Problem Set for SNIa Cosmology and SNANA

(May 24, 2019)

This problem set is intended for beginning astrophysics researchers in the sub-field of using type Ia supernova to measure properties of dark energy. The problems are based on experience from several cosmology analyses, and also from developing the SNANA software. Answers are provided, but there is no guarantee that answers are correct. Feedback is welcome on how to phrase the problems more clearly, and to propose new problems.

A few of the problems can be solved analytically. Most require a numerical code to solve an integral or minimize a function with a grid search. Software beyond SNANA is not needed.

The goal in solving these problems is to gain a more intuitive understanding of a few key analysis issues, to write better analysis software, and to make better use of public software tools .

For some of the problems we use the distance modulus (μ) as follows:

$$D_L = \frac{c}{H_0}(1+z_{\rm hel})\int_0^{z_{\rm cmb}} dz/E(z) \tag{1}$$

$$E(z) = \left[\Omega_{\rm DE}(1+z)^{3(1+w)} + \Omega_{\rm M}(1+z)^3\right]^{1/2} \tag{2}$$

$$\mu = 5\log_{10}(D_L/10\ {\rm pc}) \tag{3}$$

where $z_{\rm hel}$ is the heliocentric redshift, $z_{\rm cmb}$ is the CMB-frame redshift, $c = 2.998 \times 10^5$ km/s is the speed of light, $H_0 = 70$ km/s/Mpc is today's Hubble constant,[1] $\Omega_{\rm M}$ is today's matter density. $\Omega_{\rm DE}$ is today's dark-energy density, and w is the dark energy equation of state parameter (pressure-to-density ratio). A flat universe ($\Omega_{\rm k} = 0$) is also assumed here for $E(z)$.

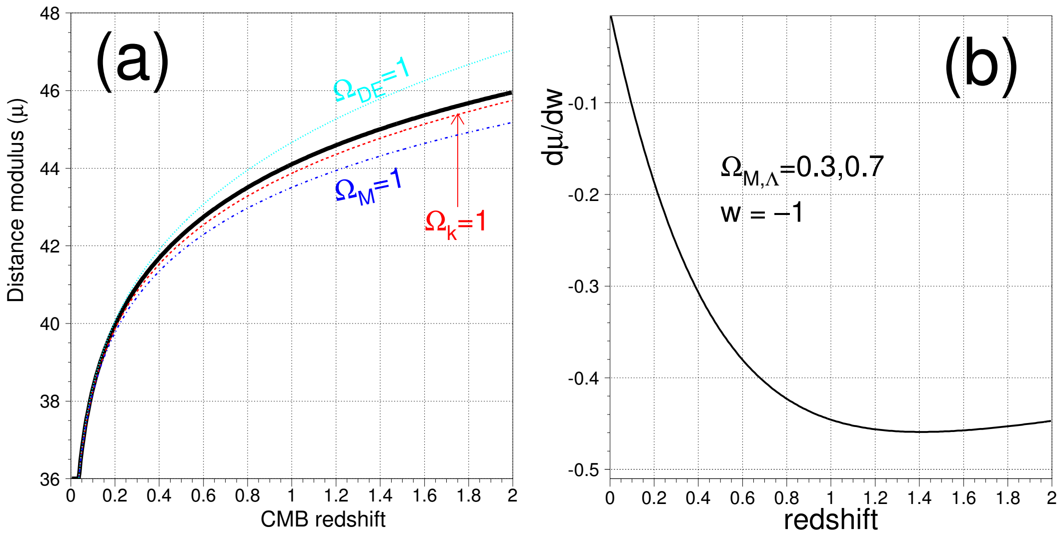

Figure 1: Panel (a) shows μ vs. CMB redshift (Eq. 3) for standard cosmological model (solid line, $\Omega_{\rm DE}, \Omega_{\rm M}, w = 0.7, 0.3, -1$), and for models with only a single component (dashed lines). Flatness assumed in all models. Panel (b) shows $d\mu/dw$ vs. CMB redshift for standard model.

[1] Here we take the average of local and CMB-based H_0 measurements and do not consider the $\sim 4\sigma$ discrepancy.

1. **Distance Modulus:**

   (a) Use Eq. 3 to numerically compute the standard cosmology curve (solid black line) in Fig. 1a. (use common approximation with $z_{\rm hel} \to z_{\rm cmb}$ in the "$1+z$" prefactor).

