalγ[τtrion], γ → 1.};
PhysicalParameters250  = {Ωe → ExperimentalΩe[0.25] / Experimentalγ[τtrion],
    Ωg → ExperimentalΩg[0.25] / Experimentalγ[τtrion], γ → 1.};
PhysicalParameters350  = {Ωe → ExperimentalΩe[0.35] / Experimentalγ[τtrion],
    Ωg → ExperimentalΩg[0.35] / Experimentalγ[τtrion], γ → 1.};
PhysicalParameters450  = {Ωe → ExperimentalΩe[0.45] / Experimentalγ[τtrion],
    Ωg → ExperimentalΩg[0.45] / Experimentalγ[τtrion], γ → 1.};
PhysicalParameters900 = {Ωe → ExperimentalΩe[0.90] / Experimentalγ[τtrion],
    Ωg → ExperimentalΩg[0.90] / Experimentalγ[τtrion], γ → 1.};
```

```
PhysicalParameters150 =
  {Ω → ExperimentalΩe[0.15] / Experimentalγ[τtrion],
   λ → (ExperimentalΩg[0.15] / Experimentalγ[τtrion]) /
     (ExperimentalΩe[0.15] / Experimentalγ[τtrion]), γ → 1.};

PhysicalParameters250 = {Ω → ExperimentalΩe[0.25] / Experimentalγ[τtrion],
   λ → (ExperimentalΩg[0.25] / Experimentalγ[τtrion]) /
     (ExperimentalΩe[0.25] / Experimentalγ[τtrion]), γ → 1.};

PhysicalParameters350 = {Ω → ExperimentalΩe[0.35] / Experimentalγ[τtrion],
   λ → (ExperimentalΩg[0.35] / Experimentalγ[τtrion]) /
     (ExperimentalΩe[0.35] / Experimentalγ[τtrion]), γ → 1.};

PhysicalParameters450 = {Ω → ExperimentalΩe[0.45] / Experimentalγ[τtrion],
   Ωg → (ExperimentalΩg[0.45] / Experimentalγ[τtrion]) /
     (ExperimentalΩe[0.45] / Experimentalγ[τtrion]), γ → 1.};

Exp1physicalparameterslist =
  {(*PhysicalParameters0,PhysicalParameters30,*)PhysicalParameters150 ,
   PhysicalParameters250, PhysicalParameters350, PhysicalParameters450 };

MagneticFieldDirection = {nx → 1, ny → 0., nz → 0};

texperimentalg2 = 57;
DataTimeRescalingFactorg2 = (2.65 / 0.48);
(*ratio between the maximal points of model and data*)
```

I. Hilbert space and operators definition☑

In this section I:

1) Define the Hilbert space structure and the operators, it follows the convention $\mathcal{H}$tot = $\mathcal{H}$en⊗$\mathcal{H}$sp. ;
2) Obtain the eigenvalues of the density matrix at time t rotated by a general unitary;

Defining the subspaces basis:

For the spin and energy subspaces I have ket0= ketUp = e and ket1= ketDw =g.

In[52]:=

```
ket[0] = ket["upm"] = ket["e"] = Basis[2, 1];
ket[1] = ket["dwm"] = ket["g"] = Basis[2, 2];
(*I added upm and dwm to emphasize that these
 are not the physical spin but the mathematucal ones *)
bra[ket_] := ket* // cf;
```

Defining the basis elements of the spin-energy Hilbert space: $\mathcal{H}$tot = $\mathcal{H}$en⊗$\mathcal{H}$sp.

In[55]:=
```
Do[
  ket[i, j] = kron[ket[i], ket[j]],
  {i, {"e", "g"}}, {j, {"upm", "dwm"}}
 ];
```

Defining the operators acting on the total Hilbert space: $\mathcal{H}$tot = $\mathcal{H}$en⊗$\mathcal{H}$sp.
I use the same notation as in the thesis.

In[56]:=
```
(*Energy subspace operators*)
(*Remark: Here I added an "E" after σ,
so that the σs are the pure Pauli operators always and it
 makes it explicity that I'm working on the energy subspace.*)
σE  = kron[σm, σ0];
σEx = kron[σx, σ0];
σEy = kron[σy, σ0];
σEz = kron[σz, σ0];

(*Spin subspace operators*)
s = kron[σ0, σm];
sx = kron[σ0, σx];
sy = kron[σ0, σy];
sz = kron[σ0, σz];
```

II. Rotation operators definition & Coefficients of the wave-function☑

In this section I:
  1) Define the ground and excited state rotation operators

1) Definition of the propagators: spontaneous emission, general and simplified☑

In[201]:=
```
(*I'll assume interaction picture with respect to ω0,
but I'll carry this factor in all the expressions,
in case we want the general expressions containing it,
it's just a matter of commenting this line*)
ω0 = 0;
Ce  = ω0 Eye[4] + Ω/2 sx;
Cg = λ Ω/2 (nx sx + ny sy + nz sz);
```

In[204]:=
```
Clear[ℛg, ℛe]
ℛg[t_] := MatrixExp[-ⅈ t Cg] // cf;
ℛe[t_] := MatrixExp[-ⅈ t Ce] // cf;
```

In[209]:=
```
(ℛg[t].ℛg[t]† // cf) == Eye[Length@ℛg[t]]
(ℛe[t]†.ℛe[t] // cf) == Eye[Length@ℛe[t]]
```

Out[209]= True

Out[210]= True

In[211]:=
```
ℛe[t] // mf;
```

$$\begin{pmatrix} \text{Cos}\left[\frac{t\,\Omega}{2}\right] & -i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] & 0 & 0 \\ -i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] & \text{Cos}\left[\frac{t\,\Omega}{2}\right] & 0 & 0 \\ 0 & 0 & \text{Cos}\left[\frac{t\,\Omega}{2}\right] & -i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] \\ 0 & 0 & -i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] & \text{Cos}\left[\frac{t\,\Omega}{2}\right] \end{pmatrix}$$

In[212]:=
```
ℛg[t] // mf;
```

$$\begin{pmatrix} \text{Cos}\left[\frac{t\,\lambda\,\Omega}{2}\right] - i\,\text{nz}\,\text{Sin}\left[\frac{t\,\lambda\,\Omega}{2}\right] & (-i\,\text{nx} - \text{ny})\,\text{Sin}\left[\frac{t\,\lambda\,\Omega}{2}\right] & 0 & 0 \\ (-i\,\text{nx} + \text{ny})\,\text{Sin}\left[\frac{t\,\lambda\,\Omega}{2}\right] & \text{Cos}\left[\frac{t\,\lambda\,\Omega}{2}\right] + i\,\text{nz}\,\text{Sin}\left[\frac{t\,\lambda\,\Omega}{2}\right] & 0 & 0 \\ 0 & 0 & \text{Cos}\left[\frac{t\,\lambda\,\Omega}{2}\right] - i\,\text{nz}\,\text{Sin}\left[\frac{t\,\lambda\,\Omega}{2}\right] & (-i\,\text{nx} - \text{ny})\,\text{Si} \\ 0 & 0 & (-i\,\text{nx} + \text{ny})\,\text{Sin}\left[\frac{t\,\lambda\,\Omega}{2}\right] & \text{Cos}\left[\frac{t\,\lambda\,\Omega}{2}\right] + i\,\text{nz}\,\text{S} \end{pmatrix}$$

In[213]:=
```
aveaRaL[υ_] := γ Exp[-γ t]
  (ket["e", "upm"].ℛe[t].ket["e", υ])* (ket["e", "dwm"].ℛe[t].ket["e", υ])
```

In[214]:=
```
(ket["e", "dwm"].ℛe[t].ket["e", "upm"])
(ket["e", "dwm"].ℛe[t].ket["e", "dwm"])
```

Out[214]= $-i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right]$

Out[215]= $\text{Cos}\left[\frac{t\,\Omega}{2}\right]$

In[216]:=
```
(ket["e", "upm"].ℛe[t].ket["e", "upm"])
(ket["e", "upm"].ℛe[t].ket["e", "dwm"])
```

Out[216]= $\text{Cos}\left[\frac{t\,\Omega}{2}\right]$

Out[217]= $-i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right]$

In[218]:=

```
Clear[M0, U0, M, U](*ne→-i γ/2*)
(*Matrices without drive: Spontaneous emission case*)
M0 = (Ce.σE†.σE - Cg.σE.σE†) + ne σE†.σE // mf;(*ne is just a dummy variable,
this is related to the 'bug' explained in the appendix*)
U0[t_] := MatrixExp[-i t M0] /. ne → -i γ/2 // cf;
U0::usage =
  "Takes the evolution time t as an argument. The internal parameters of
    this function that must be defined are: the Larmor frequencies
    Ωe and Ωg, the drirection of the magnetic field in the ground
    state (nx,ny,nz), the decay rate γ (usually taken to be 1).";

(*Matrices with drive*)
M = (Ce.σE†.σE - Cg.σE.σE†) + (ΩL/2 s.s† + ΩR/2 s†.s).σEy + ne σE†.σE // mf;
U[t_] := MatrixExp[-i t M] /. ne → (-i γ/2);
U::usage =
  "Takes the evolution time t as an argument. The internal parameters of
    this function that must be defined are: the Larmor frequencies
    Ωe and Ωg, the drirection of the magnetic field in the ground
    state (nx,ny,nz), the decay rate γ (usually taken to be 1) and
    the drive amplitudes ΩL and ΩR. For ΩL=ΩR=0, i.e. in the absence
    of a drive, we obtain U0, the spontaneous emission case.";

(*Matrices with drive neglecting Overhauser*)
Mmod = M /. λ → 1 /. ΩR → ΩH /. ΩL → ΩH /. nx → 1 /. ny → 0 /. nz → 0 // cf // mf;
Umod[s_] := MatrixExp[- i s Mmod] /. ne → -i γ/2;
```

$$\begin{pmatrix} ne & \frac{\Omega}{2} & 0 & 0 \\ \frac{\Omega}{2} & ne & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\,nz\,\lambda\,\Omega & -\frac{1}{2}\,(nx - i\,ny)\,\lambda\,\Omega \\ 0 & 0 & -\frac{1}{2}\,(nx + i\,ny)\,\lambda\,\Omega & \frac{nz\,\lambda\,\Omega}{2} \end{pmatrix}$$

$$\begin{pmatrix} ne & \frac{\Omega}{2} & -\frac{i\,\Omega R}{2} & 0 \\ \frac{\Omega}{2} & ne & 0 & -\frac{i\,\Omega L}{2} \\ \frac{i\,\Omega R}{2} & 0 & -\frac{1}{2}\,nz\,\lambda\,\Omega & -\frac{1}{2}\,(nx - i\,ny)\,\lambda\,\Omega \\ 0 & \frac{i\,\Omega L}{2} & -\frac{1}{2}\,(nx + i\,ny)\,\lambda\,\Omega & \frac{nz\,\lambda\,\Omega}{2} \end{pmatrix}$$

$$\begin{pmatrix} ne & \frac{\Omega}{2} & -\frac{i\,\Omega H}{2} & 0 \\ \frac{\Omega}{2} & ne & 0 & -\frac{i\,\Omega H}{2} \\ \frac{i\,\Omega H}{2} & 0 & 0 & -\frac{\Omega}{2} \\ 0 & \frac{i\,\Omega H}{2} & -\frac{\Omega}{2} & 0 \end{pmatrix}$$

2) Coefficients for spontaneous emission☑

In[79]:=
```
Clear[f0SE];
f0SE[finalenergy_, finalspin_, initialenergy_, initialspin_, t_] :=
 ket[finalenergy, finalspin].U0[t].ket[initialenergy, initialspin]
f0SE::usage =
  "The order of the arguments is: the system final state: energy and spin,
    the system initial state: energy and spin, and the time variable t.
    The internal parameters of this function that must be defined are:
    the Larmor frequencies Ωe and Ωg, the drirection of the magnetic
    field in the ground state (nx,ny,nz), the decay rate γ (by default
    1) and the drive amplitudes ΩL and ΩR. For ΩL=ΩR=0, i.e. in the
    absence of a drive, we obtain U0, the spontaneous emission case.";
```

In[82]:=
```
Clear[f1RSE];
f1RSE[finalenergy_, finalspin_, initialenergy_, initialspin_, t_, t1_,
  γ_ : 1] := √γ (ket[finalenergy, finalspin].U0[t - t1].ket["g", "upm"]) ×
  (ket["e", "upm"].U0[t1].ket[initialenergy, initialspin])
f1RSE::usage = "The order of the arguments is: the system final state:
    energy and spin, the system initial state: energy and spin, the
    time variable t, and the time of photon emission event t1. The
    internal parameters of this function that must be defined are:
    the Larmor frequencies Ωe and Ωg, the drirection of the magnetic
    field in the ground state (nx,ny,nz), the decay rate γ (default
    1) and the drive amplitudes ΩL and ΩR. For ΩL=ΩR=0, i.e. in the
    absence of a drive, we obtain U0, the spontaneous emission case.";
```

In[85]:=
```
Clear[f1LSE];
f1LSE[finalenergy_, finalspin_, initialenergy_, initialspin_, t_, t1_,
  γ_ : 1] := √γ (ket[finalenergy, finalspin].U0[t - t1].ket["g", "dwm"]) ×
  (ket["e", "dwm"].U0[t1].ket[initialenergy, initialspin])
f1LSE::usage = "The order of the arguments is: the system final state:
    energy and spin, the system initial state: energy and spin, the
    time variable t, and the time of photon emission event t1. The
    internal parameters of this function that must be defined are: the
    Larmor frequencies Ωe and Ωg, the drirection of the magnetic field
    in the ground state (nx,ny,nz), the decay rate γ (usually taken to
    be 1) and the drive amplitudes ΩL and ΩR. For ΩL=ΩR=0, i.e. in the
    absence of a drive, we obtain U0, the spontaneous emission case.";
```

In[ ]:=
```
(*Spontaneous emission case coefficients*)
TextGrid[
 {{"Intial Spin", "f0e↑", "f0e↓", "f1Rg↑", "f1Rg↓", "f1Lg↑", "f1Rg↓"},

