Grafos

Programação Dinâmica em DAGs

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Relação entre DAGs e programação dinâmica

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- ⋆ **É possível utilizar algoritmos de programação (DP) dinâmica em um grafo G, se G é direcionado e acíclico (DAG – *directed acyclic graph*)**

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⋆ **De fato, há uma relação direta entre uma DP e um DAG**

⋆ **Uma DP induz um DAG cujos vértices são os estados e as arestas indicam as transições entre estados**

Aplicação: número de caminhos de u a v

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- **É possível computar o número de caminhos distintos entre os vértices $u, v \in V$ de um DAG $G(V, E)$**

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⋆ **Assim, $p_u(u) = 1$ é o caso base**

Aplicação: número de caminhos de u a v

- **Para os demais vértices $x \in V$, vale que**

$$p_u(x) = \sum_{(v_i,x)\in E} p_u(v_i)$$

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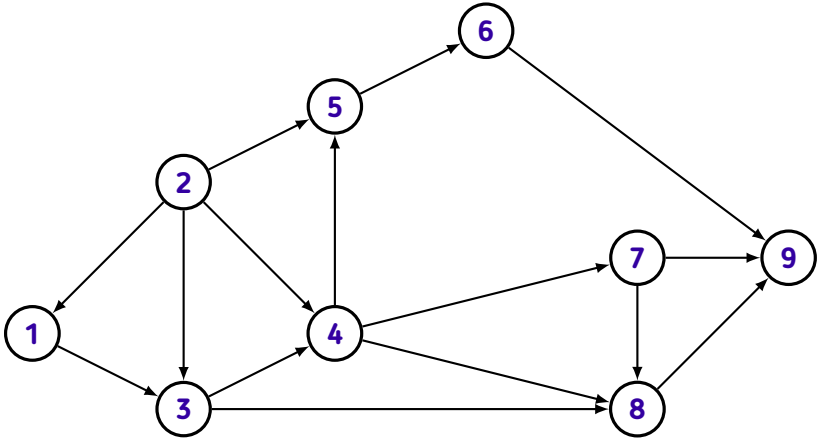
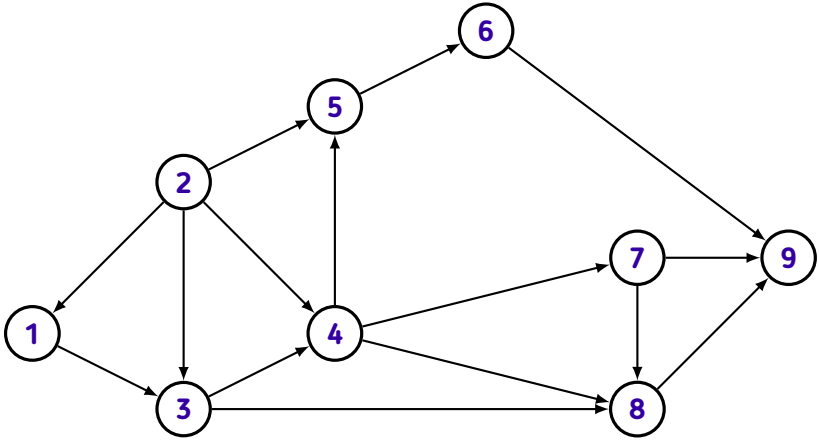
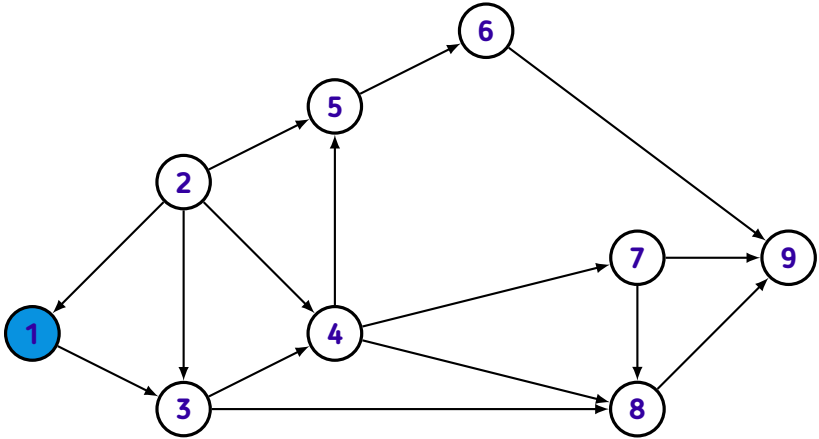
- ⋆ **O cálculo de $p_u(x)$ depende da ordem de processamento dos vértices**
- ⋆ **Como G é um DAG, basta usar a ordenação topológica para computar $p_u(x)$**


6
5
2
7
9
1
4
3
8

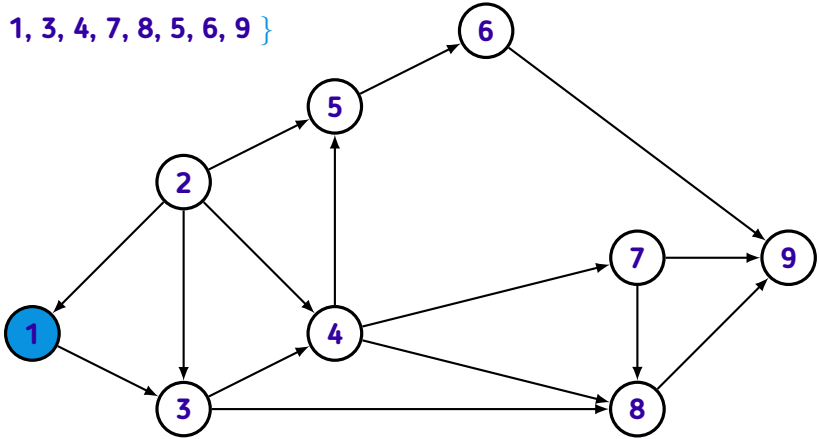


	1	2	3	4	5	6	7	8	9
$p_1(x) =$	-	-	-	-	-	-	-	-	-



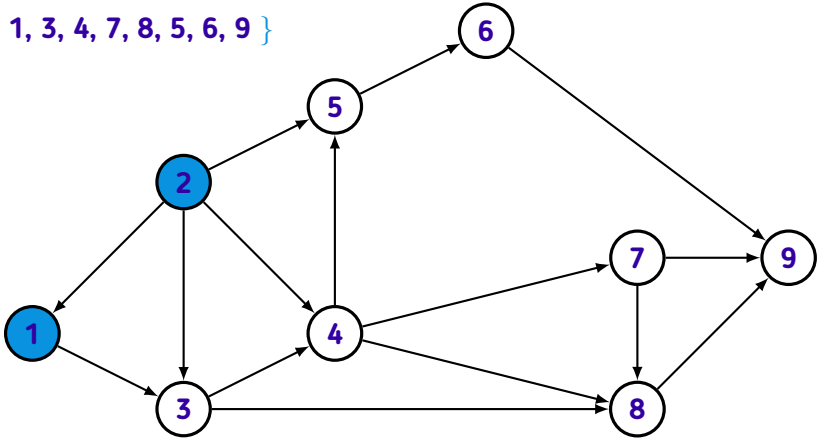
	1	2	3	4	5	6	7	8	9
$p_1(x) =$	1	-	-	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



