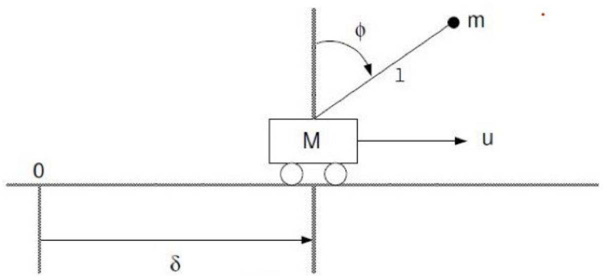
Caso Péndulo:



aceleración, aceleración, Velocidad

Modelo matemático no Lineal = $\begin{cases} (M+m)\ddot{\delta} + ml\ddot{\phi}\cos\phi - ml\dot{\phi}^2 \operatorname{sen}\phi + F\dot{\delta} = u \\ l\ddot{\phi} - g\operatorname{sen}\phi + \ddot{\delta}\cos\phi = 0 \end{cases}$ Ecinética

Epotencial, Fricción

$x = [\delta \quad \dot{\delta} \quad \phi \quad \dot{\phi}]^T,$

0 Asignación de las variables de estado

$$X = \begin{bmatrix} \delta \\ \dot{\delta} \\ \phi \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \Rightarrow \dot{X} = \begin{bmatrix} \dot{\delta} \\ \ddot{\delta} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix}$$

imponiendo salida: $Y = \begin{bmatrix} \delta \\ \phi \end{bmatrix}$ → posición/distancia, → ángulo de "ataque"

1 Defino Punto de Operación

$X_{op} = \phi = 0$ = equilibrio inestable

$$\Rightarrow X_{op} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad u_{op} = [0]$$

$$\cos\phi \Big|_{\phi \to 0} = 1$$

$$\operatorname{sen}\phi \Big|_{\phi \to 0} = \phi \longrightarrow \text{aproximación auxiliar}$$

2 Simplificación del modelo en el punto de operación en función de las variables de estado:

$$(M+m)\ddot{\delta} + ml\ddot{\phi}\cos\phi - ml\dot{\phi}^2 \operatorname{sen}\phi + F\dot{\delta} = u$$

$$l\ddot{\phi} - g\operatorname{sen}\phi + \ddot{\delta}\cos\phi = 0$$

$$\Downarrow$$

= Aproximaciones

$$(M+m)\dot{X}_2 + ml\dot{X}_4 \overbrace{\cos(X_3)}^{1} - ml X_4^2 \overbrace{\operatorname{sen}(X_3)}^{X_3} + F.X_2 = u$$

$$l.\dot{X}_4 - g\underbrace{\operatorname{sen}(X_3)}_{X_3} + \dot{X}_2 \underbrace{\cos(X_3)}_{1} = 0$$

Utilizando la aproximación válida en el punto de operación X_{op}:

$$(M+m)\dot{X}_2 + ml\dot{X}_4 - ml X_4^2 X_3 + F X_2 = u \quad (1)$$

$$l.\dot{X}_4 - g X_3 + \dot{X}_2 = 0 \quad (2)$$

③ Dejando todo en función de x_1, x_2, x_3, x_4, u obtengo las 4 funciones de estado:

$$\left.\begin{aligned} x_1 &= \delta \\ \dot{x}_1 &= \dot{\delta} \\ x_2 &= \dot{\delta} \end{aligned}\right\} \Rightarrow \boxed{\dot{x}_1 = x_2 \Rightarrow \dot{x}_1 = f_1(x_1, x_2, x_3, x_4, u)} \quad f_1$$

$$\left.\begin{aligned} x_3 &= \phi \\ \dot{x}_3 &= \dot{\phi} \\ x_4 &= \dot{\phi} \end{aligned}\right\} \Rightarrow \boxed{\dot{x}_3 = x_4 \Rightarrow \dot{x}_3 = f_3(x_1, x_2, x_3, x_4, u)} \quad f_3$$

$\dot{x}_2 = \ddot{\delta}$ = máxima derivada $\Rightarrow$ Despejo $\dot{x}_2$:

$$\dot{x}_4 = (g x_3 - \dot{x}_2) \cdot \frac{1}{\ell} \quad (3)$$

(3) en (1): $(M+m) \cdot \dot{x}_2 + m \cdot \cancel{\ell} \cdot \left[\frac{(g x_3 - \dot{x}_2)}{\cancel{\ell}} \right] - m \cdot \ell x_4^2 x_3 + F x_2 = u$

$$(M+m) \dot{x}_2 + m g x_3 - m \dot{x}_2 - m \ell x_4^2 x_3 + F x_2 = u$$

$$\dot{x}_2 \cdot [M + m - m] + m g x_3 - m \ell x_4^2 x_3 + F x_2 = u$$

$$\dot{x}_2 \cdot M + m g x_3 - m \ell x_4^2 x_3 + F x_2 = u$$

$$\boxed{\dot{x}_2 = -\frac{mg}{M} x_3 + \frac{m}{M} \ell x_4^2 x_3 - \frac{F}{M} x_2 + \frac{u}{M} \Rightarrow \dot{x}_2 = f_2(x_1, x_2, x_3 x_4, u)} \quad (4) \quad f_2$$

