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Identification for the second-order systems based on the step response[☆]

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ABSTRACT

Based on the step response data, an identification method is presented to estimate the parameters of second-order inertial systems. Using special data points, the transcendental equations are changed into algebraic equations which are easy to solve for computing the parameters of the transfer function models. The numerical examples indicate that the proposed approaches can estimate the parameters of the systems.

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1. Introduction

A system may be described by the mapping relationship (i.e., the transfer function) from the input to the output. For a linear system, its transfer function is defined as the ratio of the Laplace transforms of the input to the output and is not related to the system's input and output. The so-called system identification or parameter estimation is to identify/estimate the parameters of systems or transfer function models from available input–output data.

Parameter estimation have had important applications in system modelling, signal processing, filtering, adaptive control [1–8]. Systems are divided into continuous-time systems and discrete-time systems. Many identification methods discuss parameter estimation problems of discrete-time systems, e.g., the least squares algorithms [9,10], the stochastic gradient algorithms [11–13], the multi-innovation algorithms [14–25], the auxiliary model based identification algorithms [26–31], the hierarchical identification algorithms [32–37], the iterative algorithms [38–41] which can be used to find the iterative solutions of matrix equations [42–49], and the identification algorithms of non-stationary or nonlinear systems [50–54].

If the input of a continuous-time system is taken as a step signal, the corresponding output is called the step response. Identification of continuous-time linear systems may be defined as estimating the parameters of the transfer function models obtained by means of the Laplace transform. In the areas of continuous-time system identification, Wang, Guo and Zhang presented a direct identification approach of continuous-time delay systems from the step response [55]; Bi et al. studied robust identification problems of first-order plus a dead-time model from the step response [56]; Ahmed, Huang and Shah considered identification from step responses with transient initial conditions [57]. This paper studies identification methods of the parameters of the transfer functions from the step response data. Such identification methods are called the classical identification ones.

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This paper is organized as follows. Section 2 introduces some preliminary facts related to the Laplace transform. Section 3 derives the transfer function models from linear differential equations with constant coefficients. Sections 4 and 5 study the parameter identification methods for second-order systems. Section 6 provides several examples to illustrate the proposed methods. Section 7 simply concludes the work of the paper.

2. Preliminary facts

The basic facts of this section can be found in many textbooks and their proofs are omitted.

The Laplace transform is used for solving differential and integral equations. In physics and engineering, it is used for the analysis of linear time-invariant systems such as electrical circuits and mechanical systems. In this analysis, the Laplace transform is often interpreted as a transformation from the time-domain, in which inputs and outputs are functions of time, to the frequency-domain. The following simply introduces the definition and properties of the Laplace transform.

For a real function $f(t)$ of time t over the interval $[0, +\infty)$, its Laplace transform is defined by

$$F(s) := \int_0^{+\infty} f(t)\mathrm{e}^{-st}\mathrm{d}t,$$

which is simply denoted by

$$F(s) = \mathscr{L}[f(t)],$$

where s is the Laplace operator (a complex variable with a large real part), $\mathscr{L}$ is the symbol of the Laplace transform, $f(t)$ is the original function. A necessary condition for existence of the integral is that $f(t)$ must be integrable on $[0, \infty)$.

The inverse Laplace transform is given by the following complex integral,

$$f(t) = \frac{1}{2\pi \mathrm{j}} \int_{c-\mathrm{j}\infty}^{c+\mathrm{j}\infty} F(s)\mathrm{e}^{st}\mathrm{d}t,$$

which is denoted by

$$f(t) = \mathscr{L}^{-1}[F(s)],$$

where $\mathscr{L}^{-1}$ is the symbol of the inverse Laplace transform, c is a large positive number.

In the analysis of control systems, one often uses several typical input signals. For example, the step function $f(t)$ with amplitude U is defined by

$$f(t) = \begin{cases} U, & t \geqslant 0, \\ 0, & t < 0. \end{cases} \tag{1}$$

When $U = 1$, $f(t)$ is called the unit step function, denoted by $1(t)$.

