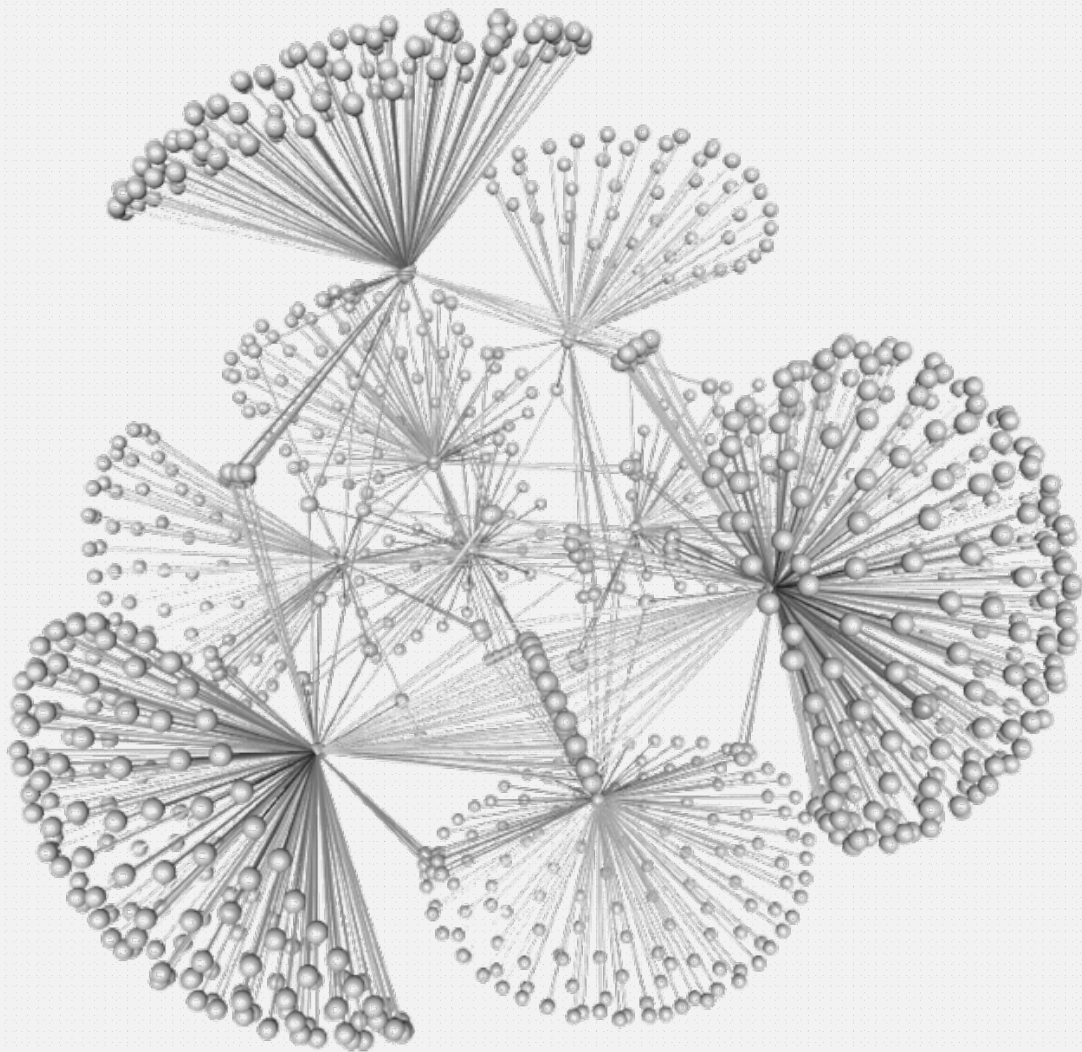


Probabilistic Programming

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ACADEMISCH JAAR 2020

H05N0A: CAPITA SELECTA : ARTIFICIAL
INTELLIGENCE

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Below's our solution for the given challenges. The questions in each section of the original assignment are answered in a section having the same title.

```
1 person(a).
2 person(b).
3 person(c).
4 0.2::stress(X) :- person(X).
5 0.1::friends(X,Y) :- person(X), person(Y).
6 0.3::smokes(X) :- stress(X).
7 0.4::smokes(X) :- friends(X,Y), smokes(Y).
8 query(smokes(a)).
```

Code snippet 1: PROLOG program used throughout the first two chapters of the report.

