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Solving crop planning and rotation problems in a sustainable agriculture perspective

Mario Benini [a], Emanuele Blasi [b], Paolo Detti [a,*], Lorenzo Fosci [b]

[a] Department of Information Engineering and Mathematics, University of Siena, Via Roma, 56, 53100 Siena, Italy
[b] Department for Innovation in Biological, Agro-food and Forest Systems, University of Tuscia, Italy

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ABSTRACT

In this paper, the problem of planning the allocation of crops to arable lands, taking into account crop rotations principles and diversification strategies promoted by sustainable agriculture is addressed. Optimization models for solving multi-period planning problems are proposed, able to decide how to allocate crops in each growing season in order to maximize the total expected profit. The allocation decisions are made considering the crop rotation benefits across seasons and the sustainable requirements stated by current regulations. A complexity analysis is performed, and polynomial special cases are presented. Integer Linear Programming models are proposed, for a case study related to structured professional Italian farms specialized in arable crops, following sustainability rules coming from public regulations and private initiatives, i.e., the Common Agricultural Policy by the European Union and "La Carta del Mulino" by the Barilla Group. Numerical experiments conducted on real data show the effectiveness of the proposed solution approaches.

1. Introduction

Climate change patterns and the uncertainty of global value chain commodities availability due to socio-economics aspects call for a viable development of resilient agricultural processes to ensure sustainability at various levels: economic (including food security), environmental (related to resource efficiency, soil and water quality and threats to habitats and biodiversity) and social (territorial and community level).

Sustainable agriculture aims to address the above challenges highlighting the benefits of crop planning decisions that include the rotation of crops across growing seasons (Boyabatli et al., 2019) and crop diversification strategies. As highlighted by Hennessy (2006), crop rotation increases crop revenues, improves soil structure and decreases farming costs owing to reduced need for fertilizers and pesticides.

Many countries and organizations promoted sustainable agriculture by ad-hoc policies followed by specific regulations. In particular, crop rotation was included as a new commitment into Good Agricultural Environmental Conditions - GAEC list published in the last Common Agricultural Policy (CAP) proposal of the European Union (EU), and several times it has been included as a tool of sustainability voluntary production schemes used by farmers, food producers and retailers (Grunert et al., 2014). As a result, attention is increasing for decision support tools that can be used to assess the impact of common policies and private initiatives from one side, and to help farmers to maximize their revenues, while respecting sustainability principles (de Frahan et al., 2016; Britz et al., 2012).

In this paper, the problem of planning the allocation of crops to arable lands is addressed, taking into account crop rotation principles and diversification strategies promoted by sustainable agriculture, and propose optimization models for solving multi-period planning problems. From one side, the models can be embedded in decision support tools that can help farmers to maximize their revenues in compliance with ecological transition pathways indications. From the other side, they can be used by decision makers to assess the efficacy of current regulations and to design future rules to promote sustainability principles in arable land use, as well as to evaluate farmers production factors allocation choices.

The optimization models proposed in this paper allow to decide how to allocate the available farmland among different crops in each growing season to maximize the total expected profit over a finite planning horizon. The allocation decisions are made considering the crop rotation benefits and the sustainable requirements stated by regulations. The proposed models, differently from other works in the literature (e.g., see Alfandari et al. (2015), Haneveld and Stegeman (2005)), allow crop successions that do not follow the *best crop rotation practices* (as defined into detail in Section 3 (see Definition 3.1)), by taking into account and quantifying the agronomic costs and constraints deriving from the adoption of the same crop species/type on the same plot.

* Corresponding author.
E-mail addresses: mbenini@diism.unisi.it (M. Benini), e.blasi@unitus.it (E. Blasi), paolo.detti@unisi.it (P. Detti), lfosci@unitus.it (L. Fosci).

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The models' outputs will provide information that could be useful to assess, via farm level quantitative economics indicators, the viability of crop rotation schemes taking into account the potential economic impact that could affect agriculture value chain actors in different pedo-climatic areas.

Numerical experiments have been conducted with real data coming from structured professional farms specialized in arable crops, located in one of the most intensive agricultural areas of production in Europe, the plain area around the Po river basin, called "Pianura Padana" Valley in northern Italy (Di Bene et al., 2022). In this pedoclimatic context, farmers plan the use of their arable land by distinguishing winter crops from summer crops. The types of crops that can be selected change along farms and across seasons according to structural features (irrigation, machines and work availability), costs and benefits prediction (potential yields, product prices, input availability, premium prices and subsidies) and specific pedo-climatic constraints (temperatures, pluviometry trends, type of soil).

The main contributions of the paper are listed below:

- A formal characterization of the crop planning and rotation problem is given, where the rotation schemes are based on sequences of k consecutive crops. The risks/benefits of all possible k-crop rotation schemes are assessed, including those not following the traditional agronomic practices.
- A complexity analysis is performed, showing that the problem is strongly NP-hard when $k \geq 3$.
- Polynomial network flow approaches for special cases are proposed.
- A real world application is presented, based on data from Italian farms and current sustainability rules coming from public (i.e., PAC) and private initiatives (promoted by the Barilla Group). So, the models are settled up for taking into account constraints and incomes coming from the adoption of different production schemes.
- Integer Linear Programming (ILP) models are developed for the case under study (Italian farms of the Pianura Padana Valley) where k is set to 3, as a consequence of best practices in Mediterranean pedo-climatic contexts.
- An experimental campaign on real data shows that the models are able to optimally solve the planning problems under different sustainability scenarios for all the real-life instances, in reasonable computational times.

The paper is organized as follows. In Section 2, a literature review on crop planning and on solution approaches is presented. In Section 3, crop rotation issues and other sustainable requirements are described into detail. A formal definition of the problem and a complexity analysis is reported in Section 4. In Section 5, a polynomial network flow approach for solving a special class of crop planning problems is proposed. In Section 6, the ILP models related to the case under study are presented. Sections 7 and 8 are devoted to describe the real data and the results of the experimental analysis, respectively. Conclusions follow in Section 9.

2. Literature review

In the literature, crop planning and rotation problems have received considerable attention from the operations management and agricultural economics communities. We refer the reader to Ahumada and Villalobos (2009), Dury et al. (2012) for reviews on the topic.

In particular, Mathematical Programming approaches have been proposed by different authors for solving crop planning and rotation problems. Haneveld and Stegeman (2005) proposed linear programming models for farm production planning with crop rotation constraints, where crop succession information is only given in the form of a set of inadmissible successions of crops. The decision variables represent the areas where a certain admissible sequence of crops is cultivated.

Several papers attempted to solve the crop rotation problem in different pedoclimatic contexts by using column generation approaches. Alfandari et al. in Alfandari et al. (2011) presented a crop planning problem arising from the Madagascan context, where the minimization of cultivated space contributed to the sustainable development of the primary forest in the long term. An ILP model is proposed for the problem, in which a plot can be cultivated with several crops in the same period, with the objective of minimizing the total cultivated land. The same authors, in Alfandari et al. (2015), motivated by the Madagascan case, presented a fully-combinatorial crop planning problem where a single crop can be cultivated on each plot at each period. An ILP formulation and a graph formulation are provided, and a branch and price and cut approach is adopted to solve the problem of minimizing the cultivated land necessary to satisfy demand constraints. Given the different pedo-climatic context, the constraints of the problem are quite different from the ones considered in this paper. Also crop rotation is handled differently and, in general, only some rotation schemes are allowed in their problem. In Santos et al. (2015), an ILP model is presented for the problem of finding the minimum land necessary to satisfy crop demand. Crop rotation is only considered by imposing a green manuring crop and a fallow period in the crop schedules. Binary variables and assignment constraints are used to model the crop schedules on single plots of land, and integer variables to minimize the total number of base-area plots assigned to the different schedules are adopted. A Branch and Price and Cut approach is presented, able to solve instances up to 20 crops and a 2 years planning horizon.

Santos et al. (2010) also presented a binary linear programming model for the Crop Rotation Scheduling Problem (CRSP) aiming to maximize the plots occupation considering demand constraints. The authors proposed a column generation procedure to solve the model. A similar procedure was also employed in Santos et al. (2011) to solve a variant of CRSP without demand constraints. In Santos et al. (2011), the authors were the first to introduce adjacency constraints for the CRSP, preventing crops of the same botanical family to be cultivated at the same time in neighboring plots. In Munari (2019), a crop planning problem in the Brazilian area is addressed, with the objective of maximizing plots' occupation and profit while dealing with adjacency constraints. The authors propose improvements in the mathematical model presented by Santos et al. (2011) and a detailed set of instances based on real-world data. In order to find bounds and solutions for the improved model, five different relaxation approaches are proposed. The results show improvements from the model proposed by Santos et al. (2011).

In Fikry and Eltawil (2019), ILP models are presented to assign crops to plots to periods with the objective of maximizing the total profit. The crop rotation is performed by simply forbidding that two crops of the same family are consecutively planted. However, in the models, the profits earned by each crop are considered as *static*, i.e., do not depend by the succession of the previous crops grown on the same piece of land as in our model. Boyabatli (Boyabatli et al., 2019) et al. proposed a stochastic dynamic program model for crop planning in sustainable agriculture, taking into account crop rotation issues. However, only succession between two crops (i.e., corn and soybeans) are considered in the study. Detlefsen and Jensen (Detlefsen and Jensen, 2007) considered the problem of finding an optimal crop rotation for a given selection of crops on a given piece of land. The problem is modeled and solved as a minimum cost network flow problem for the case in which sequences of at most three crops are considered. Bachinger and Zander (Bachinger and Zander, 2007) presented a decision support tool called ROTOR, a static rule-based model in which a set of annual crop production activities are assembled semi-automatically. However, ROTOR does not include the crop rotation benefits in the decision process, but it allow to only perform a *what if* static analysis.

Works from the literature also focus on different planning and coordination problems in agriculture. Filippi et al. (2017) addressed the problem of crop-mix selection with the objective of maximizing the farmer's profit. In particular, they focus on the working phases (and their costs) required by each crop, rather than on crop rotation issues, and develop two integer programming models: the first one to solve the problem of maximizing the farmer's profit, setting costs and prices values as the historical average; the second one where consider the risk aversion of the farmer using the maximization of the Conditional Value-at-Risk as objective function and looking for the crop-mix that allows to maximize the average expected profit under a predefined quantile of worst realizations. The models are applied to the real case of a single farm located in Italy.

Volte et al. (2023) considered the Differential harvesting problem, consisting in optimizing the harvest of different grape qualities in a vineyard so that a minimum quantity of good quality grapes is harvested and directed to a specific bin, while minimizing the total harvesting time. The authors exploit the analogies with the capacitated vehicle routing problem to develop efficient exact methods using column generation and VRPSolver™ based models.

In Yan et al. (2020), a method to coordinate a fresh agricultural product supply chain with the consideration of strategic consumer behavior is proposed. In particular, the authors developed a decision-making model established on the basis of a two-period newsvendor model under the centralized and decentralized chain.

Machine learning techniques have also been used in the literature to deal with problems in the agricultural supply chain. The reader may refer to Sharma et al. (2020) for a systematic review of such approaches in the agricultural context.

3. Crop planning issues and sustainability requirements in the European agricultural context

In this section, the concept of crop rotation and the main issues connected to it are presented. Crop rotation is one of the most important aspects in agriculture and it is well studied in the literature since it prevents problems caused by monoculture practices (see Butkevicien et al. (2021), van der Ploeg et al. (2019), Wezel et al. (2014)). In the section, different variants of the crop planning problem are also described, accounting for different sustainability scenarios. More precisely, real constraints derived from agricultural policies and agri-food value chain initiatives have been analyzed. The sustainability scenarios are defined on the basis of the current CAP regulations and the rules of private initiatives promoted by food producers. As it will be shown in Section 8, optimal solutions of the different scenarios can be used to evaluate the impact of different sustainability policies on the farmers' revenues.

Crop planning is the problem of deciding the crops to cultivate on a farmland for a given planning horizon. When dealing with this problem in sustainable agriculture, decision makers, e.g., farmers, have to take into account two main aspects: (*i*) the crop rotation principles, and (*ii*) the constraints deriving from legislative frameworks and policies, especially promoted by international agencies or governments (e.g., CAP for European Union countries) to mitigate natural resources overexploitation.

In agriculture, crop rotation refers to the succession of different crops on the same piece of land over consecutive seeding periods. In fact, monoculture schemes, in which the same crop is assigned to the entire farmland over the whole planning horizon, lead to a series of issues (such as loss of yield and soil fertility, increase of weed and pest diseases) that can be mitigated by applying suitable rotation of crops. The problems caused by intensive agriculture and monoculture practices have been the subject of numerous research over the years, both technical-agronomic and socio-economic (Butkevicien et al., 2021; van der Ploeg et al., 2019). To overcome these problems, movements of scholars, technicians and civil society proposed more sustainable arable land use patterns, inspired to agroecology principles (Wezel et al., 2014). The principles of the International Federation of Organic Agriculture Movements (IFOAM) were used worldwide to define specific regulatory framework for organic agriculture. In Europe, since the publication of the first Regulation on organic farming in 1992, the importance of crop rotation has been translated into rotation schemes that can be adopted by farmers. Indeed, it was highlighted the importance of the inclusion of leguminous plants or crops whose management had improving effects on the structure and, more generally, on the natural fertility of the soil (Baldoni and Giardini, 2002; Yigezu et al., 2019). For instance, in Italy, the viable crop rotation schemes for organic agriculture are defined by ministerial decree (Anon, 2022). The decree states that the same crop species can come back on the same land after two cycles of different crop species, that must include a nitrogen-fixing crop. As a result of the above cited literature, the best *crop rotation practice* in the Mediterranean pedo-climatic context, which is the case considered in this paper, consists in sequences of *three* crops belonging to specific *types*, in such a way that no loss of yield and soil fertility occurs. Typically, for rotation planning purposes, crops are classified into three types: *renewal* crops, *impoverishing* crops and *improver* crops. The most classical best crop rotation practice in the Mediterranean context is given by the succession of the crop types renewal, impoverishing and improver, in this order (Baldoni and Giardini, 2002). However, other rotation practices exist that are considered to be best practices, as long as they do not produce a loss in yield and soil fertility, as rigorously stated in the following.