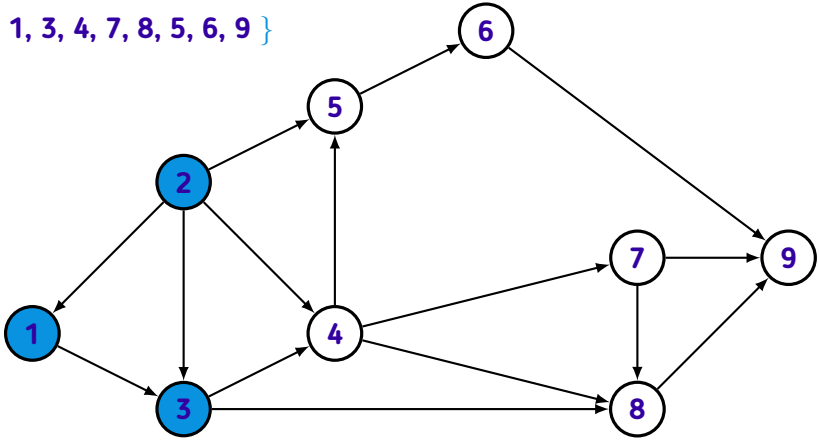
	1	2	3	4	5	6	7	8	9
$p_1(x) =$	1	-	-	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



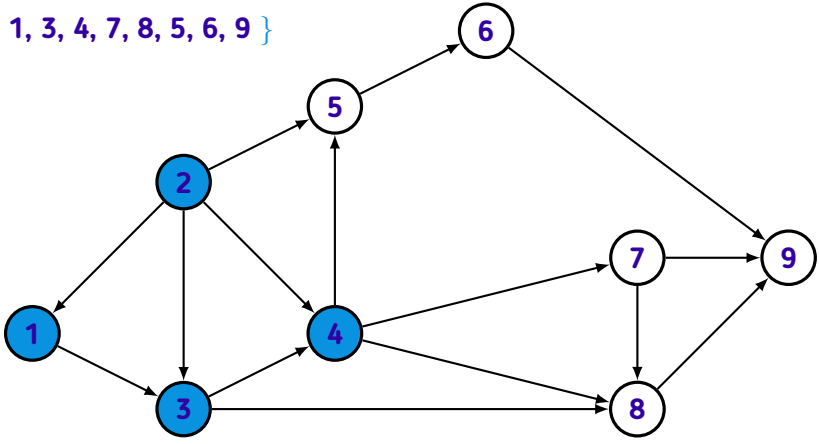
	1	2	3	4	5	6	7	8	9
$p_1(x) =$	1	**0**	-	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



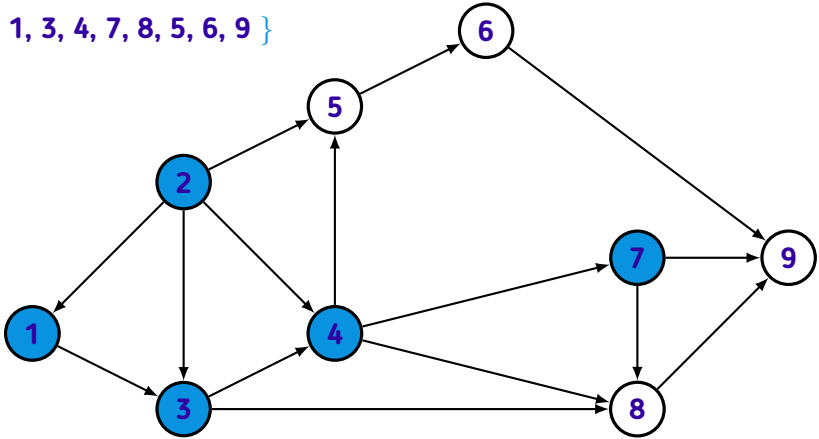
	1	2	3	4	5	6	7	8	9
$p_1(x) =$	1	0	1	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



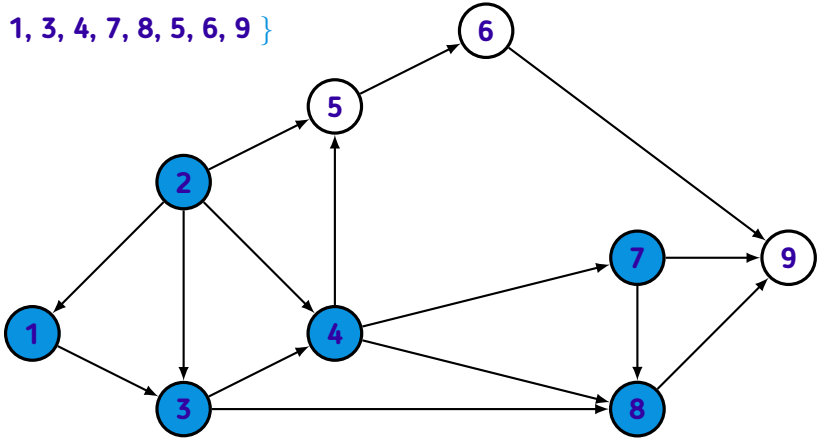
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_1(x) =$	1	0	1	**1**	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



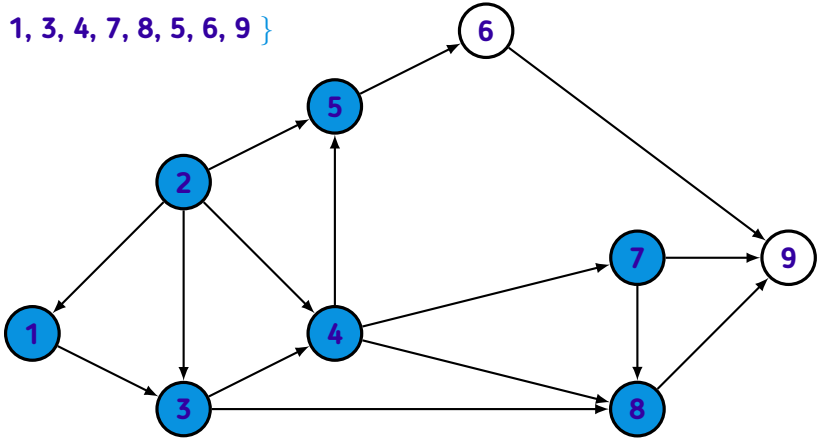
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_1(x) =$	1	0	1	1	-	-	**1**	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



