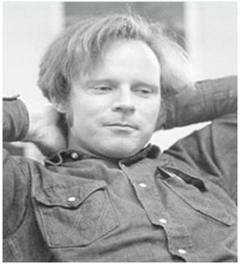
Grafos

Algoritmo de Floyd-Warshall

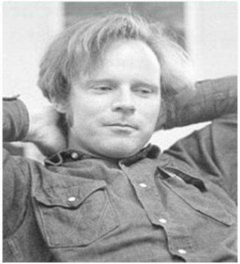
Prof. Edson Alves

Faculdade UnB Gama



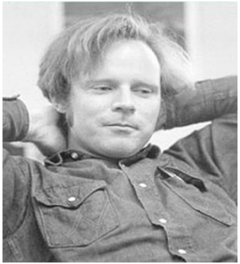
Robert W. Floyd

(1962)



Robert W. Floyd
(1962)

Stephen Warshall
(1962)



Robert W. Floyd
(1962)

Stephen Warshall
(1962)

Bernard Roy
(1959)

Características do algoritmo de Bellman-Ford

Características do algoritmo de Bellman-Ford

- ⋆ **Computa o caminho mínimo entre todos os pares de vértices de** $G(V, E)$

Características do algoritmo de Bellman-Ford

- $\star$ Computa o caminho mínimo entre todos os pares de vértices de $G(V, E)$
- $\star$ É capaz de processar arestas negativas

Características do algoritmo de Bellman-Ford

- ⋆ Computa o caminho mínimo entre todos os pares de vértices de $G(V, E)$
- ⋆ É capaz de processar arestas negativas
- ⋆ Não processa, mas identifica ciclos negativos

Características do algoritmo de Bellman-Ford

- $\star$ Computa o caminho mínimo entre todos os pares de vértices de $G(V, E)$
- $\star$ É capaz de processar arestas negativas
- $\star$ Não processa, mas identifica ciclos negativos
- $\star$ As distâncias são reduzidas por meio do uso de vértices intermediários

Características do algoritmo de Bellman-Ford

- $\star$ **Computa o caminho mínimo entre todos os pares de vértices de** $G(V, E)$
- $\star$ **É capaz de processar arestas negativas**
- $\star$ **Não processa, mas identifica ciclos negativos**
- $\star$ **As distâncias são reduzidas por meio do uso de vértices intermediários**
- $\star$ **Complexidade:** $O(V^3)$

Pseudocódigo

Pseudocódigo

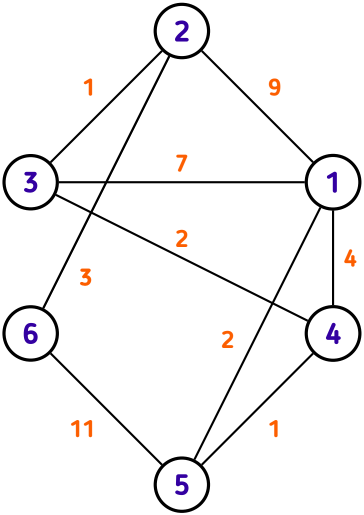
Entrada: **um grafo** $G(V, E)$
Saída: **uma matriz** d **tal que** $d[u][v]$ **é a distância mínima em** G **entre** u **e** v

1. **Faça:**
   (a) $d[u][u] = 0$, **para todos** $u \in V$
   (b) $d[u][v] = w$, **se** $(u, v, w) \in E$
   (c) $d[u][v] = \infty$, **caso contrário**

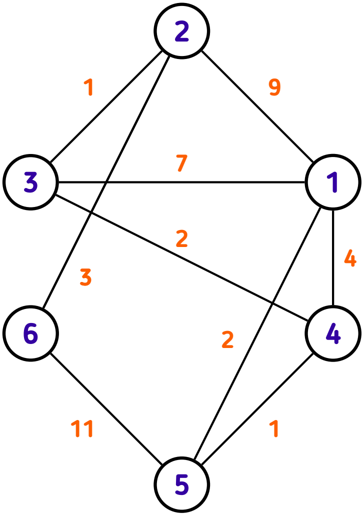
2. **Para cada vértice** k **e todos os pares** $(u, v) \in V^2$, **faça**