Definition 3.1. In the Mediterranean pedo-climatic context, each succession of three consecutive crop types that does not lead to a loss of yield and soil fertility is called *best crop rotation practice*.

The characteristics of renewal, impoverishing and improver crops are described below:

- Renewal crops: these crops (e.g., corn, sugar beet, potato, tomato, sunflower, etc.) require particular care consisting in excellent soil preparation and balanced organic fertilizations, which has a positive effect on the structure of the soil. However, in some specific contexts, also a fallow period could be considered as renewal soil quality practice.
- Impoverishing crops: they exploit the nutritional elements present in the soil and deplete it. Crops in this class are wheat, oats, barley, rye, rice, corn, sorghum and generally all grain cereals.
- Improver crops: they increase the fertility of the soil, enriching it with nutrients. Improver crops mainly are legumes, such as alfalfa or clover, which are able to fix atmospheric nitrogen.

As explained above, proper crop rotation schemes may bring many advantages to the farm, both of an agronomic and economic-managerial perspective (Bonciarelli et al., 2016). On the other hand, when crop rotation does not follow the best agronomic practice, in order to keep their yield stable over time, farmers will have to compensate for the loss of soil fertility and the increase in weeds and pests' risks by employing technical inputs, such as herbicide fertilizers and pesticides. The use of technical inputs leads to an increase in production costs, directly related on the specific crop succession performed in the rotation scheme.

Summarizing, in general, all crop successions are viable, but specific (additional) production costs may arise when crop patterns do not follow best crop rotation practices (Baldoni and Giardini, 2002). Furthermore, best practices in the Mediterranean pedo-climatic context generally assume that the production cost and the profit derived by cultivating a crop on a plot is linked to the *two* previous crops grown on the same plot, and distinguish the best rotation schemes from the others, e.g., the three crop rotation scheme renewal–impoverishing–improver. As an example, Fig. 1 reports 3-crop rotations following and not following best practices.

Another issue strongly related to the crop rotation that must be taken into account in crop planning is the *maximum crop replanting*, namely the number of times a given crop family can occur itself in a crop rotation scheme in a certain period. In fact, although a cost

Best practice rotation		
Year 1	Year 2	Year 3
Corn	Wheat	Soy
Renewal	Impoverishing	Improver

Rotation not following best practice		
Year 1	Year 2	Year 3
Corn	Wheat	Corn
Renewal	Impoverishing	Renewal

Fig. 1. 3-crop rotations following and not following best practices.

increase occurs when crop rotation does not follow the best rotation practice, crops cannot indefinitely be replanted on the same land, if their succession is not interrupted by other crops, also when this solution is technically viable.

In this paper, we address the problem of assigning crops to the arable land of a farm over a given time horizon, taking into account the crop rotation issues described above, with the objective of maximizing the total profit. In the problem, we also consider the impact of regulations arising from public sustainable policies (e.g., CAP regulation in EU) and private supply chain initiatives, that already affect European farmers' choices on arable land allocation through economic incentives. At this aim, three different production scenarios are introduced, based on different greening and sustainable constraints and different incentive levels, leading to different farmers entrepreneurship approaches. The three scenarios are denoted as "Pure farmer", "CAP farmer" and "CAP+SVC farmer" and are described in what follows.

3.1. Pure farmer

The farmer is not involved in any production scheme and does not follow any sustainable regulation. Therefore, her/his entrepreneurial choices are dictated by the objective of maximizing income and maintaining land capital. In this case, crop planning decisions are only taken on the basis of agronomic knowledge, technical features, and machinery.

3.2. CAP farmer

The farmer is involved in the greening production schemes of the Common Agricultural Policy (CAP), promoted by the European Union following the EU regulation 1307/2013. In this scenario, farmers receive payments based on a set of standard requirements defined at EU level for obtaining different kinds of incomes which, summed up, generate the CAP economic incentive. The incentive is mainly based on environmental and climate issues and aims to promote practices which are good for the environment (soil and biodiversity in particular) and for climate. More precisely, CAP regulation establishes that:

- *Crop diversification* - Farms with more than 10 ha of arable land have to grow at least two crops in each seeding period, while at least three crops are required on farms with more than 30 ha of arable land. Furthermore, a crop cannot cover more than 75% of the arable land, and the land assigned to two crops must not exceed 95% of the arable land.
- *Ecological Focus Area* - Farmers with arable land exceeding 15 ha must ensure that at least 5% of their land is an *Ecological Focus Area* (EFA). Ecological focus areas may include different kind of features linked to landscapes, grasslands or improving biodiversity crops. For the farms in the case study, located in an area characterized by the production specialization of high-income arable crops, we have chosen to introduce EFAs devoted to nitrogen fixing crops.

The above constraints forbid the adoption of certain (possibly) profitable crop plans, and the CAP economic incentives aim to compensate the possible profit losses of the farmers (see the experimental results reported in Section 8).

3.3. CAP+SVC farmer

In this scenario, an income related to a Sustainable Value Chain initiative (SVC) is added to the CAP incentive. More precisely, in the case study, CAP+SVC farmers are also involved in an initiative called *Carta del Mulino* (CdM), introduced by the international Group "Barilla" to promote crop diversification and biodiversity and to support the efforts of farmers towards sustainability through economic incentives (Anon, 2022b). In fact, the CdM incentive is an additional premium price paid for wheat, denoted as CdM crop in the following, cultivated in compliance with rules involving strong agroecological principles.

The CdM constraints related to the arable land crops allocation are of three types: (i) greening constraints, (ii) diversification constraints, (iii) repetition constraints. The greening constraints impose that, when a CdM crop is sown on a plot, then at least one legume or oilseed must be cultivated in the next succeeding four crops on the same plot. The diversification constraints state that if a CdM crop is assigned to a plot, then, at least two different crops exist in the next four succeeding crops on the same plot. The CdM regulation define a set of crops, denoted as CR, subject to repetition constraints. They state that, if a CdM crop v is assigned to a plot, then at most one repetition of crops in CR can be performed in the crop sequence containing v and the next four succeeding crops. (Namely, three crops in CR cannot be consecutively assigned in the sequence of five crops that starts from v.) Finally, to get the CdM incentives, at least 3% of the agricultural area devoted to wheat must be dedicated to flower strips.

Similarly to the CAP farmer case, the CdM economical incentives aim to compensate the possible loss that farmers may have, not being allowed to adopt certain profitable, but not sustainable, crop rotation plans.

4. Notation, problem definition and complexity

In this section, we formally define the problem of planning crop production on a farmland taking into account the crop rotation requirements reported in Section 3 and the constraints related to environmental sustainability arising from CAP regulation and the CdM initiative, reported in Sections 3.2 and 3.3, respectively.

In what follows we formally define the problem by generally considering k-crop sequences for any $k \geq 1$. Namely, due to the rotation issues introduced in Section 3, we assume that the profit earned by cultivating a crop is *affected* by the $k-1$ crops preceding it on the same piece of land. As highlighted in Section 3, we focus on the case of $k = 3$ characterizing the Mediterranean pedo-climatic context, from which the real case arise.

Let a farmland be defined as a set of *elementary* land plots $H = \{1, \dots, m\}$ to be cultivated. Each plot h has a size s_h and cannot be decomposed into sub-plots of smaller size. Furthermore, plots may have different economic values for the same crops, in terms of crop earnings, yields and costs depending on soil fertility, irrigation, lay of the land, etc. Two plots are considered *homogeneous* if they have the same size and economic values for the same crops.

Let $C = \{c_1, c_2, \dots, c_n\}$ be the set of crops available to be sown on farmland's plots during a planning horizon $T = \{1, \dots, p\}$ composed of p seeding periods. A plot can be assigned to at most one crop in each seeding period.

As we are mainly considering the European context, each agricultural year is divided into a “first” fall semester (starting in October, devoted to winter crops) and a “second” spring semester (starting in April, devoted to summer crops). Odd (even) periods in T denote the first (the second) semester of a year. We denote by T_o the set of odd periods and by T_e the set of even periods in T, with $T = T_e \cup T_o$. Note that, the last period p in T is an even period. The crops in C are classified according to their seeding periods and time requirements as *annual*, *first semester* and *second semester* crops. Let C_A, C_F and C_S be the disjoint sets of annual, first semester and second semester crops, respectively, with $C = C_A \cup C_F \cup C_S$. These three sets are formally defined in the following. The crops in C_A can only be sown in first semester periods $t \in T_o$, and need to stay on the land also in the related second semester $t + 1 \in T_e$. Moreover, each annual crop $c \in C_A$ has a *duration* d_c ($2dc$) indicating the number of years (periods) it needs to stay on the land. The crops in C_F can only be sown at the beginning of a first semester period $t \in T_o$ and the harvest is gathered at the end of the semester. Finally, the crops in C_S can only be sown in a second semester period $t \in T_e$ and only on plots where a crop in C_F was cultivated in the first semester $t - 1 \in T_o$. Given a crop $c \in C$ and a period $t \in T$, we say that c and t are *compatible* if t is a possible seeding period for c. We assume that a crop in C must be always assigned to each odd period $t \in T_o$ on each plot (fallow periods can be simply modeled by introducing a *dummy fallow crop* in C, belonging to a type depending on specific cases). On the other hand, we assume that an even period t must not be necessarily assigned to a second semester crop, when $t-1 \in T_o$ is assigned to a first semester crop. (Note that, when $t-1 \in T_o$ is assigned to an annual crop, t is not available for a second semester crop.)

As reported in Section 3, crops are also classified according to their types, i.e., *impoverishing*, *renewal* and *improver*. We denote the disjoint sets of impoverishing, renewal and improver crops as C_D, C_R and C_M, respectively, with $C = C_D \cup C_R \cup C_M$. The type is related to the effect produced by a crop on the soil, and affects the costs, profits and yields of crop successions. In fact, as stated in Section 3, the profit obtained by cultivating a crop $c \in C$ on one plot depends on the *crop rotation scheme*, i.e., on the types of the crops immediately preceding c, on that plot. In this work, we consider a k-crop rotation scheme, in which the profit obtained by assigning a crop c to a plot h depends on c and on the types of the $k-1$ crops preceding c on h, if $k-1$ preceding crops exist. (If the number of crops preceding c is smaller than $k-1$, then the profit depends on the types of all the crops cultivated before c). Furthermore, according to the *maximum crop replanting* issue (see Section 3), limits exist on the number of consecutive repetitions of some crop on the same piece of land. More precisely, crops are divided into *homogeneous replanting groups*, and those in the same group cannot be consecutively repeated on the same plot a number of times bigger than the *maximum replanting value* of the group.

In what follows, we formally define a crop assignment for our problem.

Definition 4.1. A *crop assignment* consists in:

- assigning, on each plot, an annual or a first semester crop to each odd period in T_o;
- deciding whether to assign or not, to each plot, a second semester crop to each empty even period t, i.e., the periods $t \in T_e$ such that $t-1$ is assigned to a first semester crop.

Note that a crop assignment defines a *crop sequence* $\sigma = \langle c_1, c_2, \dots, c_q \rangle$ on each plot, whose length (i.e., the number of crops in the sequence) depends on how many even periods are used for second semester crops. In fact, it is easy to see that the *longest crop sequence* assigns one crop to each period in T, and, hence, alternates first and second semester crops. On the other hand, the *shortest crop sequence* does not assign second semester crops (i.e., contains $p/2$ annual or first semester crops, where p is the last period of the time horizon T).

Let MAX_c^t and MIN_c^t be the maximum and minimum land surface that can be assigned to crop c in each period in $t \in T$, for all $c \in C$. Recalling the requirements on the maximum replanting, a *feasible crop assignment* can be formally defined as follows.

Definition 4.2. A crop assignment is *feasible* if:

- on each plot, the related crop sequence σ does not contain a subsequence in which crops of the same homogeneous replanting group are consecutively repeated a number of times bigger than the maximum replanting value;
- the total land surface assigned to a crop c in a compatible period t belongs to the interval $[MIN_c^t, MAX_c^t]$, for all $c \in C$ and $t \in T$;
- each annual crop $c \in C_A$ sown on a plot in $t \in T_o$ stays on that plot for at least $2d_c$ periods from t (its required duration).

We now formally define the profit of a feasible crop assignment. Let $h \in H$ be a plot and $\sigma = \langle c_1, c_2, \dots, c_{q-1}, c_q \rangle$ be the related crop sequence. As stated in Definition 4.1, σ assigns a first or annual crop in each odd period and, possibly, second semester crops to even periods. Hence, recalling that p is the last even period in T, c_q is assigned to $p-1$ or to p, depending on whether c_q is a first semester/annual crop or a second semester crop, respectively.

