Exercise session 7: LTL and CTL

1 Check equivalence

Which of the following pairs of CTL formulas are equivalent? For those which are not, exhibit a model of one of the pair which is not a model of the other:

1. $\mathrm{EF}\ \phi$ and $\mathrm{EG}\ \phi$
2. $\mathrm{EF}\ \phi \vee \mathrm{EF}\ \psi$ and $\mathrm{EF}\ (\phi \vee \psi)$
3. $\mathrm{AF}\ \phi \vee \mathrm{AF}\ \psi$ and $\mathrm{AF}\ (\phi \vee \psi)$
4. $\mathrm{AF}\ \neg\phi$ and $\neg\mathrm{EG}\ \phi$
5. $\mathrm{EF}\ \neg\phi$ and $\neg\mathrm{AF}\ \phi$
6. $\mathrm{A}\ (\phi_1\ \mathrm{U}\ \mathrm{A}\ (\phi_2\ \mathrm{U}\ \phi_3))$ and $\mathrm{A}\ (\mathrm{A}\ (\phi_1\ \mathrm{U}\ \phi_2)\ \mathrm{U}\ \phi_3)$, hint: it might make it simpler if you think first about models that have just one path
7. $\top$ and $\mathrm{AG}\ \phi \Rightarrow \mathrm{EG}\ \phi$
8. $\top$ and $\mathrm{EG}\ \phi \Rightarrow \mathrm{AG}\ \phi$
9. $\mathrm{A}\ [\phi\ \mathrm{U}\ \psi]$ and $\phi \wedge \mathrm{AF}\ \psi$
10. $\mathrm{A}\ [\phi\ \mathrm{U}\ \psi] \vee \mathrm{A}\ [\tau\ \mathrm{U}\ \psi]$ and $\mathrm{A}\ [(\tau \vee \phi)\ \mathrm{U}\ \psi]$

2 Express in CTL and LTL

Express the following properties in CTL and LTL whenever possible. If neither is possible, try to express the property in CTL*:

1. Whenever p is followed by q (after finitely many steps), then the system enters an 'interval' in which no r occurs until t.
2. Event p precedes s and t on all computation paths. (You may find it easier to code the negation of that specification first)
3. After p, q is never true. (Where this constraint is meant to apply on all computation paths.)
4. Between the events q and r, event p is never true.
5. Transitions to states satisfying p occur at most twice.
6. Property p is true for every second state along a path.

3 Expressable in ...

1. Give example of an LTL-formula for which equivalent translation in CTL does not exist.
2. Give example of an CTL-formula for which equivalent translation in LTL does not exist.
3. Give example of an CTL*-formula for which equivalent translation in LTL either in CTL does not exist.

4 Proof the equivalence

Given the definitions:

- $\pi \models \psi \text{ U } \phi$ iff there is some $i \geq 1$ such that $\pi^i \models \phi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \models \psi$
- $\pi \models \psi \text{ R } \phi$ iff either there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \ldots, i$ we have $\pi^j \models \phi$, or for all $k \geq 1$ we have $\pi^k \models \phi$

Proof the following theorem:

$$\neg(\psi \text{ U } \phi) \equiv \neg\psi \text{ R } \neg\phi \tag{1}$$

5 Nim game

If you have time left, and haven't made the Nim game yet last session, complete this exercise. The assignment from last week can still be found on Toledo.