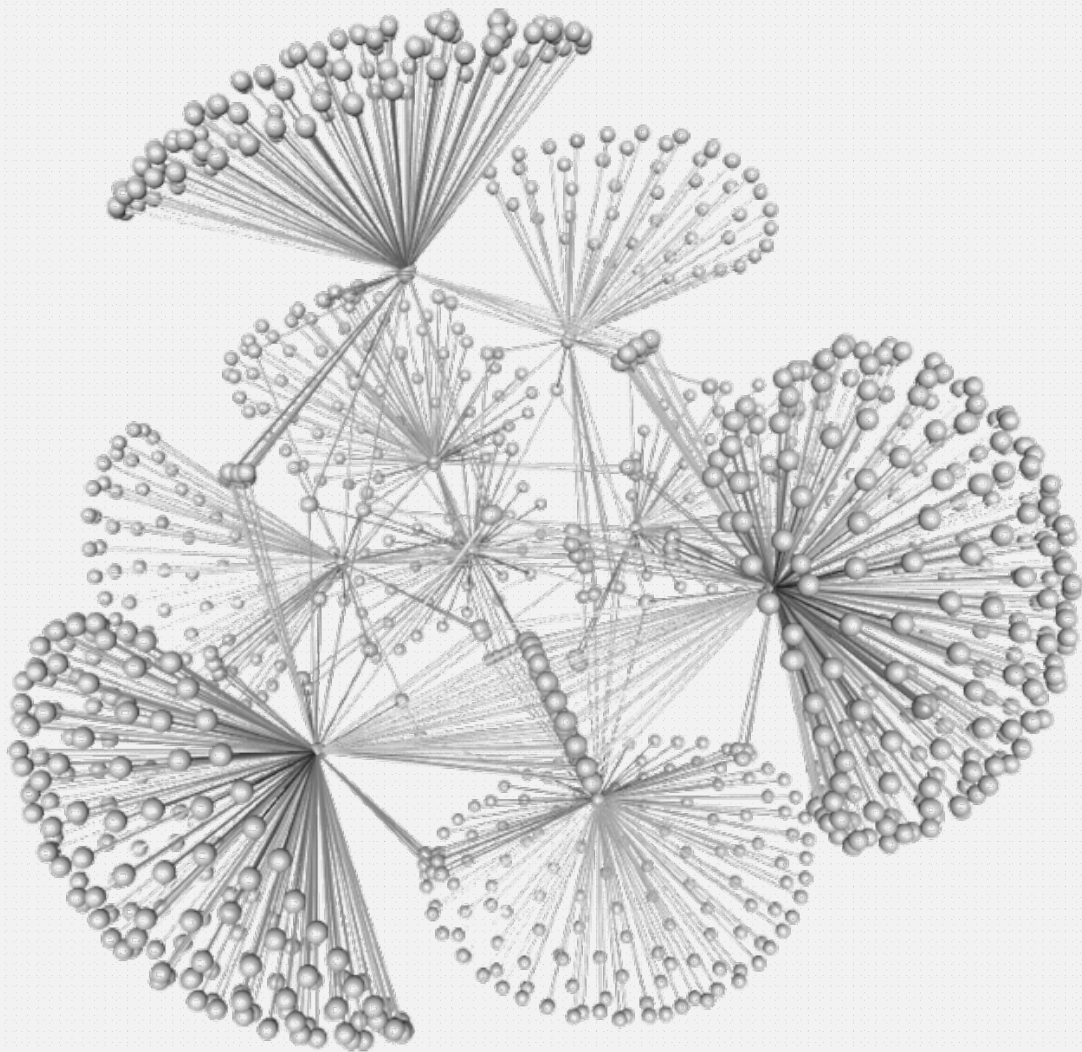


# Probabilistic Programming

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Below's our solution for the given challenges. The questions in each section of the original assignment are answered in a section having the same title.

```
1 person(a).
2 person(b).
3 person(c).
4 0.2::stress(X) :- person(X).
5 0.1::friends(X,Y) :- person(X), person(Y).
6 0.3::smokes(X) :- stress(X).
7 0.4::smokes(X) :- friends(X,Y), smokes(Y).
8 query(smokes(a)).
```

Code snippet 1: PROLOG program used throughout the first two chapters of the report.

