Lecture 15
Robust Estimation : RANSAC

RECALL: Parameter Estimation:

Let’s say we have found point matches between two images, and we think they are related by some parametric transformation (e.g. translation; scaled Euclidean; affine). How do we estimate the parameters of that transformation?

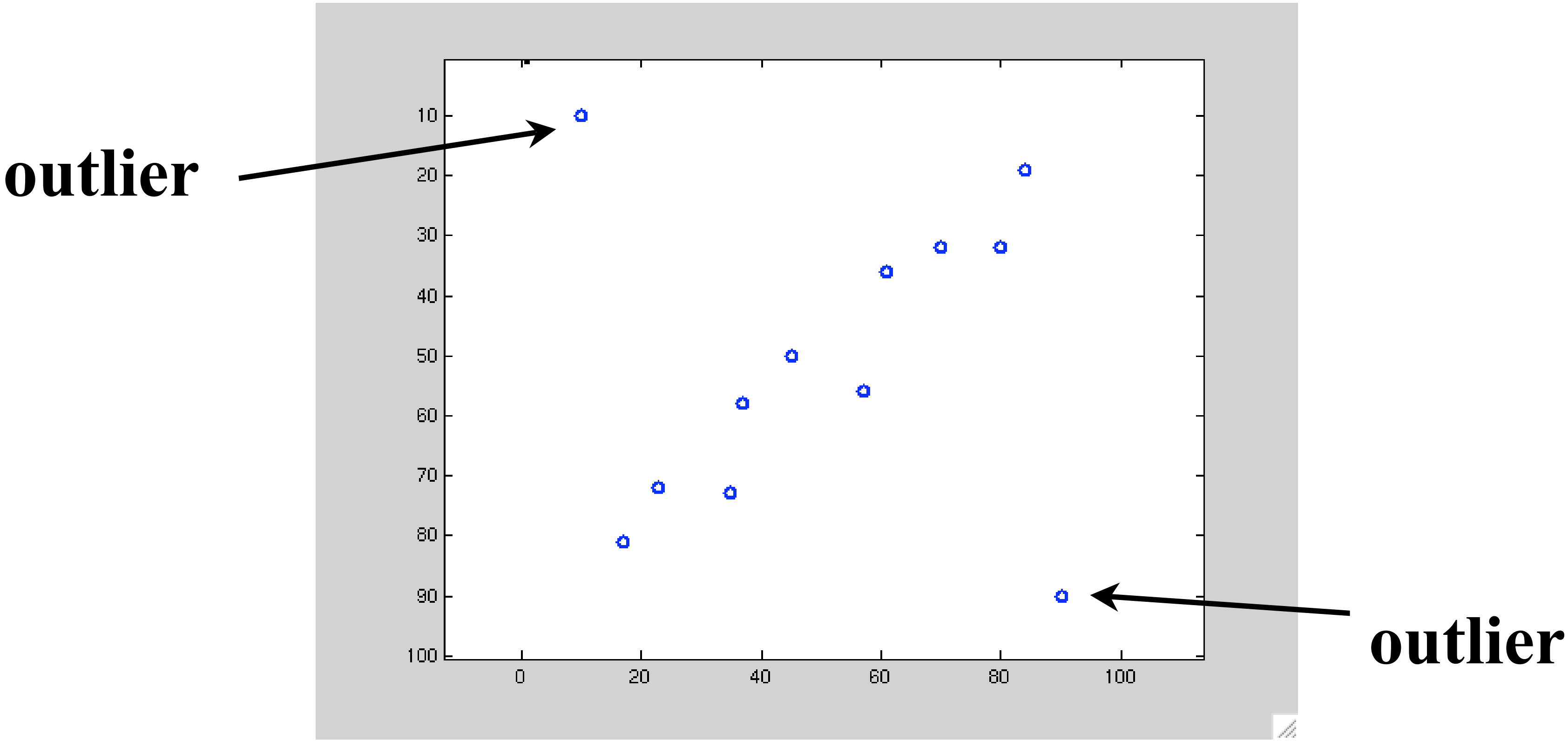
General Strategy

- Least-Squares estimation from point correspondences

But there are problems with that approach....

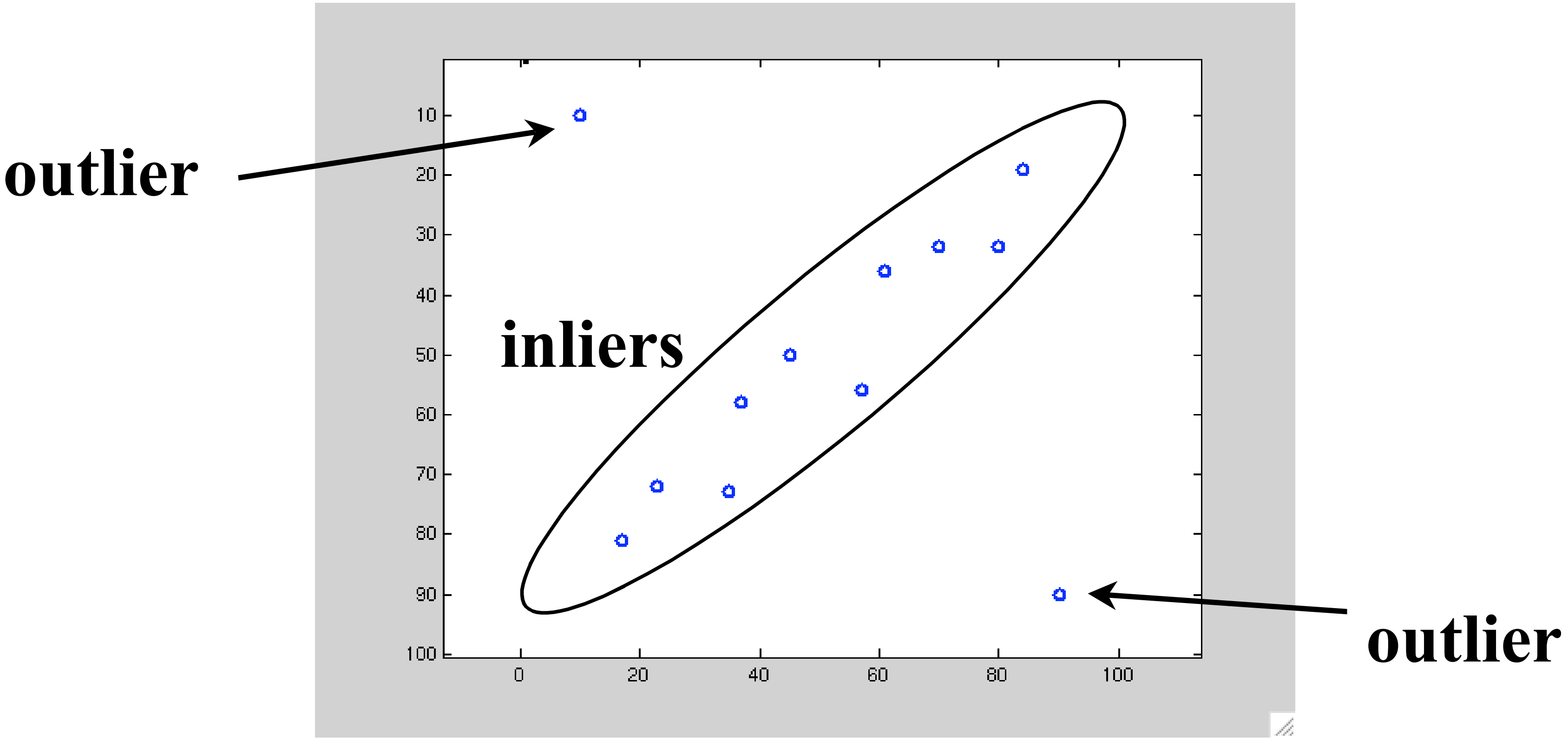
Problem : Outliers

Loosely speaking, outliers are points that don't "fit" the model.



