Itaú Asset Management

Asset Allocation and Factors

techniques for enhancing return and managing risks

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Quantitative Portfolio Management and Research

Agenda

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1 Diversification and Risk Factors

Diversification in… what ???

- **Traditional Mean-Variance Asset Allocation**

Considering a portfolio of N risky assets, an investor needs to find the allocation weights $\boldsymbol{\omega}$ from the following optimization problem:

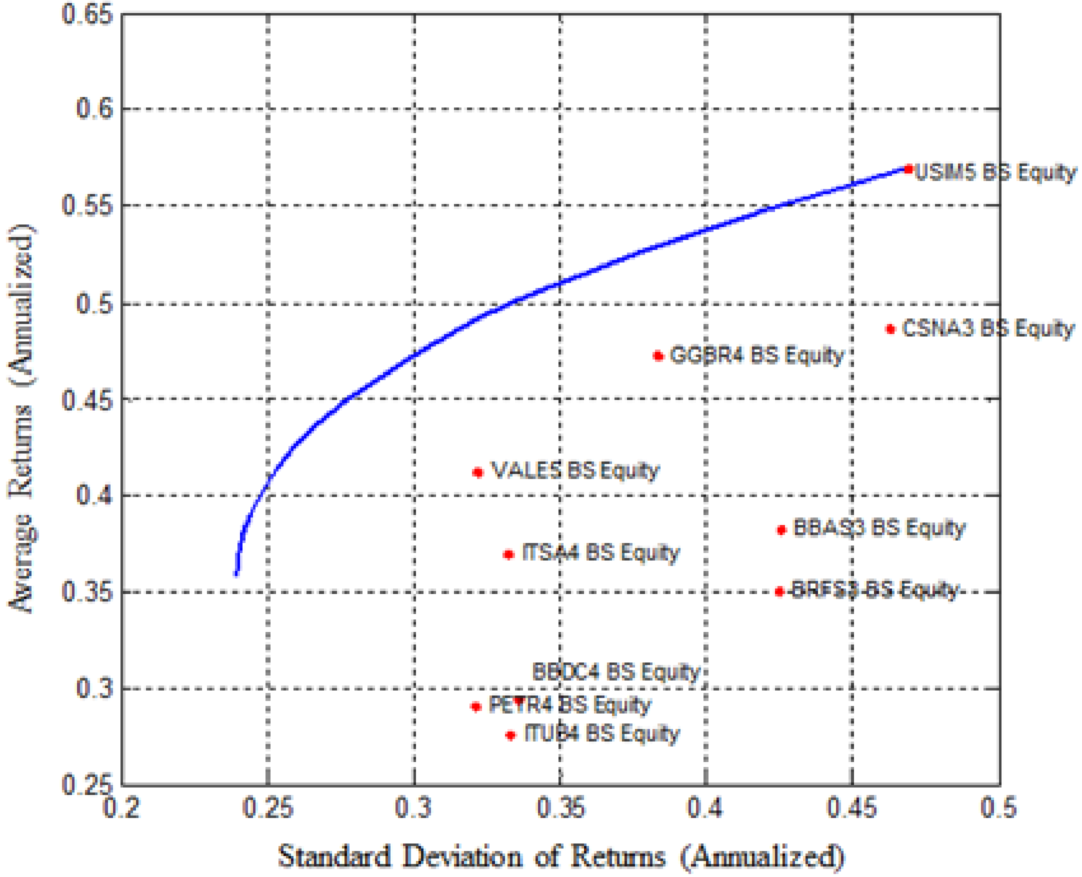
$$\max_{\boldsymbol{\omega}} \mathbb{E}\mathrm{U}(\boldsymbol{\omega}, \mathbf{R}, \lambda) = \max_{\boldsymbol{\omega}} \mathbb{E}[\boldsymbol{\omega}'\mathbf{R}] - (\lambda/2)\mathrm{Var}[\boldsymbol{\omega}'\mathbf{R}]$$

$$\mathrm{s.t.}: \boldsymbol{\omega}'\mathbf{1}_N = 1,$$

where $\mathbf{R}$ represents the excess returns of the risky assets, $\lambda \geq 0$ is the risk aversion parameter of the investor and $\mathrm{U}(\cdot)$ a utility function (the expected utility function adopted is exact for elliptical distributions of excess returns).

Diversification in… what ???

Markowitz's Efficient Frontier $\lambda \geq 0$



Input Errors???

Diversification in… what ???

- **Equal Weight – Naïve Approach**

Plyakha et al., Why Does an Equal-Weighted Portfolio Outperform Value- and Price-Weighted Portfolios? (October 16, 2012):

Metrics (per year)	Performance before transaction costs			Performance net of transaction costs		
	EW	VW	PW	EW	VW	PW
Total Return	0.1319	0.1048	0.1207	0.1279	0.1041	0.1191
Sharpe Ratio	0.4275	0.3126	0.3966	0.4048	0.3081	0.3871

Added value???

Diversification in… what ???

- **Information-based Approaches**

Considering a portfolio of N risky assets, an investor needs the allocation weights $\boldsymbol{\omega}$ in order to achieve optimum diversification (?):

Is it enough???

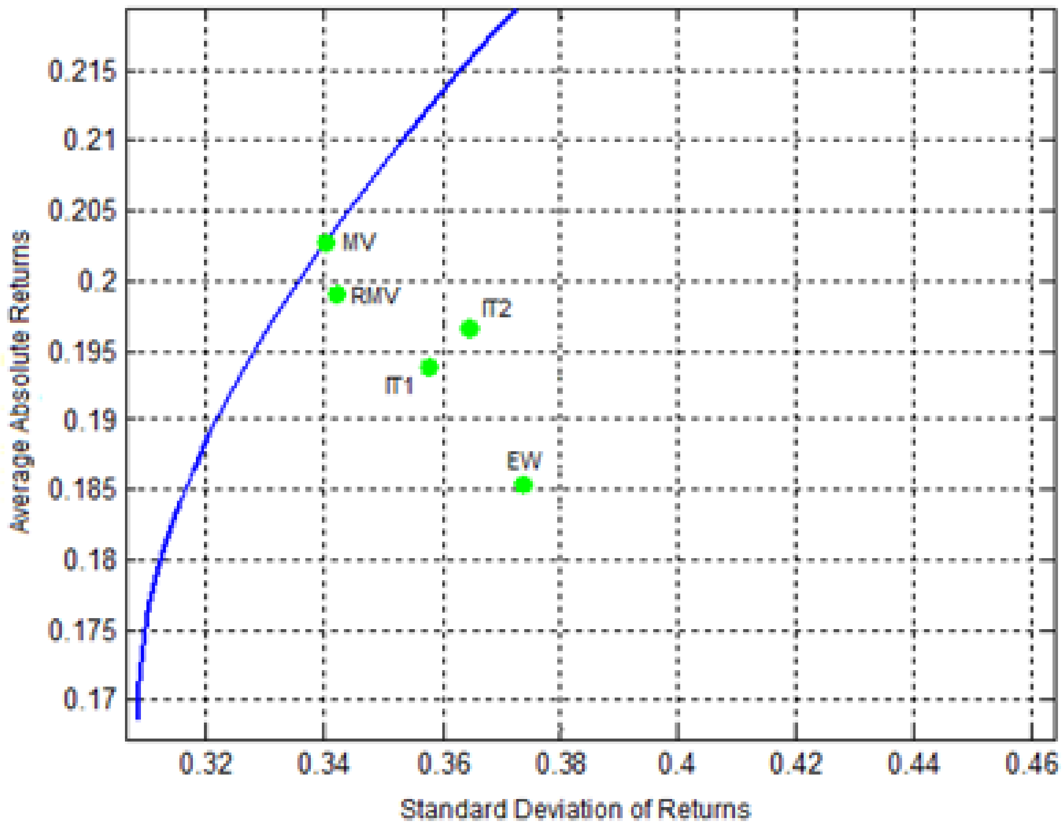
$$\max_{\boldsymbol{\omega}} - \sum_{i=1}^{N} \omega_i \ln(\omega_i)$$