$$d[u][v] = \min(d[u][v], d[u][k] + d[k][v])$$

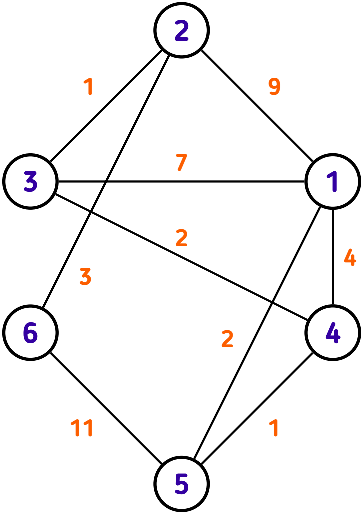
3. **Retorne** d



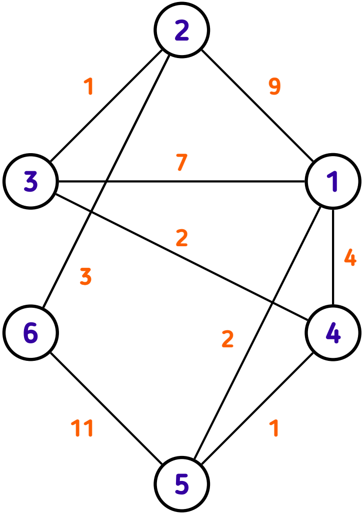
	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						



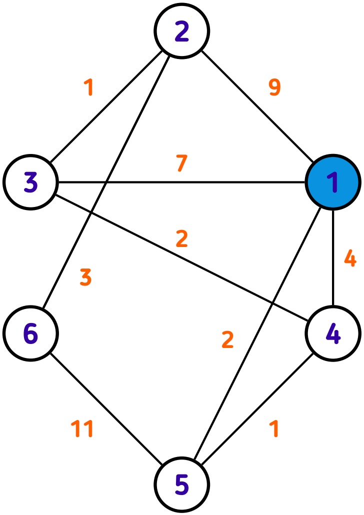
	1	2	3	4	5	6
1	0					
2		0				
3			0			
4				0		
5					0	
6						0



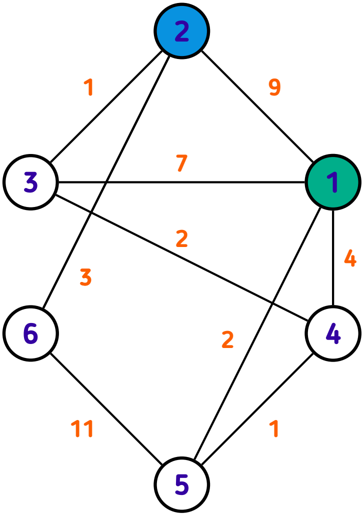
	1	2	3	4	5	6
1	0	9	7	4	2	
2	9	0	1			3
3	7	1	0	2		
4	4		2	0	1	
5	2			1	0	11
6		3			11	0



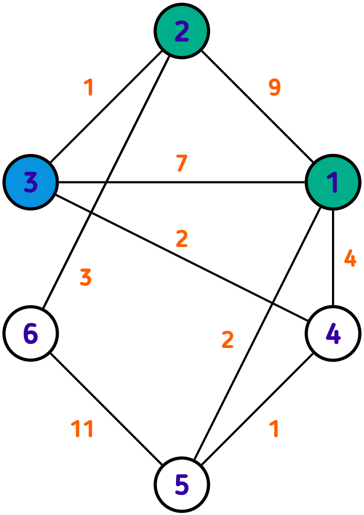
	1	2	3	4	5	6
1	0	9	7	4	2	∞
2	9	0	1	∞	∞	3
3	7	1	0	2	∞	∞
4	4	∞	2	0	1	∞
5	2	∞	∞	1	0	11
6	∞	3	∞	∞	11	0



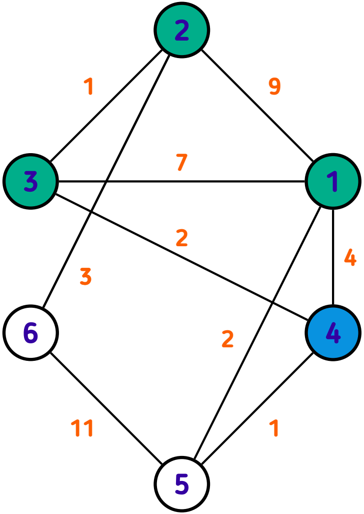
	1	2	3	4	5	6
1	0	9	7	4	2	∞
2	9	0	1	**13**	**11**	3
3	7	1	0	2	**9**	∞
4	4	**13**	2	0	1	∞
5	2	**11**	**9**	1	0	11
6	∞	3	∞	∞	11	0