Several typical functions' Laplace transforms are as follows:

$$\mathscr{L}[1(t)] = \int_0^{+\infty} 1(t)\mathrm{e}^{-st}\mathrm{d}t = \int_0^{+\infty} \mathrm{e}^{-st}\mathrm{d}t = \frac{1}{s}.$$

$$\mathscr{L}[\mathrm{e}^{-at}] = \int_0^{+\infty} \mathrm{e}^{-at}\mathrm{e}^{-st}\mathrm{d}t = \frac{1}{s+a}.$$

The Laplace transform has a number of properties that make it useful for analyzing linear dynamical systems. The most significant advantage is that differentiation and integration become multiplication and division, respectively, by s (similarly to logarithms changing multiplication of numbers to addition of their logarithms). The transform turns integral equations and differential equations to polynomial equations, which are much easier to solve. Once solved, use of the inverse Laplace transform reverts back to the time domain.

Given the functions $f_1(t)$ and $f_2(t)$, and their respective Laplace transforms $F_1(s) = \mathscr{L}[f_1(t)]$ and $F_2(s) = \mathscr{L}[f_2(t)]$, or $f_1(t) = \mathscr{L}^{-1}[F_1(s)]$ and $f_2(t) = \mathscr{L}^{-1}[F_2(s)]$. Let c_1 and c_2 be constants. Then we have the linear property:

$$\mathscr{L}[c_1 f_1(t) + c_2 f_2(t)] = c_1\mathscr{L}[f_1(t)] + c_2\mathscr{L}[f_2(t)] = c_1F_1(s) + c_2F_2(s).$$

Assuming that $F(s) = \mathscr{L}[f(t)]$, we have the differential property of the Laplace transforms:

$$\mathscr{L}\left[\frac{\mathrm{d}f(t)}{\mathrm{d}t}\right] = sF(s) - f(0). \tag{2}$$

The final value theorem of the Laplace transform indicates that

$$f(\infty) = \lim_{t\to+\infty} f(t) = \lim_{s\to 0} sF(s).$$

3. The transfer function models

A linear dynamical system can be described by a time-invariant linear differential equation:

$$y^{(n)}(t) + a_1y^{(n-1)}(t) + \cdots + a_{n-1}y^{(1)}(t) + a_ny(t) = b_1u^{(n-1)}(t) + b_2u^{(n-2)}(t) + \cdots + b_{n-1}u^{(1)}(t) + b_nu(t), \tag{3}$$

where $u(t)$ and $y(t)$ denote the input and output of the system, t represents the time variable, and

$$y^{(i)}(t)=\frac{\mathrm{d}^iy(t)}{\mathrm{d}t^i},\qquad y^{(2)}(t)=y''(t),\qquad y^{(1)}(t)=\dot{y}(t)=y'(t),\qquad y^{(0)}(t)=y(t),$$
$$u^{(i)}(t)=\frac{\mathrm{d}^iu(t)}{\mathrm{d}t^i},\qquad u^{(2)}(t)=u''(t),\qquad u^{(1)}(t)=\dot{u}(t)=u'(t),\qquad u^{(0)}(t)=u(t).$$

a_i and b_i are the parameters of this system.

Under the zero initial values, taking the Laplace transform to both sides of (3) and using the differential property in (2) give

$$(s^n+a_1s^{n-1}+\cdots+a_{n-1}s+a_n)Y(s)=(b_1s^{n-1}+b_2s^{n-2}+\cdots+b_{n-1}s+b_n)U(s),$$

where $Y(s)$ and $U(s)$ are the Laplace transforms of $y(t)$ and $u(t)$:

$$Y(s):=\mathscr{L}[y(t)],\qquad U(s):=\mathscr{L}[u(t)].$$

Hence, we have the transfer function of the system:

$$G(s):=\frac{Y(s)}{U(s)}=\frac{b_1s^{n-1}+b_2s^{n-2}+\cdots+b_{n-1}s+b_n}{s^n+a_1s^{n-1}+\cdots+a_{n-1}s^{n-2}+a_n},\tag{4}$$

which can be equivalently rewritten as

$$G(s)=\frac{K(T_{n+1}s+1)(T_{n+2}s+1)\cdots(T_{2n-1}s+1)}{(T_1s+1)(T_2s+1)(T_ns+1)}.$$

The gain is $K:=b_n/a_n$ and the time constants are $T_i\geqslant 0$ for stable systems.

