Magnetic Skyrmions at Critical Coupling [1]

Bruno Barton-Singer, Calum Ross, Bernd Schroers
Department of Mathematics
Heriot-Watt University

The model

A thin-film ferromagnet with Dzyalloshinskii-Moriya interaction, easy-plane anisotropy and an applied magnetic field can be modelled by a continuous magnetization field with energy functional

$$E[n] = \int \left\{ \frac{J}{2} (\nabla n)^2 + Dn \cdot (\nabla \times n) + B(1 - n_3) + A(n_3^2 - 1) \right\} d^2 x$$

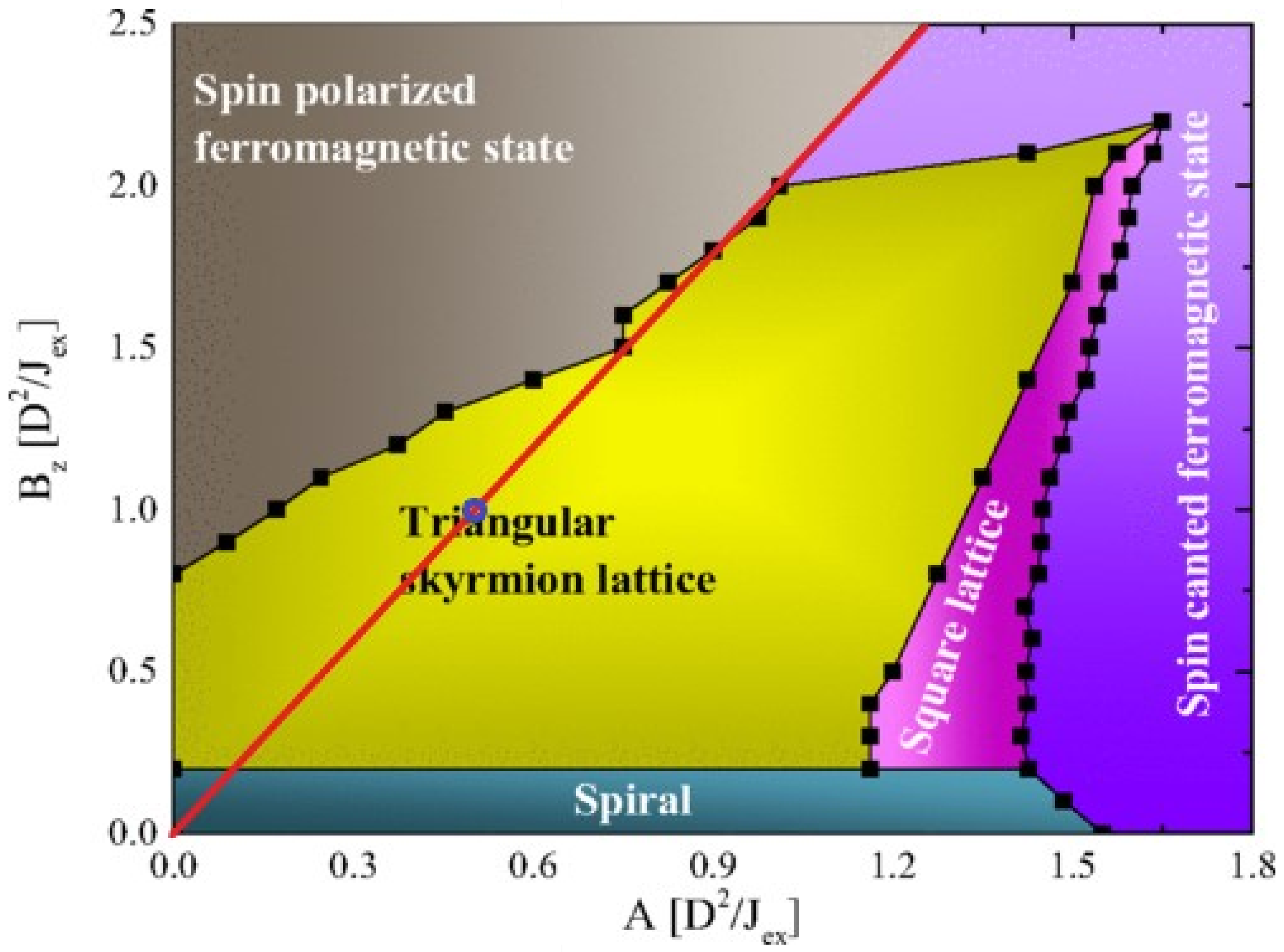

Figure 1.Numerical phase diagram [2] with critical coupling marked

Crtitical Coupling

If $A = \frac{B}{2}$, the magnetic field just balances the anisotropy so that the potential has a single minimum:

$$E[n] = \int \left\{ \frac{J}{2} (\nabla n)^2 + Dn \cdot (\nabla \times n) + B(1 - n_3)^2 \right\} d^2 x$$

This corresponds to the red 'critical line' above. Further, if $B = \frac{D^2}{J}$, we are at the 'critical point'.

