Grafos

Grafos de sucessores

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Faculdade UnB Gama

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Um grafo $G(V, E)$ é um grafo de sucessores se, para qualquer $u \in V$, o grau de saída de u é igual a 1.

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Um grafo de sucessores também é denominado grafo funcional, pois está associado a uma função $\mathsf{succ} : V \to V$ que define o conjunto de arestas

$$E = \{\ (u, \mathsf{succ}(u)) \mid u \in V\ \}$$

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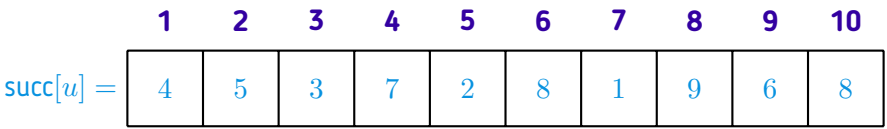
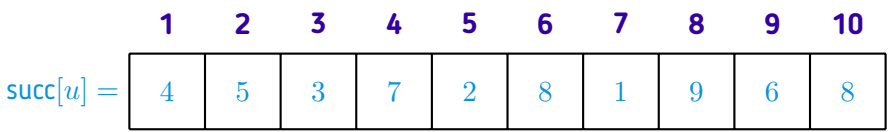
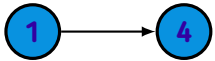
- ⋆ De fato, G é composto por k componentes, cada um deles com ao menos um ciclo e um ou mais caminhos que levam a estes ciclos
- ⋆ Cada nó u tem um sucessor $\mathrm{succ}(u)$ único


1 2 3 4 5 6 7 8 9 10
succ[u] =
4 5 3 7 2 8 1 9 6 8

1





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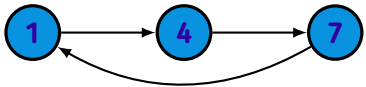

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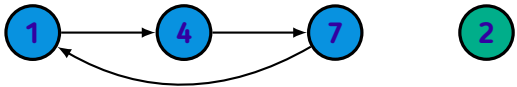

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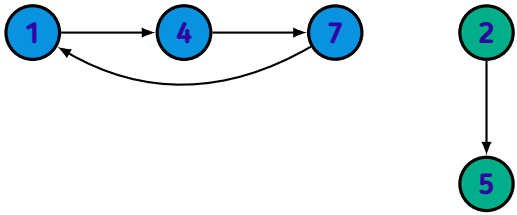

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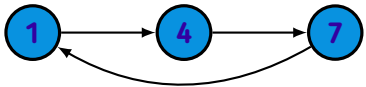

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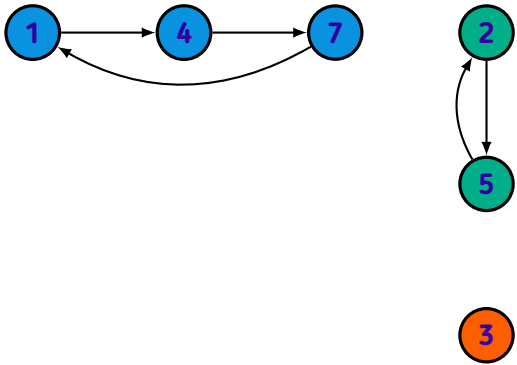

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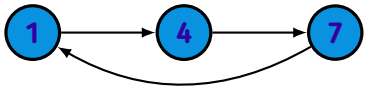

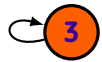

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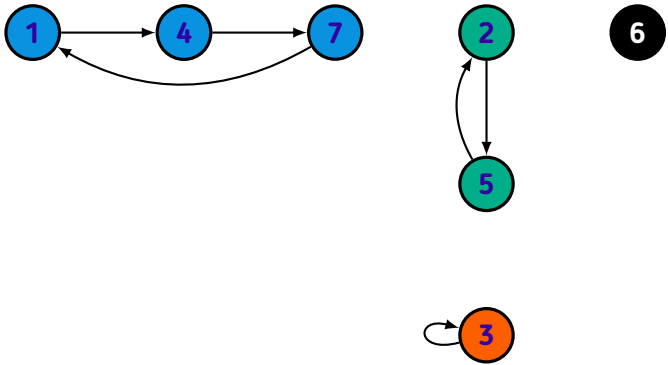

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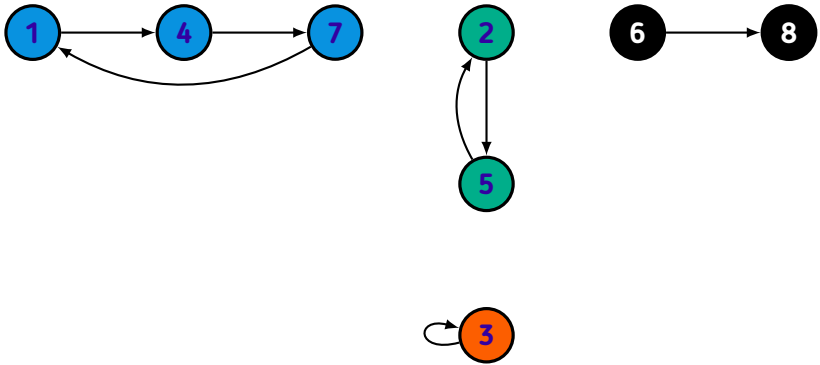

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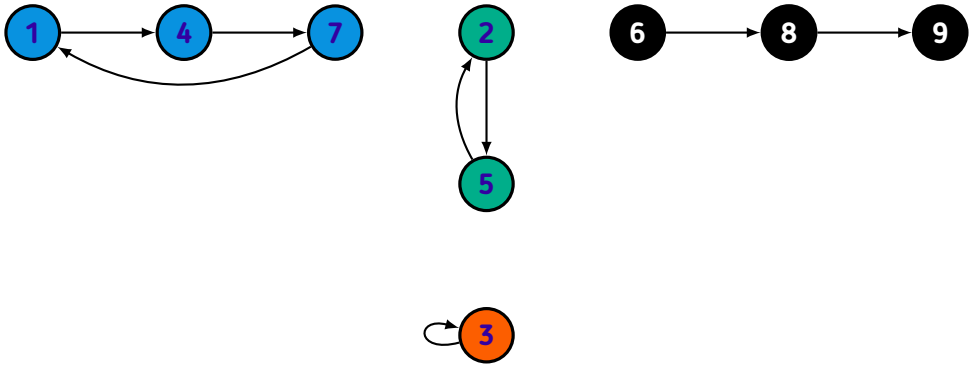

