Defense - 13/11/2023

Exactly modeling the unitary dynamics of quantum interfaces with collision models

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Introduction

- Quantum interfaces→ transfer quantum states between light and matter
  - Interactions between single (artificial) atoms & photons
  - Quantum networks
- Artificial atoms + waveguides = light matter interactions
  - Superconducting circuits
  - Quantum dots

Quantum network

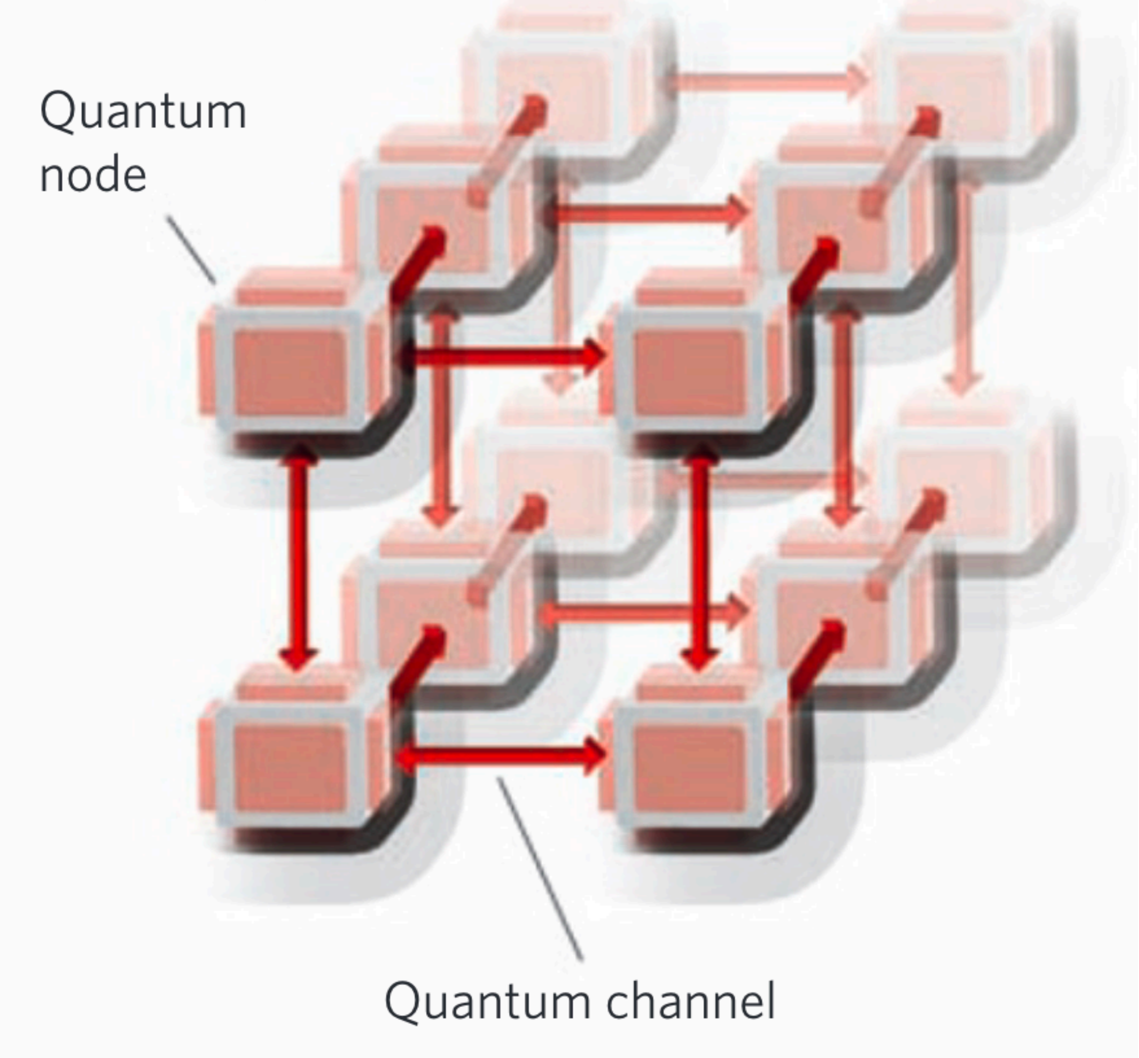


Figure taken from Ref. [1]

Quantum dot in a micropillar

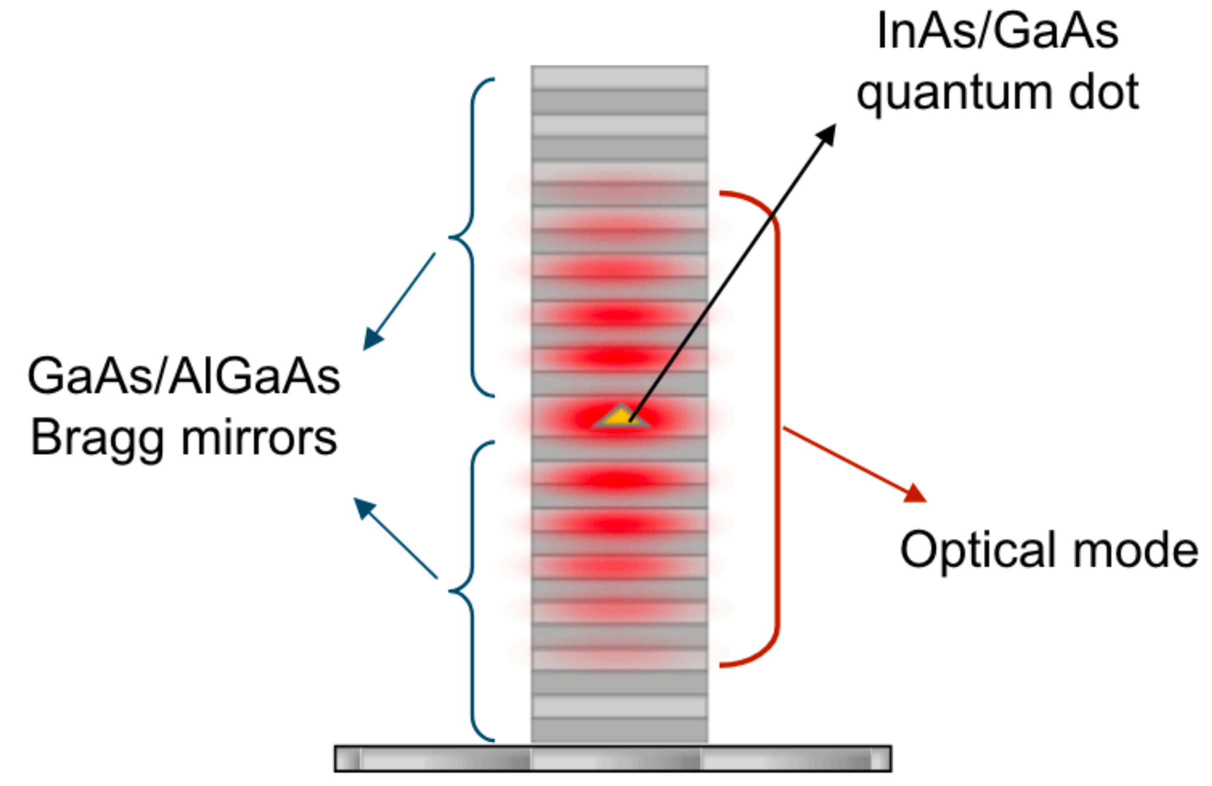


Figure taken from Ref. [2]

Superconducting circuits

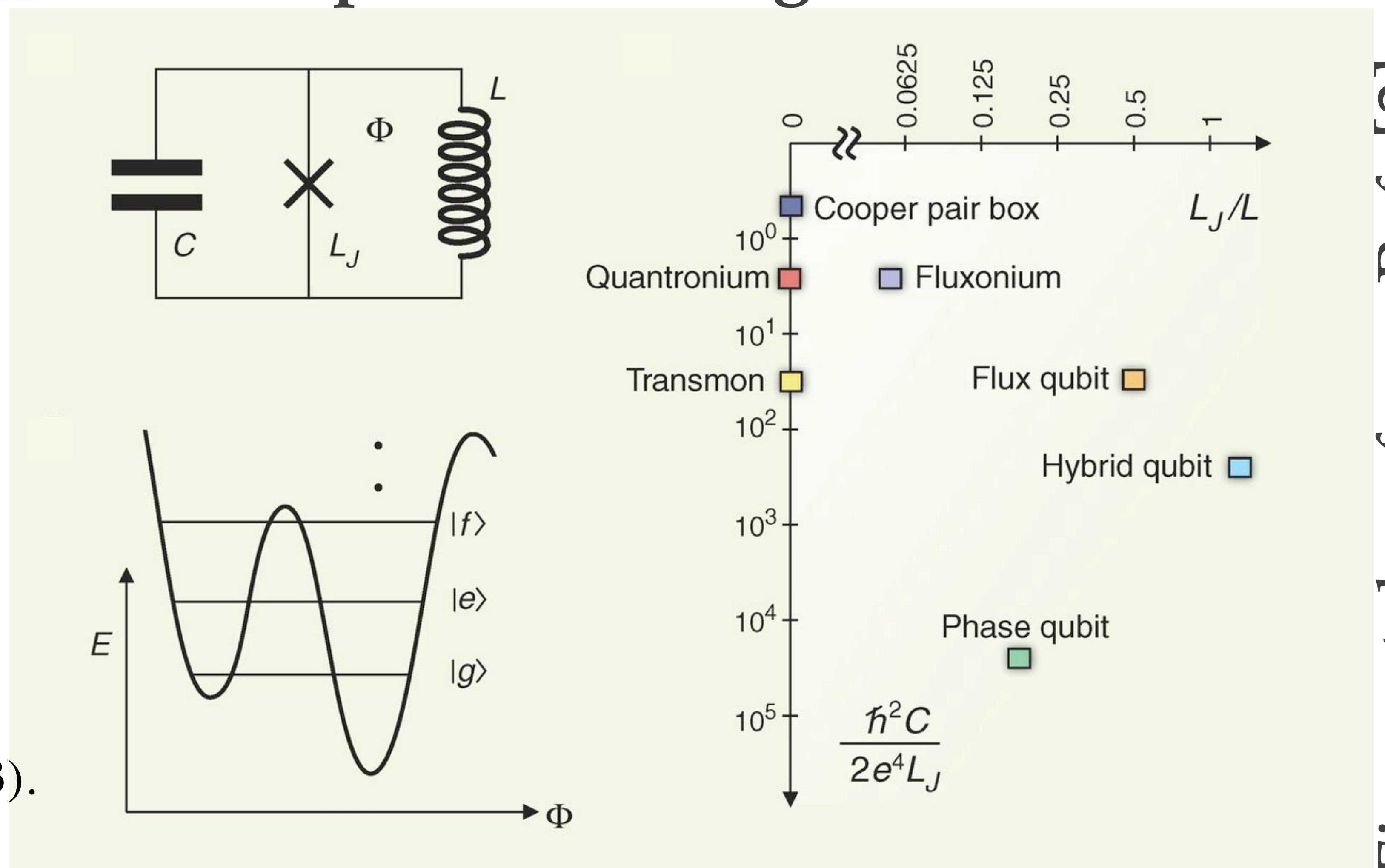


Figure taken from Ref. [3]

[1] Kimble, H. J. The quantum internet. Nature 453, 1023–1030 (2008).
[2] Lanco, L. & Senellart, P. A highly efficient single photon-single quantum dot interface. arXiv:1502.01062 (2015).
[3] Devoret, M. H. & Schoelkopf, R. J. Superconducting Circuits for Quantum Information: An Outlook. Science 339, 1169–1174 (2013).

Scope of the thesis

Quantum measurement

Fundamental

- Emergence of non-classical features?
- Non-destructive measurement, I.e., quality of the entanglement generated?

Technological applications

- Photon-photon gate
- Lindner-Rudolph protocol

Theoretical tool: Collisional model →Full-information about the entangled state of the systems

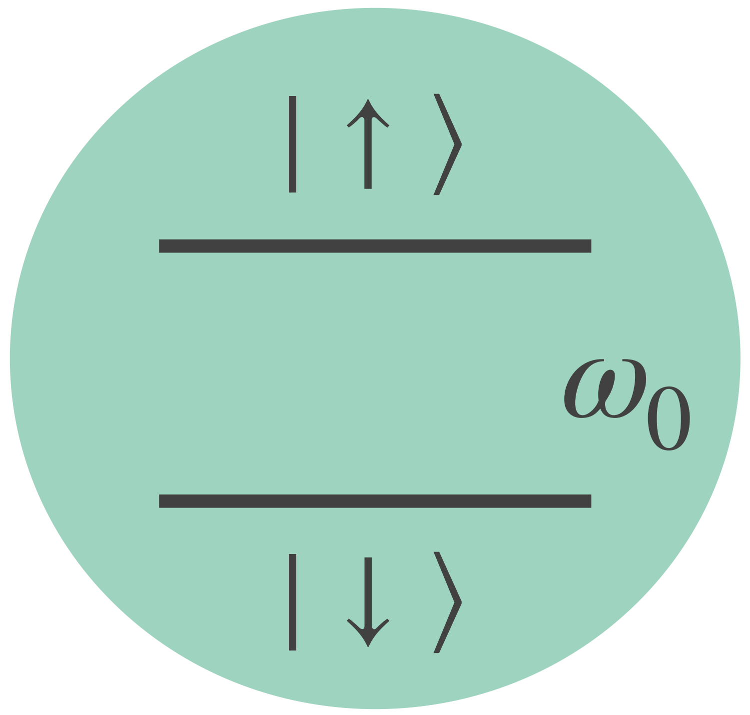
Outline

- Preliminaries
  - von Neumann measurement model
  - Collisional model or how to close open systems
- Main results
  - Anomalous energy exchanges and Wigner-function negativities in a single qubit gate
  - Energy efficient entanglement generation and readout in a spin-photon interface
  - The SPI subjected to an in-plane magnetic field
- Conclusions and perspectives

How can we **measure** the state of a qubit?

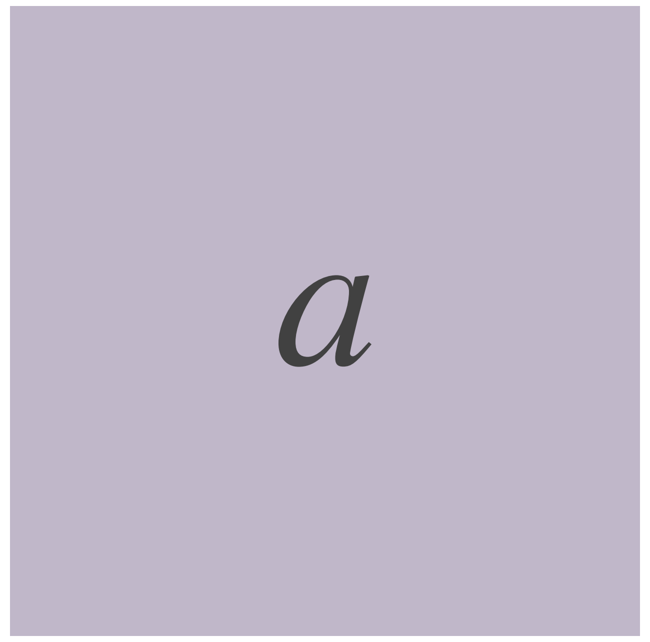
Measured system (or target): S

$$|\psi_S(0)\rangle = \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}\right)$$



$$\bigotimes$$

Meter system (or probe): M



$$|\psi_M(0)\rangle = |\varnothing\rangle$$

$$[a, a^\dagger] = 1$$

$$a|\varnothing\rangle = 0$$

$$|\Psi_{SM}(0)\rangle = \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}\right) \otimes |\varnothing\rangle_M$$

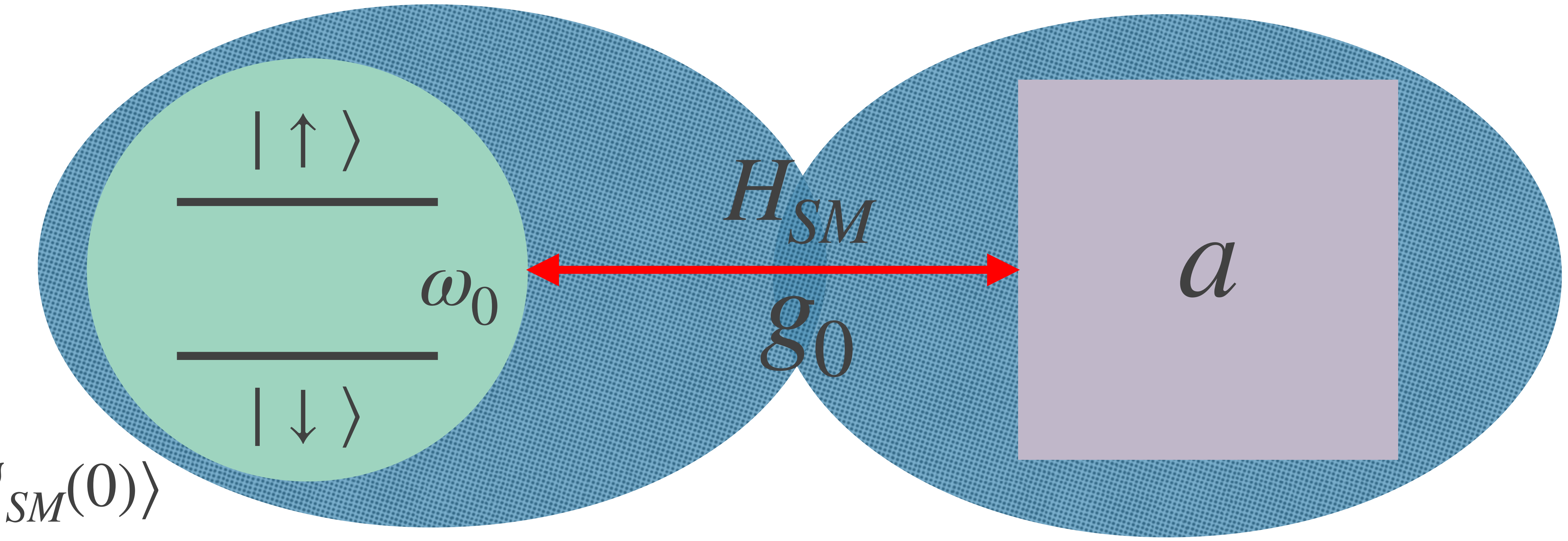
[1] J. von Neumann, Mathematical Foundations of Quantum Mechanics: New Edition. Princeton University Press, 2018.
[2] W. H. Zurek, "Decoherence, einselection, and the quantum origins of the classical," Rev. Mod. Phys., vol. 75, no. 3, pp. 715–775, May 2003

Pre-measurement is the process that generates entanglement between the target and probe.

Measured system (or target): S

Meter system (or probe): M

$$H_{SM} = ig_0\sigma_z \otimes (a^\dagger - a)$$



$$|\Psi_{SM}(t)\rangle = \exp\left\{-iH_{SM}t\right\}|\Psi_{SM}(0)\rangle$$

Displacement operator:
$D(\alpha)|\varnothing\rangle = |\alpha\rangle$
with $\alpha = g_0 tk$

Entangled state

$$|\Psi_{SM}(t)\rangle = \left(\frac{|\uparrow\rangle_S|\psi_\uparrow\rangle_M + |\downarrow\rangle_S|\psi_\downarrow\rangle_M}{\sqrt{2}}\right) \longrightarrow$$

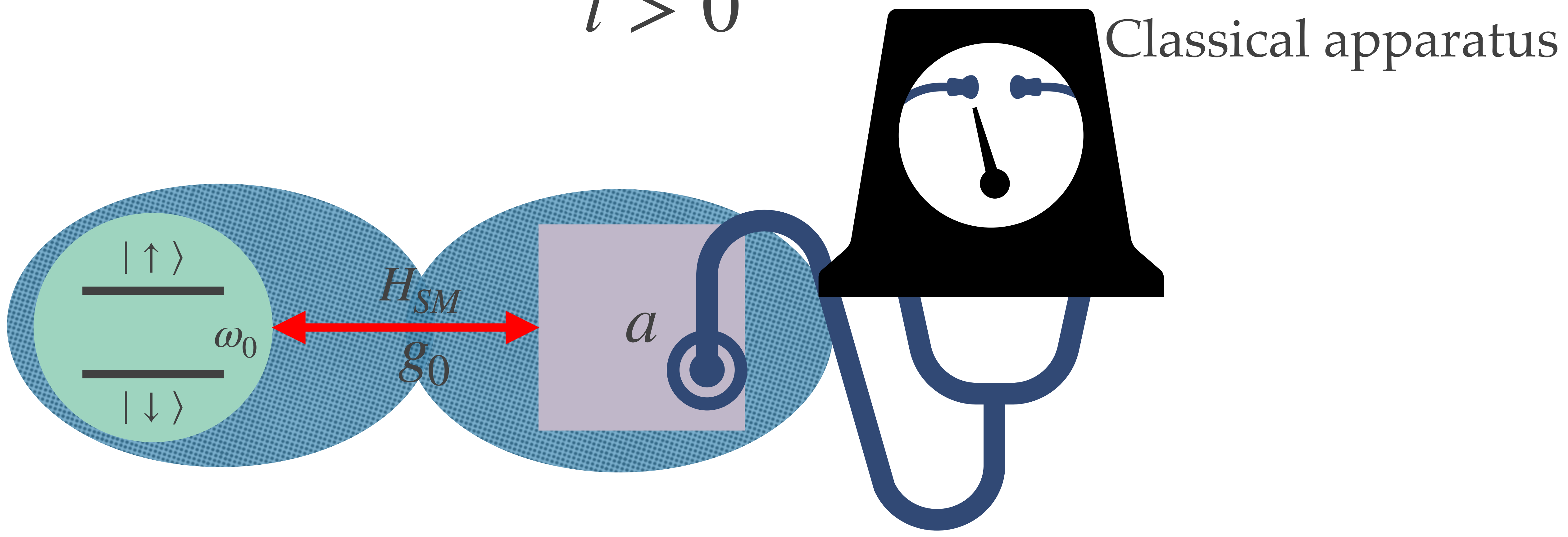
$$|\psi_\uparrow\rangle_M = D((1/2)g_0t)|\varnothing\rangle_M$$: pointer states [1,2]

$$|\psi_\downarrow\rangle_M = D((-1/2)g_0t)|\varnothing\rangle_M$$

[1] J. von Neumann, Mathematical Foundations of Quantum Mechanics: New Edition. Princeton University Press, 2018.
[2] W. H. Zurek, "Decoherence, einselection, and the quantum origins of the classical," Rev. Mod. Phys., vol. 75, no. 3, pp. 715–775, May 2003

Measurement happens when the state of the meter is collapsed with a classical apparatus.