Bad Data => Outliers

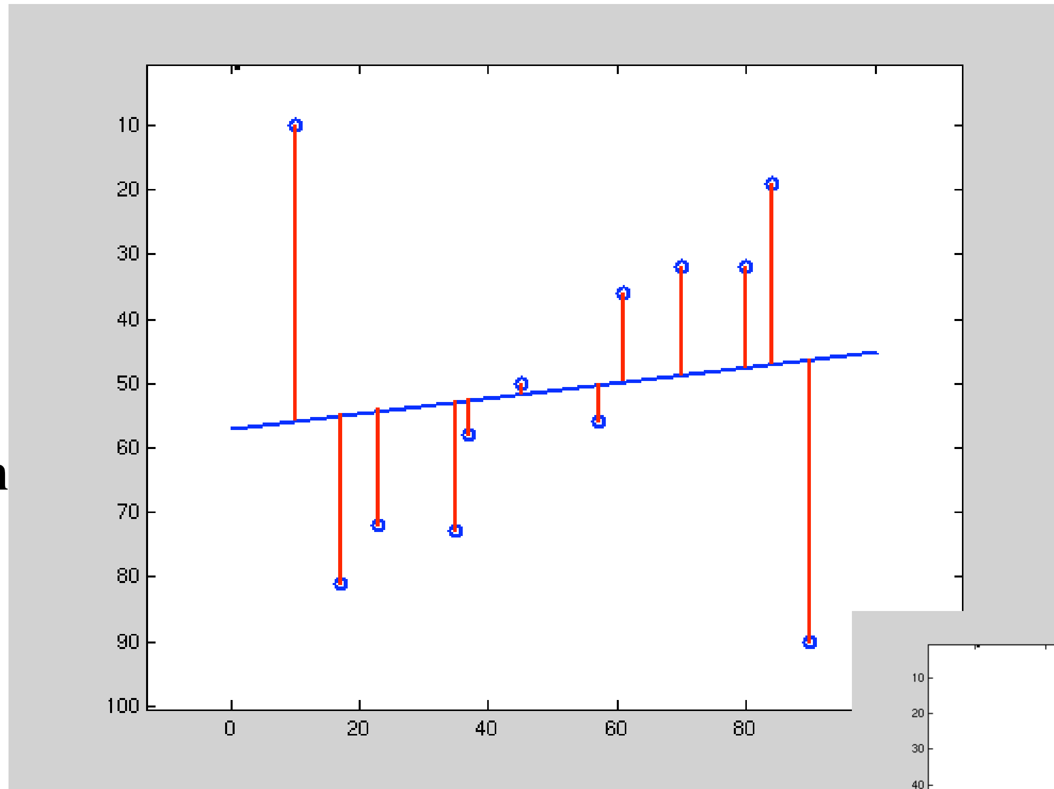
Loosely speaking, outliers are points that don't "fit" the model. Points that do fit are called "inliers"



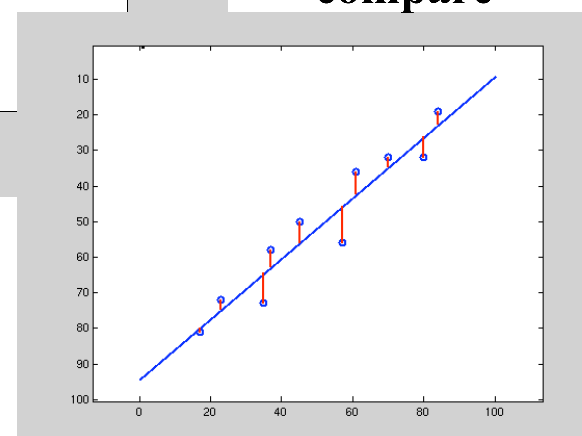
Problem with Outliers

Least squares estimation is sensitive to outliers, so that a few outliers can greatly skew the result.

Least squares regression with outliers

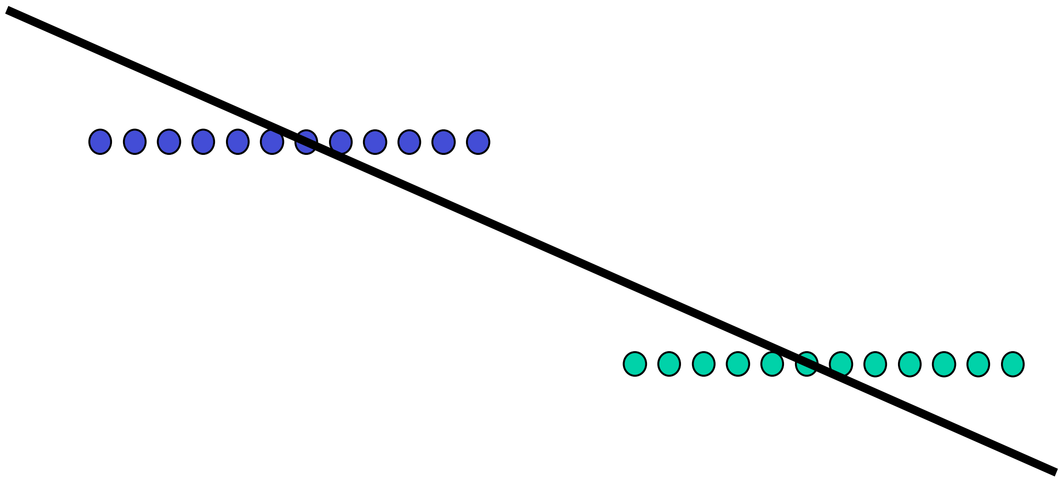


compare



Solution: Estimation methods that are robust to outliers.

Outliers aren't the Only Problem



Multiple structures can also skew the results. (the fit procedure implicitly assumes there is only one instance of the model in the data).

Robust Estimation

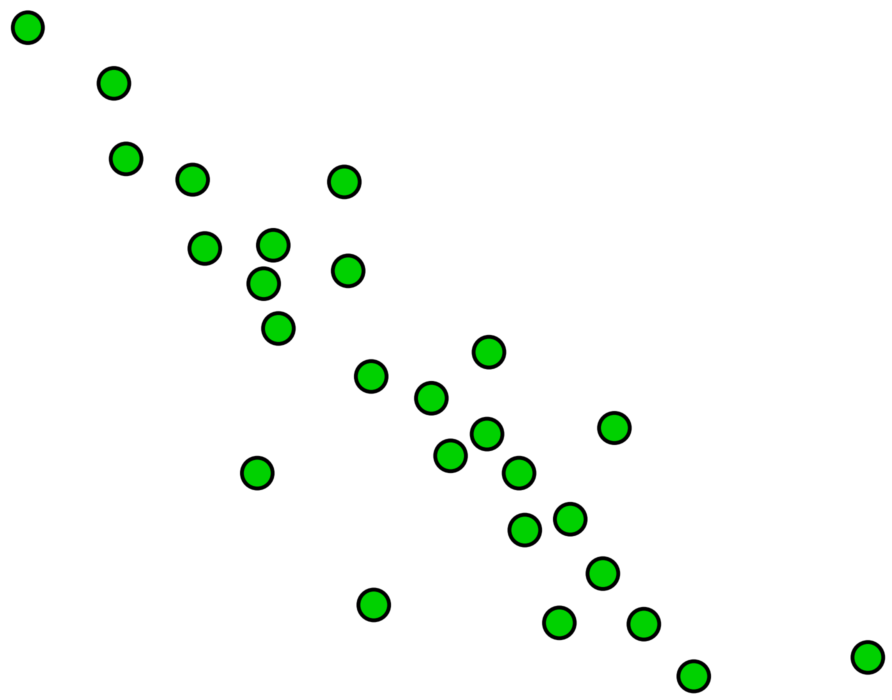
- View estimation as a two-stage process:
  - Classify data points as outliers or inliers
  - Fit model to inliers while ignoring outliers

- Example technique: RANSAC

  (**RAN**dom **SA**mple **C**onsensus)

M. A. Fischler and R. C. Bolles (June 1981). "Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography". *Comm. of the ACM* **24**: 381--395.

Ransac Procedure



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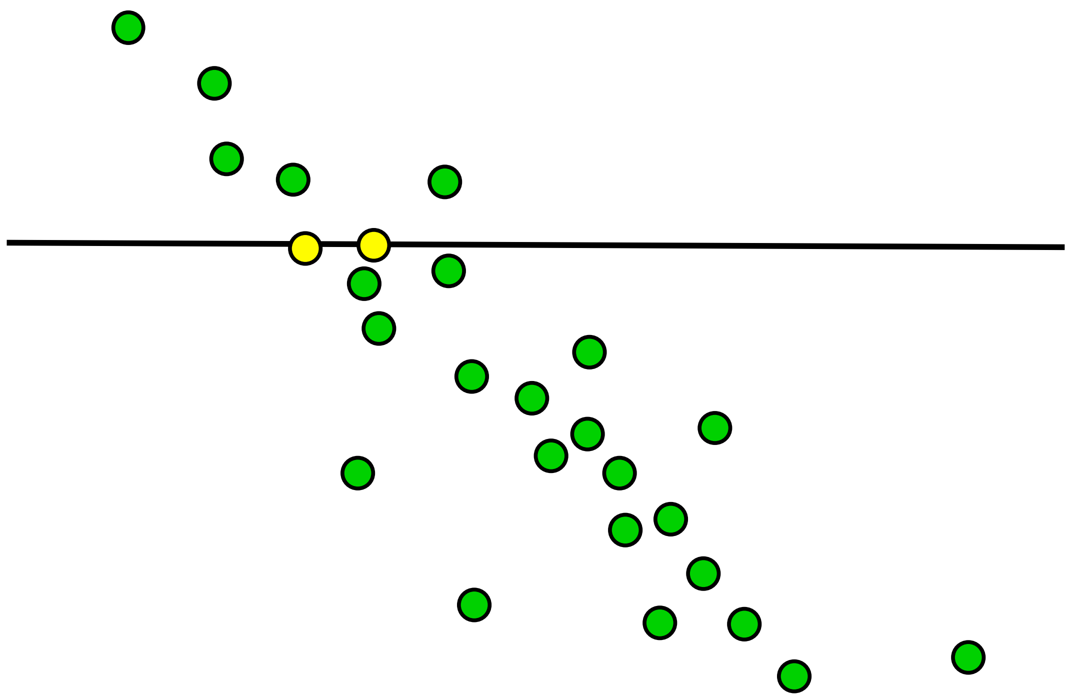
Ransac Procedure



