Probabilistic Programming

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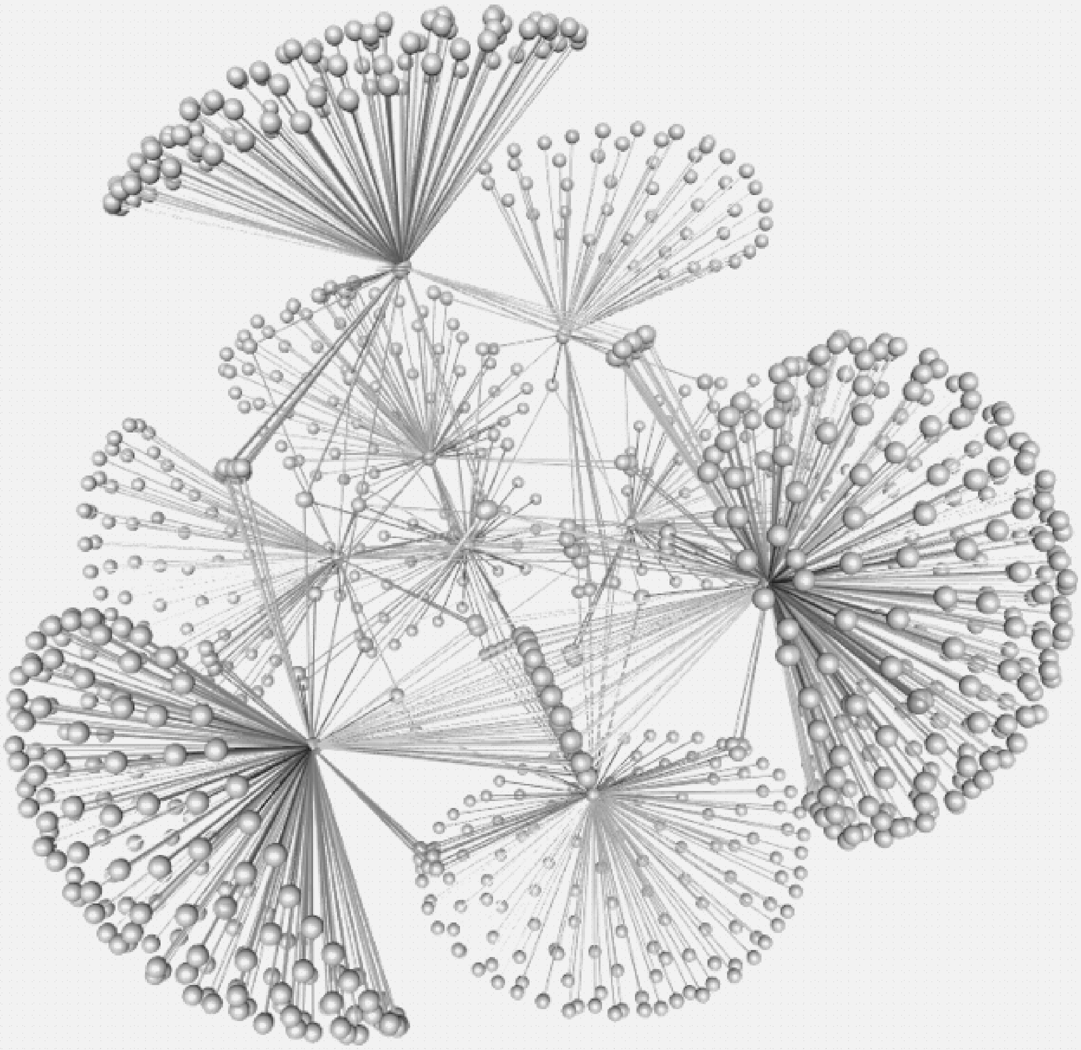

Academisch Jaar 2020

H05N0A: Capita Selecta : Artificial Intelligence

Contents

Below's our solution for the given challenges. The questions in each section of the original assignment are answered in a section having the same title.

```
person(a).
person(b).
person(c).
0.2::stress(X) :- person(X).
0.1::friends(X,Y) :- person(X), person(Y).
0.3::smokes(X) :- stress(X).
0.4::smokes(X) :- friends(X,Y), smokes(Y).
query(smokes(a)).
```

Code snippet 1: PROBLOG program used throughout the first two chapters of the report.