Infinitely many solutions at the critical point

Defining the helical derivative $D_i n = \partial_i n - \frac{D}{J} e_i \times n$, this special energy can be rewritten as

$$E[n] = \int \frac{J}{2} (D_1 n + n \times D_2 n)^2 d^2 x + 4\pi J deg[n] + b.t.$$

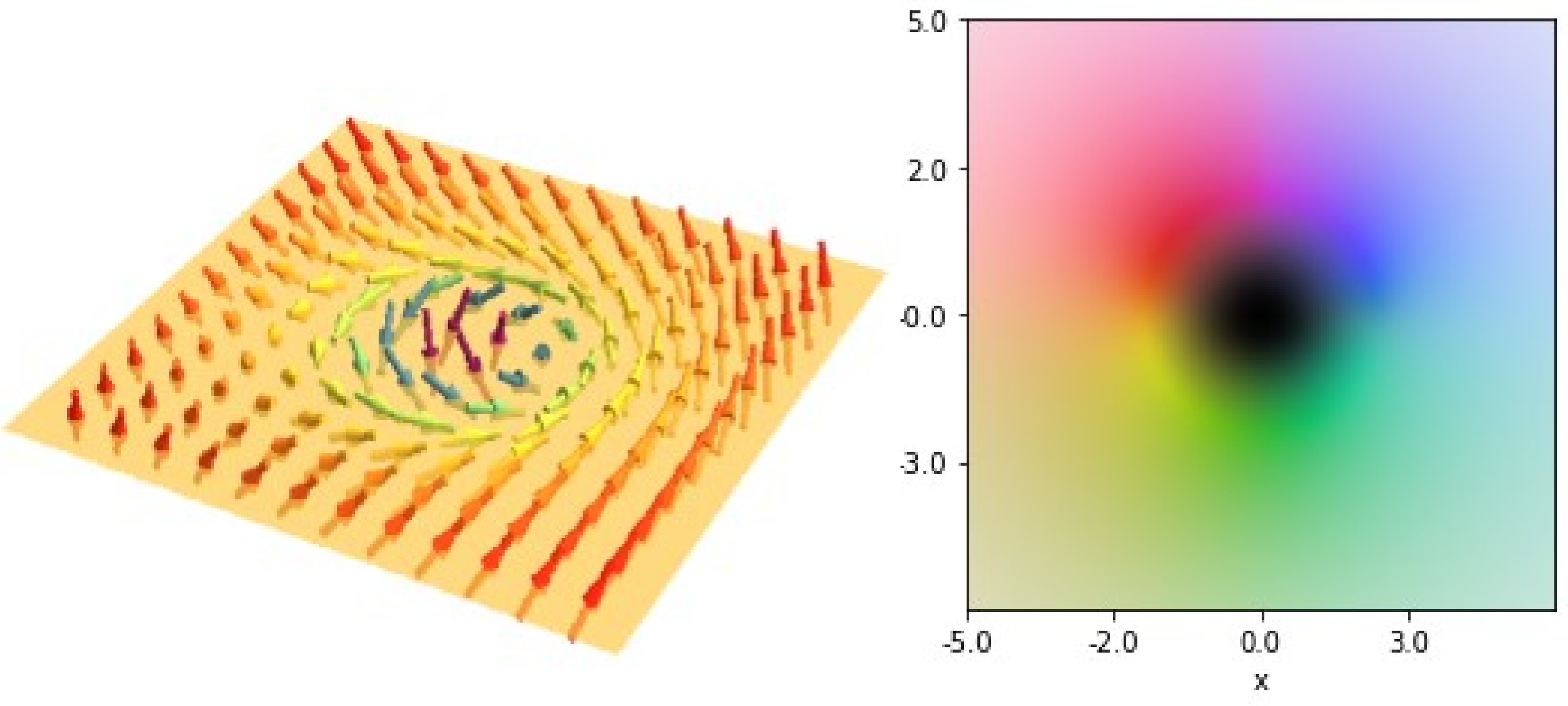

Figure 2. degree -1 minimizer (skyrmion) as arrow plot and colourmap

Up to the boundary term, this tells us that the energy is bounded below by the degree of the configuration (-1 for the basic skyrmion), and also gives us a first-order equation for solutions to the Euler-Lagrange equation that saturate this bound. We can explicitly write infinitely many solutions to this equation, some of which are shown in figures 2 and 3.