   (b) For the $z_{\rm hel} \to z_{\rm cmb}$ approximation, show that the maximum μ-error is 0.0025 mag at low-z.

   (c) Assuming flatness and a single component, show that

$$\Omega_{\rm DE} = 1 \quad \longrightarrow \quad D_L \;=\; (c/H_0)(1+z)z \tag{4}$$

$$\Omega_{\rm M} = 1 \quad \longrightarrow \quad D_L \;=\; (c/H_0)(1+z)2[1-(1+z)^{-1/2}] \tag{5}$$

$$\Omega_{\rm k} = 1 \quad \longrightarrow \quad D_L \;=\; (c/H_0)(1+z)\ln(1+z) \tag{6}$$

   and that at low redshift they are all consistent with the Hubble law. Reproduce the dashed curves in Fig. 1a. At high redshift, why is D_L largest for $\Omega_{\rm DE} = 1$ model?

   (d) Using Eq. 3, numerically compute $d\mu/dw$ vs. redshift as shown in Fig. 1b.

   (e) Suppose an experiment is constrained to measure w over a redshift range spanning no more than 0.5; show that maximum w-sensitivity is achieved for $0 < z < 0.5$.

   (f) Show that the max w-sensitivity is reduced in half by shifting the redshift range to $0.3 < z < 0.8$.

   (g) In Eq. 2 the radiation term, $\Omega_{\rm r}(1+z)^4$, is left out. Show that the μ-error from this approximation is $\sim 10^{-5}$ at low redshift, and increases to 6×10^{-5} at redshift $z = 2$.

   (h) In the Planck 2018 cosmology results,[2] the curvature term is $\Omega_{\rm k} = 0.001 \pm 0.002$, consistent with zero as assumed for $E(z)$ in Eq. 2. Allowing for a 2σ fluctuation, $\Omega_{\rm k} = 0.005$, show that the μ-shift reaches a maximum of 0.0022 mag at $z \simeq 1$, and then gradually decreases, reaching zero shift at $z \sim 3$. Note that for non-zero curvature, sinh or sin is needed in Eq, 2.

   (i) Using an empty universe approximation (and flatness) that has only curvature, $E(z) = (1+z)^{-1}$, show that a redshift uncertainty σ_z contributes a distance uncertainty of

$$\sigma_\mu^{\rm approx} = \frac{\sigma_z}{z} \cdot \frac{5}{\ln(10)} \cdot \frac{1+z}{1+z/2} \tag{7}$$

   (j) Using $\sigma_z = 0.001$ and standard cosmological parameters $(\Omega_{\rm DE}, \Omega_{\rm M}, w)$ shown in Fig. 1b, numerically compute $\sigma_\mu^{\rm exact} = \mu(z+0.001) - \mu(z)$ vs. redshift. Show that $\sigma_\mu^{\rm approx}$ is within 1% of $\sigma_\mu^{\rm exact}$ at low redshifts, is off by 8% at $z = 1$, and off by more than 20% at $z = 2$. Why is this approximation adequate for precision cosmology analyses? [3]

   (k) Use the `SNANA` simulation to plot μ vs. redshift from Eq. 3, and compare with your numerical computation. Use sim-input option "`SIMGEN_DUMP: 3 CID ZCMB DLMAG`"

   (l) The equation of state parameter (w) is -1 for DE, $+1/3$ for radiation, $-1/3$ for curvauture. For non-relativistic matter, show that $w = v^2/c^2$, and that the common approximation of $w = 0$ is valid over the relevant redshift range for SN Ia analyses.

[2] `https://arxiv.org/abs/1807.06209`

[3] Answer(1j): intrinsic scatter dominates distance uncertainty at higher redshift.

2. **Distance Bias:** Consider a simplified SN Ia model where the rest-frame magnitude (M) is wavelength-independent, and is randomly generated between $M_{\rm max} = -17.5$ and $M_{\rm min} = -19.5$ mag, with uniform probability per mag. $\overline{M} = -18.5$ is the average rest-frame mag. The observer frame mag is $m_{\rm obs} = M + \mu$, and K-corrections are ignored. Next, assume a magnitude detection limit of $m_{\rm lim} = 24$. Finally, the observed (measured) distance modulus is given by $\mu_{\rm obs} = m_{\rm obs} - \overline{M}$