$$\text{s.t.}: \boldsymbol{\omega} \geq 0, \boldsymbol{\omega}' 1_N = 1, \sqrt{\boldsymbol{\omega}'\widehat{\boldsymbol{\Sigma}}\boldsymbol{\omega}} \leq \sigma_0, \boldsymbol{\omega}'\hat{\mathbf{r}} \geq r_0,$$

where $\hat{\mathbf{r}}$ is the estimated mean and $\widehat{\boldsymbol{\Sigma}}$ is the estimated covariance of excess returns $\mathbf{R}$ of the $\mathbf{N}$ risky assets.

Diversification in… what ???

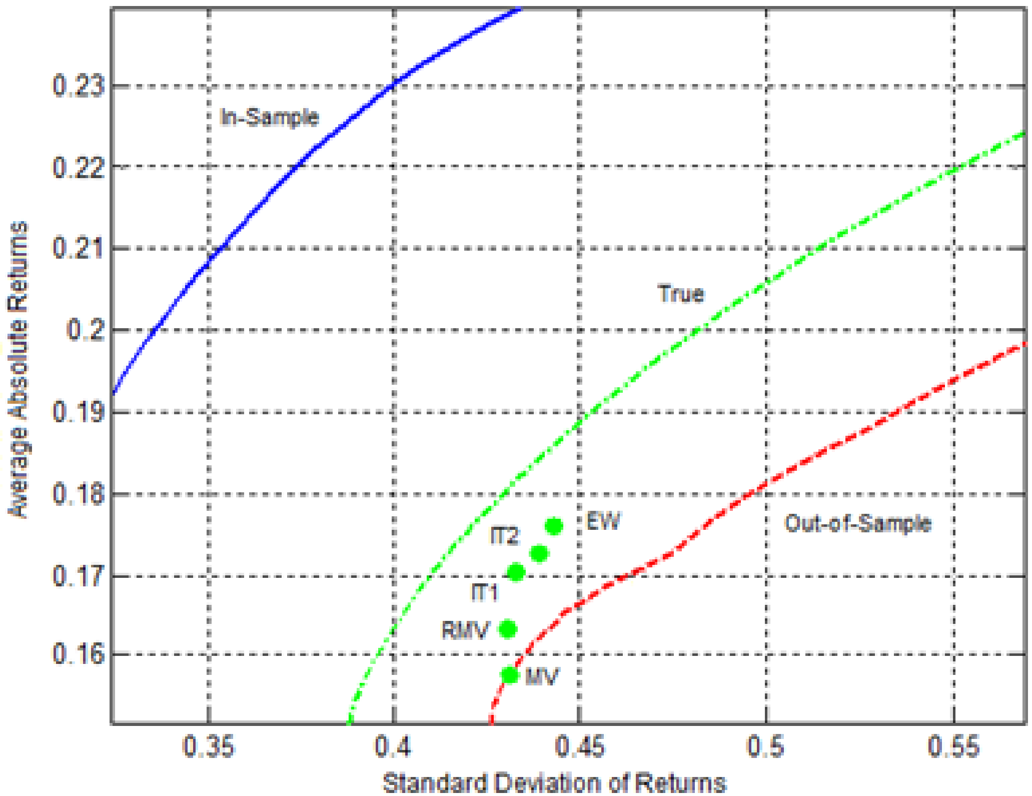
Mean-Variance Efficient Frontier and the Performance of Different Approaches using Simulated In-Sample Data:



Obvious???

Diversification in… what ???

In-Sample (blue line), True (green line) and Out-of-Sample (red line) Mean-Variance Efficient Frontiers and The Performance of Different Approaches using Simulated Out-of-Sample Data:



Diversification in… what ???

▪ Risk and Correlation Parity

It is necessary to define:

$$\mathbf{y} = \mathrm{diag}(\boldsymbol{\omega})\boldsymbol{\Sigma}\boldsymbol{\omega}$$

$$\mathbf{z} = \mathbf{C}\,\mathrm{diag}(\boldsymbol{\omega})\boldsymbol{\Sigma}\boldsymbol{\omega}$$

where $\mathbf{C}$ is the correlation matrix and $\boldsymbol{\Sigma}$ the covariance matrix.

Risk Parity: $y_1 = y_2 = \cdots = y_n$

Implementation???

Correlation Parity: $z_1 = z_2 = \cdots = z_n$

Factors… how ???

- **Techniques**
  - **Statistical vs. Non-Statistical Factors**

  - **Principal Component Analysis (PCA)**
  - **Independent Component Analysis (ICA)**
  - **Non-Negative Matrix Factorization (NNMF)**

Which one???

Factors… how ???

- **Some mathematics…**

The objective of factor analysis is to reduce the dimensionality of the original data $X = [x_{ij}] \in \mathbb{R}^{m \times p}$, $m \wedge p \in \mathbb{N}^+$, using an approximation $\tilde{X} = [\tilde{x}_{ij}] \in \mathbb{R}^{m \times p}$ such that:

$$X \approx \tilde{X} = Z\Lambda,$$

where $Z = [z_{ij}] \in \mathbb{R}^{m \times k}$ is the matrix of factors or unobserved (latent) variables, $\Lambda = [\lambda_{ij}] \in \mathbb{R}^{k \times p}$ is the matrix of factor loadings or weights, k represents the number of factors and $k \leq p$. In the literature, there are some factorization techniques to find Z and Λ.