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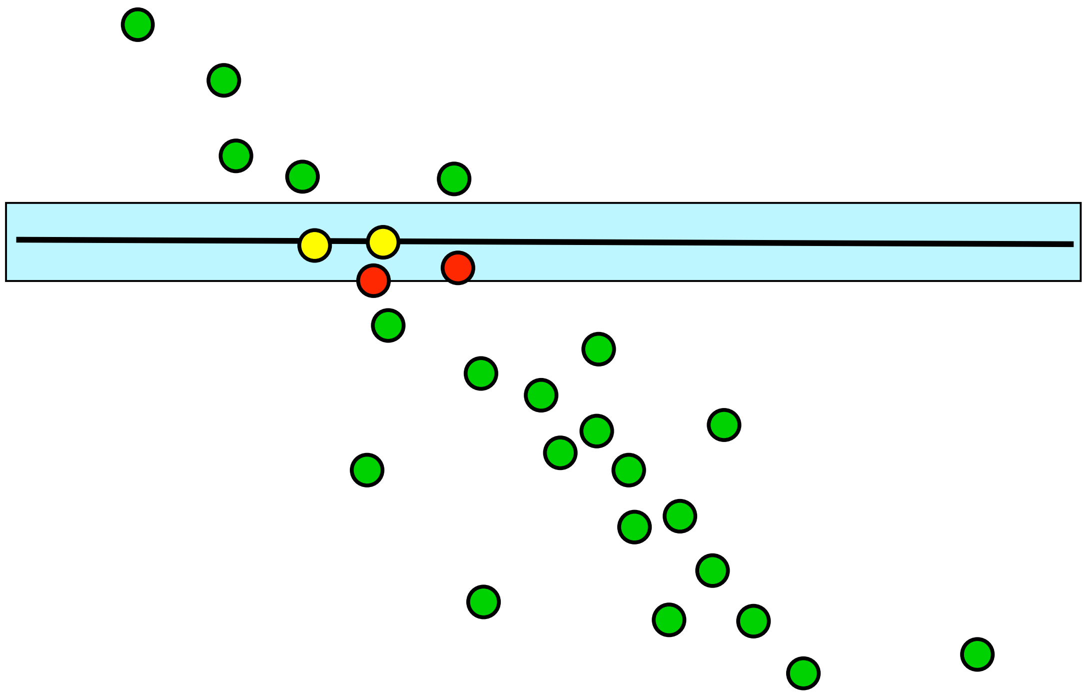
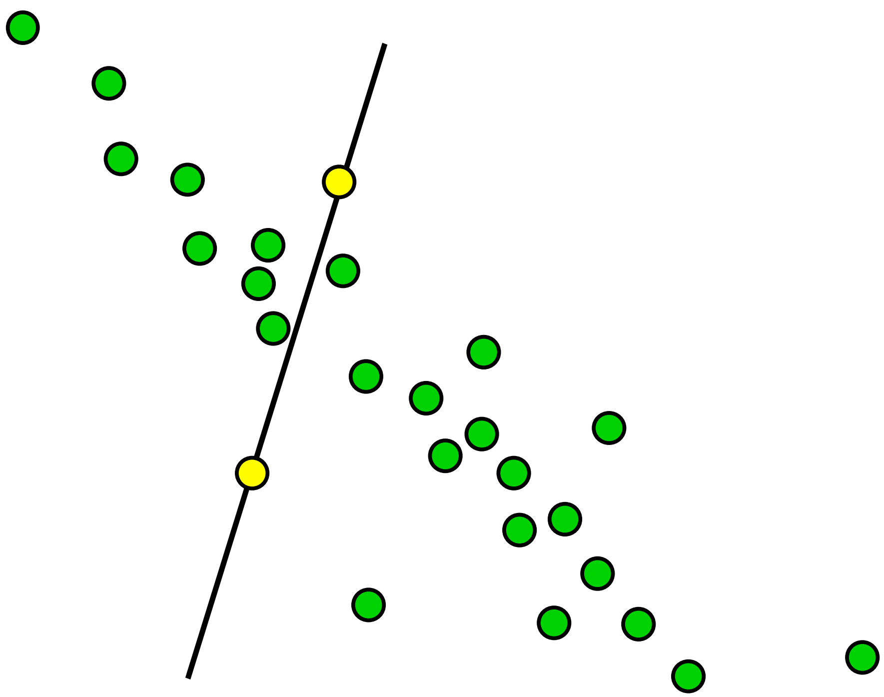
Ransac Procedure