  {"↑",
   f0SE["e", "upm", "e", "upm", t],
   f0SE["e", "dwm", "e", "upm", t],
   f1RSE["g", "upm", "e", "upm", t, t1],
   f1RSE["g", "dwm", "e", "upm", t, t1],
   f1LSE["g", "upm", "e", "upm", t, t1],
   f1LSE["g", "dwm", "e", "upm", t, t1]},


  {"↓",
   f0SE["e", "upm", "e", "dwm", t],
   f0SE["e", "dwm", "e", "dwm", t],
   f1RSE["g", "upm", "e", "dwm", t, t1],
   f1RSE["g", "dwm", "e", "dwm", t, t1],
   f1LSE["g", "upm", "e", "dwm", t, t1],
   f1LSE["g", "dwm", "e", "dwm", t, t1]}},

 Frame → All]
```

Out[ ]=

Intial Spin	f0e↑	f0e↓	f1Rg↑	f1Rg↓	f1Lg↑	f1Rg↓
↑	$e^{-\frac{t\gamma}{2}}\,\mathrm{Cos}\left[\frac{t\,\Omega}{2}\right]$	$-i\,e^{-\frac{t\gamma}{2}}\,\mathrm{Sin}\left[\frac{t\,\Omega}{2}\right]$	$e^{-\frac{t1\,\gamma}{2}}\,\mathrm{Cos}\left[\frac{t1\,\Omega}{2}\right]\left(\mathrm{Cos}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]+i\,nz\,\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]\right)$	$i\,e^{-\frac{t1\,\gamma}{2}}\,(nx+i\,ny)\,\mathrm{Cos}\left[\frac{t1\,\Omega}{2}\right]\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]$	$-i\,e^{-\frac{t1\,\gamma}{2}}\,(i\,nx+ny)\,\mathrm{Sin}\left[\frac{t1\,\Omega}{2}\right]\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]$	$-i\,e^{-\frac{t1\,\gamma}{2}}\,\mathrm{Sin}\left[\frac{t1\,\Omega}{2}\right]\left(\mathrm{Cos}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]-i\,nz\,\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]\right)$
↓	$-i\,e^{-\frac{t\gamma}{2}}\,\mathrm{Sin}\left[\frac{t\,\Omega}{2}\right]$	$e^{-\frac{t\gamma}{2}}\,\mathrm{Cos}\left[\frac{t\,\Omega}{2}\right]$	$-i\,e^{-\frac{t1\,\gamma}{2}}\,\mathrm{Sin}\left[\frac{t1\,\Omega}{2}\right]\left(\mathrm{Cos}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]+i\,nz\,\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]\right)$	$e^{-\frac{t1\,\gamma}{2}}\,(nx+i\,ny)\,\mathrm{Sin}\left[\frac{t1\,\Omega}{2}\right]\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]$	$e^{-\frac{t1\,\gamma}{2}}\,(i\,nx+ny)\,\mathrm{Cos}\left[\frac{t1\,\Omega}{2}\right]\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]$	$e^{-\frac{t1\,\gamma}{2}}\,\mathrm{Cos}\left[\frac{t1\,\Omega}{2}\right]\left(\mathrm{Cos}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]-i\,nz\,\mathrm{Sin}\left[\frac{1}{2}(t-t1)\,\lambda\,\Omega\right]\right)$

Computing the probabilities for the spontaneous emission: Assuming the spin is initially ↑

In[227]:=
```
Clear[P0SE, P1RSE, P1LSE];
Module[{ampR, ampL, initialspin},
 initialspin = "upm";
 PrettyTiming@Do[
   (*Amplitude of no-photon emission squared*)
   P0SE[ϵf, sf, "e", initialspin, t] =
    f0SE[ϵf, sf, "e", initialspin, t] × f0SE[ϵf, sf, "e", initialspin, t]*;

   (*Amplitude of R-photon emission squared and integrate over all t1*)
   ampR = f1RSE[ϵf, sf, "e", initialspin, t, t1] ×
      f1RSE[ϵf, sf, "e", initialspin, t, t1]* // cf;
   P1RSE[ϵf, sf, "e", initialspin, t] = Integrate[ampR, {t1, 0, t}];

   (*Amplitude of L-photon emission squared and integrate over all t1*)
   ampL = f1LSE[ϵf, sf, "e", initialspin, t, t1] ×
      f1LSE[ϵf, sf, "e", initialspin, t, t1]* // cf;
   P1LSE[ϵf, sf, "e", "upm", t] = Integrate[ampL, {t1, 0, t}],

   {ϵf, {"e", "g"}}, {sf, {"upm", "dwm"}}

  ]
]
```

```
0h : 2m : 16s
```

Sanity check: Is the wave-function properly normalized?

```
Sum[P0SE[ϵf, sf, "e", "upm", t] + P1RSE[ϵf, sf, "e", "upm", t] +
   P1LSE[ϵf, sf, "e", "upm", t], {ϵf, {"e", "g"}}, {sf, {"upm", "dwm"}}] // cf
```

Out[•]= 1

Yes!

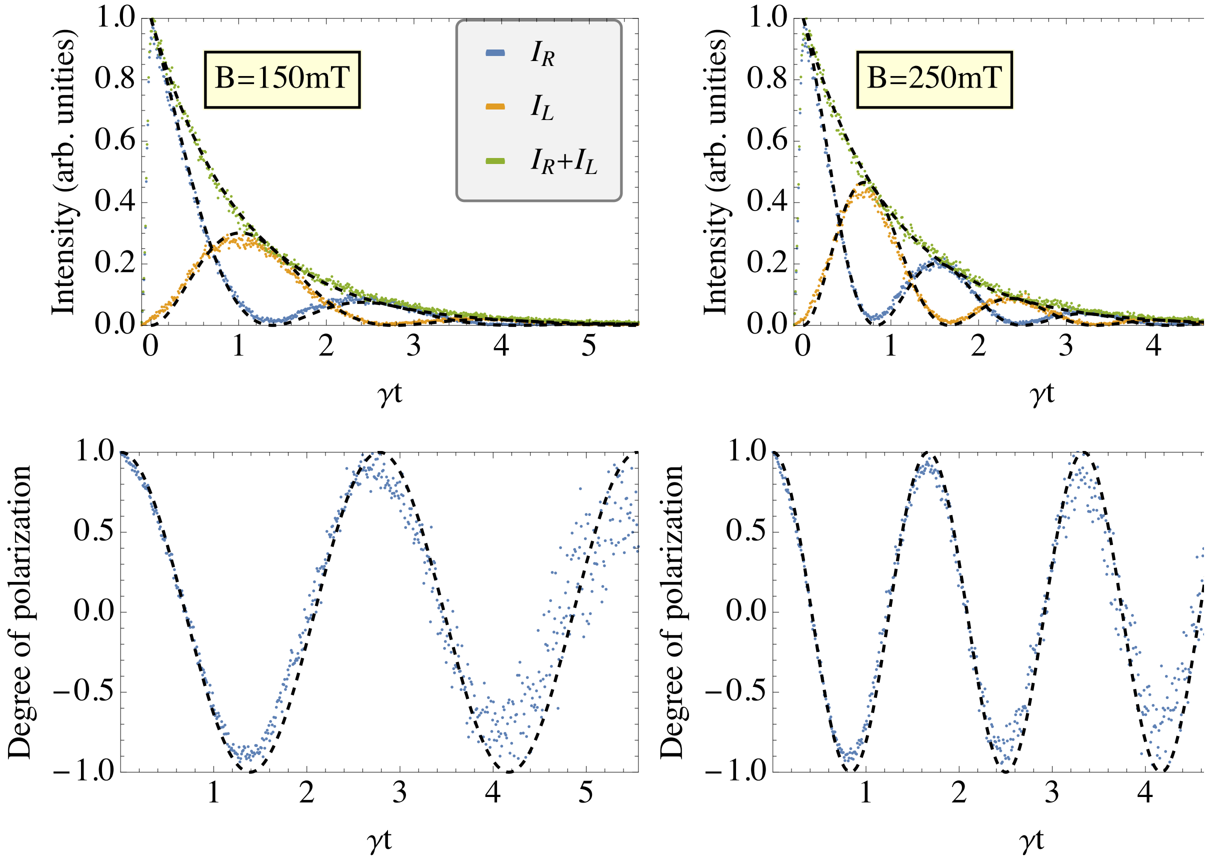
3) Model validation☑

In[800]:=
```
Clear[DCP];
texperimental = 5.56;(*Used to keep γ=1*)
(*Degre of circular polarization*)
DCP = (IRUpSE - ILUpSE) / (IRUpSE + ILUpSE) // cf;

Module[{timescaling, dataset1, dataset2, physicalparameters, epilog},
 timescaling = 5.56 / 2.5 (*Used to keep γ=1*);
 dataset1 = dataexp1list;
 dataset2 = datadcplist;
 epilog = {"B=150mT", "B=250mT", "B=350mT", "B=450mT"};
```

```
physicalparameters = Exp1physicalparameterslist;
modelValidation = Grid[{
   (*First row*)
   Table[
    Show[
     ListPlot[{
       Table[{dataset1⟦k⟧⟦i, 1⟧ * timescaling, dataset1⟦k⟧⟦i, 2⟧},
        {i, 1, Length@dataset1⟦k⟧}], Table[{dataset1⟦k⟧⟦i, 1⟧ * timescaling,
         dataset1⟦k⟧⟦i, 3⟧}, {i, 1, Length@dataset1⟦k⟧}],
       Table[{dataset1⟦k⟧⟦i, 1⟧ * timescaling, dataset1⟦k⟧⟦i, 4⟧},
        {i, 1, Length@dataset1⟦k⟧}]},
      PlotRange → {All, {0, 1}},
      PlotLegends → If[k == 1, legend[{"IR", "IL" , "IR+IL"}, {0.8, 0.7}], None],
      FrameLabel → {"γt", "Intensity (arb. unities)"},
      Epilog →
       Inset[Framed[Style[epilog⟦k⟧], Background → LightYellow], {1.5, 0.8}]],

     Plot[{IRUpSE /. physicalparameters⟦k⟧,
       ILUpSE /. physicalparameters⟦k⟧,
       IRUpSE + ILUpSE /. physicalparameters⟦k⟧},
      {t, 0, texperimental},

      PlotStyle → {{Black, Dashed}, {Black, Dashed}, {Black, Dashed}},
      PlotRange → {All, {0, 1}},
      PlotLegends → legend[{"IR", "IL" , "IR+IL"}, {0.8, 0.7}],
      FrameLabel → {"γt", "Intensity (arb. unities)"},
      Epilog → Inset[Framed[Style[k], Background → LightYellow], {1.5, 0.8}]]
    ],
    {k, 1, Length@dataset1}],

   (*Second row*)
   Table[
    Show[
     ListPlot[
      Table[{dataset2⟦k⟧⟦i, 1⟧ * (5.56 / 2.5),
             dataset2⟦k⟧⟦i, 2⟧}, {i, 1, Length@dataset2⟦k⟧}],
      PlotRange → {All, {-1, 1}},
      FrameLabel → {"γt", "Degree of polarization"}],

     Plot[DCP /. physicalparameters⟦k⟧, {t, 0, texperimental},
      PlotStyle → {{Black, Dashed}},
      PlotRange → {All, {-1, 1}},
      FrameLabel → {"γt", "Degree of polarization"}]],
    {k, 1, Length@dataset2}
   ]
  }]
]
```

```
savePlot["modelValidation-ExperimentalDatavsModel.pdf", modelValidation]
```

Out[803]=



III. Fidelity of the spin at the moment of re-excitation☑

1) Spin state at time t: Analytical calculations ☑

Ideal protocol parameters:

In[661]:=

```
idealprotocolparamaters = {γ → 1, nx → 1, ny → 0, nz → 0, λ → 1};
idealalignementparamaters = {γ → 1, nx → 1, ny → 0, nz → 0};(*λ can change*)
```

In[ ]:=