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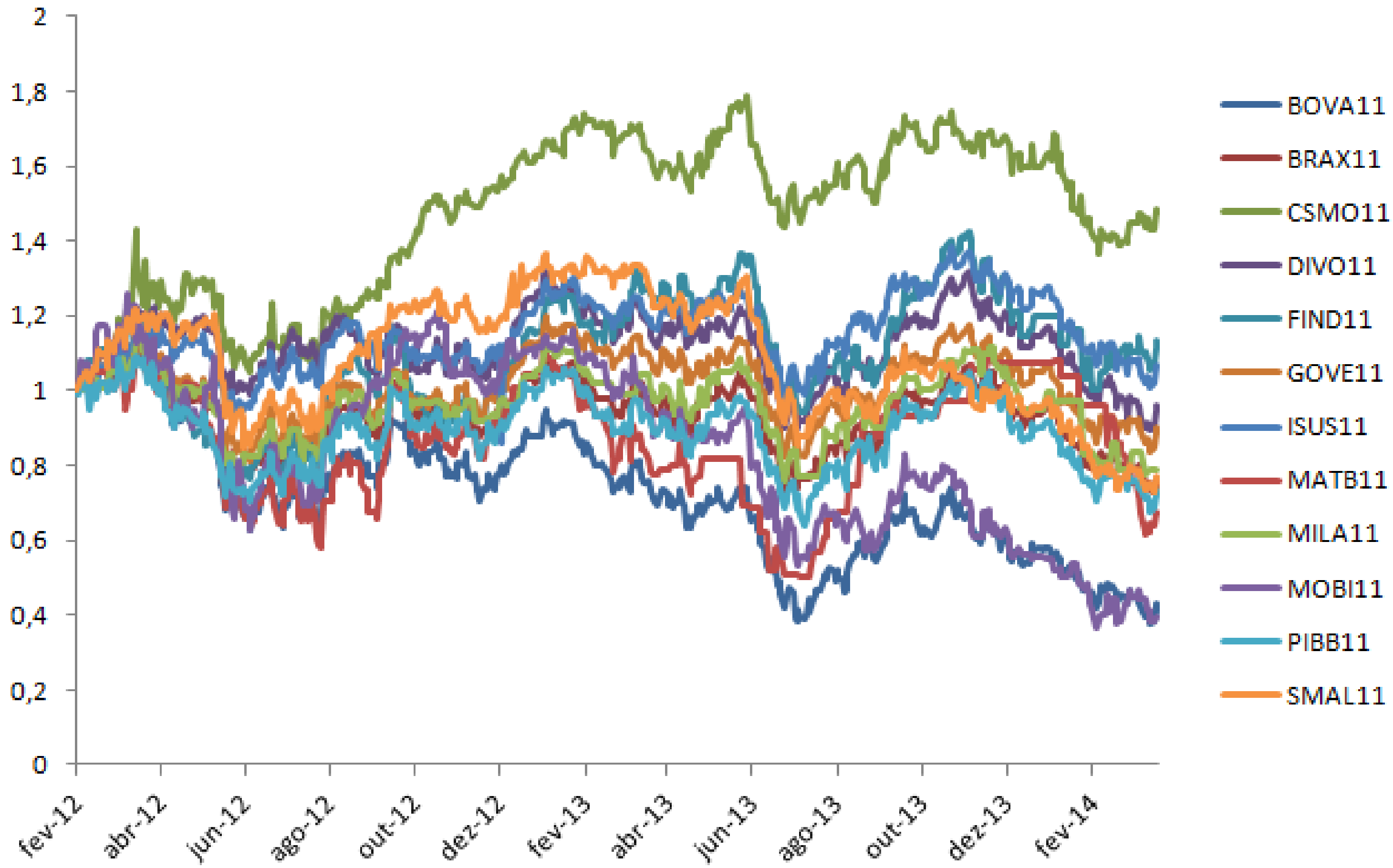
2 Common Trends in Prices

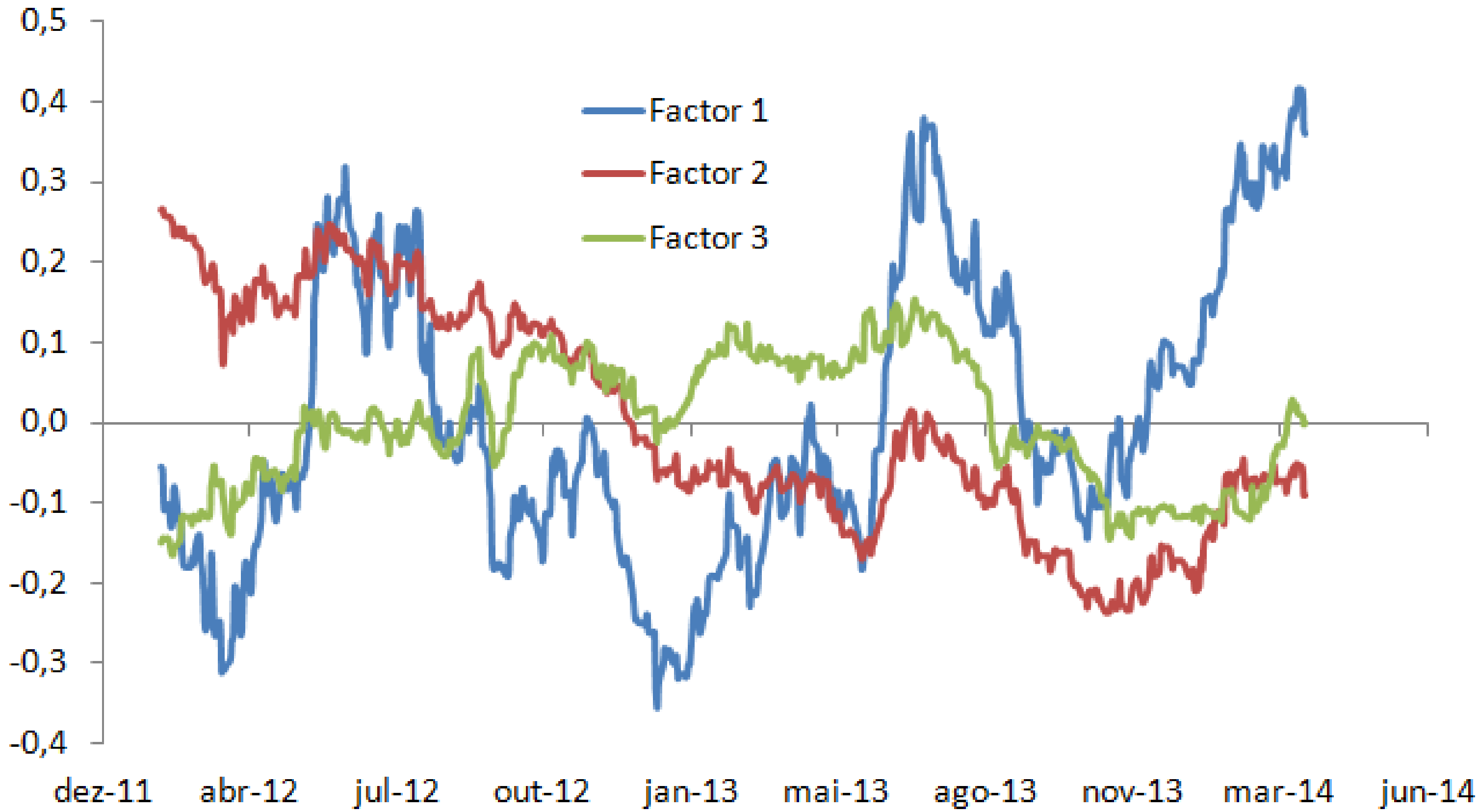
Are factors common trends?