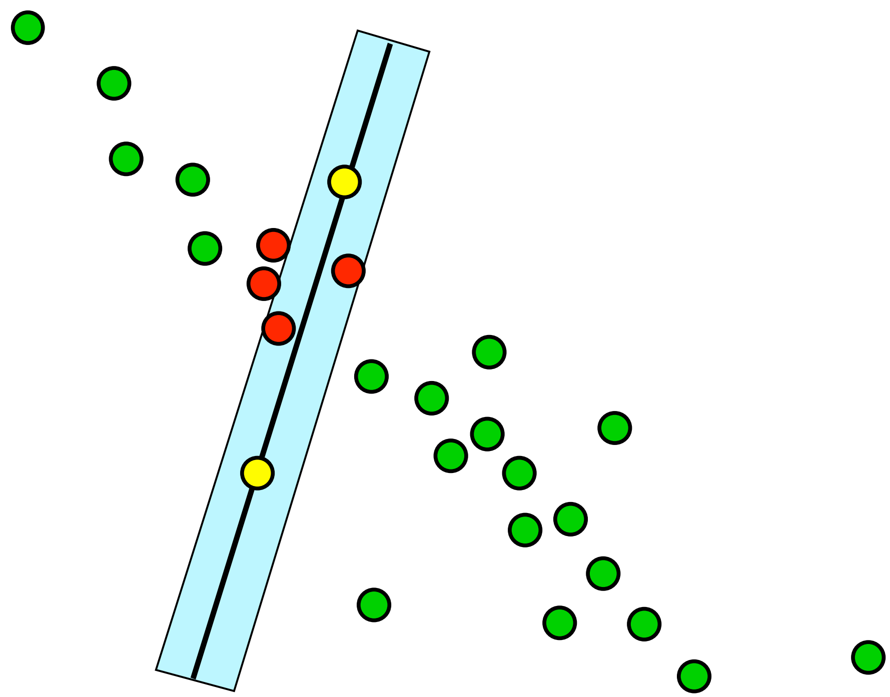


Count = 4

Ransac Procedure

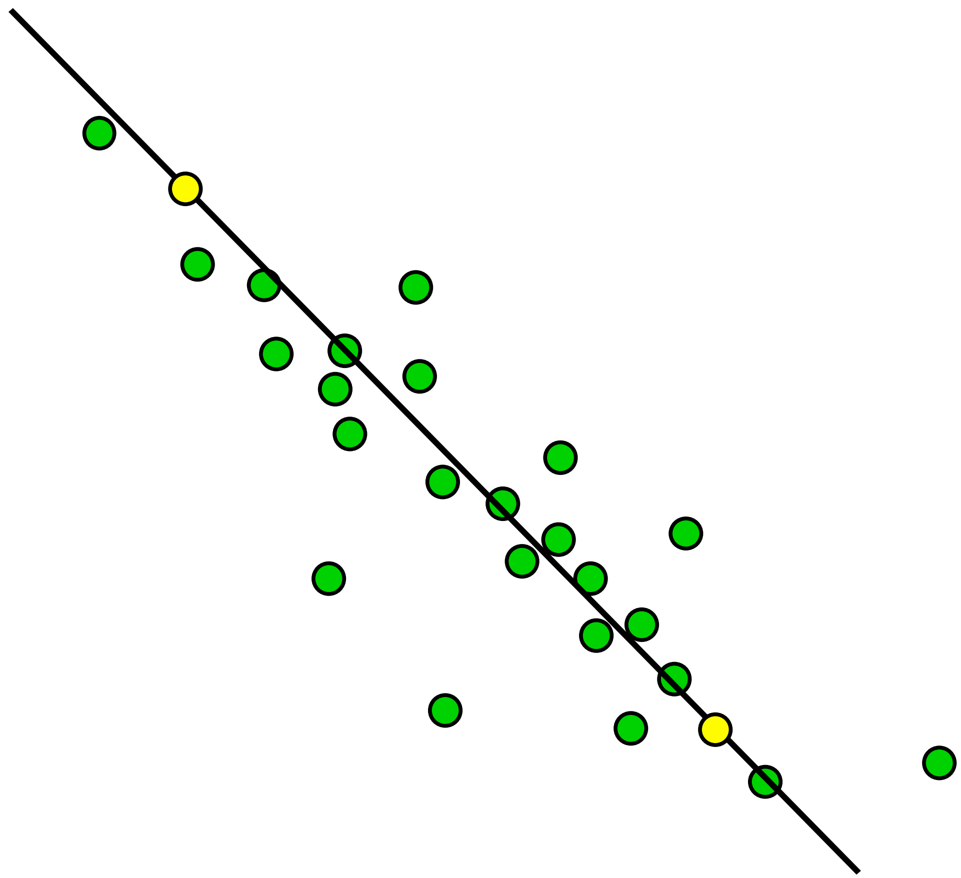


Ransac Procedure



Count = 6

Ransac Procedure



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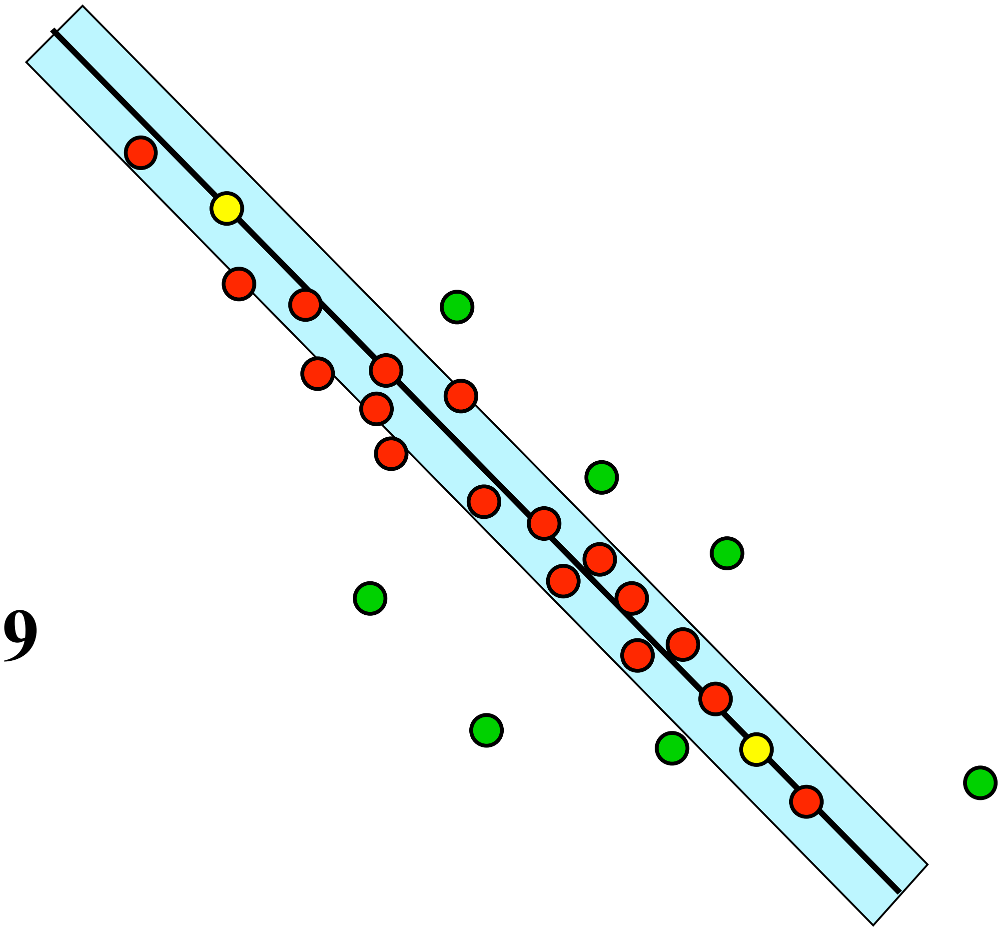
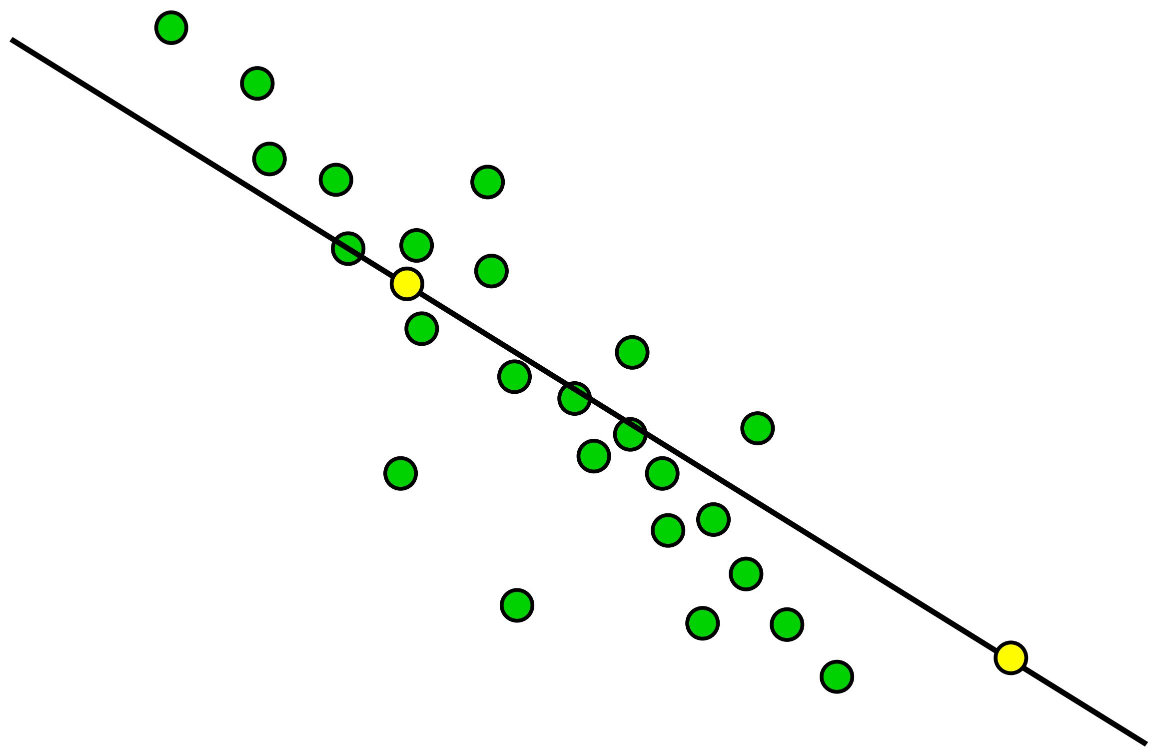
Ransac Procedure



Count = 19

Ransac Procedure



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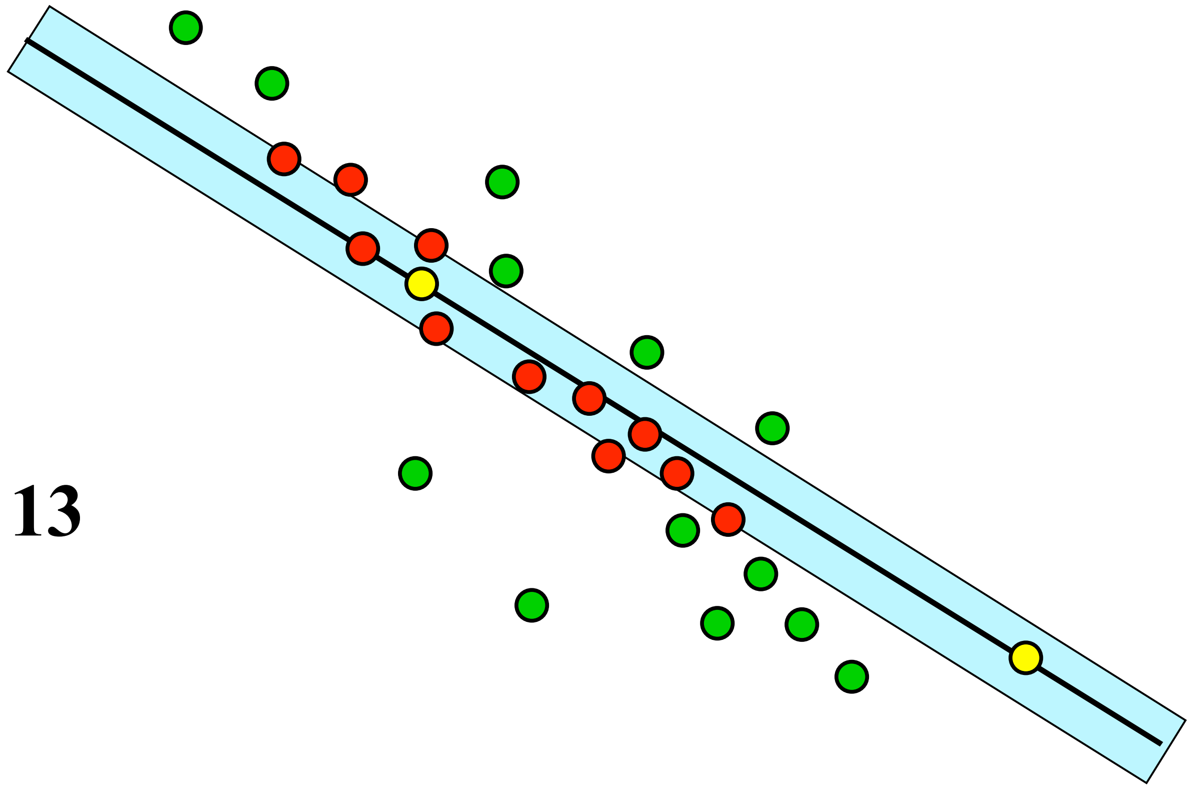
Ransac Procedure