Probabilistic Inference Using Weighted Model Counting

SRL to CNF

First the program is grounded. This is a matter of collecting all atoms involved in all proofs of the query.

```
1 0.2::stress(a).
2 0.2::stress(b).
3 0.2::stress(c).
4
5 0.1::friends(a,a).
6 0.1::friends(a,b).
7 0.1::friends(a,c).
8
9 0.1::friends(b,a).
10 0.1::friends(b,b).
11 0.1::friends(b,c).
12
13 0.1::friends(c,a).
14 0.1::friends(c,b).
15 0.1::friends(c,c).
16
17 0.3::smokes(a) :- stress(a).
18 0.3::smokes(b) :- stress(b).
19 0.3::smokes(c) :- stress(c).
```

```
20 0.4::smokes(a) :- friends(a,a), smokes(a).
21 0.4::smokes(a) :- friends(a,b), smokes(b).
22 0.4::smokes(a) :- friends(a,c), smokes(c).
23 0.4::smokes(b) :- friends(b,a), smokes(a).
24 0.4::smokes(b) :- friends(b,b), smokes(b).
25 0.4::smokes(b) :- friends(b,c), smokes(c).
26
27 0.4::smokes(c) :- friends(c,a), smokes(a).
28 0.4::smokes(c) :- friends(c,b), smokes(b).
29 0.4::smokes(c) :- friends(c,c), smokes(c).
```

Code snippet 2: Relevant ground program.

The proofs of the query make for a trie as shown in figure 1, where colourings indicate the presence of cycles. Any proof involving an atom `friends(X,X)` or `friends(Y,a)` (with $Y \in \{b,c\}$) is non-minimal and doesn't affect the final probability. These atoms are disregarded. For the remaining cycles (involving `friends(b,c)` and `friends(c,b)`) auxiliary variables can be used to obtain a cycle-free program without intensional probabilistic facts :

```
1 0.2::stress(a).
2 0.2::stress(b).
3 0.2::stress(c).
4
```

5	0.1::friends(a,b).	$\wedge (\neg \text{smokes}(a) \vee p(a) \vee p(a,b) \vee \text{friends}(a,c))$
6	0.1::friends(a,c).	$\wedge (\neg \text{smokes}(a) \vee p(a) \vee p(a,b) \vee \text{smokes}(c))$
7	0.1::friends(b,c).	$\wedge (\neg \text{smokes}(a) \vee p(a) \vee p(a,b) \vee p(a,c))$
8	0.1::friends(c,b).	$\wedge (\neg \text{stress}(a) \vee \neg p(a) \vee \text{smokes}(a))$
9		$\wedge (\neg \text{friends}(a,b) \vee \neg \text{smokes}(b) \vee \neg p(a,b) \vee \text{smokes}(a))$
10	0.3::p(a).	$\wedge (\neg \text{friends}(a,c) \vee \neg \text{smokes}(c) \vee \neg p(a,c) \vee \text{smokes}(a))$
11	0.3::p(b).	$\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee \text{friends}(b,c))$
12	0.3::p(c).	$\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee \text{stress}(c))$
13		$\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee p(c))$
14	0.4::p(a,b).	$\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee p(b,c))$
15	0.4::p(a,c).	$\wedge (\neg \text{smokes}(b) \vee p(b) \vee \text{friends}(b,c))$
16	0.4::p(b,c).	$\wedge (\neg \text{smokes}(b) \vee p(b) \vee \text{stress}(c))$
17	0.4::p(c,b).	$\wedge (\neg \text{smokes}(b) \vee p(b) \vee p(c))$
18		$\wedge (\neg \text{smokes}(b) \vee p(b) \vee p(b,c))$
19	smokes(a) :- stress(a), p(a).	$\wedge (\neg \text{stress}(b) \vee \neg p(b) \vee \text{smokes}(b))$
20	smokes(b) :- stress(b), p(b).	$\wedge (\neg \text{friends}(b,c) \vee \neg \text{stress}(c) \vee \neg p(c) \vee \neg p(b,c) \vee \text{smokes}(b))$
21	smokes(c) :- stress(c), p(c).	$\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee \text{friends}(c,b))$
22		$\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee \text{stress}(b))$
23	smokes(a) :-	$\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee p(b))$
24	friends(a,b), smokes(b), p(a,b).	$\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee p(b))$
25	smokes(a) :-	$\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee p(c,b))$
26	friends(a,c), smokes(c), p(a,c).	$\wedge (\neg \text{smokes}(c) \vee p(c) \vee \text{friends}(c,b))$
27	smokes(b) :-	$\wedge (\neg \text{smokes}(c) \vee p(c) \vee \text{stress}(b))$
28	friends(b,c), stress(c), p(c), p(b,c).	$\wedge (\neg \text{smokes}(c) \vee p(c) \vee p(b))$
29	smokes(c) :-	$\wedge (\neg \text{smokes}(c) \vee p(c) \vee p(c,b))$
30	friends(c,b), stress(b), p(b), p(c,b).	$\wedge (\neg \text{stress}(c) \vee \neg p(c) \vee \text{smokes}(c))$
31		$\wedge (\neg \text{friends}(c,b) \vee \neg \text{stress}(b) \vee \neg p(b) \vee \neg p(c,b) \vee \text{smokes}(c))$
32	query(smokes(a)).	