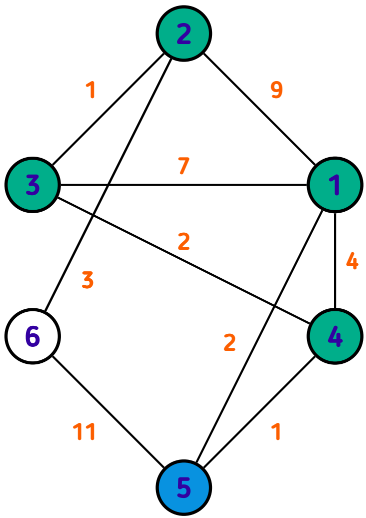
	1	2	3	4	5	6
1	0	9	7	4	2	**12**
2	9	0	1	13	11	3
3	7	1	0	2	9	**4**
4	4	13	2	0	1	**16**
5	2	11	9	1	0	11
6	**12**	3	**4**	**16**	11	0



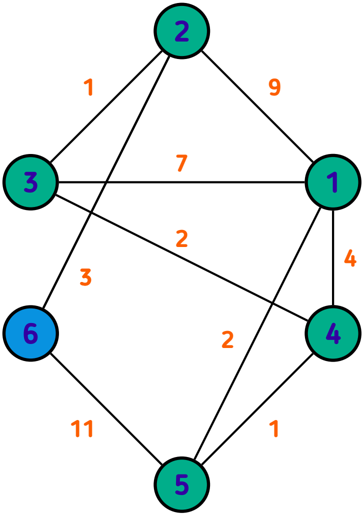
	1	2	3	4	5	6
1	0	**8**	7	4	2	**11**
2	**8**	0	1	**3**	**10**	3
3	7	1	0	2	9	4
4	4	**3**	2	0	1	**6**
5	2	**10**	9	1	0	11
6	**11**	3	4	**6**	11	0



	1	2	3	4	5	6
1	0	**7**	**6**	4	2	**10**
2	**7**	0	1	3	**4**	3
3	**6**	1	0	2	**3**	4
4	4	3	2	0	1	6
5	2	**4**	**3**	1	0	**7**
6	**10**	3	4	6	**7**	0



	1	2	3	4	5	6
1	0	**6**	**5**	**3**	2	**9**
2	**6**	0	1	3	4	3
3	**5**	1	0	2	3	4
4	**3**	3	2	0	1	6
5	2	4	3	1	0	7
6	**9**	3	4	6	7	0



	1	2	3	4	5	6
1	0	6	5	3	2	9
2	6	0	1	3	4	3
3	5	1	0	2	3	4
4	3	3	2	0	1	6
5	2	4	3	1	0	7
6	9	3	4	6	7	0

```
vector<vector<int>> floyd_warshall(int N)
{
    vector<vector<int>> dist(N + 1, vector<int>(N + 1, oo));

    for (int u = 1; u <= N; ++u)
        dist[u][u] = 0;

    for (int u = 1; u <= N; ++u)
        for (auto [v, w] : adj[u])
            dist[u][v] = w;

    for (int k = 1; k <= N; ++k)
        for (int u = 1; u <= N; ++u)
            for (int v = 1; v <= N; ++v)
                dist[u][v] = min(dist[u][v], dist[u][k] + dist[k][v]);

    return dist
}
```

Identificação de um caminho mínimo

Identificação de um caminho mínimo

- O algoritmo de Dijkstra computa as distâncias mínimas, mas não os caminhos mínimos

Identificação de um caminho mínimo

- $\star$ **O algoritmo de Dijkstra computa as distâncias mínimas, mas não os caminhos mínimos**

- $\star$ **Para determinar um caminho mínimo, é preciso definir uma matriz auxiliar** pred**, onde** $\text{pred}[u][v] =$ **antecessor de** v **no caminho mínimo de** u **a** v

Identificação de um caminho mínimo

$\star$ **O algoritmo de Dijkstra computa as distâncias mínimas, mas não os caminhos mínimos**

$\star$ **Para determinar um caminho mínimo, é preciso definir uma matriz auxiliar** pred**, onde** $\text{pred}[u][v] =$ **antecessor de** v **no caminho mínimo de** u **a** v