- **Data**



Are factors common trends?

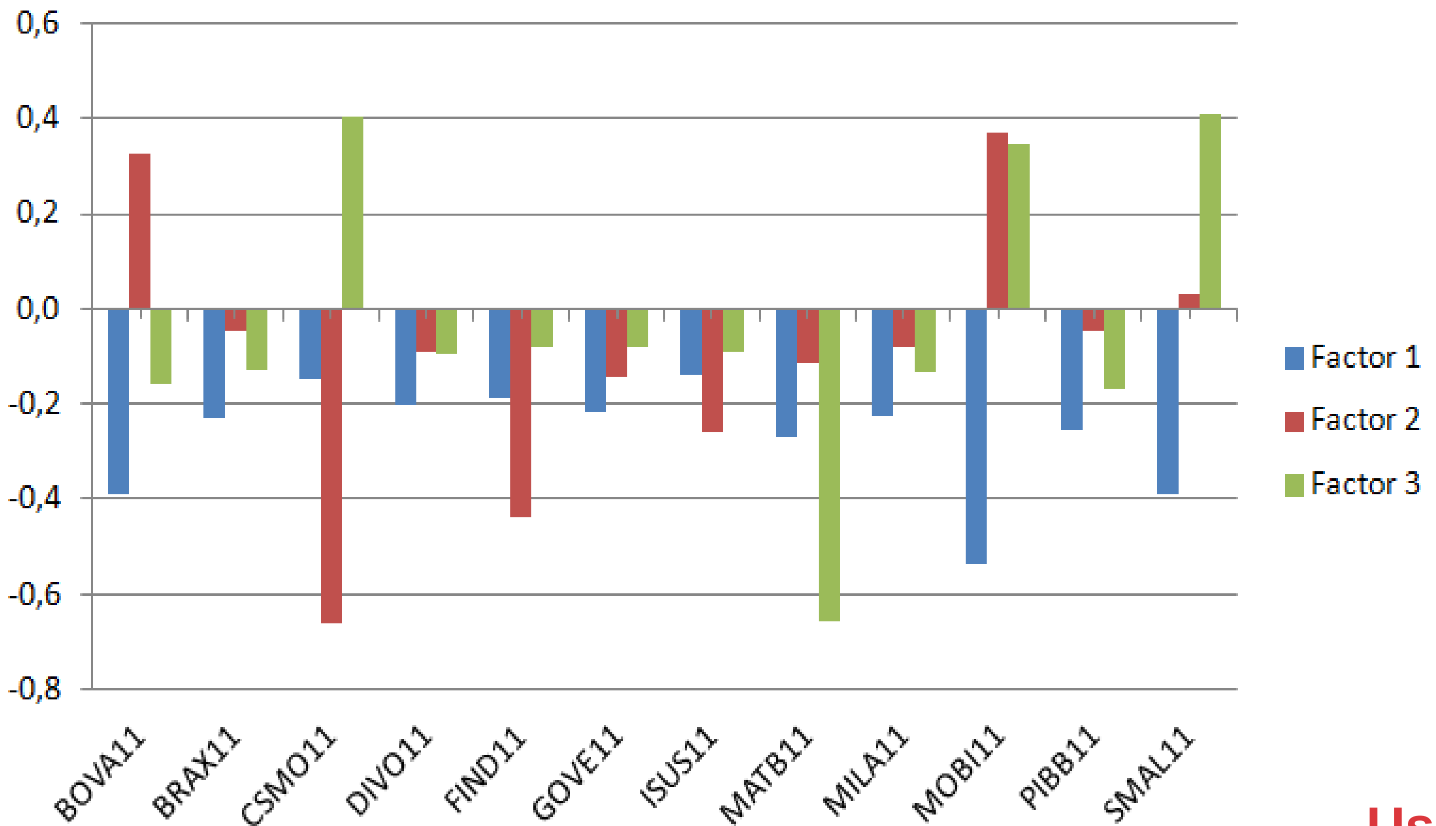
- **Identification…**



Using PCA.

Are factors common trends?