Code snippet 3: Relevant ground program without cycles.

The probabilistic literals in the CNF are assigned weights (derived literals get a weight of 1) :

The above logic program is equivalent to the following propositional formula :

$$\begin{aligned}
& (\text{smokes}(a) \leftrightarrow (\text{stress}(a) \wedge p(a)) \\
& \quad \vee (\text{friends}(a,b) \wedge \text{smokes}(b) \wedge p(a,b)) \\
& \quad \vee (\text{friends}(a,c) \wedge \text{smokes}(c) \wedge p(a,c))) \\
& \quad \wedge \\
& (\text{smokes}(b) \leftrightarrow (\text{stress}(b) \wedge p(b)) \\
& \quad \vee (\text{friends}(b,c) \wedge \text{stress}(c) \wedge p(c) \wedge p(b,c))) \\
& \quad \wedge \\
& (\text{smokes}(c) \leftrightarrow (\text{stress}(c) \wedge p(c)) \\
& \quad \vee (\text{friends}(c,b) \wedge \text{stress}(b) \wedge p(b) \wedge p(c,b)))
\end{aligned}$$

Which corresponds to the following CNF :

$$\begin{aligned}
& (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{friends}(a,b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{friends}(a,b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{friends}(a,b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{smokes}(b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{smokes}(b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{smokes}(b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee p(a,b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee p(a,b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee p(a,b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{friends}(a,b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{friends}(a,b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{friends}(a,b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{smokes}(b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{smokes}(b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{smokes}(b) \vee p(a,c))
\end{aligned}$$

Literal	Weight
stress(a)	0.2
\neg stress(a)	0.8
stress(b)	0.2
\neg stress(b)	0.8
stress(c)	0.2
\neg stress(c)	0.8
friends(a,b)	0.1
\neg friends(a,b)	0.9
friends(a,c)	0.1
\neg friends(a,c)	0.9
friends(b,c)	0.1
\neg friends(b,c)	0.9
friends(c,b)	0.1
\neg friends(c,b)	0.9
p(a)	0.3
\neg p(a)	0.7
p(b)	0.3
\neg p(b)	0.7
p(c)	0.3
\neg p(c)	0.7
p(a,b)	0.4
\neg p(a,b)	0.6
p(a,c)	0.4
\neg p(a,c)	0.6
p(b,c)	0.4
\neg p(b,c)	0.6
p(c,b)	0.4
\neg p(c,b)	0.6