For each crop $c \in \sigma$, let $\sigma_k(c)$ be the subsequence of σ containing crop c and the $k-1$ crops immediately preceding c in σ, if any, and let $t(c)$ be the period in which c is sown. If less than $k-1$ crops precede c in σ, then $\sigma_k(c)$ will contain c and all the preceding crops. We denote by $p_h(\sigma_k(c), t(c))$ the profit earned by growing crop c on plot h in period $t(c)$ when the at most $k-1$ crops immediately preceding c on h are those given in $\sigma_k(c)$. The total profit $\pi_h(\sigma)$ earned by cultivating the crop sequence σ on plot h is then given by

$$\pi_h(\sigma) = \sum_{c \in \sigma} p_h(\sigma_k(c), t(c)). \tag{1}$$

As an example, assume that $k = 3$, $p = 8$ and consider a crop assignment corresponding to the sequence $\sigma = \langle c_1, c_2, c_3, c_4 \rangle$, where c_1, c_2, c_3, c_4 are annual crops. Then, as reported above, crops c_1, c_2, c_3, c_4 are assigned to periods $1, 3, 5, 7$, respectively. The total profit (1) produced by the sequence $\sigma = \langle c_1, c_2, c_3, c_4 \rangle$ on a plot h is given by

$$\pi_h(\sigma) = p_h(\langle c_1 \rangle, 1) + p_h(\langle c_1, c_2 \rangle, 3) + p_h(\langle c_1, c_2, c_3 \rangle, 5) + p_h(\langle c_2, c_3, c_4 \rangle, 7).$$

The k-sequence Crop Rotation Problem (CRP-k) can be now formally defined.

k Crop Rotation Problem (CRP-k):

Given a set C of crops, a set H of plots, a set T of p periods, the profits $\pi_h(\sigma)$ for all $c \in C$, $h \in H$ and crop sequences σ, a minimum MIN_c^t and a maximum MAX_c^t surface to assign to each crop $c \in C$ in each period $t \in T$, the maximum replanting values of the homogeneous groups and the duration of the annual crops;

Find a feasible crop assignment, resulting in a crop sequence σ on each plot maximizing the total profit $\sum_{h \in H} \pi_h(\sigma)$.

Note that, by definition, CRP-k corresponds to the crop planning problem in the “Pure Farmer” scenario introduced in Section 3.1, i.e., sustainability requirements are not considered.

As an example, let us consider a CRP-k instance with $k = 3$, three crops, $C = \{c_1, c_2, c_3\}$, ten periods, $T = \{1, 2, \dots, 10\}$, corresponding to five years, and one plot $H = \{1\}$ of total surface s_1. Let us assume that c_1 and c_2 are annual crops with minimum duration of 1 year and c_3 a first semester crop. Let $MIN_c^t = 0$ and $MAX_c^t = s_1$ for all $c \in C$ and $t \in T$, and suppose that each crop belongs to a different homogeneous replanting group with maximum replanting equal to two. Hence, the seeding periods are all the odd periods in T, i.e., $T_o = \{1, 3, \dots, 9\}$. Let us consider the solution in which the crops c_1, c_1, c_2, c_3, c_2 are respectively assigned to the odd periods in T_o on the available plot, producing the crop sequence $\sigma = \langle c_1, c_1, c_2, c_3, c_2 \rangle$. Note that the crop assignment is feasible, since it satisfies the maximum replanting requirement (no crop is consecutively repeated more than twice), and the constraints on the minimum and maximum surface for each period and crop are trivially satisfied. By definition, the total profit of the solution is given by

$$\pi_1(\sigma) = p_1(\langle c_1 \rangle, 1) + p_1(\langle c_1, c_1 \rangle, 3) + p_1(\langle c_1, c_1, c_2 \rangle, 5) + p_1(\langle c_1, c_2, c_3 \rangle, 7) + p_1(\langle c_2, c_3, c_2 \rangle, 9).$$

The following theorem shows that CRP-k with $k = 3$ is strongly NP-hard.

Theorem 4.1. *CRP-3 is strongly NP-hard.*

Proof. The proof is by reduction from the strongly NP-hard axial 3-index assignment problem (3IAP) (Garey and Johnson, 1979). 3IAP can be defined as follows.

Instance: We are given 3 sets P, Q and R of equal size n, a profit $a_{i,j,l}$ associated to each triple $(i, j, l) \in P \times Q \times R$.

Problem: Find n triples such that: (i) each element of P, Q and R belongs to exactly one triple, (ii) the total profit of the selected triples is maximum.

Given an instance of 3IAP, we build an instance of CRP-3 with a time horizon of 3 years (six periods), a set C with n first semester crops, each crop belonging to a different replanting group, and a farmland with n plots of equal size s. Hence, crops can only be assigned to the odd periods 1, 3 and 5. Let the maximum replanting value of each crop be equal to 3, i.e., each crop $c \in C$ can be replanted in all odd periods on any hectare. Then, we set the maximum and minimum surface to assign to each crop in each period equal to the size of each plot s, i.e., $MAX_c^t = s$ and $MIN_c^t = s$. Hence, each crop $c \in C$ must be assigned to exactly one plot in each odd period. Note that, a crop assignment produces a crop sequence of length three on each plot, say $\langle i, j, l \rangle$, with $i, j, l \in C$, that we associate to the triple $(i, j, l) \in P \times Q \times R$ of the 3IAP instance. Furthermore, since a crop c can be assigned to exactly one plot in each odd period ($MAX_c^t = MIN_c^t = s$), a feasible crop assignment of the CRP-3 instance corresponds to a set of triples such that each element of P, Q and R belongs to exactly one triple of the set.

The profits $p_h(\langle i \rangle, 1)$, $p_h(\langle i, j \rangle, 3)$ and $p_h(\langle i, j, l \rangle, 5)$ obtained by assigning the crop sequence $\langle i, j, l \rangle$ on plot h to periods 1, 3 and 5, respectively, are defined in such way that the total profit of the sequence is $\pi_h(\langle i, j, l \rangle, 5) = p_h(\langle i \rangle, 1) + p_h(\langle i, j \rangle, 3) + p_h(\langle i, j, l \rangle, 5) = 2a_{i,j,l}$. Note that, this is always possible by setting: $p_h(\langle i \rangle, 1) = \min_{u \in Q, v \in R}\{a_{i,u,v}\}$; $p_h(\langle i, j \rangle, 3) = \min_{v \in R}\{a_{i,j,v}\}$; $p_h(\langle i, j, l \rangle, 5) = 2a_{i,j,l} - p_h(\langle i \rangle, 1) - p_h(\langle i, j \rangle, 3)$.

Hence, by definition of the profits, a feasible solution of 3IAP of total profit M corresponds to a feasible solution of CRP-3 with total profit $2M$, and vice versa. Such correspondence holds even for an optimal solution of 3IAP, and the thesis follows. □

4.1. Variants of the problem accounting for sustainable scenarios

According to the sustainable scenarios introduced in Sections 3.1–3.3, CRP-k basically corresponds to the "Pure Farmer" scenario, since no greening and/or sustainable constraint is explicitly considered. However, the "CAP Farmer" scenario can be formally stated by properly redefining a feasible crop assignment (given in Definition 4.2), in such a way that the CAP constraints specified in Section 3.2 are taken into account. Similarly, the "CAP+SVC farmer" scenario can be formally defined by changing Definition 4.2, in such a way that the CdM rules defined by the *Carta del Mulino* initiative (see Section 3.3) are satisfied. In order to introduced the ILP models, a formal notation for these two latter scenarios is introduced in Section 6.

5. A network flow model for solving a variant of CRP-k without assignment restrictions

In this section, we introduce a simplified version of CRP-k, denoted as SCRP-k, and show that it can be polynomially solved as a minimum cost network flow problem when $k = 3$ and, in general, for any k, with k fixed. SCRP-k is defined as CRP-k in which:

- restrictions on the minimum and maximum land surface to assign to each crop in each period do not exist (i.e., $MIN_c^t = 0$ and $MAX_c^t = \infty$ for all $c \in C$ and $t \in T$);
- the maximum replanting for each group and the duration d_c of each annual crop c is at most $k - 1$;

In Section 5.1, we show that the network flow approach described in the following for SCRP-3 can be easily extended to polynomially solve SCRP-k, when $k > 3$ is fixed.

SCRP-3 can be modeled and solved as a minimum cost network flow problem, as explained below. Recall that n is the number of crops in C and H is the set of all land plots. In what follows, we assume that all plots are homogeneous, i.e., have the same size and economic values per crop. However, the following discussion can be easily extended when plots are not homogeneous, in which case a different graph is built for each set of homogeneous plots, and a minimum cost network flow problem is solved on each graph. Let m be the number of plots and $G = (V, E)$ be a graph where V and E are the node and the arc sets, respectively. On G, we define a minimum cost network flow problem with one source and one demand node, denoted as s and d, respectively. An amount of flow equal to m has to be sent from s to d. The nodes in V are partitioned into layers, say $L_0, L_1, \dots, L_l$, where L_0 and L_l only contain the source and the demand node, respectively. The arcs in E only connect the nodes of two adjacent layers L_i, L_{i+1}, for $i = 0, \dots, l-1$, or a node of a layer with the demand node. A minimum and a maximum capacity equal to 0 and m, respectively, is assigned to each arc. Each node of an intermediate layer L_i, for $i = 1, \dots, l-1$, is denoted as $(a, b)^{t,q}$ and models the assignment of the two crops a and b in C on the two compatible periods t and q, respectively, with $t < q$, when no other crop is assigned to periods $t+1, t+2, \dots, q-1$.

An arc between nodes of layers L_i and L_{i+1}, with $1 \le i < l-1$, models a sequence of four crops, possibly not all distinct, over four periods, as detailed in the following. Let us denote as $(a, b)^{t,q}$ and $(c, d)^{u,v}$ two nodes of layers L_i and L_{i+1}, respectively, with $1 \le i < l-1$. The flow on the arc from $(a, b)^{t,q}$ to $(c, d)^{u,v}$ represents the number of plots on which the crop sequence $\langle a, b, c, d \rangle$ is assigned to the compatible set of periods $\{t, q, u, v\}$. Observe that, the arc from $(a, b)^{t,q}$ to $(c, d)^{u,v}$ allows to keep track of the two crops preceding c and d, i.e., $\langle a, b \rangle$ and $\langle b, c \rangle$, respectively. Given a node $(c, d)^{u,v}$ of an intermediate layer, it may happen that the crop d does not exist (e.g., when $u = p$ is the last period in T, or when $u = p - 1$ and c is an annual crop). In such cases, the node is written as $(c, -)^{u,-}$. The profit assigned to the arc from $(a, b)^{t,q}$ to $(c, d)^{u,v}$ is equal to the profit per plot (recall that the plots are homogeneous), obtained by assigning crops c and d to the periods u and v respectively, when the two preceding crops of c and d are those given in the 3-crop sequences $\langle a, b, c \rangle$ and $\langle b, c, d \rangle$ respectively, i.e.,

$$p(\langle a, b, c \rangle, u) + p(\langle b, c, d \rangle, v).$$

The profit assigned to an arc from the source node s to a node $(a, b)^{t,q}$ of the first intermediate Layer L_1 is equal to the profit per plot obtained by assigning the sequence $\langle a, b \rangle$ to periods $\{t, q\}$, i.e.,

$$p(\langle a \rangle, t) + p(\langle a, b \rangle, q).$$

In what follows, we derive the length of shortest and longest paths on G. Note that, given a node $(a, b)^{t,q}$ of G, we have either $q = t + 1$ or $q = t + 2$. In fact, if a and b are a first and a second semester crop, respectively, then t is an odd period and $q = t + 1$. While if nor a nor b are second semester crops, then t and q are odd periods and

$q = t + 2$. By the above observation, and recalling that the length of a crop sequence in a crop assignment (from period 1 to period p) depends on how may even periods are assigned to second semester crops, we can easily determine the length of a shortest and of a longest path on G, in terms of number of nodes. In fact, a path on G cannot be shorter than a path only containing nodes $(a,b)^{t,q}$, in which a and b are annual or first semester crops (and consequently in which t and q are consecutive odd periods). Hence, such a path is a *shortest path* on G. On the other hand, a path on G cannot be longer than a path only containing nodes $(a,b)^{t,q}$ representing a succession of first and second semester crops in which $q = t + 1$. Hence, such a path is a *longest path* on G.

The two following lemmas provide the number of layers and the number of nodes of each layer in G.

Lemma 5.1. *The number of node layers in graph G is $\frac{p}{2} + 2$, where p is the last period of the time horizon T.*

Proof. Recall that the time periods in the time horizon $T = \{1, \dots, p\}$ correspond to first and second semesters of a given number of consecutive years, where period 1 is the first semester of the first year and the last period p is the second semester of the last year. Since each node of a path in G belongs to a different layer, the number of layers is equal to the length of a longest path on G. As already observed, a path visiting nodes $(a,b)^{t,q}$, where a and b are first and second semester crops, respectively, assigned to periods $t \in T_o$ and $q = t + 1$, is a longest path on G. Then, it is easy to see that a longest path from the source node to the demand node must contain $p/2+2$ nodes, that by definition of G belong to distinct layers. □

The following lemma provides the maximum number of nodes contained in each intermediate layer of G. (Recall that the first and the last layer only contain one node.)

Lemma 5.2. *The maximum number of nodes contained in each Layer L_i of G is $4in^2$ where n is the number of crops in C, for $i = 1, \dots, \frac{p}{2} + 1$.*

Proof. By definition, a node $(a,b)^{t,q}$ in each Layer L_i, for $i = 1, \dots, l-1$, is related to a crop pair $a, b \in C$ and to a pair of periods $t, q \in T$, with $q \in \{t+1, t+2\}$. We first compute the number of possible period pairs t, q associated to nodes in each layer L_i. At this aim, given a node $(a,b)^{t,q}$ of layer L_i, we determine the minimum and the maximum values taken by t and q.