## Probabilistic Inference Using Weighted Model Counting

### SRL to CNF

First the program is grounded. This is a matter of collecting all atoms involved in all proofs of the query.

```
1 0.2::stress(a).
2 0.2::stress(b).
3 0.2::stress(c).
4
5 0.1::friends(a,a).
6 0.1::friends(a,b).
7 0.1::friends(a,c).
8
9 0.1::friends(b,a).
10 0.1::friends(b,b).
11 0.1::friends(b,c).
12
13 0.1::friends(c,a).
14 0.1::friends(c,b).
15 0.1::friends(c,c).
16
17 0.3::smokes(a) :- stress(a).
18 0.3::smokes(b) :- stress(b).
19 0.3::smokes(c) :- stress(c).
```

```
20 0.4::smokes(a) :- friends(a,a), smokes(a).
21 0.4::smokes(a) :- friends(a,b), smokes(b).
22 0.4::smokes(a) :- friends(a,c), smokes(c).
23 0.4::smokes(b) :- friends(b,a), smokes(a).
24 0.4::smokes(b) :- friends(b,b), smokes(b).
25 0.4::smokes(b) :- friends(b,c), smokes(c).
26
27 0.4::smokes(c) :- friends(c,a), smokes(a).
28 0.4::smokes(c) :- friends(c,b), smokes(b).
29 0.4::smokes(c) :- friends(c,c), smokes(c).
```

Code snippet 2: Relevant ground program.

The proofs of the query make for a trie as shown in figure 1, where colourings indicate the presence of cycles. Any proof involving an atom `friends(X,X)` or `friends(Y,a)` (with  $Y \in \{b,c\}$ ) is non-minimal and doesn't affect the final probability. These atoms are disregarded. For the remaining cycles (involving `friends(b,c)` and `friends(c,b)`) auxiliary variables can be used to obtain a cycle-free program without intensional probabilistic facts :

```
1 0.2::stress(a).
2 0.2::stress(b).
3 0.2::stress(c).
4
```

