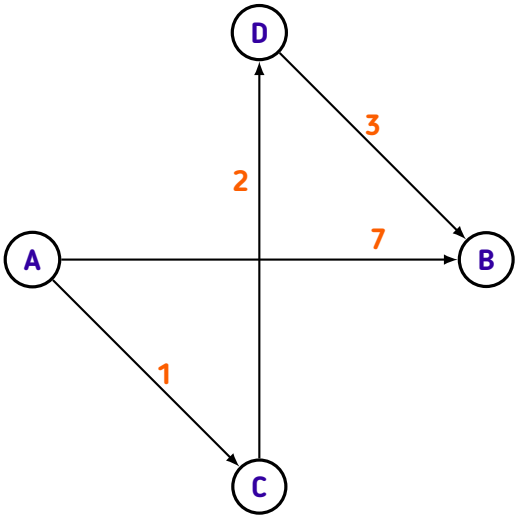
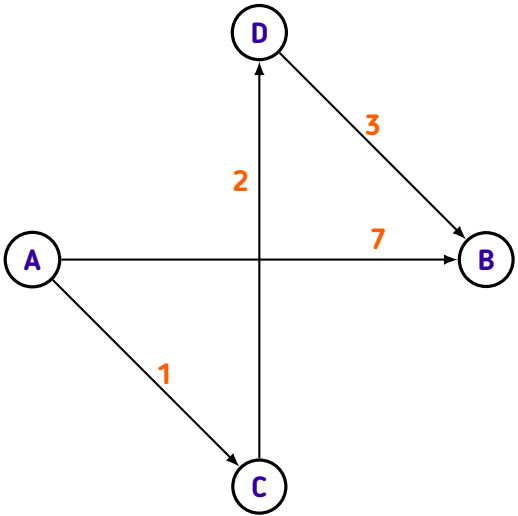
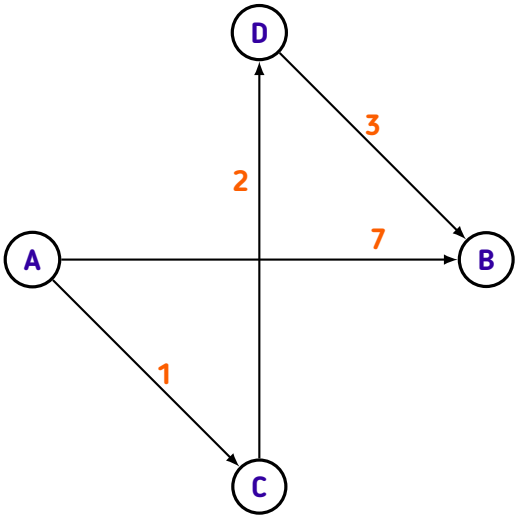