```
<< overlapsforSpinState.mx
```

In this section I:
1) Perform the integrals to obtain the matrix elements of the spin density matrix analytically;
2) Obtain the eigenvalues of the density matrix at time t rotated by a general unitary;

Mathematica has some problem with understanding that some parameters are real, so when we conjugate something it considers everything as a complex number. That’s why I write all the expressions explicitly and use the cf command (from Melt!) to

FullSimplify considering the parameters real. If I don't do this, Mathematica simply doesn't perform the necessary integrals.

Below I compute all the overlaps for all times, that's to say, I'm not simplifying to the long time limit, so all the expressions below DO HAVE the excited state component.

a) Computing the overlaps

In[240]:=
```
PrettyTiming[
 Module[{FirstTerm, SecondTerm, ThirdTerm,
   Product1, Product2, FirstIntegral, SecondIntegral},

  FirstTerm = Exp[-γ t] (ket["e", "upm"]*.ℛe[t].ket["e", "upm"])*×
     ket["e", "upm"]*.ℛe[t].ket["e", "upm"] // cf;
  (*✓*)

  SecondTerm = (ket["g", "upm"]*.ℛg[t - u].ket["g", "upm"])*×
     ket["g", "upm"]*.ℛg[t - u].ket["g", "upm"] // cf;
  (*✓*)
  Product1 = f0SE["e", "upm", "e", "upm", u]*×
     f0SE["e", "upm", "e", "upm", u] // cf;
  (*✓*)

  ThirdTerm = (ket["g", "upm"]*.ℛg[t - u].ket["g", "dwm"])*×
     ket["g", "upm"]*.ℛg[t - u].ket["g", "dwm"] // cf;
  (*✓*)
  Product2 = f0SE["e", "dwm", "e", "upm", u]*×
     f0SE["e", "dwm", "e", "upm", u] // cf;
  (*✓*)

  PrettyTiming[
   FirstIntegral = Integrate[SecondTerm Product1 // cf, {u, 0, t}]];
  PrettyTiming[SecondIntegral =
    Integrate[ThirdTerm Product2 // cf, {u, 0, t}]];

  oUpUp = FirstTerm + γ (FirstIntegral + SecondIntegral) /.
     (nx^2 + ny^2) → 1 - nz^2 // cf (*✓*);
  oUpUpg = γ (FirstIntegral + SecondIntegral) /. (nx^2 + ny^2) → 1 - nz^2 // cf (*✓*);
 ]
]
```

```
0h : 0m : 26s
0h : 0m : 39s
0h : 6m : 12s
```

In[241]:= oUpUp

Out[241]=
$$\frac{1}{4}\times\left(2+\frac{e^{i t\lambda\Omega}\left(-1+nz^2\right)\gamma\left(-i\gamma+\lambda\Omega\right)}{\left(i\gamma+\Omega-\lambda\Omega\right)\left(\gamma+i\left(1+\lambda\right)\Omega\right)}+\frac{2\,nz^2\gamma^2}{\gamma^2+\Omega^2}-\frac{e^{-i t\lambda\Omega}\left(-1+nz^2\right)\gamma\left(\gamma-i\lambda\Omega\right)}{\gamma^2-2i\gamma\lambda\Omega+\Omega^2-\lambda^2\Omega^2}+\frac{e^{-t(\gamma-i\Omega)}\gamma\left(-\gamma^2+2i\gamma\Omega+\Omega^2-nz^2\lambda^2\Omega^2\right)}{\left(\gamma-i\Omega\right)\left(\gamma^2-2i\gamma\Omega+\left(-1+\lambda^2\right)\Omega^2\right)}-\frac{e^{-t(\gamma+i\Omega)}\gamma\left(\gamma^2+2i\gamma\Omega+\left(-1+nz^2\lambda^2\right)\Omega^2\right)}{\left(\gamma+i\Omega\right)\left(\gamma^2+2i\gamma\Omega+\left(-1+\lambda^2\right)\Omega^2\right)}+2e^{-t\gamma}\mathrm{Cos}[t\Omega]\right)$$

In[242]:= oUpUpg

Out[242]=
$$\frac{1}{8}\times\left(4-4e^{-t\gamma}+\frac{2e^{i t\lambda\Omega}\left(-1+nz^2\right)\gamma\left(-i\gamma+\lambda\Omega\right)}{\left(i\gamma+\Omega-\lambda\Omega\right)\left(\gamma+i\left(1+\lambda\right)\Omega\right)}+\frac{4\,nz^2\gamma^2}{\gamma^2+\Omega^2}-\frac{2e^{-i t\lambda\Omega}\left(-1+nz^2\right)\gamma\left(\gamma-i\lambda\Omega\right)}{\gamma^2-2i\gamma\lambda\Omega-\left(-1+\lambda^2\right)\Omega^2}-\frac{2e^{-t(\gamma-i\Omega)}\gamma\left(\gamma^2-2i\gamma\Omega+\left(-1+nz^2\lambda^2\right)\Omega^2\right)}{\left(\gamma-i\Omega\right)\left(\gamma^2-2i\gamma\Omega+\left(-1+\lambda^2\right)\Omega^2\right)}-\frac{2e^{-t(\gamma+i\Omega)}\gamma\left(\gamma^2+2i\gamma\Omega+\left(-1+nz^2\lambda^2\right)\Omega^2\right)}{\left(\gamma+i\Omega\right)\left(\gamma^2+2i\gamma\Omega+\left(-1+\lambda^2\right)\Omega^2\right)}\right)$$

In[243]:=
```
PrettyTiming[
 Module[{FirstTerm, SecondTerm, ThirdTerm,
   Product1, Product2, FirstIntegral, SecondIntegral},

  FirstTerm = Exp[-γ t] (ket["e", "dwm"]*.ℛe[t].ket["e", "upm"])*×
     ket["e", "dwm"]*.ℛe[t].ket["e", "upm"] // cf;
  (*✓*)

  SecondTerm = (ket["g", "dwm"]*.ℛg[t - u].ket["g", "upm"])*×
     ket["g", "dwm"]*.ℛg[t - u].ket["g", "upm"] // cf;
  (*✓*)
  Product1 = f0SE["e", "upm", "e", "upm", u]*×
     f0SE["e", "upm", "e", "upm", u] // cf;
  (*✓*)

  ThirdTerm = (ket["g", "dwm"]*.ℛg[t - u].ket["g", "dwm"])*×
     ket["g", "dwm"]*.ℛg[t - u].ket["g", "dwm"] // cf;
  (*✓*)
  Product2 = f0SE["e", "dwm", "e", "upm", u]*×
     f0SE["e", "dwm", "e", "upm", u] // cf;
  (*✓*)
  FirstIntegral = Integrate[(SecondTerm Product1) // cf, {u, 0, t}];
  SecondIntegral = Integrate[(ThirdTerm Product2) // cf, {u, 0, t}];

  oDwDw = FirstTerm + γ (FirstIntegral + SecondIntegral) /.
     (nx^2 + ny^2) → 1 - nz^2 // cf(*✓*);

  oDwDwg = γ (FirstIntegral + SecondIntegral) /. (nx^2 + ny^2) → 1 - nz^2 // cf(*✓*);
 ]
]
```

0h : 7m : 5s

In[244]:= oDwDw

Out[244]=

$$\frac{1}{8}\times\Bigg(4-\frac{4\,nz^2\,\gamma^2}{\gamma^2+\Omega^2}-4\,e^{-t\,\gamma}\,\mathrm{Cos}[t\,\Omega]+\Big(4\,e^{-t\,\gamma}\,\gamma\,\big(\gamma\,\big(\gamma^4+\gamma^2\,\big(2+\big(1+nz^2\big)\,\lambda^2\big)\,\Omega^2+\big(1+\lambda^2+nz^2\,\lambda^2\,\big(-3+\lambda^2\big)\big)\,\Omega^4\big)\,\mathrm{Cos}[t\,\Omega]-\Omega\,\big(\gamma^4+\gamma^2\,\big(2+\big(-1+3\,nz^2\big)\,\lambda^2\big)\,\Omega^2+\big(-1+\lambda^2\big)\times\big(-1+nz^2\,\lambda^2\big)\,\Omega^4\big)\,\mathrm{Sin}[t\,\Omega]+e^{t\,\gamma}\,\big(-1+nz^2\big)\,\big(\gamma^2+\Omega^2\big)\,\big(\gamma\,\big(\gamma^2+\big(1+\lambda^2\big)\,\Omega^2\big)\,\mathrm{Cos}[t\,\lambda\,\Omega]+\lambda\,\Omega\,\big(\gamma^2+\big(-1+\lambda^2\big)\,\Omega^2\big)\,\mathrm{Sin}[t\,\lambda\,\Omega]\big)\big)\Big)\Big/\Big(\big(\gamma^2+\Omega^2\big)\,\big(\gamma^2+(-1+\lambda)^2\,\Omega^2\big)\,\big(\gamma^2+(1+\lambda)^2\,\Omega^2\big)\Big)\Bigg)$$

In[245]:= 𝑜DwDwg

Out[245]=
$$\frac{1}{4}\times\left(2-2\,e^{-t\gamma}-\frac{2\,nz^2\gamma^2}{\gamma^2+\Omega^2}+\frac{e^{-i\,t\,\lambda\,\Omega}\left(-1+nz^2\right)\gamma\,(\gamma-i\,\lambda\,\Omega)}{\gamma^2-2\,i\,\gamma\,\lambda\,\Omega-\left(-1+\lambda^2\right)\Omega^2}+\frac{i\,e^{i\,t\,\lambda\,\Omega}\left(-1+nz^2\right)\gamma\,(-i\,\gamma+\lambda\,\Omega)}{\gamma^2+2\,i\,\gamma\,\lambda\,\Omega-\left(-1+\lambda^2\right)\Omega^2}+\frac{e^{-t\,(\gamma-i\,\Omega)}\,\gamma\left(\gamma^2-2\,i\,\gamma\,\Omega+\left(-1+nz^2\lambda^2\right)\Omega^2\right)}{(\gamma-i\,\Omega)\left(\gamma^2-2\,i\,\gamma\,\Omega+\left(-1+\lambda^2\right)\Omega^2\right)}+\frac{e^{-t\,(\gamma+i\,\Omega)}\,\gamma\left(\gamma^2+2\,i\,\gamma\,\Omega+\left(-1+nz^2\lambda^2\right)\Omega^2\right)}{(\gamma+i\,\Omega)\left(\gamma^2+2\,i\,\gamma\,\Omega+\left(-1+\lambda^2\right)\Omega^2\right)}\right)$$

In[246]:=
```
PrettyTiming[

 Module[{FirstTerm, SecondTerm, ThirdTerm,
   Product1, Product2, FirstIntegral, SecondIntegral},

  FirstTerm = Exp[-γ t] (ket["e", "dwm"]*.ℛe[t].ket["e", "upm"])*×
     ket["e", "upm"]*.ℛe[t].ket["e", "upm"] // cf;
  (*✓*)

  SecondTerm = (ket["g", "dwm"]*.ℛg[t - u].ket["g", "upm"])*×
     ket["g", "upm"]*.ℛg[t - u].ket["g", "upm"] // cf;
  Product1 = f0SE["e", "upm", "e", "upm", u]*×
     f0SE["e", "upm", "e", "upm", u] // cf;
  (*✓*)

  ThirdTerm = (ket["g", "dwm"]*.ℛg[t - u].ket["g", "dwm"])*×
     ket["g", "upm"]*.ℛg[t - u].ket["g", "dwm"] // cf;
  (*✓-There was a correction here! The not conjugated
    part was swaped the Up and Dw*)
  Product2 = f0SE["e", "dwm", "e", "upm", u]*×
     f0SE["e", "dwm", "e", "upm", u] // cf;
  (*✓*)

  FirstIntegral  = Integrate[(SecondTerm Product1) // cf, {u, 0, t}];
  SecondIntegral  = Integrate[(ThirdTerm Product2) // cf, {u, 0, t}];

  𝑜DwUp  = FirstTerm + γ (FirstIntegral + SecondIntegral) // cf;

  𝑜DwUpg  = γ (FirstIntegral + SecondIntegral) // cf;