Recall that any arc connecting nodes of two intermediate layers L_i and L_{i+1} connects a node $(a,b)^{t,q}$ with a node $(c,d)^{u,v}$. As already observed, there exists on G a shortest path P_s only containing nodes related to annual and first semester crops. Let $(a,b)^{t,q}$ and $(c,d)^{u,v}$ be the two (consecutive) nodes on path P_s belonging to layers L_i and L_{i+1}, respectively. We have that a, b, c and d are first semester or annual crops, and t, q, u, v, are consecutive odd periods. On such a path, an arc from $(a,b)^{t,q}$ to $(c,d)^{u,v}$ "moves" from period $t \in T_o$ to $v = t+6 \in T_o$. Note that, no other succession of four consecutive crops produces a move bigger than this, in terms of number of periods. As a consequence, the node of layer L_i belonging to path P_s, i.e., $(a,b)^{t,q}$, has the biggest period values t and q. In fact, we have $t = 4i - 3$ and $q = 4i - 1$.

Similarly, as already observed, a longest path exists on G, say P_l, only containing nodes related to first and second semester crops. Hence, given two consecutive nodes $(a,b)^{t,q}$ and $(c,d)^{u,v}$ on P_l, we have that t, q, u, v are consecutive periods (with $q = t + 1$, $u = t + 2$, and $v = t + 3$), and the arc from $(a,b)^{t,q}$ to $(c,d)^{u,v}$ "moves" from period t to $v = t + 3$. Note that, no other succession of four consecutive crops produces a move smaller than this, in terms of number of periods. Hence, the minimum values taken by the periods associated to nodes of layer L_i are the periods t and q of the node $(a,b)^{t,q}$ on P_l, for which we have $t = 2i - 1$ and $q = 2i$, respectively.

Let $(i,j)^{t_1,t_{max}}$ and $(f,g)^{t_2,t_{min}}$ be the nodes of the intermediate layer L_i belonging to the shortest path P_s and to the longest path P_l of G, respectively, where t_{max} and t_{min} are the maximum and minimum period where the second crop of a node can be sown in that layer. By the above discussion, we have that $t_{max} = 4i - 1$ and $t_{min} = 2i$ (hence, $t_{max} - t_{min} = 2i - 1$). In fact, $t_{max} = 4i - 1 \in T_o$ is the seeding period of the $2i$-th crop of the shortest crop sequence (containing annual or first semester crops only), while $t_{min} \in T_e$ is the seeding period of the $2i$-th crop of the longest crop sequence (alternating first and second semester crops). As a consequence, the last period q of each node $(a,b)^{t,q}$ in Layer L_i can take at most $2i$ values, i.e., $2i \le q \le 4i - 1$. Since, in $(a,b)^{t,q}$, the first period $t \in \{q-2, q-1\}$, the number of possible period pairs associated to nodes of L_i is $2 \times 2i$. Recalling that, in the intermediate layer L_i, at most two nodes exist for each crop pair a, b and period pair t, q, i.e., $(a,b)^{t,q}$ and $(b,a)^{t,q}$, the nodes in L_i are at most $4in^2$. □

Lemma 5.3 provides the total number of nodes in G.

Lemma 5.3. *The number of nodes in G is $O(n^2p^2)$ for $f = 1, \dots, \tau$, where n is the number of crops in C and p is the number of time periods in T.*

Proof. The thesis easily follows by Lemmas 5.1 and 5.2 and since $\sum_{i=1}^{p/2} 4in^2 = 4n^2(p/4)(p/2+1)$. □

A consequence of Lemma 5.3 is that, when $MIN_c^t = 0$ and $MAX_c^t = \infty$ for all $c \in C$ and $t \in T$, CRP-3 can be optimally solved in polynomial time by solving a minimum cost network flow problem on graph G. In fact, SCRP-3 can be converted to a minimum cost network flow problem with negative arc lengths by simply changing the sign of the profits on all arcs in G. Such a problem can be solved in polynomial time when the graph is acyclic or does not contain directed negative cost cycles of infinite capacities (see Ahuja et al. (1993)). Observe that the graph G is acyclic and has finite arc capacities (bounded between 0 and m) and, hence, SCRP-3 can be solved in polynomial time as a minimum cost network flow problem. (Furthermore, recall that, when all arc capacities are bounded, a minimum cost network flow problem with negative arc lengths can be easily converted to a minimum cost network flow problem with non-negative arc lengths (Ahuja et al., 1993).)

In Fig. 2, the graph G is reported for a SCRP-3 instance with a three year planning horizon $T = \{1,2,3,4,5,6\}$ and 3 crops $C = \{a,b,c\}$, where a is an annual crop and b and c are first and second semester crops, respectively. Hence, a and b can be assigned to periods in $T_o = \{1,3,5\}$, and c to periods in $T_e = \{2,4,6\}$. Note that, a shortest path on G alternating annual or first semester crops is $s \to (a,b)^{1,3} \to (a,-)^{5,-} \to t$. On the other hand, a longest path on G, alternating first and second semester crops, is $s \to (b,c)^{1,2} \to (b,c)^{3,4} \to (b,c)^{5,6} \to t$.

5.1. Extending the network flow approach to SCRP-k

The graph G described in the above section can be suitably modified to model SCRP-k when $k > 3$. The graph G used for modeling SCRP-k is a multi-layer graph too, with the difference that a node in each intermediate layer models the assignment of a subsequence of $k-1$ crops to $k-1$ compatible periods. Such a node can be denoted as $(c_1, c_2, \dots, c_{k-1})^{t_1,t_2,\dots,t_{k-1}}$.

By exploiting the concepts of shortest and longest paths on the graph G introduced for SCRP-3, it can be proved that the number of layers and nodes in each layer is polynomial with respect to the number of crops and periods. This implies that SCRP-k can be polynomially solved as a minimum cost network flow problem when k is fixed, i.e., it is not part of the input.

6. An integer linear programming formulation for CRP-3

In this section, an ILP formulation for CRP-k with $k = 3$ is presented. As already stated in Section 3, the rationale of considering 3-crop rotation schemes is a consequence of the best agronomic practice in mediterranean pedo-climatic contexts (Baldoni and Giardini, 2002), that is based on the consecutive succession of three crop species. Hence,

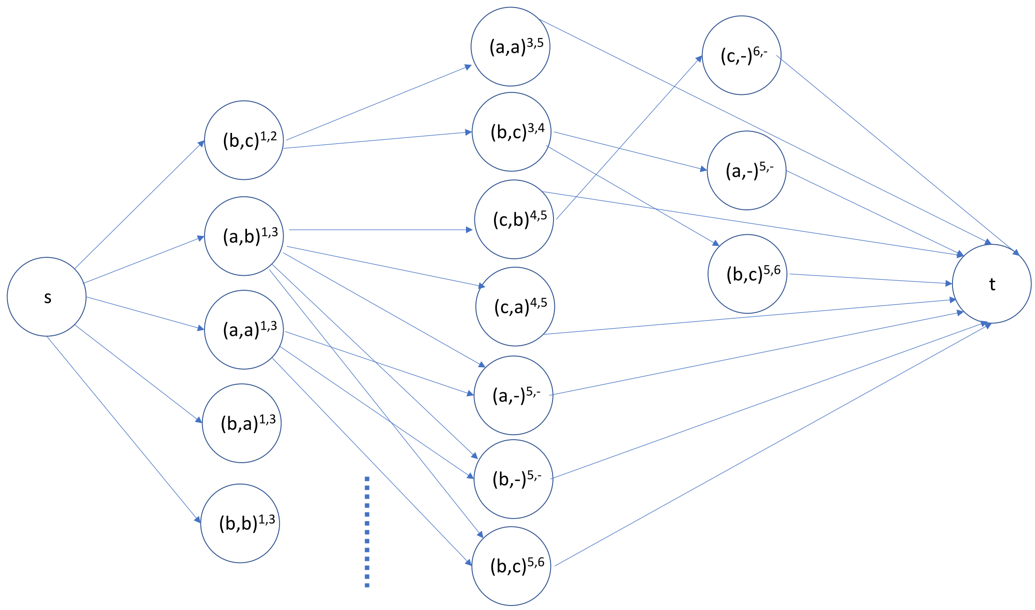

Fig. 2. The network flow of the VCRP-3 instance.

in what follows we assume that the profits earned by cultivating a crop on a piece of land are affected by the species of the two crops preceding it on the same piece of land.

Recall that each crop can be classified into the *crop types*: renewal (set C_R), impoverishing (set C_D) or improver (set C_M). As reported in Section 3, all possible successions of crop types are allowed, but for each assigned crop c a profit loss may occur depending on the types of c and of the two immediately preceding crops, if they exist. Hence, since $k = 3$, to determine the profit of a crop c assigned to a given time period $t \geq 3$ we have to take into account the types of the two crops immediately preceding c, if any. When c is only preceded by one crop b, the profit of c is determined by considering the two crop sequence $\langle b, c\rangle$, only.

From now on, we call *3-rotation* (*2-rotation*) the succession of three (two) crop types. Let R_3 and R_2 be the sets of all possible 3-rotations and 2-rotations, respectively. Obviously, the number of possible 3-rotations (2-rotations) of the three (two) types is 27 (is 9). Given a three crop sequence $\sigma_3 = \langle c_1, c_2, c_3\rangle$, let $r = \langle \alpha, \beta, \gamma\rangle \in R_3$ be the corresponding 3-rotation, where α, β, γ are the types of the crops c_1, c_2 and c_3, respectively. Similarly, given a two crop sequence $\sigma_2 = \langle c_1, c_2\rangle$, let $r = \langle \alpha, \beta\rangle \in R_2$ be the corresponding 2-rotation, where α, β are the types of crops c_1 and c_2, respectively. In what follows, given a three crop sequence $\sigma_3 = \langle c_1, c_2, c_3\rangle$ and the corresponding 3-rotation $\langle \alpha, \beta, \gamma\rangle \in R_3$, we denote as C_α, C_β and C_γ the sets of all crops of type α, β, γ, respectively. Similarly, we denote as C_α and C_β the sets of types α and β, respectively, corresponding to the 2-rotation $\langle \alpha, \beta\rangle \in R_2$.

In the following, the decision variables used in the ILP model are listed:

- $x^t_{ch} \in \{0, 1\}$ equal to 1 if crop $c \in C$ is assigned to plot $h \in H$ in period t and 0 otherwise.
- $y^t_{rch} \in \{0, 1\}$ equal to 1 if crop $c \in C$ is assigned to plot $h \in H$ in an odd period $t \geq 5$, activating the 3-rotation scheme $r \in R_3$ and 0 otherwise.
- $y_{rch} \in \{0, 1\}$ equal to 1 if crop $c \in C$ is assigned to plot $h \in H$ in period 3 activating the 2-rotation scheme $r \in R_2$ and 0 otherwise.
- $w^t_{rch} \in \{0, 1\}$ equal to 1 if the second semester crop $c \in C_S$ is assigned to plot $h \in H$ in the even period t, with $t \geq 4$, activating the 3-rotation scheme $r \in R_3$ and 0 otherwise.
- $w_{rch} \in \{0, 1\}$ equal to 1 if the second semester crop $c \in C_S$ is assigned to plot $h \in H$ in period 2, activating the 2-rotation scheme $r \in R_2$ and 0 otherwise.

Let P^{ht}_c be the *nominal* profit attained by assigning crop $c \in C$ on plot $h \in H$ to period t, when the best crop rotation practice is followed (i.e. no profit loss occurs). Note that, the profit P^{ht}_c is different for each scenario and higher for the CAP and the CAP+SVC variants, since it includes the CAP incentives for the former, and both the CAP and "Carta del Mulino" incentives for the latter.

Given a sequence $\sigma_3 = \langle c_1, c_2, c_3\rangle$, let $r \in R_3$ be the 3-rotation activated by the sequence. Then, the profit $p_h(\sigma_3, t)$ earned by crop $c_3 \in C$ when it is assigned to time t and preceded by crops $\langle c_1, c_2\rangle$ on plot h is $p_h(\sigma_3, t) = P^{ht}_{c_3} - L^{ht}_{rc_3}$, where $L^{ht}_{rc_3}$ is the profit loss (possibly 0 if r is a best rotation practice) produced by rotation r when crop c_3 is assigned to plot h at time t. Similarly, we can define the profit earned by assigning the last crop $c_2 \in C$ of a 2-crop sequence $\sigma_2 = \langle c_1, c_2\rangle$ to plot h at time t, with $t = 2, 3$, activating the 2-rotation $r \in R_2$, as $p_h(\sigma_2, t) = P^{ht}_{c_2} - L^{ht}_{rc_2}$.