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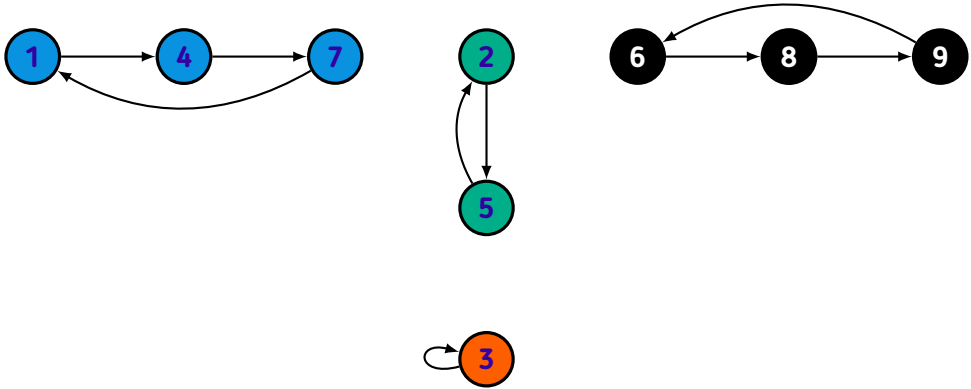

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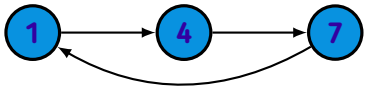

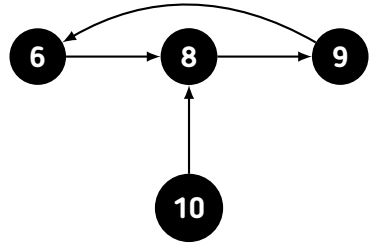

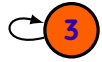

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$$\mathsf{succ}(u, k) = \mathsf{succ}^k(u) = \mathsf{succ}(\mathsf{succ}(\ldots \mathsf{succ}(u)))$$

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$$\text{succ}(u, 2^i) = \begin{cases} \text{succ}(u), & \text{se i = 0} \\ \text{succ}(\text{succ}(u, 2^{i-1}), 2^{i-1}), & \text{caso contrário} \end{cases}$$

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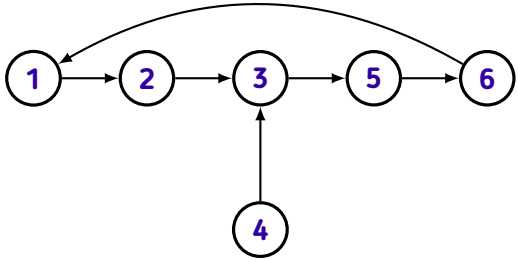
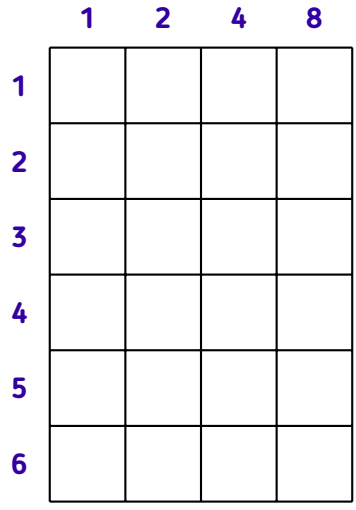
- ⋆ **Estes valores podem ser pré-computados em** $O(|V| \log k)$
- ⋆ **De posse destes valores,** $\mathsf{succ}(u, k)$ **é dado por**

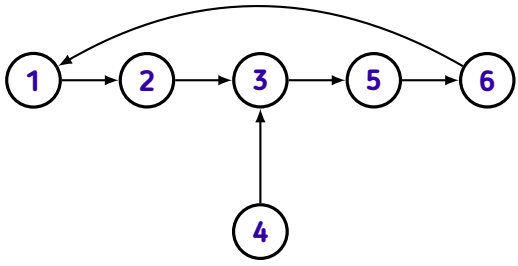
$$\mathsf{succ}(u, k) = \mathsf{succ}(\mathsf{succ}(\mathsf{succ}(\mathsf{succ}(u, 2^{\alpha}), 2^{\beta}), \ldots), 2^{\omega}),$$

onde $k = 2^{\alpha} 2^{\beta} \ldots 2^{\omega}$


1
2
3
5
6
4



succ(u, k)



	1	2	4	8
1	2			
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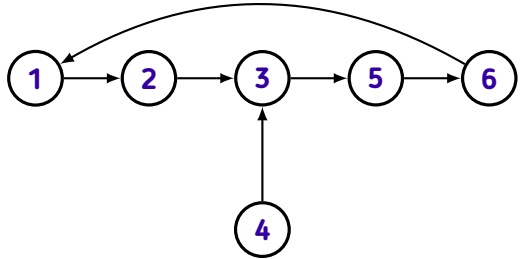

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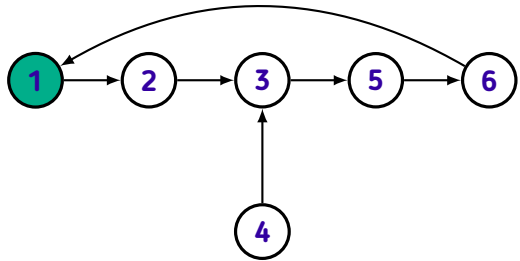

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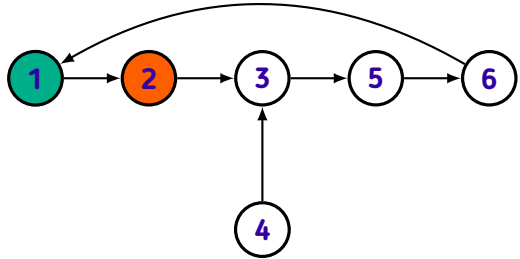

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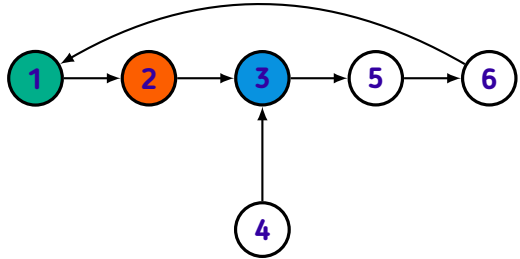