Assume that $u(t)$ is a step function with amplitude U. Its Laplace transform is

$$U(s)=\mathscr{L}[u(t)]=\begin{cases}\dfrac{U}{s}, & t\geqslant 0,\\ 0, & t<0.\end{cases}$$

The Laplace transform of the output $y(t)$ is

$$Y(s)=G(s)U(s).$$

According to the final value theorem, we have

$$y(\infty)=\lim_{s\to 0}Y(s)s=\lim_{s\to 0}G(s)U(s)s=KU.\tag{5}$$

Or

$$K=\frac{y(\infty)}{U}.$$

This implies that the gain K is equal to the ratio of the stable output $y(\infty)$ to the amplitude U of the input of the system. Without loss of generality, let $U=1$.

4. The second-order systems with the same poles

Consider a second-order system with the same poles plus a zero, whose transfer function is given by

$$G(s)=\frac{K(T_2s+1)}{(T_1s+1)^2},\quad T_1\neq T_2.\tag{6}$$

Here, the time constants T_1 and T_2 and the system gain K are the parameters to be identified from the input–output data $\{u(t),y(t)\}$.

When the input is taken as a unit step function, i.e., $u(t)=1$ for $t\geqslant 0$, the Laplace transform of the system output is

$$\begin{aligned}Y(s)&=G(s)U(s)=\frac{K(T_2s+1)}{(T_1s+1)^2}\frac{1}{s}\\&=K\left[\frac{1}{s}-\frac{T_1}{T_1s+1}-\frac{T_1-T_2}{T_1^2}\frac{T_1^2}{(T_1s+1)^2}\right].\end{aligned}\tag{7}$$

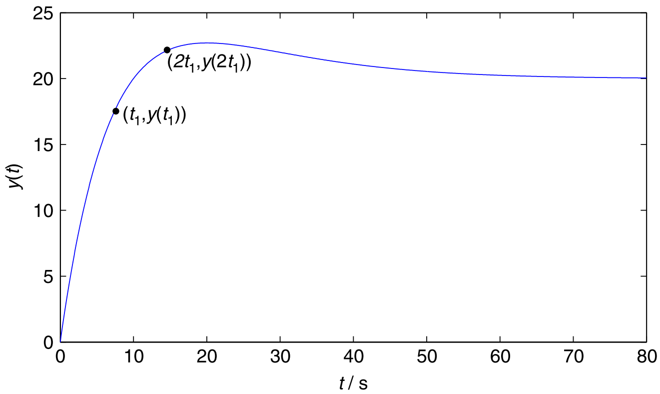


Fig. 1. The step response curve.

Its inverse Laplace transform (i.e., the system response) is given by

$$y(t) = \mathscr{L}^{-1}[Y(s)] = K\left[1 - \mathrm{e}^{-t/T_1} - \frac{T_1 - T_2}{T_1^2} t\mathrm{e}^{-t/T_1}\right]$$
$$= K[1 - \mathrm{e}^{-t/T_1} + \beta t\mathrm{e}^{-t/T_1}], \tag{8}$$

where

$$\beta := -\frac{T_1 - T_2}{T_1^2}. \tag{9}$$

From (8), we can see that $K = y(\infty)$.

We can take two points $(t_1, y(t_1))$ and $(t_2, y(t_2))$ on the step response curve in Fig. 1 and substitute them into (8) and then get two equations

$$\begin{cases} y(t_1) = K[1 - \mathrm{e}^{-t_1/T_1} + \beta t_1\mathrm{e}^{-t_1/T_1}], \\ y(t_2) = K[1 - \mathrm{e}^{-t_2/T_1} + \beta t_2\mathrm{e}^{-t_2/T_1}]. \end{cases}$$

This is a transcendental equation, so we cannot obtain the solution of T_1 and T_2 using the algebraic method. In order to avoid the transcendental equation, we take two special points $(t_1, y(t_1))$ and $(2t_1, y(2t_1))$ on the step response curve and substitute them into (8) and then get two equations:

$$\begin{cases} y(t_1) = K[1 - \mathrm{e}^{-t_1/T_1} + \beta t_1\mathrm{e}^{-t_1/T_1}], \\ y(2t_1) = K[1 - \mathrm{e}^{-2t_1/T_1} + \beta 2t_1\mathrm{e}^{-2t_1/T_1}]. \end{cases}$$

