The skyrmion ‘zoo’

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The chiral magnet

Consider a 2D magnet with a crystal structure that breaks reflection symmetry. The magnetisation field (n_1, n_2, n_3), $\boldsymbol{n}\cdot\boldsymbol{n} = 1$, has the effective continuum (classical) energy functional:

$$E(\boldsymbol{n}) = \int \underbrace{\frac{1}{2}|\partial_i \boldsymbol{n}|^2}_{\text{Heisenberg}} + \underbrace{\boldsymbol{A}_i \cdot ((\boldsymbol{n}-\boldsymbol{n}_0)\times\partial_i \boldsymbol{n})}_{\text{DMI (with boundary)}} + \underbrace{h_z(1-\boldsymbol{e}_h\cdot\boldsymbol{n})}_{\text{Zeeman}} + \underbrace{h_a(1-n_3^2)}_{\text{anisotropy}} d^2x$$

- Heisenberg: spins want to align (ferromagnet)
- Dzyaloshinskii-Moriya Interaction (DMI): spins want to be at right angles
- Potential: spins have a preferred orientation $\boldsymbol{n}_0$ (usually north pole, $n_3 = 1$)

The competition of Heisenberg and DMI leads to spins preferring twisting with a given lengthscale and handedness. This twisting cannot be satisfied in all directions at once - frustration.

Finite-energy magnetisation configurations classified by topological degree $Q(\boldsymbol{n})$ - it cannot change under continuous evolution of the spins*. What is the minimizer for $Q \neq 0$?

Colour labelling gives scaling. Heisenberg invariant, DMI linear and potential terms quadratic.

This means that localised minimisers are not forbidden, but if they exist they must have DMI < 0.

Magnetic skyrmions

This system has two continuous symmetries, translation and vectorial rotation $\boldsymbol{n}(\vec{x}) \mapsto R\boldsymbol{n}(R^{-1}\vec{x})$.

The simplest localised solution we can look for breaks translation symmetry and retains rotation*. Gives form

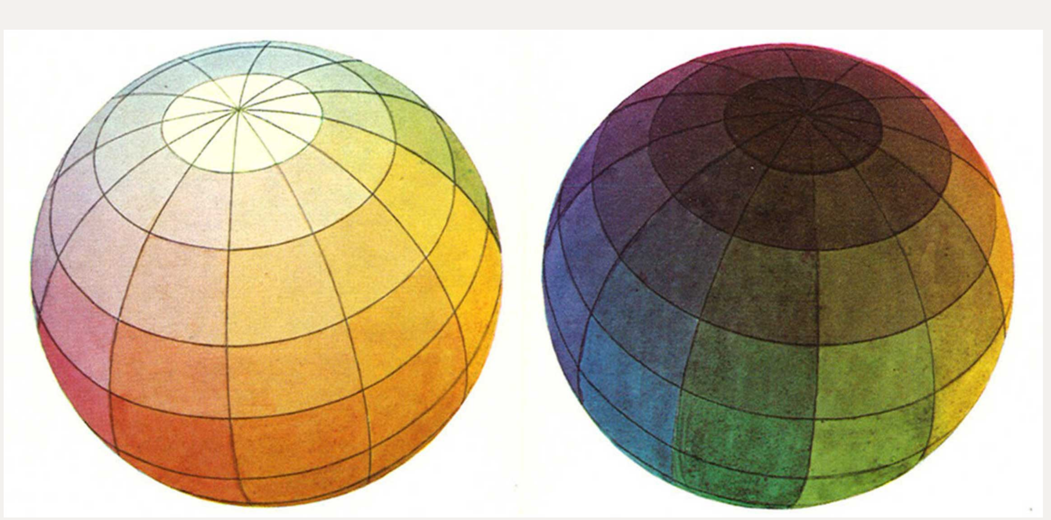
$$\boldsymbol{n}(r,\phi) = (\sin\Theta(r)\cos(\phi+\gamma), \sin\Theta(r)\sin(\phi+\gamma), \cos\Theta(r)) \,,$$