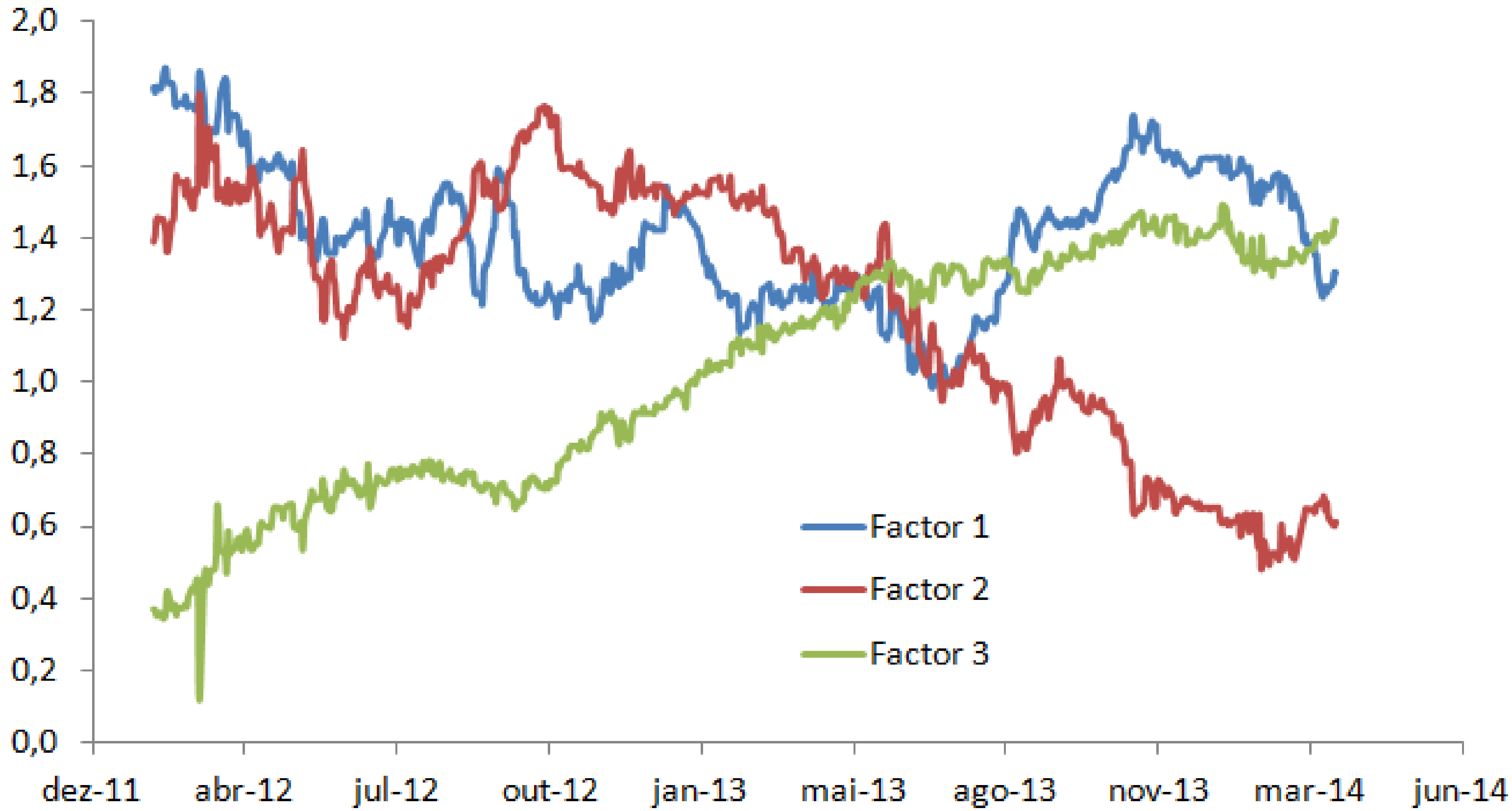
- **Interpretation…**



Using PCA.

Are factors common trends?

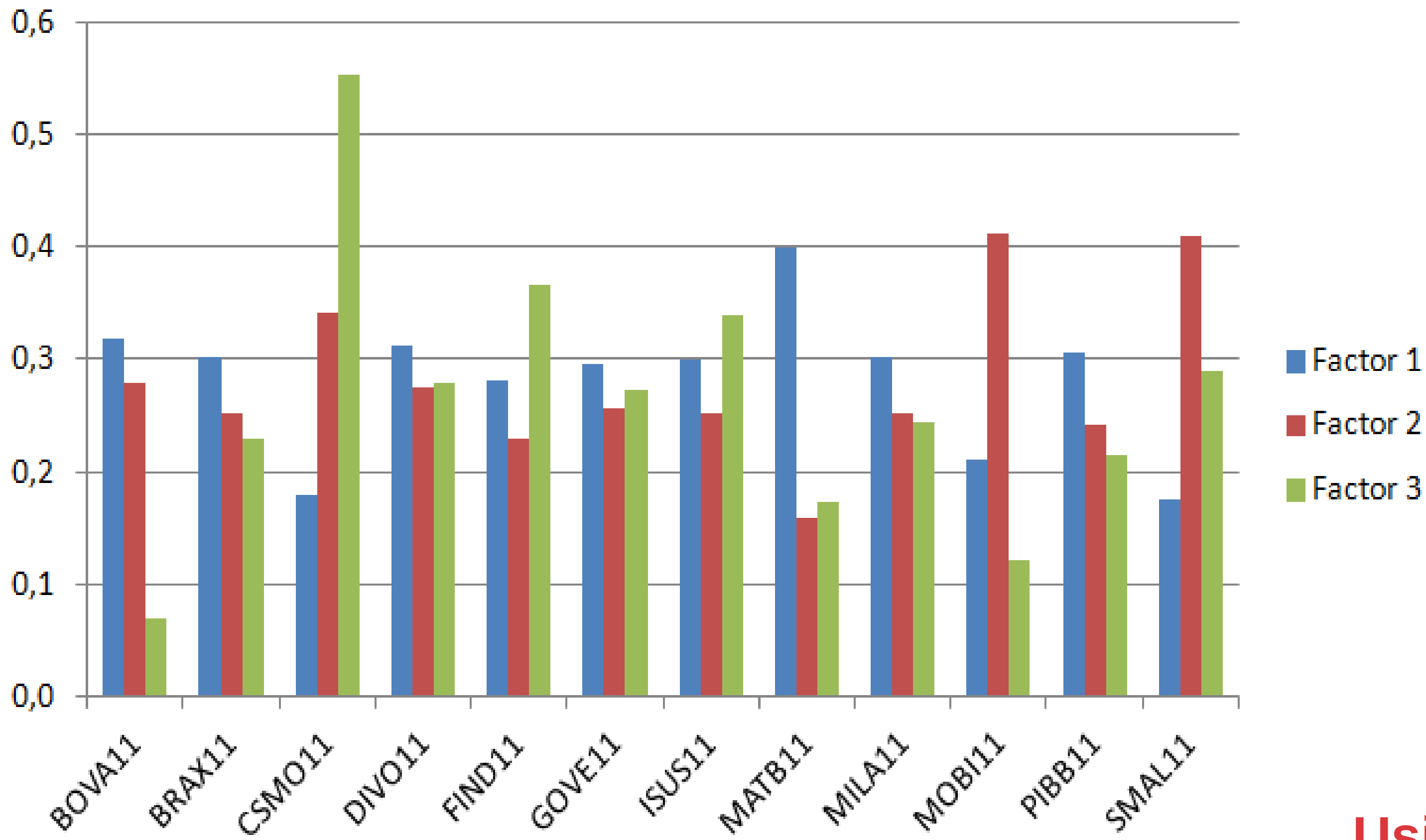
- **Other factors…**



Using NNMF.

Are factors common trends?

