BRUNO BORGES DE SOUZA

LISTA 5

Exercício 26.3

Queremos encontrar a expressão (26.3.22) da expressão (26.3.19).

Temos:

$$\frac{dL}{dw_{ij}} = \sum_{t=1}^{T-1} \frac{1}{2}\left(V_i(t+1) - \langle V_i(t+1)\rangle_{p(V_i(t+1)|a_i(t))}\right) V_j(t) \quad (26.3.22)$$

$$\frac{dL}{dw_{ij}} = \beta \sum_{t=1}^{T-1} \gamma_i(t) V_i(t+1) V_j(t) \quad (26.3.19)$$

Primeiro, temos que $\langle V_i(t+1)\rangle_{p(V_i(t+1)|a_i(t))} = (+1)\sigma(a_i(t)) + (-1)(1-\sigma(a_i(t)))$

Temos também que, sabendo o fato de $\gamma_i(t) = 1 - \sigma_\beta(V_i(t+1)a_i(t))$

$\Rightarrow 1 - p(V_i(t+1)|v(t))$, $1 - \sigma_\beta(x) = \sigma_\beta(-x)$

$$\gamma_i(t) = 1 - \left(\frac{V_i(t+1)+1}{2}\sigma(a_i(t)) - \frac{V_i(t+1)-1}{2}(1-\sigma(a_i(t)))\right) \quad (1)$$

Calculando $\sigma(a_i(t))$, temos: $\sigma(a_i(t)) = \frac{1}{2}(1+\langle V_i(t+1)\rangle)$ (2)

Substituindo (2) em (1):

$V = V_i(t+1)$, $\sigma(a_i(t)) = \sigma$, $\langle V_i(t+1)\rangle = \langle V\rangle$ $\therefore$

$$\gamma_i(t) = 1 - \frac{V+1}{2}\sigma + \frac{V-1}{2}(1-\sigma) = \frac{1}{2}(2 - V\sigma - \cancel{\sigma} + V - 1 - V\sigma + \cancel{\sigma})$$

$$= \frac{1}{2}(2 - 2V\sigma + V - 1) = \frac{1}{2}\left(1 - \cancel{2}V\left(\cancel{\tfrac{1}{2}}(1+\langle V_i\rangle)\right) + V - \cancel{1}\right)$$

$$\gamma_i(t) = \frac{1}{2} - \frac{1}{2}V(1+\langle V_i\rangle) + \frac{1}{2}V - \cancel{\tfrac{1}{2}} = \frac{1}{2}(1 - V\langle V_i\rangle) \quad (3)$$

Substituindo (3) em 26.3.19: ($V^2 = 1$, $\cancel{\beta = 1}$)

$$\frac{dL}{dw_{ij}} = \sum_{t=1}^{T-1} \frac{1}{2}(V - \langle V\rangle) V(t-1) \Leftrightarrow \sum_{t=1}^{T-1} \frac{1}{2}\left(V_i(t+1) - \langle V_i(t+1)\rangle_{p(V_i(t+1)|a_i(t))}\right) V_j(t)$$