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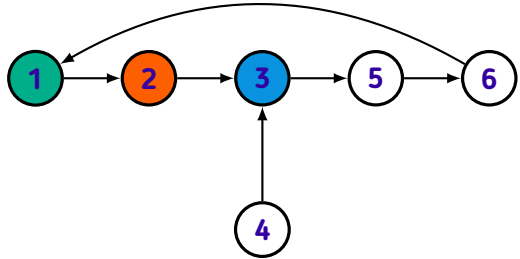

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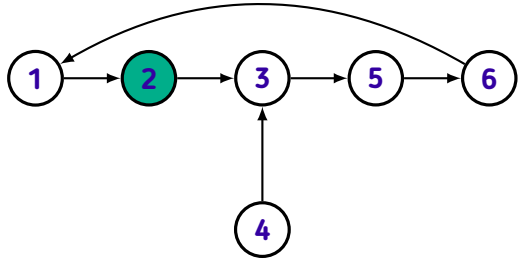

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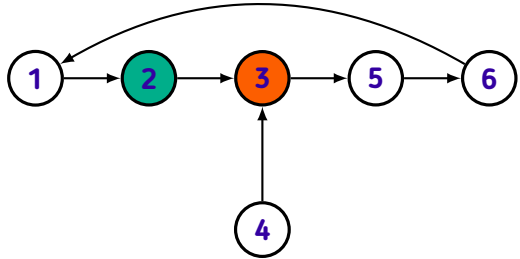

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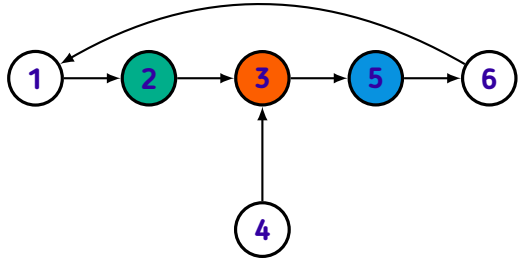

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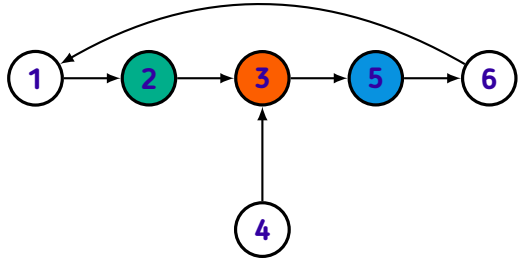

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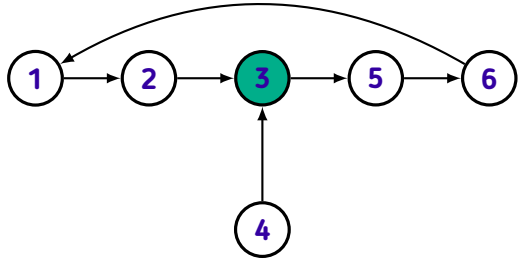

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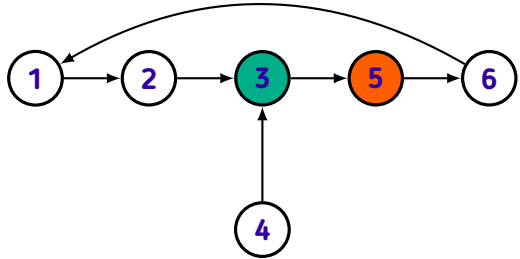

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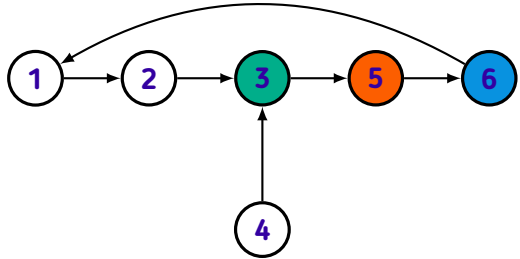

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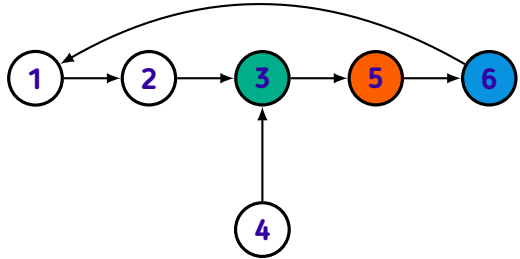

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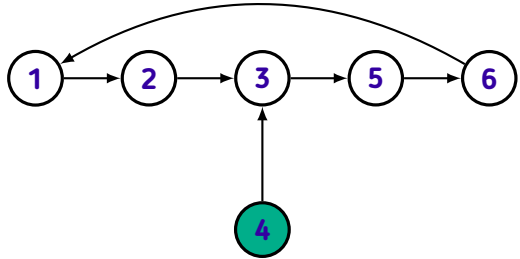

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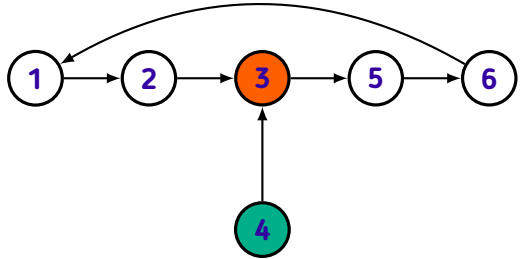

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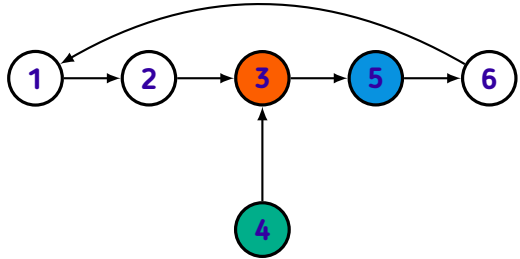

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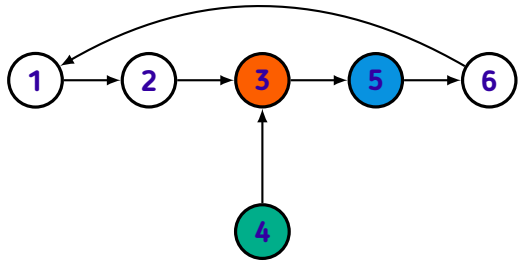