Since in CRP-3 the objective is to maximize the total profit, the objective function is:

$$\max \sum_{t\in T}\sum_{h\in H}\sum_{c\in C} P^{ht}_c x^t_{ch} - \sum_{h\in H}\sum_{c\in C}\sum_{r\in R_3}\Big(\sum_{t\in T_o:\,t\geq 5} L^{ht}_{rc} y^t_{rch} + \sum_{t\in T_e:\,t\geq 4} L^{ht}_{rc} w^t_{rch}\Big) - \sum_{h\in H}\sum_{c\in C}\sum_{r\in R_2}\big(L^{h3}_{rc} y_{rch} + L^{h2}_{rc} w_{rch}\big) \tag{2}$$

The constraints of the problem are reported in the following.

$$\sum_{c\in C} x^t_{ch} \leq 1, \forall h \in H, \forall t \in T \tag{3}$$

$$\sum_{h\in H} s_h x^t_{ch} \leq MAX^t_c \quad \forall c \in C, \forall t \in T \tag{4}$$

$$\sum_{h\in H} s_h x^t_{ch} \geq MIN^t_c \quad \forall c \in C, \forall t \in T \tag{5}$$

$$\sum_{c\in C_F\cup C_A} x^t_{ch} = 1, \forall h \in H, \forall t \in T_o \tag{6}$$

$$x^t_{ch} = 0, \forall c \in C_F, \forall h \in H, \forall t \in T_e \tag{7}$$

$$x^t_{ch} = 0, \forall c \in C_S, \forall h \in H, \forall t \in T_o \tag{8}$$

$$x^t_{ch} \leq \sum_{k\in C_F} x^{t-1}_{kh}, \forall c \in C_S, \forall h \in H, \forall t \in T_e \tag{9}$$

$$x_{ch}^{t+1} \geq x_{ch}^{t} \quad \forall c \in C_A, \forall h \in H, \forall t \in T_o \tag{10}$$

$$x_{ch}^{t+1} \leq x_{ch}^{t} \quad \forall c \in C_A, \forall h \in H, \forall t \in T_o \tag{11}$$

$$x_{ch}^{t+2q} \geq x_{ch}^{t} - x_{ch}^{t-1} \quad \forall c \in C_A, \forall h \in H, \forall t \in T_o, \forall q = 1, \dots, d_c - 1 \tag{12}$$

Constraints (3) state that, in each period t, i.e., semester, at most one crop can be assigned to each plot. Constraints (4) and (5) impose the minimum and maximum land surface assigned to each crop in each period. Constraints (6) state that a first semester or annual crop must be assigned to each odd period on each plot. Constraints (7)–(11) set the seeding periods for annual, first semester and second semester crops. In particular, Constraints (10) and (11) impose that an annual crop occupies both semesters t and $t+1$, with $t \in T_o$, of a given year. Constraints (12) state that if the annual crop c is assigned to period t but not to period $t-1$, then it must be assigned to periods $\{t, t+1, \dots, 2d_c - 2\}$ (it must stay on the plot for its duration, corresponding to $2d_c$ consecutive periods).

The Constraints (13) and (14) reported below impose requirements on the maximum replanting. In the constraints, let ng be the number of homogeneous replanting groups, RG_i be the set of crops of the ith group and rep_i be the maximum replanting value of RG_i, for $i = 1, \dots, ng$. Hence, rep_i is the maximum number of times that crops in RG_i can be consecutively assigned to the same plot.

More precisely, Constraints (13) state that, for each homogeneous replanting group RG_i, the number of crops in RG_i cultivated in odd periods from t to $t + 2rep_i$ must not be greater than the maximum replanting value rep_i of the group RG_i. The second term on the right hand side of the constraints reflects the fact that, if there are second semester crops cultivated in the even periods between $t+1$ and $t + 2rep_i - 1$, than the constraint is relaxed, because the second semester crops interrupt the sequence of consecutive crops belonging to RG_i. Constraints (14) impose that, for each homogeneous replanting group RG_i containing second semester crops, the sum of second semester crops in RG_i cultivated in the even periods from t to $t+p$ and of the crops in RG_i grown in the odd periods in $\{t, t+1, \dots, t+p\}$ must not be greater than rep_i. The sum in the right hand side of the constraints regards all the crops not belonging to RG_i that are cultivated from t to $t+p$. If this sum is bigger than zero, the crop sequence from t to $t+p$ contains at least a crop not in RG_i and the constraints must be deactivated. The value $rep_i + 1$ works as a big-M with the minimum value needed to deactivate the constraints. The extremes of the interval of p cover the case in which crops of the group RG_i are grown in all consecutive periods (i.e., odd and even periods) of the interval $t, t+1, \dots, t+p$ and the case in which crops in RG_i are grown only in the odd periods (i.e., no second semester crops in RG_i are assigned in the interval).

$$\sum_{c \in RG_i} \sum_{q=t:q \in T_o}^{t+2rep_i} x_{ch}^{q} \leq rep_i + \sum_{c \in C_S} \sum_{q=t+1:q \in T_e}^{t+2rep_i-1} x_{ch}^{q}$$
$$\forall i : RG_i \cap C_S = \emptyset, i = 1 \dots, ng, t \in T_o \tag{13}$$

$$\sum_{c \in RG_i \setminus C_S} \sum_{q=t:q \in T_o}^{t+p} x_{ch}^{q} + \sum_{c \in RG_i \cap C_S} \sum_{q=t:q \in T_e}^{t+p} x_{ch}^{q} \leq rep_i + (rep_i + 1) \sum_{c \in C \setminus RG_i} \sum_{q=t}^{t+p} x_{ch}^{q} \tag{14}$$
$$\forall i : RG_i \cap C_S \neq \emptyset, i = 1 \dots, ng, p = rep_i, \dots, 2rep_i, \forall h \in H, t \in T$$

In the following, the constraints on the crop rotation are presented. Recall that a 3-rotation $r \in R_3$ is the succession $r = \langle \alpha, \beta, \gamma \rangle$ of three crops of types α, β and γ, respectively, while a 2-rotation $r \in R_2$ is the succession $r = \langle \alpha, \beta \rangle$ of two crops of types α and β. Crops of types α, β and γ are denoted as C_α, C_β and C_γ, respectively. These constraints force variables y and w to be 1 when specific rotation schemes occur, accounting for different cases that depend on how many second semester crops are cultivated in a given crop sequence. Constraints (15)–(17) consider odd periods $t \in T_o$ only, and account for all combinations of the two preceding crops. More precisely, Constraints (15) are related to the case in which no second semester crop is cultivated in the even periods $t-1$ and $t-3$ and, so, the two crops preceding c in t are annual or first semester crops assigned to the periods $t-2$ and $t-4$. In this case, y_{rch}^{t} is forced to be 1 when $x_{ch}^{t} = 1$. Constraints (16) force y_{rch}^{t} to be 1 when the crop $c \in C_\gamma$ is cultivated in the odd period t, a second semester crop in C_β is assigned to the (even) period $t-1$, and a crop in C_α is grown in $t-2$. Constraints (17) state that $y_{rch}^{t} = 1$ if: (i) $c \in C_\gamma$ is cultivated in the odd period t, (ii) a second semester crop of C_β is assigned to $t-2$, (iii) a crop in C_α is in $t-3$, and (iv) no second semester crop is assigned to $t-1$.

On the other hand, Constraints (18) and (19) consider even periods $t \in T_e$ only, and consider all possible combinations of the two preceding crops. Constraints (18) impose that if $c \in C_\gamma$ is cultivated in t, a crop in C_β is in $t-1$, a crop in C_α is in $t-3$, and there is no crop in C_S in $t-2$, then $w_{rch}^{t} = 1$. Similarly, Constraints (19) state that if $c \in C_\gamma$ is cultivated in t, a crop in C_β is in $t-1$, a crop in C_α is in $t-2$, and there is a crop in C_S in $t-2$, then $w_{rch}^{t} = 1$.

Finally, Constraints (20) and (21) are used to activate the variables related to the 2-rotations. Constraints (20) impose that, if $c \in C_\beta$ is cultivated in $t = 3$, a crop in C_α is in $t-2$, and there is no crop in C_S in $t-1$, then $y_{rch} = 1$. Constraints (21) state that if $c \in C_\beta$ is cultivated in $t = 2$, and a crop in C_α is in $t-1$, then $y_{rch} = 1$.

$$y_{rch}^{t} \geq x_{ch}^{t} + \sum_{i \in C_\alpha} x_{ih}^{t-4} + \sum_{i \in C_\beta} x_{ih}^{t-2} - \sum_{i \in C_S} x_{ih}^{t-1} - \sum_{i \in C_S} x_{ih}^{t-3} - 2,$$
$$\forall r = \langle \alpha, \beta, \gamma \rangle \in R_3, \forall c \in C_\gamma, \forall h \in H, t \in T_o, t \geq 5 \tag{15}$$

$$y_{rch}^{t} \geq x_{ih}^{t} + \sum_{i \in C_\beta} x_{ih}^{t-1} + \sum_{i \in C_\alpha} x_{ih}^{t-2} + \sum_{i \in C_S} x_{ih}^{t-1} - 3,$$
$$\forall r = \langle \alpha, \beta, \gamma \rangle \in R_3, \forall c \in C_\gamma, \forall h \in H, \forall t \in T_o, t \geq 3 \tag{16}$$

$$y_{rch}^{t} \geq x_{ih}^{t} + \sum_{i \in C_\beta} x_{ih}^{t-2} + \sum_{i \in C_\alpha} x_{ih}^{t-3} + \sum_{i \in C_S} x_{ih}^{t-3} - \sum_{i \in C_S} x_{ih}^{t-1} - 3,$$
$$\forall r = \langle \alpha, \beta, \gamma \rangle \in R_3, \forall c \in C_\gamma, \forall h \in H, \forall t \in T_o, t \geq 5 \tag{17}$$

$$w_{rch}^{t} \geq x_{ch}^{t} + \sum_{i \in C_\beta} x_{ih}^{t-1} + \sum_{i \in C_\alpha} x_{ih}^{t-3} - \sum_{i \in C_S} x_{ih}^{t-2} - 2,$$
$$\forall r = \langle \alpha, \beta, \gamma \rangle \in R_3, \forall c \in C_\gamma, c \in C_S, \forall h \in H, \forall t \in T_e, t \geq 4 \tag{18}$$

$$w_{rch}^{t} \geq x_{ch}^{t} + \sum_{i \in C_\beta} x_{ih}^{t-1} + \sum_{i \in C_\alpha} x_{ih}^{t-2} + \sum_{i \in C_S} x_{ih}^{t-2} - 3,$$
$$\forall r = \langle \alpha, \beta, \gamma \rangle \in R_3, \forall c \in C_\gamma, c \in C_S, \forall h \in H, \forall t \in T_e, t \geq 4 \tag{19}$$

$$y_{rch} \geq x_{ch}^{t} + \sum_{i \in C_\alpha} x_{ih}^{t-2} - \sum_{i \in C_S} x_{ih}^{t-1} - 1, \forall r = \langle \alpha, \beta \rangle \in R_2, \forall c \in C_\beta, \forall h \in H, t = 3 \tag{20}$$

$$w_{rch} \geq x_{ch}^{t} + \sum_{i \in C_\alpha} x_{ih}^{t-1} - 1, \forall r = \langle \alpha, \beta \rangle \in R_2, \forall c \in C_\beta, c \in C_S, \forall h \in H, t = 2 \tag{21}$$

6.1. Constraints of the CAP farmer scenario

As already mentioned in Section 3.2, farmers observing CAP regulation have benefits and incentives, but have to follow strict ecological and diversification rules. These rules are described into detail in Section 3.2.

The diversification constraints can be written as

$$\sum_{h \in H} s_h x_{ch}^{t} \leq 0.75 \sum_{h \in H} s_h, \forall c \in C, \forall t \in T_o. \tag{22}$$

Constraints (22) state that, in each odd period, no crop can be assigned to more than the 75% of the farmland. Recall that, as reported in Section 3.2, such constraints hold only for farms with total land between 10 and 30 ha. Note that, since we assume that a crop is assigned to each odd period, Constraints (22) also imply that at least two crops will be cultivated in each odd period (which is another CAP requirement).

Now let C_{leg} be the set of legume crops. The following Constraints (23) only exist for farmlands with more than 15 ha, while Constraints (24) are imposed for farmlands with more than 30 ha.

$$\sum_{c\in C_{leg}} \sum_{h\in H} s_h x^t_{ch} \geq 0.05 \sum_{h\in H} s_h, \forall t \in T_o \tag{23}$$

$$\sum_{h\in H} s_h (x^t_{c_1 h} + x^t_{c_2 h}) \leq 0.95 \sum_{h\in H} s_h, \forall c_1, c_2 \in C, c_1 \neq c_2, \forall t \in T_o \tag{24}$$

Constraints (23) impose a minimum of 5% of the whole land to be cultivated in legumes each year. Constraints (24) state that the two most cultivated crops cannot exceed the 95% of the farmland, each year.

6.2. CAP+SVC scenario

In this scenario, in addition to the CAP constraints, farmers have to respect the policies introduced by the Carta del Mulino initiative (see Section Section 3.3). CdM identifies the set of crops of interest of the initiative (i.e., wheat), in what follows denoted as C_{CdM}.