A family of hedgehog solutions along the critical line

Along the entire critical line, explicit solutions of the hedgehog form can be given. In polar co-ordinates, they have the following form:

$$\Theta(r) = 2arctan\left(\frac{2D}{Br}\right)$$

Generalizations

This same trick can be repeated for general Dzyaloshinskii-Moriya interactions, i.e. of the form

$$DMI = \int \mathbb{D}_{ij} (n \times \partial_i n)_j d^2 x$$

So in particular solutions can be found for interfacial (Neel) DMI as well as crystalline (Bloch) DMI. In fact it can even be repeated for arbitrary surfaces [3].

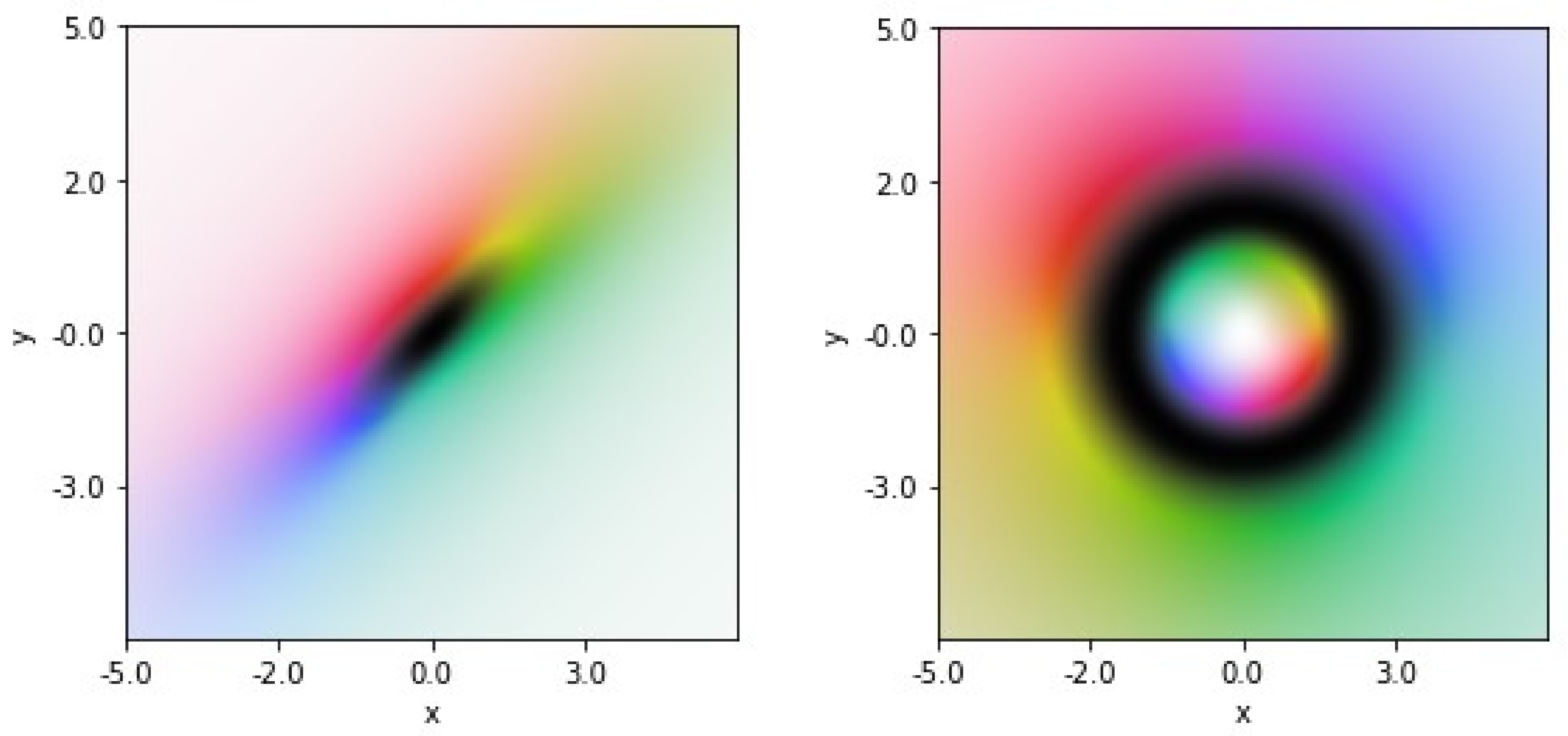

Figure 3. Elliptical distortion and 'bag'

Conclusion

For a certain range of parameters of the field theory model for magnetic skyrmions, there exist explicit minimizers of the energy for given degree. These solutions include the basic hedgehog skyrmion, as well as multiple anti-skyrmion configurations, 'bags' and elliptical distortions.

References

1. arxiv:1812.07268
2. Shi-Zeng Lin, Avadh Saxena, and Cristian D. Batista. Skyrmion fractionalization and merons in chiral magnets with easy-plane anisotropy. Phys. Rev. B, 91:224407, Jun 2015.
3. Bernd Schroers. Gauged sigma models and magnetic skyrmions, 2019; arxiv:1905.06285