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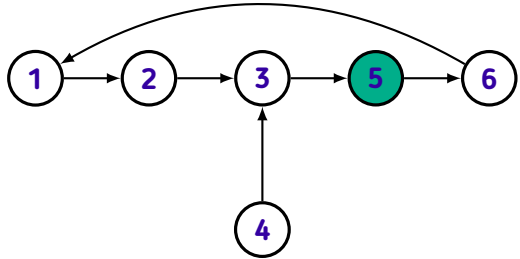
6	1			

succ(u, k)



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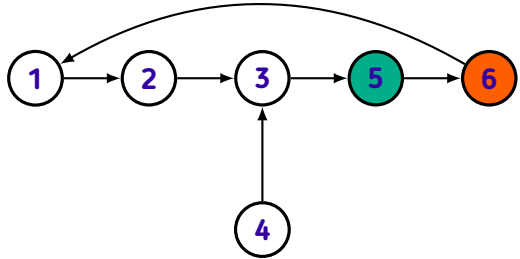

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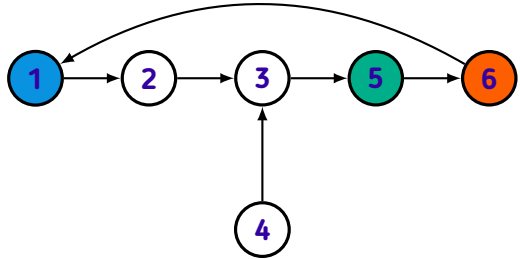

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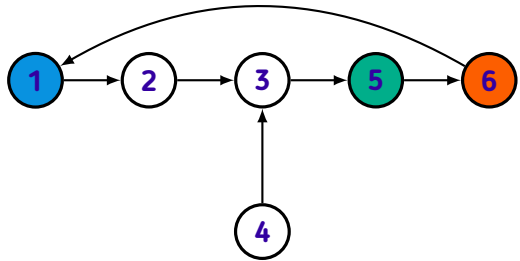

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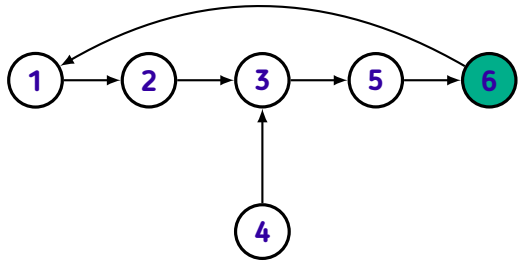

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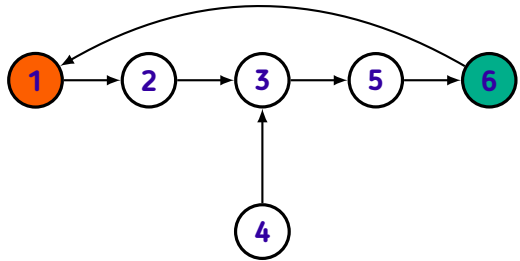

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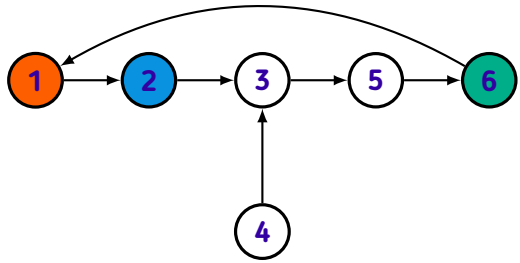

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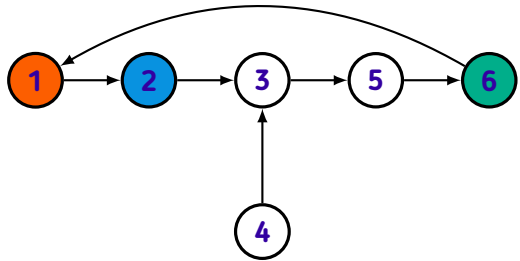

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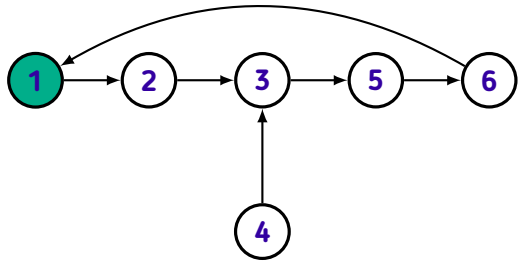

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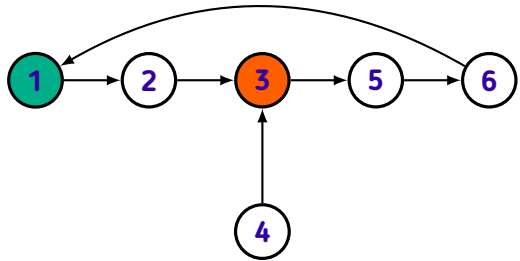

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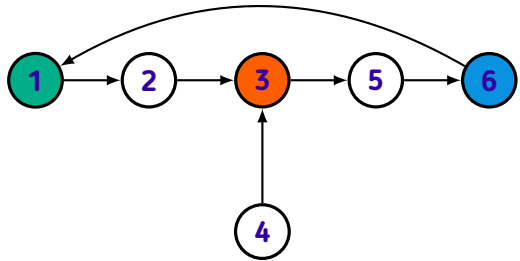

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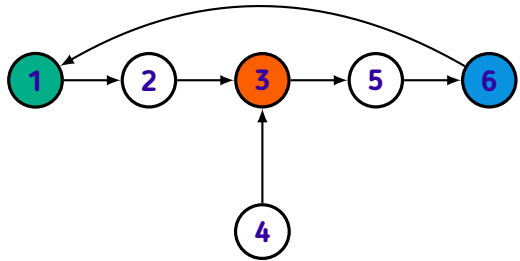

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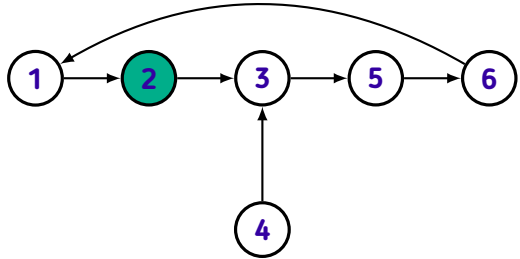
succ(u, k)