$t > 0$



$$|\Psi_{SM}(t)\rangle = \left(\frac{|\uparrow\rangle_S|\psi_\uparrow\rangle_M + |\downarrow\rangle_S|\psi_\downarrow\rangle_M}{\sqrt{2}}\right)$$

- Projection of the meter in $|\psi_\uparrow\rangle$:
  - $$\langle\psi_\uparrow|\Psi_{SM}(t)\rangle = \left(\frac{|\uparrow\rangle_S + |\downarrow\rangle_S\langle\psi_\uparrow|\psi_\downarrow\rangle_M}{\sqrt{2}}\right)$$
- One can infer the target is in state $|\uparrow\rangle_S$ if $\langle\psi_\uparrow|\psi_\downarrow\rangle_M = 0$
  - Information about the system is fully encoded in the meter.

[1] J. von Neumann, Mathematical Foundations of Quantum Mechanics: New Edition. Princeton University Press, 2018.
[2] W. H. Zurek, "Decoherence, einselection, and the quantum origins of the classical," Rev. Mod. Phys., vol. 75, no. 3, pp. 715–775, May 2003

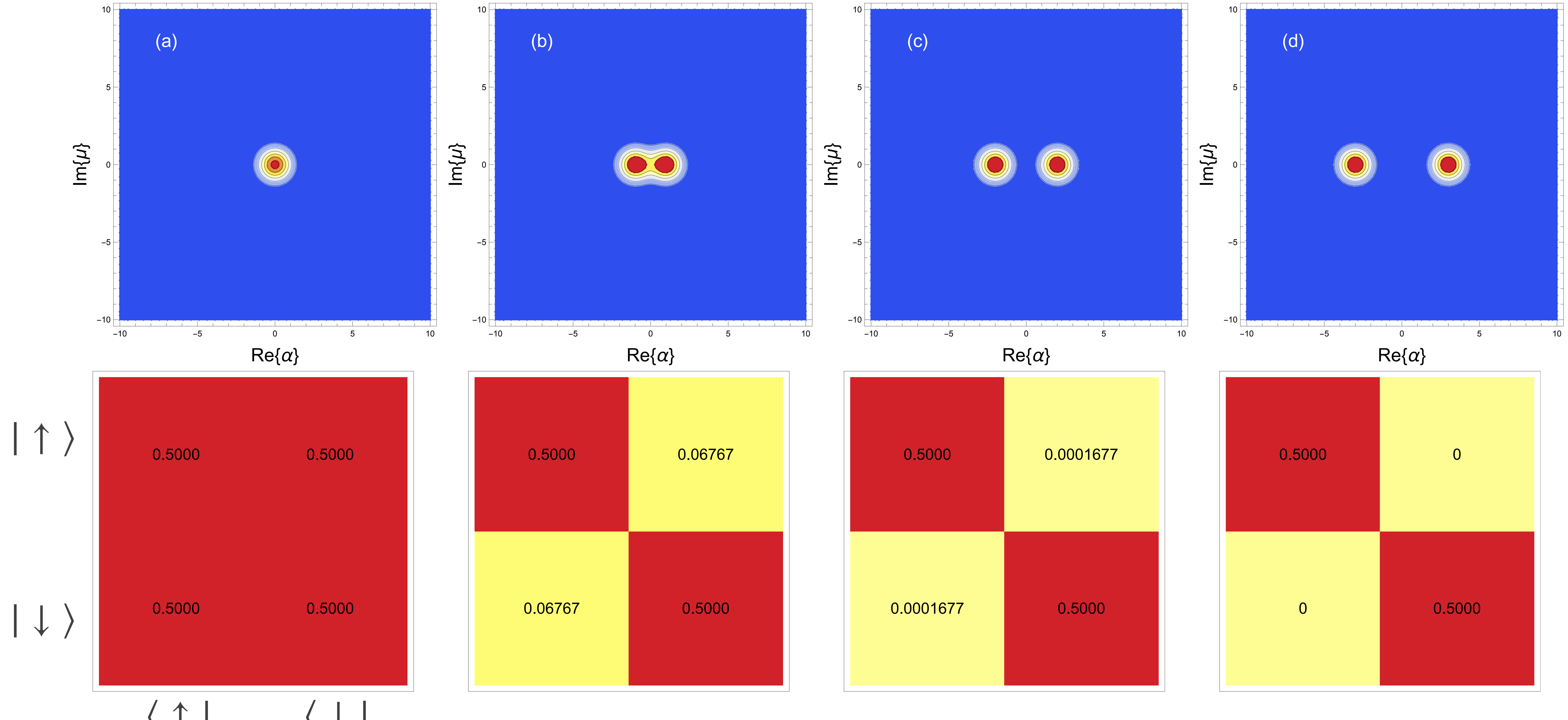
Example: measuring the spin of a 1/2 particle

- System density matrix:

$$\rho_S = \sum_{k,q} c_k c_q^* \exp\left\{ -\frac{(g_0 t)^2}{2}(k-q)^2 \right\} |k\rangle_S\langle q|$$

- Meter density matrix:

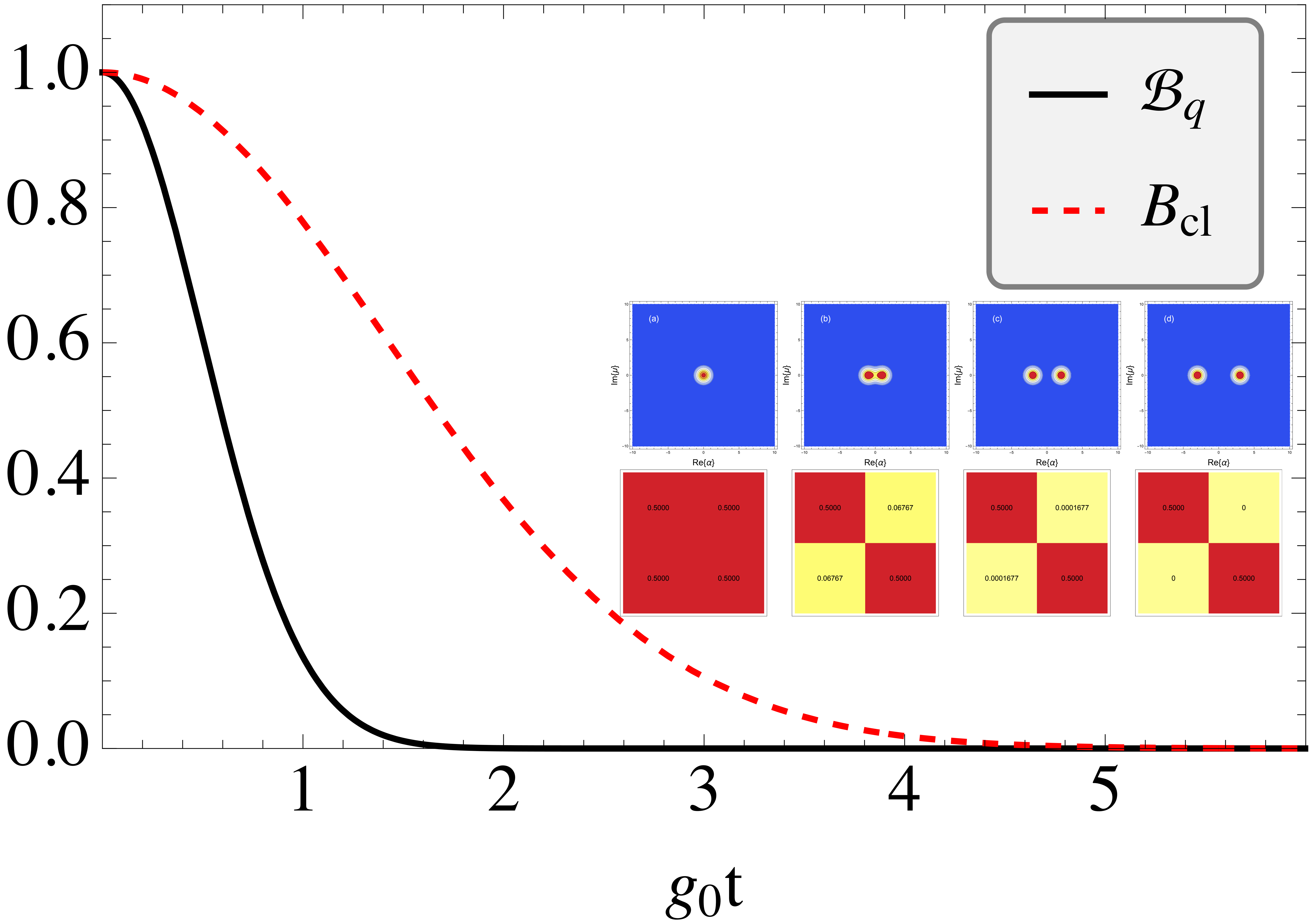
$$\rho_M = \sum_k |c_k|^2 |g_0 k t\rangle_M \langle g_0 k t| \longrightarrow Q_M(\mu,\bar{\mu}) = \frac{1}{\pi}\langle \mu | \rho_M | \mu \rangle$$



→ Disjoint probability distributions.

→ Mixed state.

Quantifying the quality of pre-measurement and collapse



Bhattacharyya coefficients [3]

Quantum Bhattacharyya coefficient (qBhat)

$$\mathcal{B}_q(t) = |\langle \psi_\uparrow(t) | \psi_\downarrow(t) \rangle|$$

Classical Bhattacharyya coefficient (cBhat)

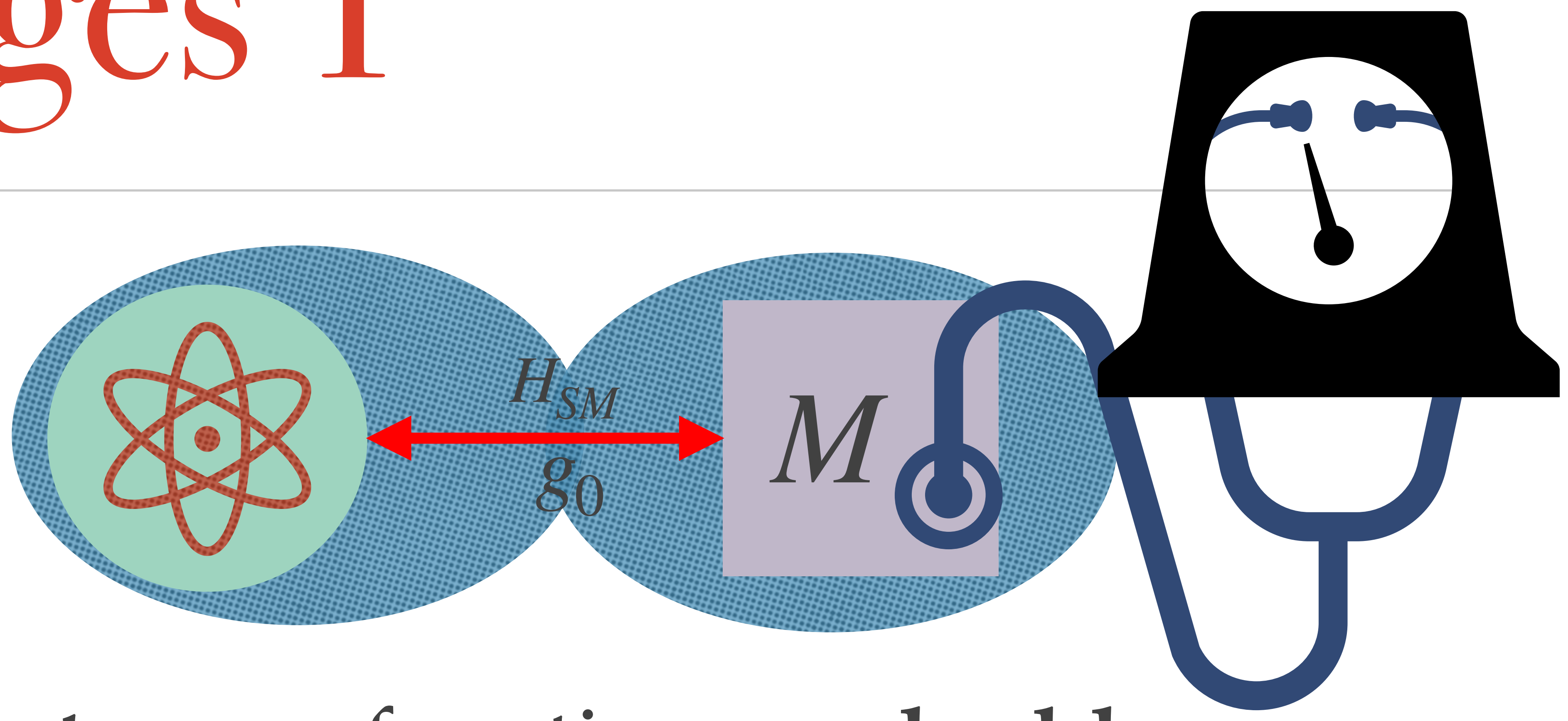
$$B_{cl}(p_l, p_m) = \sum_{x \in \mathcal{X}} \sqrt{p_l(x) p_m(x)}$$

Relation between the coefficients

$$\mathcal{B}_q(t) \leq B_{cl}(p_i, p_j)$$

[3] Fuchs, C. A. & Caves, C. M. Mathematical techniques for quantum communication theory. Open Syst Inf Dyn 3, 345–356 (1995).

Take away messages 1



- When measurement is concerned, having **access to the joint** wave-function a **valuable information (here enters the Collisional Model to be discussed next)**
- The pre-measurement is successful if the pointer states are orthogonal!
- The collapse gives information about the target when the probability distributions of the outcomes are disjoint!

[1] J. von Neumann, Mathematical Foundations of Quantum Mechanics: New Edition. Princeton University Press, 2018.
[2] W. H. Zurek, "Decoherence, einselection, and the quantum origins of the classical," Rev. Mod. Phys., vol. 75, no. 3, pp. 715–775, May 2003
[3] Fuchs, C. A. & Caves, C. M. Mathematical techniques for quantum communication theory. Open Syst Inf Dyn 3, 345–356 (1995).

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  - Collisional model or how to close open systems
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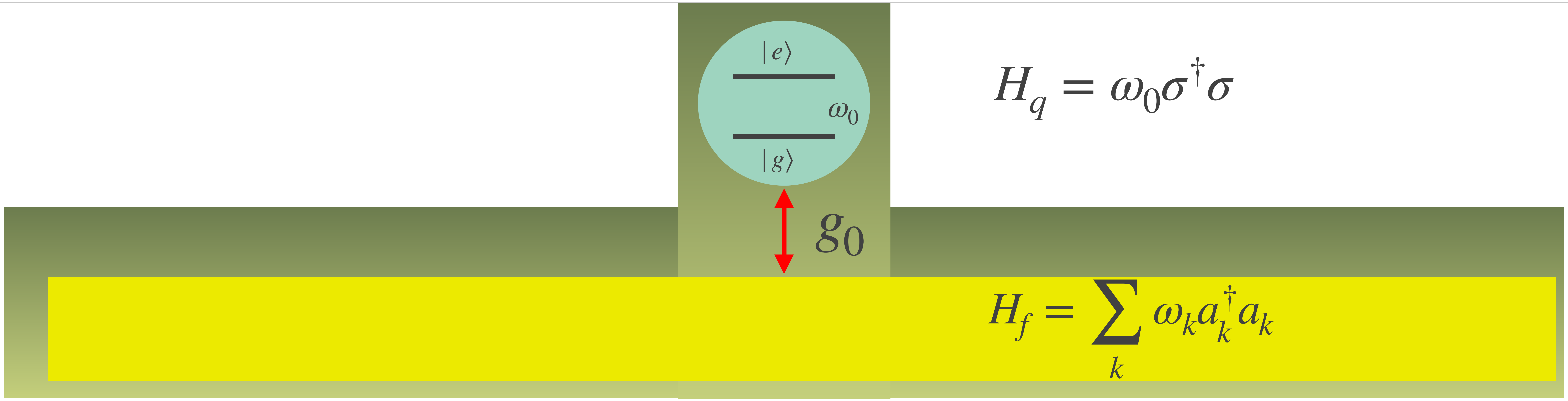
Qubit + multimode electromagnetic field in a 1D waveguide



$$V_{qf} = i g_0 \sum_{k=0}^{\infty} \left(\sigma^\dagger a_k - a_k^\dagger \sigma \right)$$

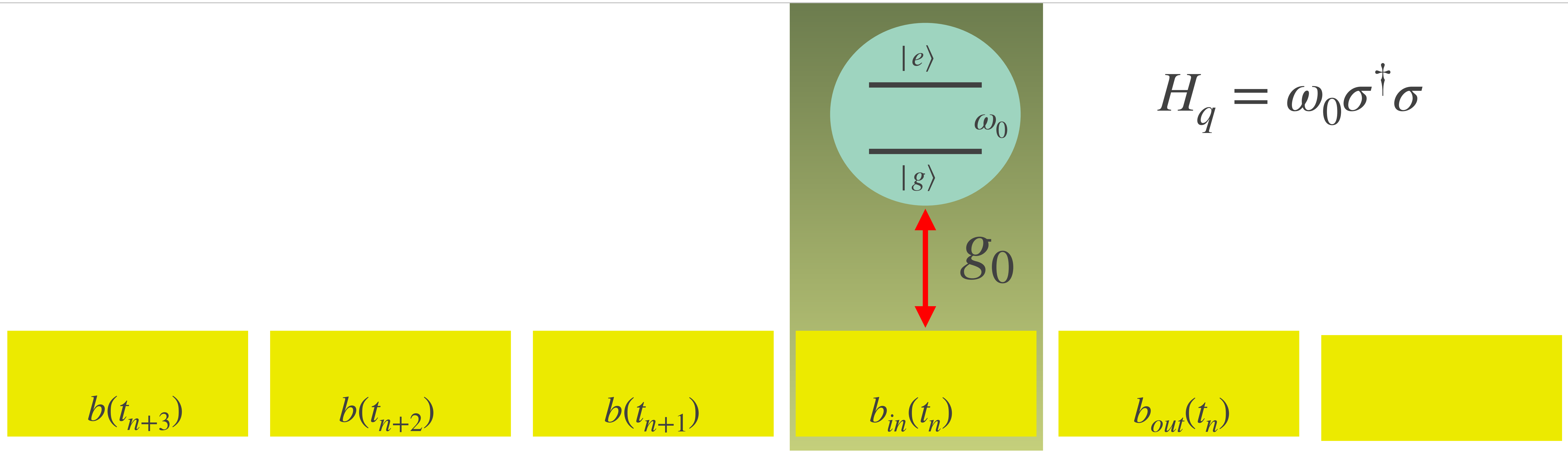
The emitter interacts with **all the modes** at the same time.

$$H = \boxed{H_0} + V_{qf} = \boxed{\omega_0 \sigma^\dagger \sigma + \sum_k \omega_k a_k^\dagger a_k} + i g_0 \sum_k (\sigma^\dagger a_k - \sigma a_k^\dagger)$$

How to solve the joint dynamics?

[1] F. Ciccarello, S. Lorenzo, V. Giovannetti, and G. M. Palma, "Quantum collision models: open system dynamics from repeated interactions," *arXiv:2106.11974 [quant-ph]*
[2] F. Ciccarello, "Collision models in quantum optics," doi: 10.1515/qmetro-2017-0007.

Interaction picture & time discretization

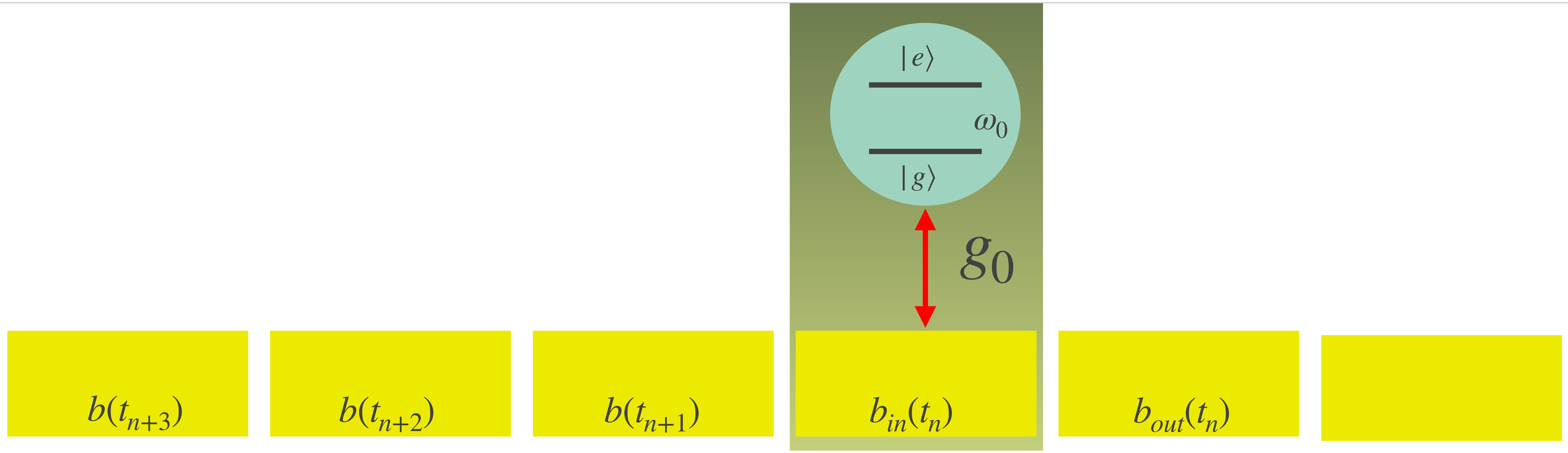


$$H_q = \omega_0 \sigma^\dagger \sigma$$

$$V_{qf} \overset{H_0}{\rightarrow} V_{qf}(t) = i g_0 \sum_{k=0}^{\infty} \left(\sigma^\dagger(t) e^{-i\omega_k t} a_k - e^{i\omega_k t} a_k^\dagger \sigma(t) \right) \xrightarrow{t_n = n\Delta t} V_{qf}(t_n) = i\sqrt{\frac{\gamma}{\Delta t}} (\sigma^\dagger(t) b(0,t_n) - \sigma(t) b^\dagger(0,t_n))$$

[3] M. Maffei, P. A. Camati, and A. Auffèves, “Closed-System Solution of the 1D Atom from Collision Model,” Entropy, vol. 24, no. 2, p. 151, Jan. 2022