D
3
2
A
7
B
1
C

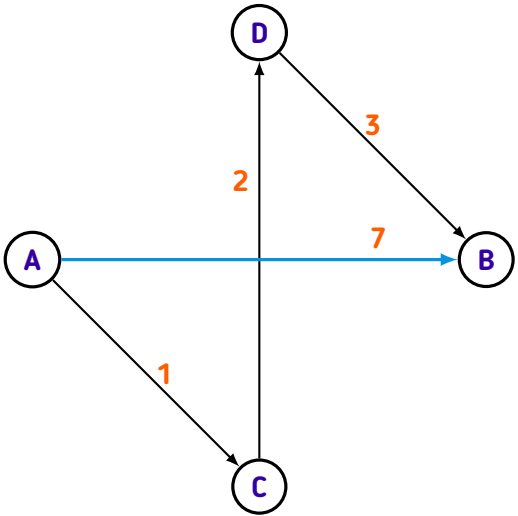


	A	B	C	D
$\text{dist}(u, \mathbf{A})$	0	∞	∞	∞



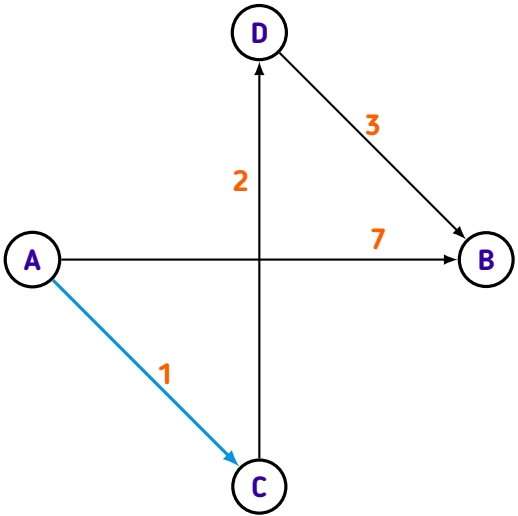
	A	B	C	D
dist$(u, \mathbf{A})$	0	∞	∞	∞

	A	B	C	D
pred(u)	A	-	-	-



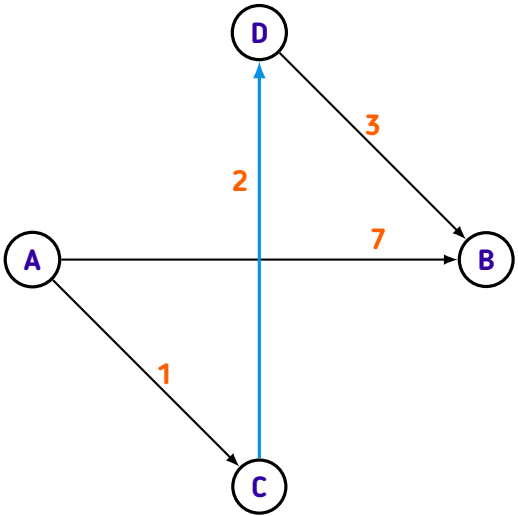
	A	B	C	D
dist$(u, \mathbf{A})$	0	7	∞	∞

	A	B	C	D
pred(u)	A	A	-	-



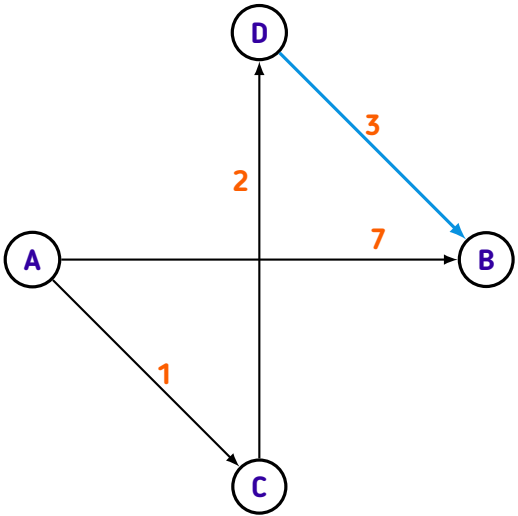
	A	B	C	D
dist$(u, \mathbf{A})$	0	7	**1**	∞

	A	B	C	D
pred(u)	**A**	**A**	**A**	-



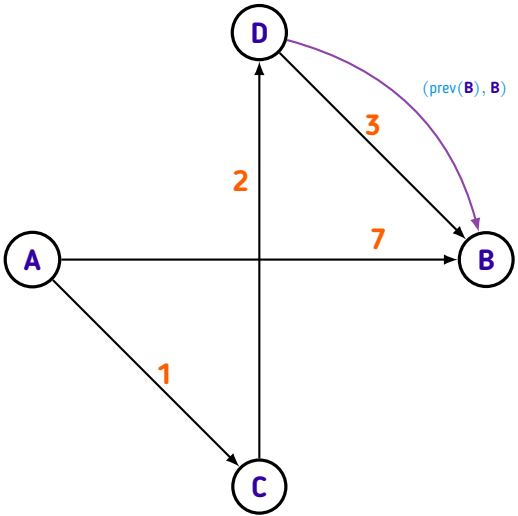
	A	B	C	D
dist$(u, \mathbf{A})$	0	7	1	3

	A	B	C	D
pred(u)	A	A	A	C



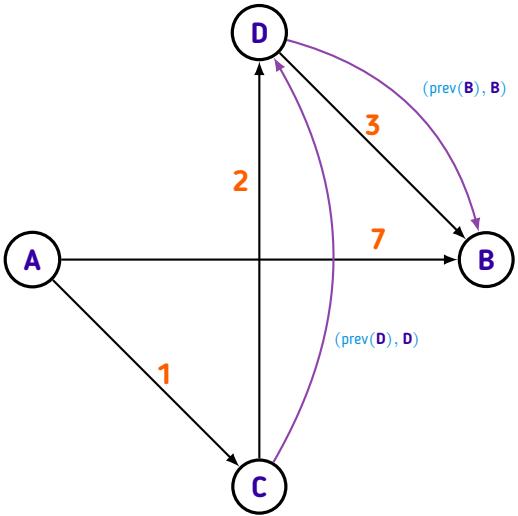
	A	B	C	D
$\text{dist}(u, \mathbf{A})$	0	**6**	1	3