Or

$$\begin{cases} y(t_1) = K(1 - \alpha + \beta\alpha t_1), \\ y(2t_1) = K(1 - \alpha^2 + 2\beta\alpha^2 t_1), \end{cases} \tag{10}$$

where

$$\alpha := \mathrm{e}^{-t_1/T_1}. \tag{11}$$

Let

$$k_1 := \frac{y(t_1)}{K} - 1, \qquad k_2 := \frac{y(2t_1)}{K} - 1. \tag{12}$$

Eq. (10) gives

$$-\alpha + \beta\alpha t_1 = k_1, \tag{13}$$

$$-\alpha^2 + 2\beta\alpha^2 t_1 = k_2. \tag{14}$$

Squaring both sides of (13) plus (14) gives

$$\beta\alpha t_1 = \pm\sqrt{k_1^2 + k_2}.$$

Consider $\alpha > 0$ and substituting into (13), we have

$$\alpha = -k_1 + \sqrt{k_1^2 + k_2}. \tag{15}$$

Substituting into (13) gives

$$\beta = \frac{\sqrt{k_1^2 + k_2}}{\alpha t_1}. \tag{16}$$

Substituting (15) and (16) into (9) and (11) gives

$$\hat{T}_1 = \frac{-t_1}{\ln \alpha},$$
$$\hat{T}_2 = \beta \hat{T}_1^2 + \hat{T}_1.$$

To enhance the estimation accuracy, we may take $(t_2, y(t_2))$ and $(2t_2, y(2t_2))$, $(t_3, y(t_3))$ and $(2t_3, y(2t_3)), \ldots, (t_N, y(t_N))$ and $(2t_N, y(2t_N))$, respectively, repeating the above procedure gives a series of the estimates $\hat{T}_{ij}$ of T_i, we take their average

$$\hat{T}_1 = \frac{\hat{T}_{11} + \hat{T}_{12} + \cdots + \hat{T}_{1N}}{N} \quad \text{and} \quad \hat{T}_2 = \frac{\hat{T}_{21} + \hat{T}_{22} + \cdots + \hat{T}_{2N}}{N}$$

as the estimates of T_1 and T_2.

5. The second-order systems with different poles

Consider a second-order system with different poles plus a zero:

$$G(s) = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)}, \quad T_1 < T_2,\ T_3 \neq T_1,\ T_3 \neq T_2, \tag{17}$$

where K is the gain, and T_1, T_2 and T_3 are the time constants.

Let $u(t)$ be a unit step function. We have

$$Y(s) = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)} \frac{1}{s}.$$

The step response is

$$y(t) = K\left[1 + \frac{T_3 - T_1}{T_1 - T_2} \mathrm{e}^{-t/T_1} - \frac{T_3 - T_2}{T_1 - T_2} \mathrm{e}^{-t/T_2}\right]. \tag{18}$$

Notice $\hat{K} = y(\infty)$. Taking three points $(t_1, y(t_1))$, $(2t_1, y(2t_1))$ and $(3t_1, y(3t_1))$ on the step response curve in Fig. 2 and substituting them into (18) give

$$\begin{cases} y(t_1) = K\left[1 + \dfrac{T_3 - T_1}{T_1 - T_2} \mathrm{e}^{-t_1/T_1} - \dfrac{T_3 - T_2}{T_1 - T_2} \mathrm{e}^{-t_1/T_2}\right], \\ y(2t_1) = K\left[1 + \dfrac{T_3 - T_1}{T_1 - T_2} \mathrm{e}^{-2t_1/T_1} - \dfrac{T_3 - T_2}{T_1 - T_2} \mathrm{e}^{-2t_1/T_2}\right], \\ y(3t_1) = K\left[1 + \dfrac{T_3 - T_1}{T_1 - T_2} \mathrm{e}^{-3t_1/T_1} - \dfrac{T_3 - T_2}{T_1 - T_2} \mathrm{e}^{-3t_1/T_2}\right]. \end{cases}$$