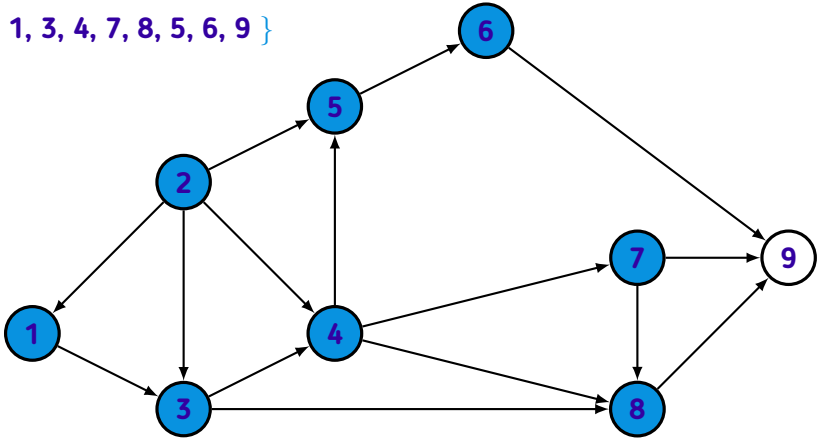
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_1(x) =$	1	0	1	1	-	-	1	**3**	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



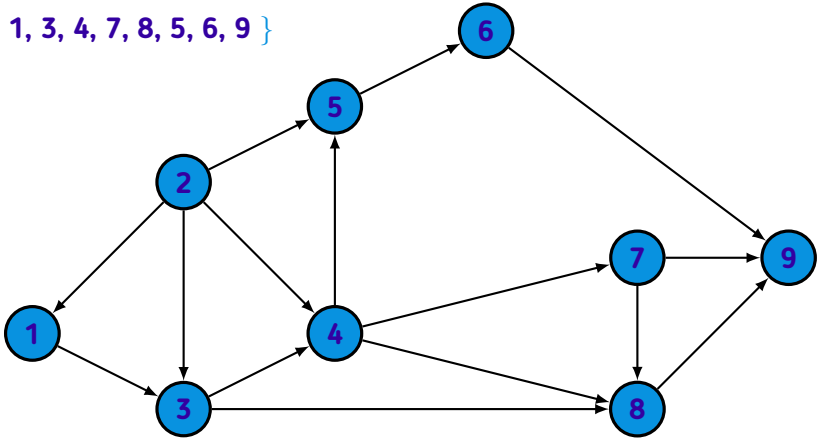
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_1(x) =$	1	0	1	1	**1**	-	1	3	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



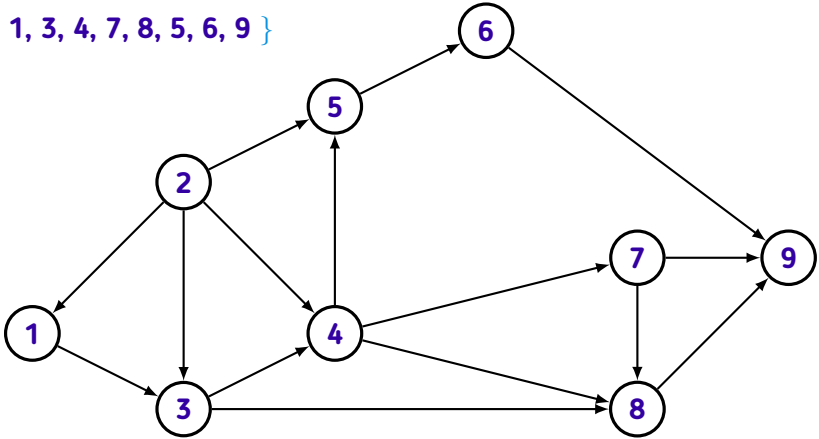
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_1(x) =$	1	0	1	1	1	**1**	1	3	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



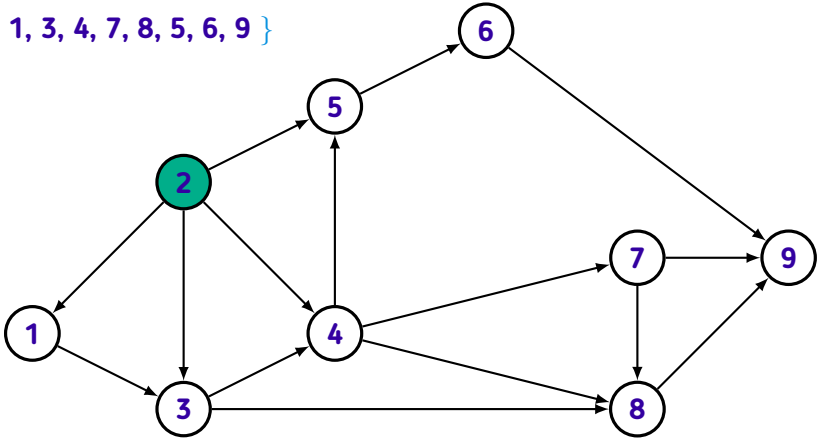
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_1(x) =$	1	0	1	1	1	1	1	3	**5**

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



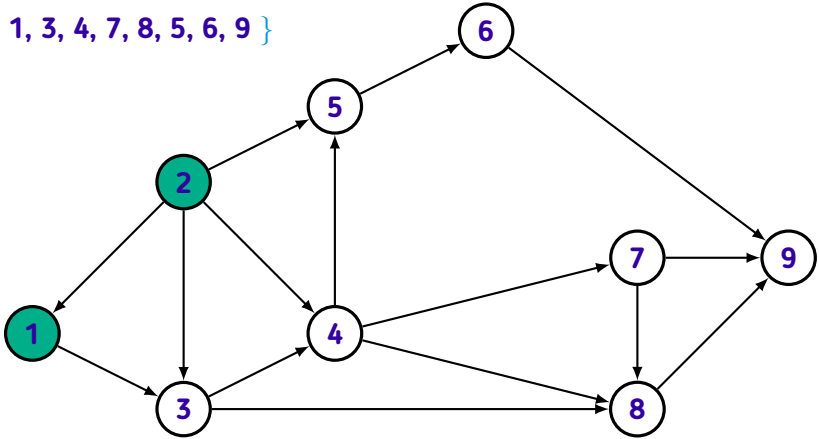
	1	2	3	4	5	6	7	8	9
$p_2(x) =$	-	-	-	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