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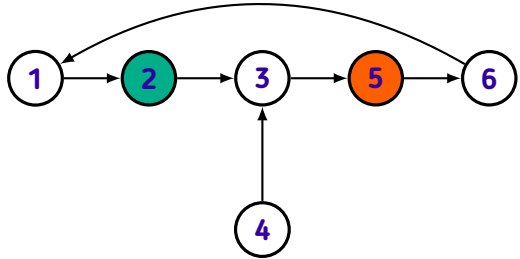

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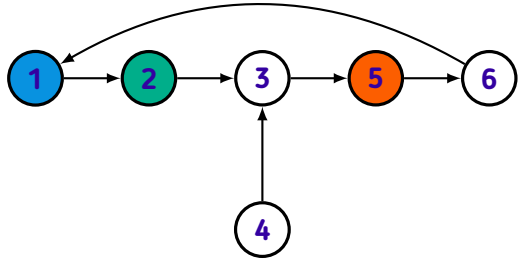

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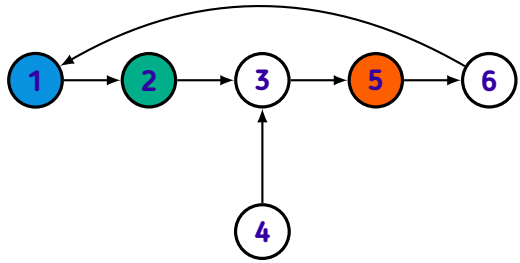
$\mathrm{succ}(u, k)$



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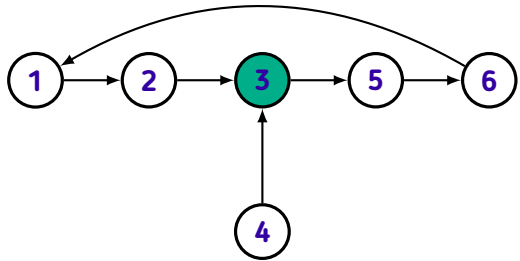

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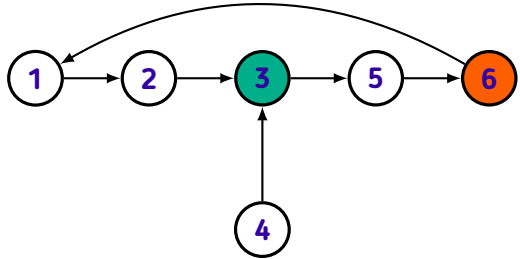

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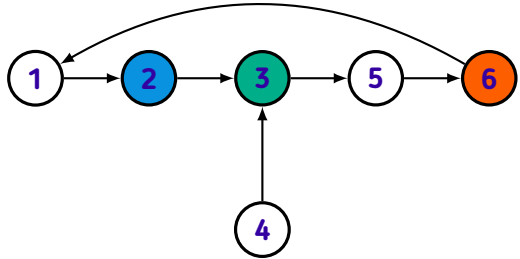

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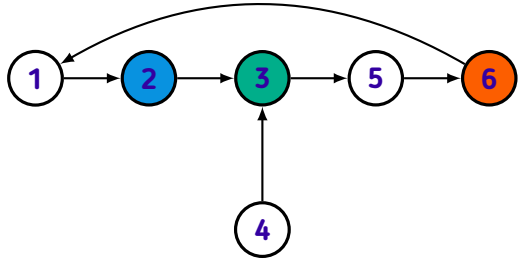

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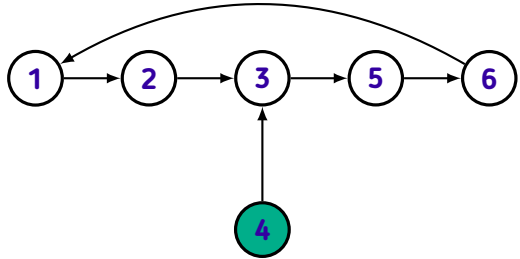

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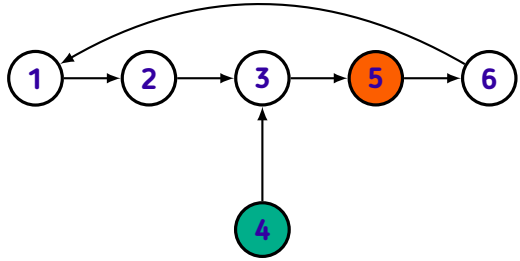

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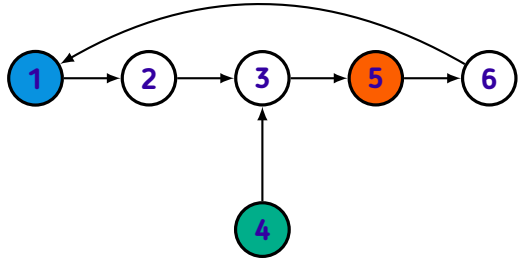

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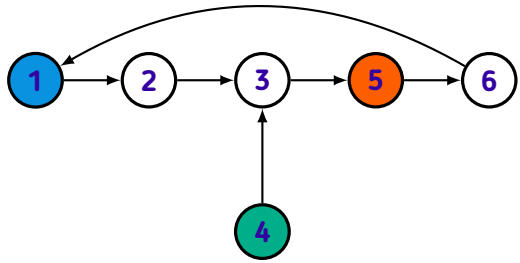

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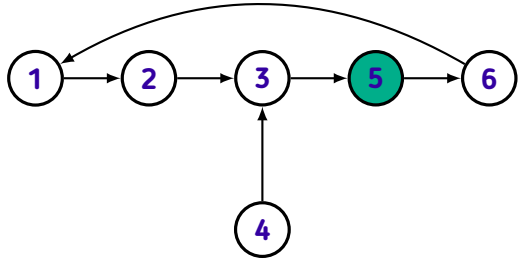

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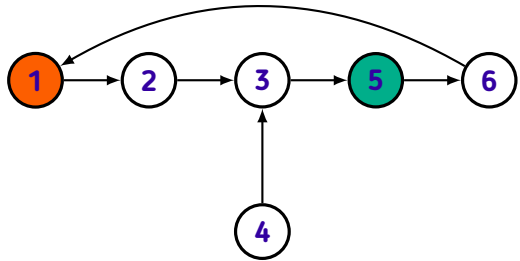

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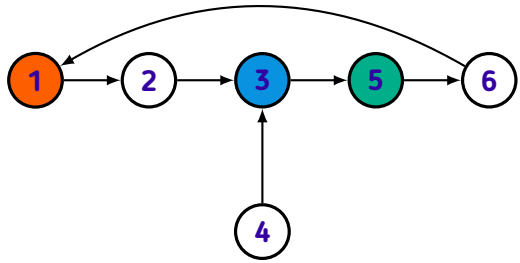