 ]
]
```

0h : 10m : 43s

In[247]:= ρDwUp

Out[247]=
$$\frac{1}{4}\times\Big(2\,i\,e^{-t\gamma}\,\mathrm{Sin}[t\,\Omega]-\Big(2\,(nx-i\,ny)\,\gamma\,\Big(nz\,\gamma\,\Big(-\gamma^4-2\,\gamma^2\,(1+\lambda^2)\,\Omega^2-(-1+\lambda^2)^2\,\Omega^4\Big)+(\gamma^2+\Omega^2)\,(i\,\lambda\,\Omega\,(\gamma^2+(-1+\lambda^2)\,\Omega^2)+nz\,\gamma\,(\gamma^2+(1+\lambda^2)\,\Omega^2))\,\mathrm{Cos}[t\,\lambda\,\Omega]+e^{-t\gamma}\,\lambda\,\Omega\,\Big(\big(-i\,\gamma^4+nz\,\gamma^3\,\lambda\,\Omega-i\,\gamma^2\,\lambda^2\,\Omega^2+nz\,\gamma\,\lambda\,(-3+\lambda^2)\,\Omega^3-i\,(-1+\lambda^2)\,\Omega^4\big)\,\mathrm{Cos}[t\,\Omega]+\Omega\,\big(2\,i\,\gamma^3-3\,nz\,\gamma^2\,\lambda\,\Omega+2\,i\,\gamma\,\Omega^2-nz\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\Omega]\Big)+(\gamma^2+\Omega^2)\,\big(-i\,\gamma^3+nz\,\gamma^2\,\lambda\,\Omega-i\,\gamma\,(1+\lambda^2)\,\Omega^2+nz\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\lambda\,\Omega]\Big)\Big)\Big/\Big(\gamma^6+\gamma^4\,(3+2\,\lambda^2)\,\Omega^2+\gamma^2\,(3+\lambda^4)\,\Omega^4+(-1+\lambda^2)^2\,\Omega^6\Big)\Big)$$

In[248]:= ρDwUpg

Out[248]=
$$-\Big(\Big(e^{-2\,t\,\gamma+i\,t\,(-1+\lambda)\,\Omega}\,(nx-i\,ny)\,\gamma\,\Big(e^{2\,t\,\gamma+i\,t\,\Omega}\,(-1+nz)\,(\gamma-i\,\Omega)\,(\gamma+i\,\Omega)\,(\gamma-i\,(-1+\lambda)\,\Omega)\,(\gamma+i\,\lambda\,\Omega)\,(\gamma-i\,(1+\lambda)\,\Omega)+e^{t\,(2\,\gamma+i\,(1-2\,\lambda)\,\Omega)}\,(1+nz)\,(\gamma-i\,\Omega)\,(\gamma+i\,\Omega)\,(\gamma+i\,(-1+\lambda)\,\Omega)\,(\gamma-i\,\lambda\,\Omega)\,(\gamma+i\,(1+\lambda)\,\Omega)-2\,e^{t\,(2\,\gamma-i\,(-1+\lambda)\,\Omega)}\,nz\,\gamma\,\big(\gamma^2+(-1+\lambda)^2\,\Omega^2\big)\,\big(\gamma^2+(1+\lambda)^2\,\Omega^2\big)+e^{t\,(\gamma-i\,\lambda\,\Omega)}\,\lambda\,(\gamma-i\,\Omega)\,\Omega\,(-i\,\gamma+\Omega+nz\,\lambda\,\Omega)\,\big(\gamma^2-2\,i\,\gamma\,\Omega+(-1+\lambda^2)\,\Omega^2\big)+e^{t\,(\gamma-i\,(-2+\lambda)\,\Omega)}\,\lambda\,(\gamma+i\,\Omega)\,\Omega\,(-i\,\gamma+(-1+nz\,\lambda)\,\Omega)\,\big(\gamma^2+2\,i\,\gamma\,\Omega+(-1+\lambda^2)\,\Omega^2\big)\Big)\Big)\Big/\Big(4\,(\gamma^2+\Omega^2)\,\big(\gamma^4+2\,\gamma^2\,(1+\lambda^2)\,\Omega^2+(-1+\lambda^2)^2\,\Omega^4\big)\Big)\Big)$$

In[249]:= ρUpDw = Conjugate[ρDwUp]

Out[249]=
$$\frac{1}{4}\,\mathrm{Conjugate}\Big[2\,i\,e^{-t\gamma}\,\mathrm{Sin}[t\,\Omega]-\Big(2\,(nx-i\,ny)\,\gamma\,\Big(nz\,\gamma\,\Big(-\gamma^4-2\,\gamma^2\,(1+\lambda^2)\,\Omega^2-(-1+\lambda^2)^2\,\Omega^4\Big)+(\gamma^2+\Omega^2)\,(i\,\lambda\,\Omega\,(\gamma^2+(-1+\lambda^2)\,\Omega^2)+nz\,\gamma\,(\gamma^2+(1+\lambda^2)\,\Omega^2))\,\mathrm{Cos}[t\,\lambda\,\Omega]+e^{-t\gamma}\,\lambda\,\Omega\,\Big(\big(-i\,\gamma^4+nz\,\gamma^3\,\lambda\,\Omega-i\,\gamma^2\,\lambda^2\,\Omega^2+nz\,\gamma\,\lambda\,(-3+\lambda^2)\,\Omega^3-i\,(-1+\lambda^2)\,\Omega^4\big)\,\mathrm{Cos}[t\,\Omega]+\Omega\,\big(2\,i\,\gamma^3-3\,nz\,\gamma^2\,\lambda\,\Omega+2\,i\,\gamma\,\Omega^2-nz\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\Omega]\Big)+(\gamma^2+\Omega^2)\,\big(-i\,\gamma^3+nz\,\gamma^2\,\lambda\,\Omega-i\,\gamma\,(1+\lambda^2)\,\Omega^2+nz\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\lambda\,\Omega]\Big)\Big)\Big/\Big(\gamma^6+\gamma^4\,(3+2\,\lambda^2)\,\Omega^2+\gamma^2\,(3+\lambda^4)\,\Omega^4+(-1+\lambda^2)^2\,\Omega^6\Big)\Big]$$

Yeah, Mathematica doesn't conjugate smartly. We have to outsmart Mathematica, below I do it by hand:

In[250]:=

$$o\mathrm{UpDw} = \frac{1}{4}\times\Big(-2\,\mathrm{i}\,\mathrm{e}^{-t\gamma}\,\mathrm{Sin}[t\,\Omega] - \Big(2\,(\mathrm{nx}+\mathrm{i}\,\mathrm{ny})\,\gamma\,\Big(\mathrm{nz}\,\gamma\,\big(-\gamma^4-2\,\gamma^2\,(1+\lambda^2)\,\Omega^2-(-1+\lambda^2)^2\,\Omega^4\big) + (\gamma^2+\Omega^2)\,\big(-\mathrm{i}\,\lambda\,\Omega\,(\gamma^2+(-1+\lambda^2)\,\Omega^2)+\mathrm{nz}\,\gamma\,(\gamma^2+(1+\lambda^2)\,\Omega^2)\big)\,\mathrm{Cos}[t\,\lambda\,\Omega] + \mathrm{e}^{-t\gamma}\,\lambda\,\Omega\,\Big(\big(\mathrm{i}\,\gamma^4+\mathrm{nz}\,\gamma^3\,\lambda\,\Omega+\mathrm{i}\,\gamma^2\,\lambda^2\,\Omega^2+\mathrm{nz}\,\gamma\,\lambda\,(-3+\lambda^2)\,\Omega^3+\mathrm{i}\,(-1+\lambda^2)\,\Omega^4\big)\,\mathrm{Cos}[t\,\Omega] + \Omega\,\big(-2\,\mathrm{i}\,\gamma^3-3\,\mathrm{nz}\,\gamma^2\,\lambda\,\Omega-2\,\mathrm{i}\,\gamma\,\Omega^2-\mathrm{nz}\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\Omega]\Big) + (\gamma^2+\Omega^2)\,\big(\mathrm{i}\,\gamma^3+\mathrm{nz}\,\gamma^2\,\lambda\,\Omega+\mathrm{i}\,\gamma\,(1+\lambda^2)\,\Omega^2+\mathrm{nz}\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\lambda\,\Omega]\Big)\Big) \Big/ \big(\gamma^6+\gamma^4\,(3+2\,\lambda^2)\,\Omega^2+\gamma^2\,(3+\lambda^4)\,\Omega^4+(-1+\lambda^2)^2\,\Omega^6\big)\Big)$$

Out[250]=

$$\frac{1}{4}\times\Big(-2\,\mathrm{i}\,\mathrm{e}^{-t\gamma}\,\mathrm{Sin}[t\,\Omega] - \Big(2\,(\mathrm{nx}+\mathrm{i}\,\mathrm{ny})\,\gamma\,\Big(\mathrm{nz}\,\gamma\,\big(-\gamma^4-2\,\gamma^2\,(1+\lambda^2)\,\Omega^2-(-1+\lambda^2)^2\,\Omega^4\big) + (\gamma^2+\Omega^2)\,\big(-\mathrm{i}\,\lambda\,\Omega\,(\gamma^2+(-1+\lambda^2)\,\Omega^2)+\mathrm{nz}\,\gamma\,(\gamma^2+(1+\lambda^2)\,\Omega^2)\big)\,\mathrm{Cos}[t\,\lambda\,\Omega] + \mathrm{e}^{-t\gamma}\,\lambda\,\Omega\,\Big(\big(\mathrm{i}\,\gamma^4+\mathrm{nz}\,\gamma^3\,\lambda\,\Omega+\mathrm{i}\,\gamma^2\,\lambda^2\,\Omega^2+\mathrm{nz}\,\gamma\,\lambda\,(-3+\lambda^2)\,\Omega^3+\mathrm{i}\,(-1+\lambda^2)\,\Omega^4\big)\,\mathrm{Cos}[t\,\Omega] + \Omega\,\big(-2\,\mathrm{i}\,\gamma^3-3\,\mathrm{nz}\,\gamma^2\,\lambda\,\Omega-2\,\mathrm{i}\,\gamma\,\Omega^2-\mathrm{nz}\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\Omega]\Big) + (\gamma^2+\Omega^2)\,\big(\mathrm{i}\,\gamma^3+\mathrm{nz}\,\gamma^2\,\lambda\,\Omega+\mathrm{i}\,\gamma\,(1+\lambda^2)\,\Omega^2+\mathrm{nz}\,\lambda\,(-1+\lambda^2)\,\Omega^3\big)\,\mathrm{Sin}[t\,\lambda\,\Omega]\Big)\Big) \Big/ \big(\gamma^6+\gamma^4\,(3+2\,\lambda^2)\,\Omega^2+\gamma^2\,(3+\lambda^4)\,\Omega^4+(-1+\lambda^2)^2\,\Omega^6\big)\Big)$$

In[251]:=

$$o\mathrm{UpDwg} = -\Big(\Big(\mathrm{e}^{-2t\gamma-\mathrm{i}\,t\,(-1+\lambda)\,\Omega}\,(\mathrm{nx}+\mathrm{i}\,\mathrm{ny})\,\gamma\,\Big(\mathrm{e}^{2t\gamma-\mathrm{i}\,t\,\Omega}\,(-1+\mathrm{nz})\,(\gamma+\mathrm{i}\,\Omega)\,(\gamma-\mathrm{i}\,\Omega)\,(\gamma+\mathrm{i}\,(-1+\lambda)\,\Omega)\,(\gamma-\mathrm{i}\,\lambda\,\Omega)\,(\gamma+\mathrm{i}\,(1+\lambda)\,\Omega) + \mathrm{e}^{t\,(2\gamma-\mathrm{i}\,(1-2\lambda)\,\Omega)}\,(1+\mathrm{nz})\,(\gamma+\mathrm{i}\,\Omega)\,(\gamma-\mathrm{i}\,\Omega)\,(\gamma-\mathrm{i}\,(-1+\lambda)\,\Omega)\,(\gamma+\mathrm{i}\,\lambda\,\Omega)\,(\gamma-\mathrm{i}\,(1+\lambda)\,\Omega) - 2\,\mathrm{e}^{t\,(2\gamma+\mathrm{i}\,(-1+\lambda)\,\Omega)}\,\mathrm{nz}\,\gamma\,\big(\gamma^2+(-1+\lambda)^2\,\Omega^2\big)\,\big(\gamma^2+(1+\lambda)^2\,\Omega^2\big) + \mathrm{e}^{t\,(\gamma+\mathrm{i}\,\lambda\,\Omega)}\,\lambda\,(\gamma+\mathrm{i}\,\Omega)\,\Omega\,(\mathrm{i}\,\gamma+\Omega+\mathrm{nz}\,\lambda\,\Omega)\,\big(\gamma^2+2\,\mathrm{i}\,\gamma\,\Omega+(-1+\lambda^2)\,\Omega^2\big) + \mathrm{e}^{t\,(\gamma+\mathrm{i}\,(-2+\lambda)\,\Omega)}\,\lambda\,(\gamma-\mathrm{i}\,\Omega)\,\Omega\,(\mathrm{i}\,\gamma+(-1+\mathrm{nz}\,\lambda)\,\Omega)\,\big(\gamma^2-2\,\mathrm{i}\,\gamma\,\Omega+(-1+\lambda^2)\,\Omega^2\big)\Big)\Big) \Big/ \Big(4\,(\gamma^2+\Omega^2)\,\big(\gamma^4+2\,\gamma^2\,(1+\lambda^2)\,\Omega^2+(-1+\lambda^2)^2\,\Omega^4\big)\Big)\Big)$$

Out[251]=

$$-\Big(\Big(\mathrm{e}^{-2t\gamma-\mathrm{i}\,t\,(-1+\lambda)\,\Omega}\,(\mathrm{nx}+\mathrm{i}\,\mathrm{ny})\,\gamma\,\Big(\mathrm{e}^{t\,(2\gamma-\mathrm{i}\,(1-2\lambda)\,\Omega)}\,(1+\mathrm{nz})\,(\gamma-\mathrm{i}\,\Omega)\,(\gamma+\mathrm{i}\,\Omega)\,(\gamma-\mathrm{i}\,(-1+\lambda)\,\Omega)\,(\gamma+\mathrm{i}\,\lambda\,\Omega)\,(\gamma-\mathrm{i}\,(1+\lambda)\,\Omega) + \mathrm{e}^{2t\gamma-\mathrm{i}\,t\,\Omega}\,(-1+\mathrm{nz})\,(\gamma-\mathrm{i}\,\Omega)\,(\gamma+\mathrm{i}\,\Omega)\,(\gamma+\mathrm{i}\,(-1+\lambda)\,\Omega)\,(\gamma-\mathrm{i}\,\lambda\,\Omega)\,(\gamma+\mathrm{i}\,(1+\lambda)\,\Omega) - 2\,\mathrm{e}^{t\,(2\gamma+\mathrm{i}\,(-1+\lambda)\,\Omega)}\,\mathrm{nz}\,\gamma\,\big(\gamma^2+(-1+\lambda)^2\,\Omega^2\big)\,\big(\gamma^2+(1+\lambda)^2\,\Omega^2\big) + \mathrm{e}^{t\,(\gamma+\mathrm{i}\,(-2+\lambda)\,\Omega)}\,\lambda\,(\gamma-\mathrm{i}\,\Omega)\,\Omega\,(\mathrm{i}\,\gamma+(-1+\mathrm{nz}\,\lambda)\,\Omega)\,\big(\gamma^2-2\,\mathrm{i}\,\gamma\,\Omega+(-1+\lambda^2)\,\Omega^2\big) + \mathrm{e}^{t\,(\gamma+\mathrm{i}\,\lambda\,\Omega)}\,\lambda\,(\gamma+\mathrm{i}\,\Omega)\,\Omega\,(\mathrm{i}\,\gamma+\Omega+\mathrm{nz}\,\lambda\,\Omega)\,\big(\gamma^2+2\,\mathrm{i}\,\gamma\,\Omega+(-1+\lambda^2)\,\Omega^2\big)\Big)\Big) \Big/ \Big(4\,(\gamma^2+\Omega^2)\,\big(\gamma^4+2\,\gamma^2\,(1+\lambda^2)\,\Omega^2+(-1+\lambda^2)^2\,\Omega^4\big)\Big)\Big)$$

As it takes a lot of time to compute these overlaps I'll save them:

In[252]:=