(4) en (3) para calcular $\dot{x}_4 = f(x_1, x_2, x_3, x_4, u)$:

$$\boxed{\dot{x}_4 = \frac{g}{\ell} . x_3 + \frac{mg}{M\ell} x_3 - \frac{m\cancel{\ell}}{M\cancel{\ell}} x_4^2 x_3 + \frac{F}{M\ell} x_2 - \frac{u}{M\ell} \Rightarrow \dot{x}_4 = f_4(x_1, x_2, x_3, x_4, u)} \quad f_4$$

④ Aplico Linealización por Taylor:

$$\dot{\mathbf{X}}_i = f(\mathbf{X}_0, \mathbf{U}_0) + \sum_{j=1}^{n} \left. \frac{\partial f_i(\mathbf{X}_t, \mathbf{U}_t)}{\partial X_j} \right|_{X_0, U_0} (X_j - X_{0j}) + \sum_{j=1}^{r} \left. \frac{\partial f_i(\mathbf{X}_t, \mathbf{U}_t)}{\partial U_j} \right|_{X_0, U_0} (\mathbf{U}_j - U_{0j}), \quad i = 1,2 \ldots n.$$

$$\mathbf{Y}_h = g(\mathbf{X}_0, \mathbf{U}_0) + \sum_{j=1}^{n} \left. \frac{\partial g_h(\mathbf{U}_t, \mathbf{U}_t)}{\partial X_j} \right|_{X_0, U_0} (X_j - X_{0j}) + \sum_{j=1}^{r} \left. \frac{\partial g_h(\mathbf{X}_t, \mathbf{U}_t)}{\partial \mathbf{U}_j} \right|_{X_0, U_0} (\mathbf{U}_j - \mathbf{U}_{0j}), \quad h = 1,2, \ldots . m.$$

$$\downarrow$$

$$\begin{aligned} \dot{\mathbf{x}}_t &= \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t \\ \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \mathbf{D}\mathbf{u}_t \end{aligned} \qquad \text{con} \quad \dot{x}_t = \dot{x} - f(x_0, u_0)$$

Cálculo de las matrices del modelo lineal:

$$f_1 \begin{cases} \left.\dfrac{\partial f_1}{\partial x_1}\right|_{X_{op}} = 0 \quad \left.\dfrac{\partial f_1}{\partial x_2}\right|_{X_{op}} = 1 \quad \left.\dfrac{\partial f_1}{\partial x_3}\right|_{X_{op}} = 0 \quad \left.\dfrac{\partial f_1}{\partial x_4}\right|_{X_{op}} = 0 \\ \left.\dfrac{\partial f_1}{\partial u}\right|_{X_{op}} = 0 \end{cases}$$

$$f_2 \begin{cases} \left.\dfrac{\partial f_2}{\partial x_1}\right|_{X_{op}} = 0 \quad \left.\dfrac{\partial f_2}{\partial x_2}\right|_{X_{op}} = -\dfrac{F}{M} \quad \left.\dfrac{\partial f_2}{\partial x_3}\right|_{X_{op}} = -\dfrac{mg}{M} + \dfrac{m}{M} l \overbrace{x_4}^{X_{4op}\ 0}{}^2 = -\dfrac{mg}{M} \quad \left.\dfrac{\partial f_2}{\partial x_4}\right|_{X_{op}} = 2 \cdot \dfrac{m}{M} l x_4 x_3 = 0 \\ \left.\dfrac{\partial f_2}{\partial u}\right|_{X_{op}} = \dfrac{1}{M} \end{cases}$$

$$f_3 \begin{cases} \left.\dfrac{\partial f_3}{\partial x_1}\right|_{X_{op}} = 0 \quad \left.\dfrac{\partial f_3}{\partial x_2}\right|_{X_{op}} = 0 \quad \left.\dfrac{\partial f_3}{\partial x_3}\right|_{X_{op}} = 0 \quad \left.\dfrac{\partial f_3}{\partial x_4}\right|_{X_{op}} = 1 \\ \left.\dfrac{\partial f_1}{\partial u}\right|_{X_{op}} = 0 \end{cases}$$

$$f_4 \begin{cases} \left.\dfrac{\partial f_4}{\partial x_1}\right|_{X_{op}} = 0 \quad \left.\dfrac{\partial f_4}{\partial x_2}\right|_{X_{op}} = \dfrac{F}{Ml} \quad \left.\dfrac{\partial f_4}{\partial x_3}\right|_{X_{op}} = \dfrac{(M+m)}{Ml} \cdot g \quad \left.\dfrac{\partial f_4}{\partial x_4}\right|_{X_{op}} = -\dfrac{m}{M} \cdot 2 \cdot x_4 \cdot x_3 = 0 \\ \dfrac{\partial f_4}{\partial u} = -\dfrac{1}{Ml} \end{cases}$$

⇒ Matricialmente:

Modelo Lineal en el punto de operación:

$$\begin{bmatrix} \dot{\delta} \\ \ddot{\delta} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{F}{M} & -\dfrac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{F}{lM} & \dfrac{g(m+M)}{lM} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{M} \\ 0 \\ -\dfrac{1}{lM} \end{bmatrix} \cdot u$$

$$Y = \begin{bmatrix} \delta \\ \phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{array}{c} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \end{array} \begin{array}{l} \delta \\ \dot{\delta} \\ \phi \\ \dot{\phi} \end{array}$$

* Diferencia entre sistema lineal y sistema no lineal radica en el truncamiento realizado al expresar f como una serie de Taylor