Count = 13

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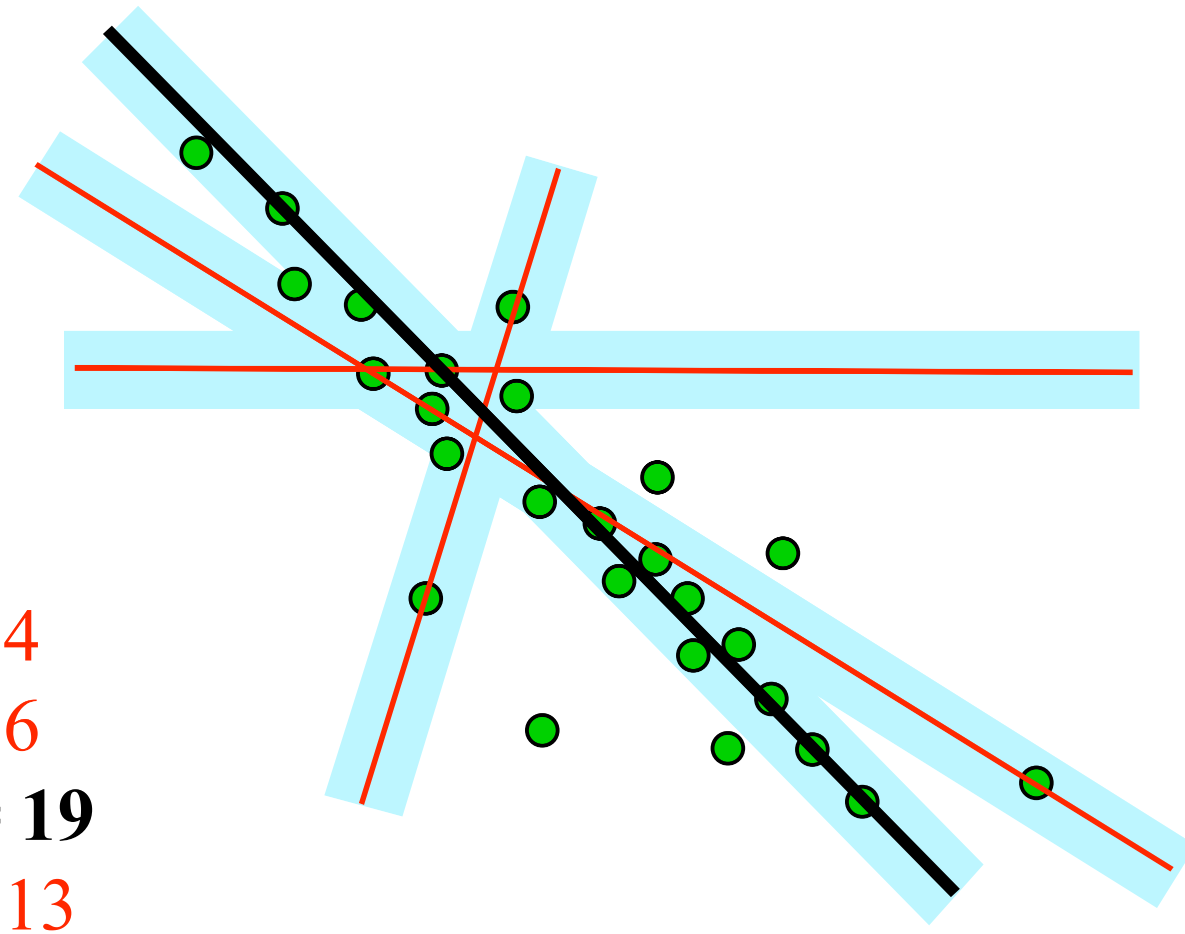
Ransac Procedure



Count = 4

Count = 6

Count = 19

Count = 13

Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:
- **s** — the smallest number of points required
- **N** — the number of iterations required
- **d** — the threshold used to identify a point that fits well
- **T** — the number of nearby points required to assert a model fits well

Until **N** iterations have occurred
  Draw a sample of **s** points from the data uniformly and at random
  Fit to that set of **s** points
  For each data point outside the sample
    Test the distance from the point to the line against **d** if the distance from the point to the line is less than **d** the point is close
  end
  If there are **T** or more points close to the line then there is a good fit. Refit the line using all these points.
end
Use the best fit from this collection, using the fitting error as a criterion

(Forsyth & Ponce)

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

Solve the following for N:

$$1 - (1 - (1 - e)^s)^N = p$$

Where in the world did that come from? ….

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

$$1 - (1 - (\underbrace{1 - e}_{\text{Probability that choosing one point yields an inlier}})^s)^N = p$$

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

$$1 - (1 - \underbrace{(1 - e)}_{} {}^{s})^{N} = p$$

Probability of choosing s inliers in a row (sample only contains inliers)

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

$$1 - (\underbrace{1 - (1 - e)^s}\,)^N = p$$

Probability that one or more points in the sample were outliers (sample is contaminated).

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

$$1 - \underbrace{(1 - (1 - e)^s)^N}_{\text{Probability that N samples were contaminated.}} = p$$

Probability that N samples were contaminated.

How Many Samples to Choose?

e = probability that a point is an outlier
s = number of points in a sample
N = number of samples (we want to compute this)
p = desired probability that we get a good sample

$$\underbrace{1 - (1 - (1 - e)^s)^N}_{\text{Probability that at least one sample was not contaminated}} = p$$

Probability that at least one sample was not contaminated (at least one sample of s points is composed of only inliers).

How many samples?

Choose N so that, with probability p, at least one random sample is free from outliers. e.g. p=0.99

$$(1-(1-e)^s)^N = 1-p$$

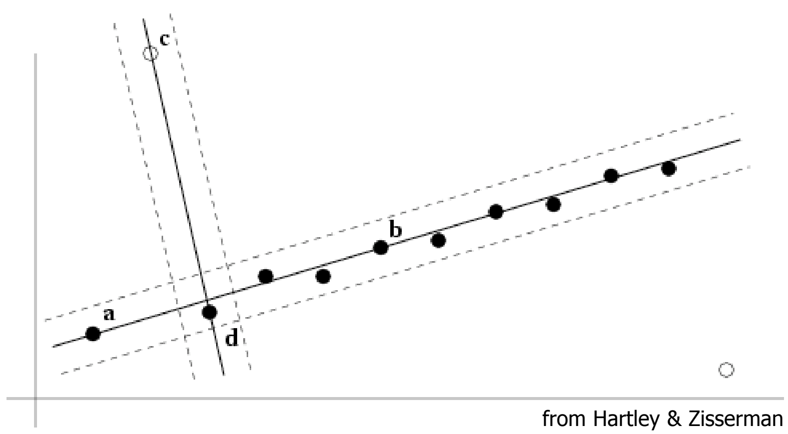
$$N = \frac{\log(1-p)}{log\left(1-(1-e)^s\right)}$$

	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Example: N for the line-fitting problem

- n = 12 points
- Minimal sample size s = 2
- 2 outliers: e = 1/6 => 20%
- So N = 5 gives us a 99% chance of getting a pure-inlier sample
  - Compared to N = 66 by trying every pair of points

66 = (12 * 11) / 2 (Brute Force)



from Hartley & Zisserman

Acceptable consensus set?

- We have seen that we don't have to exhaustively sample subsets of points, we just need to randomly sample N subsets.

- However, typically, we don't even have to sample N sets!

- <u>Early termination</u>: terminate when inlier ratio reaches expected ratio of inliers

$$T = (1 - e) * (\textit{total number of data points})$$

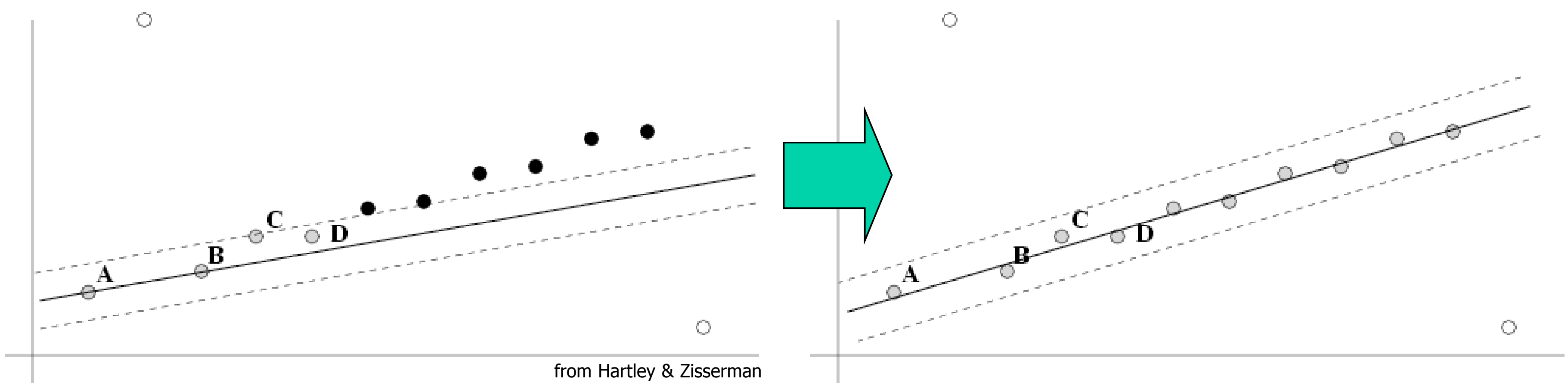
RANSAC: Picking Distance Threshold d

- Usually chosen empirically
- But…when measurement error is known to be Gaussian with mean 0 and variance s^2:
  - Sum of squared errors follows a χ^2 distribution with m DOF, where m is the DOF of the error measure (the *codimension*)
  - (*dimension* + *codimension*) = *dimension of parameter space*
    - E.g., $m = 1$ for line fitting because error is perpendicular distance
    - E.g., $m = 2$ for point distance
- Examples for probability $p = 0.95$ that point is inlier

m	**Model**	d^2
1	Line, fundamental matrix	$3.84\ s^2$
2	Homography, camera matrix	$5.99\ s^2$

After RANSAC

- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers with greatest support
- Improve this initial estimate with Least Squares estimation over all inliers (i.e., standard minimization)
- Find inliers wrt that L.S. line, and compute L.S. one more time.



from Hartley & Zisserman

Practical Example

- Stabilizing aerial imagery using RANSAC
  - find corners in two images
  - hypothesize matches using NCC
  - do RANSAC to find matches consistent with an affine transformation
  - take the inlier set found and estimate a full projective transformation (homography)

Stabilization Application

Input: two images from an aerial video sequence.