$\star$ **No início do algoritmo,**

(a) $\text{pred}[u][u] = u, \forall u \in V$

(b) $\text{pred}[u][v] = u$**, se** $(u, v) \in E$

(c) $\text{pred}[u][v] = \text{undef}$**, caso contrário**

Identificação de um caminho mínimo

Identificação de um caminho mínimo

- $\star$ **Se** (u, v) **atualizar** $d[v]$**, faça** $\text{pred}[v] = u$

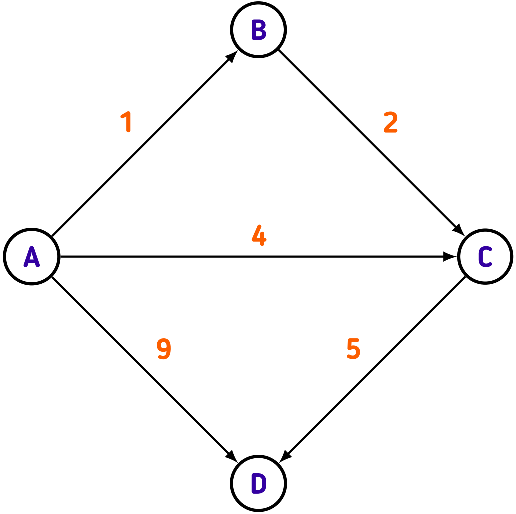
Identificação de um caminho mínimo

- $\star$ **Se** (u, v) **atualizar** $d[v]$**, faça** $\mathsf{pred}[v] = u$

- $\star$ **A sequência**

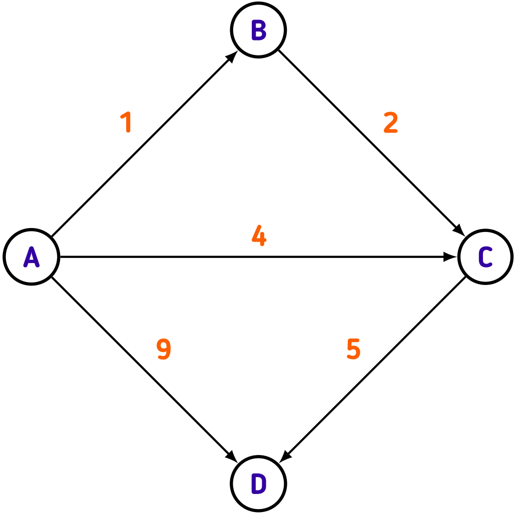
$$p = \{(u, \mathsf{pred}^{k-1}[u][v]), \dots, (\mathsf{pred}[\mathsf{pred}[u][v]], \mathsf{pred}[u][v]), (\mathsf{pred}[u][v], v)\}$$

é um caminho mínimo de u **a** v **composto por** k **arestas e tamanho** $d[u][v]$



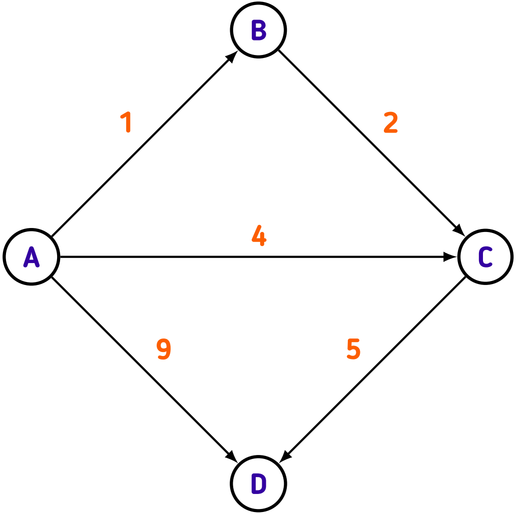
$$M = \begin{array}{|c|c|c|c|c}
\multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \\
\hline
\quad & \quad & \quad & \quad & A \\
\hline
 & & & & B \\
\hline
 & & & & C \\
\hline
 & & & & D \\
\hline
\end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