Probabilistic Inference Using Weighted Model Counting

SRL to CNF

First the program is grounded. This is a matter of collecting all atoms involved in all proofs of the query.

```
0.2::stress(a).
0.2::stress(b).
0.2::stress(c).

0.1::friends(a,a).
0.1::friends(a,b).
0.1::friends(a,c).

0.1::friends(b,a).
0.1::friends(b,b).
0.1::friends(b,c).

0.1::friends(c,a).
0.1::friends(c,b).
0.1::friends(c,c).

0.3::smokes(a) :- stress(a).
0.3::smokes(b) :- stress(b).
0.3::smokes(c) :- stress(c).
0.4::smokes(a) :- friends(a,a), smokes(a).
0.4::smokes(a) :- friends(a,b), smokes(b).
0.4::smokes(a) :- friends(a,c), smokes(c).
0.4::smokes(b) :- friends(b,a), smokes(a).
0.4::smokes(b) :- friends(b,b), smokes(b).
0.4::smokes(b) :- friends(b,c), smokes(c).

0.4::smokes(c) :- friends(c,a), smokes(a).
0.4::smokes(c) :- friends(c,b), smokes(b).
0.4::smokes(c) :- friends(c,c), smokes(c).
```

Code snippet 2: Relevant ground program.

The proofs of the query make for a trie as shown in figure 1, where colourings indicate the presence of cycles. Any proof involving an atom `friends(X,X)` or `friends(Y,a)` (with $Y \in \{b, c\}$) is non-minimal and doesn't affect the final probability. These atoms are disregarded. For the remaining cycles (involving `friends(b,c)` and `friends(c,b)`) auxiliary variables can be used to obtain a cycle-free program without intensional probabilistic facts :

```
0.2::stress(a).
0.2::stress(b).
0.2::stress(c).

```

```
0.1::friends(a,b).
0.1::friends(a,c).
0.1::friends(b,c).
0.1::friends(c,b).

0.3::p(a).
0.3::p(b).
0.3::p(c).

0.4::p(a,b).
0.4::p(a,c).
0.4::p(b,c).
0.4::p(c,b).

smokes(a) :- stress(a), p(a).
smokes(b) :- stress(b), p(b).
smokes(c) :- stress(c), p(c).

smokes(a) :-
    friends(a,b), smokes(b), p(a,b).
smokes(a) :-
    friends(a,c), smokes(c), p(a,c).
smokes(b) :-
    friends(b,c), stress(c), p(c), p(b,c).
smokes(c) :-
    friends(c,b), stress(b), p(b), p(c,b).

query(smokes(a)).
```

Code snippet 3: Relevant ground program without cycles.

The above logic program is equivalent to the following propositional formula :

$$\begin{array}{l}
(smokes(a) \leftrightarrow \quad (stress(a) \wedge p(a)) \\
\quad \vee\ (friends(a,b) \wedge smokes(b) \wedge p(a,b)) \\
\quad \vee\ (friends(a,c) \wedge smokes(c) \wedge p(a,c))) \\
\qquad\qquad\qquad\qquad \wedge \\
(smokes(b) \leftrightarrow \quad (stress(b) \wedge p(b)) \\
\vee\ (friends(b,c) \wedge stress(c) \wedge p(c) \wedge p(b,c))) \\
\qquad\qquad\qquad\qquad \wedge \\
(smokes(c) \leftrightarrow \quad (stress(c) \wedge p(c)) \\
\vee\ (friends(c,b) \wedge stress(b) \wedge p(b) \wedge p(c,b)))
\end{array}$$

Which yields the following CNF :

$$\begin{array}{l}
(\neg smokes(a) \vee stress(a) \vee friends(a,b) \vee friends(a,c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee friends(a,b) \vee smokes(c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee friends(a,b) \vee p(a,c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee smokes(b) \vee friends(a,c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee smokes(b) \vee smokes(c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee smokes(b) \vee p(a,c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee p(a,b) \vee friends(a,c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee p(a,b) \vee smokes(c)) \\
\wedge (\neg smokes(a) \vee stress(a) \vee p(a,b) \vee p(a,c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee friends(a,b) \vee friends(a,c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee friends(a,b) \vee smokes(c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee friends(a,b) \vee p(a,c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee smokes(b) \vee friends(a,c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee smokes(b) \vee smokes(c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee smokes(b) \vee p(a,c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee p(a,b) \vee friends(a,c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee p(a,b) \vee smokes(c)) \\
\wedge (\neg smokes(a) \vee p(a) \vee p(a,b) \vee p(a,c)) \\
\wedge (\neg stress(a) \vee \neg p(a) \vee smokes(a)) \\
\wedge (\neg friends(a,b) \vee \neg smokes(b) \vee \neg p(a,b) \vee smokes(a)) \\
\wedge (\neg friends(a,c) \vee \neg smokes(c) \vee \neg p(a,c) \vee smokes(a)) \\
\wedge (\neg smokes(b) \vee stress(b) \vee friends(b,c)) \\
\wedge (\neg smokes(b) \vee stress(b) \vee stress(c)) \\
\wedge (\neg smokes(b) \vee stress(b) \vee p(b)) \\
\wedge (\neg smokes(b) \vee stress(b) \vee p(b)c) \\
\wedge (\neg smokes(b) \vee p(b) \vee friends(b,c)) \\
\wedge (\neg smokes(b) \vee p(b) \vee stress(c)) \\
\wedge (\neg smokes(b) \vee p(b) \vee p(b)) \\
\wedge (\neg smokes(b) \vee p(b) \vee p(b)c) \\
\wedge (\neg stress(b) \vee \neg p(b) \vee smokes(b)) \\
\wedge (\neg friends(b,c) \vee \neg stress(c) \vee \neg p(b) \vee \neg p(b)c \vee smokes(b)) \\
\wedge (\neg smokes(c) \vee stress(c) \vee friends(c,b)) \\
\wedge (\neg smokes(c) \vee stress(c) \vee stress(b)) \\
\wedge (\neg smokes(c) \vee stress(c) \vee p(b)) \\
\wedge (\neg smokes(c) \vee stress(c) \vee p(c,b)) \\
\wedge (\neg smokes(c) \vee p(b) \vee friends(c,b)) \\
\wedge (\neg smokes(c) \vee p(b) \vee stress(b)) \\
\wedge (\neg smokes(c) \vee p(b) \vee p(b)) \\
\wedge (\neg smokes(c) \vee p(b) \vee p(c,b)) \\
\wedge (\neg stress(c) \vee \neg p(b) \vee smokes(c)) \\
\wedge (\neg friends(c,b) \vee \neg stress(b) \vee \neg p(b) \vee \neg p(c,b) \vee smokes(c))
\end{array}$$