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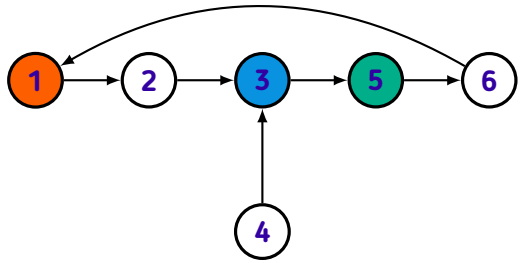

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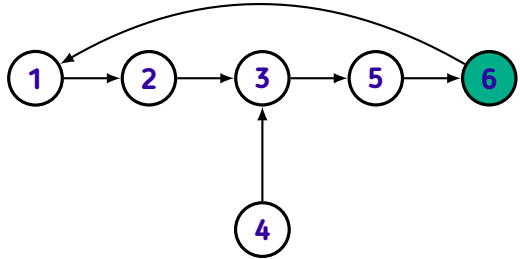

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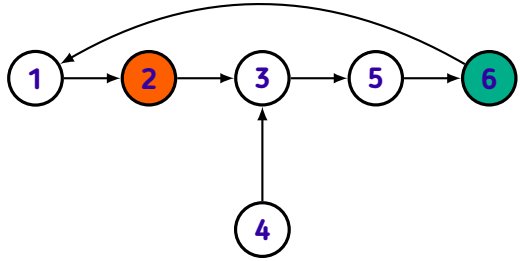

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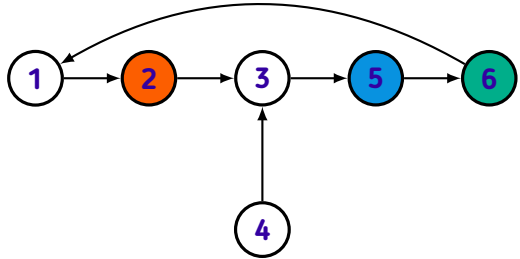
succ(u, k)



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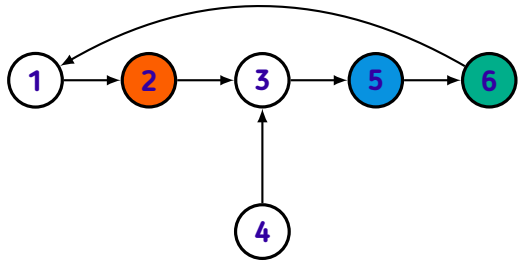
succ(u, k)



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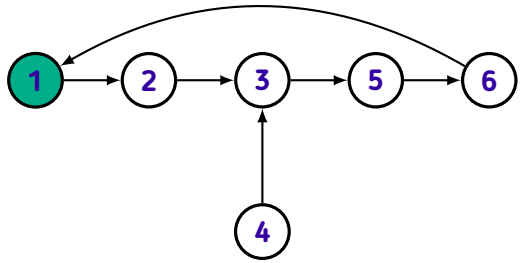

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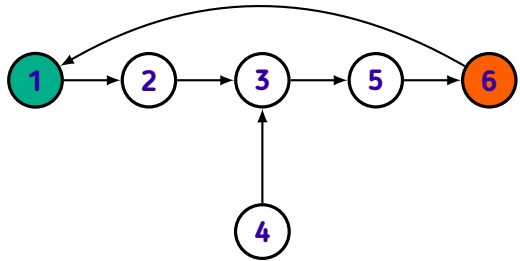

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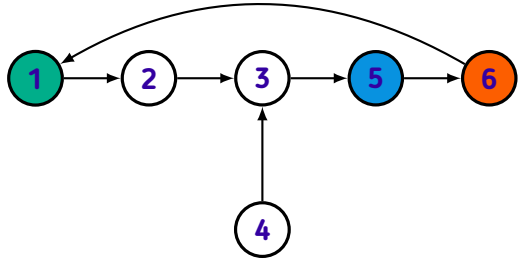

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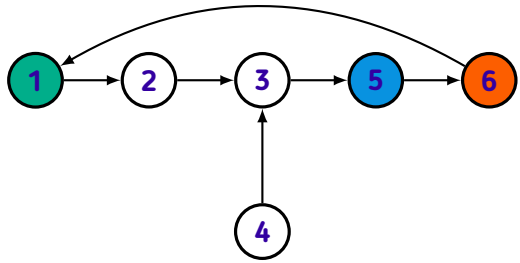
succ(u, k)



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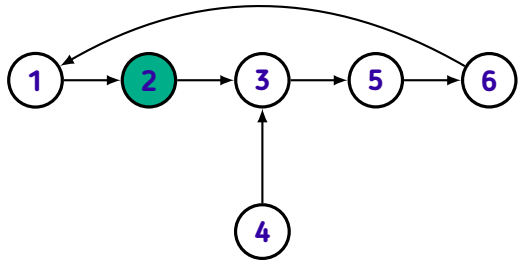

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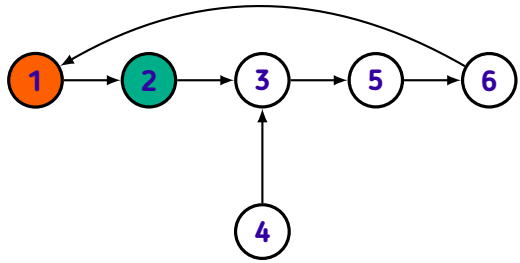

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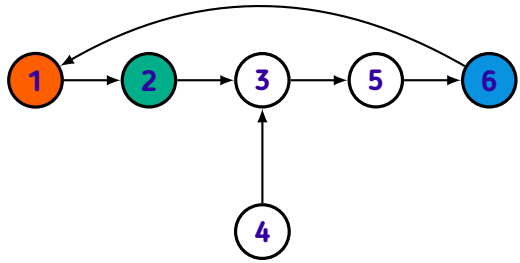
succ(u, k)



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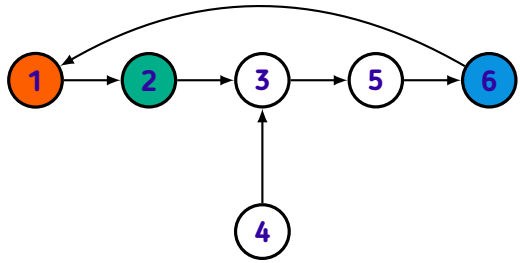
succ(u, k)