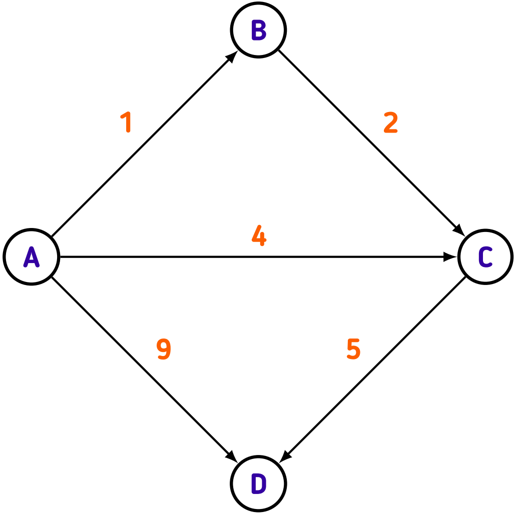
$$M = \begin{array}{cccc|c} A & B & C & D & \\ \hline 0_A & & & & A \\ & 0_B & & & B \\ & & 0_C & & C \\ & & & 0_D & D \end{array}$$

$$m_{ij} = \mathrm{dist}[i][j]_{\mathrm{pred}[i][j]}$$



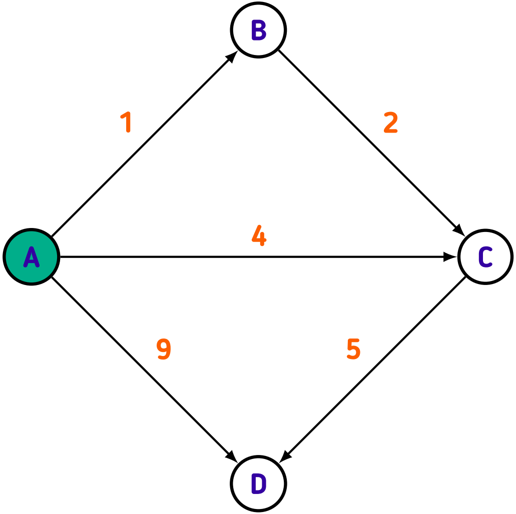
$$M = \begin{array}{cc} \begin{array}{cccc} A & B & C & D \end{array} & \\ \left[\begin{array}{cccc} 0_A & 1_A & 4_A & 9_A \\ & 0_B & 2_B & \\ & & 0_C & 5_C \\ & & & 0_D \end{array}\right] & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



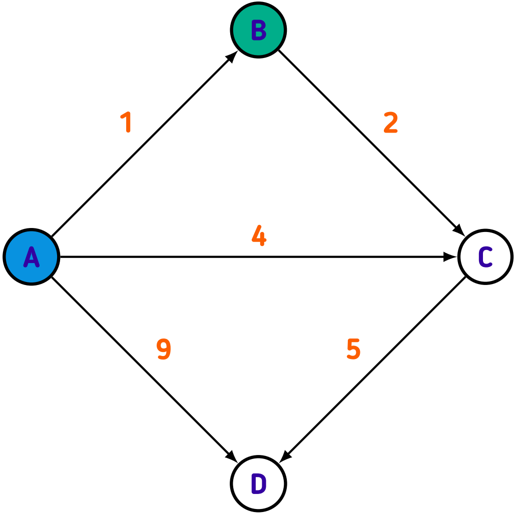
$$
M = \begin{array}{cccc|c}
\text{A} & \text{B} & \text{C} & \text{D} & \\
\hline
0_{\text{A}} & 1_{\text{A}} & 4_{\text{A}} & 9_{\text{A}} & \text{A} \\
\infty_{\text{-}} & 0_{\text{B}} & 2_{\text{B}} & \infty_{\text{-}} & \text{B} \\
\infty_{\text{-}} & \infty_{\text{-}} & 0_{\text{C}} & 5_{\text{C}} & \text{C} \\
\infty_{\text{-}} & \infty_{\text{-}} & \infty_{\text{-}} & 0_{\text{D}} & \text{D}
\end{array}
$$