Recall that, the CdM initiative impose three types of requirements: (i) greening constraints, (ii) diversification constraints, (iii) repetition constraints. For the three types, each set of constraints covers a specific case, characterized by the set of (even) periods assigned to second semester crops. In the following, as an example, one constraint set for each of the above three types is presented. The constraints related to the other cases are reported in Appendix.

$$\sum_{i\in C_L} \sum_{q=t+1}^{t+4} x^q_{ih} \geq x^t_{ch} + \sum_{i\in C_S} x^{t+1}_{ih} + \sum_{i\in C_S} x^{t+3}_{ih} - 2,\ \forall c \in C_{CdM}, \forall h \in H, \forall t \in T_o. \tag{25}$$

Constraints (25) are greening constraints related to the case in which a CdM crop is assigned to period t and second semester crops are cultivated in $t+1$ and $t+3$. In (25), C_{leg} and C_{oil} are the sets of legume and oil crops, respectively, and $C_L = C_{leg} \cup C_{oil}$. They impose that at least a crop in C_L must be assigned on h from period $t+1$ to $t+4$.

$$\begin{aligned}&\sum_{i\in\{c,v\}} (x^{t+1}_{ih} + x^{t+2}_{ih} + x^{t+4}_{ih} + x^{t+6}_{ih}) \leq 3 + (1 - x^t_{vh}) + (1 - \sum_{i\in C_S} x^{t+1}_{ih})\\ &\quad + \sum_{c\in C_S} x^{t+3}_{ih} + \sum_{i\in C_S} x^{t+5}_{ih}\ \forall v \in C_{CdM}, c \in C, h \in H, t \in T_o.\end{aligned} \tag{26}$$

Constraints (26) are the CdM diversification constraints related to the case in which a CdM crop v is assigned to plot h in the odd period t and a second semester crop is assigned in $t+1$, but no second semester crop is assigned to $t+3$ and $t+5$. Hence, the seeding periods for the four crops succeeding v are $t+1, t+2, t+4, t+6$. The constraints state that, for each crop $c \in C$, the crops v and c cannot be assigned in more than 3 periods in $\{t+1, t+2, t+4, t+6\}$ (allowing the assignment of a crop different from v and c in these periods).

$$\begin{aligned}&\sum_{c\in CR} (x^{t_1}_{ch} + x^{t_2}_{ch} + x^{t_3}_{ch}) \leq 2 + (1 - \sum_{i\in C_S} x^{t+1}_{ih}) + \sum_{c\in C_S} x^{t+3}_{ih}\\ &\quad + \sum_{i\in C_S} x^{t+5}_{ih} + (1 - x^t_{vh}) \forall v \in C_{CdM}, \forall h \in H, \forall t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega\end{aligned} \tag{27}$$

Constraints (27) are the CdM repetition constraints covering the case in which a CdM crop v is assigned to plot h in period t, and second semester crops are cultivated in $t+1$, but not in $t+3$ and $t+5$. In (27), let CR be the set of crops subject to the repetition constraints. The constraints impose that crops in CR can be repeated at most once in the seeding periods $t, t+1, t+2, t+4, t+6$. In the constraints, Ω denotes the set of all triples $\{t_1, t_2, t_3\}$ of 3 consecutive periods chosen among the seeding periods considered in each constraint (i.e., $\{t, t+1, t+2, t+4, t+6\}$ in this specific case). Hence, in Constraints (27), Ω is the set of triples $\{\{t, t+1, t+2\}, \{t+1, t+2, t+4\}, \{t+2, t+4, t+6\}\}$.

7. Instance description and real data

The real data used in this study have been collected from a panel of Italian farms involved in the CdM (Anon, 2022b) initiative (described in Section 3.3 into detail), promoted by the Barilla Group in collaboration with WWF Italy, University of Bologna, University of Tuscia and Open Fields. The farms have been identified in the Italian provinces with the greater arable land cultivated with soft wheat for the year 2018 (1st year of the CdM initiative), compared to the total arable land involved in crop rotation. In total, 23 farms have been selected, with a land surface ranging from 19 to 229 hectares. In all the instances, the land is homogeneous, and we assume the farmland in each instance divided in homogeneous plots with a size of 1 hectare (even if all the developed models and solution approaches hold for farmlands with heterogeneous plots, too). Hence, in what follows, the words “plot” and “hectare” are interchangeable. In the experimental campaign, the set of crops C considered for each farm includes the crops already grown during the years of data collection and other crops that could potentially be cultivated. In fact, our models can be also employed to evaluate the potential benefits arising from the cultivation of crops currently not considered by the farmers. In the instances, the number of crops available for each farm ranges from 5 to 10. The details of each instance, in terms of number of considered crops and number of hectares of the arable land, are reported in the Columns 1–3 of Table 2.

Since the real data collected from the farms include the crops already planned during the first year of the CdM initiative (i.e., year 2018), two different sets of experiments have been conducted for the CAP and CAP+SVC scenarios (denoted as *Set 1* and *Set 2* in the following). In Set 1, crop planning decisions are only taken from the second year on, keeping the planning of the first year fixed according to the real data. In fact, the seeding plan for the first year was established and declared by the farmers, in order to satisfy the CAP regulations and join the CdM initiative. In Set 2, also the first year is included in the crop planning decision process. Note that, since the Pure farmer scenario is not actually followed by any of the selected farms, no real data are available for the first year of planning. Hence, in all the experiments on this scenario we always include the first year in the decision process.

The analysis of the “Used Agricultural Area” of the farms showed differences in terms of production specialization of the arable land. These differences are due to the different cultivation areas and the different cultivation techniques adopted. All the farms are specialized in cereals production (wheat and corn), but all of them have a legume crop or an oil crop. However, the distribution of crop areas within farms is not uniform.

In order to reconstruct, for each farm, the profits obtained by the cultivation of the arable land with the available crops, economic surveys and official data sources have been used. More precisely, the crop profits are determined by factors such as crop prices, yields, cultivation costs and incentives. The main data sources exploited are summarized below. The crop prices were obtained by using: annual prices available on official commodities exchange (AGER – Bologna (Anon, 2022a)); prices annually fixed by contracts for industrial crops; official Italian Ministry of Agriculture decree. Yields are set according to the National Agricultural Information System (Anon, 2022c). The system records all the crops yields in Italy for each year and municipality. For the CAP incentives, the real farms’ data for all the years have been used. For the cultivation costs, composed by technical inputs and operation costs, farms’ real figures collected during surveys have been employed.

Table 1
Cost increase of the rotations.

Crop rotation scheme			Cost increase		Crop rotation scheme			Cost increase	
Period 0	Period 1	Period 2	Period 1	Period 2	Period 0	Period 1	Period 2	Period 1	Period 2
Impoverishing	Impoverishing	Impoverishing	20%	30%	Renewal	Renewal	Improver	10%	0
Impoverishing	Impoverishing	Renewal	20%	0%	Renewal	Improver	Impoverishing	0	0
Impoverishing	Impoverishing	Improver	20%	0%	Renewal	Improver	Renewal	0	0
Impoverishing	Renewal	Impoverishing	0	10%	Renewal	Improver	Improver	0	10%
Impoverishing	Renewal	Renewal	0	10%	Improver	Impoverishing	Impoverishing	0	20%
Impoverishing	Renewal	Improver	0	0	Improver	Impoverishing	Renewal	0	0
Impoverishing	Improver	Impoverishing	0	0	Improver	Impoverishing	Improver	0	0
Impoverishing	Improver	Renewal	0	0	Improver	Renewal	Impoverishing	0	0
Impoverishing	Improver	Improver	0	10%	Improver	Renewal	Renewal	0	10%
Renewal	Impoverishing	Impoverishing	0	20%	Improver	Renewal	Improver	0	0
Renewal	Impoverishing	Renewal	0	10%	Improver	Improver	Impoverishing	10%	0
Renewal	Impoverishing	Improver	0	0	Improver	Improver	Renewal	10%	0
Renewal	Renewal	Impoverishing	10%	0	Improver	Improver	Improver	0	10%
Renewal	Renewal	Renewal	10%	20%					

As mentioned in Section 3, when crop rotation does not follow the best agronomic practices, a cost increase occurs depending on the specific crop succession in the rotation scheme. In our experiments, we assume a cost increase ranging from 0% (best rotation practice) to 30%, depending on the specific rotation of crop types (i.e., renewal, improver, impoverishing). The cost increases considered in the experimental campaign are reported in Table 1. As an example, when three impoverishing crops are consecutively assigned to the same plot (the first rotation in Table 1), a cost increase of 30% (20%) occurs when the third (second) crop is cultivated.

The cost increases shown in Table 1 derive from specific agronomic considerations for each sequence of 3-crop species. The classical agronomic literature (Baldoni and Giardini, 2002) defines as best rotation practices sequences containing crops of the three types, renewal, impoverishing and improver. However, as already stated in Definition 3.1, there are other crop sequences that do not necessarily lead to crop cost increases over the observed crop planning period. For example, the presence of leguminous crops (i.e., improver crops) always mitigates the increase in costs because they have improving effects on the structure and, more generally, on the natural fertility of the soil (Baldoni and Giardini, 2002; Yigezu et al., 2019). On the other hand, monoculture sequences lead to a decrease in crops' yield, due to the risk increase of pests and diseases, and in loss of soil fertility, as demonstrated by numerous long-term experiments. The percentage values shown in Table 1 are based on the experience and primary data collected by surveys involving farmers and technicians in the Po Valley (located in the north of Italy) (Fosci, 2022). However, these values closely depend on the peculiarity of the single land plot and on choices applied to previous crop cultivation and land allocation. As a consequence these values can be adapted according to specific information collected in other real contexts. In practice, the single farmer could select the cost increase according to own specific experience about arable land fields.

8. Experimental results

In this section, the results of the computational campaign on the MIP models introduced in Section 6 are presented on the real instances described in the previous section. The experiments have been performed using the Gurobi Optimizer version 9.0.1 on a 2.5 GHz Quad-Core computer equipped with 16 GB of RAM. In particular, all the real world instances have been solved in the three scenarios of the problem: Pure farmer, CAP farmer and CAP+SVC farmer (see Sections 3.1–3.3). The aim of the experimental campaign is twofold: (1) to evaluate the effectiveness of the sustainable policies proposed by the CAP and CdM regulations, with respect to the case in which no sustainable initiative is followed (recall that, this last case corresponds to the Pure farmer scenario); (2) to assess the ability of the proposed MIP models to solve real-life instances.

A five year planning horizon has been considered in all the instances. Hence, $T = \{1, 2, \dots, 10\}$. As already stated in the above section, two sets of experiments have been performed for both the CAP and CAP+SVC scenarios. In Set 1, the crop planning of the first year is already given and set according to the real data. In Set 2, the CAP and CAP+SVC scenarios were solved on the overall 5-year planning horizon, including the first year in the planning process. Recall that, in the experiments, the crop planning of the first year in the Pure farmer scenario is always included in the decision process, i.e., not given.

Table 2 reports the results of the Pure farmer scenario and those of Set 1 for the CAP and CAP+SVC farmer scenarios. Columns 1, 2 and 3 of the table respectively report the data of the instances: id, number of hectares and number of crops of each farm. Columns 4–6, 7–10 and 11–12 report the results for the three scenarios of the problem. For each scenario, the optimal solution value (the profit in Euros), called "Value", the computing time (in seconds) and the number of Branch & Bound nodes explored by Gurobi are reported. Furthermore, for the CAP and CAP+SVC scenarios, we also report the profit increases (in %) with respect to the Pure farmer. Such increases are denoted as $\Delta(P-C)$ and $\Delta(P-SVC)$ and computed as $(Value_{CAP}-Value_{Pure})/Value_{Pure}\times 100$ and $(Value_{CAP+SVC} - Value_{Pure})/Value_{Pure} \times 100$, respectively.

In the table, the instances are ordered by increasing number of hectares, ranging from 19 to 229 hectares. Note that the number of the available crops does not vary as much in the instances, ranging from 5 to 10, implying that the instance dimension is mainly determined by the number of hectares. The last row of the table reports the average values on all the instances.

In Table 2, a comparison of the "Value" (i.e., the profit) of the three scenarios and of the profit increases $\Delta(P - C)$ and $\Delta(P - SVC)$ show that the CAP and CAP+SVC attain higher profits than the Pure farmer. Note that, with respect to the Pure farmer, the profits obtained with the CAP farmer (CAP+SVC farmer) are 73.5% (74.5%) higher in Set 1, on average. These results show that the incentives introduced by the EU and by Barilla (for farmers joining their sustainability initiatives) overcompensate the potential loss due to the additional constraints introduced by the new regulations. Hence, such initiatives can be profitable and encourage farmers to join them.

Regarding the computational times, they generally increase with the number of hectares, even though there are some exceptions that may depend on the particular structure of some instances. On average, the computing time increases by 320% going from the Pure farmer scenario to the CAP farmer scenario and by a further 20% in the CAP+SVC farmer case. These increases are probably due to the higher number of constraints added to model the CAP and CAP+SVC scenarios, as shown in Section 6. As expected, a general increase can be also observed in the number of nodes explored by the Branch & Bound procedure performed by Gurobi. The maximum computation time is registered on the instance "EMR_BO2": The CAP+SVC scenario is solved in about 1 min, while the CAP and the Pure farmer scenarios require around

Table 2
Results of set 1 of experiments.