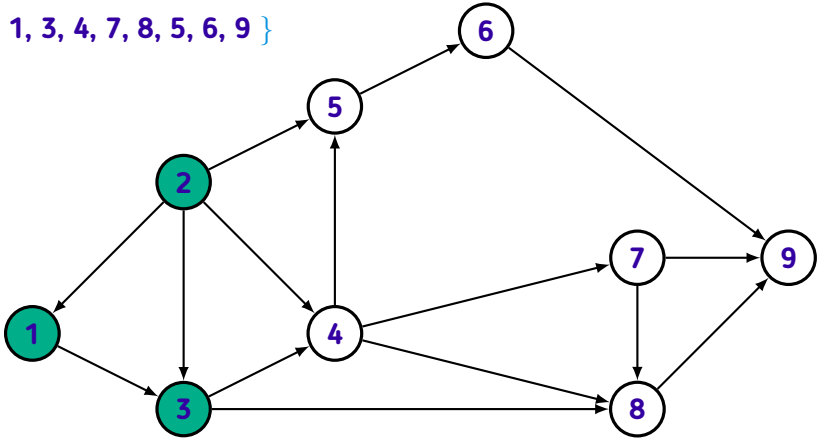
	1	2	3	4	5	6	7	8	9
$p_2(x) =$	-	1	-	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



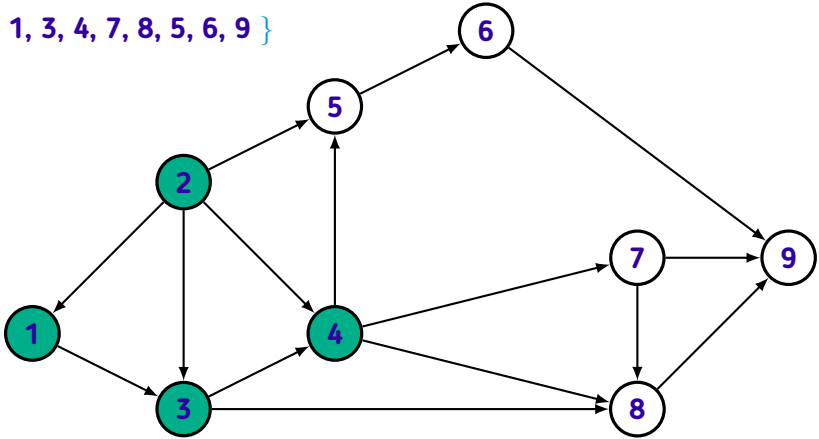
	1	2	3	4	5	6	7	8	9
$p_2(x) =$	**1**	1	-	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



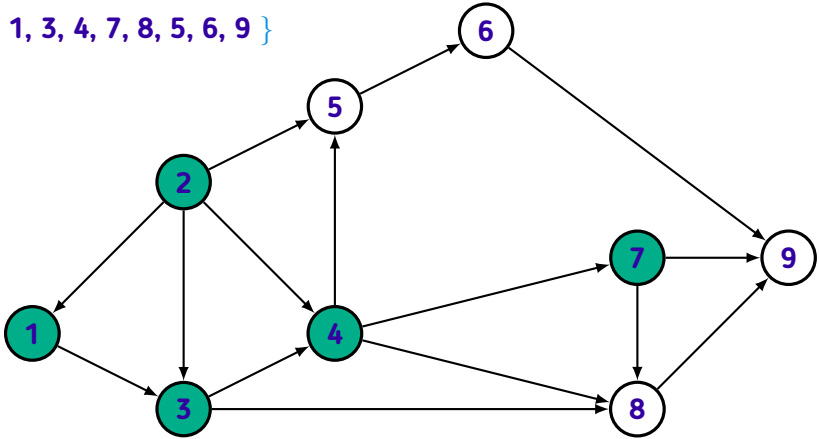
	1	2	3	4	5	6	7	8	9
$p_2(x) =$	1	1	**2**	-	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



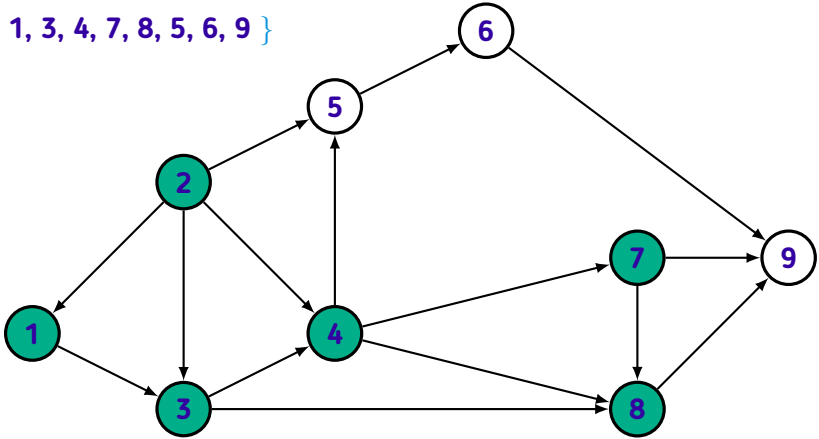
	1	2	3	4	5	6	7	8	9
$p_2(x) =$	1	1	2	**3**	-	-	-	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



	1	2	3	4	5	6	7	8	9
$p_2(x) =$	1	1	2	3	-	-	3	-	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_2(x) =$	1	1	2	3	-	-	3	**8**	-

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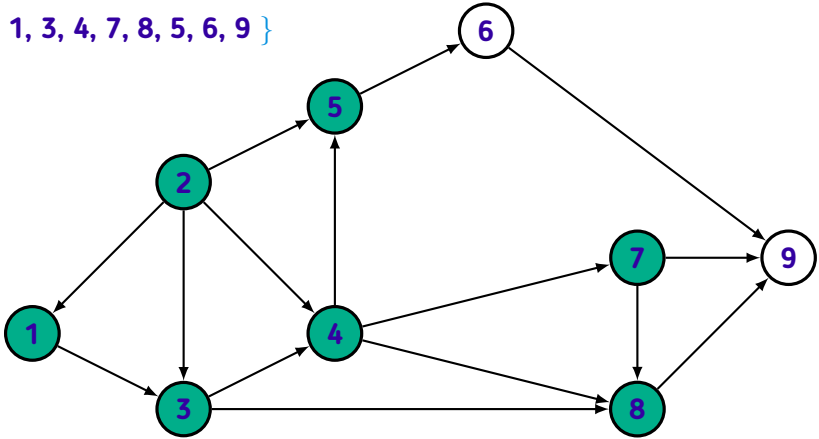

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_2(x) =$	1	1	2	3	**4**	-	3	8	-