Note that the motion of the camera is "disturbing"

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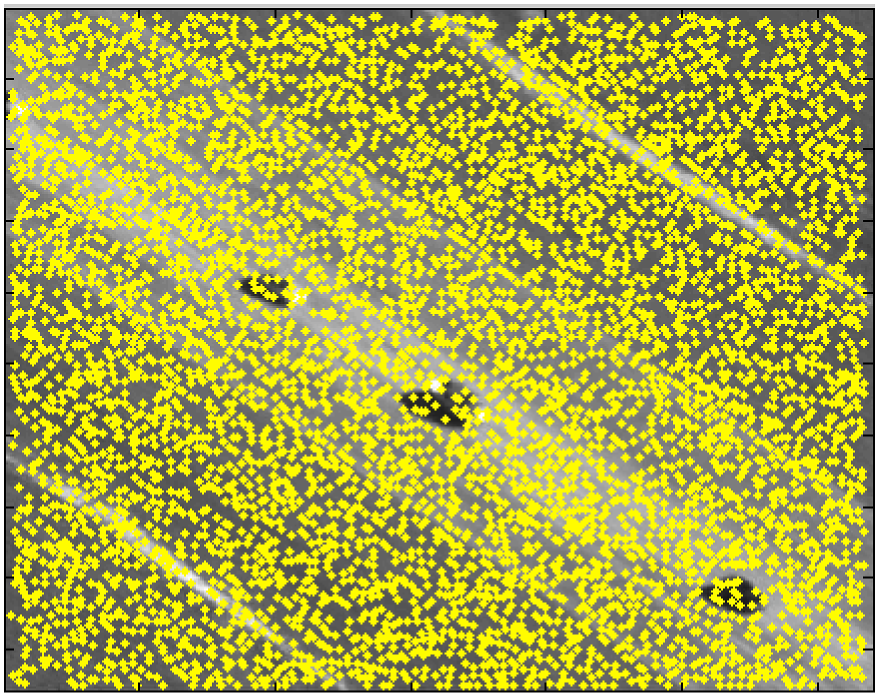
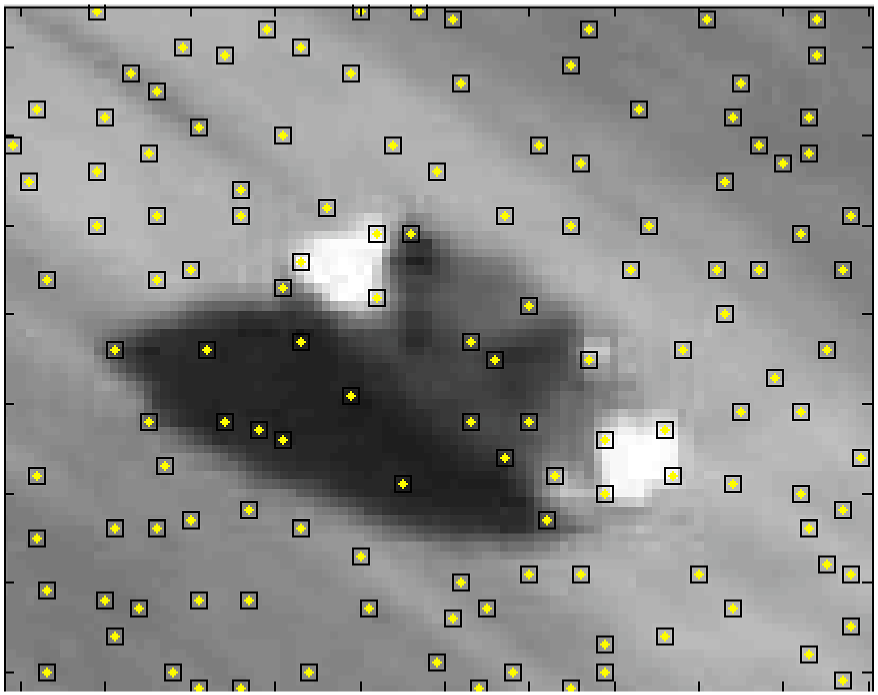
Stabilization Application

Step1: extract Harris corners from both frames. We use a small threshold for R because we want LOTS of corners (fodder for our next step, which is matching).



Harris corners for first frame



Detailed view

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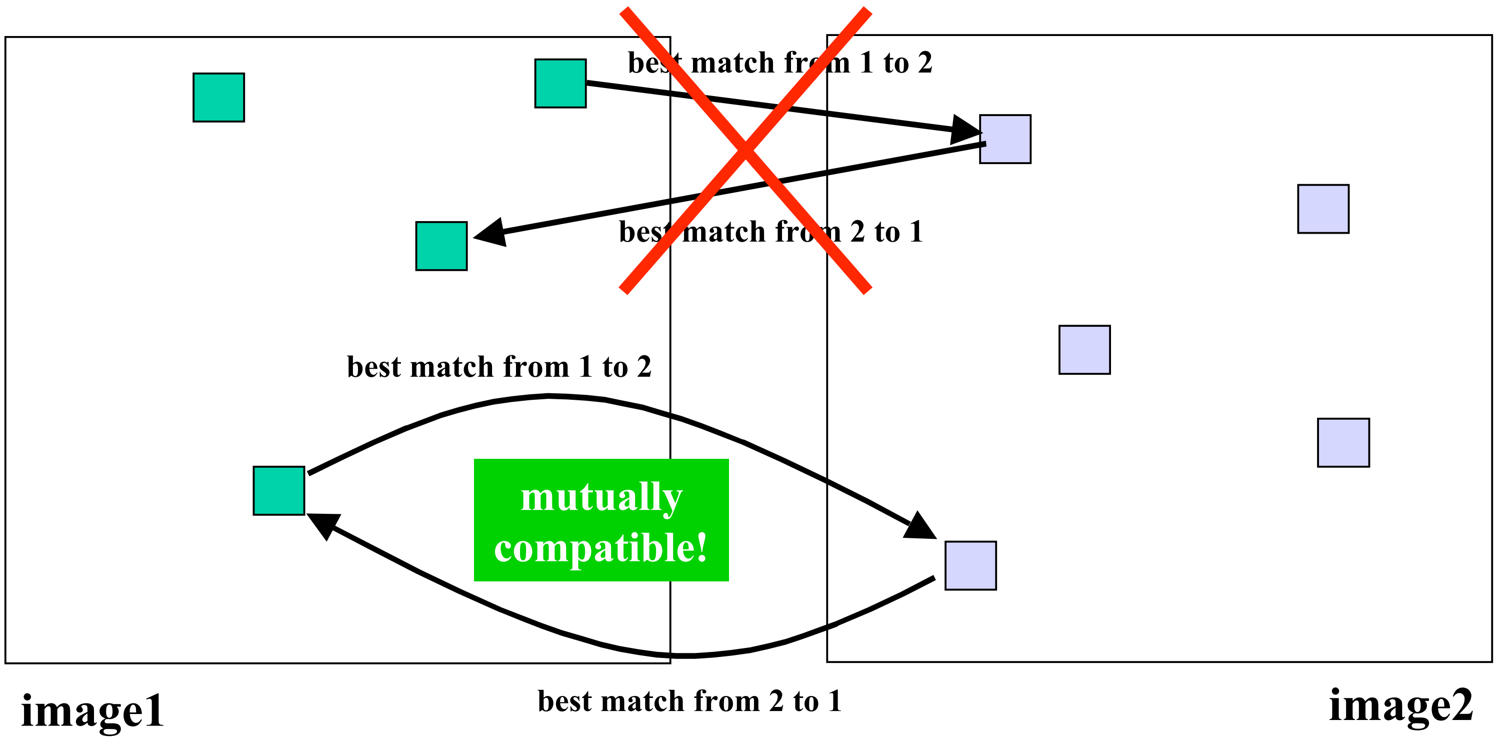
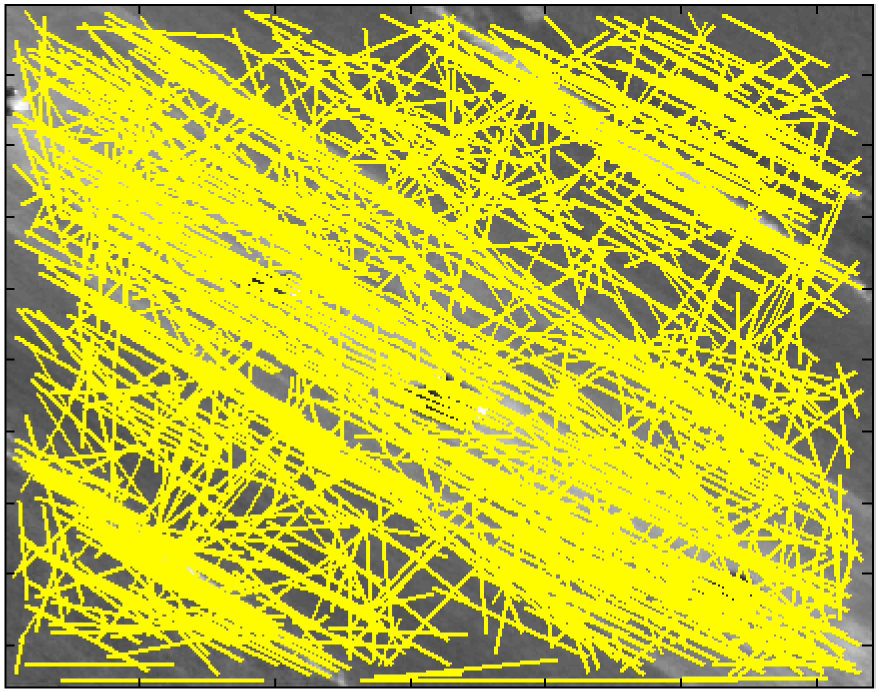
Stabilization Application

Step2: hypothesize matches. For each corner in image 1, look for matching intensity patch in image2 using NCC. Make sure matching pairs have highest NCC match scores in BOTH directions.