Interaction picture & time discretization



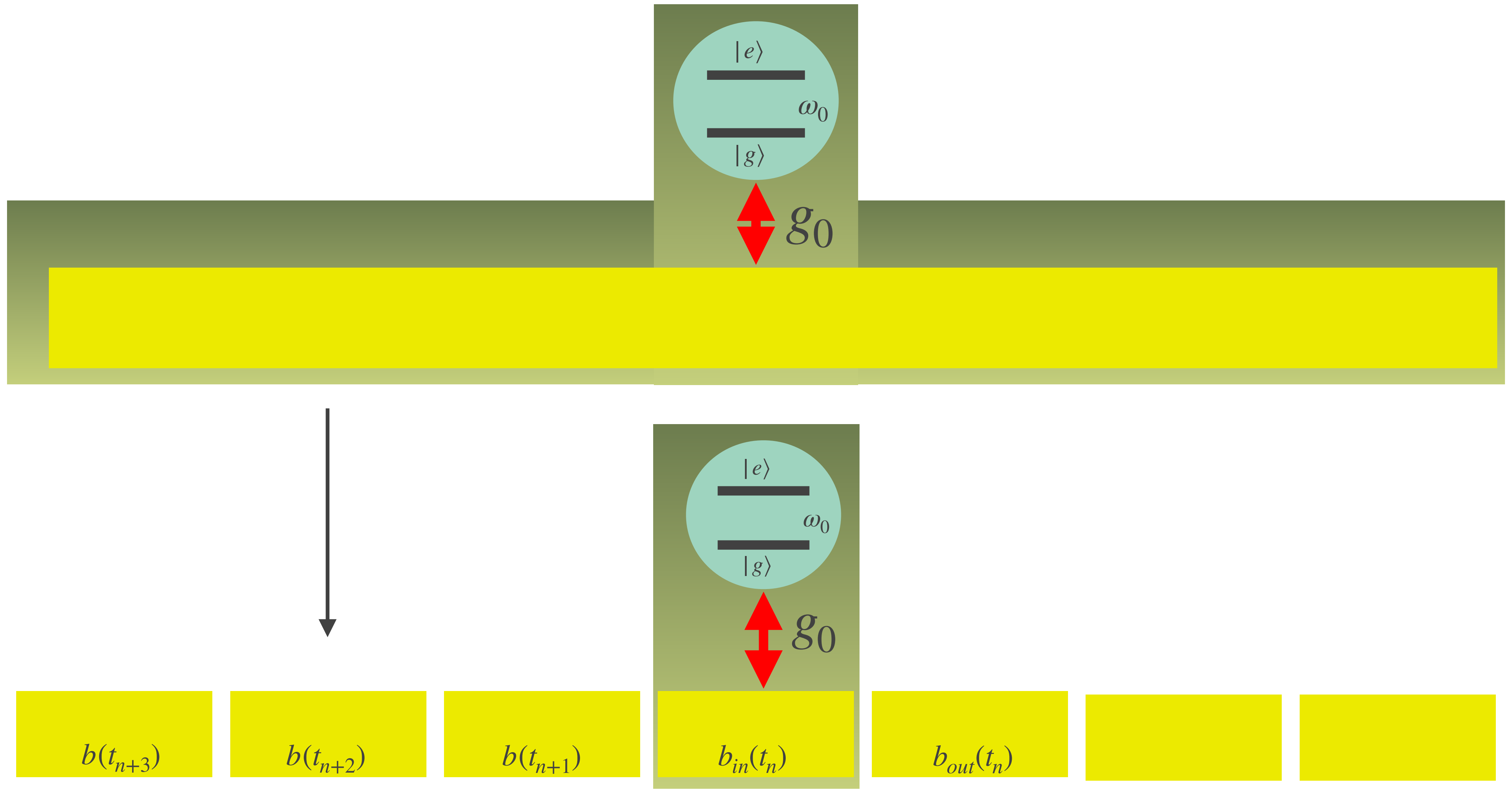
$$V_{qf}(t_n) = V_n = i\sqrt{\frac{\gamma}{\Delta t}}\left[\sigma^\dagger(t_n)b(t_n) - b^\dagger(t_n)\sigma(t_n)\right] \quad \text{With} \quad b(0,t_n) = b(t_n) \equiv \frac{b_n}{\sqrt{\Delta t}} = \sqrt{\frac{\Delta t}{\varrho}}\sum_{k=0}^{\infty} e^{-i\omega_k t_n} a_k$$

The emitter interacts with one *temporal mode* at a time, repeatedly →Collisional model interpretation.

[3] M. Maffei, P. A. Camati, and A. Auffèves, "Closed-System Solution of the 1D Atom from Collision Model," Entropy, vol. 24, no. 2, p. 151, Jan. 2022
[4] Gross, J. A., Caves, C. M., Milburn, G. J. & Combes, J. Qubit models of weak continuous measurements: markovian conditional and open-system dynamics. Quantum Sci. Technol. 3, 024005 (2018).

CM developed in the group

- Suzuki-Trotter formula [5]
  $|\Psi_N\rangle = e^{-i(N-1)\Delta t V_{N-1}} e^{-i(N-2)\Delta t V_{N-2}} \dots e^{-iM\Delta t V_M} |\Psi_0\rangle$
- **Joint emitter+field wave-function solution**
- Input-output relation for the average values [3]
  - $\langle b_{out}\rangle = \langle b_{in}\rangle - \sqrt{\gamma}\langle\sigma\rangle$
- Similar to the known textbook [6]: $b_{out} = b_{in} - \sqrt{\gamma}\sigma$.



[3] M. Maffei, P. A. Camati, and A. Auffèves, "Closed-System Solution of the 1D Atom from Collision Model," Entropy, vol. 24, no. 2, p. 151, Jan. 2022

[4] Gross, J. A., Caves, C. M., Milburn, G. J. & Combes, J. Qubit models of weak continuous measurements: markovian conditional and open-system dynamics. Quantum Sci. Technol. 3, 024005 (2018).

[5] M. Suzuki, "Generalized Trotter's formula and systematic approximants of exponential operators and inner derivations with applications to many-body problems," Commun.Math. Phys., vol. 51, no. 2, pp. 183–190, Jun. 1976

[6] Gardiner, C. W. & Collett, M. J. Input and output in damped quantum systems: Quantum stochastic differential equations and the master equation. Phys. Rev. A 31, 3761–3774 (1985).

Coherent field solution

$$|\Psi_{CS}(0)\rangle = D(\alpha_d)|\zeta\rangle \otimes |\varnothing\rangle, \zeta = g, e$$

Initially uncorrelated: Markovian dynamics (Lindblad)

$$|\Psi_{\beta}^{\zeta}(t_N)\rangle = \sqrt{P_g(t)}\,|g\rangle|\phi_g(t)\rangle + \sqrt{P_e(t)}\,|e\rangle|\phi_e(t)\rangle$$

Technical detail: displaced frame.

$$|\phi_\varepsilon\rangle = \frac{1}{\sqrt{P_\varepsilon(t)}}\left[\sqrt{p_{0,\varepsilon}}\tilde{f}^{(0)}_{\varepsilon,\zeta}(t) + \sum_{m=1}^{\infty}\sqrt{p_{m,\varepsilon}(t)}\int_0^t d\mathbf{s}_m \tilde{f}^{(m)}_{\varepsilon,\zeta}(t,\mathbf{s})\prod_{i=1}^{m} b_m^\dagger\right]|\varnothing\rangle$$

Suffices to find the coefficients: possible to do analytically.

[3] M. Maffei, P. A. Camati, and A. Auffèves, "Closed-System Solution of the 1D Atom from Collision Model," Entropy, vol. 24, no. 2, p. 151, Jan. 2022

Single photon solution

$$|1\rangle = \sum_{n=0}^{\infty} \sqrt{\Delta t}\,\xi(t_n) b_n^\dagger |\varnothing\rangle \qquad \sum_{n=0}^{\infty} \Delta t |\xi(t_n)| = 1$$

Initially correlated: Non-markovian dynamics

$$|\Psi_{SP}(t_N)\rangle = \left(\sqrt{\gamma}\sqrt{\Delta t} e^{-\frac{\gamma}{2}t_N} \sum_{n=0}^{N-1} \sqrt{\Delta t} e^{\left(\frac{\gamma}{2}+i\omega_0\right)t_n} \xi(t_n) |\varnothing\rangle\right) \otimes |e\rangle$$

$$+\left(\sum_{n=0}^{N-1}\left[\sqrt{\Delta t}\,\xi(t_n) - \gamma \Delta t e^{-\left(\frac{\gamma}{2}+i\omega_0\right)t_n} \sum_{m=0}^{n}\left(e^{\left(\frac{\gamma}{2}+i\omega_0\right)t_m}\sqrt{\Delta t}\,\xi(t_m)\right)\right] b_n^\dagger |\varnothing\rangle\right) \otimes |g\rangle + \left(\sum_{n=N}^{\infty} \sqrt{\Delta t}\,\xi(t_n) b_n^\dagger |\varnothing\rangle\right) \otimes |g\rangle .$$

Already interacted.

Part that is yet to interact.

[3] M. Maffei, P. A. Camati, and A. Auffèves, "Closed-System Solution of the 1D Atom from Collision Model," Entropy, vol. 24, no. 2, p. 151, Jan. 2022

Take away messages 2

- The collisional model allows to solve the the unitary dynamics of the 1D waveguide and emitter
  - Analytical light-matter wave functions are derived
- Important in the context of measurement and quantum information processing

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  - SPI under a magnetic field
- Conclusions and perspectives

Theoretical background

PRL **113,** 200401 (2014) PHYSICAL REVIEW LETTERS week ending 14 NOVEMBER 2014

Anomalous Weak Values Are Proofs of Contextuality

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(Received 5 September 2014; published 12 November 2014)

The average result of a weak measurement of some observable A can, under postselection of the measured quantum system, exceed the largest eigenvalue of A. The nature of weak measurements, as well as the presence of postselection and hence possible contribution of measurement disturbance, has led to a long-running debate about whether or not this is surprising. Here, it is shown that such "anomalous weak values" are nonclassical in a precise sense: a sufficiently weak measurement of one constitutes a proof of contextuality. This clarifies, for example, which features must be present (and in an experiment, verified) to demonstrate an effect with no satisfying classical explanation.

DOI: 10.1103/PhysRevLett.113.200401 PACS numbers: 03.65.Ta, 03.67.-a

PHYSICAL REVIEW LETTERS **129,** 230401 (2022)

Contextuality and Wigner Negativity Are Equivalent for Continuous-Variable Quantum Measurements

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Quantum computers promise considerable speedups with respect to their classical counterparts. However, the identification of the innately quantum features that enable these speedups is challenging. In the continuous-variable setting—a promising paradigm for the realization of universal, scalable, and fault-tolerant quantum computing—contextuality and Wigner negativity have been perceived as two such distinct resources. Here we show that they are in fact equivalent for the standard models of continuous-variable quantum computing. While our results provide a unifying picture of continuous-variable resources for quantum speedup, they also pave the way toward practical demonstrations of continuous-variable contextuality and shed light on the significance of negative probabilities in phase-space descriptions of quantum mechanics.

DOI: 10.1103/PhysRevLett.129.230401

[1] Aharonov, Y., Albert, D. Z. & Vaidman, L. How the result of a measurement of a component of the spin of a spin- 1/2 particle can turn out to be 100. Phys. Rev. Lett. 60, 1351–1354 (1988).
[2] Dressel, J., Malik, M., Miatto, F. M., Jordan, A. N. & Boyd, R. W. Colloquium : Understanding quantum weak values: Basics and applications. Rev. Mod. Phys. 86, 307–316 (2014).
[3] Kenfack, A. & Yczkowski, K. Negativity of the Wigner function as an indicator of non-classicality. J. Opt. B: Quantum Semiclass. Opt. 6, 396–404 (2004).

Experimental background

PHYSICAL REVIEW LETTERS **129,** 110601 (2022)

Energetics of a Single Qubit Gate

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[2]*CNRS and Université Grenoble Alpes, Institut Néel, F-38042 Grenoble, France*
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[4]*Institute for Quantum Studies, Chapman University, 1 University Drive, Orange, California 92866, USA*
[5]*Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, USA*

(Received 21 October 2021; revised 19 August 2022; accepted 24 August 2022; published 9 September 2022)

Qubits are physical, a quantum gate thus not only acts on the information carried by the qubit but also on its energy. What is then the corresponding flow of energy between the qubit and the controller that implements the gate? Here we exploit a superconducting platform to answer this question in the case of a quantum gate realized by a resonant drive field. During the gate, the superconducting qubit becomes entangled with the microwave drive pulse so that there is a quantum superposition between energy flows. We measure the energy change in the drive field conditioned on the outcome of a projective qubit measurement. We demonstrate that the drive's energy change associated with the measurement backaction can exceed by far the energy that can be extracted by the qubit. This can be understood by considering the qubit as a weak measurement apparatus of the driving field.

DOI: 10.1103/PhysRevLett.129.110601

- Anomalous weak values appear
  - Post-selection on the ground state
- Continuous monitoring of the scattered field
- The joint dynamics of the system is obtained via collisional model
  - qubit+waveguide with intense coherent state

Anomalous energy exchanges and Wigner-function negativities in a single qubit gate

- **Question:** Two hallmarks of non-classicality:
  - Foundational perspective
  - Anomalous weak values→Contextuality
  - Contextuality →Negativity of the Wigner function (for some set of measurements)
  - **DO THE PRESENCE OF ANOMALOUS WEAK VALUES INDICATE THE NEGATIVITY OF THE CONDITIONED WIGNER FUNCTION?**
- **Tool: Analytical** calculation of the **field's weak values** from the joint-state solution
  - → CM spot how the weak values appear analytically
  - → Compute the conditional Wigner function

10.1103/PhysRevA.107.023710

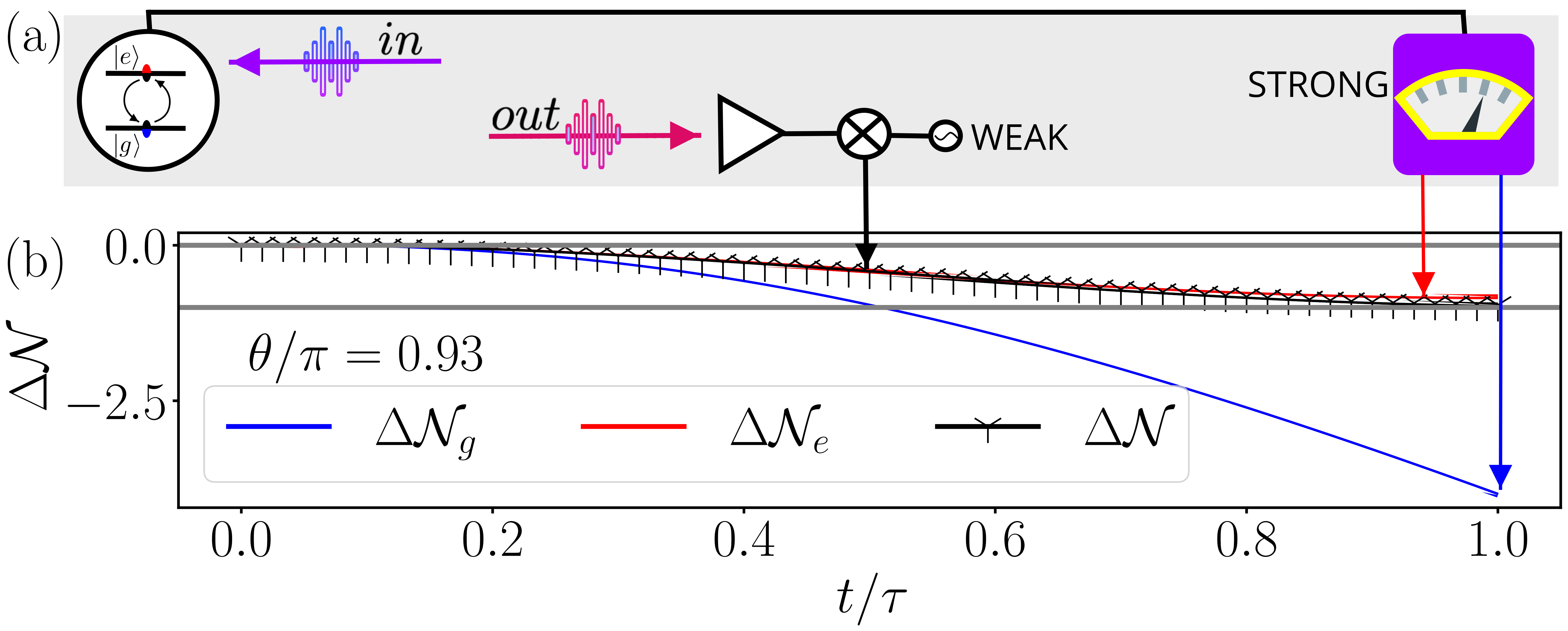
Measurement protocol

[4] P. Campagne-Ibarcq, L. Bretheau, E. Flurin, A. Auffèves, F. Mallet, and B. Huard, “Observing interferences between past and future quantum states in resonance fluorescence,” 10.1103/PhysRevLett.112.180402.

[5] J. Stevens et al., “Energetics of a Single Qubit Gate,” doi: 10.1103/PhysRevLett.129.110601.

Weak value interpretation

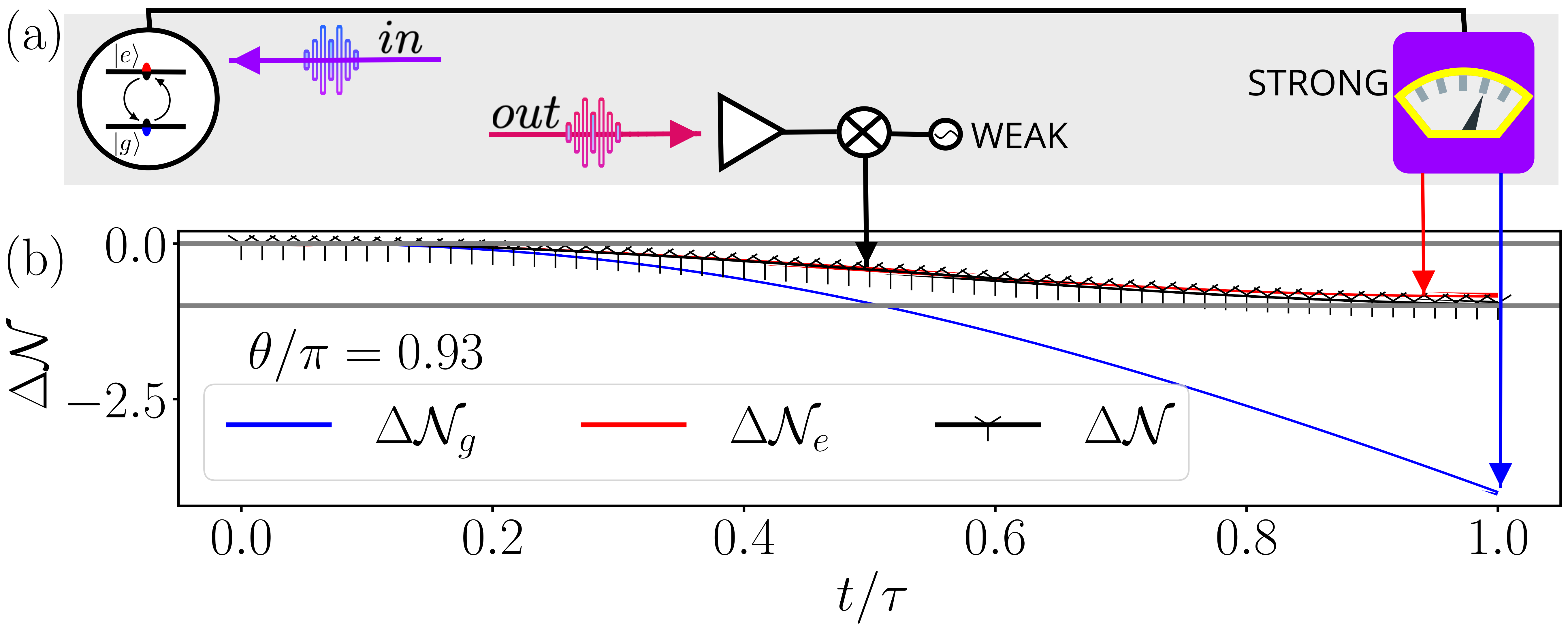
$$H_D(t) = -\frac{\Omega}{2}\sigma_y$$



$$\Omega = g\alpha \longrightarrow \theta = \Omega\tau$$

- $|g\rangle \otimes |\alpha\rangle \xrightarrow{\text{int}(\tau)} \left[\cos(\theta/2)\,|g\rangle + \sin(\theta/2)\,|e\rangle\right] \otimes |\alpha\rangle$

Weak value interpretation



- $|g\rangle \otimes |\alpha\rangle \xrightarrow{\text{int}(\tau)} [\cos(\theta/2)\,|g\rangle + \sin(\theta/2)\,|e\rangle] \otimes |\alpha\rangle,$
- $\theta < \pi$, rotation around y-axis — **single qubit gate**.
- **Weak values interpretation**:
  - **unlikeness** of finding the qubit in the post-selected state: $|g\rangle$

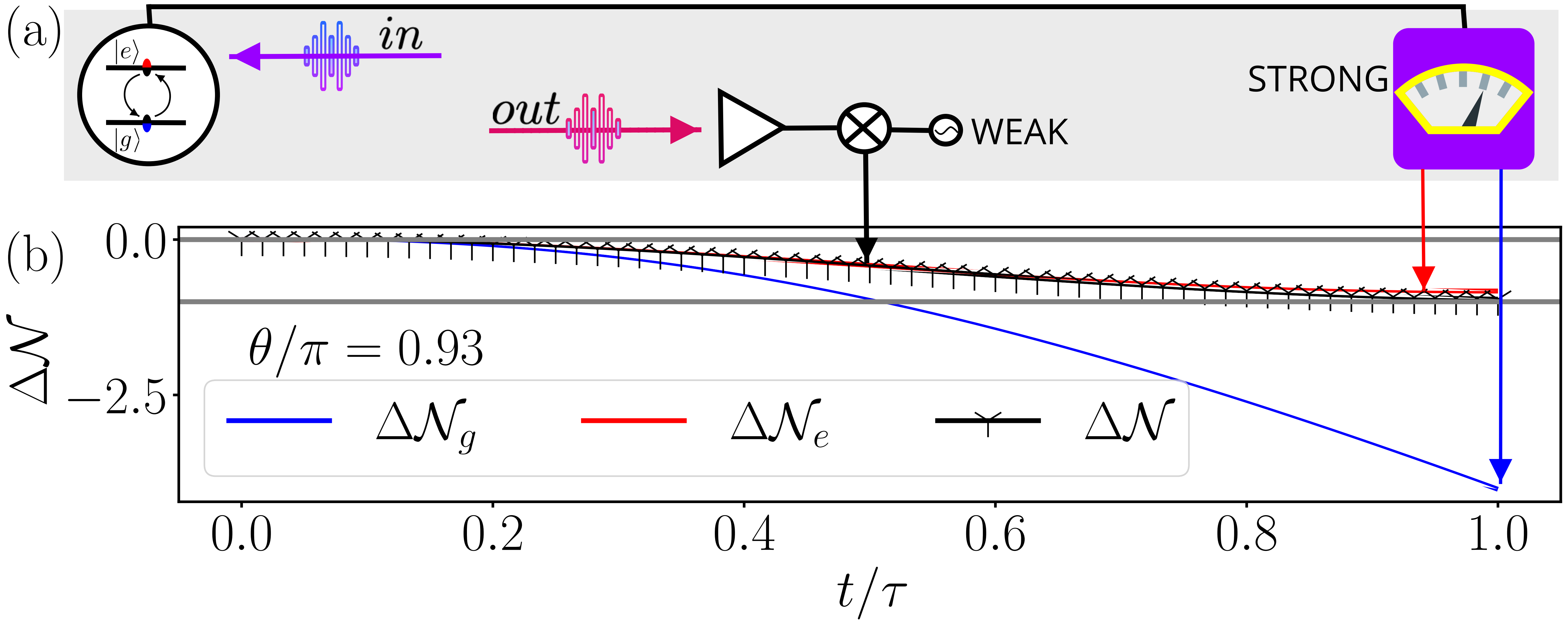
Change of the field's number of excitations vs. time