Instance			Pure farmer			CAP farmer				CAP+SVC farmer			
Id	# Hectares	# Crops	Value	Time	B&B nodes	Value	Time	$\Delta(P-C)(\%)$	B&B nodes	Value	Time	$\Delta(P-SVC)(\%)$	B&B nodes
PIE_TO1	19	5	78953.93	0.27	1	106746.82	0.47	35.2	1	107852.47	0.49	36.6	1
EMR_FE3	23	7	89596.04	0.41	1	136005.36	0.36	51.8	1	140252.79	0.3	56.54	1
EMR_FE4	27	9	102788.35	2.52	403	172293.69	2.241	67.62	1383	177466.44	1.8	72.65	1
EMR_PR1	29	10	198626.22	0.53	1	165290.27	0.85	−16.78	1	166551.05	1.06	−16.15	1
PIE_CN2	34	10	103718.36	0.47	1	175718.9	0.96	69.42	1	177950.46	0.67	71.57	1
VEN_RO1	49	6	70545.3	0.57	1	186722.88	3.68	164.69	1520	189916.02	6.81	169.21	2428
LMB_MI2	51	7	136158.78	0.44	1	289198.29	1.62	112.4	102	289967.38	8.03	112.96	4551
LMB_MI1	53	8	103948.9	0.46	1	256200.01	0.83	146.47	32	257995.15	0.96	148.19	32
PIE_AL3	54	8	127342.8	0.74	1	254348.29	12.12	99.74	2457	260248.43	7.64	104.37	1820
PIE_CN1	62	7	272821.7	0.48	1	502097.02	3.17	84.04	1	456235.65	1.68	67.23	1
LMB_MN2	66	9	210945.24	9.78	2910	410982.3	8.66	94.83	3037	418645	11.77	98.45	1551
EMR_BO3	72	7	186914.88	0.54	1	340864.76	1.33	82.36	24	347632.28	1.19	85.98	20
PIE_AL4	84	5	289413.6	0.4	1	398178.06	1.41	37.58	1	392440.63	1.63	35.6	1
LMB_MN3	84	10	1187961.57	1.14	1	1412452.51	7.34	18.9	302	1418220.35	6.76	19.38	1
LMB_MN1	86	10	984338.56	1.68	1	1085012.45	1.73	10.23	1	1092714.29	1.72	11.01	1
EMR_FE2	89	9	293788.11	0.77	1	454837.92	10.15	54.82	1965	465995.281	29.21	58.62	1645
PIE_AL2	120	7	607236	1.26	1	1250272.16	6.08	105.9	1	1256405.84	4.01	106.91	1
VEN_RO2	134	8	2784483.82	0.81	1	2701647.35	16.1	−2.97	9901	2702264.75	15.57	−2.95	6916
PIE_AL1	154	10	1141090.72	2.52	1	1587527.65	17.78	39.12	1	1588214.07	11.71	39.18	13
EMR_BO1	161	10	494902.73	24.68	2326	1082081.88	25.76	118.65	4016	1086159.58	38.81	119.47	1753
EMR_FE1	177	9	681327.87	1.42	1	1089875.38	40.59	59.96	2409	1093902.78	32.55	60.55	2123
VEN_VE1	211	7	819954.44	2.2	1	1666298.71	6.77	103.22	18	1669041.69	7.28	103.55	1
EMR_BO2	229	10	673816.47	14.29	135	1701880.12	53.17	152.57	1523	1721257.04	61.87	155.45	1706
		Average	506116.28	2.97	251.87	757675.34	9.70	73.47	1247.74	759883.16	11.02	74.54	1068.22

Table 3
Results of set 2 of experiments.

Instance			Pure farmer			CAP farmer				CAP+SVC farmer			
Id	# Hectares	# Crops	Value	Time	B&B nodes	Value	Time	$\Delta(P-C)(\%)$	B&B nodes	Value	Time	$\Delta(P-SVC)(\%)$	B&B nodes
PIE_TO1	19	5	78953.93	0.27	1	114458.93	0.42	44.97	1	114458.93	0.43	44.97	1
EMR_FE3	23	7	89596.04	0.41	1	137409.17	0.43	53.37	1	140463.11	0.39	56.77	1
EMR_FE4	27	9	102788.35	2.52	403	178538.45	3.44	73.7	812	183656.45	4.61	78.67	471
EMR_PR1	29	10	198626.22	0.53	1	216608.02	3.05	9.05	13	216814.66	7.73	9.16	1529
PIE_CN2	34	10	103718.36	0.47	1	185834.7	2.65	79.17	1	187871.7	3.14	81.14	87
VEN_RO1	49	6	70545.3	0.57	1	195225.7	8.39	176.74	1879	195225.7	53.4	176.74	44368
LMB_MI2	51	7	136158.78	0.44	1	295682.29	5.02	117.16	868	295682.29	14.38	117.16	3140
LMB_MI1	53	8	103948.9	0.46	1	276281.55	9.02	165.79	4229	276292.83	51.1	165.8	42575
PIE_AL3	54	8	127342.8	0.74	1	260415.19	19.14	104.5	2156	266285.59	44.04	109.11	16985
PIE_CN1	62	7	272821.7	0.48	1	528035.5	4.65	93.55	653	491212.14	5.64	80.05	1
LMB_MN2	66	9	210945.24	9.78	2910	417575.48	27.56	97.95	8560	426448.08	169.26	102.16	73733
EMR_BO3	72	7	186914.88	0.54	1	351828.16	2.66	88.23	55	353861.36	2.98	89.32	13
PIE_AL4	84	5	289413.6	0.4	1	423140.48	2.38	46.21	1	428068.96	1.85	47.91	1
LMB_MN3	84	10	1187961.57	1.14	1	1587311.8	48.92	33.62	4172	1589201.22	90.01	33.78	6198
LMB_MN1	86	10	984338.56	1.68	1	1226554.67	9.36	24.61	10	1233986.27	11.13	25.36	1
EMR_FE2	89	9	293788.11	0.77	1	469323.98	12.29	59.75	705	479512.94	138.08	63.22	3589
PIE_AL2	120	7	607236	1.26	1	1283437.44	47.57	111.36	7493	1282917.12	60.18	111.27	1626
VEN_RO2	134	8	2784483.82	0.81	1	2931218.51	20.18	5.27	1674	2931218.51	15.3	5.27	820
PIE_AL1	154	10	1141090.72	2.52	1	1778159.32	147.7	55.83	8165	1778159.32	128.36	55.83	5140
EMR_BO1	161	10	494902.73	24.68	2326	1143805.91	247.82	131.12	1749	1147046.18	364.61	131.77	9321
EMR_FE1	177	9	681327.87	1.42	1	1130892.09	118.79	65.98	1644	1130892.09	115.01	65.98	1620
VEN_VE1	211	7	819954.44	2.2	1	1856487.73	124.17	126.41	1536	1859614.81	106.02	126.79	1518
EMR_BO2	229	10	673816.47	14.29	135	1786250.55	261.29	165.09	2995	1806878.75	357.5	168.16	1748
		Average	506116.28	2.97	251.87	816281.55	49.00	83.89	2146.61	818076.91	75.88	84.63	9325.48

50 and 14 seconds, respectively. Note that these computation times are reasonable, proving a good ability of the models to solve this kind of problems for real-life instances.

Table 3 shows the results of the experiments of Set 2 on the CAP and CAP+SVC scenarios (to simplify the reading we also retrieve the results of the Pure farmer scenario already reported in Table 2). Recall that, in these experiments, the CAP and CAP+SVC scenarios are solved with the crop planning of the first year not set, but included in the decision process. From one side, these results allow to evaluate the benefits of fully joining sustainability initiatives. In fact, since the first year is included in the decision process, it is possible to establish the actual convenience of joining them. On the other side, the results of Set 2 allow to assess the additional computational effort required for solving larger instances. As expected, the objective solution values (i.e., the profits) in Set 2 are higher than in Set 1, since the problem is less constrained, being the first year included in the decision process. Furthermore, as shown by the $\Delta(P-C)$ and $\Delta(P-SVC)$ values, in Set 2 the CAP and the CAP+SVC scenarios attain profits that are, on average, 83.9% and 84.6% higher than those obtained with the Pure farmer scenario. These results suggest that the optimization models can help farmers to increase their profits when they attempt to join sustainability programs.

The computational times of CAP and CAP+SVC in Set 2 are about 50 and 76 seconds, on average, higher than those of CAP and CAP+SVC in Set 1. Nevertheless, all the instances in Set 2 are optimally solved with maximum computational times smaller than 262 and 365 s for CAP and CAP+SVC respectively, meaning that the models are still able to solve real world instances in reasonable time. Note that, in

Set 2, the solution values of some instances are the same in the CAP and CAP+SVC scenarios. This happens when the CdM crop is never cultivated in the optimal solution of the CAP+SVC scenario (recall that the CdM crop is the only crop that affects the decision process in the CAP+SVC scenario), implying that the optimal solution of the two scenarios is indeed the same.

To conclude, the models perform well on real-life instances, implying that the models could be embedded in a decision support tool that can be practically used by farmers to plan their production and by regulatory bodies to tune the incentives of sustainability programs.

As an example, in Fig. 3, the optimal crop sequences on each hectare on the three scenarios are reported for instance "PIE_CN2". For each scenario, the hectares are given on the x-axis and the periods are reported on the y-axis (where "I" and "II" are the first and second period of each of the five years). The color of each block is related to a specific crop, as showed in the legend. The figure highlights the main differences among the three scenarios. The pattern characterizing the Pure farmer scenario (on the top of the figure) is a monoculture scheme that we can observe over the entire planning horizon: in each period, the whole land is cultivated with a single crop. In particular, the land is mainly dedicated to the most high-income crops (soft wheat and corn). However, in the third year, the land is cultivated with a first semester (soft wheat) and a second semester crop (soy second harvest), which is an improver crop, so that the land fertility is enhanced and costs do not increase (see Table 1). In the CAP scenario (in the middle of the figure) more diversified patterns exist. In fact, this variant is characterized by the introduction of crop diversification constraints for each year: since the farmland is bigger than 30 hectares of land, at least 3 different crops must be grown each period. There is still a high presence of cereals (corn and soft wheat), but it is not possible anymore to grow only corn or wheat each year. In particular, the presence of soft wheat is higher and this crop is always followed by second harvest soy which, being an improver crop, plays a fundamental role in a virtuous crop rotation planning. Thus, the introduction of crop diversification within the single periods has also a positive impact on the rotation schemes on the single hectares. In the CAP+SVC scenario (at the bottom of the figure), the soft wheat (called here "SVC soft wheat") is valued with a premium price when the constraints prescribed by the "Carta del Mulino" are satisfied. This variant introduces diversification constraints on the single hectares over the years, so that more crops and legumes must be grown after the SVC soft wheat. In fact, in the solution, we can observe a more diversified pattern on the hectares, with a higher use of the annual soy, which is a legume and improver crop.

9. Conclusions

In this paper, the decision problem of crop planning in sustainable agriculture taking into account crop rotation benefits across growing seasons is considered. A rigorous problem characterization is given and a complexity analysis is performed. A polynomial minimum cost network flow approach is proposed for special cases. Integer Linear Programming models including all the problem characteristics have been developed for the case in which the rotation is based on sequences of $k = 3$ crops, and tested on real data. (Recall that, as stated by classical agronomic literature, $k = 3$ is identified as best agronomic practice in Mediterranean pedo-climatic contexts and prescribed by Italian regulations on organic agriculture (Anon, 2022).)

An experimental campaign on real-world instances shows that the proposed approaches could be embedded in a decision support tool that can be practically used by farmers to plan their production and to evaluate the convenience of joining sustainability initiatives. In the particular case under study, the results show that the incentives introduced for farmers joining the CAP and the SVC initiative "Carta del Mulino" overcompensate the farmers' potential loss due to the strict regulations introduced. In fact, joining such initiatives seems to be much more profitable than deciding not to join them.

In the case under study, no investment is required by farmers that want to join CAP and SVC programs. However, the same methodology and solution approaches could be adapted to cases where adherence to particular planning rotation schemes involves the purchase of machinery and other forms of technology. In such cases, the models should include the fixed costs resulting from the depreciation of acquired assets. Then, as in the case under study, the output of the model could be used to inform farmers on the convenience of adhering to given production programs. When the transition to more complex production models involves major changes in the capital structure of small and medium-sized farms, farmers could use the results of the models to become more aware of their medium-term investment choices.

The models proposed in the paper can be also employed by regulatory bodies to tune the incentives of their sustainability initiatives. The policy makers will be able to use the results to assess the suitability of rules for farm sustainability transition. In fact, the comparison of the farmers' profitability among different scenarios will facilitate the design of adequate business tools by agri-food value-chain actors aimed to pull farmers into more sustainable and profitable arable land management.

Furthermore, although the presented approaches are based on CAP and SVC policies, they can be extended and adapted for ex-ante evaluation and monitoring activities, related to safeguard soil fertility and agricultural systems productivity, in other regions and pedo-climatic contexts. At this aim, the data collected for quite homogeneous areas by single states and/or international government agencies, e.g., see the FAO project "Agro-Ecological Zones" (Anon, 2023), can be used to detect "optimal" crop rotation patterns tailored on socio-economic and pedoclimatic features. As stated by United Nations - Agenda 2030 (United Nations, 2015), synergies between public&private actors are welcome when they promote Sustainable Development Goals (SDGs) oriented actions (Singh et al., 2021). As in the case analyzed, CAP and SVC could address food systems transformation coherently to SDGs 2, 13 and 14, providing data and information useful to improve farmers and food value chain operators awareness about new production model profitability. Hence, based on estimated data for land uses, expected yields and costs references, models similar to those proposed in the paper can be devised and used to design of SDGs targeted policy tools in EU and in different agrarian regions (Matthews, 2020).

As future research, we devise to develop new and more effective solution approaches for the addressed problems. At this aim, as many solution methods for crop rotation planning proposed in the literature are based on column generation schemes (see Section 2), where each column is generally associated to a given crop sequence on the whole planning horizon, we leave as future research the study of solution methods based on column generation. Other future research directions include (i) the use of the network flow approach presented in Section 5 for the solution of more general cases (e.g., by employing Lagrangian Relaxation techniques for handling the constraints of the CAP and CAP+SVC scenarios), (ii) the development of ad-hoc models for solving CRP-k in other pedo-climatic contexts.

CRediT authorship contribution statement

Mario Benini: Conceptualization, Analysis, Methodology, Software, Writing. **Emanuele Blasi:** Conceptualization, Analysis, Methodology, Writing. **Paolo Detti:** Conceptualization, Analysis, Methodology, Software, Writing. **Lorenzo Fosci:** Conceptualization, Analysis, Methodology, Writing.

Data availability

Data will be made available on request.