Stabilization Application

Step2: hypothesize matches.

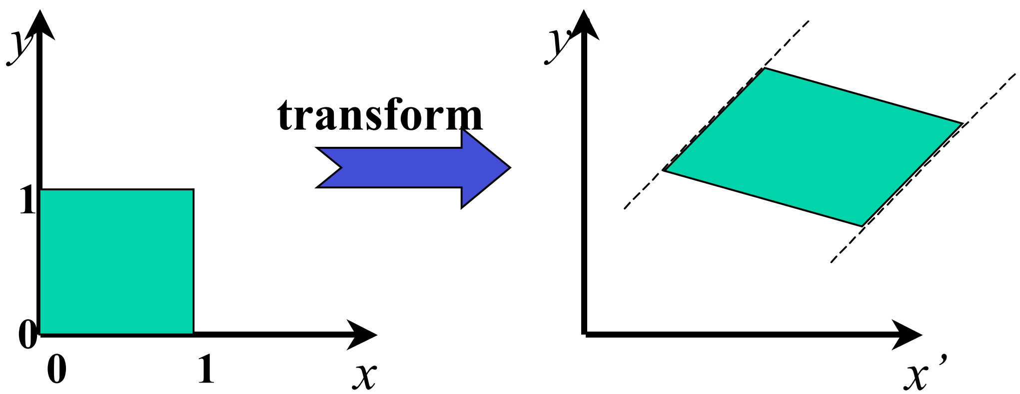


yikes!

As you can see, a lot of false matches get hypothesized. The job of RANSAC will be to clean this mess up.

Stabilization Application

Step3: Use RANSAC to robustly fit best affine transformation to the set of point matches.



$$x_i' = a\,x_i + c\,y_i + c$$
$$y_i' = d\,x_i + e\,y_i + f$$

How many unknowns?
How many point matches are needed?

Stabilization Application

Step3: Use RANSAC to robustly fit best affine transformation to the set of point matches.

Affine transformation has 6 degrees of freedom.
We therefore need 3 point matches [each gives 2 equations]

Randomly sample sets of 3 point matches. For each, compute the unique affine transformation they define. How?

Stabilization Application

How to compute affine transformation from 3 point matches?
Use Least Squares! (renewed life for a nonrobust approach)

$$
\begin{bmatrix}
\Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i & 0 & 0 & 0 \\
\Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i & 0 & 0 & 0 \\
\Sigma x_i & \Sigma y_i & \Sigma 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \Sigma x_i^2 & \Sigma x_i y_i & \Sigma x_i \\
0 & 0 & 0 & \Sigma x_i y_i & \Sigma y_i^2 & \Sigma y_i \\
0 & 0 & 0 & \Sigma x_i & \Sigma y_i & \Sigma 1
\end{bmatrix}
\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}
=
\begin{bmatrix} \Sigma x_i x_i' \\ \Sigma y_i x_i' \\ \Sigma x_i' \\ \Sigma x_i y_i' \\ \Sigma y_i y_i' \\ \Sigma y_i' \end{bmatrix}
$$

Then transform all points from image1 to image2 using that computed transformation, and see how many other matches confirm the hypothesis.

Repeat N times.

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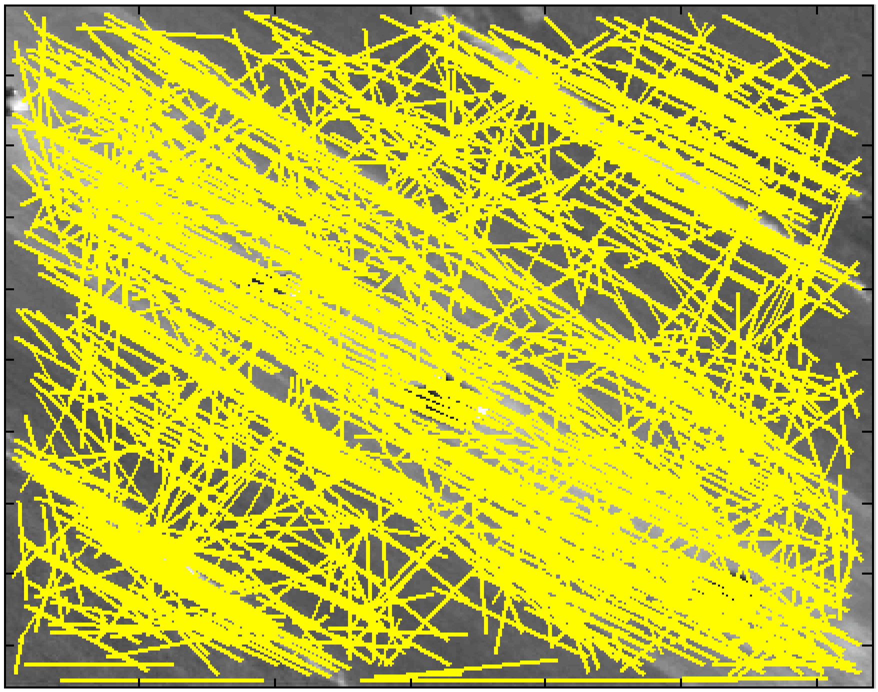
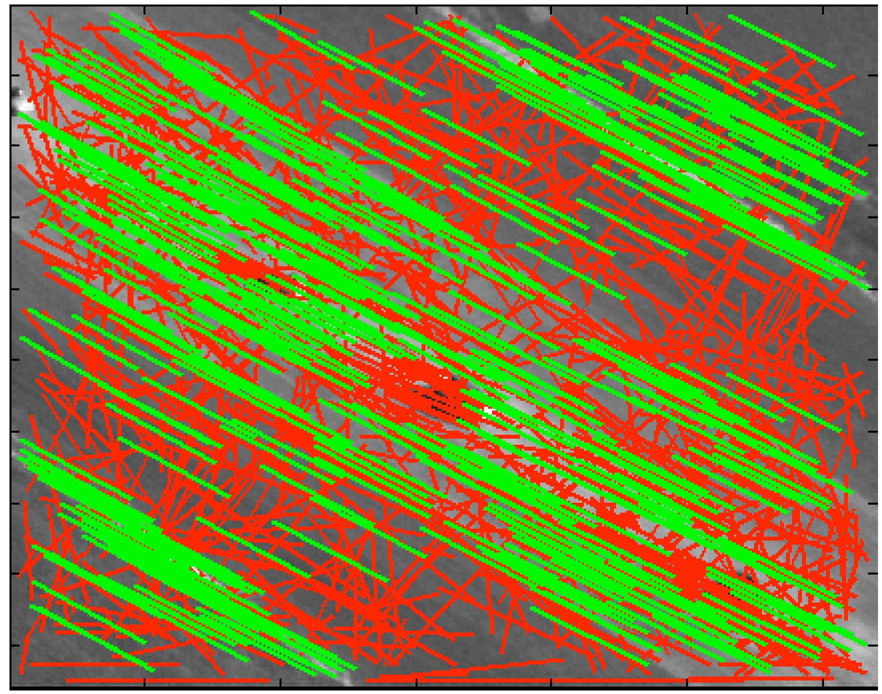
Stabilization Application



original point matches



labels from RANSAC
green: inliers
red: outliers

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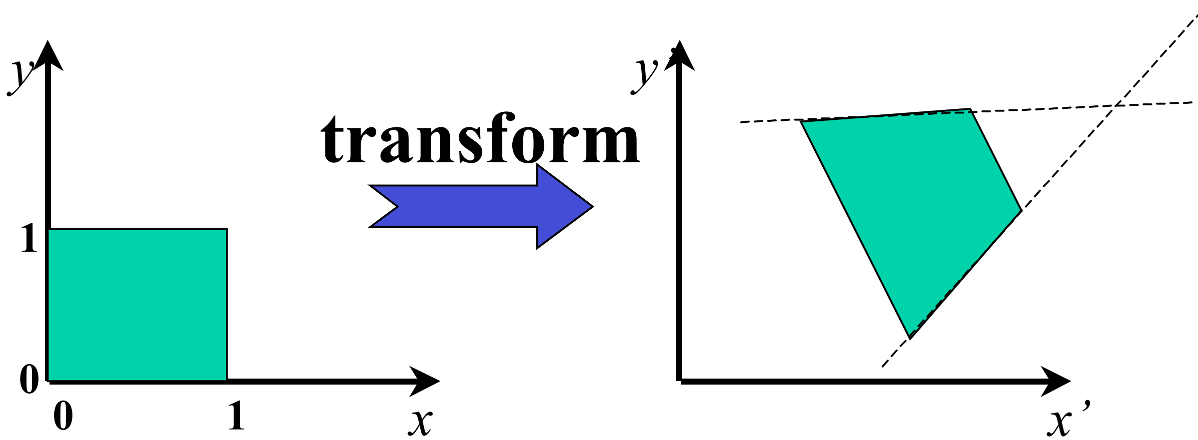
Stabilization Example

Step4: Take inlier set labeled by RANSAC, and now use least squares to estimate a projective transformation that aligns the images. (we will discuss this ad nauseum in a later lecture).