- **Common trends?**



Using NNMF.

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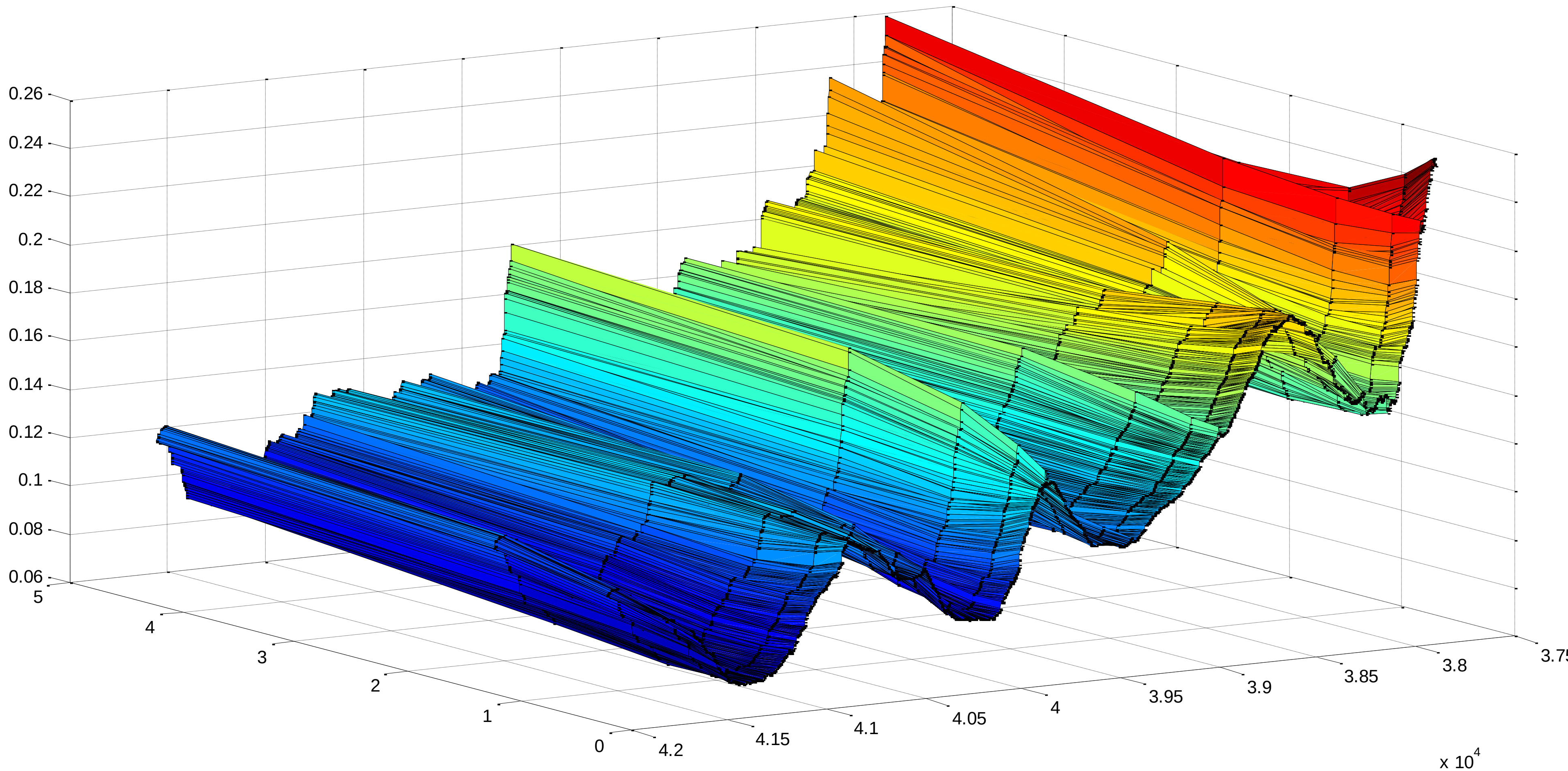
3 Trends in Fixed Income

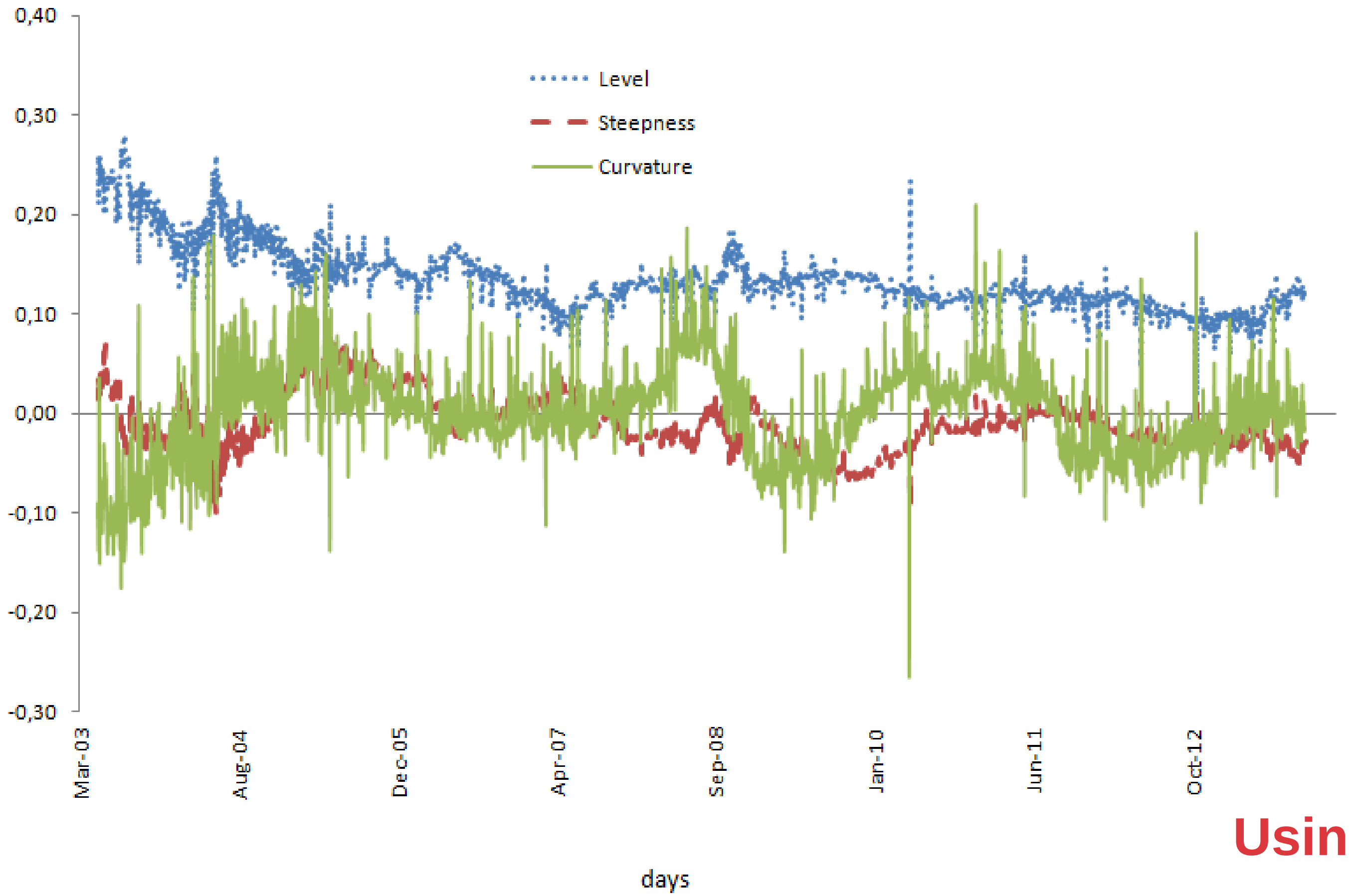
Factors in fixed income?