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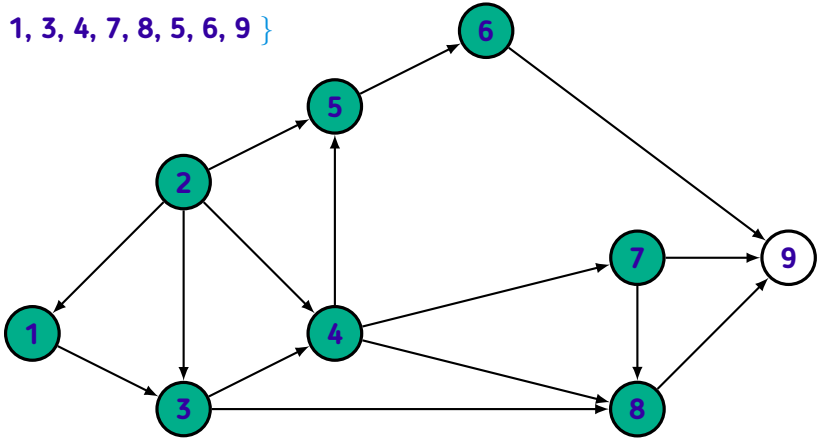
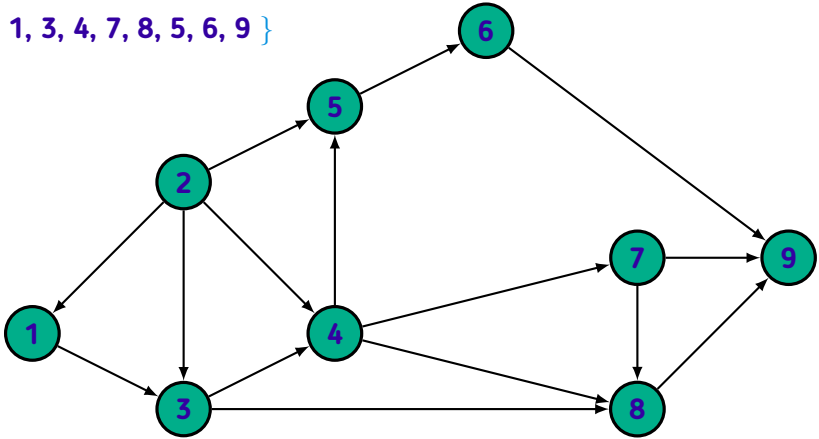


	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_2(x) =$	1	1	2	3	4	**4**	3	8	-

$O = \{$ **2, 1, 3, 4, 7, 8, 5, 6, 9** $\}$



	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$p_2(x) =$	1	1	2	3	4	4	3	8	**15**

```cpp
vector<int> paths(int u, int N)
{
    auto o = topological_sort(N);

    vector<int> ps(N + 1, 0);
    ps[u] = 1;

    for (auto x : o)
        for (auto v : in[x])
            ps[x] += ps[v];

    return ps;
}
```

Aplicação: número de caminhos mínimos de u a v

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- O algoritmo de Dijkstra induz um DAG que indica, para cada vértice do grafo original, como atingir os demais vértices por caminhos mínimos

Aplicação: número de caminhos mínimos de u a v

- ⋆ O algoritmo de Dijkstra induz um DAG que indica, para cada vértice do grafo original, como atingir os demais vértices por caminhos mínimos

- ⋆ Assim, pode-se usar DP neste grafo

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⋆ Assim seria possível, por exemplo, computar o número de caminhos mínimos de u a v

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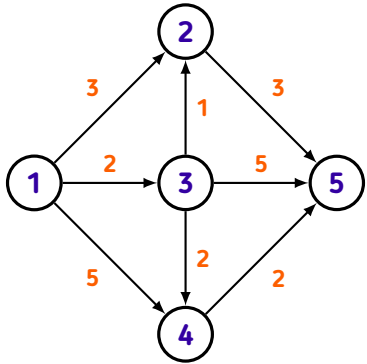
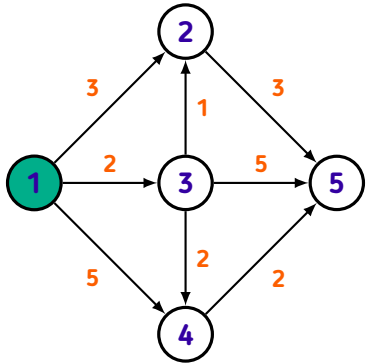
⋆ Assim seria possível, por exemplo, computar o número de caminhos mínimos de u a v

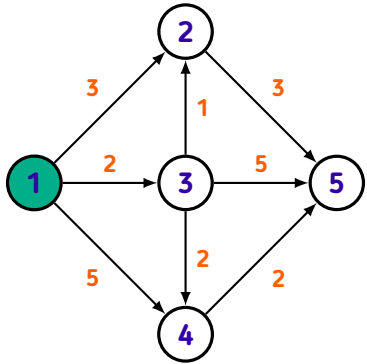
⋆ O algoritmo seria o mesmo usado para computar o número de caminhos, com a ordenação O sendo substituída pela ordenação de Dijkstra