Let

$$\alpha_1 := \exp(-t_1/T_1), \qquad \alpha_2 := \exp(-t_1/T_2), \qquad \beta := \frac{T_3 - T_1}{T_1 - T_2}. \tag{19}$$

Then we have

$$\begin{cases} y(t_1) = K[1 + \beta\alpha_1 - (1 + \beta)\alpha_2], \\ y(2t_1) = K[1 + \beta\alpha_1^2 - (1 + \beta)\alpha_2^2], \\ y(3t_1) = K[1 + \beta\alpha_1^3 - (1 + \beta)\alpha_2^3]. \end{cases}$$

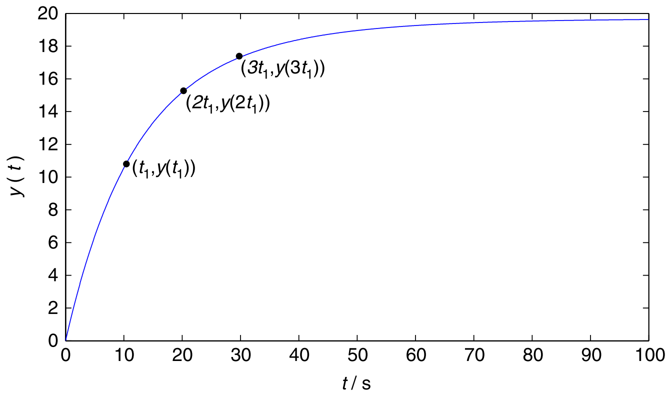


Fig. 2. The step response curve.

Let

$$k_1 := \frac{y(t_1)}{K} - 1, \qquad k_2 := \frac{y(2t_1)}{K} - 1, \qquad k_3 := \frac{y(3t_1)}{K} - 1.$$

Hence, we have the following equations:

$$\begin{aligned} &\beta(\alpha_1 - \alpha_2) - \alpha_2 = k_1, \\ &\beta(\alpha_1^2 - \alpha_2^2) - \alpha_2^2 = k_2, \\ &\beta(\alpha_1^3 - \alpha_2^3) - \alpha_2^3 = k_3, \end{aligned}$$

or

$$\begin{aligned} &\beta(\alpha_1 - \alpha_2) = k_1 + \alpha_2, \\ &\beta(\alpha_1^2 - \alpha_2^2) = k_2 + \alpha_2^2, \\ &\beta(\alpha_1^3 - \alpha_2^3) = k_3 + \alpha_2^3. \end{aligned} \tag{20}$$

Eliminating β gives

$$\begin{aligned} &k_1(\alpha_1 + \alpha_2) + \alpha_1\alpha_2 = k_2, \\ &k_1[(\alpha_1 + \alpha_2)^2 - \alpha_1\alpha_2] + \alpha_1\alpha_2(\alpha_1 + \alpha_2) = k_3. \end{aligned}$$

Thus, we have

$$\alpha_1\alpha_2 = \frac{k_2^2 - k_1k_3}{k_1^2 + k_2}, \qquad \alpha_1 + \alpha_2 = \frac{k_3 + k_1k_2}{k_1^2 + k_2}.$$

Since $T_1 < T_2$, we have $\alpha_1 < \alpha_2$, and

$$\alpha_1 = \frac{k_1k_2 + k_3 - \sqrt{b}}{2(k_1^2 + k_2)}, \tag{21}$$

$$\alpha_2 = \frac{k_1k_2 + k_3 + \sqrt{b}}{2(k_1^2 + k_2)}, \tag{22}$$

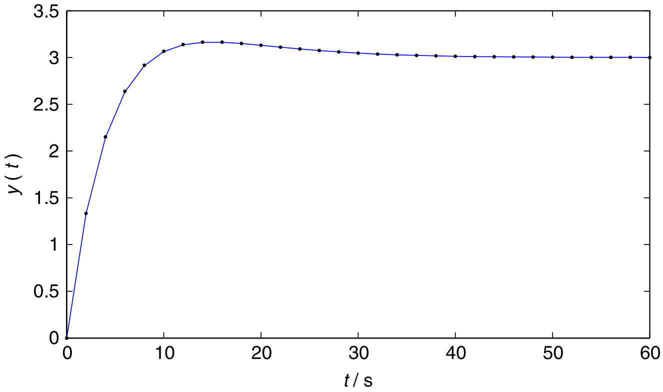
$$b := 4k_1^3k_3 - 3k_1^2k_2^2 - 4k_2^3 + k_3^2 + 6k_1k_2k_3. \tag{23}$$