```
DumpSave["overlapsforSpinState.mx",
  {oUpUp, oDwDw, oDwUp, oUpDw, oUpUpg, oDwDwg, oDwUpg, oUpDwg}];
```

b) Density matrix and fidelity: general analytics

In[ ]:=

```
<< AnalyticalExpressionOfTheFidelity.mx
```

In[733]:= ρmathematicalspin = $\begin{pmatrix} o\text{UpUp} & o\text{DwUp} \\ o\text{UpDw} & o\text{DwDw} \end{pmatrix}$ // mf;

$$\left(\begin{array}{l} \frac{1}{4}\times\left(2+\frac{e^{i t \lambda \Omega}\left(-1+nz^2\right)\gamma\left(-i\gamma+\lambda\Omega\right)}{\left(i\gamma+\Omega-\lambda\Omega\right)\left(\gamma+i\left(1+\lambda\right)\Omega\right)}+\frac{2\,nz^2\gamma^2}{\gamma^2+\Omega^2}-\frac{e^-}{} \right. \\ \frac{1}{4}\times\left(-2\, i\, e^{-t\gamma}\,\text{Sin}[t\,\Omega]-\frac{2\left(nx+i\,ny\right)\gamma\left(nz\,\gamma\left(-\gamma^4-2\gamma^2\left(1+\lambda^2\right)\Omega^2-\left(-1+\lambda^2\right)^2\Omega^4\right)+\left(\gamma^2+\Omega^2\right)\left(-i\lambda\Omega\left(\gamma^2+\left(-1+\lambda^2\right)\Omega^2\right)+nz\,\gamma\left(\gamma^2+\left(1+\lambda^2\right)\Omega^2\right.\right.\right.}{} \right. \end{array}\right.$$

The density matrix for the mathematical spin, i.e., the one considering the decay as well, is given by:

In[734]:= ρmathematicalspin = $\begin{pmatrix} o\text{UpUp} & o\text{DwUp} \\ o\text{UpDw} & o\text{DwDw} \end{pmatrix}$ // mf;

$$\left(\begin{array}{l} \frac{1}{4}\times\left(2+\frac{e^{i t \lambda \Omega}\left(-1+nz^2\right)\gamma\left(-i\gamma+\lambda\Omega\right)}{\left(i\gamma+\Omega-\lambda\Omega\right)\left(\gamma+i\left(1+\lambda\right)\Omega\right)}+\frac{2\,nz^2\gamma^2}{\gamma^2+\Omega^2}-\frac{e^-}{} \right. \\ \frac{1}{4}\times\left(-2\, i\, e^{-t\gamma}\,\text{Sin}[t\,\Omega]-\frac{2\left(nx+i\,ny\right)\gamma\left(nz\,\gamma\left(-\gamma^4-2\gamma^2\left(1+\lambda^2\right)\Omega^2-\left(-1+\lambda^2\right)^2\Omega^4\right)+\left(\gamma^2+\Omega^2\right)\left(-i\lambda\Omega\left(\gamma^2+\left(-1+\lambda^2\right)\Omega^2\right)+nz\,\gamma\left(\gamma^2+\left(1+\lambda^2\right)\Omega^2\right.\right.\right.}{} \right. \end{array}\right.$$

The density matrix of the spin state, i.e., the decay is completed is given by:

In[735]:= ρspin = $\begin{pmatrix} o\text{UpUpg} & o\text{DwUpg} \\ o\text{UpDwg} & o\text{DwDwg} \end{pmatrix}$ // mf;

$$\left(\begin{array}{l} \frac{1}{8}\times\left(4-4\,e^{-t\gamma}+\frac{2\,e^{i t\lambda\Omega}\left(-1+nz^2\right)\gamma\left(-i\gamma+\lambda\right.}{\left(i\gamma+\Omega-\lambda\Omega\right)\left(\gamma+i\left(1+\lambda\right)\Omega\right)}\right. \\ -\frac{e^{-2t\gamma-i t\left(-1+\lambda\right)\Omega}\left(nx+i\,ny\right)\gamma\left(e^{t\left(2\gamma-i\left(1-2\lambda\right)\Omega\right)}\left(1+nz\right)\left(\gamma-i\Omega\right)\left(\gamma+i\Omega\right)\left(\gamma-i\left(-1+\lambda\right)\Omega\right)\left(\gamma+i\lambda\Omega\right)\left(\gamma-i\left(1+\lambda\right)\Omega\right)+e^{2t\gamma-it\Omega}\left(-1+nz\right)\left(\gamma-i\Omega\right)\left(\gamma+i\right.\right.}{} \end{array}\right.$$

In[736]:= Tr[%] // cf

Out[736]= $1-e^{-t\gamma}$

It total sense. Now we have to normalize the density matrix in the ground state.

In[737]:= ρspin = $\frac{\rho\text{spin}}{\text{Tr}[\rho\text{spin}]}$;

In[257]:= Tr[%] // cf

Out[257]= 1

Target state:

Assuming we had a click in R, the spin in initially up. Under the action of the magnetic field alingned in the x-direction, without any perturbation, the state of the system in a time t=$\pi/(2\lambda\,\Omega)$ evolves to the target state:

In[258]:= σz.Basis[2, 1]

Out[258]= {1, 0}

In[259]:= ψtarget = MatrixExp$\left[-i\,\frac{\lambda\,\Omega}{2}\,\sigma x\;t\right]$.Basis[2, 1] /. t → π / (2 λ Ω) // mf;

ρtarget = out[ψtarget, ψtarget†] // mf;

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} \end{pmatrix}$$

A generic density matrix of a qubit can be written as:

In[261]:=
```
ρgen = (Eye[2] + rx σx + ry σy + rz σz)/2 // mf;
```

$$\begin{pmatrix} \frac{1+rz}{2} & \frac{1}{2}(rx - i\, ry) \\ \frac{1}{2}(rx + i\, ry) & \frac{1-rz}{2} \end{pmatrix}$$

The fidelity is easy computed between our target (at time t=$\pi/2r\Omega$, with r=1) and the general state:

In[262]:=
```
ℱ = QuantumFidelity[ρgen, ρtarget] // cf
```

Out[262]= $\frac{1 - ry}{2}$

where

In[263]:=
```
Tr[σy.ρgen] // cf
```

Out[263]= ry

In[266]:=
```
ρtest = {{ρuu, ρud}, {ρdu, ρdd}};
Tr[σy.ρtest] /. ρud → reρud + i imρud /. ρdu → reρud − i imρud // cf
```

Out[267]= −2 imρud

Hence, ry = - 2 Im{ρud}
So we have a way of obtaining an analytic expression for the fidelity, which is given by:

In[268]:=
```
Im[Tr[σy.ρspin] // ComplexExpand]
```

Out[268]= Im[… 1 …] + Re[− (… 1 …)/(4 … 2 … (… 1 …)) + … 12 874 …]

large output | show less | show more | show all | set size limit...

In[269]:= $\mathcal{F}\text{analytical} = \dfrac{1 - \text{Tr}[\sigma y.\rho\text{spin}]}{2}$ // cf

Out[269]=
$$\Big(e^{-t(\gamma+i\lambda\Omega)}\Big(e^{2t(\gamma+i\lambda\Omega)}(-i\,nx+ny\,nz)\,\gamma\,(\gamma-i\Omega)(\gamma+i\Omega)(\gamma-i(-1+\lambda)\Omega)(\gamma+i\lambda\Omega)(\gamma-i(1+\lambda)\Omega)+ e^{2t\gamma}(nx-i\,ny\,nz)\,\gamma\,(\gamma-i\Omega)(\gamma+i\Omega)(\gamma+i(-1+\lambda)\Omega)(i\gamma+\lambda\Omega)(\gamma+i(1+\lambda)\Omega)- 2\,e^{2t\gamma+it\lambda\Omega}\big((-1+ny\,nz)\gamma^2-\Omega^2\big)\big(\gamma^2+(-1+\lambda)^2\Omega^2\big)\big(\gamma^2+(1+\lambda)^2\Omega^2\big)- 2\,e^{t(\gamma+i\lambda\Omega)}\big(\gamma^2+\Omega^2\big)\big(\gamma^2+(-1+\lambda)^2\Omega^2\big)\big(\gamma^2+(1+\lambda)^2\Omega^2\big)+ e^{t(\gamma+i(-1+\lambda)\Omega)}\gamma\lambda(\gamma-i\Omega)\Omega\,(nx(\gamma+i\Omega)+ny\,nz\,\lambda\Omega)\big(\gamma^2-2i\gamma\Omega+(-1+\lambda^2)\Omega^2\big)+ e^{t(\gamma+i(1+\lambda)\Omega)}\gamma\lambda(\gamma+i\Omega)\Omega\,(nx(\gamma-i\Omega)+ny\,nz\,\lambda\Omega)\big(\gamma^2+2i\gamma\Omega+(-1+\lambda^2)\Omega^2\big)\Big)\Big)\Big/ \Big(4\times\big(-1+e^{t\gamma}\big)\big(\gamma^2+\Omega^2\big)\big(\gamma^4+2\gamma^2(1+\lambda^2)\Omega^2+(-1+\lambda^2)^2\Omega^4\big)\Big)$$

In[270]:= $\mathcal{F}\text{tg} = \mathcal{F}\text{analytical} /.\ t \to \dfrac{\pi}{2\lambda\Omega}$ // cf

Out[270]=
$$-\Bigg(\bigg(i\,e^{-\frac{\pi\gamma}{2\lambda\Omega}}\Big(e^{\frac{\pi(\gamma-i\Omega)}{2\lambda\Omega}}\gamma\lambda\Omega(i\gamma+\Omega)(nx(\gamma+i\Omega)+ny\,nz\,\lambda\Omega)\big(\gamma^2-2i\gamma\Omega+(-1+\lambda^2)\Omega^2\big)- 2i\,e^{\frac{\pi\gamma}{2\lambda\Omega}}\big(\gamma^2+\Omega^2\big)\big(\gamma^4+2\gamma^2(1+\lambda^2)\Omega^2+(-1+\lambda^2)^2\Omega^4\big)+2i\,e^{\frac{\pi\gamma}{\lambda\Omega}} \Big((1+nx-ny\,nz)\gamma^6+ny\,nz\,\gamma^5\lambda\Omega+\gamma^4\big(3+2\lambda^2-2\,ny\,nz(1+\lambda^2)+nx(2+\lambda^2)\big)\Omega^2+ ny\,nz\,\gamma^3\lambda^3\Omega^3+\gamma^2\big(3+\lambda^4-ny\,nz(-1+\lambda^2)^2+nx(1+\lambda^2)\big)\Omega^4+ ny\,nz\,\gamma\lambda(-1+\lambda^2)\Omega^5+(-1+\lambda^2)^2\Omega^6\Big)+ e^{\frac{\pi(\gamma+i\Omega)}{2\lambda\Omega}}\gamma\lambda(\gamma+i\Omega)\Omega\big(\gamma^2+2i\gamma\Omega+(-1+\lambda^2)\Omega^2\big)(i\,ny\,nz\,\lambda\Omega+nx(i\gamma+\Omega))\Big)\bigg)\Big/ \bigg(4\times\Big(-1+e^{\frac{\pi\gamma}{2\lambda\Omega}}\Big)\big(\gamma^2+\Omega^2\big)\big(\gamma^4+2\gamma^2(1+\lambda^2)\Omega^2+(-1+\lambda^2)^2\Omega^4\big)\bigg)\Bigg)$$

Let’s save this nice expression:

In[271]:= `DumpSave["AnalyticalExpressionOfTheFidelity.mx", {ℱanalytical, ℱtg}];`

2) Fidelity for $t_{pulse} = t_g = \pi/(2\,\Omega_g) = \pi/(2\,r_{ge}\Omega)$☑

3) Fidelity for $t_{pulse} = t_{optimal}$☑

a) Finding $t_{optimal}$

In[807]:=
$$\text{Fidelity} = \Bigg(-\frac{ny\,nz\,\gamma^6}{2\left(\gamma^2+\Omega e^2\right)\left(\gamma^2+(\Omega e-\Omega g)^2\right)\left(\gamma^2+(\Omega e+\Omega g)^2\right)}- \frac{ny\,nz\,\gamma^4\,\Omega e^2}{\left(\gamma^2+\Omega e^2\right)\left(\gamma^2+(\Omega e-\Omega g)^2\right)\left(\gamma^2+(\Omega e+\Omega g)^2\right)}- \frac{ny\,nz\,\gamma^2\,\Omega e^4}{2\left(\gamma^2+\Omega e^2\right)\left(\gamma^2+(\Omega e-\Omega g)^2\right)\left(\gamma^2+(\Omega e+\Omega g)^2\right)}- \frac{ny\,nz\,\gamma^4\,\Omega g^2}{\left(\gamma^2+\Omega e^2\right)\left(\gamma^2+(\Omega e-\Omega g)^2\right)\left(\gamma^2+(\Omega e+\Omega g)^2\right)}+$$

$$\frac{ny\ nz\ \gamma^2\ \Omega e^2\ \Omega g^2}{(\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}-$$
$$\frac{ny\ nz\ \gamma^2\ \Omega g^4}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{ny\ nz\ \gamma^6\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{ny\ nz\ \gamma^4\ \Omega e^2\ \mathrm{Cos}[T\ \Omega g]}{(\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{ny\ nz\ \gamma^2\ \Omega e^4\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}-$$
$$\frac{nx\ \gamma^5\ \Omega g\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{nx\ \gamma\ \Omega e^4\ \Omega g\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{ny\ nz\ \gamma^4\ \Omega g^2\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{ny\ nz\ \gamma^2\ \Omega e^2\ \Omega g^2\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}-$$
$$\frac{nx\ \gamma^3\ \Omega g^3\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}-$$
$$\frac{nx\ \gamma\ \Omega e^2\ \Omega g^3\ \mathrm{Cos}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{nx\ \gamma^6\ \mathrm{Sin}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{nx\ \gamma^4\ \Omega e^2\ \mathrm{Sin}[T\ \Omega g]}{(\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{nx\ \gamma^2\ \Omega e^4\ \mathrm{Sin}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{ny\ nz\ \gamma^5\ \Omega g\ \mathrm{Sin}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}-$$
$$\frac{ny\ nz\ \gamma\ \Omega e^4\ \Omega g\ \mathrm{Sin}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{nx\ \gamma^4\ \Omega g^2\ \mathrm{Sin}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$
$$\frac{nx\ \gamma^2\ \Omega e^2\ \Omega g^2\ \mathrm{Sin}[T\ \Omega g]}{2\ (\gamma^2+\Omega e^2)\ (\gamma^2+(\Omega e-\Omega g)^2)\ (\gamma^2+(\Omega e+\Omega g)^2)}+$$

$$\frac{\text{ny nz}\,\gamma^3\,\Omega g^3\,\text{Sin}[T\,\Omega g]}{2\left(\gamma^2+\Omega e^2\right)\left(\gamma^2+(\Omega e-\Omega g)^2\right)\left(\gamma^2+(\Omega e+\Omega g)^2\right)}+\frac{\text{ny nz}\,\gamma\,\Omega e^2\,\Omega g^3\,\text{Sin}[T\,\Omega g]}{2\left(\gamma^2+\Omega e^2\right)\left(\gamma^2+(\Omega e-\Omega g)^2\right)\left(\gamma^2+(\Omega e+\Omega g)^2\right)}\Bigg)\ /.\ \gamma\to 1\ //\ \text{Simplify}$$

Out[807]= $$\Big(-\text{ny nz}\left(\Omega e^4-2\,\Omega e^2\left(-1+\Omega g^2\right)+\left(1+\Omega g^2\right)^2\right)+\left(1+\Omega e^2\right)\left(\text{nx}\,\Omega g\left(-1+\Omega e^2-\Omega g^2\right)+\text{ny nz}\left(1+\Omega e^2+\Omega g^2\right)\right)\text{Cos}[T\,\Omega g]+\left(1+\Omega e^2\right)\left(\text{nx}\left(1+\Omega e^2+\Omega g^2\right)+\text{ny nz}\left(\Omega g-\Omega e^2\,\Omega g+\Omega g^3\right)\right)\text{Sin}[T\,\Omega g]\Big)\Big/\left(2\times\left(1+\Omega e^2\right)\times\left(1+\Omega e^2-2\,\Omega e\,\Omega g+\Omega g^2\right)\times\left(1+\Omega e^2+2\,\Omega e\,\Omega g+\Omega g^2\right)\right)$$

In[811]:= approxf = Fidelity /. $\{\Omega e^2\to 0,\ \Omega g^2\to 0,\ \Omega e\,\Omega g\to 0,\ \Omega e^4\to 0\}$
(*Get rid of all the higher order terms*)

Out[811]= $$\frac{1}{2}\left(-\text{ny nz}+(\text{ny nz}-\text{nx}\,\Omega g)\,\text{Cos}[T\,\Omega g]+\left(\text{nx}+\text{ny nz}\left(\Omega g+\Omega g^3\right)\right)\text{Sin}[T\,\Omega g]\right)$$

sol = Solve[D[approxf, T] == 0, T]

$$\Bigg\{\Bigg\{T\to\text{ConditionalExpression}\Bigg[\frac{1}{\Omega g}\Bigg(\text{ArcTan}\Bigg[-\frac{\text{ny nz}-\text{nx}\,\Omega g}{\sqrt{\text{nx}^2+\text{ny}^2\,\text{nz}^2+\text{nx}^2\,\Omega g^2+\text{ny}^2\,\text{nz}^2\,\Omega g^2}},\ \frac{\frac{\text{nx ny nz}}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}-\frac{\text{nx}^2\,\Omega g}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}+\frac{\text{ny}^2\,\text{nz}^2\,\Omega g}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}-\frac{\text{nx ny nz}\,\Omega g^2}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}}{-\text{ny nz}+\text{nx}\,\Omega g}\Bigg]+2\,\pi\,C[1]\Bigg),\ C[1]\in\text{Integers}\Bigg]\Bigg\},$$

$$\Bigg\{T\to\text{ConditionalExpression}\Bigg[\frac{1}{\Omega g}\Bigg(\text{ArcTan}\Bigg[\frac{\text{ny nz}-\text{nx}\,\Omega g}{\sqrt{\text{nx}^2+\text{ny}^2\,\text{nz}^2+\text{nx}^2\,\Omega g^2+\text{ny}^2\,\text{nz}^2\,\Omega g^2}},\ \frac{-\frac{\text{nx ny nz}}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}+\frac{\text{nx}^2\,\Omega g}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}-\frac{\text{ny}^2\,\text{nz}^2\,\Omega g}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}+\frac{\text{nx ny nz}\,\Omega g^2}{\sqrt{(\text{nx}^2+\text{ny}^2\,\text{nz}^2)(1+\Omega g^2)}}}{-\text{ny nz}+\text{nx}\,\Omega g}\Bigg]+2\,\pi\,C[1]\Bigg),\ C[1]\in\text{Integers}\Bigg]\Bigg\}\Bigg\}$$