- **Anomalous** weak values when conditioned on the ground state.

$$\Delta \mathcal{N}_\epsilon = \int_0^\tau dt \left(\boxed{\gamma \mathcal{J}_\epsilon(t)} \boxed{- \Omega \mathrm{Re}\{\langle \sigma(t) \rangle_\epsilon\}} \right) .$$

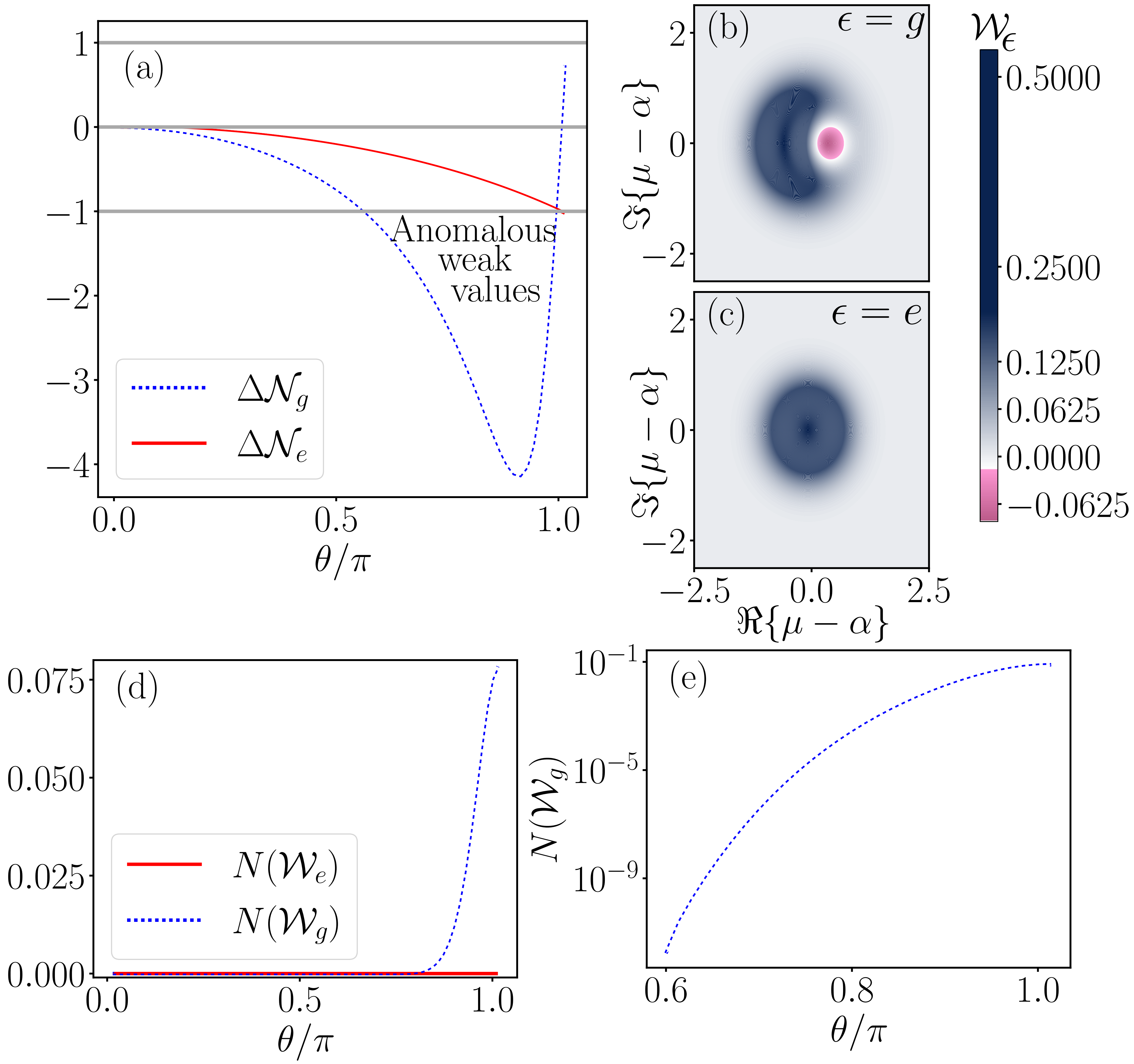
Total probability of spontaneous emission along the evolution $|g\rangle \to |\epsilon\rangle$

Interference between input and emitter's fluorescence **post-selected** over $|\epsilon\rangle \to$ module can exceed 1→**Anomalous values!**



Wigner function negativity

$t = \tau$



Take away messages 3

- Analytical expression of atom-field wave-function→ Weak value of field's energy change & field's conditional Wigner functions
- Weak value **exceeding single quantum** → anomalous values → **contextuality**
- Anomalous weak values of energy change → Wigner negativity

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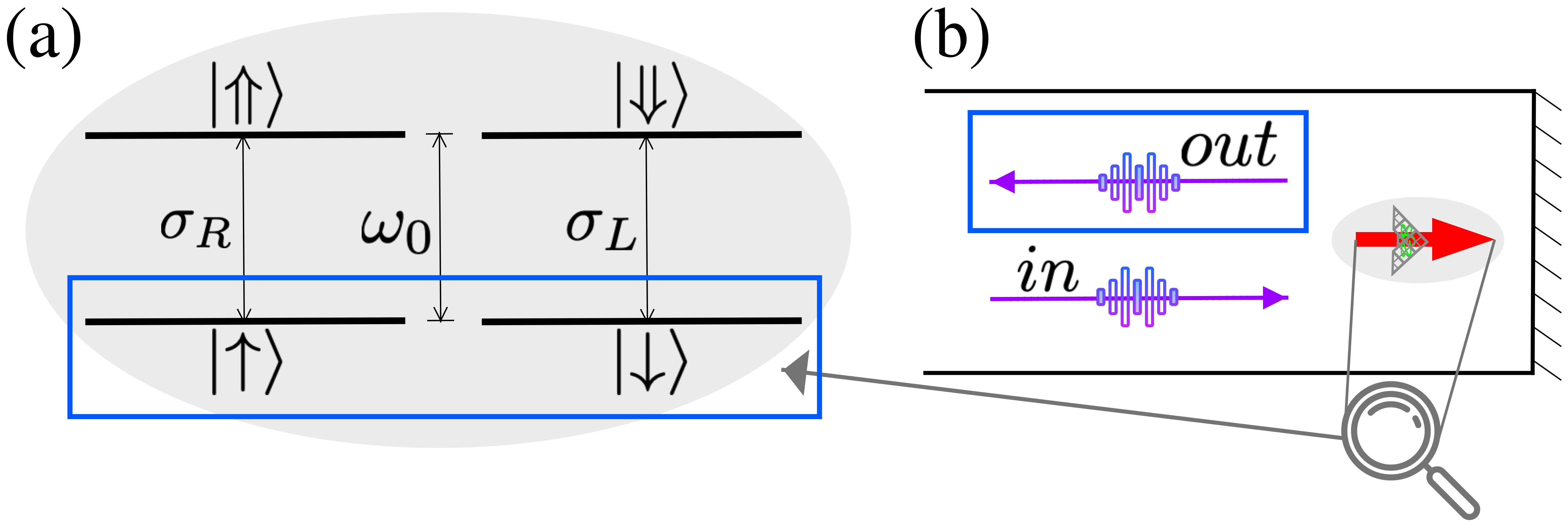
  - Conclusions and perspectives

Measuring the spin state

- **Set-up:**



- **Questions:**
  - “Light battle”: Superposition of **number states vs.** low energy **coherent pulses**
    - **Budget**: 1 quanta → Get entanglement between spin state and output field.
    - **1) WHICH ONE ENTANGLES BETTER? (qBhat)**
    - **2) DOES BETTER ENTANGLEMENT TRANSLATE INTO BETTER READOUT? (cBhat)**

Spin-light entanglement

- Long-time limit state:

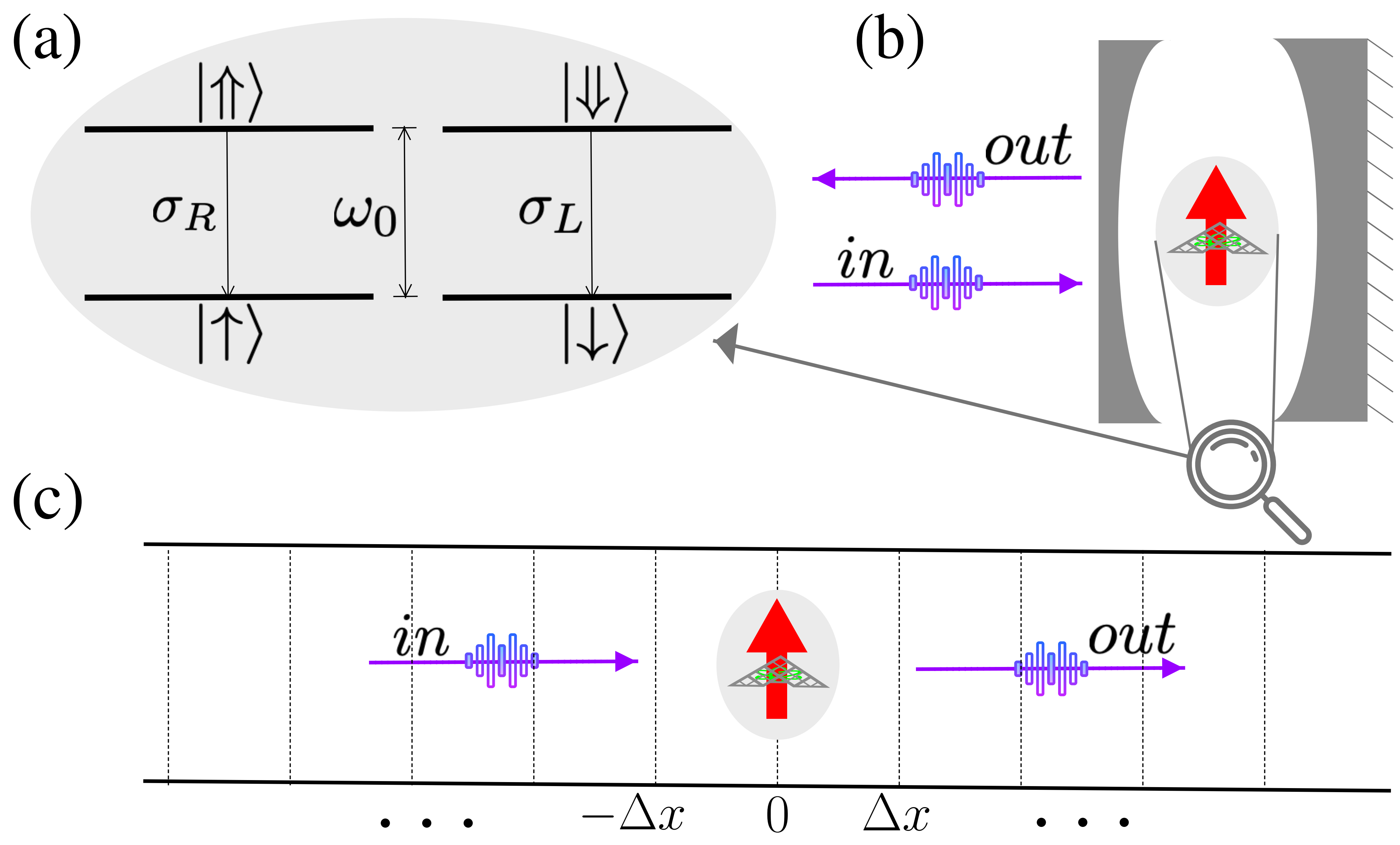
$$|\Psi(t\to\infty)\rangle = c_\uparrow|\psi_\uparrow\rangle|\uparrow\rangle + c_\downarrow|\psi_\downarrow\rangle|\downarrow\rangle$$

$|\psi^{cs}(0)\rangle_H = |\beta\rangle_H$ a) Coherent state

$$\longrightarrow |\psi^{cs}_{\uparrow(\downarrow)}\rangle = D_H(\beta)\left[\sqrt{p_0} + \sqrt{p_1}\int dt f^{(1)}(t) b^\dagger_{R(L)}(t) + \ldots\right]|\varnothing\rangle$$

$|\psi^{qs}(0)\rangle_H = c_0|0\rangle + c_1|1\rangle_H$ b) Superposition of number states

$$\longrightarrow |\psi^{qs}_{\uparrow(\downarrow)}(t)\rangle = c_0|\varnothing\rangle + \frac{c_1}{\sqrt{2}}\int dt\left[\Upsilon(t) b^\dagger_{R(L)}(t) + \beta(t) b^\dagger_{L(R)}(t)\right]|\varnothing\rangle$$



Spin-light entanglement: qBhat

$$|\psi^{cs}_{\uparrow(\downarrow)}\rangle = D_H(\beta)\left[\sqrt{p_0} + \sqrt{p_1}\int dt f^{(1)}(t) b^\dagger_{R(L)}(t) + \ldots\right]|\varnothing\rangle$$

$$\mathscr{B}^{cs}_q = p_0$$

a) Coherent state

$$|\psi^{qs}_{\uparrow(\downarrow)}(t)\rangle = c_\varnothing|\varnothing\rangle + \frac{c_1}{\sqrt{2}}\int dt\left[\Upsilon(t) b^\dagger_{R(L)}(t) + \beta(t) b^\dagger_{L(R)}(t)\right]|\varnothing\rangle$$

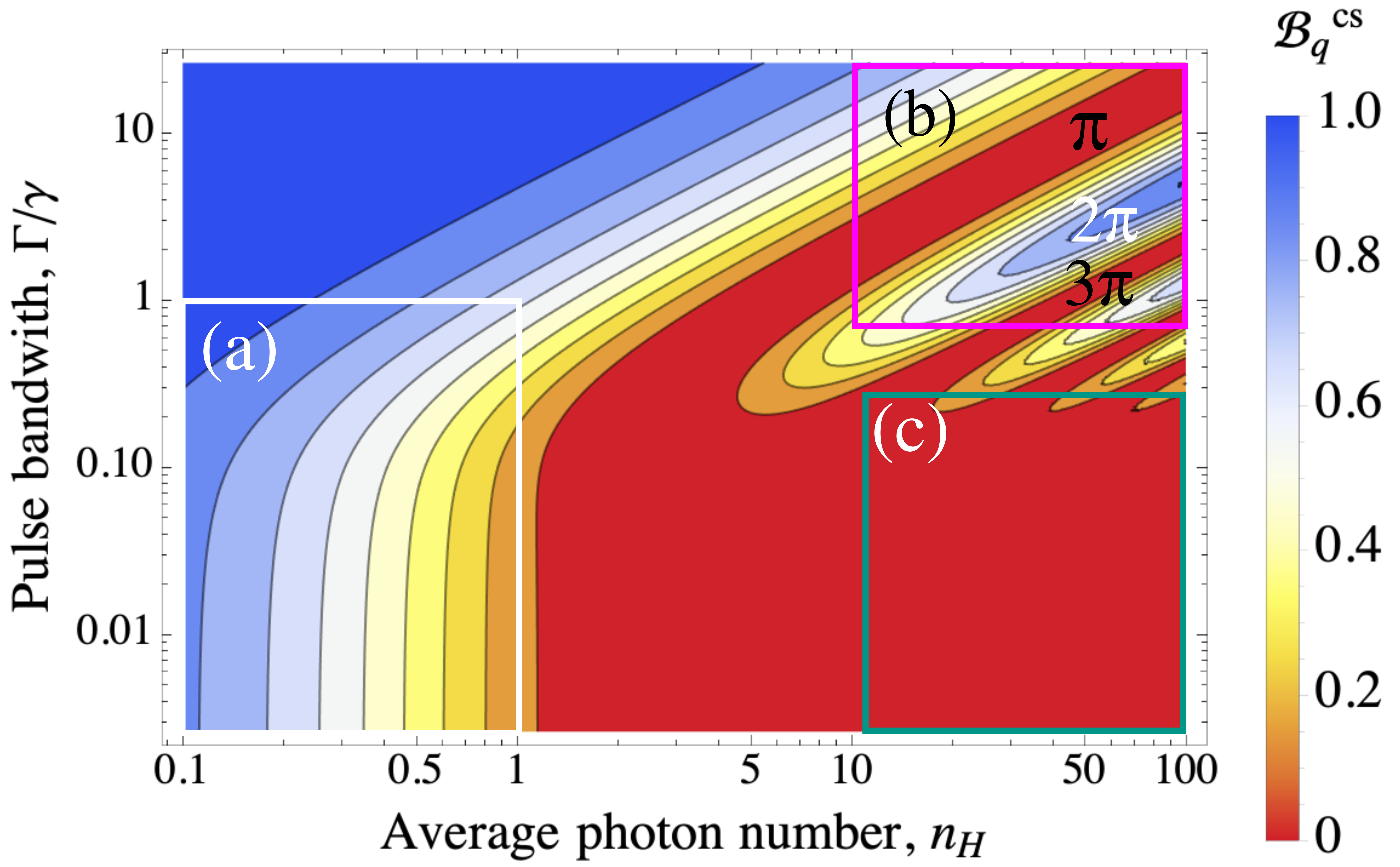
$$p_H = |c_1|^2$$

$$\mathscr{B}^{qs}_q = \left|1 - \gamma p_H \int dt\, \Re\left\{\beta(t)\tilde{\beta}(t)\right\}\right|$$

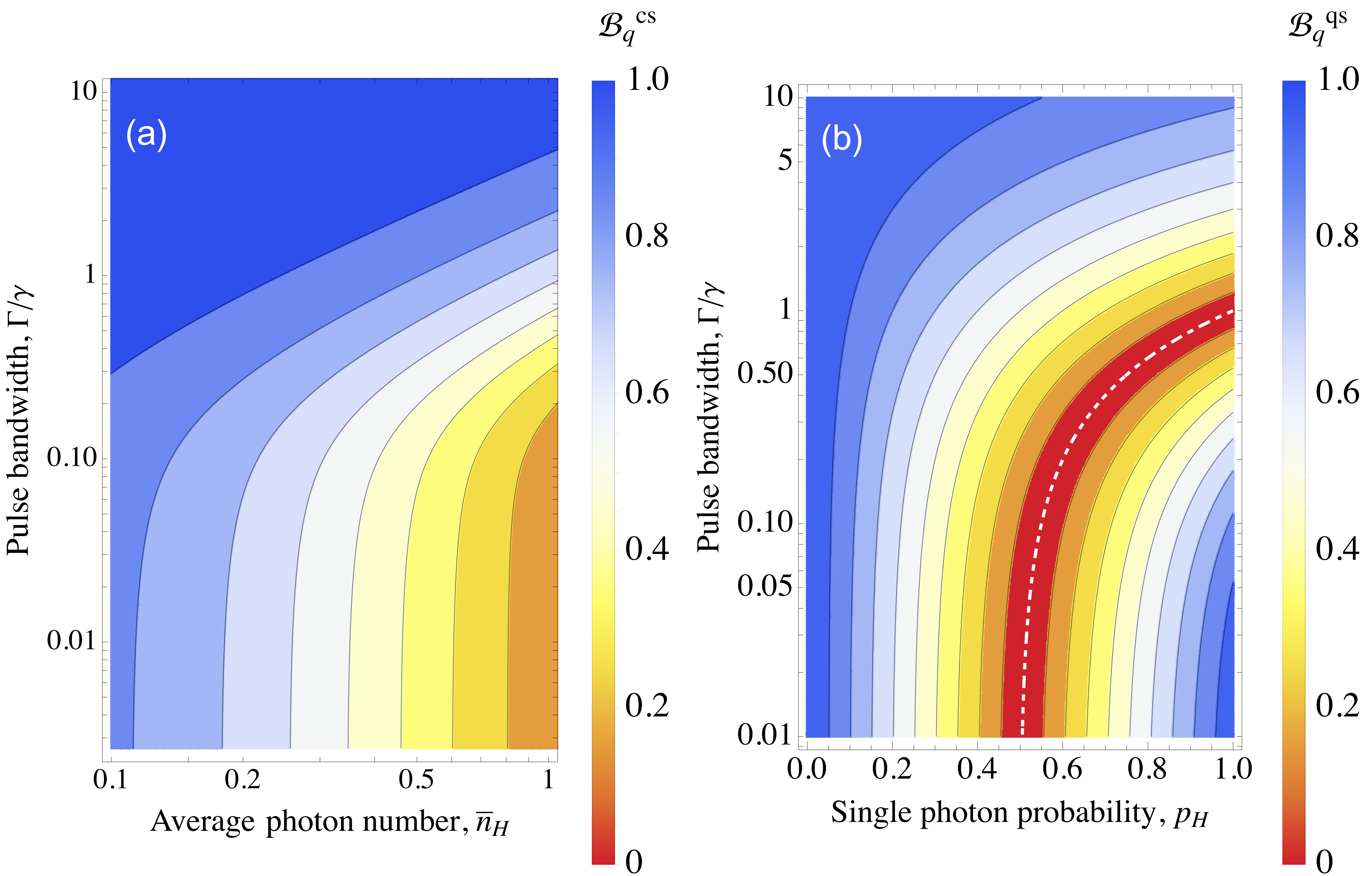
b) Number states

- $\mathscr{B}_q = |\langle\psi_\downarrow|\psi_\uparrow\rangle|$
- $p_0 \rightarrow$ no photon re-emission probability
  - Fundamental limitation for the coherent state
- For a given β it is possible to tune p_H to vanish the qBhat.

Spin-light entanglement: Coherent state



Quantum advantage: the superposition of number states creates orthogonal pointer states of the meter.