   (a) Show that the distance bias, $\Delta\mu_{\rm bias} \equiv \mu_{\rm obs} - \mu_{\rm true}$, is

$$\Delta\mu_{\rm bias} = \left[\int_{m_0}^{m_1} dm' P(m')m'\right] / \left[\int_{m_0}^{m_1} dm' P(m')\right] - \overline{M} - \mu_{\rm true} = \left[(m_0 + m_1)/2 - \overline{M} - \mu_{\rm true}\right] \tag{8}$$

   where $m_0 = M_{\rm min} + \mu$, $m_1 = \min(M_{\rm max} + \mu_{\rm true}, m_{\rm lim})$, and $P(m') = 1$ if $M_{\rm min} < m' - \mu_{\rm true} < M_{\rm max}$, and zero otherwise.

   (b) Reproduce Fig. 2 using the `SNANA` simulation with

```
GENMODEL:     FIXMAG -19.5:-17.5
SIMGEN_DUMP:  4 CID ZCMB DLMAG PEAKMAG_g
```

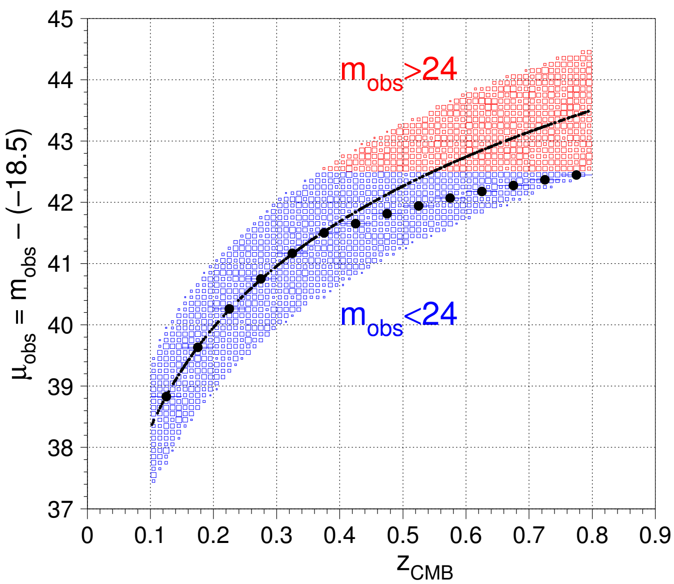

Figure 2: $m_{\rm obs}$ vs. CMB redshift. The black line is the unbiased $\mu_{\rm true}$ vs. redshift, while the black filled circles are from $\mu_{\rm obs} = \langle m_{\rm obs}\rangle - \overline{M}$.

(c) Check the simulation by computing $\Delta\mu_{\text{bias}}$ vs. redshift in Fig. 2.

(d) In a more realistic analysis using a standardized magnitude (e.g. $m_{\text{obs}} = m_B - \alpha x_1 + \beta c$ for SALT2) the line between rejected and accepted events is very blurry. Give three reasons for the blurring. [4]

(e) To estimate the real spread in SN Ia magnitudes, consider the SALT2 color and stretch parameters, and the high-z G10 row of Table 1 in SK16[5]. With $\beta = 3.2$ and $\alpha = 0.14$, show that 95% of the events span 0.9 mag in brightness due to color variation, and also span 0.5 mag due to stretch variation. Identify another source of variation in the observed magnitude. [6]

---

[4] Answer(2d): Poisson noise, color & stretch dependence, weather variations.

[5] `https://arxiv.org/abs/1603.01559`

[6] Answer(2e): galactic extinction.

3. **Wavelength Calibration of Filters:** Consider filter transmission $T_\lambda(\lambda) = 1$ for $L_0 < \lambda < L_1$, and zero otherwise. Ignore atmospheric components. The primary reference is the AB system with $F_{\rm AB}(\lambda) = F_{\rm AB}/\lambda^2$, and the source spectrum is $F(\lambda) = F_{\rm src}\lambda^n$, where n is an arbitrary index. Both $F_{\rm AB}(\lambda)$ and $F(\lambda)$ have physical units of erg/s/cm^2/Å.