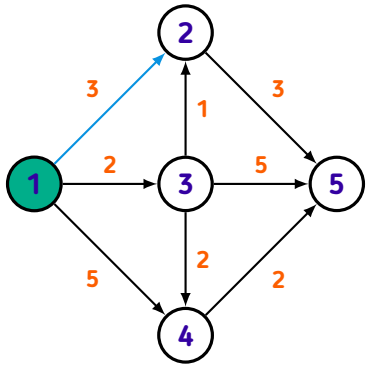
	1	2	3	4	5
$d_1(x) =$	-	-	-	-	-



	1	2	3	4	5
$d_1(x) =$	0	-	-	-	-

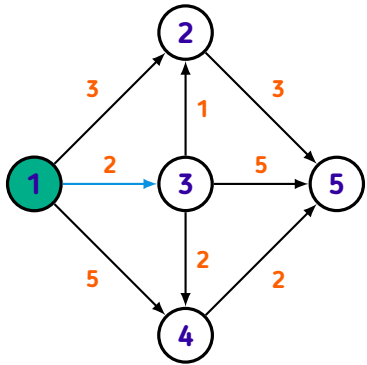


	1	2	3	4	5
$d_1(x) =$	0	-	-	-	-



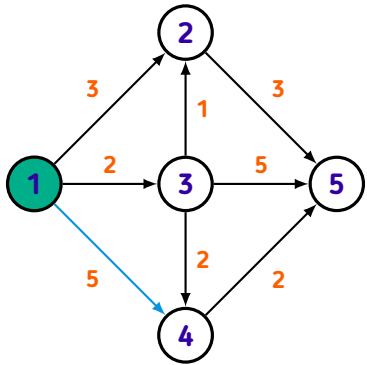
1^1

	1	2	3	4	5
$d_1(x) =$	0	3	-	-	-



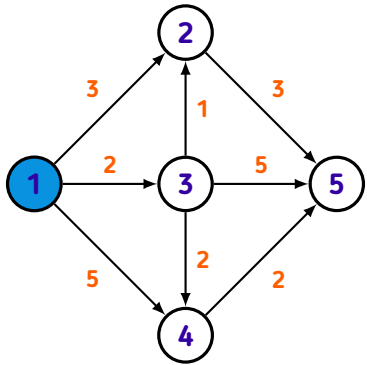
1^1

	1	2	3	4	5
$d_1(x) =$	0	3	2	-	-



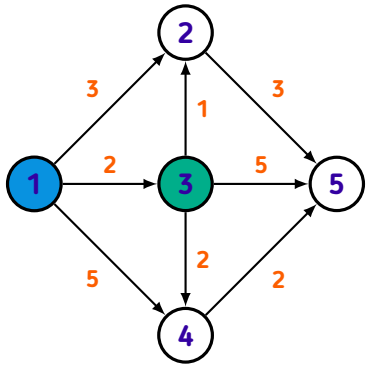
1^1

	1	2	3	4	5
$d_1(x) =$	0	3	2	5	-



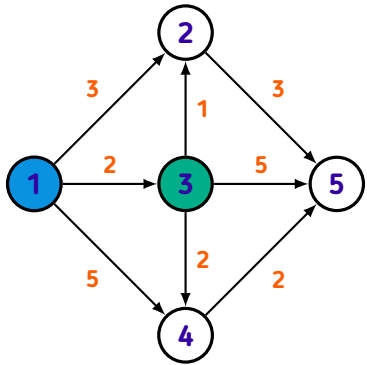
1^1

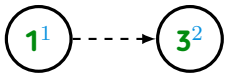
	1	2	3	4	5
$d_1(x) =$	0	3	2	5	-



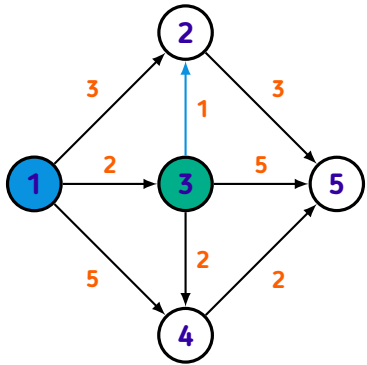
1^1

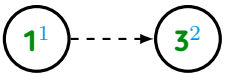
	1	2	3	4	5
$d_1(x) =$	0	3	2	5	-



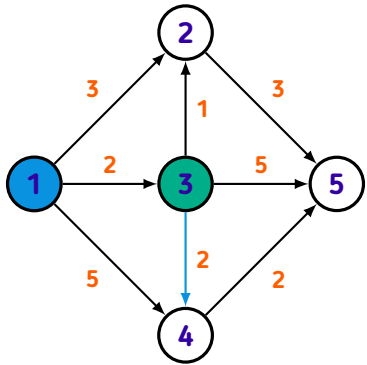


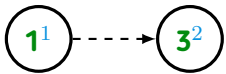
	1	2	3	4	5
$d_1(x) =$	0	3	2	5	-





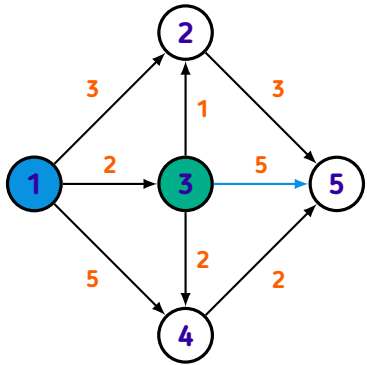
	1	2	3	4	5
$d_1(x) =$	0	3	2	5	-

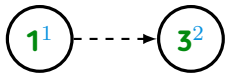




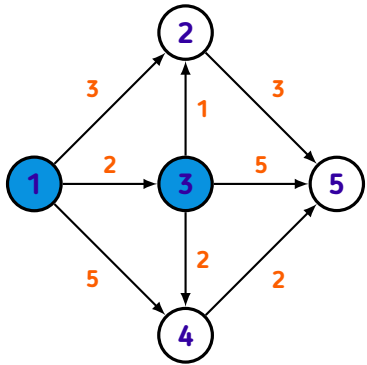
$d_1(x) =$

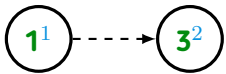
1	2	3	4	5
0	3	2	4	-



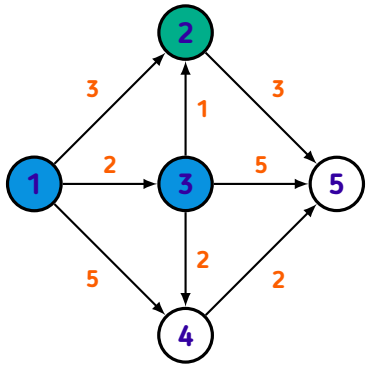


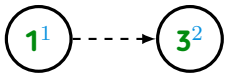
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	**7**



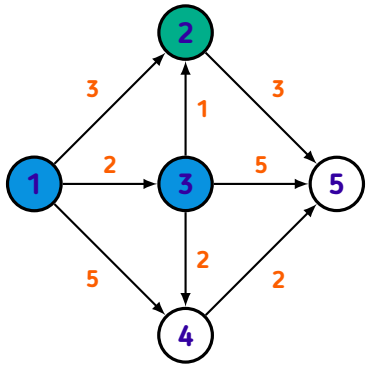


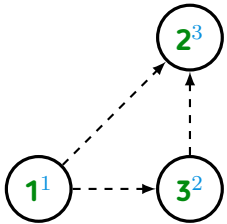
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	7



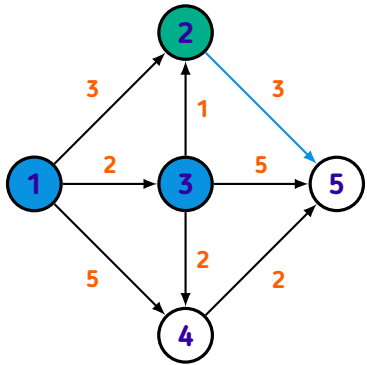


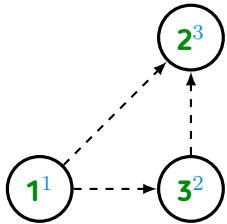
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	7



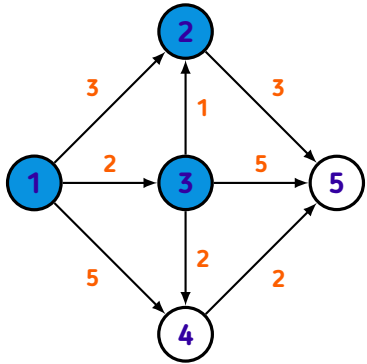


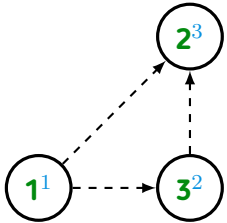
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	7



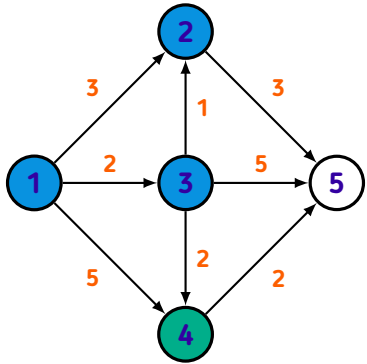


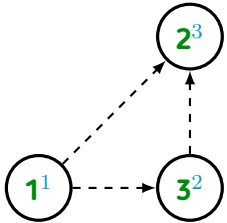
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	**6**