The probabilistic literals CNF are assigned weights (derived literals get a weight of 1) :

Literal	Weight
stress(a)	0.2
¬stress(a)	0.8
stress(b)	0.2
¬stress(b)	0.8
stress(c)	0.2
¬stress(c)	0.8
friends(a,b)	0.1
¬friends(a,b)	0.9
friends(a,c)	0.1
¬friends(a,c)	0.9
friends(b,c)	0.1
¬friends(b,c)	0.9
friends(c,b)	0.1
¬friends(c,b)	0.9
p(a)	0.3
¬p(a)	0.7
p(b)	0.3
¬p(b)	0.7
p(c)	0.3
¬p(c)	0.7
p(a,b)	0.4
¬p(a,b)	0.6
p(a,c)	0.4
¬p(a,c)	0.6
p(b,c)	0.4
¬p(b,c)	0.6
p(c,b)	0.4
¬p(c,b)	0.6

SRL to PGM

PGM to CNF

Weighted Model Counting

Lifted Inference

Parameter Learning

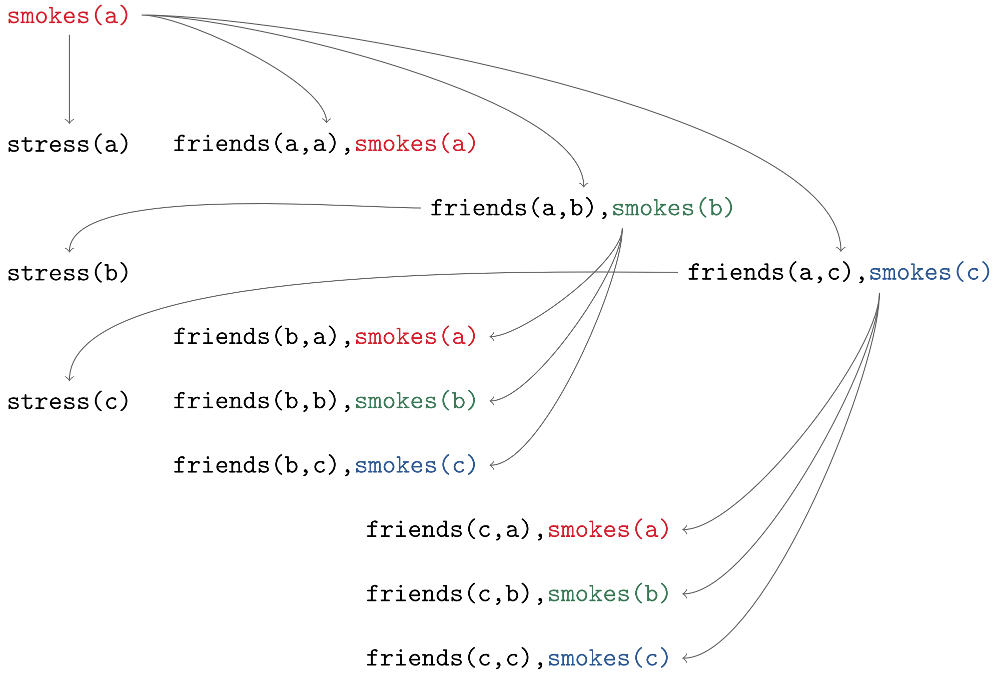

Figure 1: SLG-tree produced while turning the ground program into a boolean formula. Coloured atoms indicate the presence of cycles.