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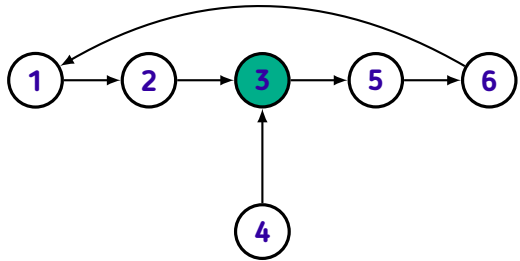
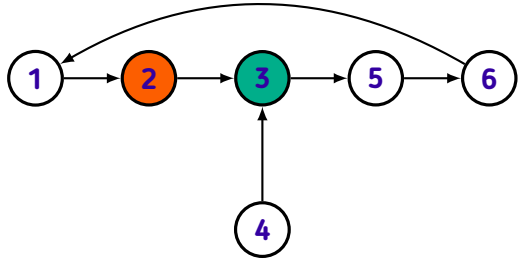
6	1	2	5	

succ(u, k)



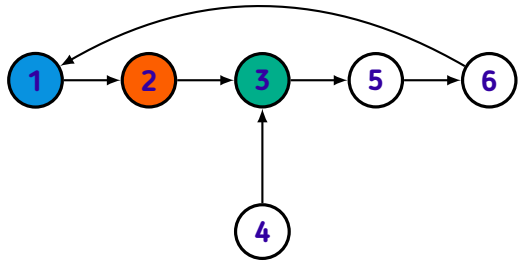
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	
4	3	5	1	
5	6	1	3	
6	1	2	5	

succ(u, k)



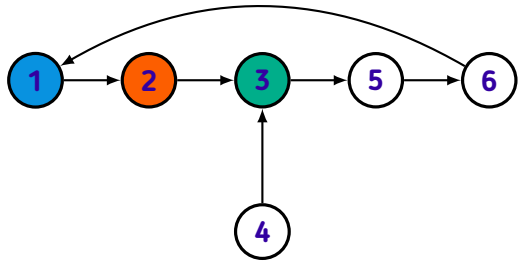
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	
4	3	5	1	
5	6	1	3	
6	1	2	5	

succ(u, k)



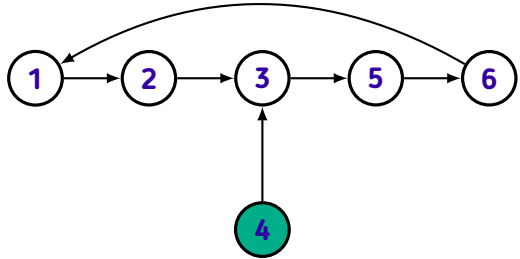
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	
5	6	1	3	
6	1	2	5	

succ(u, k)



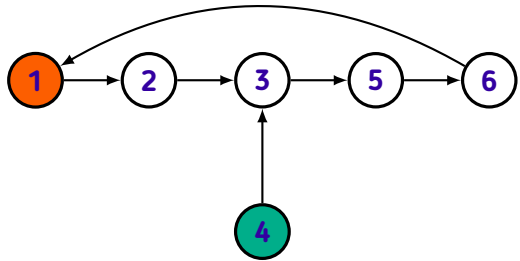
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	
5	6	1	3	
6	1	2	5	

succ(u, k)



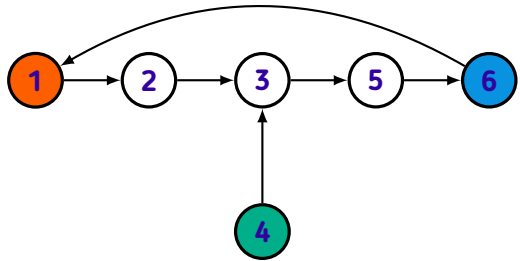
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	
5	6	1	3	
6	1	2	5	

succ(u, k)



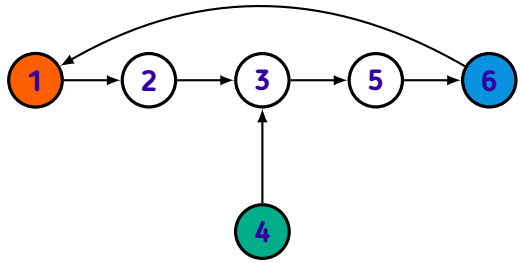
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	
5	6	1	3	
6	1	2	5	

$\mathsf{succ}(u, k)$



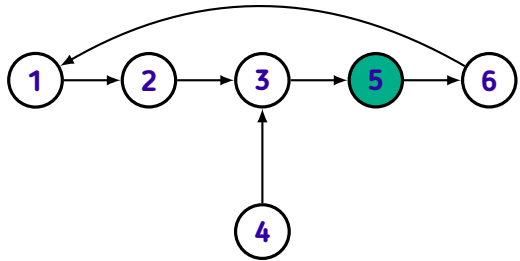
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	
6	1	2	5	

succ(u, k)



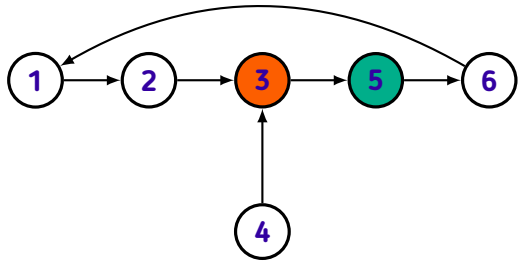
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	
6	1	2	5	

$\mathsf{succ}(u, k)$



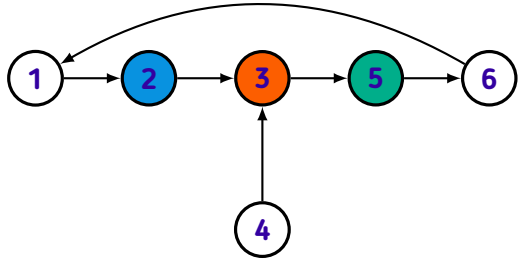
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	
6	1	2	5	

succ(u, k)