|    |  |  |
|----|--|--|
| 5  | 0.1::friends(a,b).                     | $\wedge (\neg \text{smokes}(a) \vee p(a) \vee p(a,b) \vee \text{friends}(a,c))$                                      |
| 6  | 0.1::friends(a,c).                     | $\wedge (\neg \text{smokes}(a) \vee p(a) \vee p(a,b) \vee \text{smokes}(c))$   |
| 7  | 0.1::friends(b,c).                     | $\wedge (\neg \text{smokes}(a) \vee p(a) \vee p(a,b) \vee p(a,c))$   |
| 8  | 0.1::friends(c,b).                     | $\wedge (\neg \text{stress}(a) \vee \neg p(a) \vee \text{smokes}(a))$  |
| 9  |  | $\wedge (\neg \text{friends}(a,b) \vee \neg \text{smokes}(b) \vee \neg p(a,b) \vee \text{smokes}(a))$                |
| 10 | 0.3::p(a).                             | $\wedge (\neg \text{friends}(a,c) \vee \neg \text{smokes}(c) \vee \neg p(a,c) \vee \text{smokes}(a))$                |
| 11 | 0.3::p(b).                             | $\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee \text{friends}(b,c))$                                      |
| 12 | 0.3::p(c).                             | $\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee \text{stress}(c))$   |
| 13 |  | $\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee p(c))$   |
| 14 | 0.4::p(a,b).                           | $\wedge (\neg \text{smokes}(b) \vee \text{stress}(b) \vee p(b,c))$   |
| 15 | 0.4::p(a,c).                           | $\wedge (\neg \text{smokes}(b) \vee p(b) \vee \text{friends}(b,c))$  |
| 16 | 0.4::p(b,c).                           | $\wedge (\neg \text{smokes}(b) \vee p(b) \vee \text{stress}(c))$   |
| 17 | 0.4::p(c,b).                           | $\wedge (\neg \text{smokes}(b) \vee p(b) \vee p(c))$   |
| 18 |  | $\wedge (\neg \text{smokes}(b) \vee p(b) \vee p(b,c))$   |
| 19 | smokes(a) :- stress(a), p(a).          | $\wedge (\neg \text{stress}(b) \vee \neg p(b) \vee \text{smokes}(b))$  |
| 20 | smokes(b) :- stress(b), p(b).          | $\wedge (\neg \text{friends}(b,c) \vee \neg \text{stress}(c) \vee \neg p(c) \vee \neg p(b,c) \vee \text{smokes}(b))$ |
| 21 | smokes(c) :- stress(c), p(c).          | $\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee \text{friends}(c,b))$                                      |
| 22 |  | $\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee \text{stress}(b))$   |
| 23 | smokes(a) :-                           | $\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee p(b))$   |
| 24 | friends(a,b), smokes(b), p(a,b).       | $\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee p(b))$   |
| 25 | smokes(a) :-                           | $\wedge (\neg \text{smokes}(c) \vee \text{stress}(c) \vee p(c,b))$   |
| 26 | friends(a,c), smokes(c), p(a,c).       | $\wedge (\neg \text{smokes}(c) \vee p(c) \vee \text{friends}(c,b))$  |
| 27 | smokes(b) :-                           | $\wedge (\neg \text{smokes}(c) \vee p(c) \vee \text{stress}(b))$   |
| 28 | friends(b,c), stress(c), p(c), p(b,c). | $\wedge (\neg \text{smokes}(c) \vee p(c) \vee p(b))$   |
| 29 | smokes(c) :-                           | $\wedge (\neg \text{smokes}(c) \vee p(c) \vee p(c,b))$   |
| 30 | friends(c,b), stress(b), p(b), p(c,b). | $\wedge (\neg \text{stress}(c) \vee \neg p(c) \vee \text{smokes}(c))$  |
| 31 |  | $\wedge (\neg \text{friends}(c,b) \vee \neg \text{stress}(b) \vee \neg p(b) \vee \neg p(c,b) \vee \text{smokes}(c))$ |
| 32 | query(smokes(a)).                      |  |

Code snippet 3: Relevant ground program without cycles.

The above logic program is equivalent to the following propositional formula :

$$\begin{aligned}
& (\text{smokes}(a) \leftrightarrow (\text{stress}(a) \wedge p(a)) \\
& \quad \vee (\text{friends}(a,b) \wedge \text{smokes}(b) \wedge p(a,b)) \\
& \quad \vee (\text{friends}(a,c) \wedge \text{smokes}(c) \wedge p(a,c))) \\
& \quad \wedge \\
& (\text{smokes}(b) \leftrightarrow (\text{stress}(b) \wedge p(b)) \\
& \quad \vee (\text{friends}(b,c) \wedge \text{stress}(c) \wedge p(c) \wedge p(b,c))) \\
& \quad \wedge \\
& (\text{smokes}(c) \leftrightarrow (\text{stress}(c) \wedge p(c)) \\
& \quad \vee (\text{friends}(c,b) \wedge \text{stress}(b) \wedge p(b) \wedge p(c,b)))
\end{aligned}$$

Which yields the following CNF :

$$\begin{aligned}
& (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{friends}(a,b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{friends}(a,b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{friends}(a,b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{smokes}(b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{smokes}(b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee \text{smokes}(b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee p(a,b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee p(a,b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee \text{stress}(a) \vee p(a,b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{friends}(a,b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{friends}(a,b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{friends}(a,b) \vee p(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{smokes}(b) \vee \text{friends}(a,c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{smokes}(b) \vee \text{smokes}(c)) \\
& \wedge (\neg \text{smokes}(a) \vee p(a) \vee \text{smokes}(b) \vee p(a,c))
\end{aligned}$$