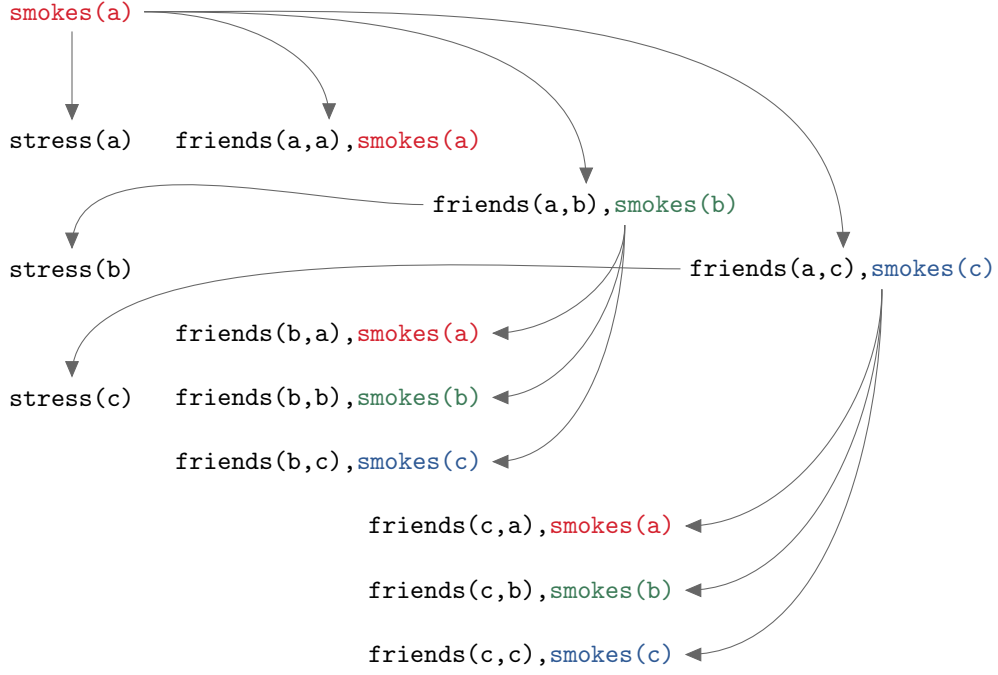


Figure 1: Trie representing proofs of the query. Coloured atoms indicate the presence of cycles.