```
T /. sol /. {nx^2 ny^2 → 0, nx^2 nz^2 → 0, nz^2 ny^2 → 0}
```

$$\left\{\text{ConditionalExpression}\left[\frac{\text{ArcTan}\left[-\frac{\text{ny nz}-\text{nx }\Omega g}{\sqrt{\text{nx}^2+\text{nx}^2\,\Omega g^2}},\ \frac{\frac{\text{nx ny nz}}{\sqrt{\text{nx}^2(1+\Omega g^2)}}-\frac{\text{nx}^2\,\Omega g}{\sqrt{\text{nx}^2(1+\Omega g^2)}}-\frac{\text{nx ny nz }\Omega g^2}{\sqrt{\text{nx}^2(1+\Omega g^2)}}}{-\text{ny nz}+\text{nx }\Omega g}\right]+2\pi\,C[1]}{\Omega g},\ C[1]\in\text{Integers}\right],\ \text{ConditionalExpression}\left[\frac{\text{ArcTan}\left[\frac{\text{ny nz}-\text{nx }\Omega g}{\sqrt{\text{nx}^2+\text{nx}^2\,\Omega g^2}},\ \frac{-\frac{\text{nx ny nz}}{\sqrt{\text{nx}^2(1+\Omega g^2)}}+\frac{\text{nx}^2\,\Omega g}{\sqrt{\text{nx}^2(1+\Omega g^2)}}+\frac{\text{nx ny nz }\Omega g^2}{\sqrt{\text{nx}^2(1+\Omega g^2)}}}{-\text{ny nz}+\text{nx }\Omega g}\right]+2\pi\,C[1]}{\Omega g},\ C[1]\in\text{Integers}\right]\right\}$$

$$\textbf{trial} = \frac{\text{ArcTan}\left[-\frac{\text{ny nz}-\text{nx }\Omega g}{\sqrt{\text{nx}^2+\text{nx}^2\,\Omega g^2}},\ \frac{\frac{\text{nx ny nz}}{\sqrt{\text{nx}^2(1+\Omega g^2)}}-\frac{\text{nx}^2\,\Omega g}{\sqrt{\text{nx}^2(1+\Omega g^2)}}-\frac{\text{nx ny nz }\Omega g^2}{\sqrt{\text{nx}^2(1+\Omega g^2)}}}{-\text{ny nz}+\text{nx }\Omega g}\right]}{\Omega g}\ \textbf{// FullSimplify}$$

$$\frac{\text{ArcTan}\left[\frac{-\text{ny nz}+\text{nx }\Omega g}{\sqrt{\text{nx}^2(1+\Omega g^2)}},\ -\frac{\text{nx}\left(\text{nx }\Omega g+\text{ny nz}\left(-1+\Omega g^2\right)\right)}{\left(-\text{ny nz}+\text{nx }\Omega g\right)\sqrt{\text{nx}^2(1+\Omega g^2)}}\right]}{\Omega g}$$