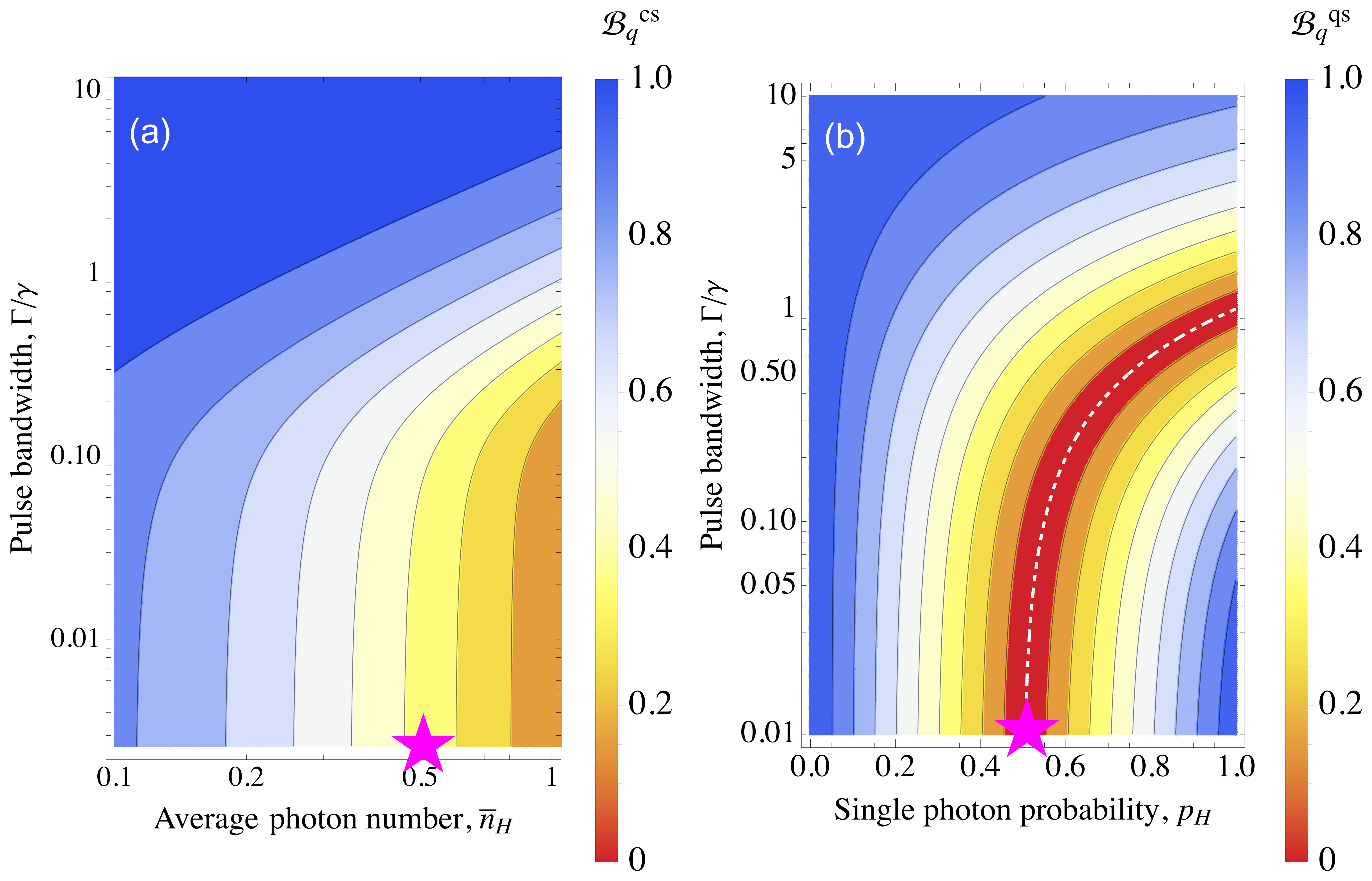
Quantum advantage:
$\mathcal{B}_q = 0 \rightarrow$ Superposition of number states
$p_H = (1 + \Gamma/\gamma)/2$

Phase measurement

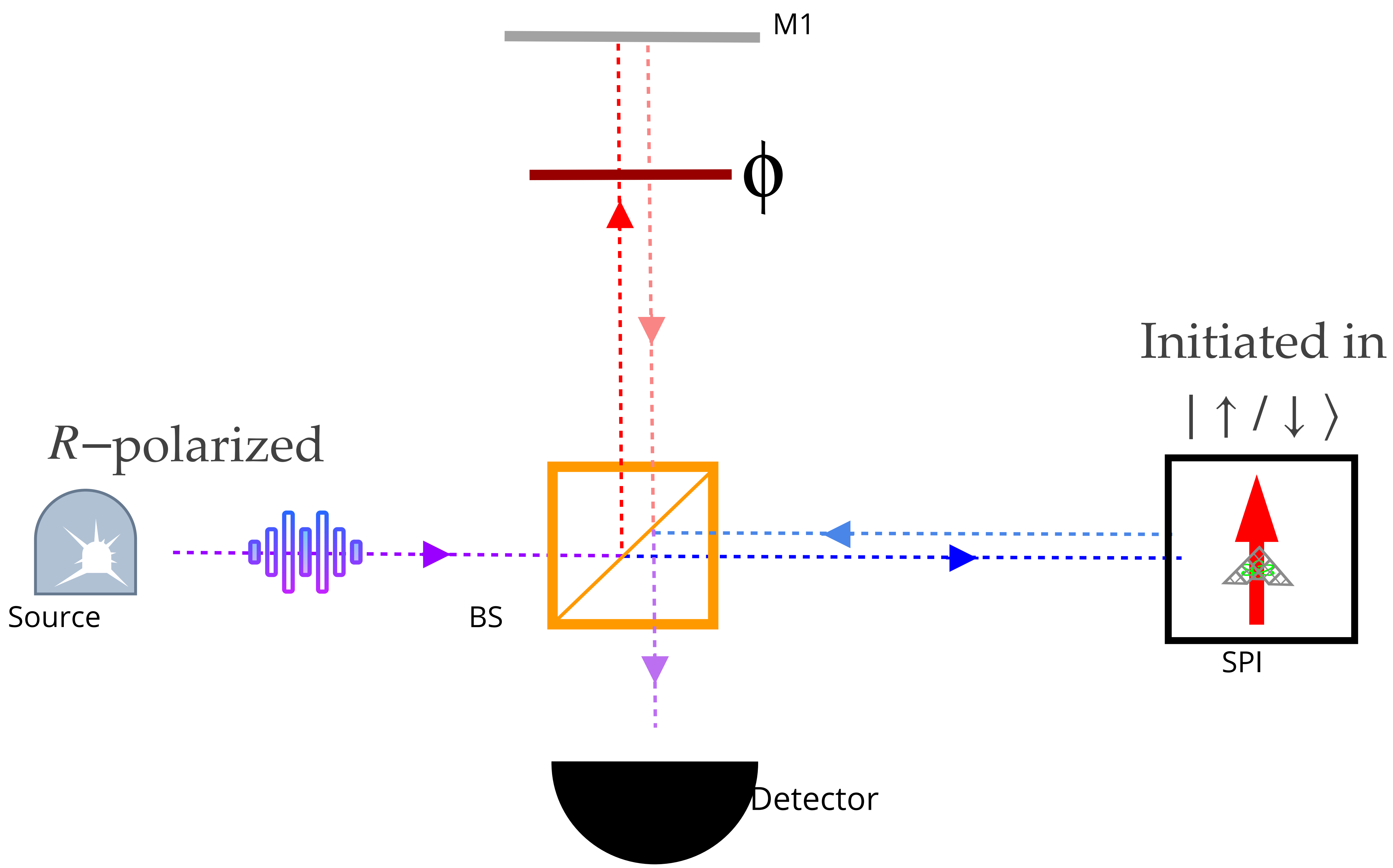
- Regime: $\bar{n} = p_H = 0.5$, and
- $\Gamma = 10^{-2}\gamma$

$$|\uparrow(\downarrow), 1_{\mathrm{R(L)}}\rangle \rightarrow -|\uparrow(\downarrow), 1_{\mathrm{R(L)}}\rangle$$

$$|\downarrow(\uparrow), 1_{\mathrm{R(L)}}\rangle \rightarrow |\downarrow(\uparrow), 1_{\mathrm{R(L)}}\rangle$$

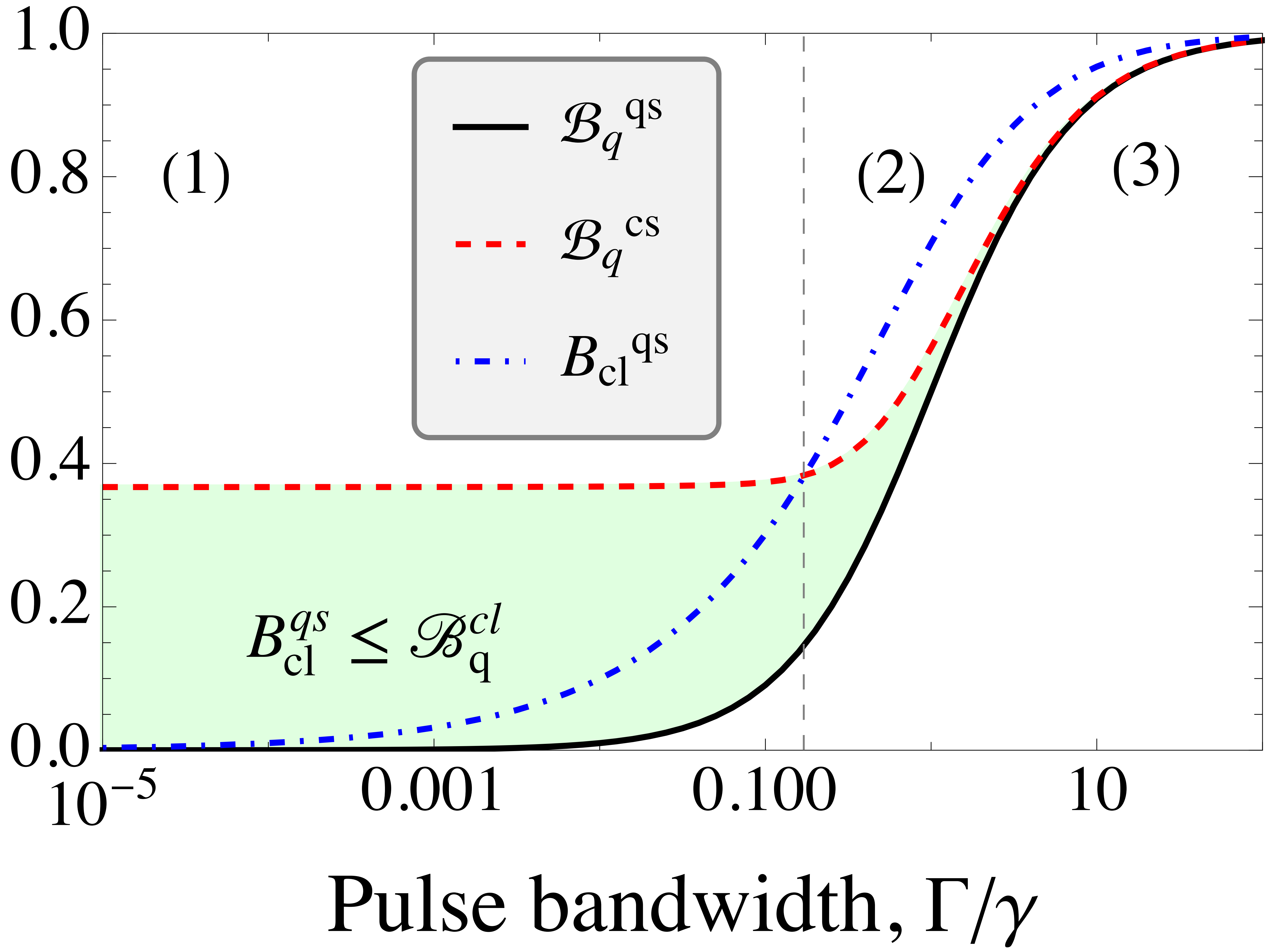


Michelson interferometer



Phase measurement

- Perfect measurement→$B_{cl} = 0$



- Region (1) quantum advantage: the cBhat of the quantum state is below the qBhat of the classical state!
- Region (2) the advantage is unclear: could exist another classical measurement with coherent field that performs better than the quantum.
- Region (3) → Unnacessible

Take away messages 4

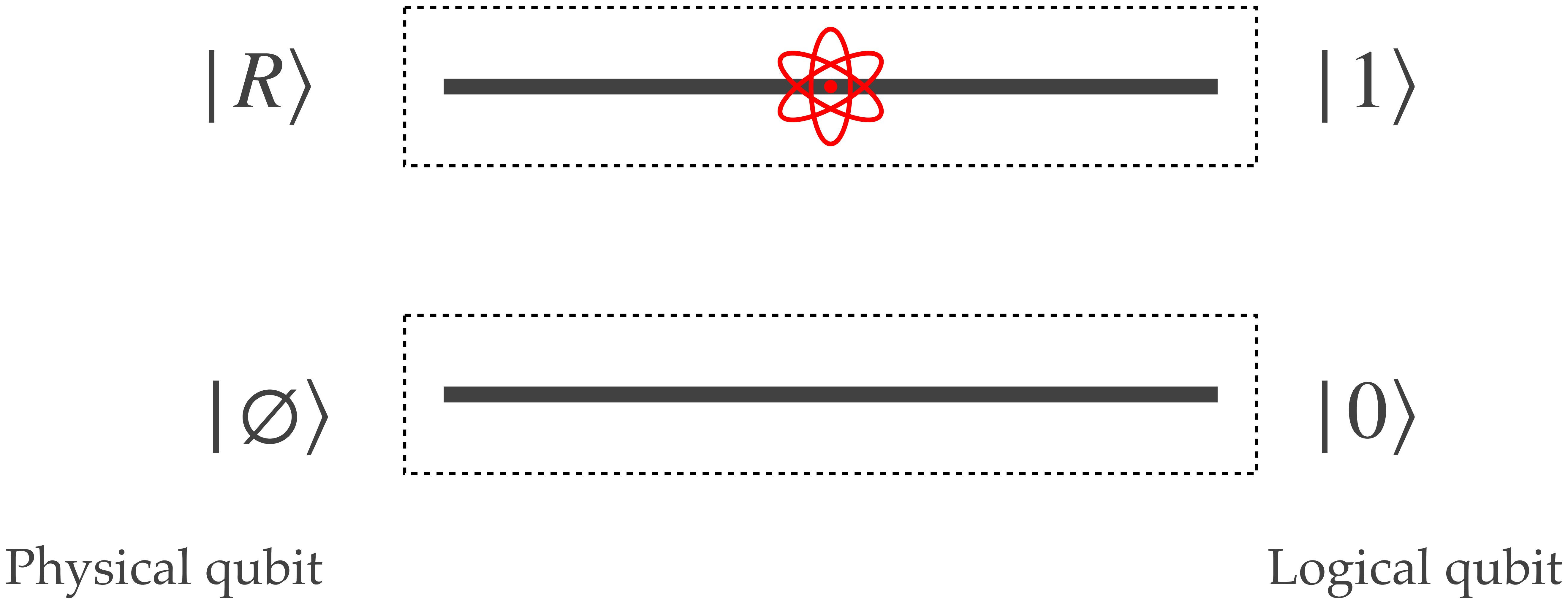
- **Low-energy regime** →restricted budget: one excitation on average
  - Compared coherent field with quantum superposition of zero and single photon states
- **Quantum advantage 1:** superposition of zero and single photon produces **better entanglement**
- **Quantum advantage 2:** maintained when spin state information extracted at classical level
- Relevant for technological applications such as photon-photon gates and cluster states.

Outline

- Preliminaries
  - von Neumann measurement model
  - Collisional model or how to close open systems
- Main results
  - Anomalous energy exchanges and Wigner-function negativities in a single qubit gate
  - Energy efficient entanglement generation and readout in a spin-photon interface
    - Photon-photon controlled phase-gate and error analysis
  - The SPI subjected to an in-plane magnetic field
- Conclusions and perspectives

C-Phase gate and Error

- **Goal**: perform a controlled phase gate [1] and characterize the error induced **solely due** to the **finite duration of the pulse.**
- Single rail basis:

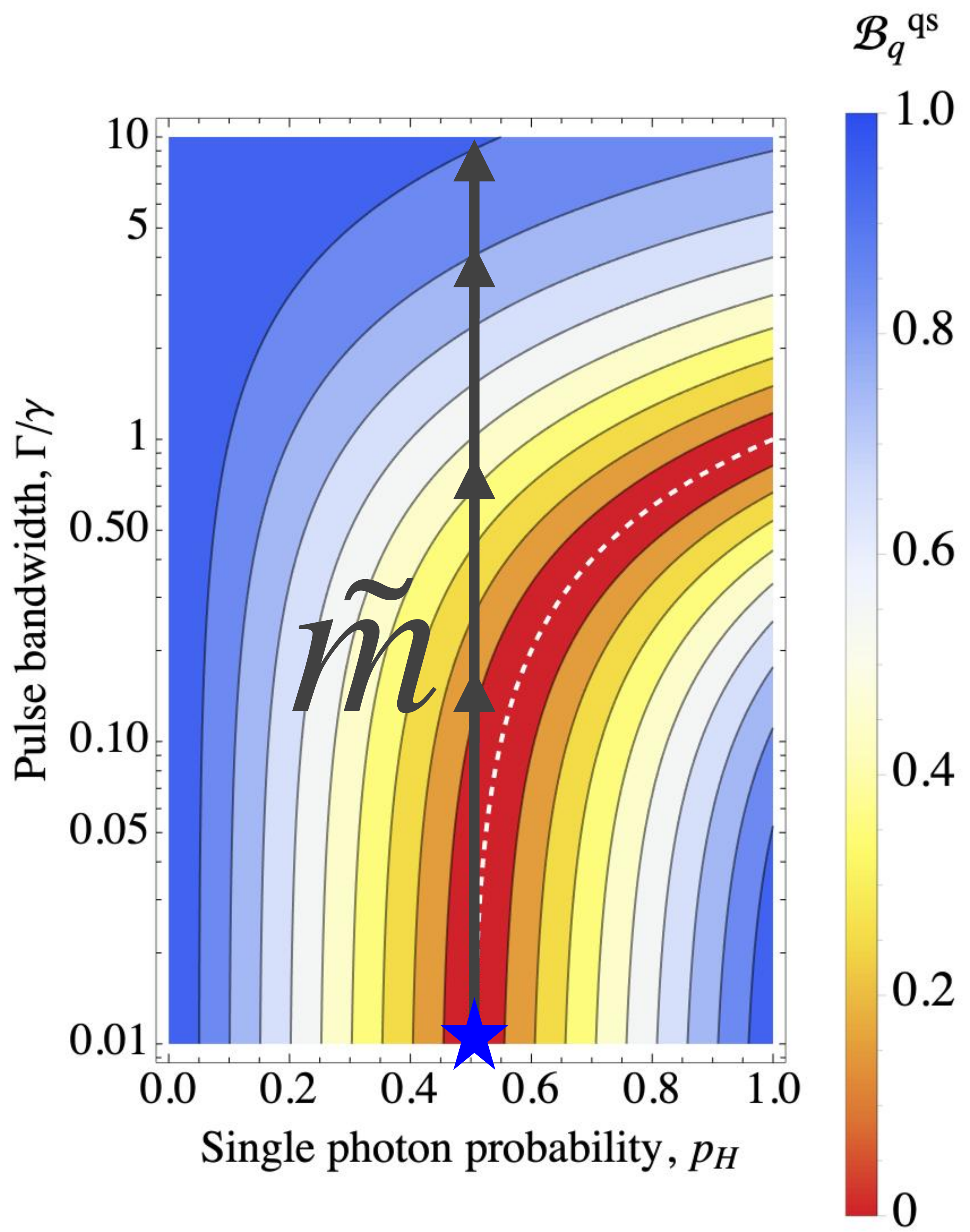


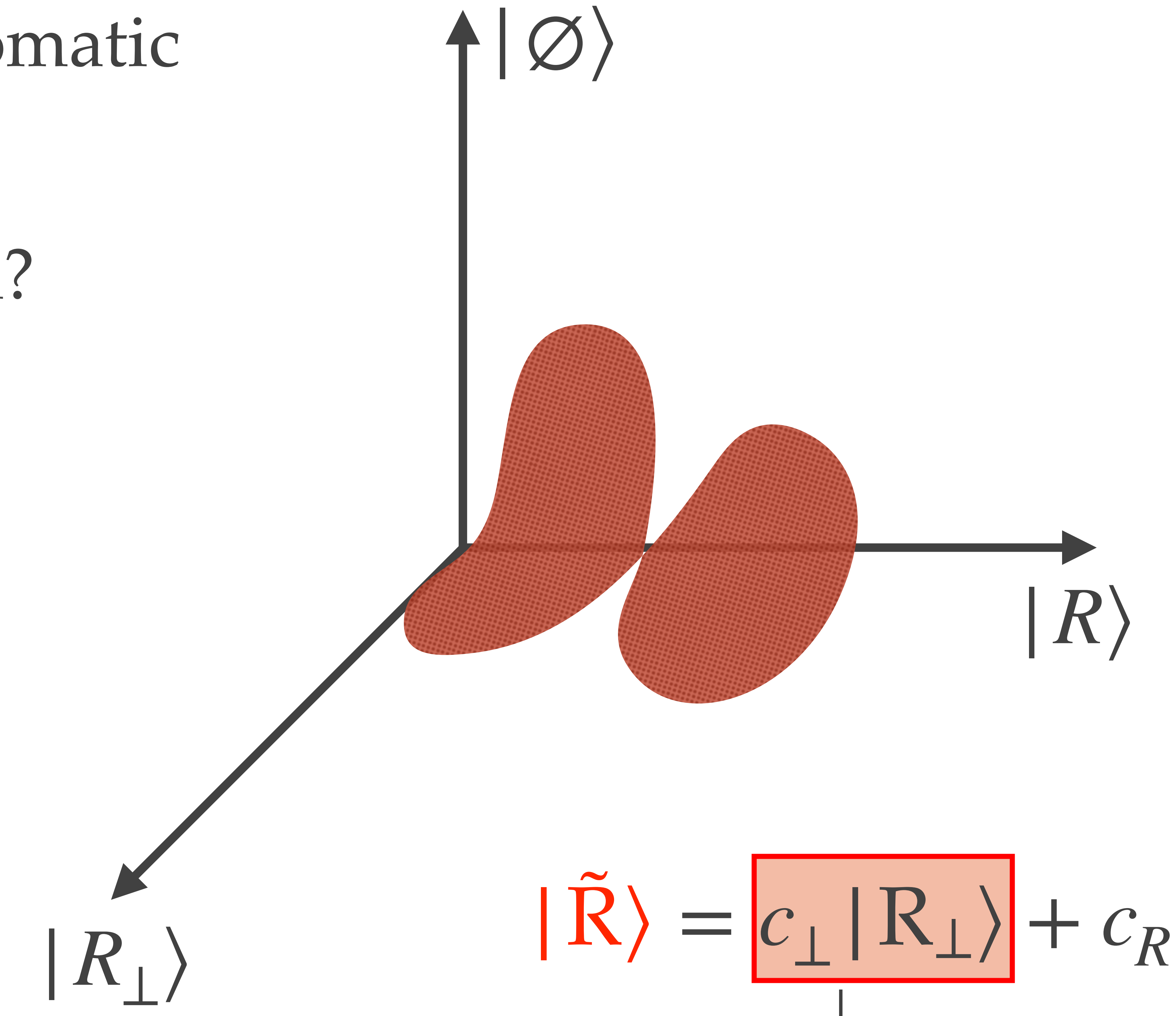
$$
\begin{aligned}
|\varnothing\rangle|\uparrow\rangle &\to |\varnothing\rangle|\uparrow\rangle \\
|\varnothing\rangle|\downarrow\rangle &\to |\varnothing\rangle|\downarrow\rangle \\
|\mathrm{R}\rangle|\uparrow\rangle &\to -|\tilde{\mathrm{R}}\rangle|\uparrow\rangle \xrightarrow{\text{Monochromatic}} -|\mathrm{R}\rangle \\
|\mathrm{R}\rangle|\downarrow\rangle &\to |\mathrm{R}\rangle|\downarrow\rangle
\end{aligned}
$$

[1] M. A. Nielsen and I. L. Chuang, "Quantum Computation and Quantum Information: 10th Anniversary Edition".