Substituting α_1 and α_2 into (20) gives

$$\beta = \frac{2k_1^3 + 3k_1k_2 + k_3 - \sqrt{b}}{\sqrt{b}}. \tag{24}$$

Substituting (21)–(24) into (19) gives the estimates of the time constants T_1 and T_2 as follows:

$$\begin{aligned} &\hat{T}_1 = -\frac{t_1}{\ln \alpha_1}, \qquad \hat{T}_2 = -\frac{t_1}{\ln \alpha_2}, \\ &\hat{T}_3 = \beta(\hat{T}_1 - \hat{T}_2) + \hat{T}_1. \end{aligned}$$



The Solid line : the output of $G(s)$, dots: the output of $\hat{G}(s)$

Fig. 3. The step responses.

Table 1
The step response data.

t (s)	$y(t)$	t (s)	$y(t)$	t (s)	$y(t)$
0	0.00	12	12.54	24	12.37
2	5.31	14	12.65	26	12.30
4	8.58	16	12.65	28	12.24
6	10.53	18	12.60	...	...
8	11.65	20	12.52	70	12.00
10	12.25	22	12.44		

To enhance the estimation accuracy, we may take $(t_2, y(t_2))$, $(2t_2, y(2t_2))$ and $(3t_2, y(3t_2))$, $(t_3, y(t_3))$, $(2t_3, y(2t_3))$ and $(3t_3, y(3t_3))$, ..., $(t_N, y(t_N))$, $(2t_N, y(2t_N))$ and $(3t_N, y(3t_N))$, respectively, repeating the above procedure gives a series of the estimates $\hat{T}_{ij}$ of T_i, we take their average

$$\hat{T}_i = \frac{\hat{T}_{i1} + \hat{T}_{i2} + \cdots + \hat{T}_{iN}}{N}, \quad i = 1, 2, 3$$

as the estimates of T_i.

This method of determining the parameters of the second-order systems can be extended to higher-order systems.

About the choice of t_1, three scales t_1 and $t_2 = 2t_1$ (and $t_3 = 3t_1$) should generally be chosen uniformly from the dynamic process instead of the steady process in order to enhance the accuracy of the parameter estimates.

6. Examples

Example 1. Consider the following second-order system:

$$G(s) = \frac{3(10s + 1)}{(6s + 1)^2}.$$

In simulation, the input is taken as a step signal with amplitude 4, and considering the rounding errors (i.e., the noise), the step response data are shown in Table 1, keeping 2 decimal places.

From Table 1, we have $y(\infty) = 12$ and the gain $\hat{K} = y(\infty)/U = 3$.

We take $t_1 = 4$ s, $t_2 = 6$ s and $t_3 = 8$ s and use the data in Table 1 and the proposed method to determine the parameters of this example system, the estimated transfer function is

$$\hat{G}(s) = \frac{3(9.98s + 1)}{(5.98s + 1)^2}.$$

The step responses of $G(s)$ and $\hat{G}(s)$ are shown in Fig. 3.

Example 2. Consider the following second-order system:

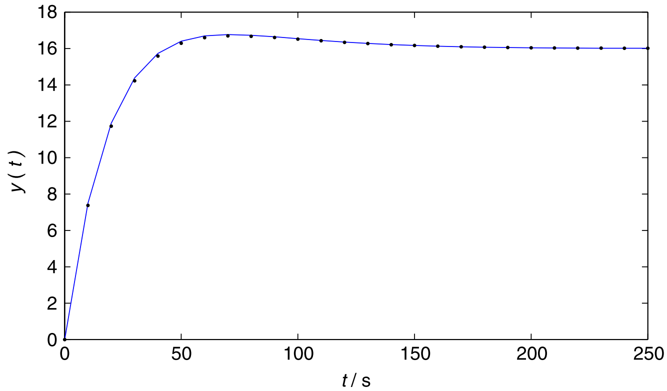
$$G(s) = \frac{16(45s + 1)}{(25s + 1)(30s + 1)}.$$