```
Fidelity /. {T → Pi / Ωg - ArcTan[nx / (nx Ωg - ny nz)] / Ωg}
```

$$\Bigg(-\text{ny nz}\left(\Omega e^4-2\,\Omega e^2\left(-1+\Omega g^2\right)+\left(1+\Omega g^2\right)^2\right)+\left(1+\Omega e^2\right)\left(\text{nx }\Omega g\left(-1+\Omega e^2-\Omega g^2\right)+\text{ny nz}\left(1+\Omega e^2+\Omega g^2\right)\right)\text{Cos}\left[\Omega g\left(\frac{\pi}{\Omega g}-\frac{\text{ArcTan}\left[\frac{\text{nx}}{-\text{ny nz}+\text{nx }\Omega g}\right]}{\Omega g}\right)\right]+\left(1+\Omega e^2\right)\left(\text{ny nz }\Omega g\left(1-\Omega e^2+\Omega g^2\right)+\text{nx}\left(1+\Omega e^2+\Omega g^2\right)\right)\text{Sin}\left[\Omega g\left(\frac{\pi}{\Omega g}-\frac{\text{ArcTan}\left[\frac{\text{nx}}{-\text{ny nz}+\text{nx }\Omega g}\right]}{\Omega g}\right)\right]\Bigg)\Big/\left(2\times\left(1+\Omega e^2\right)\times\left(1+\Omega e^2-2\,\Omega e\,\Omega g+\Omega g^2\right)\times\left(1+\Omega e^2+2\,\Omega e\,\Omega g+\Omega g^2\right)\right)$$

b) Analytical expression and plots

Components of the fidelity as a function of time is given by:

In[738]:=

$$\Omega s = \left\{\Delta\Omega m \to \frac{(\Omega g - \Omega e)}{2}, \Delta\Omega p \to \frac{(\Omega g + \Omega e)}{2}\right\};$$

$$fxt = \frac{1 + (\Omega e / \gamma)^2 + (\Omega g / \gamma)^2}{\left(1 + 4\left(\frac{\Delta\Omega m}{\gamma}\right)^2\right) \times \left(1 + 4\left(\frac{\Delta\Omega p}{\gamma}\right)^2\right)} \mathrm{Sin}[\Omega g\, t] - \frac{1 - (\Omega e / \gamma)^2 + (\Omega g / \gamma)^2}{\left(1 + 4\left(\frac{\Delta\Omega m}{\gamma}\right)^2\right) \times \left(1 + 4\left(\frac{\Delta\Omega p}{\gamma}\right)^2\right)} \frac{\Omega g}{\gamma} \mathrm{Cos}[\Omega g\, t] \;/.\; \Omega s \;/.\; \Omega e \to \Omega \;/.\; \Omega g \to \lambda\,\Omega \;//\; cf;$$

$$fyzt = \frac{(\Omega g / \gamma)\,\mathrm{Sin}[\Omega g\, t] - 1 - 2\left(\frac{\Omega e}{\gamma}\right)^2 - 2\left(\frac{\Omega g}{\gamma}\right)^2}{\left(1 + \left(\frac{\Omega e}{\gamma}\right)^2\right) \times \left(1 + 4\left(\frac{\Delta\Omega m}{\gamma}\right)^2\right) \times \left(1 + 4\left(\frac{\Delta\Omega p}{\gamma}\right)^2\right)} + \frac{1 + \left(\frac{\Omega e}{\gamma}\right)^2 + \left(\frac{\Omega g}{\gamma}\right)^2}{\left(1 + 4\left(\frac{\Delta\Omega m}{\gamma}\right)^2\right) \times \left(1 + 4\left(\frac{\Delta\Omega p}{\gamma}\right)^2\right)} \mathrm{Cos}[\Omega g\, t] \;/.\; \Omega s \;/.\; \Omega e \to \Omega \;/.\; \Omega g \to \lambda\,\Omega \;//\; cf;$$

In[741]:=

$$toptimal = \frac{\pi}{\Omega g} - \frac{1}{\Omega g} \mathrm{ArcTan}\left[\frac{nx}{nx\,\Omega g - ny\,nz}\right] \;/.\; \Omega e \to \Omega \;/.\; \Omega g \to \lambda\,\Omega;$$

The fidelity as a function of time is given by:

In[742]:=

$$\mathcal{F}t = \frac{1}{2} - \frac{ny\,nz}{2} fyzt + \frac{nx}{2} fxt \;//\; cf$$

Out[742]=

$$\frac{1}{2} \times \left(1 + \frac{nx\,\gamma\left(-\lambda\,\Omega\left(\gamma^2 + \left(-1 + \lambda^2\right)\Omega^2\right)\mathrm{Cos}[t\,\lambda\,\Omega] + \gamma\left(\gamma^2 + \left(1 + \lambda^2\right)\Omega^2\right)\mathrm{Sin}[t\,\lambda\,\Omega]\right)}{\gamma^4 + 2\,\gamma^2\left(1 + \lambda^2\right)\Omega^2 + \left(-1 + \lambda^2\right)^2\Omega^4} + \left(ny\,nz\,\gamma^2\left(-\left(\left(\gamma^2 + \Omega^2\right)\left(\gamma^2 + \left(1 + \lambda^2\right)\Omega^2\right)\mathrm{Cos}[t\,\lambda\,\Omega]\right) + \gamma^2\left(\gamma^2 + 2 \times \left(1 + \lambda^2\right)\Omega^2 - \gamma\,\lambda\,\Omega\,\mathrm{Sin}[t\,\lambda\,\Omega]\right)\right)\right) \Big/ \left(\left(\gamma^2 + \Omega^2\right)\left(\gamma^2 + (-1 + \lambda)^2\,\Omega^2\right)\left(\gamma^2 + (1 + \lambda)^2\,\Omega^2\right)\right)\right)$$

In[743]:=

$$\mathcal{F}toptimal = \mathcal{F}t \;/.\; t \to toptimal;$$

In[744]:=

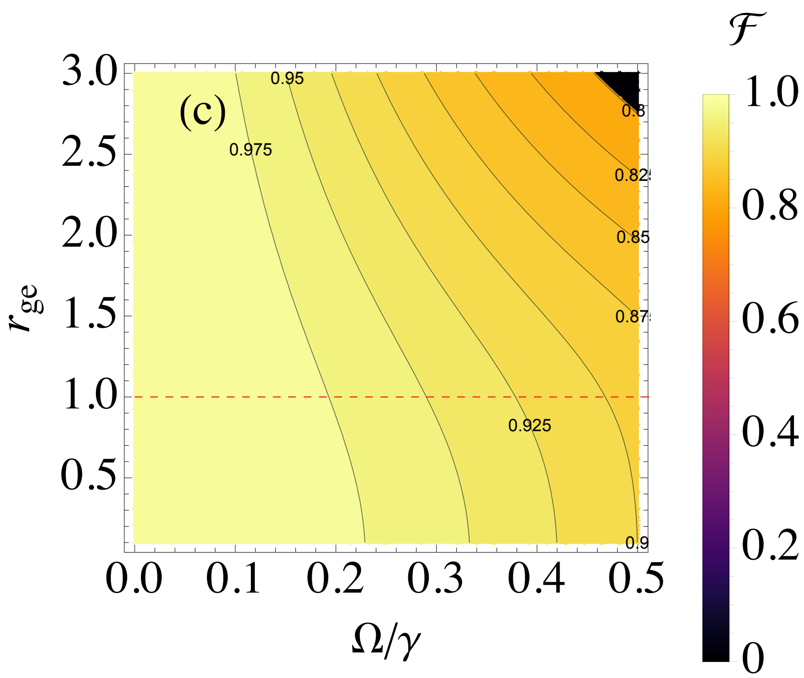
$$\mathcal{F}\lambda\Omega 2 = \mathcal{F}toptimal \;/.\; idealalignementparamaters \;//\; cf$$

Out[744]=

$$\frac{1}{2} \times \left(1 + \frac{1 + \Omega^2\left(1 + \lambda^2\left(2 + \left(-1 + \lambda^2\right)\Omega^2\right)\right)}{\sqrt{1 + \lambda^2\,\Omega^2}\left(1 + 2 \times \left(1 + \lambda^2\right)\Omega^2 + \left(-1 + \lambda^2\right)^2\Omega^4\right)}\right)$$

In[745]:=
```
Module[{cf},

 cf = MPLColorMap["Inferno"];

 plot3 =
  ContourPlot[ℱλΩ2, {Ω, 0., 0.5}, {λ, 0.1, 3.},
   PlotRange → All,
   AspectRatio → 7 / 7.5,
   FrameLabel → {Style["Ω/γ", FontFamily → "Times", FontSize → fontsize, Black],
     Style["r_ge", FontFamily → "Times", FontSize → fontsize, Black]},
   LabelStyle → Directive[FontFamily → "Times", FontSize → fontsize, Black],
   ColorFunction → cf, ContourLabels → All, ColorFunctionScaling → False,
   PlotLegends → Placed[BarLegend[{cf, {0, 1}}, LegendLabel → Style["ℱ",
        FontFamily → "Times", FontSize → fontsize, Black]], Right], Epilog → {
     {Text[Style["(c)", FontSize → fontsize, FontFamily → "Times",
        FontColor → Black], Scaled[{0.15, 0.9}]]},

     (*Horizontal line*){Red, Dashed, Line[{{0, 1}, {1, 1}}]}},
   ImageSize → 350]
]
```

Out[745]=

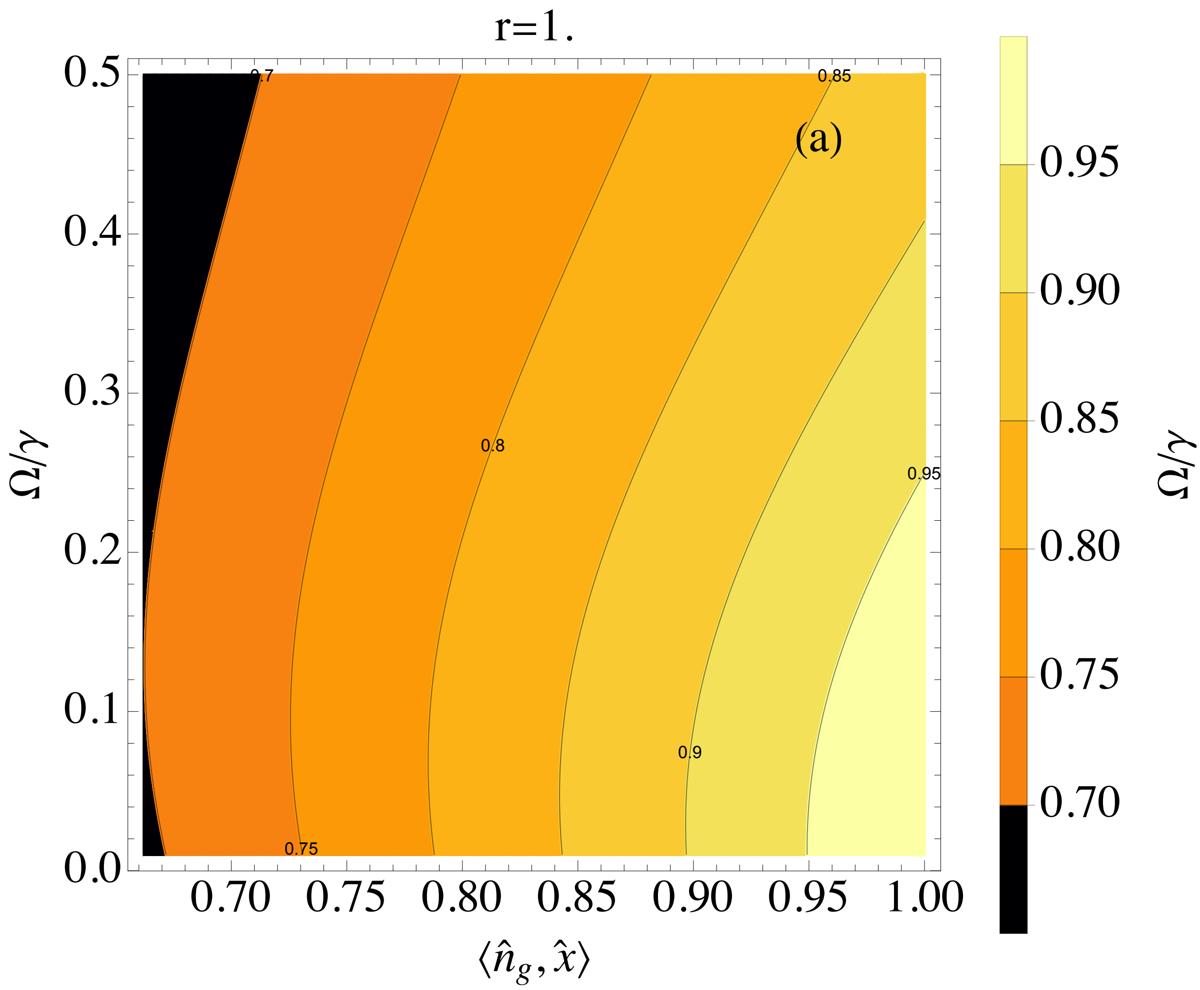


In[559]:=
```
λλlist = {1.0, 1.5, 2.5};
Module[{nhat, parOptimal, xhat},
 nhat = Normalize@{1, nn, nn};
 xhat = {1, 0, 0};
 parOptimal = {γ → 1, t → toptimal, λ → λλ,
   nx → N[nhat[[1]]], ny → N[nhat[[2]]], nz → N[nhat[[3]]], Ω → ΩΩ};
 Do[

  fidelitytOptimalcontplot[λλ] =
   Flatten[Table[{nhat.xhat, N@ΩΩ, Chop@N[ℱtoptimal /. parOptimal]},
     {ΩΩ, 10^-2, 0.5, 10^-3}, {nn, 0, 0.8, 10^-2}], 1]