The probabilistic literals in the CNF are assigned weights (derived literals get a weight of 1) :

| Literal             | Weight |
|---------------------|--------|
| stress(a)           | 0.2    |
| $\neg$ stress(a)    | 0.8    |
| stress(b)           | 0.2    |
| $\neg$ stress(b)    | 0.8    |
| stress(c)           | 0.2    |
| $\neg$ stress(c)    | 0.8    |
| friends(a,b)        | 0.1    |
| $\neg$ friends(a,b) | 0.9    |
| friends(a,c)        | 0.1    |
| $\neg$ friends(a,c) | 0.9    |
| friends(b,c)        | 0.1    |
| $\neg$ friends(b,c) | 0.9    |
| friends(c,b)        | 0.1    |
| $\neg$ friends(c,b) | 0.9    |
| p(a)                | 0.3    |
| $\neg$ p(a)         | 0.7    |
| p(b)                | 0.3    |
| $\neg$ p(b)         | 0.7    |
| p(c)                | 0.3    |
| $\neg$ p(c)         | 0.7    |
| p(a,b)              | 0.4    |
| $\neg$ p(a,b)       | 0.6    |
| p(a,c)              | 0.4    |
| $\neg$ p(a,c)       | 0.6    |
| p(b,c)              | 0.4    |
| $\neg$ p(b,c)       | 0.6    |
| p(c,b)              | 0.4    |
| $\neg$ p(c,b)       | 0.6    |

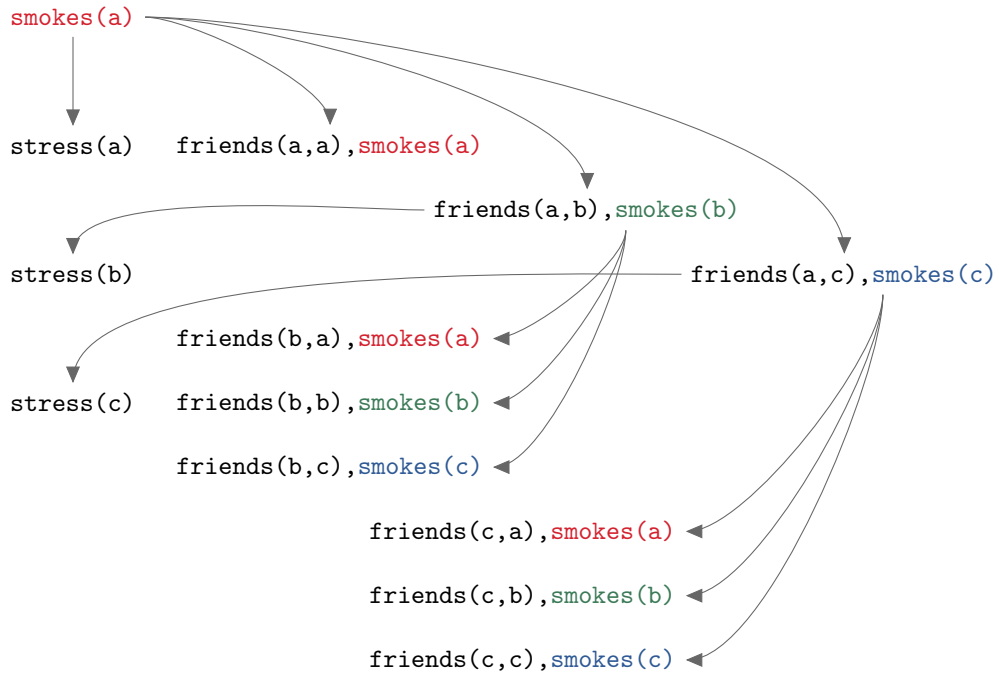


Figure 1: Trie representing proofs of the query. Coloured atoms indicate the presence of cycles.

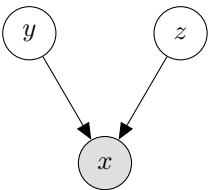


Figure 2: Bayes network representation of the program.

SRL to PGM

PGM to CNF

Weighted Model Counting

Lifted Inference

Parameter Learning

