1.2 Finite volume in fluid mechanics *(14 values out of 20)*

Consider the linearized potential flow equations and the small perturbations approach.

The linearized potential flow equation for subsonic and supersonic flows reads as:

$$\frac{\partial^2 \phi}{\partial x^2}(1 - M_\infty^2) + \frac{\partial^2 \phi}{\partial y^2} = 0$$

The linearized potential flow equation for transonic flows reads as:

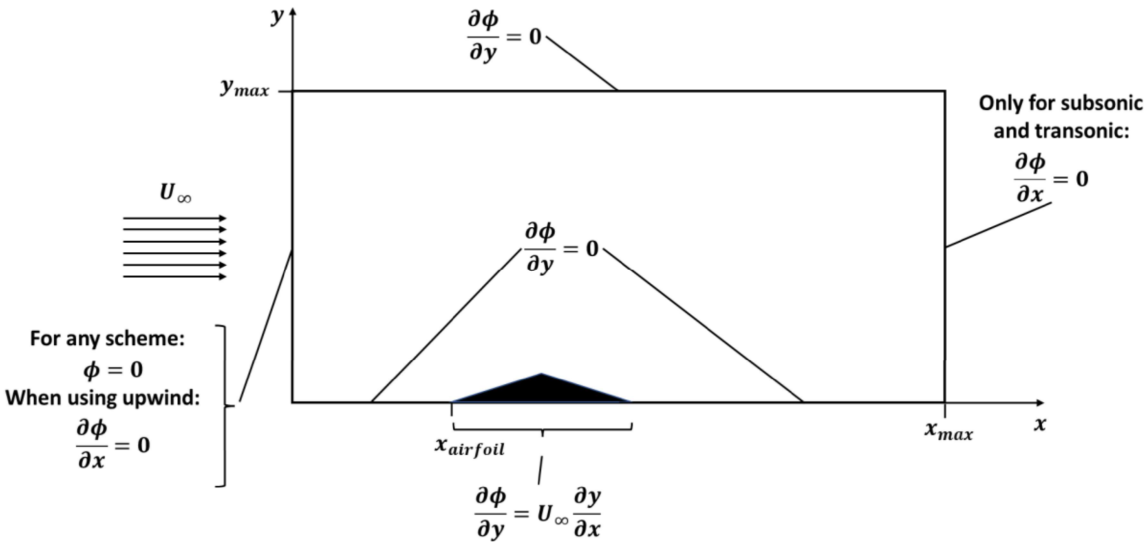
$$\frac{\partial^2 \phi}{\partial x^2}\left(1 - M_\infty^2 - (1+\gamma)\frac{M_\infty^2}{U_\infty}\frac{\partial \phi}{\partial x}\right) + \frac{\partial^2 \phi}{\partial y^2} = 0$$

The goal is to solve the problem illustrated in the next figure to obtain the potential flow field of the perturbed flow and thus to obtain the potential flow field. The proposed domain may be adapted to the actual data.

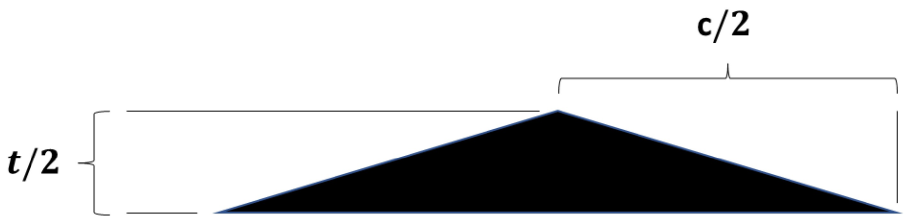
Remember that the potential flow field is $U = U_\infty + u$ and $V = V_\infty + v$, with $V_\infty = 0$, and u and v the field perturbations to be solved. Also remember that $u = \frac{\partial \phi}{\partial x}$ and $v = \frac{\partial \phi}{\partial y}$.

In the left boundary of the domain $\phi = 0$ and $u = u_{left} = u_\infty = 0$. The far field approaching flow velocity, U_∞, is only considered to the profile definition in the bottom boundary.

Also remember that, under this theory, the geometry of the airfoil is not to be defined in the mesh. The mesh covers the computational domain, which is a rectangle. The existence/influence of the airfoil in the flow field is only considered through the boundary condition $\frac{\partial \phi}{\partial y} = U_\infty \frac{dy}{dx}$.



For the diamond airfoil below, with half thickness $t/2 = 0.025c$ (c is the airfoil's chord, $c = 1\ m$), at zero angle of attack:



- Solve the subsonic flow equation for $M_\infty = 0.35$
- Solve the supersonic flow equation for $M_\infty = 1.8$
- Solve the transonic flow equation for a transonic $M_\infty = 0.95$

Consider reference values for the air temperature and pressure as $T = 288\ K$ and $p = 1\ atm$.
In the case of supersonic flow, the derivatives in the *x* direction must be regressive while the derivatives in the *y* direction may be central.
In the case of transonic flow, the derivatives in the *x* direction must be regressive or central according to the sign of $\left(1 - M_\infty^2 - (1+\gamma)\frac{M_\infty^2}{U_\infty}\frac{\partial \phi}{\partial x}\right)$. The derivatives in the *y* direction may be central.
For the subsonic case, use central differences for both directions.
For the supersonic case, in the left boundary, since we do not have any regressive point, the information used to close the problem can be the Dirichlet boundary condition $\phi = 0$ together with the Neumann boundary condition $\frac{\partial \phi}{\partial x} = 0$. You should discuss this boundary problem in your report.
Clearly indicate the iterative (or direct) method selected to solve the system of linear equations.

For the supersonic case, check if the shock angle and the Mach angle coincide: $\beta = arcsin\left(\frac{1}{Ma}\right)$.

To compare and analyse the results:

- Show the results of the potential field
- Show the Mach flow field
- Show the density field

A critical analysis of this problem is encouraged and plays an important role in the evaluation of (**1.2**).
In the following, find some suggestions of items that may be addressed:

- grid size influences
- boundary conditions
- uniform vs non-uniform grids
- domain size and airfoil relative position (x_{max}, y_{max} and $x_{airfoil}$)
- numerical schemes used
- solvers used for the solution of the algebraic systems of equations
- residuals decay for linear / non-linear problems
- how to address the non-linearities
- computational efforts related to different possible strategies
- etc.