  , {λλ, λλlist}]
]
```

In[561]:=
```
λλlist = {1.0, 1.5, 2.5};
Module[{nhat, partg, xhat},
 nhat = Normalize@{1, nn, nn};
 xhat = {1, 0, 0};
 partg = {γ → 1, t → π / (2 λ Ω), λ → λλ,
   nx → N[nhat[[1]]], ny → N[nhat[[2]]], nz → N[nhat[[3]]], Ω → ΩΩ};
 Do[

  fidelitytgcontplot[λλ] = Flatten[Table[{nhat.xhat, N@ΩΩ, Chop@N[ℱch /. partg]},
     {ΩΩ, 10^-2, 0.5, 10^-3}, {nn, 0, 0.8, 10^-2}], 1]

  , {λλ, λλlist}]
]
```

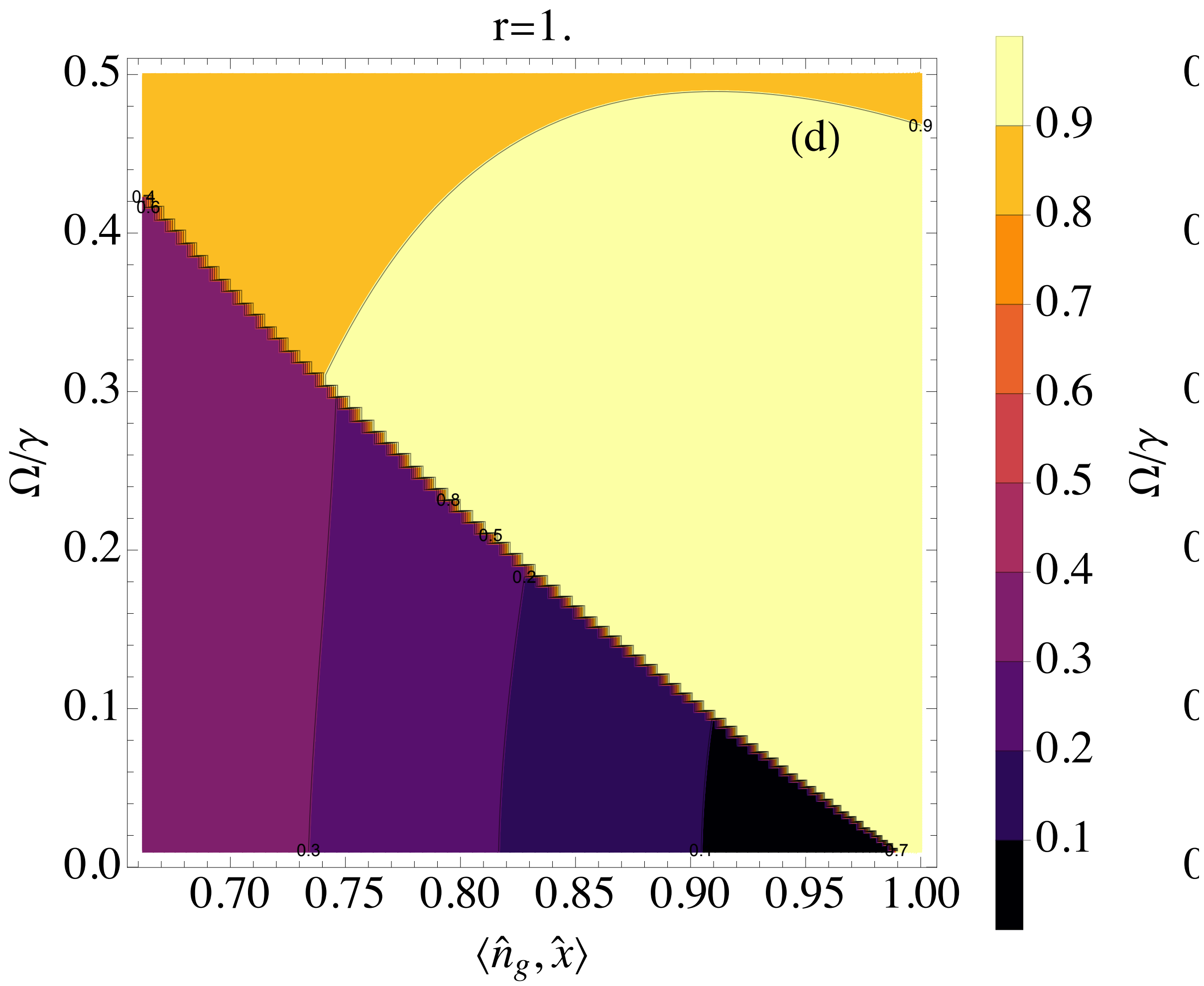
```
In[796]:= Module[{cf, plotSize, frameLabelStyle},

   cf = MPLColorMap["Inferno"];
   plotSize = 500;
   frameLabelStyle =
    {FontSize → fontsize, FontColor → Black, FontFamily → "Times"};
   contourplotsInnerProdTg = Grid[{Table[
        ListContourPlot[fidelitytgcontplot[λλlist[[j]]],
         PlotRange → All, FrameLabel →
          {Style["⟨n̂g, x̂⟩", FontFamily → "Times", FontSize → fontsize, Black],
           Style["Ω/γ", FontFamily → "Times", FontSize → fontsize, Black]},
         LabelStyle → Directive[FontFamily → "Times", FontSize → fontsize, Black],
         ColorFunction → cf, ContourLabels → All, ColorFunctionScaling → False,
         ImageSize → plotSize,
         Epilog → {Text[Style[alphabetlabel[[j]], FontSize → fontsize,
             FontFamily → "Times", FontColor → Black], Scaled[{0.85, 0.9}]]},
         PlotLegends → Automatic, PlotLabel → Style["r=" <> ToString[λλlist[[j]]],
           FontSize → fontsize, FontFamily → "Times", FontColor → Black]
  ]
        , {j, 1, Length@λλlist}]}]
  ]
  savePlot["Fidelity-ContPlot-InnerProdTg.jpg", contourplotsInnerProdTg]
```

Out[796]=


r=1.
(a)
0.7
0.75
0.8
0.85
0.9
0.95
Ω/γ
⟨n̂g, x̂⟩
0.0
0.1
0.2
0.3
0.4
0.5
0.70
0.75
0.80
0.85
0.90
0.95
1.00

In[798]:=
```
Module[{cf, plotSize, frameLabelStyle},

  cf = MPLColorMap["Inferno"];
  plotSize = 500;
  frameLabelStyle =
   {FontSize → fontsize, FontColor → Black, FontFamily → "Times"};
  contourplotsInnerProdTOptimal = Grid[{Table[
       ListContourPlot[fidelitytOptimalcontplot[λλlist〚j〛],
        PlotRange → All, FrameLabel →
         {Style["⟨n̂g, x̂⟩", FontFamily → "Times", FontSize → fontsize, Black],
          Style["Ω/γ", FontFamily → "Times", FontSize → fontsize, Black]},
        LabelStyle → Directive[FontFamily → "Times", FontSize → fontsize, Black],
        ColorFunction → cf, ContourLabels → All, ColorFunctionScaling → False,
        ImageSize → plotSize,
        Epilog → {Text[Style[alphabetlabel〚j + 3〛, FontSize → fontsize,
             FontFamily → "Times", FontColor → Black], Scaled[{0.85, 0.9}]]},
        PlotLegends → Automatic, PlotLabel → Style["r=" <> ToString[λλlist〚j〛],
          FontSize → fontsize, FontFamily → "Times", FontColor → Black]
 ]
       , {j, 1, Length@λλlist}]}]
 ]
savePlot["Fidelity-ContPlot-InnerProdTOptimal.jpg",
  contourplotsInnerProdTOptimal]
```

Out[798]=



IV. Spin decoupling☑

1) Analytics for Intensities and cBhat ☑

In[577]:=

```
IRUpSE = γ Abs[f0SE["e", "upm", "e", "upm", t]]^2 // cf
IRDwSE = γ Abs[f0SE["e", "upm", "e", "dwm", t]]^2 // cf

ILUpSE = γ Abs[f0SE["e", "dwm", "e", "upm", t]]^2 // cf
ILDwSE = γ Abs[f0SE["e", "dwm", "e", "dwm", t]]^2 // cf
```

Out[577]= $e^{-t\gamma}\,\gamma\,\mathrm{Cos}\left[\frac{t\,\Omega}{2}\right]^2$

Out[578]= $e^{-t\gamma}\,\gamma\,\mathrm{Sin}\left[\frac{t\,\Omega}{2}\right]^2$

Out[579]= $e^{-t\gamma}\,\gamma\,\mathrm{Sin}\left[\frac{t\,\Omega}{2}\right]^2$

Out[580]= $e^{-t\gamma}\,\gamma\,\mathrm{Cos}\left[\frac{t\,\Omega}{2}\right]^2$

In[581]:= IRUpSE

Out[581]= $e^{-t\gamma}\,\gamma\,\mathrm{Cos}\left[\frac{t\,\Omega}{2}\right]^2$

In[582]:= pRUp = Integrate[IRUpSE, {t, 0, τ}]

Out[582]= $\frac{1}{4}\gamma\left(\frac{2-2\,e^{-\gamma\tau}}{\gamma}+\frac{1-e^{-\tau(\gamma-i\,\Omega)}}{\gamma-i\,\Omega}+\frac{1-e^{-\tau(\gamma+i\,\Omega)}}{\gamma+i\,\Omega}\right)$

In[583]:= % // cf

Out[583]= $\frac{1}{2\left(\gamma^2+\Omega^2\right)}\,e^{-\gamma\tau}\left(\left(-1+2\,e^{\gamma\tau}\right)\gamma^2+\left(-1+e^{\gamma\tau}\right)\Omega^2-\gamma^2\,\mathrm{Cos}[\tau\,\Omega]+\gamma\,\Omega\,\mathrm{Sin}[\tau\,\Omega]\right)$

In[584]:= % // Expand

Out[584]= $\frac{\gamma^2}{\gamma^2+\Omega^2}-\frac{e^{-\gamma\tau}\gamma^2}{2\left(\gamma^2+\Omega^2\right)}+\frac{\Omega^2}{2\left(\gamma^2+\Omega^2\right)}-\frac{e^{-\gamma\tau}\Omega^2}{2\left(\gamma^2+\Omega^2\right)}-\frac{e^{-\gamma\tau}\gamma^2\,\mathrm{Cos}[\tau\,\Omega]}{2\left(\gamma^2+\Omega^2\right)}+\frac{e^{-\gamma\tau}\gamma\,\Omega\,\mathrm{Sin}[\tau\,\Omega]}{2\left(\gamma^2+\Omega^2\right)}$

Assuming $\gamma\,\tau >> 1$, we neglect the exponential terms (SS stands for steady state):

In[585]:= pRUpSS = $\frac{\gamma^2}{\gamma^2+\Omega^2}+\frac{\Omega^2}{2\left(\gamma^2+\Omega^2\right)}$ (*$-\frac{e^{-\gamma\tau}\gamma^2\,\mathrm{Cos}[\tau\,\Omega]}{2\left(\gamma^2+\Omega^2\right)}+\frac{e^{-\gamma\tau}\gamma\,\Omega\,\mathrm{Sin}[\tau\,\Omega]}{2\left(\gamma^2+\Omega^2\right)}-\frac{e^{-\gamma\tau}\gamma^2}{2\left(\gamma^2+\Omega^2\right)}-\frac{e^{-\gamma\tau}\Omega^2}{2\left(\gamma^2+\Omega^2\right)}$*) // cf

Out[585]= $\frac{1}{2}\times\left(1+\frac{\gamma^2}{\gamma^2+\Omega^2}\right)$

In[586]:= cBhatSE = 2 $\sqrt{\text{pRUpSS (1 - pRUpSS)}}$ // cf

Out[586]= $\frac{\Omega\sqrt{2\,\gamma^2+\Omega^2}}{\gamma^2+\Omega^2}$

Expanding in Taylor series for Ω close to 0:

In[588]:= Series[cBhatSE, {Ω, 0, 10}]

Out[588]= $\frac{\sqrt{2}\,\Omega}{\gamma}-\frac{3\,\Omega^3}{2\left(\sqrt{2}\,\gamma^3\right)}+\left(-\frac{9}{16\sqrt{2}\,\gamma^5}+\frac{\sqrt{2}}{\gamma^5}\right)\Omega^5+\left(\frac{37}{64\sqrt{2}\,\gamma^7}-\frac{\sqrt{2}}{\gamma^7}\right)\Omega^7+\left(-\frac{597}{1024\sqrt{2}\,\gamma^9}+\frac{\sqrt{2}}{\gamma^9}\right)\Omega^9+O[\Omega]^{11}$

In[714]:=
```
cBhatPlot = Plot[cBhatSE /. γ → 1, {Ω, 0, 0.5},
  AspectRatio → 10 / 20,
  PlotRange → {{0, 0.51}, {0, 1}},
  PlotStyle → {red},
  FrameLabel → {"Ω/γ", "B_cl"}]
(*Save the figure in every project it is used.*)
savePlot["cBhatPlot.pdf", cBhatPlot]
```

Out[714]=