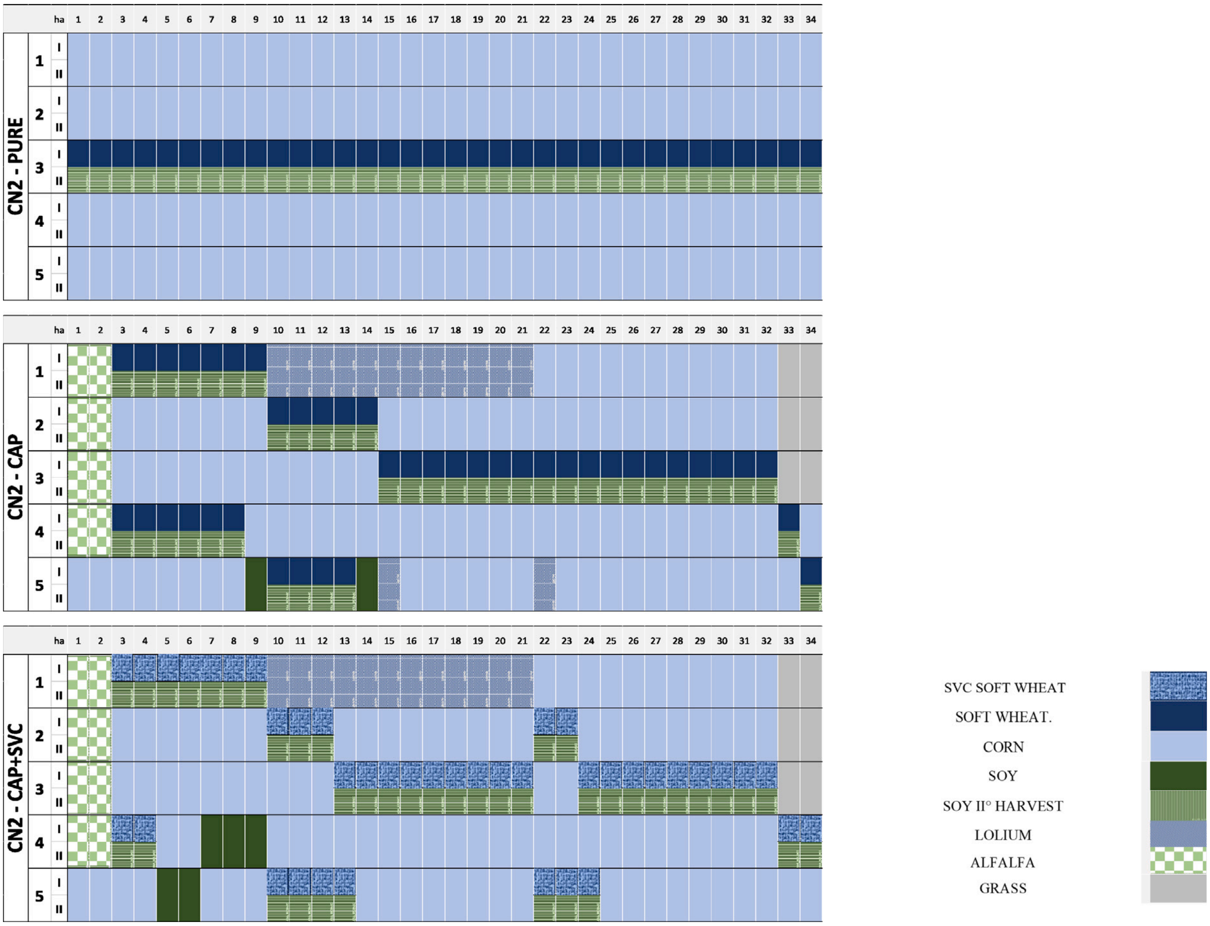

Fig. 3. Optimal crop sequences on each hectare of the instance PIE_CN2 on the three scenarios.

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Appendix. Remaining constraints of the CAP+SVC scenario

In this section, the remaining greening, diversification and repetition constraints of the CAP+SVC scenario are reported.

The greening constraints read as follows:

$$\sum_{i\in C_L}\sum_{q=t+1}^{t+5} x_{ih}^{q} \ge x_{ch}^{t} + \sum_{i\in C_S} x_{ih}^{t+1} - \sum_{i\in C_S} x_{ih}^{t+3} + \sum_{i\in C_S} x_{ih}^{t+5} - 2,$$
$$\forall c\in C_{CdM}, \forall h\in H, \forall t\in T_o \tag{28}$$

$$\sum_{i\in C_L}\sum_{q=t+1}^{t+5} x_{ih}^{q} \ge x_{ch}^{t} - \sum_{i\in C_S} x_{ih}^{t+1} + \sum_{i\in C_S} x_{ih}^{t+3} + \sum_{i\in C_S} x_{ih}^{t+5} - 2,$$
$$\forall c\in C_{CdM}, \forall h\in H, \forall t\in T_o \tag{29}$$

$$\sum_{i\in C_L}\sum_{q=t+1}^{t+6} x_{ih}^{q} \ge x_{ch}^{t} - \sum_{i\in C_S} x_{ih}^{t+1} - \sum_{i\in C_S} x_{ih}^{t+3} + \sum_{i\in C_S} x_{ih}^{t+5} - 1,$$
$$\forall c\in C_{CdM}, \forall h\in H, \forall t\in T_o \tag{30}$$

$$\sum_{i\in C_L}\sum_{q=t+1}^{t+7} x_{ih}^{q} \ge x_{ch}^{t} - \sum_{i\in C_S} x_{ih}^{t+1} - \sum_{i\in C_S} x_{ih}^{t+3} - \sum_{i\in C_S} x_{ih}^{t+5} + \sum_{i\in C_S} x_{ih}^{t+7} - 1,$$
$$\forall c\in C_{CdM}, \forall h\in H, \forall t\in T_o \tag{31}$$

$$\sum_{i\in C_L}\sum_{q=t+1}^{t+8} x_{ih}^{q} \ge x_{ch}^{t} - \sum_{i\in C_S} x_{ih}^{t+1} - \sum_{i\in C_S} x_{ih}^{t+3} - \sum_{i\in C_S} x_{ih}^{t+5} - \sum_{i\in C_S} x_{ih}^{t+7},$$
$$\forall c\in C_{CdM}, \forall h\in H, \forall t\in T_o \tag{32}$$

The diversification constraints read as follows:

$$\sum_{i\in\{c,v\}} (x_{ih}^{t+1} + x_{ih}^{t+2} + x_{ih}^{t+4} + x_{ih}^{t+5}) \le 3 + (1 - x_{vh}^{t})$$
$$+ (1 - \sum_{i\in C_S} x_{ih}^{t+1}) + \sum_{c\in C_S} x_{ih}^{t+3} + (1 - \sum_{i\in C_S} x_{ih}^{t+5}) \tag{33}$$
$$\forall v\in CdM, \forall c\in C, \forall h\in H, t\in T_o$$

$$\sum_{i\in\{c,v\}} (x_{ih}^{t+1} + x_{ih}^{t+2} + x_{ih}^{t+3} + x_{ih}^{t+4}) \le 3 + (1 - x_{vh}^{t})$$
$$+ (1 - \sum_{i\in C_S} x_{ih}^{t+1}) + (1 - \sum_{c\in C_S} x_{ih}^{t+3}) \tag{34}$$
$$\forall v\in CdM, \forall c\in C, \forall h\in H, t\in T_o$$

$$\sum_{i\in\{c,v\}} (x_{ih}^{t+2} + x_{ih}^{t+3} + x_{ih}^{t+4} + x_{ih}^{t+6}) \le 3 + (1 - x_{vh}^{t}) + \sum_{i\in C_S} x_{ih}^{t+1}$$
$$+ (1 - \sum_{c\in C_S} x_{ih}^{t+3}) + \sum_{i\in C_S} x_{ih}^{t+5} \tag{35}$$
$$\forall v\in CdM, \forall c\in C, \forall h\in H, t\in T_o$$

$$\sum_{i\in\{c,v\}} (x_{ih}^{t+2} + x_{ih}^{t+3} + x_{ih}^{t+4} + x_{ih}^{t+5}) \le 3 + (1 - x_{vh}^{t}) + \sum_{i\in C_S} x_{ih}^{t+1}$$
$$+ (1 - \sum_{c\in C_S} x_{ih}^{t+3}) + (1 - \sum_{i\in C_S} x_{ih}^{t+5}) \tag{36}$$

$\forall v \in CdM, \forall c \in C, \forall h \in H, t \in T_o$

$$\sum_{i \in \{c,v\}} (x_{ih}^{t+2} + x_{ih}^{t+4} + x_{ih}^{t+5} + x_{ih}^{t+6}) \leq 3 + (1 - x_{vh}^{t}) + \sum_{i \in C_S} x_{ih}^{t+1} + \sum_{c \in C_S} x_{ih}^{t+3} + (1 - \sum_{i \in C_S} x_{ih}^{t+5}) \quad (37)$$
$\forall v \in CdM, \forall c \in C, \forall h \in H, t \in T_o$

$$\sum_{i \in \{c,v\}} (x_{ih}^{t+2} + x_{ih}^{t+4} + x_{ih}^{t+6} + x_{ih}^{t+7}) \leq 3 + (1 - x_{vh}^{t}) + \sum_{i \in C_S} x_{ih}^{t+1} + \sum_{c \in C_S} x_{ih}^{t+3} + \sum_{i \in C_S} x_{ih}^{t+5} + (1 - \sum_{i \in C_S} x_{ih}^{t+7}) \quad (38)$$
$\forall v \in CdM, \forall c \in C, \forall h \in H, t \in T_o$

$$\sum_{i \in \{c,v\}} (x_{ih}^{t+2} + x_{ih}^{t+4} + x_{ih}^{t+6} + x_{ih}^{t+8}) \leq 3 + (1 - x_{vh}^{t}) + \sum_{i \in C_S} x_{ih}^{t+1} + \sum_{c \in C_S} x_{ih}^{t+3} + \sum_{i \in C_S} x_{ih}^{t+5} + \sum_{i \in C_S} x_{ih}^{t+7} \quad (39)$$
$\forall v \in CdM, \forall c \in C, \forall h \in H, t \in T_o$

The repetition constraints are reported in the following.

$$\sum_{c \in CR} (x_{ch}^{t_1} + x_{ch}^{t_2} + x_{ch}^{t_3}) \leq 2 + (1 - \sum_{i \in C_S} x_{ih}^{t+1}) + \sum_{c \in C_S} x_{ih}^{t+3} + (1 - \sum_{i \in C_S} x_{ih}^{t+5}) + (1 - x_{vh}^{t}) \quad (40)$$
$\forall v \in CdM, \forall h \in H, t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega$

$$\sum_{c \in CR} (x_{ch}^{t_1} + x_{ch}^{t_2} + x_{ch}^{t_3}) \leq 2 + (1 - \sum_{i \in C_S} x_{ih}^{t+1}) + (1 - \sum_{c \in C_S} x_{ih}^{t+3}) + (1 - x_{vh}^{t}) \quad (41)$$
$\forall v \in CdM, \forall h \in H, t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega$

$$\sum_{c \in CR} (x_{ch}^{t_1} + x_{ch}^{t_2} + x_{ch}^{t_3}) \leq 2 + \sum_{i \in C_S} x_{ih}^{t+1} + (1 - \sum_{c \in C_S} x_{ih}^{t+3}) + \sum_{i \in C_S} x_{ih}^{t+5} + (1 - x_{vh}^{t}) \quad (42)$$
$\forall v \in CdM, \forall h \in H, t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega$

$$\sum_{c \in CR} (x_{ch}^{t_1} + x_{ch}^{t_2} + x_{ch}^{t_3}) \leq 2 + \sum_{i \in C_S} x_{ih}^{t+1} + (1 - \sum_{c \in C_S} x_{ih}^{t+3}) + (1 - \sum_{i \in C_S} x_{ih}^{t+5}) + (1 - x_{vh}^{t}) \quad (43)$$
$\forall v \in CdM, \forall h \in H, t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega$

$$\sum_{c \in CR} (x_{ch}^{t_1} + x_{ch}^{t_2} + x_{ch}^{t_3}) \leq 2 + \sum_{i \in C_S} x_{ih}^{t+1} + \sum_{c \in C_S} x_{ih}^{t+3} + (1 - \sum_{i \in C_S} x_{ih}^{t+5}) + (1 - x_{vh}^{t}) \quad (44)$$
$\forall v \in CdM, \forall h \in H, t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega$

$$\sum_{c \in CR} (x_{ch}^{t_1} + x_{ch}^{t_2} + x_{ch}^{t_3}) \leq 2 + \sum_{i \in C_S} x_{ih}^{t+1} + \sum_{c \in C_S} x_{ih}^{t+3} + \sum_{i \in C_S} x_{ih}^{t+5} + (1 - \sum_{i \in C_S} x_{ih}^{t+7}) + (1 - x_{vh}^{t}) \quad (45)$$
$\forall v \in CdM, \forall h \in H, t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega$

$$\sum_{c \in CR} (x_{ch}^{t_1} + x_{ch}^{t_2} + x_{ch}^{t_3}) \leq 2 + \sum_{i \in C_S} x_{ih}^{t+1} + \sum_{c \in C_S} x_{ih}^{t+3} + \sum_{i \in C_S} x_{ih}^{t+5} + \sum_{i \in C_S} x_{ih}^{t+7} + (1 - x_{vh}^{t}) \quad (46)$$
$\forall v \in CdM, \forall h \in H, t \in T_o, \forall \{t_1, t_2, t_3\} \in \Omega$

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