$$m_{ij} = \text{dist}[i][j]_{\text{pred}[i][j]}$$



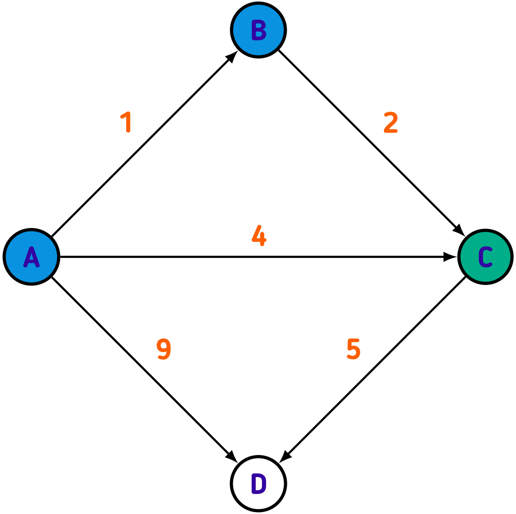
$$M = \begin{array}{cccc|c} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} & \\ \hline 0_{\mathsf{A}} & 1_{\mathsf{A}} & 4_{\mathsf{A}} & 9_{\mathsf{A}} & \mathsf{A} \\ \infty_{-} & 0_{\mathsf{B}} & 2_{\mathsf{B}} & \infty_{-} & \mathsf{B} \\ \infty_{-} & \infty_{-} & 0_{\mathsf{C}} & 5_{\mathsf{C}} & \mathsf{C} \\ \infty_{-} & \infty_{-} & \infty_{-} & 0_{\mathsf{D}} & \mathsf{D} \end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



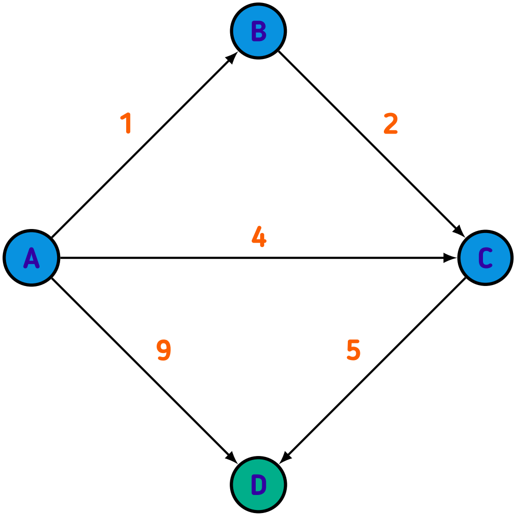
$$M = \begin{array}{c} \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \begin{array}{|c|c|c|c|} \hline 0_{\mathsf{A}} & 1_{\mathsf{A}} & \mathbf{3_{B}} & 9_{\mathsf{A}} \\ \hline \infty_{-} & 0_{\mathsf{B}} & 2_{\mathsf{B}} & \infty_{-} \\ \hline \infty_{-} & \infty_{-} & 0_{\mathsf{C}} & 5_{\mathsf{C}} \\ \hline \infty_{-} & \infty_{-} & \infty_{-} & 0_{\mathsf{D}} \\ \hline \end{array} \end{array} \begin{array}{c} \\ \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



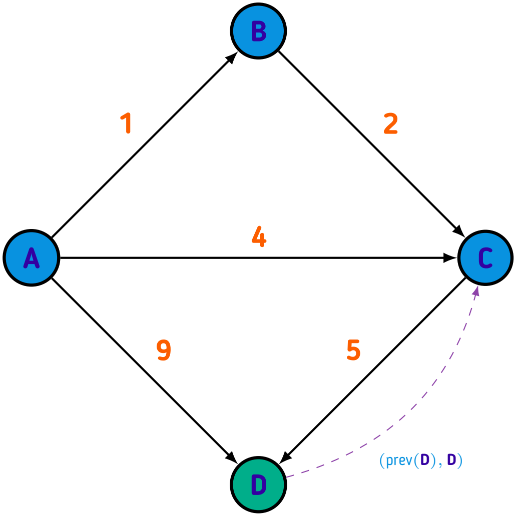
$$M = \begin{array}{c c}
\begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} & \\
\begin{array}{|c|c|c|c|} \hline
0_{\mathsf{A}} & 1_{\mathsf{A}} & 3_{\mathsf{B}} & \mathbf{8}_{\mathsf{C}} \\ \hline
\infty_{-} & 0_{\mathsf{B}} & 2_{\mathsf{B}} & \mathbf{7}_{\mathsf{C}} \\ \hline
\infty_{-} & \infty_{-} & 0_{\mathsf{C}} & 5_{\mathsf{C}} \\ \hline
\infty_{-} & \infty_{-} & \infty_{-} & 0_{\mathsf{D}} \\ \hline
\end{array} & \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array}
\end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



