

Homework (Session 5)

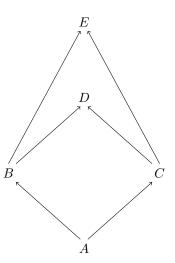
Bruno Vandekerkhove

Contents

| Pushout in C++ | 1 |
|----------------------------------|---|
| Limit of the identity functor | 1 |
| Pullback, pushout, in Set | 2 |
| Coequalizer | 2 |
| Pullback towards terminal object | 2 |
| Pushout from initial object | 2 |

Pushout in C++

We talked about products and co-products in posets before, where the term supremum and infimum applied. In this more general case applied on C++ types a diagram of a pushout D is as follows:



Any superclass E of B and C is a superclass of D. Which means that D corresponds to a strict superclass, one that encompasses all the functionality that the subclasses have in common¹.

 $^{^{1}}$ One has to consider all possible superclasses of B and C. Then D inherits from all of those. Some superclasses may only declare a part of the functionality that's present in both the classes B and C, yet together they declare all of it and nothing more or some wouldn't be superclasses.

Limit of the identity functor

The identity functor Id maps a category to itself. The initial object is the object for which there's exactly one morphism to every object. When constructing the limit for Id any apex will correspond to an initial object (or there wouldn't be a natural transformation from Δ_c to Id). If there are many apexes (or initial objects) there will be unique transformations between them. Indeed, initial objects are unique up to isomorphism, which we knew already.

Pullback, pushout, ... in Set

Coequalizer

Pullback towards terminal object

Pushout from initial object