Light-matter interaction map: plots

- The protocol works in the quasi-monochromatic regime.
- What is the effect of a finite pulse duration?

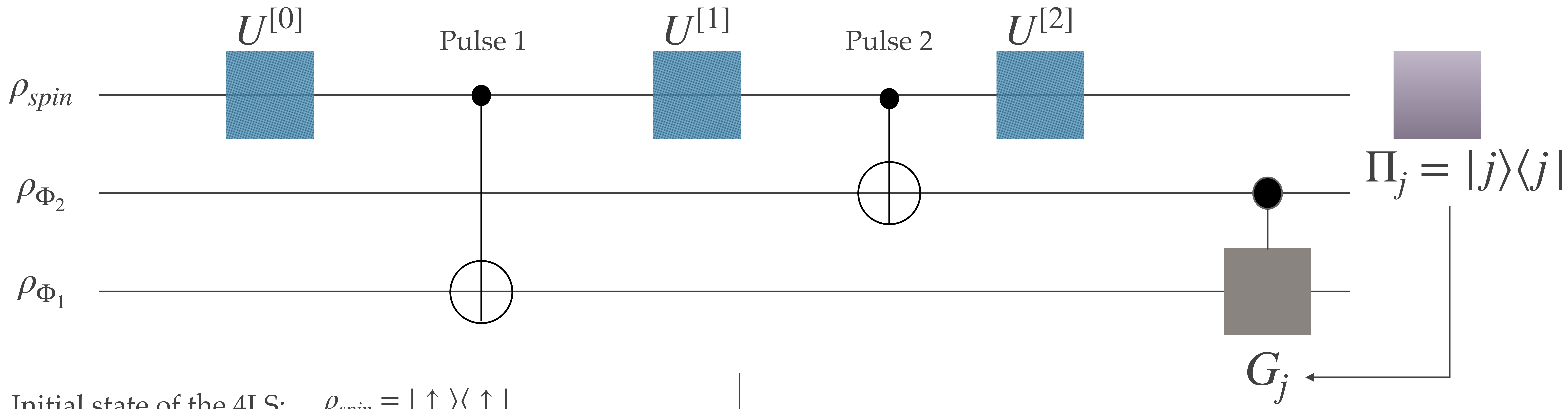




$$|\tilde{\mathrm{R}}\rangle = \boxed{c_\perp|\mathrm{R}_\perp\rangle} + c_R|\mathrm{R}\rangle$$

Leak out of the logical space.

Gate protocol



Initial state of the 4LS: $\rho_{spin} = |\uparrow\rangle\langle\uparrow|$

Light-pulses

$$\rho_{\Phi_1} = \rho_{\Phi_2} = \left(\frac{|\varnothing\rangle + |\mathrm{R}\rangle}{\sqrt{2}}\right)\left(\frac{\langle\varnothing| + \langle\mathrm{R}|}{\sqrt{2}}\right)$$

Spin rotations

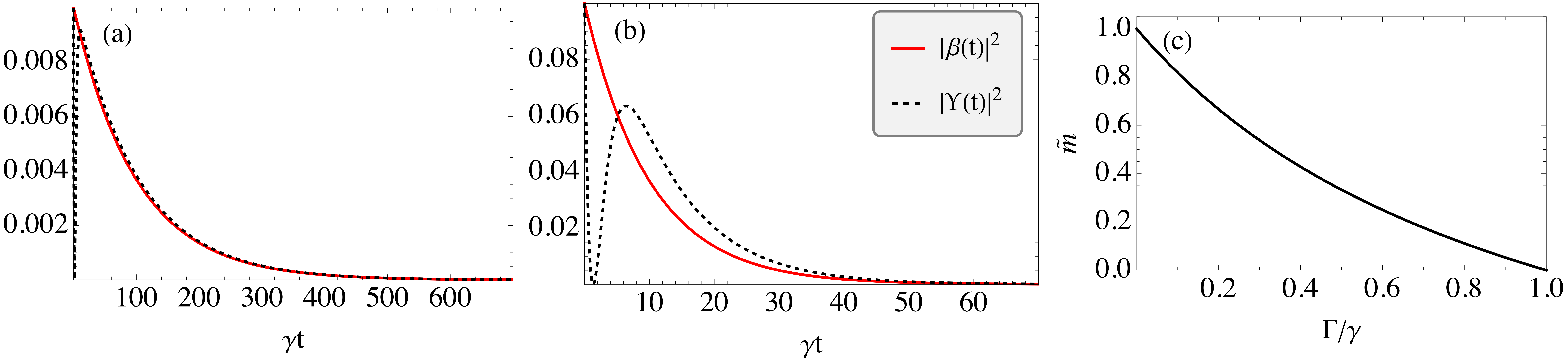
$$U^{[0]} = U^{[1]} = U^{[2]} = R_y\left(\frac{\pi}{2}\right) = \exp\left\{-i\frac{\pi}{4}\sigma_y\right\}$$

Light-matter interaction map: non-monochromaticity

- **"Non-monochromaticity"** of the scattering process:
  - $\tilde{m} = \langle \mathrm{R} | \tilde{\mathrm{R}} \rangle$

$$\tilde{m}(\xi, \Upsilon) = \int_0^\infty dt' \; \xi^*(t')\Upsilon(t')$$

$$|\mathrm{R}\rangle = \int dt \; \xi(t) b_{\mathrm{R}}^\dagger(t) |0\rangle \to |\tilde{\mathrm{R}}\rangle = \int dt \; \Upsilon(t) b_{\mathrm{R}}^\dagger(t) |0\rangle$$



[2] M. Maffei, B. O. Goes, S. C. Wein, A. N. Jordan, L. Lanco, and A. Auffèves, "Energy-efficient entanglement generation and readout in a spin-photon interface."

Gate action

Gate action for $|\uparrow\rangle$:

Real

$$|00\rangle \xrightarrow{G_{\uparrow}^{[1]}} |00\rangle$$
$$|01\rangle \xrightarrow{G_{\uparrow}^{[1]}} -\tilde{m}\,|01\rangle \equiv -e^{i\varepsilon_{\downarrow}}$$
$$|10\rangle \xrightarrow{G_{\uparrow}^{[1]}} |10\rangle$$
$$|11\rangle \xrightarrow{G_{\uparrow}^{[1]}} \frac{1}{2}(\tilde{m}^2+1)\,|11\rangle \equiv e^{i\delta_{\downarrow}}$$

$\xrightarrow{\tilde{m}=1}$

Ideal

$$|00\rangle \xrightarrow{G_{\uparrow}^{[1]}} |00\rangle$$
$$|01\rangle \xrightarrow{G_{\uparrow}^{[1]}} -|01\rangle$$
$$|10\rangle \xrightarrow{G_{\uparrow}^{[1]}} |10\rangle$$
$$|11\rangle \xrightarrow{G_{\uparrow}^{[1]}} |11\rangle$$

Gate errors:

$$\varepsilon_{\uparrow} = 2\pi - i\mathrm{Log}\left\{-\frac{(\tilde{m}^2+2\tilde{m}-1)}{2}\right\}$$

$$\delta_{\downarrow} = 2\pi - i\mathrm{Log}\left\{\frac{\tilde{m}+1}{2}\right\}$$

Gate action for $|\downarrow\rangle$:

Real

$$|00\rangle \xrightarrow{G_{\uparrow}^{[1]}} -|00\rangle$$
$$|01\rangle \xrightarrow{G_{\uparrow}^{[1]}} -|01\rangle$$
$$|10\rangle \xrightarrow{G_{\uparrow}^{[1]}} -|10\rangle$$
$$|11\rangle \xrightarrow{G_{\uparrow}^{[1]}} \frac{1}{2}(\tilde{m}^2+2\tilde{m}-1)\,|11\rangle \equiv -e^{i\varepsilon_{\uparrow}}$$

$\xrightarrow{\tilde{m}=1}$

Ideal

$$|00\rangle \xrightarrow{G_{\uparrow}^{[1]}} -|00\rangle$$
$$|01\rangle \xrightarrow{G_{\uparrow}^{[1]}} -|01\rangle$$
$$|10\rangle \xrightarrow{G_{\uparrow}^{[1]}} -|10\rangle$$
$$|11\rangle \xrightarrow{G_{\uparrow}^{[1]}} |11\rangle$$

Gate error:

$$\varepsilon_{\downarrow} = 2\pi - i\mathrm{Log}\,\{-\tilde{m}\}$$

Error matrix

$$G_j^{\text{real}} = E_j G_j^{\text{ideal}} \rightarrow E_j = G_j^{\text{real}} \left(G_j^{\text{ideal}} \right)^{-1}$$

Pauli error model

Decompose the error in a Pauli basis

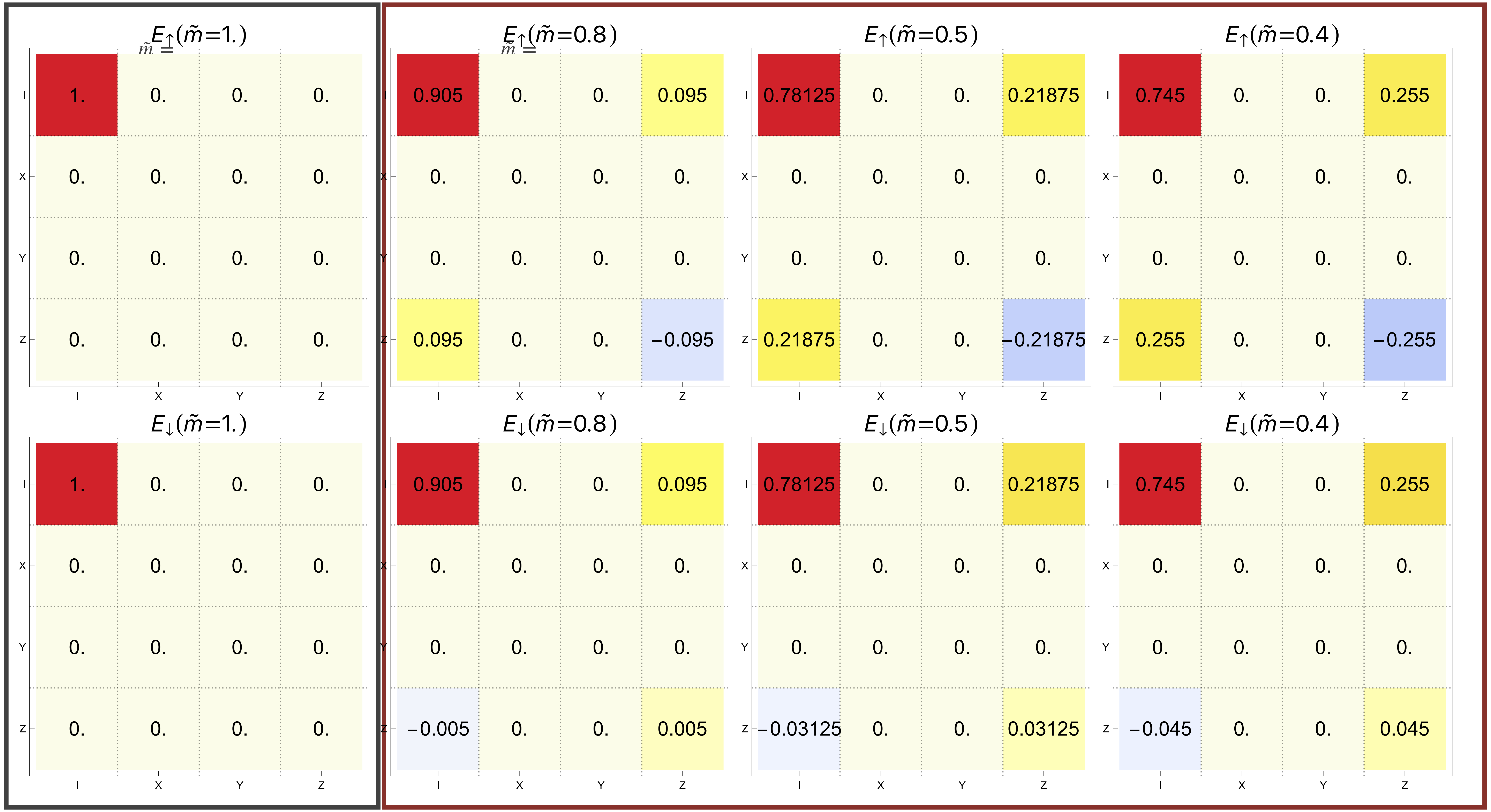
$$\mathscr{B}_P = \left\{ \underbrace{1 \equiv I}_{\text{No error.}},\ \underbrace{\sigma_x \equiv X}_{\text{Bit flip.}},\ \underbrace{\sigma_y \equiv Y}_{\text{Bit flip \& Phase error.}},\ \underbrace{\sigma_z \equiv Z}_{\text{Phase error.}} \right\}$$

$$\mathscr{B}_2 = \left(\mathscr{B}_P \right)^{\otimes 2}$$

[3] A. N. Korotkov, "Error matrices in quantum process tomography."

Error matrices: Hinton diagram

Ideal process:
only one peak [3]



Imperfect processes

[3] A. N. Korotkov, "Error matrices in quantum process tomography."

Take away messages 5

- Exploiting our analytical knowledge of the state of the electromagnetic field we found how the gate's fidelity depends on the duration and shape of the light pulses;
- Our detailed modeling of the gate also allowed us to analyze the gate's error;
- We aim to propose protocols of error correction based on well-established techniques of pulse manipulation.

Outline

- Preliminaries
  - von Neumann measurement model
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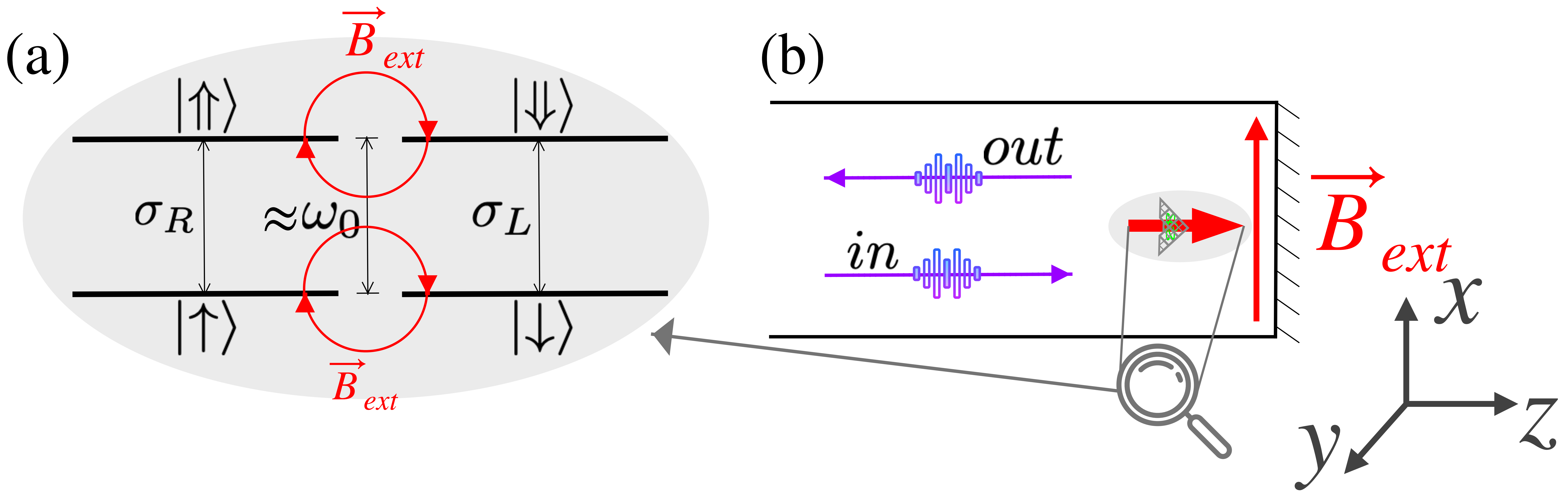
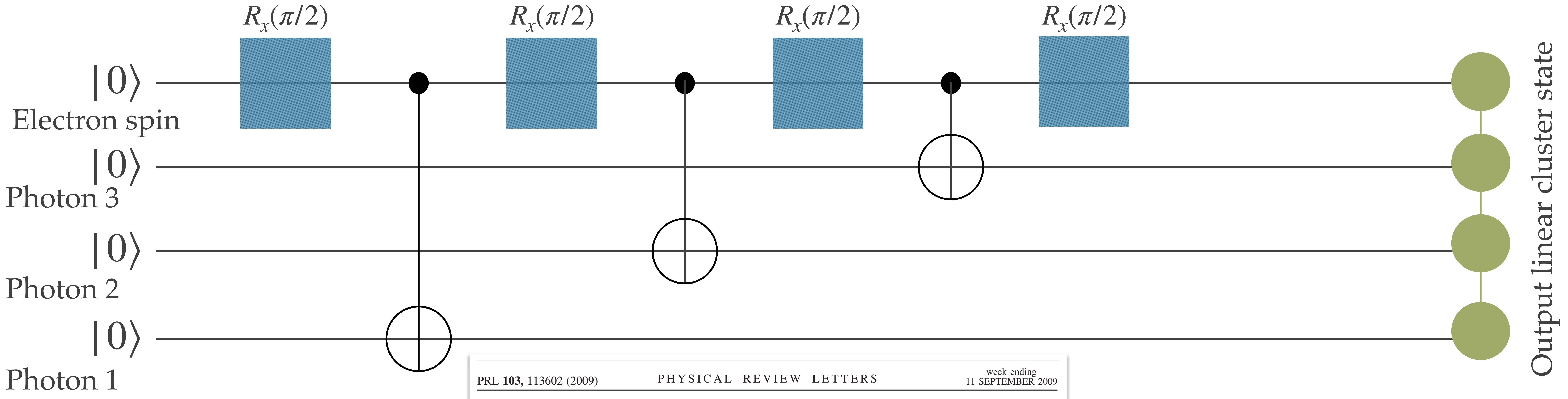
  - SPI under a magnetic field
- Conclusions and perspectives

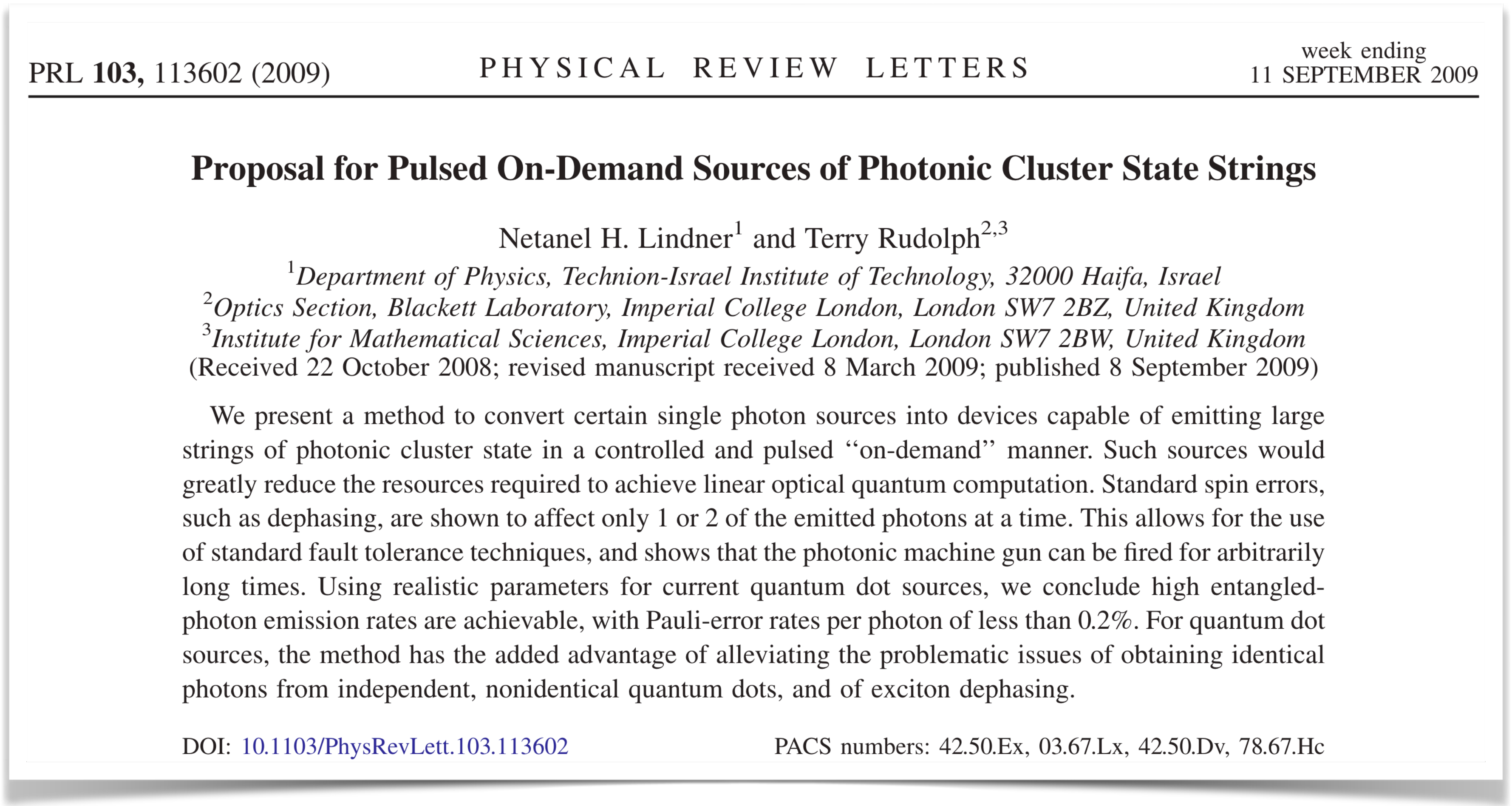
Measuring the spin state



- **Set-up:** Same as before + perpendicular magnetic field (Voigt configuration)
  - Necessary to perform the **Lindner Rudolph protocol (LRP)** → Generation of highly photonic entangled states, precisely cluster states

What is the LRP and how does it work?





