

Geometria Computacional

Polígonos: Trelças – problemas resolvidos

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1. UVA 10088 – Trees on My Island

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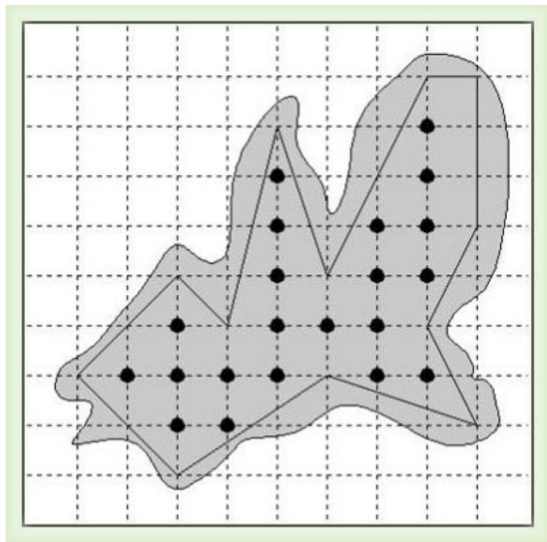
Problema

I have bought an island where I want to plant trees in rows and columns. So, the trees will form a rectangular grid and each of them can be thought of having integer coordinates by taking a suitable grid point as the origin.

But, the problem is that the island itself is not rectangular. So, I have identified a simple polygonal area inside the island with vertices on the grid points and have decided to plant trees on grid points lying strictly inside the polygon.

Now, I seek your help for calculating the number of trees that can be planted on my island.

Problema



Input

The input file may contain multiple test cases. Each test case begins with a line containing an integer N ($3 \leq N \leq 1,000$) identifying the number of vertices of the polygon. The next N lines contain the vertices of the polygon either in clockwise or in anti-clockwise direction. Each of these N lines contains two integers identifying the x and y -coordinates of a vertex. You may assume that none of the coordinates will be larger than 1,000,000 in absolute values.

A test case containing a zero for N in the first line terminates the input.

Output

For each test case in the input print a line containing the number of trees that can be planted inside the polygon.

Exemplo de entradas e saídas

Sample Input

```
12
3 1
6 3
9 2
8 4
9 6
9 9
8 9
6 5
5 8
4 4
3 5
1 3
12
1000 1000
2000 1000
4000 2000
6000 1000
8000 3000
8000 8000
7000 8000
5000 4000
4000 5000
3000 4000
3000 5000
1000 3000
0
```

Sample Output

```
21
25990001
```

Solução $O(TN)$

- A tentativa de testar todos os pontos dentro do retângulo que delimita o polígono gera um algoritmo $O(N^2)$ que leva ao TLE
- Contudo, este problema pode ser resolvido com complexidade $O(N)$ para cada um dos T casos de teste
- Basta utilizar o Teorema de Pick, que relaciona o número de pontos com coordenadas inteiras que estão no interior (I) e na borda (B) do polígono P com sua área A
- A área A pode ser computada em $O(N)$ a partir das coordenadas de seus vértices, utilizando a expressão

$$A = \frac{1}{2} \left| \sum_{i=1}^N x_i y_{i+1} - \sum_{j=1}^N y_j x_{j+1} \right|,$$

com $(x_N, y_N) = (x_0, y_0)$

Solução $O(TN)$

- Cada aresta QR de P intercepta $d + 1$ pontos com coordenadas inteiras, onde $d = (b, h)$ é o maior divisor comum entre $b = |Q_x - R_x|$ e $h = |Q_y - R_y|$
- Como os vértices são contados duas vezes cada, segue que

$$B = -N + \sum_i^N \gcd(b_i, h_i),$$

- Assim, pelo Teorema de Pick,

$$2I = 2A - B + 2,$$

o que permite computar o valor de I a partir de A e B

Solução $O(TN)$

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4 using ll = long long;
5 using ii = pair<ll, ll>;
6
7 #define x first
8 #define y second
9
10 ll gcd(ll a, ll b) { return b ? gcd(b, a % b) : a; }
11
12 ll area(int N, const vector<ii>& ps)
13 {
14     ll A = 0;
15
16     for (int i = 0; i < N; ++i)
17     {
18         A += ps[i].x * ps[i + 1].y;
19         A -= ps[i].y * ps[i + 1].x;
20     }
21 }
```

Solução $O(TN)$

```
22     return llabs(A);
23 }
24
25 // Teorema de Pick:  $A = I + B/2 - 1$ 
26 ll solve(int N, vector<ii>& ps)
27 {
28     ps.push_back(ps.front());
29
30     ll B = 0;
31
32     for (int i = 0; i < N; ++i)
33     {
34         auto b = llabs(ps[i].x - ps[i + 1].x);
35         auto h = llabs(ps[i].y - ps[i + 1].y);
36         auto d = gcd(b, h);
37
38         B += (d + 1);
39     }
40
41     // Desconta os vértices, contados em duplicidade
42     B -= N;
```

Solução $O(TN)$

```
43
44     auto _2A = area(N, ps);
45     auto I = (_2A - B + 2)/2;
46
47     return I;
48 }
49
50 int main()
51 {
52     ios::sync_with_stdio(false);
53
54     int N;
55
56     while (cin >> N, N)
57     {
58         vector<ii> ps(N);
59
60         for (int i = 0; i < N; ++i)
61             cin >> ps[i].x >> ps[i].y;
62
63         auto ans = solve(N, ps);
```

```
64  
65     cout << ans << '\n';  
66 }  
67  
68 return 0;  
69 }
```

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