- **Data**



Factors in fixed income?

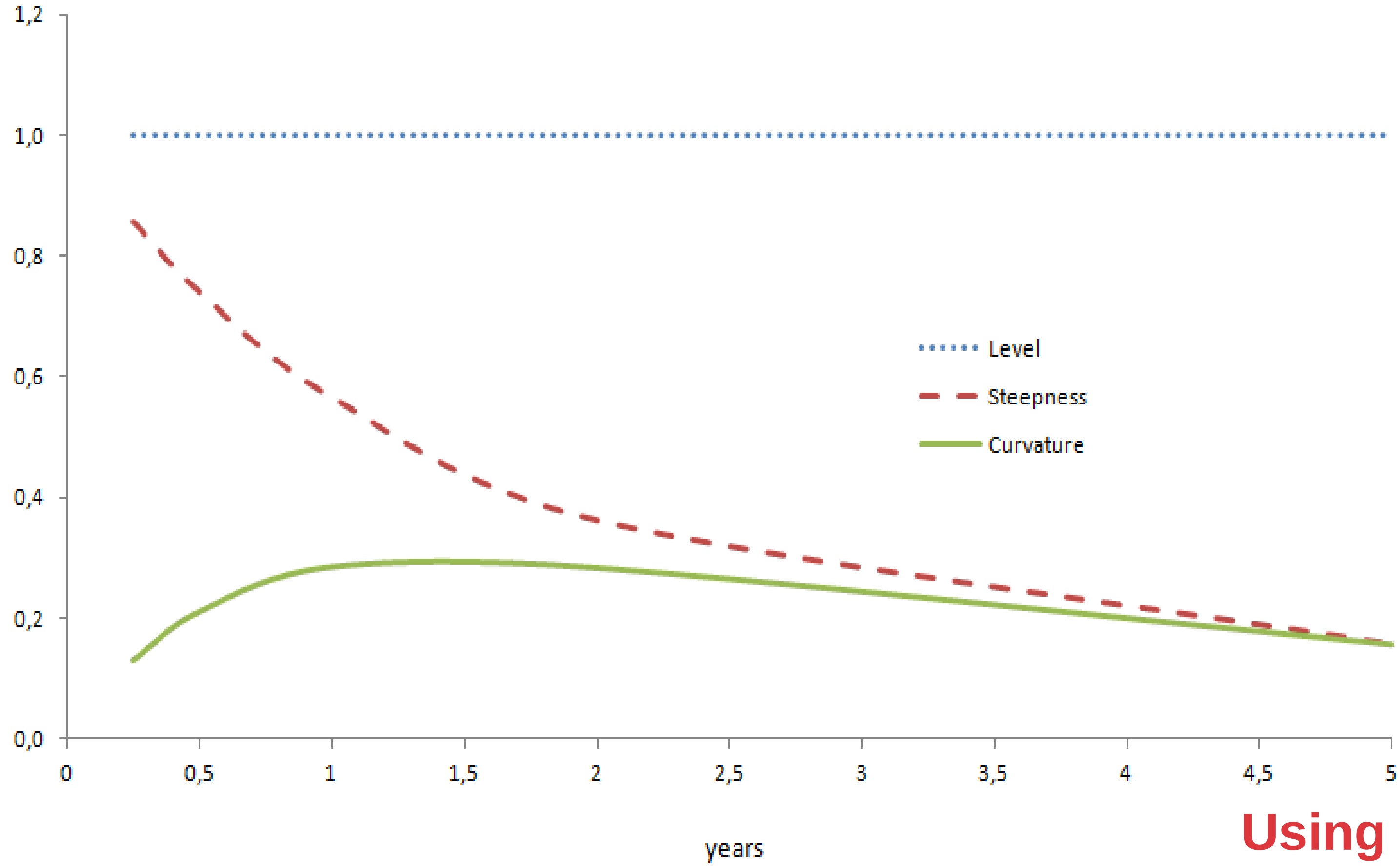
- **Parametric factors…**



Using Nelson-Siegel.

Factors in fixed income?