PRL **103,** 113602 (2009) PHYSICAL REVIEW LETTERS week ending 11 SEPTEMBER 2009

Proposal for Pulsed On-Demand Sources of Photonic Cluster State Strings

Netanel H. Lindner[1] and Terry Rudolph[2,3]

[1]*Department of Physics, Technion-Israel Institute of Technology, 32000 Haifa, Israel*
[2]*Optics Section, Blackett Laboratory, Imperial College London, London SW7 2BZ, United Kingdom*
[3]*Institute for Mathematical Sciences, Imperial College London, London SW7 2BW, United Kingdom*
(Received 22 October 2008; revised manuscript received 8 March 2009; published 8 September 2009)

We present a method to convert certain single photon sources into devices capable of emitting large strings of photonic cluster state in a controlled and pulsed "on-demand" manner. Such sources would greatly reduce the resources required to achieve linear optical quantum computation. Standard spin errors, such as dephasing, are shown to affect only 1 or 2 of the emitted photons at a time. This allows for the use of standard fault tolerance techniques, and shows that the photonic machine gun can be fired for arbitrarily long times. Using realistic parameters for current quantum dot sources, we conclude high entangled-photon emission rates are achievable, with Pauli-error rates per photon of less than 0.2%. For quantum dot sources, the method has the added advantage of alleviating the problematic issues of obtaining identical photons from independent, nonidentical quantum dots, and of exciton dephasing.

DOI: 10.1103/PhysRevLett.103.113602 PACS numbers: 42.50.Ex, 03.67.Lx, 42.50.Dv, 78.67.Hc

[1] Lindner, N. H. & Rudolph, T. Proposal for Pulsed On-Demand Sources of Photonic Cluster State Strings. Phys. Rev. Lett. 103, 113602 (2009).

Experimental implementation

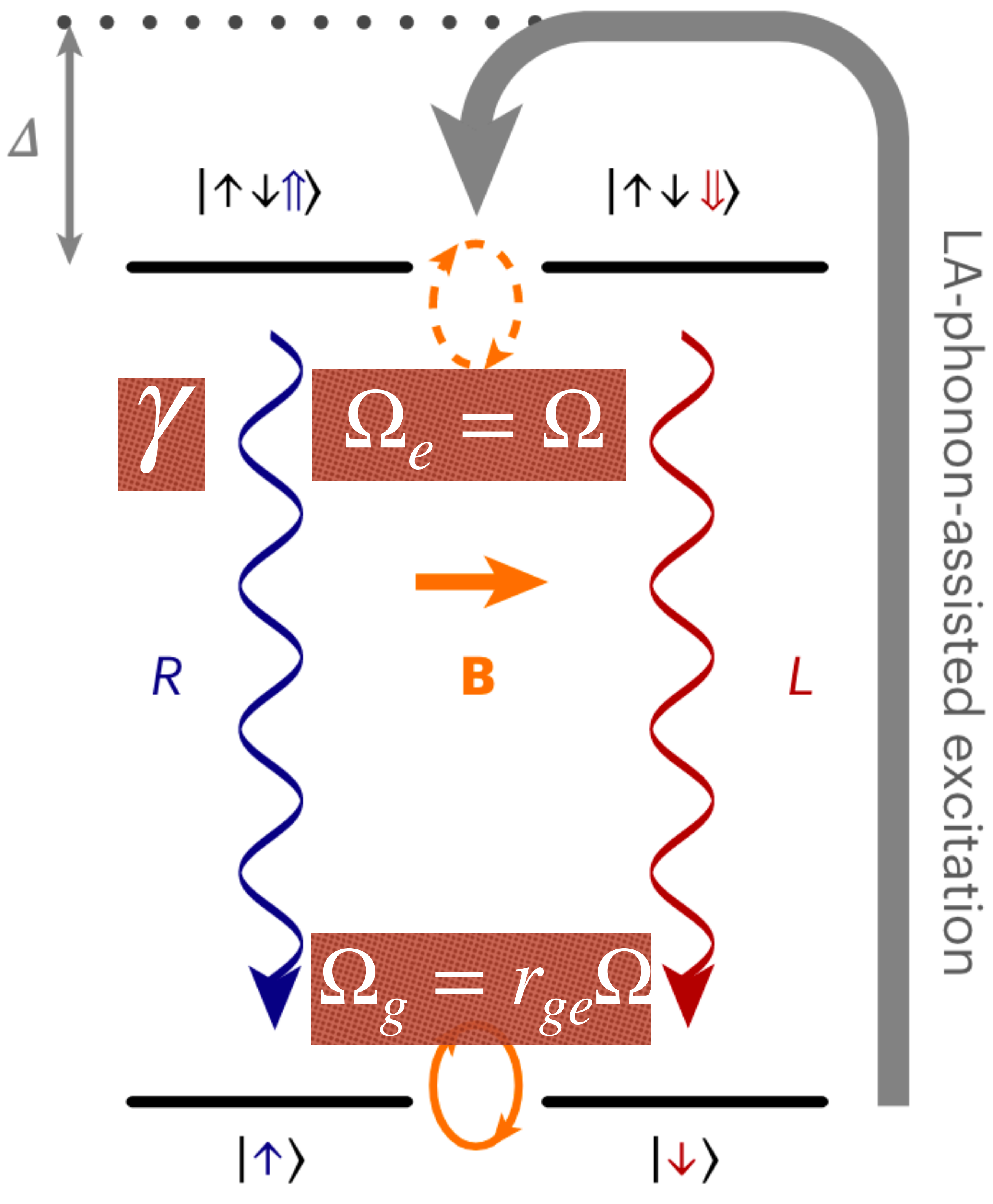
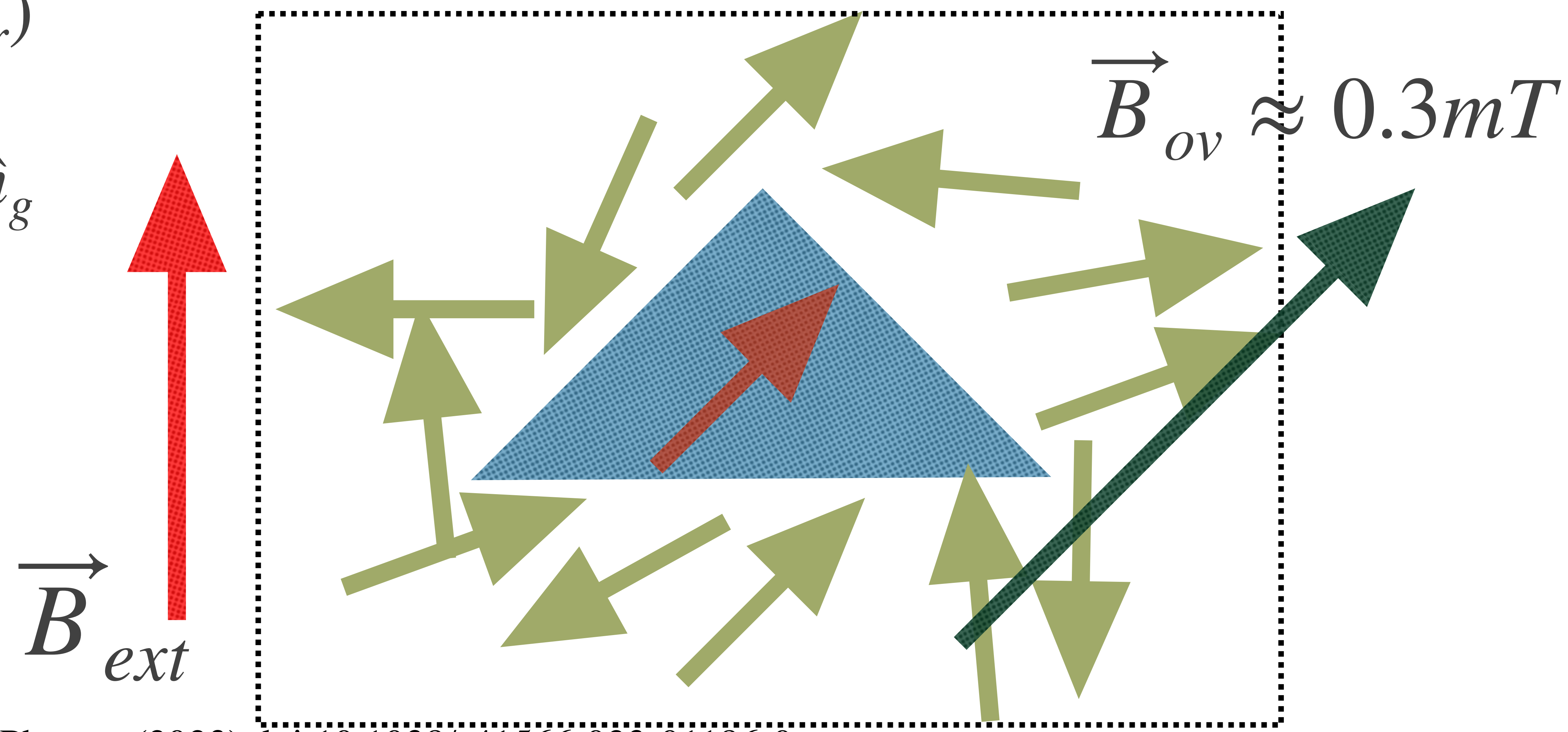


Figure taken from Ref. [2]

- $\Omega = \mu_B g_h \overrightarrow{B}_{ext}$
- $\gamma^{-1} \ll \Omega^{-1}$
- $r_{\Omega\gamma} = \Omega/\gamma \ll 1$
- $\Omega_g = \mu_B g_e(\overrightarrow{B}_{ext} + \delta\overrightarrow{B}_{par})$
- $\overrightarrow{B}_{ext} + \delta\overrightarrow{B}_{par} = \overrightarrow{B}_g = B_g \hat{n}_g$
- $r_{ge} = \Omega_g/\Omega$
- $\langle \overrightarrow{B}_g, \hat{x} \rangle$

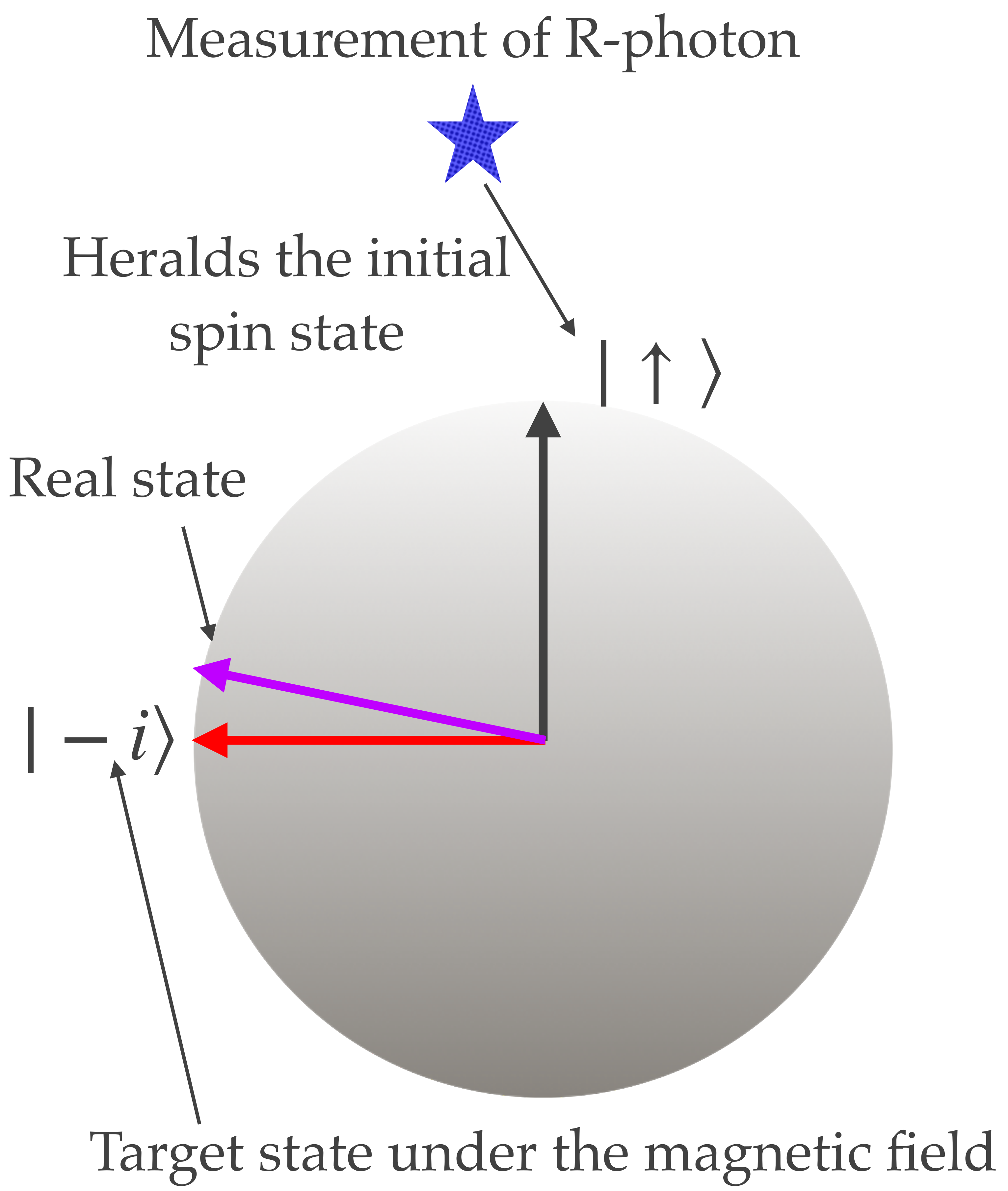
- Ideal scenario:
- $r_{\Omega\gamma} = \Omega/\gamma \ll 1$
- $r_{ge} = 1$
- $\langle \hat{n}_g, \hat{x} \rangle = 1$

What is the effect of the deviations from this functioning?



[2] Coste, N. et al. High-rate entanglement between a semiconductor spin and indistinguishable photons. Nat. Photon. (2023) doi:10.1038/s41566-023-01186-0.
[3] Coste, N. et al. Probing the dynamics and coherence of a semiconductor hole spin via acoustic phonon-assisted excitation. Preprint at http://arxiv.org/abs/2207.05981 (2022).

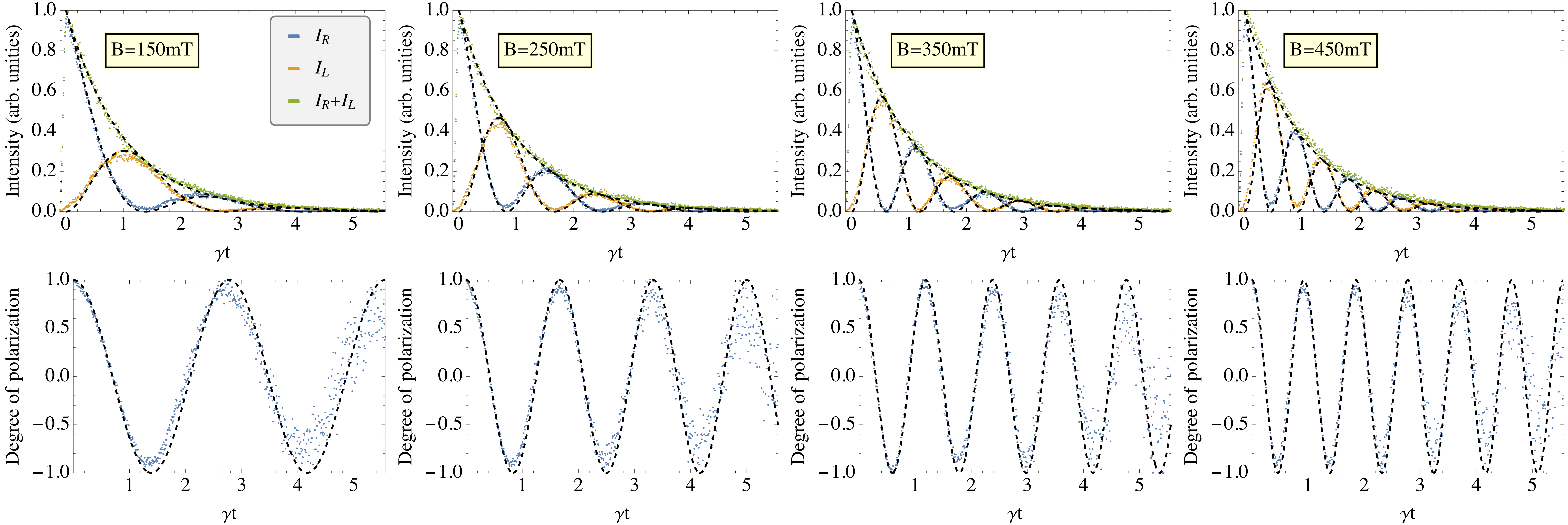
Solution



$$
\begin{aligned}
|\Psi^{\zeta}(t)\rangle = &\left(\exp\left\{-\frac{\gamma}{2}t\right\}\langle\uparrow_z|\mathscr{R}_e|\zeta\rangle\right)|e\rangle|\uparrow_z\rangle|\varnothing\rangle|\varnothing\rangle \\
&+\left(\exp\left\{-\frac{\gamma}{2}t\right\}\langle\downarrow_z|\mathscr{R}_e|\zeta\rangle\right)|e\rangle|\downarrow_z\rangle|\varnothing\rangle|\varnothing\rangle \\
&+\left(\int_0^t du\,\langle\uparrow_z|\mathscr{R}_g(t-u)|\uparrow_z\rangle\sqrt{\gamma}b_{\mathrm{R}}^{\dagger}\exp\left\{-\frac{\gamma}{2}u\right\}\langle\uparrow_z|\mathscr{R}_e(t-u)|\zeta\rangle\right)|g\rangle|\uparrow_z\rangle|\varnothing\rangle|\varnothing\rangle \\
&+\left(\int_0^t du\,\langle\uparrow_z|\mathscr{R}_g(t-u)|\downarrow_z\rangle\sqrt{\gamma}b_{\mathrm{L}}^{\dagger}\exp\left\{-\frac{\gamma}{2}u\right\}\langle\downarrow_z|\mathscr{R}_e(t-u)|\zeta\rangle\right)|g\rangle|\uparrow_z\rangle|\varnothing\rangle|\varnothing\rangle \\
&+\left(\int_0^t du\,\langle\downarrow_z|\mathscr{R}_g(t-u)|\uparrow_z\rangle\sqrt{\gamma}b_{\mathrm{R}}^{\dagger}\exp\left\{-\frac{\gamma}{2}u\right\}\langle\uparrow_z|\mathscr{R}_e(t-u)|\zeta\rangle\right)|g\rangle|\downarrow_z\rangle|\varnothing\rangle|\varnothing\rangle \\
&+\left(\int_0^t du\,\langle\downarrow_z|\mathscr{R}_g(t-u)|\downarrow_z\rangle\sqrt{\gamma}b_{\mathrm{L}}^{\dagger}\exp\left\{-\frac{\gamma}{2}u\right\}\langle\downarrow_z|\mathscr{R}_e(t-u)|\zeta\rangle\right)|g\rangle|\downarrow_z\rangle|\varnothing\rangle|\varnothing\rangle
\end{aligned}
$$

$$\mathscr{F}(\rho_{tar},\rho_{real}) = \mathrm{tr}\left\{\sqrt{\sqrt{\rho_{tar}}\rho_{real}\sqrt{\rho_{tar}}}\right\}$$

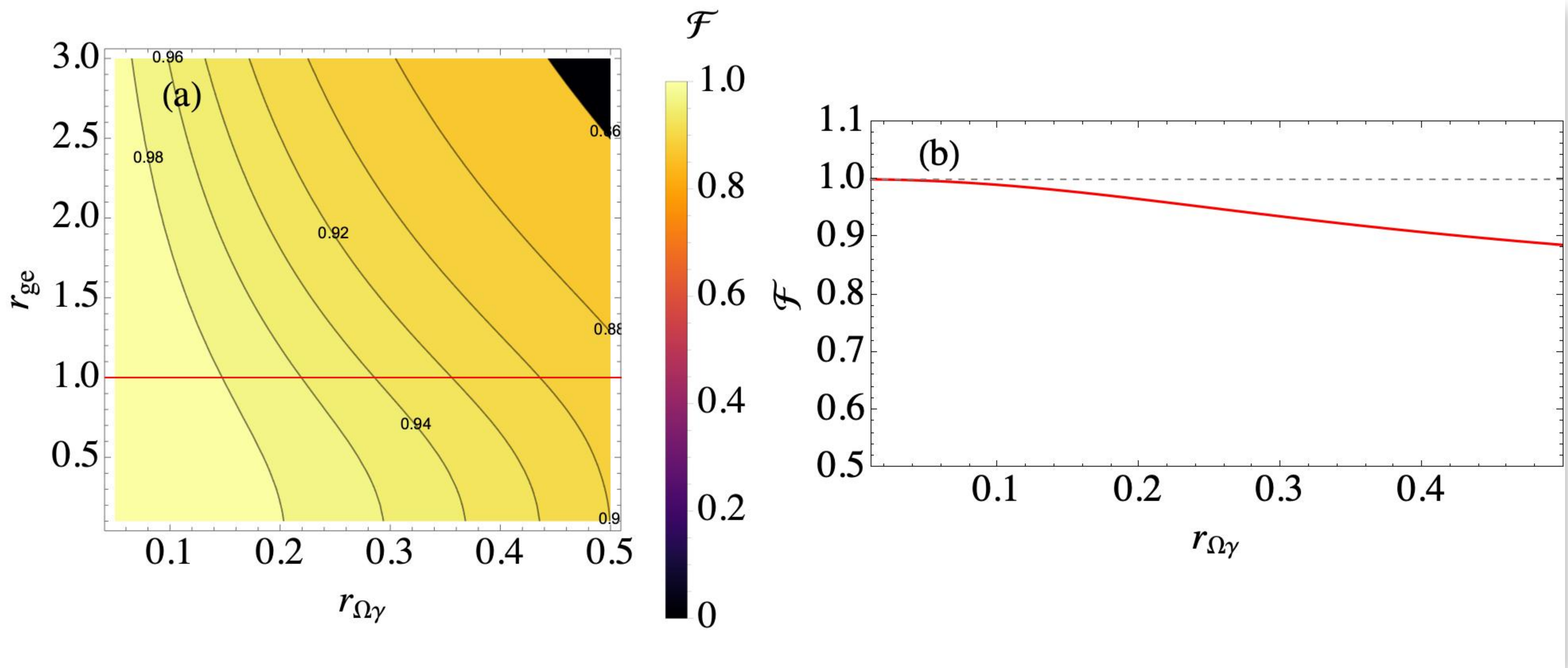
The model fits experimental results.



The closed dynamics solution replicates experimental data from Ref. [2] (provided by the authors)

[2] Coste, N. et al. High-rate entanglement between a semiconductor spin and indistinguishable photons. Nat. Photon. (2023) doi:10.1038/s41566-023-01186-0.
[3] Coste, N. et al. Probing the dynamics and coherence of a semiconductor hole spin via acoustic phonon-assisted excitation. Preprint at http://arxiv.org/abs/2207.05981 (2022).