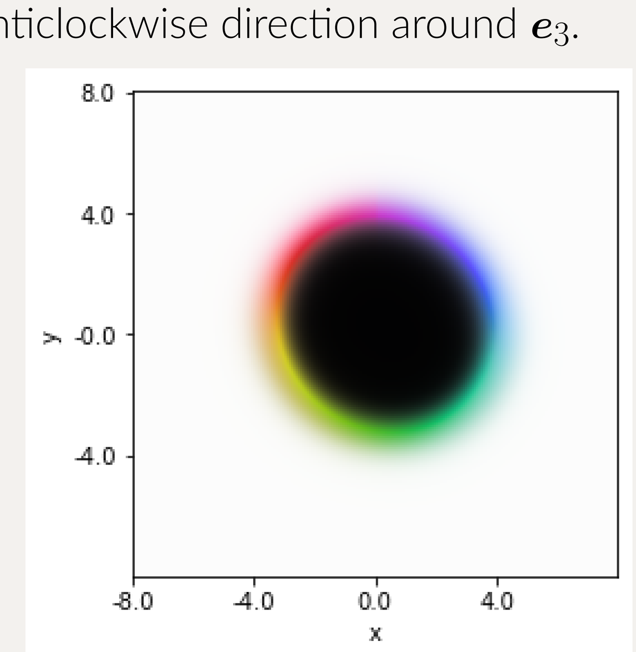
where $\Theta(r)$ solves an ODE and interpolates between* $\Theta = \pi$ and $\Theta = 0$. This has $Q = -1$. It is called the magnetic skyrmion, and it is a robust particle-like excitation observed in a variety of real materials.

Many applications are foreseen for the magnetic skyrmion - but is it the only non-trivial energy minimiser? **Other minimizers besides the skyrmion may fulfill the functions of a skyrmion better - or provide new, unforeseen roles.**

Plotting magnetisation with a colour field

The magnetisation field $\boldsymbol{n}(x,y)$ can be plotted using the ‘Runge sphere’ colouring convention shown below, where white corresponds to $\boldsymbol{n} = \boldsymbol{e}_3$, black to $\boldsymbol{n} = -\boldsymbol{e}_3$ and the azimuthal angle is coloured by the rainbow, red-yellow-green-blue in the anticlockwise direction around $\boldsymbol{e}_3$.





Exact solutions at specially tuned ‘critical’ coupling

An exact balance of (axisymmetric) DMI, Zeeman and anisotropy terms* gives

$$E(\boldsymbol{n}) = \int \frac{1}{2}|\partial_i \boldsymbol{n}|^2 + \underbrace{k(\boldsymbol{n}-\boldsymbol{e}_3)\cdot(\nabla\times\boldsymbol{n})}_{\text{axisymmetric Bloch DMI}} + \underbrace{\frac{1}{2}k^2(1-n_3)^2}_{\text{tuned Zeeman+anisotropy}} \quad d^2x$$

This can be understood as a gauged $O(3)$ sigma model with fixed background gauge,

$$E(\boldsymbol{n}) = \int \frac{1}{2}|D_i\boldsymbol{n}|^2 - \boldsymbol{F}\cdot\boldsymbol{n})d^2x + \int_{\partial^2} \boldsymbol{A}_i \cdot (\boldsymbol{n}_0 \times \boldsymbol{n}) d\nu_i,$$

where $\boldsymbol{F}$ is the curvature $\partial_1 \boldsymbol{A}_2 - \partial_2 \boldsymbol{A}_1 + \boldsymbol{A}_1 \times \boldsymbol{A}_2$, and in the particular case above $\boldsymbol{A}_i = -k\boldsymbol{e}_i$.

Such a model has a Bogomol’nyi argument; it can be rearranged as follows:

$$E(\boldsymbol{n}) = \int \frac{1}{2}|D_1\boldsymbol{n} + \boldsymbol{n}\times D_2\boldsymbol{n}|^2 d^2x + \int \boldsymbol{n}\cdot(\partial_1\boldsymbol{n}\times\partial_2\boldsymbol{n})d^2x.$$

The second integral is equal to $4\pi Q(\boldsymbol{n})$ and thus invariant under variations. So solutions of

$$D_1\boldsymbol{n} + \boldsymbol{n}\times D_2\boldsymbol{n} = \boldsymbol{0}$$

are minimizers. Moreover, we can solve this equation explicitly, and its energy will be $4\pi Q$.

At this coupling, we have an infinite family of explicit minimizers of the energy functional.

Zoo of solutions

There is a large moduli space of solutions with $Q \geq -1$. The dimension of the moduli space* of charge Q minimizers increases linearly, as $\sim 4Q$.