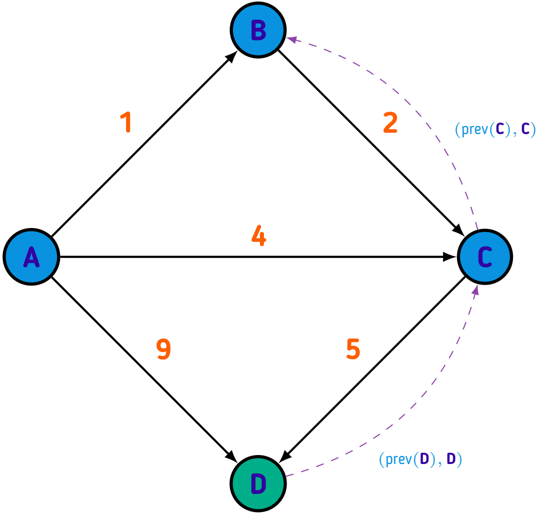
$$M = \begin{array}{cccc c} A & B & C & D & \\ \begin{array}{|c|c|c|c|} \hline 0_{A} & 1_{A} & 3_{B} & 8_{C} \\ \hline \infty_{-} & 0_{B} & 2_{B} & 7_{C} \\ \hline \infty_{-} & \infty_{-} & 0_{C} & 5_{C} \\ \hline \infty_{-} & \infty_{-} & \infty_{-} & 0_{D} \\ \hline \end{array} & & & & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



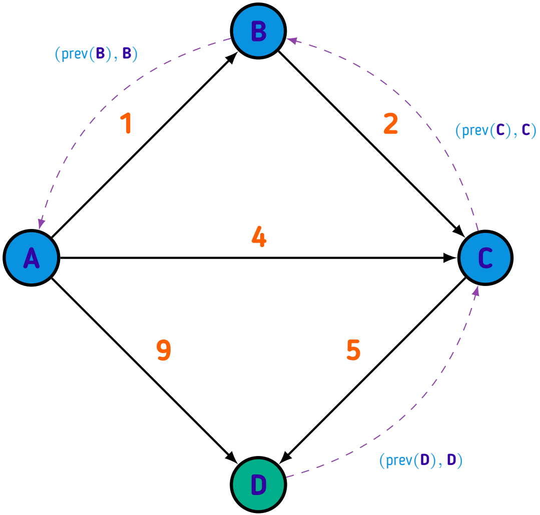
$$M = \begin{array}{cccc|c} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} & \\ \hline 0_{\mathsf{A}} & 1_{\mathsf{A}} & 3_{\mathsf{B}} & 8_{\mathsf{C}} & \mathsf{A} \\ \infty_{-} & 0_{\mathsf{B}} & 2_{\mathsf{B}} & 7_{\mathsf{C}} & \mathsf{B} \\ \infty_{-} & \infty_{-} & 0_{\mathsf{C}} & 5_{\mathsf{C}} & \mathsf{C} \\ \infty_{-} & \infty_{-} & \infty_{-} & 0_{\mathsf{D}} & \mathsf{D} \end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



$$M = \begin{array}{cccccc} & \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} & \\ & \left[\begin{array}{cccc} 0_{\mathsf{A}} & 1_{\mathsf{A}} & 3_{\mathsf{B}} & 8_{\mathsf{C}} \\ \infty_{-} & 0_{\mathsf{B}} & 2_{\mathsf{B}} & 7_{\mathsf{C}} \\ \infty_{-} & \infty_{-} & 0_{\mathsf{C}} & 5_{\mathsf{C}} \\ \infty_{-} & \infty_{-} & \infty_{-} & 0_{\mathsf{D}} \end{array}\right] & & & & \begin{array}{c} \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array} \end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$



$$M = \begin{array}{c} \begin{array}{cccc} \mathsf{A} & \mathsf{B} & \mathsf{C} & \mathsf{D} \end{array} \\ \left[\begin{array}{cccc} 0_{\mathsf{A}} & 1_{\mathsf{A}} & 3_{\mathsf{B}} & 8_{\mathsf{C}} \\ \infty_{-} & 0_{\mathsf{B}} & 2_{\mathsf{B}} & 7_{\mathsf{C}} \\ \infty_{-} & \infty_{-} & 0_{\mathsf{C}} & 5_{\mathsf{C}} \\ \infty_{-} & \infty_{-} & \infty_{-} & 0_{\mathsf{D}} \end{array}\right] \end{array} \begin{array}{c} \\ \mathsf{A} \\ \mathsf{B} \\ \mathsf{C} \\ \mathsf{D} \end{array}$$

$$m_{ij} = \mathsf{dist}[i][j]_{\mathsf{pred}[i][j]}$$