Fidelity analysis: impact of finite pulse duration $r_{\Omega\gamma}$ and different precession frequencies r_{ge}

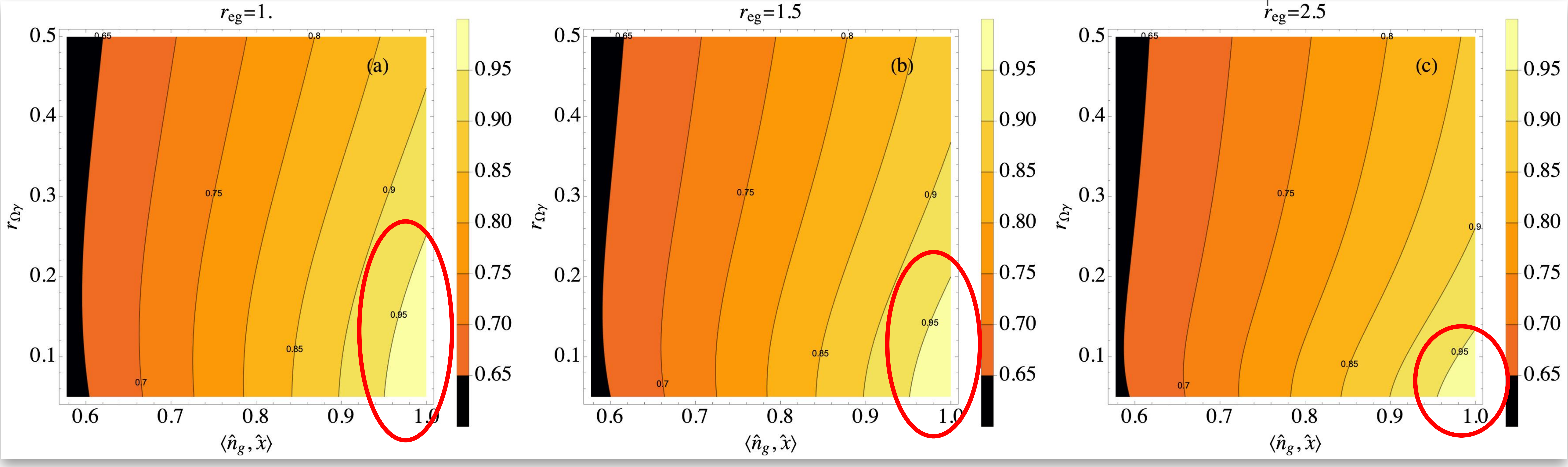


Perfect aligned magnetic field: overall we have a high fidelity.

$r_{\Omega\gamma}$ plays a bigger role.

Fidelity analysis for different r_{ge}: impact of finite pulses $r_{\Omega\gamma}$ and imperfect alignment of the magnetic field $\langle \hat{n}_g, \hat{x} \rangle$

Closer to experimental situation



Tilted magnetic field: the region of high fidelity **shrinks** as the r_{ge}

Outline

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  - SPI under a magnetic field
- Conclusions and perspectives

Conclusions and perspectives

- **Comprehensive Hamiltonian Solution**: We employed a comprehensive Hamiltonian solution to solve system of technological interest.
    - **Closing Quantum Systems**: The technique closes systems that are usually treated as open quantum systems or cQED, providing complete information about the entangled state.
- **Contributions:**
    - **Quantum Contextuality**: Explored the emergence of non-classical behaviors linked with contextulity in a single qubit gate: anomalous weak values and Wigner negativity.
    - **Spin-Photon Interface**: Achieved quantum advantage in non-destructive spin state measurements, relevant for quantum information protocols.
    - **Photon-Photon Gate**: Proposed a single-rail photon-photon gate, analyzed error characteristics thanks to the dynamics resolution.
    - **Modeling the LRP**: Extended solutions to include in-plane magnetic fields to characterize the protocol.
- **Perspectives**
    - **Spin-Photon Entanglement**: Investigate spin-spin entanglement generation via the spin-photon entanglement generated.
    - **Non-Classical Light States**: Develop new protocols to generate non-classical states of light.
    - **Quantum Thermodynamics**: Use the approach to study the thermodynamics of quantum systems with quantum optics.

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Publications

PHYSICAL REVIEW A **107**, 023710 (2023)

Anomalous energy exchanges and Wigner-function negativities in a single-qubit gate

Maria Maffei,[1] Cyril Elouard,[2] Bruno O. Goes,[1] Benjamin Huard,[3] Andrew N. Jordan,[4,5] and Alexia Auffèves[1,6]

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Quantum

the open journal for quantum science

PAPERS

Energy-efficient quantum non-demolition measurement with a spin-photon interface

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High-rate entanglement between a semiconductor spin and indistinguishable photons

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Check for updates

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Participation in events

- QUDOT-TECH + QLUSTER workshop! - Palaiseau. Talk. 2023. (Workshop)
- QUDOT-TECH 3rd summer school - Oxford. Poster presentation. 2023. (School).
- (Post)modern thermodynamics school + workshop - Luxembourg. Poster presentation. Dec. 2022 (School & workshop).
- QUDOT-TECH 2nd summer school - Denmark. Poster presentation. 2022. (School).
- QUDOT-TECH 1st summer school - Online. Poster presentation. 2021. (School).
- Quantum Thermodynamics Summer School - Switzerland. (Summer school).
- Quantum nanophotonics - Centro de Ciencias de Benasque Pedro Pascual - Online. Poster presentation. 2021. (Workshop)

Thank you!

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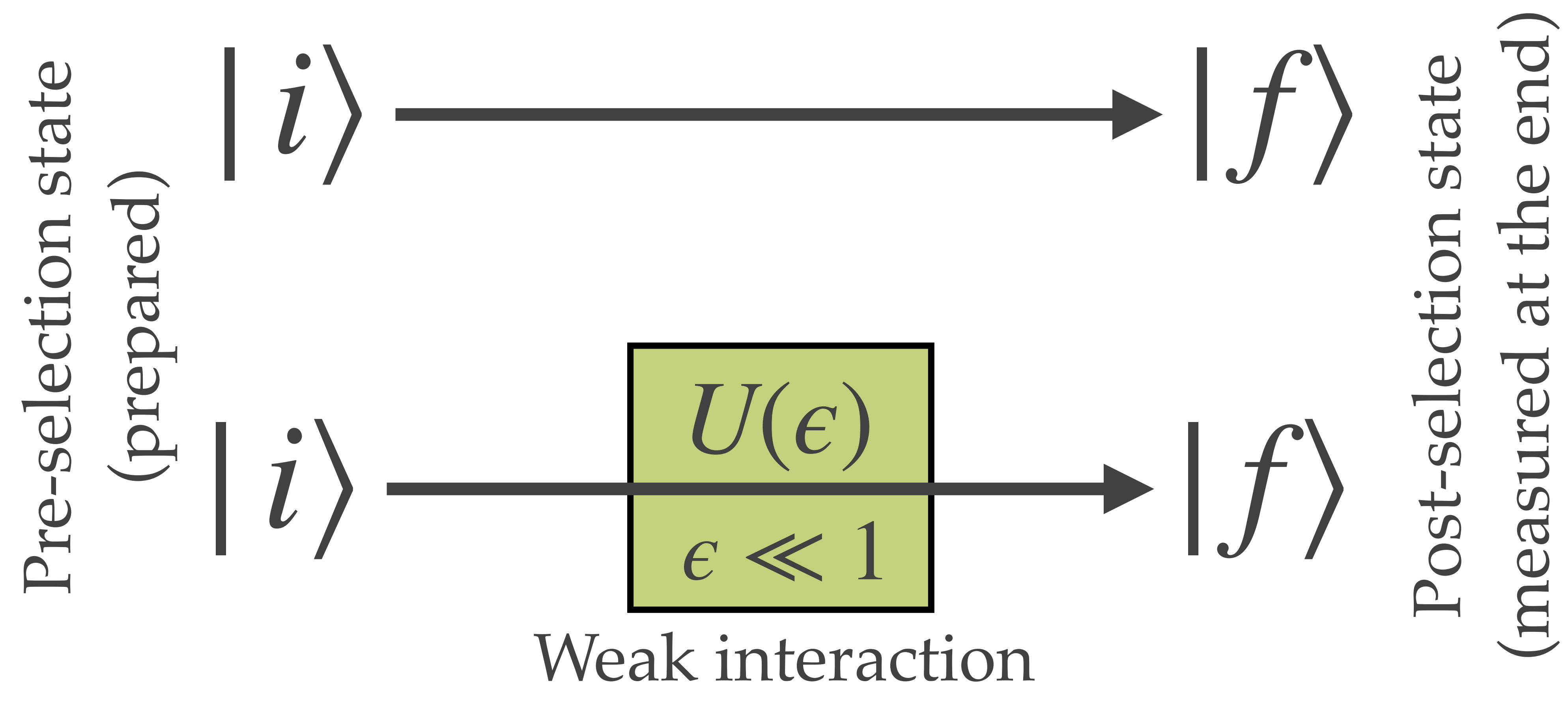
Backup slides

Quantum weak values



$$P_{i\to f} = |\langle f|i\rangle|^2$$

$$P_{i\to f}(\epsilon) = |\langle f|U(\epsilon)|i\rangle|^2$$

$$\frac{P_{i\to f}(\epsilon)}{P_{i\to f}} = 1 + 2\epsilon\Im\{A_w^1\} + \mathcal{O}(\epsilon^2)$$

The nth order weak value of A has the form:

$$\boxed{A_w^n = \frac{\langle f|A^n|i\rangle}{\langle f|i\rangle}}$$

- Weak measurements → **minimal disturbance to the system.**

$$U(\epsilon) = \exp\{-i\epsilon A\} \approx 1 - i\epsilon A + \ldots$$

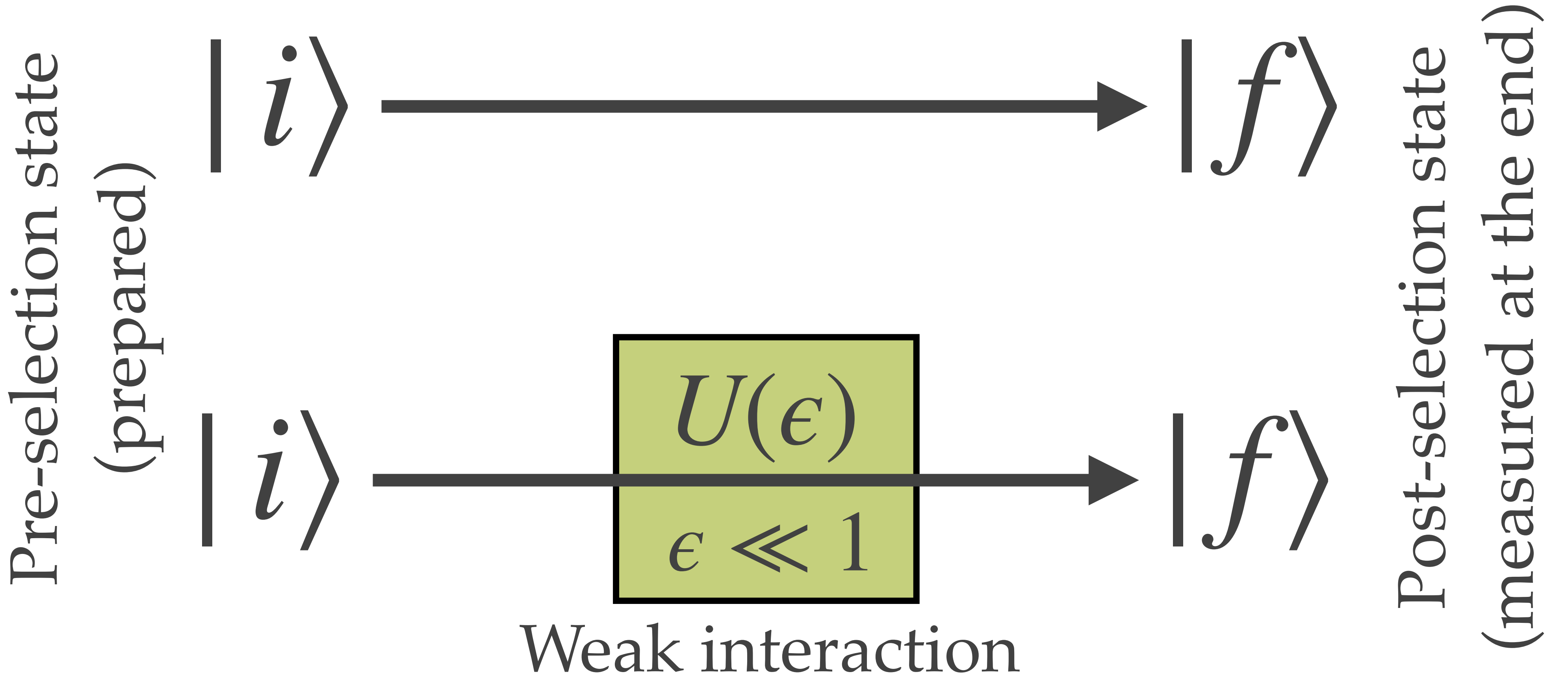
$$P_{i\to f}(\epsilon) = |\langle f|U(\epsilon)|i\rangle|^2 = P_{i\to f} + 2\epsilon\Im\{\langle i|f\rangle\langle f|A|i\rangle\} + \mathcal{O}(\epsilon^2)$$

$|i\rangle$=pre-selection state

$|f\rangle$=post-selection state

Y. Aharonov, D. Albert, and L. Vaidman, "How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100," Physical Review Letters 60, 1351 (1988).
J. Dressel, M. Malik, F. M. Miatto, A. N. Jordan, and R. W. Boyd, "*Colloquium*: Understanding quantum weak values: Basics and applications," *Rev. Mod. Phys.*, vol. 86, no. 1, pp. 307–316, Mar. 2014

Quantum weak values



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- The weak value → **shift in the measured value** of the observable due to a weak measurement
- They can be:
  - **Outside the range of eigenvalues→Anomalous weak values→nonclassical effect .**
  - Complex Values

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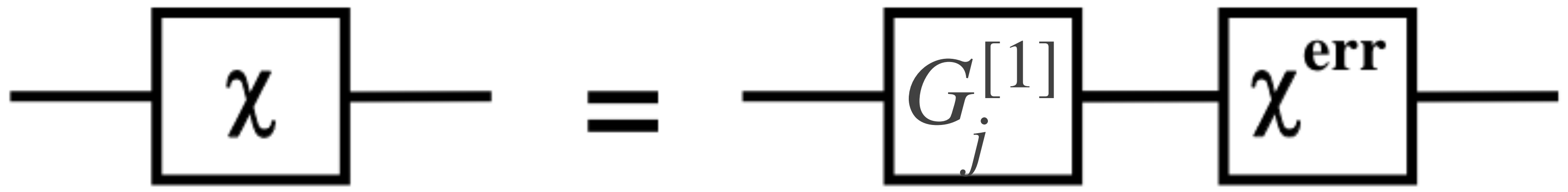
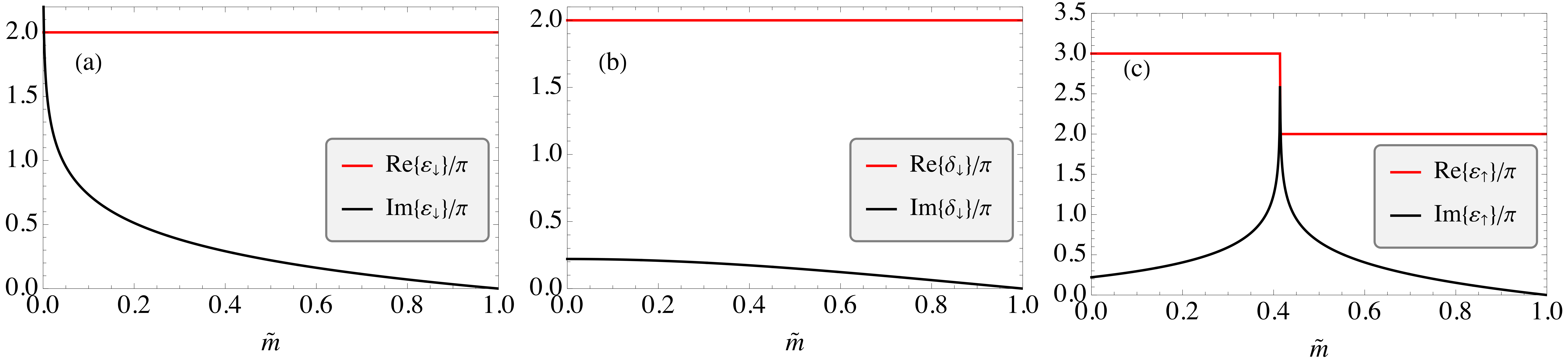
Brief review of quantum operations and process matrix

- $\rho \rightarrow \frac{\mathscr{E}(\rho)}{tr\{\mathscr{E}(\rho)\}}$, where $\mathscr{E}$ is the quantum operation describing the dynamics of the system;
- One can write $\mathscr{E}(\rho) = \sum_i K_i \rho K_i^\dagger = \sum_{mn} \chi_{mn} \tilde{K}_m \rho \tilde{K}_n^\dagger$
- χ_{mn} is the process matrix, which can be accessed experimentally, we consider the process as composed of the desired unitary $G_j^{[1]}$ and the **error process** χ^{error} [3];
- In the error matrix only one peak (at the top left corner) corresponds to the desired operation, while **other peaks indicate imperfections.**



[3] A. N. Korotkov, "Error matrices in quantum process tomography."

Error analysis: Gate errors



Protocol - place holder

Error matrix

$$G_j^{\text{real}} = E_j G_j^{\text{ideal}} \to E_j = G_j^{\text{real}} \left(G_j^{\text{ideal}} \right)^{-1}$$

Pauli error model

$$\mathscr{B}_P = \left\{ 1 \equiv I, \sigma_x \equiv X, \sigma_y \equiv Y, \sigma_z \equiv Z \right\}$$

Decompose the error in a Pauli basis

$$\mathscr{B}_2 = \left(\mathscr{B}_P \right)^{\otimes 2}$$

Initial photonic state $\rho_0^{\text{photonic}} \to \sum_{B \in \mathscr{B}_2} \boxed{p_B} B \rho B^\dagger$

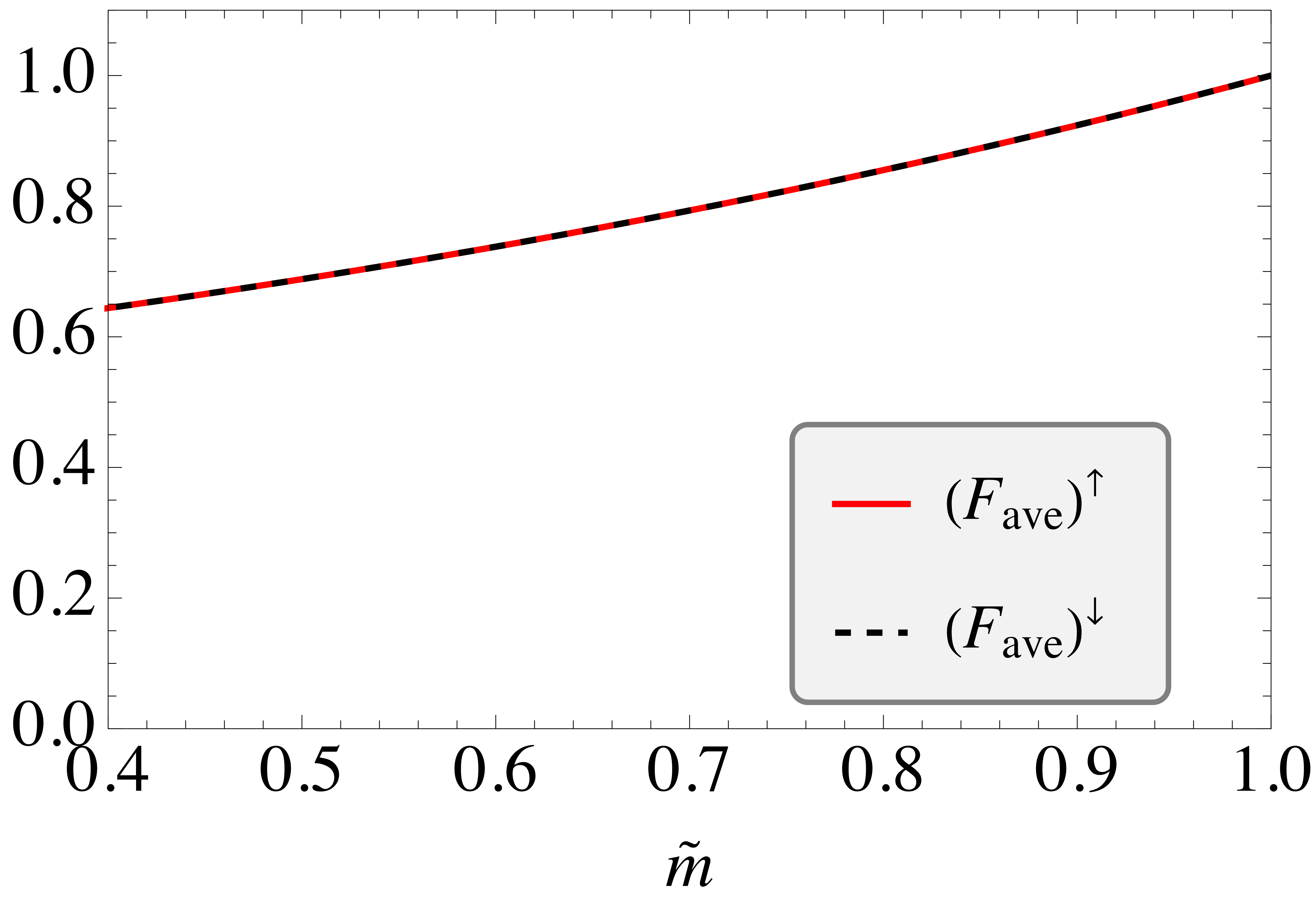
Pauli error probabilities

[3] A. N. Korotkov, "Error matrices in quantum process tomography."

Error analysis: Gate fidelity

$$F^j_{\mathrm{ave}}(G_j^{\mathrm{real}}, G_j^{\mathrm{target}}) \equiv \frac{\mathrm{tr}\left\{(G_j^{\mathrm{real}})^\dagger G_j^{\mathrm{real}}\right\} + |\,\mathrm{tr}\left\{(G_j^{\mathrm{target}})^\dagger G_j^{\mathrm{real}}\right\}|^2}{20}$$

$$F^j_{\mathrm{ave}} = \frac{1}{20}\left(4 + \frac{1}{4}\left(5 + \tilde{m}(2+\tilde{m})\right)^2\right)$$



Spin read-out benchmark vs. $r_{\Omega\gamma}$