

Timus 1146

Tobby and Query

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Problema

Given a 2-dimensional array of positive and negative integers, find the sub-rectangle with the largest sum. The sum of a rectangle is the sum of all the elements in that rectangle. In this problem the sub-rectangle with the largest sum is referred to as the maximal sub-rectangle. A sub-rectangle is any contiguous sub-array of size 1×1 or greater located within the whole array.

As an example, the maximal sub-rectangle of the array:

| | | | |
|----|----|----|----|
| 0 | -2 | -7 | 0 |
| 9 | 2 | -6 | 2 |
| -4 | 1 | -4 | 1 |
| -1 | 8 | 0 | -2 |

is in the lower-left-hand corner and has the sum of 15.

Input

The input consists of an $N \times N$ array of integers. The input begins with a single positive integer N on a line by itself indicating the size of the square two dimensional array. This is followed by N^2 integers separated by white-space (newlines and spaces). These N^2 integers make up the array in row-major order (i.e., all numbers on the first row, left-to-right, then all numbers on the second row, left-to-right, etc.). N may be as large as 100. The numbers in the array will be in the range $[-127, 127]$.

Output

The output is the sum of the maximal sub-rectangle.

Exemplo de entradas e saídas

Sample Input

```
4
0 -2 -7 0
9 2 -6 2
-4 1 -4 1
-1 8 0 -2
```

Sample Output

```
15
```

Solução $O(N^3)$

- Uma solução de força bruta computaria a soma todas as N^4 submatrizes, sendo que cada soma é feita em $O(N^2)$, de modo que a solução teria complexidade $O(N^6)$
- Contudo, o uso de combinado de somas prefixadas e o algoritmo de Kadane permite identificar a submatriz de soma máxima com complexidade $O(N^3)$
- Para cada par de colunas (i, j) , deve ser computado, por meio do algoritmo de Kadane nas somas $p_k(i, j)$, para $1 \leq k \leq N$, o intervalo de maior soma, onde

$$p_k(i, j) = \sum_{t=i}^j a_{kt}$$

- Veja que, dados os limites do problema, mesmo nos casos extremos a soma máxima ainda pode ser armazenada em variáveis inteiras

Solução $O(N^3)$

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4 const int oo { 1000000010 };
5
6 int kadane(int N, const vector<int>& as)
7 {
8     vector<int> s(N + 1);
9     s[1] = as[1];
10
11     for (size_t i = 2; i < as.size(); ++i)
12         s[i] = max(as[i], s[i - 1] + as[i]);
13
14     return *max_element(s.begin() + 1, s.end());
15 }
16
17 int solve(int N, const vector<vector<int>>& A)
18 {
19     vector<vector<int>> p(N + 1, vector<int>(N + 1, 0));
20     int ans = -oo;
```

Solução $O(N^3)$

```
22  for (int i = 1; i <= N; ++i)
23  {
24      vector<int> r(N + 1, 0);
25
26      for (int j = i; j <= N; ++j)
27      {
28          for (int k = 1; k <= N; ++k)
29              r[k] += A[k][j];
30
31          ans = max(ans, kadane(N, r));
32      }
33  }
34
35  return ans;
36 }
37
38 int main()
39 {
40     ios::sync_with_stdio(false);
```

Solução $O(N^3)$

```
42  int N;  
43  cin >> N;  
44  
45  vector<vector<int>> A(N + 1, vector<int>(N + 1));  
46  
47  for (int i = 1; i <= N; ++i)  
48      for (int j = 1; j <= N; ++j)  
49          cin >> A[i][j];  
50  
51  auto ans = solve(N, A);  
52  
53  cout << ans << endl;  
54  
55  return 0;  
56 }
```