Table 2
The step response data.

t (s)	$y(t)$	t (s)	$y(t)$	t (s)	$y(t)$
0	0.00	30	71.91	60	83.45
5	21.16	35	75.83	65	83.73
10	37.47	40	78.66	70	83.81
15	49.95	45	80.66	...	...
20	59.43	50	82.02	400	80.00
25	66.58	55	82.91		

Table 3
The step response data.

t (s)	$y(t)$	t (s)	$y(t)$	t (s)	$y(t)$
0	0.17491	12	10.07839	24	10.22430
2	4.23428	14	10.23160	26	10.24584
4	6.82100	16	9.96224	28	10.20419
6	8.24238	18	10.11159	...	...
8	9.24654	20	10.33812	50	10.02477
10	9.66473	22	10.19159		



The Solid line : the output of $G(s)$, dots: the output of $\hat{G}(s)$

Fig. 4. The step responses.

In the simulation, the input is taken as a step signal with amplitude 5, and the step response data are shown in Table 2, keeping 2 decimal places. From Table 2, we have $y(\infty) = 80$ and the gain $\hat{K} = y(\infty)/U = 16$.

We take $t_1 = 10$ s, $t_2 = 15$ s and $t_3 = 20$ s and use the data in Table 2 and the proposed method to determine the parameters of this example system, the estimated transfer function is

$$\hat{G}(s) = \frac{16(45.71s + 1)}{(24.58s + 1)(31.06s + 1)}.$$

The step responses of $G(s)$ and $\hat{G}(s)$ are shown in Fig. 4.

Example 3. Consider the following second-order system:

$$G(s) = \frac{2(8s + 1)}{(5s + 1)(6s + 1)}.$$

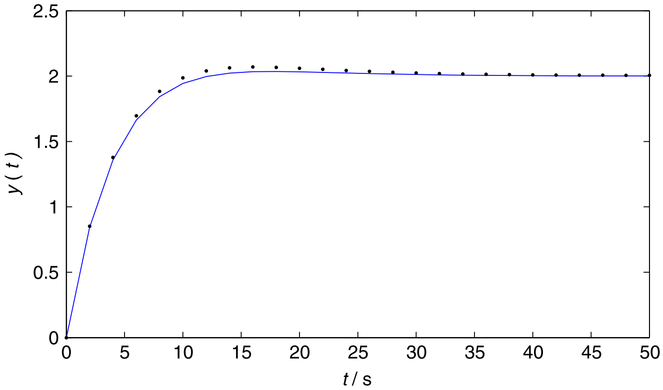
In simulation, the input is taken as a step signal with amplitude 5, the measured output $y(t)$ is contaminated by a white noise $v(t)$ with zero mean and variance $\sigma^2 = 0.10^2$, and the step response data are shown in Table 3.

From Table 3, we have $y(\infty) = 10.02477$ and the gain $\hat{K} = y(\infty)/U = 2.00495$.

We use the data with $t_1 = 4$ s, $t_2 = 6$ s and $t_3 = 8$ s in Table 3 and apply the proposed method to determine the parameters of this example system, the estimated transfer function is

$$\hat{G}(s) = \frac{2.00495(8.42536s + 1)}{(5.38064s + 1)(5.89253s + 1)}.$$

The step responses of $G(s)$ and $\hat{G}(s)$ are shown in Fig. 5.



The Solid line : the output of $G(s)$, dots: the output of $\hat{G}(s)$

Fig. 5. The step responses.

From Figs. 3–5, the step responses of the estimated transfer functions are very close to those of the example systems. This implies that the estimated models can capture the system's dynamics.

7. Conclusions

This paper uses the algebraic method to estimate the parameters of the transfer function models of second-order systems from step response data, avoiding the difficulty of solving transcendental equations. The numerical examples show that the proposed methods are effective for identifying the second-order systems.

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