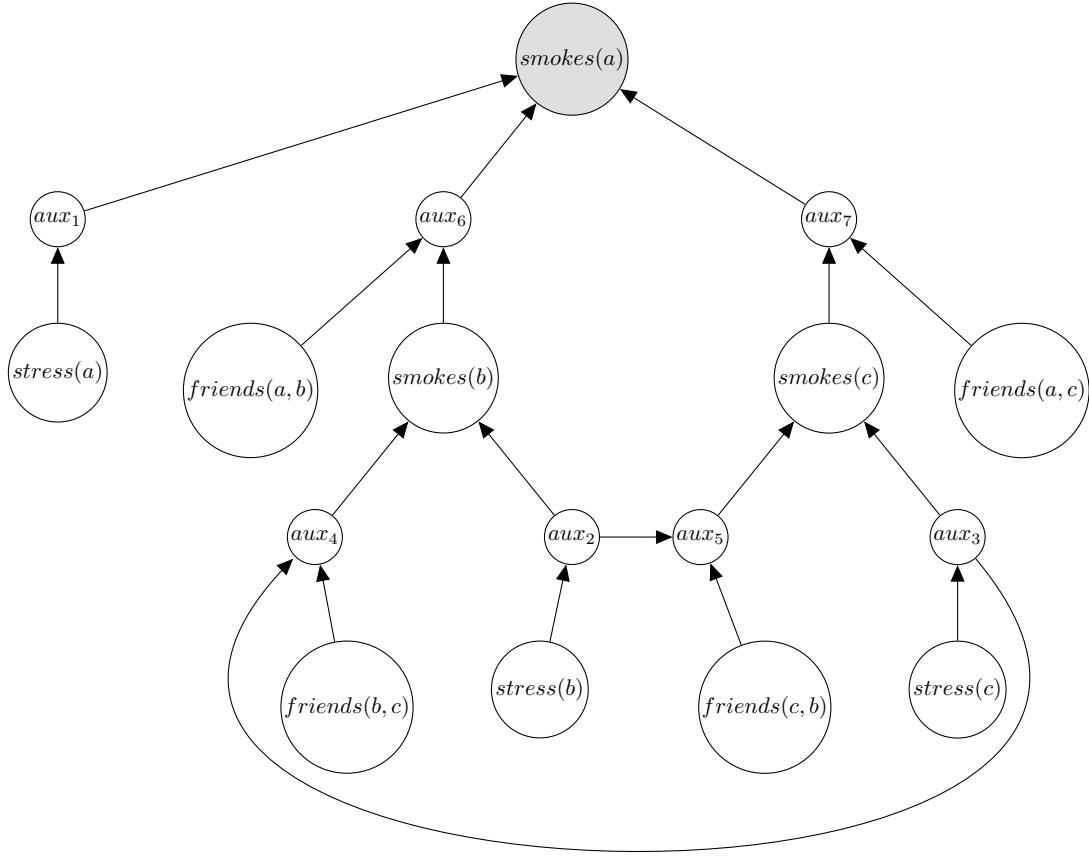


Figure 2: Bayesian network corresponding to the ground acyclic program.

SRL to PGM

A Bayesian network is shown in figure 2. The conditional probability tables (CPTs) for the nodes are given below. Note that every table represents multiple identical tables in the network (like those of the $stress(a)$, $stress(b)$ and $stress(c)$ nodes for example).

\top	$\frac{stress(\{a, b, c\})}{0.2}$	\top	$\frac{friends(\{a, b, c\}, \{b, c\})}{0.1}$
\perp	0.8	\perp	0.9

$\frac{stress(\{a, b, c\})}{\top}$	$\frac{aux_{\{1,2,3\}}}{\top}$	\perp
\perp	0.3	0.7
\perp	0.0	1.0

$\frac{friends(\{b, c\}, \{c, b\})}{\top}$	$\frac{aux_{2,3}}{\top}$	$\frac{aux_{4,5}}{\top}$	\perp
\perp	\perp	0.4	0.6
\perp	\perp	0.0	1.0
\perp	\perp	0.0	1.0
\perp	\perp	0.0	1.0

$\frac{friends(\{a\}, \{b, c\})}{\top}$	$\frac{smokes(\{b, c\})}{\top}$	$\frac{aux_{6,7}}{\top}$	\perp
\perp	\perp	0.4	0.6
\perp	\perp	0.0	1.0
\perp	\perp	0.0	1.0
\perp	\perp	0.0	1.0

$\frac{aux_{4,5}}{\top}$	$\frac{aux_{2,3}}{\top}$	$\frac{smokes(\{b, c\})}{\top}$	\perp
\perp	\perp	1.0	0.0
\perp	\perp	0.0	1.0
\perp	\perp	0.0	1.0
\perp	\perp	0.0	1.0

$\frac{aux_1}{\top}$	$\frac{smokes(b)}{\top}$	$\frac{smokes(c)}{\top}$	$\frac{smokes(a)}{\top}$	\perp
\perp	\perp	\perp	1.0	0.0
\perp	\perp	\perp	0.0	1.0
\perp	\perp	\perp	0.0	1.0
\perp	\perp	\perp	0.0	1.0
\perp	\perp	\perp	0.0	1.0
\perp	\perp	\perp	0.0	1.0
\perp	\perp	\perp	0.0	1.0
\perp	\perp	\perp	0.0	1.0
\perp	\perp	\perp	0.0	1.0

to avoid unnecessary cluttering. Instead just one atom is introduced for each network variable. This also makes the indicator clauses redundant.

In ENC1 each row in each CPT is encoded by a parameter clause.

In ENC2 an order is assumed over each variable's values. Then, each row of each CPT is encoded by an equivalence.

Encoding the deterministic noisy-OR makes either of these more compact. For example, the CPT of $smokes(a)$ can be represented by the following one-liner :

$$smokes(a) \Leftrightarrow aux_1 \vee aux_6 \vee aux_7$$

Weighted Model Counting

Lifted Inference

Parameter Learning

PGM to CNF

The above Bayesian network can subsequently be encoded by a logical formula. Since it consists of nothing but boolean variables, indicator variables representing each value of each variable (e.g. $\lambda_{stress(a)=true}$ and $\lambda_{stress(a)=false}$) are not used in any of the DIMACS files corresponding to encodings presented below. This