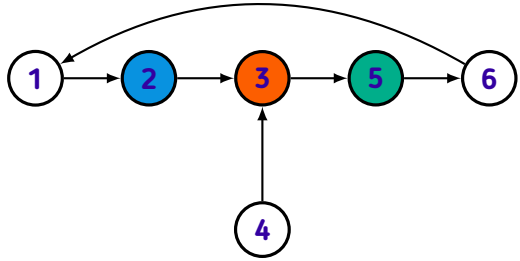
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	
6	1	2	5	

succ(u, k)



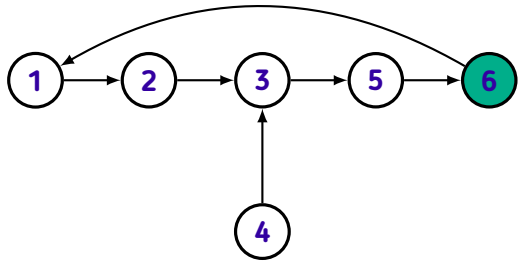
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	2
6	1	2	5	

succ(u, k)



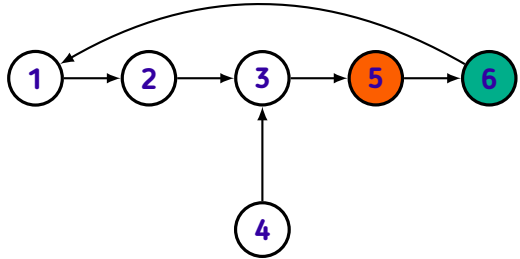
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	2
6	1	2	5	

succ(u, k)



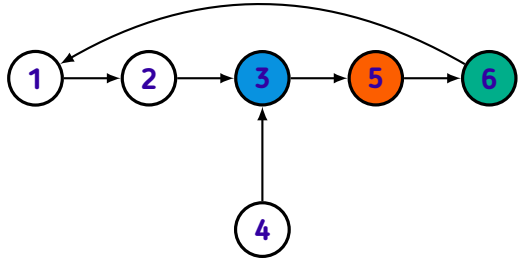
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	2
6	1	2	5	

succ(u, k)



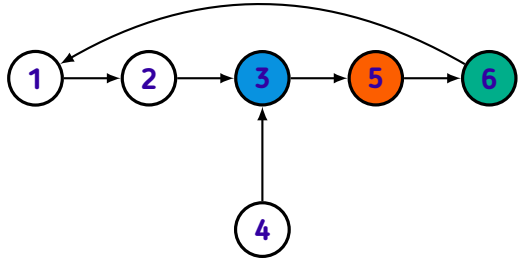
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	2
6	1	2	5	

succ(u, k)



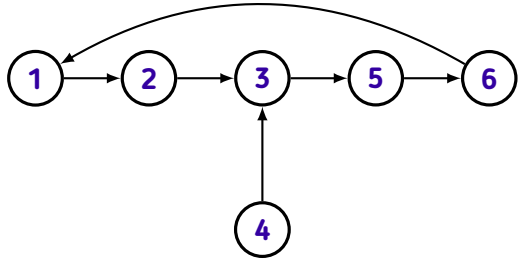
	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	2
6	1	2	5	3

$\mathsf{succ}(u, k)$



	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	2
6	1	2	5	3

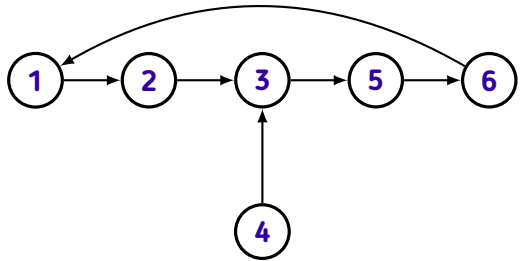
$\mathsf{succ}(u, k)$



$\mathsf{succ}(4, 14) = \mathsf{succ}(\mathsf{succ}(\mathsf{succ}(4, 2), 4), 8)$

	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	**5**	1	6
5	6	1	3	2
6	1	2	5	3

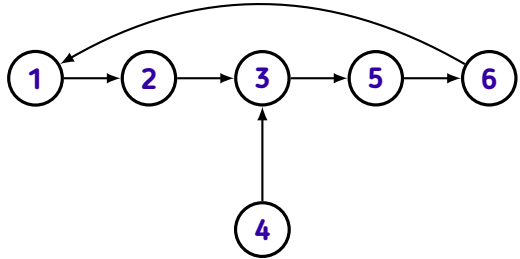
$\mathsf{succ}(u, k)$



$\mathsf{succ}(4, 14) = \mathsf{succ}(\mathsf{succ}(5, 4), 8)$

	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	**3**	2
6	1	2	5	3

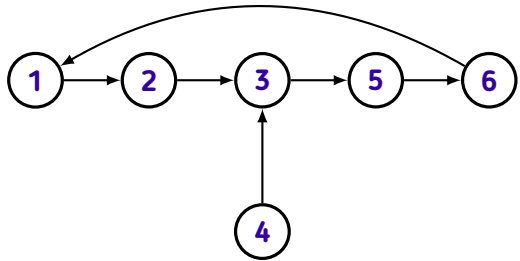
$\mathsf{succ}(u, k)$



$\mathsf{succ}(4, 14) = \mathsf{succ}(3, 8)$

	1	2	4	8
1	2	3	6	5
2	3	5	1	6
3	5	6	2	1
4	3	5	1	6
5	6	1	3	2
6	1	2	5	3

$\mathsf{succ}(u, k)$



$\mathsf{succ}(4, 14) = 1$

```cpp
void precomp(int N, int M, const vector<int>& s)
{
    for (int u = 1; u <= N; ++u)
        S[u][0] = s[u];

    for (int i = 1; i <= M; ++i)
        for (int u = 1; u <= N; ++u)
            S[u][i] = S[S[u][i - 1]][i - 1];
}

int succ(int u, int k)
{
    for (int i = 0; (1 << i) <= k; ++i)
        if (k & (1 << i))
            u = S[u][i];

    return u;
}
```

Problemas sugeridos

1. **CSES 1750 - Planet Queries I**
2. **CSES 1160 - Planty Queries II**
3. **USACO 2017 December Contest, Silver - Problem 3: The Bovine Shuffle**

Referências

1. **HALIM**, **Felix**; **HALIM**, **Steve**. ***Competitive Programming 3*, 2010.**

2. **LAAKSONEN**, **Antti**. ***Competitive Programmer's Handbook*, 2018.**

2. **USACO**, **Guide.** ***Introduction to Functional Graphs***, **acesso em 21/09/2021.**