Projective Transformation

Stabilization Example

Step4: estimate projective transformation that aligns the images.

Now it is easier for people (and computers) to see the moving objects.

Stabilization Examples

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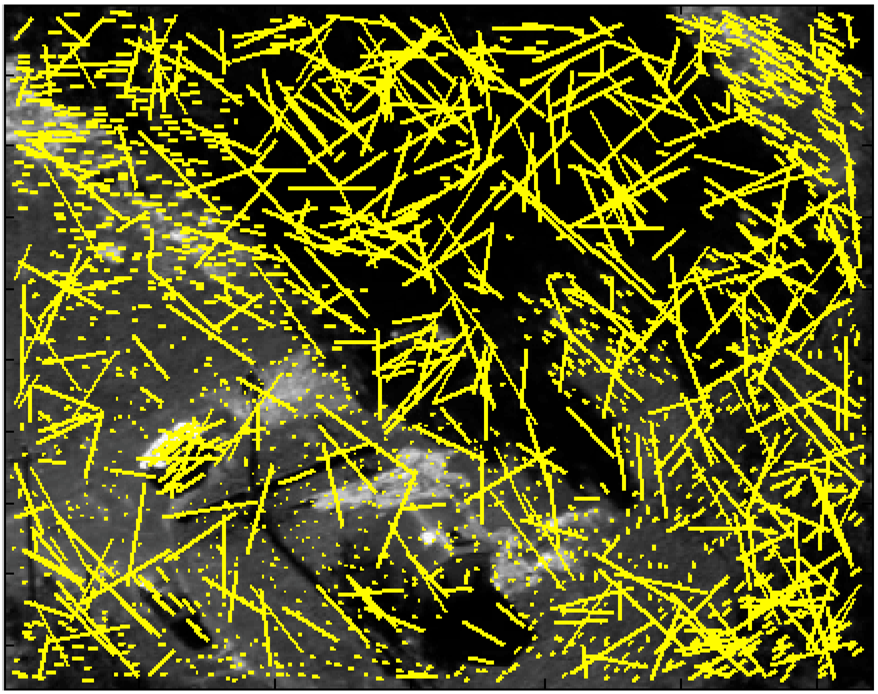
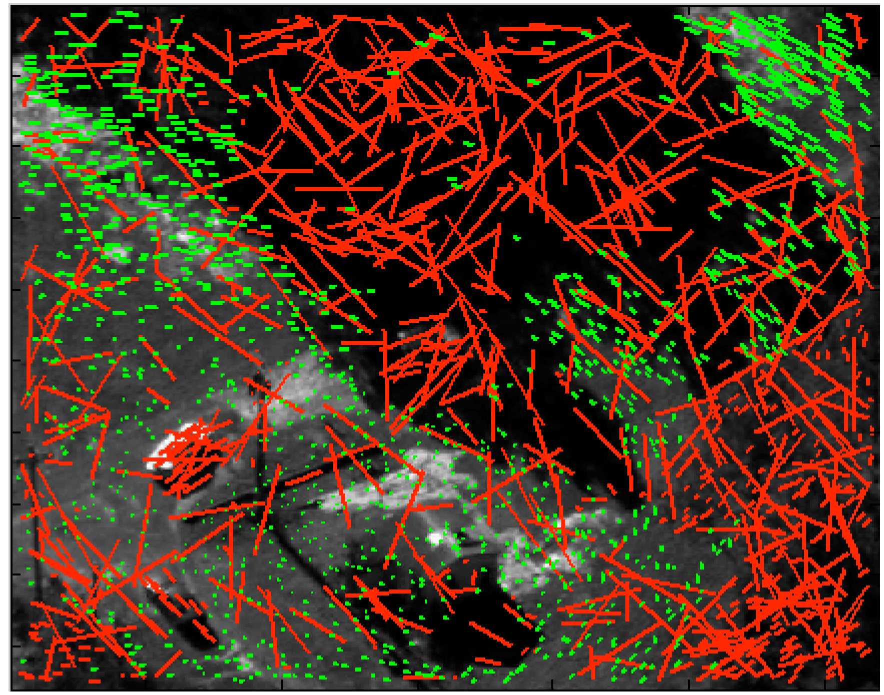
Stabilization Examples



original point matches



labels from RANSAC
green: inliers
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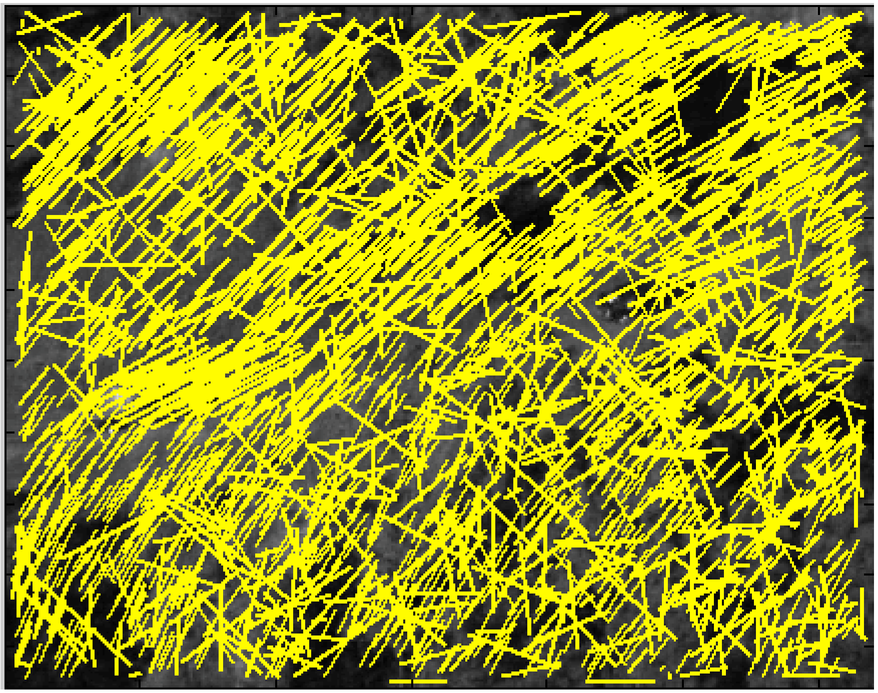
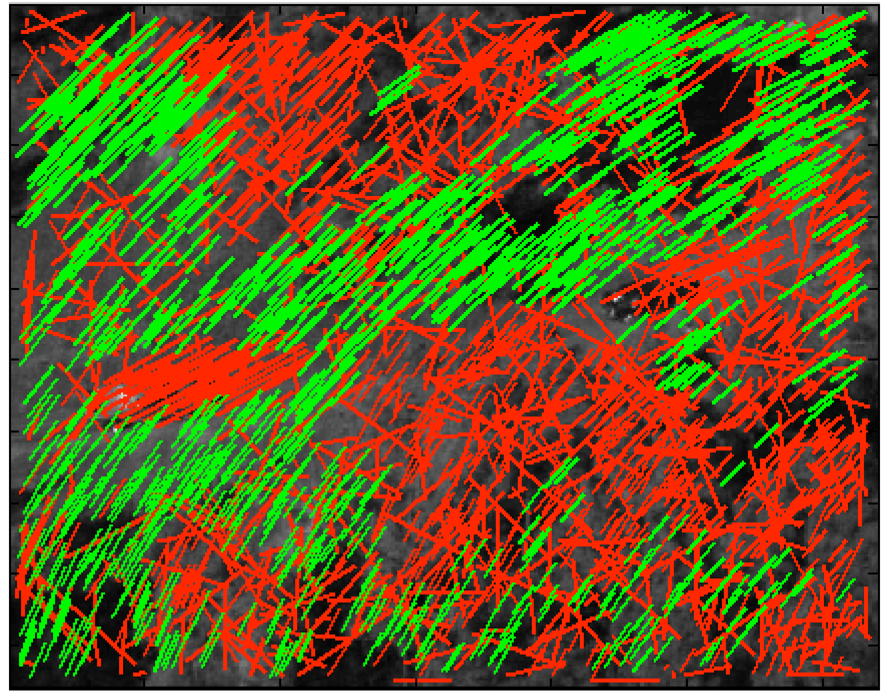
Stabilization Examples

Stabilization Examples

Stabilization Examples



original point matches



labels from RANSAC
green: inliers
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Stabilization Examples