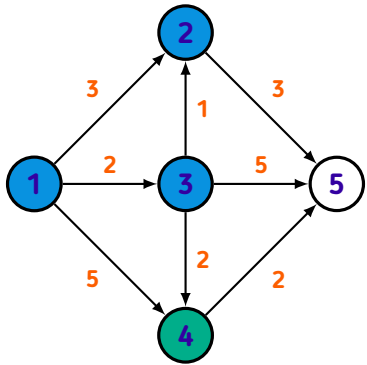


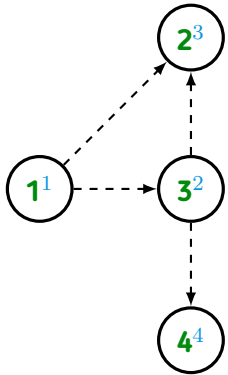
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6



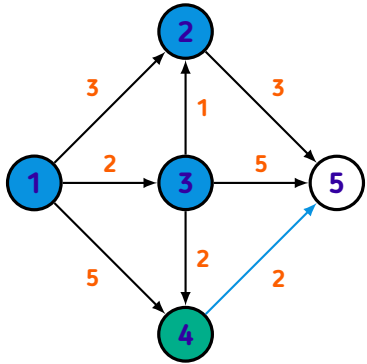


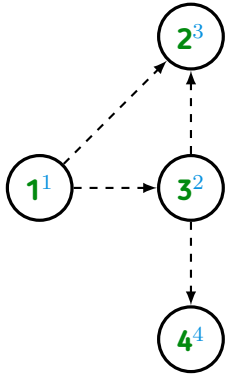
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6



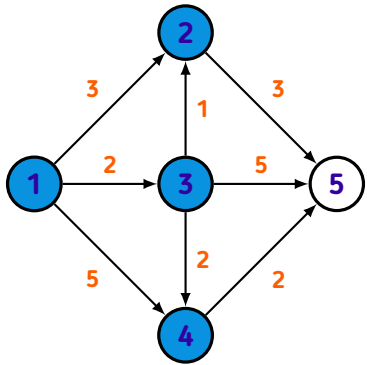


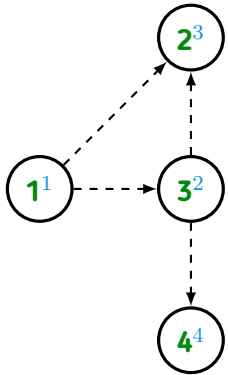
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6



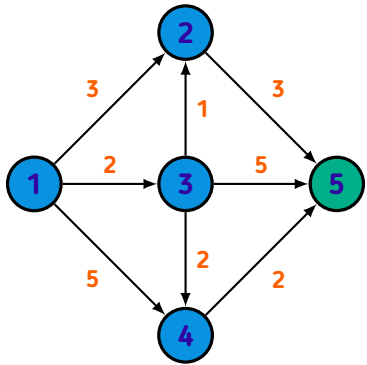


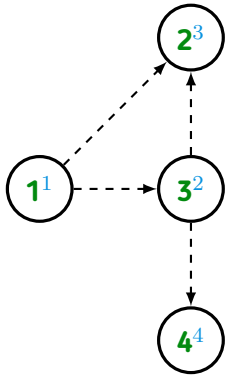
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6



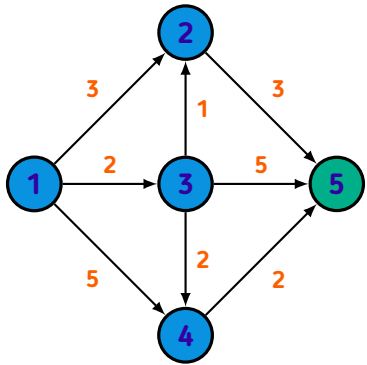


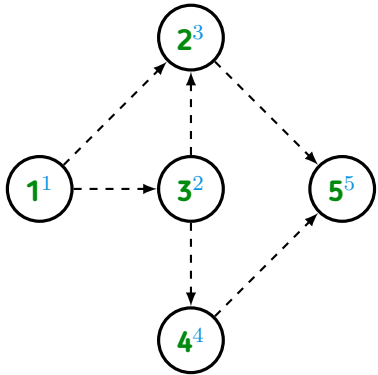
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6



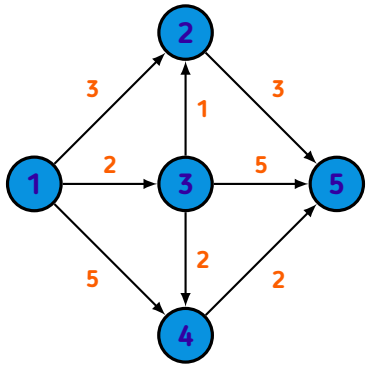


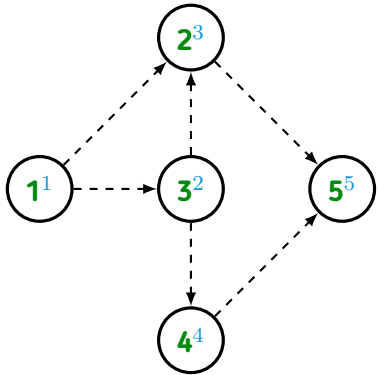
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6



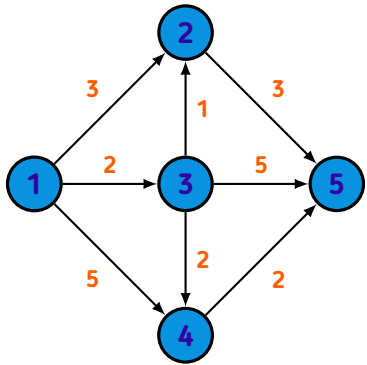


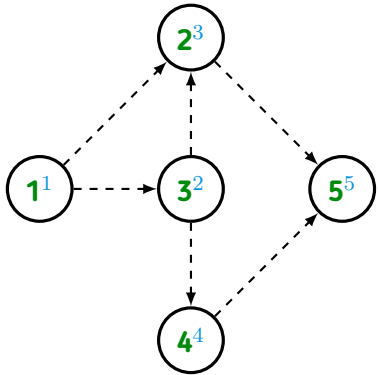
	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6