Among the many solutions observed, **previously unknown topological solitons in chiral magnets have been found.**

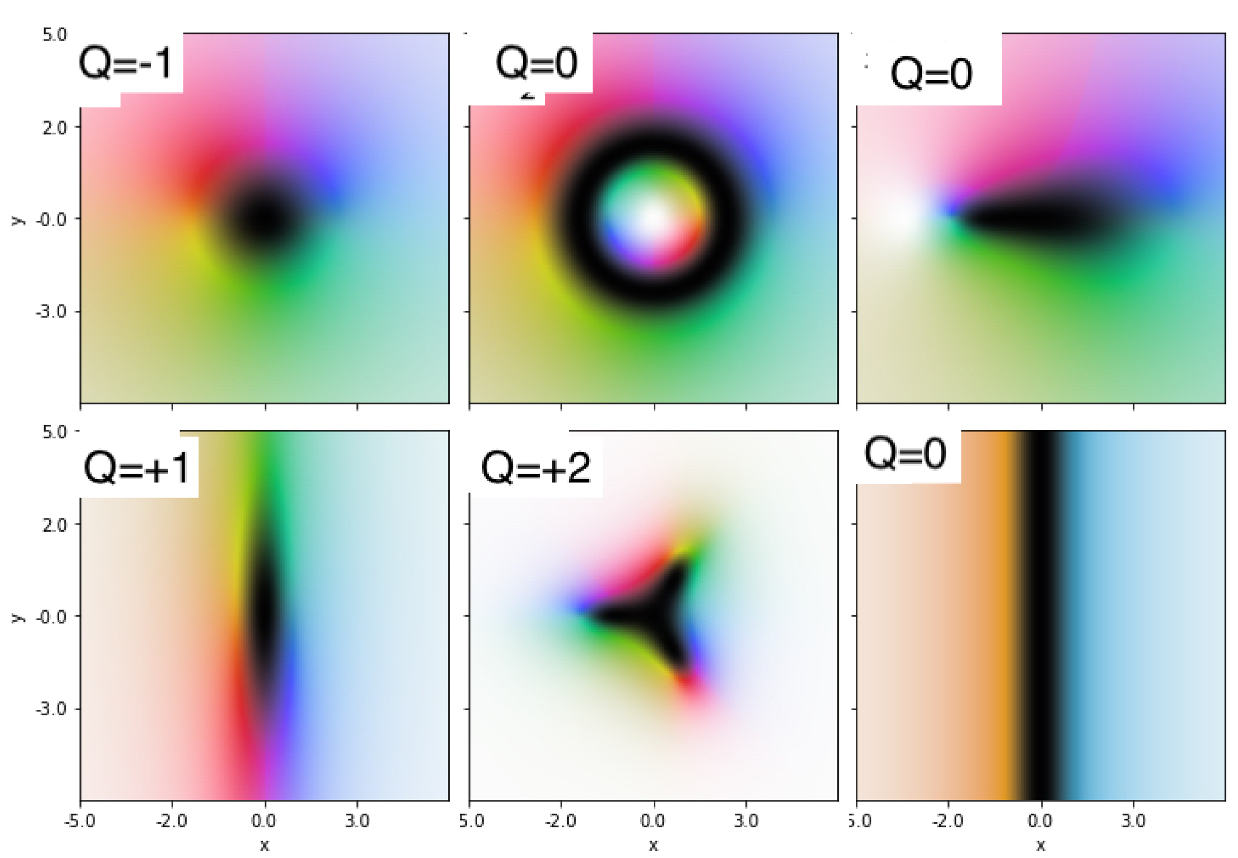


Figure 1. Various explicit minimizers of the critically coupled chiral magnet energy functional

The $Q = -1$ minimisers (inc. skyrmion) have negative energy. This tells us that the true ground state of the material at this coupling is an infinite skyrmion lattice, which we cannot find explicitly.

Applications 1: Insights on stability

By considering certain paths within these moduli spaces, we see that various solitons are neutrally stable to extension to infinity (known as the elliptical instability), shrinking to a singularity (collapse instability), and unwinding of 2π domain walls (previously not observed).

