BRUNO BORGES DE SOUZA

LISTA 5

Exercício 27.1

- De acordo com o exercício $x_1 \sim U(x_1|[0,1])$, $x_2 \sim U(x_2|[0,1])$ e $y_1 = \sqrt{-2\log x_1}\cos 2\pi x_2$, $y_2 = \sqrt{-2\log x_1}\sin 2\pi x_2$. Conforme exercício 8.10, se fizermos $\vec{y} = f(\vec{x})$, $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \sqrt{-2\log x_1}\cos(2\pi x_2) \\ \sqrt{-2\log x_1}\sin(2\pi x_2) \end{bmatrix}$ (1)

e $P(y) = P(x = f^{-1}(y))\left[\det\left(\frac{\partial f}{\partial x}\right)\right]^{-1}$.

Calculando $|J_x(f)|$, temos:

$$\begin{vmatrix} \frac{\partial}{\partial x_1} y_1 & \frac{\partial}{\partial x_2} y_1 \\ \frac{\partial}{\partial x_1} y_2 & \frac{\partial}{\partial x_2} y_2 \end{vmatrix} = \begin{vmatrix} -(-2\log x_1)^{-\frac{1}{2}} \frac{\cos 2\pi x_2}{x_1} & -2\pi(-2\log x_1)^{\frac{1}{2}} \sin 2\pi x_2 \\ -(-2\log x_1)^{-\frac{1}{2}} \frac{\sin 2\pi x_2}{x_1} & 2\pi(-2\log x_1)^{1/2}\cos 2\pi x_2 \end{vmatrix}$$

$$= -2\pi(-2\log x_1)^0 \frac{\cos^2 2\pi x_2}{x_1} - 2\pi(-2\log x_1)^0 \frac{\sin^2 2\pi x_2}{x_1}$$

$$= -\frac{2\pi}{x_1}(\cos^2 2\pi x_2 + \sin^2 2\pi x_2) = -\frac{2\pi}{x_1} \quad (2)$$

$\hookrightarrow \left|\det\left(\frac{\partial f}{\partial x}\right)\right|^{-1} = \frac{x_1}{2\pi}$

Elevando ao quadrado em (1):

$$y_1^2 = -2\log x_1 \cos^2(2\pi x_2), \quad y_2^2 = -2\log x_1 \sin^2(2\pi x_2)$$

$$y_1^2 + y_2^2 = -2\log x_1 (\underbrace{\cos^2(2\pi x_2) + \sin^2(2\pi x_2)}_{1})$$

$$x_1 = e^{-\frac{1}{2}(y_1^2 + y_2^2)}$$

Sendo $P(x_1) = P(x_2) = 1$, temos então:

$$P(y_1, y_2) = \int P(y_1|x_1,x_2) P(y_2|x_1,x_2) P(x_1) P(x_2)\, dx_1 dx_2 = \frac{x_1}{2\pi} = \frac{1}{2\pi} e^{-\frac{1}{2}(y_1^2 + y_2^2)}$$

$$= N(y_1|0,1)\, N(y_2|0,1)$$

Um idéia de um algoritmo seria basicamente:

1. plotar $u_1, u_2 \sim \text{unif}(0,1) = U([0,1])$;
2. Fazer a transformação, conforme ex. 8.10: $y_1 = \sqrt{-2\log x_1}\cos\theta$, $y_2 = \sqrt{-2\log x_1}\sin\theta$, $\theta = 2\pi x_2$ (coordenadas polares);
3. Fazer a transformação de coordenadas polares em $\frac{1}{2\pi} e^{-\frac{1}{2}(y_1^2 + y_2^2)}$ e plotar as duas normais.