   (a) For $n \neq 0$ show that the AB photon-count magnitude m is

   $$m = -2.5\log_{10}\left[\frac{F(0)}{F_{\rm AB}(0)} \cdot \frac{2}{n} \cdot \frac{L_1^n - L_0^n}{L_0^{-2} - L_1^{-2}}\right] \tag{9}$$

   (b) Consider a small uncertainty (σ_L) in the average filter wavelength, $\bar{L} = (L_0 + L_1)/2 \to \bar{L} + \sigma_L$, while the filter width $\Delta_L \equiv L_1 - L_0$ remains constant. Using Taylor expansions to first-order in σ_L and to 2nd-order in $\Delta_L/\bar{L}$, show that the mag-uncertainty is

   $$\sigma_m \simeq \frac{2.5}{\ln(10)} |n+2| \sigma_L/\bar{L} \tag{10}$$

   (c) Why does $\sigma_m = 0$ for $n = -2$? [7]

   (d) For $n = -1$ and $\sigma_L = 10$ Å, show that $\sigma_m = 3.6$ mmag for a UV filter with $\bar{L} = 3000$ Å, and drops to 1.2 mmag for a NIR filter with $\bar{L} = 9000$ Å.

   (e) For $n = +1$ and $\sigma_L = 10$ Å, show that $\sigma_m = 11$ mmag for the UV filter.

   (f) Why is there a large uncertainty difference between $n = \pm 1$? [8]

   (g) Next consider an uncertainty on the filter width, σ_Δ, keeping $\bar{L}$ constant. Use Taylor expansions to show that the mag-uncertainty is

   $$\sigma_m \simeq \frac{2.5}{\ln(10)} \cdot \frac{\sigma_\Delta \Delta_L}{4\bar{L}^2} \left| \frac{(n-1)(n-2)}{3} - 4 \right| \tag{11}$$

   (h) For $\sigma_\Delta = 10$ Å, $\bar{L} = 3000$ Å, $\Delta_L = 1000$ Å, and $n = -1$, show that $\sigma_m = 0.6$ mmag.

   (i) Why is the mag-uncertainty for σ_L much larger than for σ_Δ ? [9]

---

[7] Answer(3c): there is no uncertainty if primary reference and source SEDs have the same λ dependence.

[8] Answer(3f): difference w.r.t. primary ($n = -2$) drives the error, so $n = +1$ has larger error.

[9] Answer(3i): for Δ_L change, fraction flux change for any n is $\sim\sigma_\Delta/\Delta_L$, and thus no mag change. For $\bar{L}$ change, primary and source flux s

4. w **Bias:**

Consider an ideal redshift-binned Hubble diagram with ideal measurements in 0.1 bins: the redshift bins are $\{0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95\}$. Each measured distance modulus has the exact value from Eq. 3 ($\Omega_{\rm M}, \Omega_{\rm DE}, w = 0.3, 0.7, -1$), and an uncertainty of $\sigma_\mu = 0.005$ mag. Assume a Gaussian matter prior, $\Omega_{\rm M} = 0.300 \pm 0.005$.

(a) Write a χ^2 minimizer using a grid-search in the space of w and $\Omega_{\rm M}$, and impose a flatness constraint. Show that the minimized w is -1.000. The function to minimize is

$$\chi^2 = [\mu(z) - \mu(z, w, \Omega_{\rm M})]^2/\sigma_\mu^2 + (\Omega_{\rm M} - 0.3)^2/0.005^2 \qquad (12)$$

(b) Consider a μ-bias, $\Delta\mu_{\rm bias} = 0.01$ for $z > z_{\rm min}$. Try $z_{\rm min} = 0.1, 2, 0.3$ ... 0.9, and find the minimized w for each $z_{\rm min}$. Show that the max w-bias is $\Delta w_{\rm bias} = 0.036$ for $z_{\rm min} = 0.3$.

(c) Identify two sources of uncertainty that qualitatively contribute a μ-bias that is essentially zero at low-redshift, and starts increasing at a higher redshift. [10]

(d) Consider a μ bias linear with redshift, $\Delta\mu_{\rm bias} = 0.01 \times z$: show that $\Delta w_{\rm bias} = 0.02$.

(e) For each Hubble diagram above, construct an input file for the `SNANA` program "`wfit.exe`." and verify your results.

(f) Remove the explicit $\Delta\mu_{\rm bias}$, and instead add a redshift bias of 10^{-4} in all redshift bins. Show that $\Delta w_{\rm bias} = 0.011$.

(g) Remove the explicit redshift bias and instead apply μ-shifts from Eq. 7 with $\sigma_z = 10^{-4}$. Show that $\Delta w_{\rm bias} = 0.011$ is the same as from explicitly shifting the z values.