	1	2	3	4	5
$d_1(x) =$	0	3	2	4	6



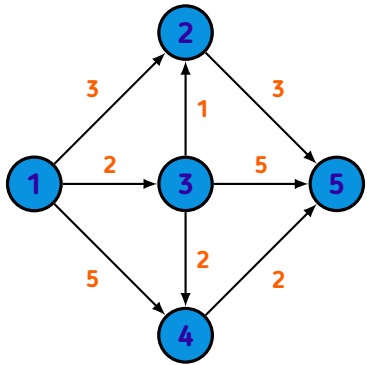


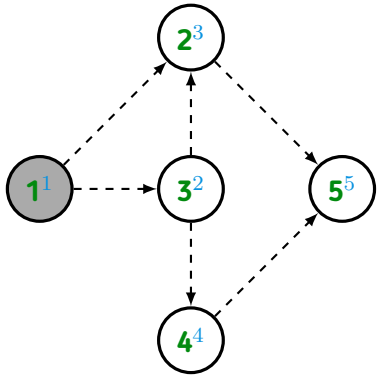
$d_1(x) =$

1	2	3	4	5
0	3	2	4	6

$p_1(x) =$

1	2	3	4	5
-	-	-	-	-



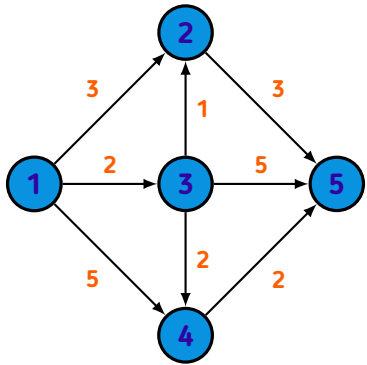


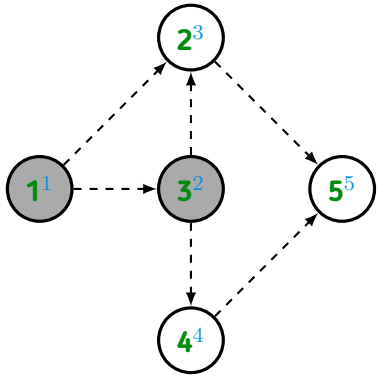
$d_1(x) =$

1	2	3	4	5
0	3	2	4	6

$p_1(x) =$

1	2	3	4	5
1	-	-	-	-



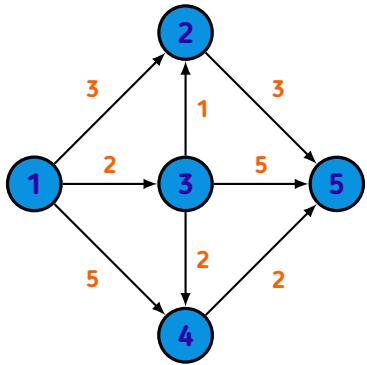


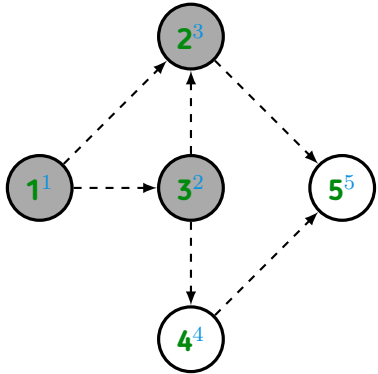
$d_1(x) =$

1	2	3	4	5
0	3	2	4	6

$p_1(x) =$

1	2	3	4	5
1	-	1	-	-



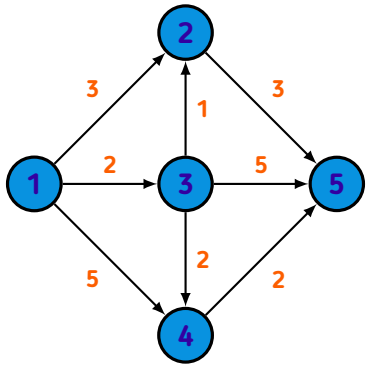


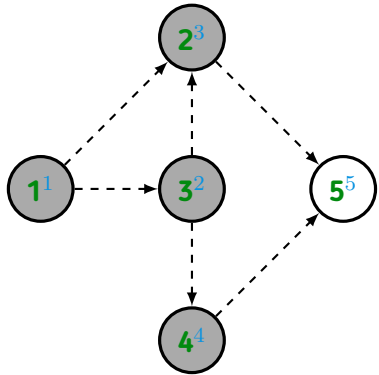
$d_1(x) =$

1	2	3	4	5
0	3	2	4	6

$p_1(x) =$

1	2	3	4	5
1	**2**	1	-	-



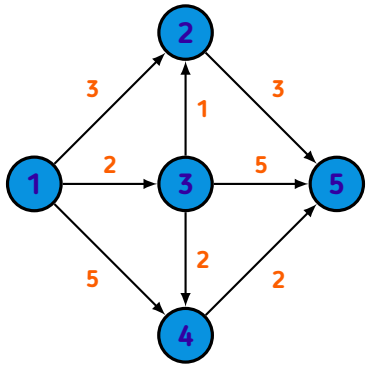


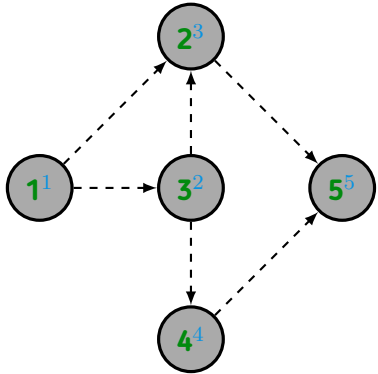
$d_1(x) =$

1	2	3	4	5
0	3	2	4	6

$p_1(x) =$

1	2	3	4	5
1	2	1	1	-





$d_1(x) =$

1	2	3	4	5
0	3	2	4	6

$p_1(x) =$

1	2	3	4	5
1	2	1	1	**3**

```
vector<int> min_paths(int s, int N)
{
    vector<int> ps(N + 1, 0);
    ps[s] = 1;

    auto o = dijkstra_order(s, N);

    for (auto x : o)
        for (auto v : in[x])
            ps[x] += ps[v];

    return ps;
}
```

```
vector<int> dijkstra_order(int s, int N)
{
    vector<int> dist(N + 1, oo), order;
    dist[s] = 0;

    processed.reset();

    priority_queue<ii, vector<ii>, greater<ii>> pq;
    pq.emplace(0, s);

    while (not pq.empty())
    {
        auto [d, u] = pq.top();
        pq.pop();

        if (processed[u])
            continue;
```

```
        order.emplace_back(u);
        processed[u] = true;

        for (auto [v, w] : adj[u])
        {
            if (dist[v] > d + w)
            {
                dist[v] = d + w;
                pq.emplace(dist[v], v);
                in[v] = { u };
            } else if (dist[v] == d + w)
                in[v].emplace(u);
        }
    }

    return order;
}
```

Problemas sugeridos

1. **CSES 1202 – Investigation**
2. **HackerEarth – Counting on Tree**
3. **SPOJ DAGCNT2 – Counting in a DAG**
4. **Timus 1018 – Binary Apple Tree**

Referências

1. **Codeforces.** ***DP on Trees Tutorial*****, acesso em 14/09/2021.**
2. **DUMOL**, **Tim.** ***IOI Trainning Week 5 – DP on Trees and DAGs***.
3. **HALIM**, **Felix**; **HALIM**, **Steve**. ***Competitive Programming 3*****, 2010.**
4. **LAAKSONEN**, **Antti**. ***Competitive Programmer's Handbook*****, 2018.**