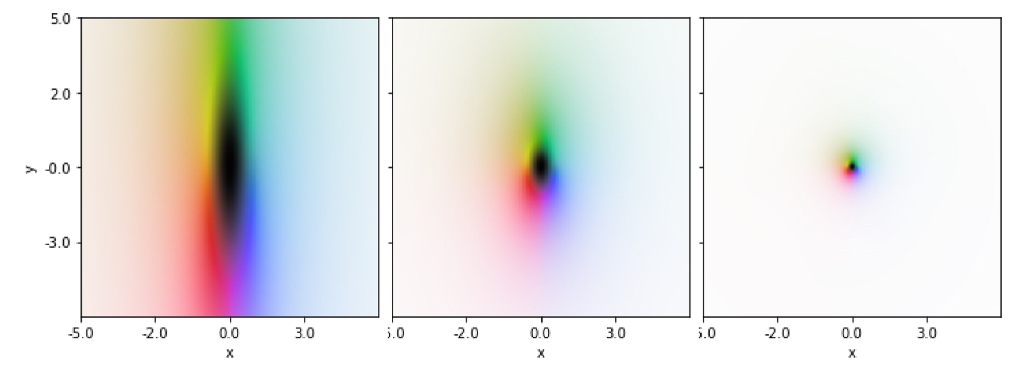


Figure 2. The antiskyrmion demonstrates both elliptical and collapse zero modes

We can use these families of minimizers to model soliton instability, away from critical coupling

Applications 2: New numerical solutions away from specially tuned coupling

Using these solutions as starting ansätze, **many new solutions have been found numerically**, stabilised against both collapse and elliptical distortion in a narrow range of parameters.

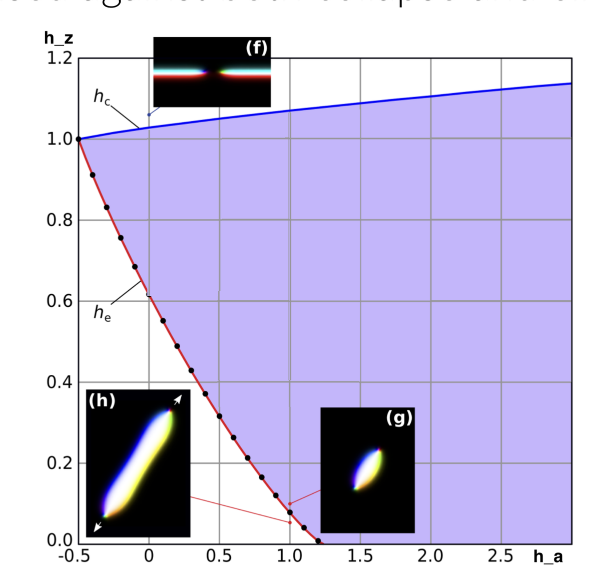


Figure 3. Region of stability of novel solutions

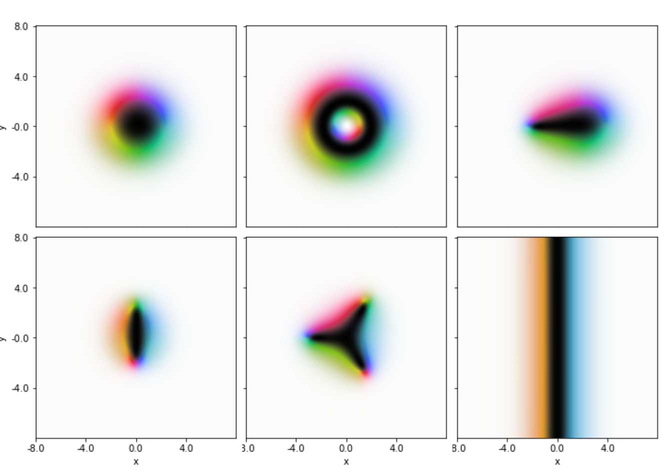


Figure 4. Solutions at $h_z = 0.61k^2$, $h_a = 0$

Applications 3: Classical and quantum dynamics

Moduli spaces of explicit solutions are also a useful tool for understanding adiabatic classical dynamics (work in progress) - and quantum dynamics (work to come).

In real chiral magnets, skyrmions are $\tilde{1}00nm$ large and getting smaller; quantum effects have so far been neglected, but may be important.

Summary

Chiral magnets host a larger variety of static solutions than previously thought possible. We arrive at these solutions via a critical coupling of the chiral magnet, where they are explicit. These exact solutions allow us to understand various dynamical processes of topological solitons.

References

[1] Magnetic skyrmions at critical coupling, Communications in Mathematical Physics, 375(3), 2020

[2] Magnetic skyrmions, chiral kinks, and holomorphic functions. Phys. Rev. B, 102:144422, Oct 2020.

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