(h) Increase the redshift bias to 10^{-3} and show that Eq. 7 still works very well in predicting $\Delta w_{\rm bias}$. Why does this empty-universe approximation work so well ?

[10] Answer(4c): calibration, selection bias, peculiar velocity.

5. **Signal-to-noise (SNR):**

(a) During the SN search for the Dark Energy Survey (DES), artificial sources with magnitude $m = 20$ were overlaid on images and processed through the difference imaging pipeline. For a given exposure, the recovered flux is F_{20} (photoelectrons) and the SNR is SNR_{20}. For this exposure, show that the limiting 5σ magnitude, $m_{5\sigma}$, can be computed as

$$m_{5\sigma} = 20 + 2.5\log_{10}(F_{20}/F_5), \qquad \text{where} \tag{13}$$

$$F_5 = \frac{25}{2}\left[1 + \sqrt{1 + 4V_{\mathrm{sky}}/25}\right], \qquad \text{and} \quad V_{\mathrm{sky}} = (F_{20}/\mathrm{SNR}_{20})^2 - F_{20} \tag{14}$$

(b) For $F_{20} = 250,000$ and $\mathrm{SNR}_{20} = 250$, show that $m_{5\sigma} = 24.4$ mag.

(c) Validate Eq. 13 with the SNANA simulation using the following options:

```
GENMODEL:  fixmag 20
SMEARFLAG_FLUX:    1     # Poisson noise from source and sky
SMEARFLAG_ZEROPT:  0     # turn off zeropoint smearing.
```

From the sim data files, compute the average F_{20} and SNR_{20} at an arbitrary epoch for a few dozen events. Next, compute $m_{5\sigma}$. Finally, re-run the simulation with $20 \rightarrow m_{5\sigma}$ and check that SNR$= 5$. Why average the SNR over a few dozen events ?

(d) Consider a search image and template image, each with the same zero point and same sky variance (V_{sky}) within an aperture used to measure flux. Difference imaging results in a 5σ limiting magnitude, $m_{5\sigma}$. Next, replace the single template image with a co-added template image constructed from N images that each have the same depth and sky variance as the search image. Using the co-added template, show that the increase in difference-imaging depth is

$$\Delta m_{5\sigma} = -2.5\log_{10}\left[\frac{1 + \sqrt{1+(4/25)V_{\mathrm{sky}}(1+1/N))}}{1+\sqrt{1+(8/25)V_{\mathrm{sky}}}}\right] \tag{15}$$

$$\xrightarrow{V_{\mathrm{sky}}\gg 1} -2.5\log_{10}\left[\sqrt{(1+1/N)/2}\right] \xrightarrow{N\rightarrow\infty} -2.5\log_{10}\left[\sqrt{1/2}\right] \tag{16}$$

(e) Qualitatively explain the limit with $V_{\mathrm{sky}} \gg 1$ and $N \rightarrow \infty$. [11]

[11] Answer(5e): Going from $N = 1$ to $N = \infty$ templates, sky variance is reduced by 2, and thus noise is reduced by $\sqrt{2}$.

6. **Construct Approximate Hubble Diagram with Simulated LSST Sample:**
This exercise is based on simulated LSST data from the "Photometric LSST Astronomical Time Series Classification Challenge," known as `PLAsTiCC`. Start by downloading the *training* data from the Kaggle platform,[12] in which all events have a known type. There is no need to download the large *test* set. There are two files to download: `training_set_metadata.csv` with one row of information per event, and `training_set.csv` with the light curves. Extract light curves from the Deep Drilling Fields (ddf=1) that are SNIa (target=90); there are 837 events. The CMB-frame redshift ($z_{\rm cmb}$) is the "`hostgal_specz`" column. Model names for all `target` types can be found in the post-challenge unblinded data release.[13]

   (a) For each event, find the observation with the maximum r-band flux. Define $T_{r,\max}$ to be the MJD of this observation, and $F_{r,\max}$ to be the max flux.

   (b) For each event, compute an approximate distance modulus,
$$\mu_{r,\max} = Z_{\rm cal} - 2.5\log_{10}(F_{r,\max}) - M_0 + 2.5\log_{10}(1+z_{\rm cmb}) \ , \tag{17}$$
where $Z_{\rm cal} = 27.5$ is an arbitrary zeropoint to which the fluxes are calibrated, and $M_0 = -19$ is the rest-frame magnitude.