	A	B	C	D
$\text{pred}(u)$	A	D	A	C



	A	B	C	D
dist$(u, \mathbf{A})$	0	6	1	3

	A	B	C	D
pred(u)	A	D	A	C

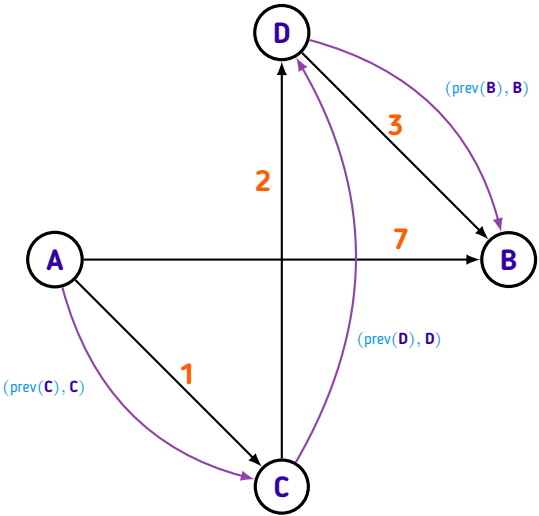


$\text{dist}(u, \mathbf{A})$

A	B	C	D
0	6	1	3

$\text{pred}(u)$

A	B	C	D
A	D	A	C



	A	B	C	D
dist$(u, \mathbf{A})$	0	6	1	3

	A	B	C	D
pred(u)	A	D	A	C

```cpp
pair<vector<int>, vector<int>>
bellman_ford(int s, int N, const vector<edge>& edges)
{
    vector<int> dist(N + 1, oo), pred(N + 1, oo);

    dist[s] = 0;
    pred[s] = s;

    for (int i = 1; i <= N - 1; i++)
        for (auto [u, v, w] : edges)
            if (dist[v] > dist[u] + w) {
                dist[v] = dist[u] + w;
                pred[v] = u;
            }

    return { dist, pred };
}
```

```
vector<ii> path(int s, int u, const vector<int>& pred)
{
    vector<ii> p;
    int v = u;

    do {
        p.push_back(ii(pred[v], v));
        v = pred[v];
    } while (v != s);

    reverse(p.begin(), p.end());

    return p;
}
```

Caminhos mínimos e ciclos

Caminhos mínimos e ciclos

Seja

$$p = \{(a, u_1), (u_1, u_2), \ldots, (v, u_r), \ldots, (u_s, v), \ldots, (u_t, b)\}$$

um caminho de a **a** b **e** $\omega(c)$ **o custo do ciclo** $c = \{(v, u_r), \ldots, (u_s, v)\}$**, isto é**

$$\omega(c) = \sum_{e \in c} w(e)$$

Caminhos mínimos e ciclos

Seja

$$p = \{(a, u_1), (u_1, u_2), \ldots, (v, u_r), \ldots, (u_s, v), \ldots, (u_t, b)\}$$

um caminho de a a b e $\omega(c)$ o custo do ciclo $c = \{(v, u_r), \ldots, (u_s, v)\}$, isto é

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custo da aresta e

Caminhos mínimos e ciclos

Seja

$$p = \{(a, u_1), (u_1, u_2), \ldots, (v, u_r), \ldots, (u_s, v), \ldots, (u_t, b)\}$$

um caminho de a a b e $\omega(c)$ o custo do ciclo $c = \{(v, u_r), \ldots, (u_s, v)\}$, isto é

$$\omega(c) = \sum_{e \in c} w(e)$$

custo da aresta e

Se p é caminho mínimo de a a b então $\omega(c) = 0$.

Caminhos mínimos e ciclos

Seja

$$p = \{(a, u_1), (u_1, u_2), \ldots, (v, u_r), \ldots, (u_s, v), \ldots, (u_t, b)\}$$

um caminho de a a b e $\omega(c)$ o custo do ciclo $c = \{(v, u_r), \ldots, (u_s, v)\}$, isto é

$$\omega(c) = \sum_{e \in c} w(e)$$

custo da aresta e

Se p é caminho mínimo de a a b então $\omega(c) = 0$.

Caminhos mínimos e ciclos positivos

Caminhos mínimos e ciclos positivos

Seja $\omega(c) > 0$ **e**

$$q = \{(a, u_1), (u_1, u_2), \ldots, (u_{r-1}, v), (v, u_{s+1}), \ldots, (u_t, b)\},$$

o caminho resultante da exclusão do ciclo c **de** p**. Então** $\omega(q) < \omega(p)$**, pois**

$$\omega(p) = \sum_{e_i \in p} w(e_i) = \sum_{e_j \in q} w(e_j) + \sum_{e_k \in c} w(e_k) = \omega(q) + \omega(c) > \omega(q)$$

Problemas sugeridos

1. **AtCoder Beginner Contest 088 – Problem D: Repainting**
2. **Codeforces Beta Round #3 – Problem A: Shortest path of the king**
3. **OJ 10000 – Longest Paths**
4. **OJ 10959 – The Party, Part I**

Referências

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2. **LAAKSONEN**, **Antti**. ***Competitive Programmer's Handbook*, 2018.**
3. **SKIENA**, **Steven**; **REVILLA**, **Miguel**. ***Programming Challenges*, 2003.**
4. **Wikipédia**, ***Bellman–Ford algorithm.*** **Acesso em 07/07/2021.**
5. **Wikipédia**, ***L. R. Ford Jr.*** **Acesso em 07/07/2021.**
6. **Wikipédia**, ***Richard E. Bellman.*** **Acesso em 07/07/2021.**