   (c) The "$-2.5\log_{10}(1+z_{\rm cmb})$" term in Eq. 17 is part of a K-correction,[14] where the flux-integrals have been ignored for simplicity. What is the physical interpretation of this term? What is the interpretation of the ignored K-correction terms?

   (d) Plot $\mu_{r,\max}$ vs. $z_{\rm cmb}$, and overlay the theory curve as shown in Fig. 3a.

   (e) To reduce catastrophic outliers (upper left corner), apply sampling cuts: at least one measurement between 5 and 20 days prior to $T_{r,\max}$, and at least one measurement between 10 and 40 days after $T_{r,\max}$. Plot $\mu_{r,\max}$ vs. $z_{\rm cmb}$ with sampling cuts as shown in Fig. 3b. Why do these sampling cuts reduce catastrophic outliers?

   (f) Even after sampling cuts, catastrophic outliers remain. Visually examine a few outlier light curves and explain why they remain.[15]

   (g) In a real SNIa-cosmology analysis, how are these catastrophic outliers removed? [16]

   (h) Plot $\mu_{r,\max}$ vs. $z_{\rm cmb}$ without the K-correction term, as shown in Fig. 3c. At higher redshift, why are the $\mu_{r,\max}$ below the theory curve (instead of above it)?

   (i) Using the sample cuts from Fig. 3b, plot a 1-dimensional Hubble residual histogram of $\mu_{r,\max} - \mu_{\rm true}$. Fit a Gaussian to the core (i.e., ignore the outliers) and show that the Hubble scatter is 0.27 mag.

   (j) List three analysis techniques to reduce the Hubble scatter and bias.[17]

[12] `https://www.kaggle.com/c/PLAsTiCC-2018`
[13] see note2 in `https://zenodo.org/record/2539456`
[14] Full K-correction formula can be found in Eq. 4 of `https://arxiv.org/abs/astro-ph/0205351`
[15] Answer(6f): outliers from explosion between seasons, and max flux is an upward fluctuation on the light curve tail.
[16] Answer(6g): Light curve fit, fit-χ^2 cut, and sampling cuts.
[17] Answer(6j): 1) color and stretch corrections, 2) proper K-correction, 3) bias-correction.

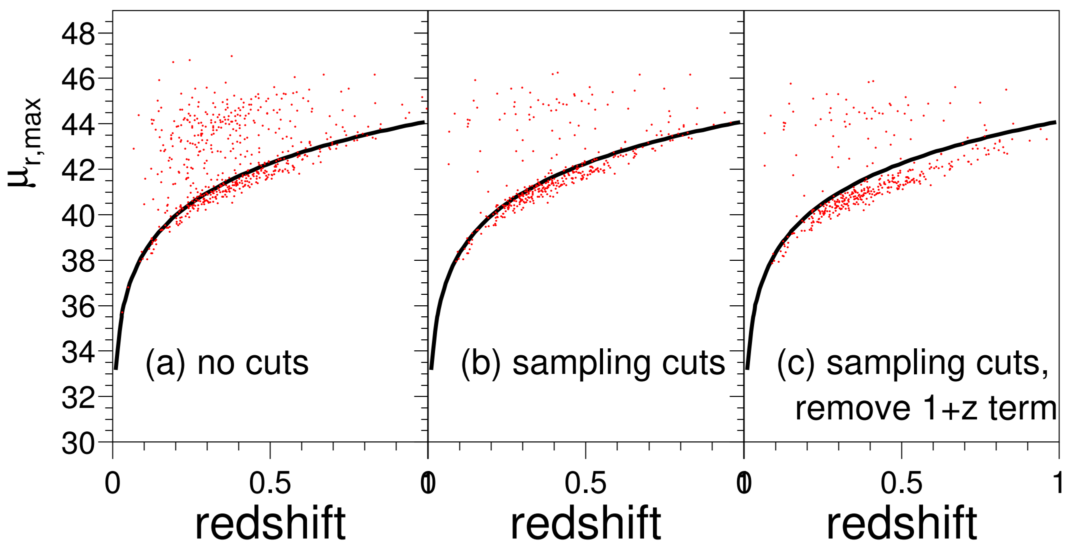

Figure 3: $\mu_{r,\max}$ vs. CMB redshift from `PLAsTiCC`, using tagged SNIa from ddf (red points). Black solid curve is ΛCDM model with $\Omega_{\rm M}, \Omega_{\Lambda}, w = 0.3, 0.7, -1$.