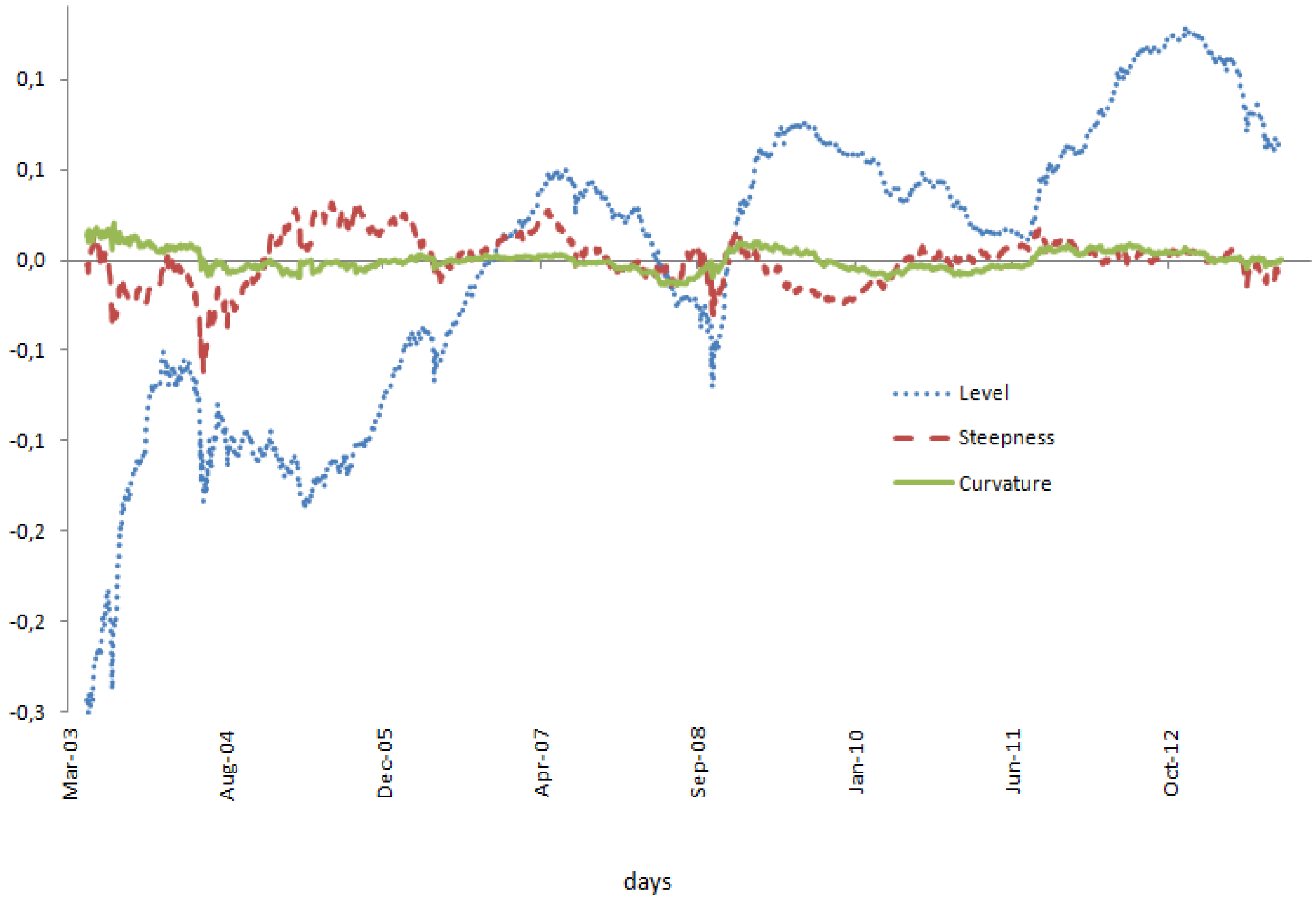
- **… and factor loadings:**



Using Nelson-Siegel.

Factors in fixed income?

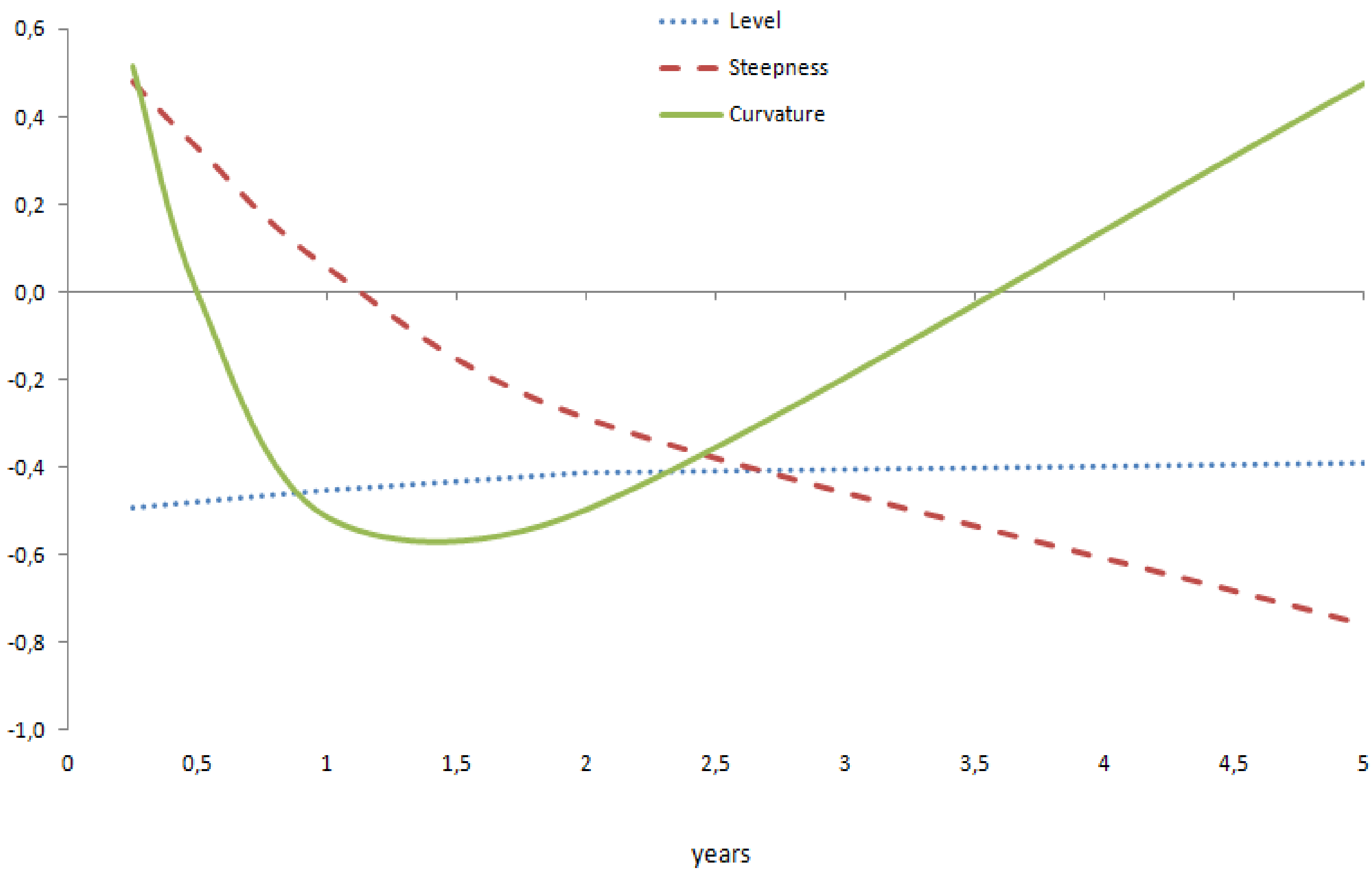
- **Traditional factors…**



Using PCA.

Factors in fixed income?

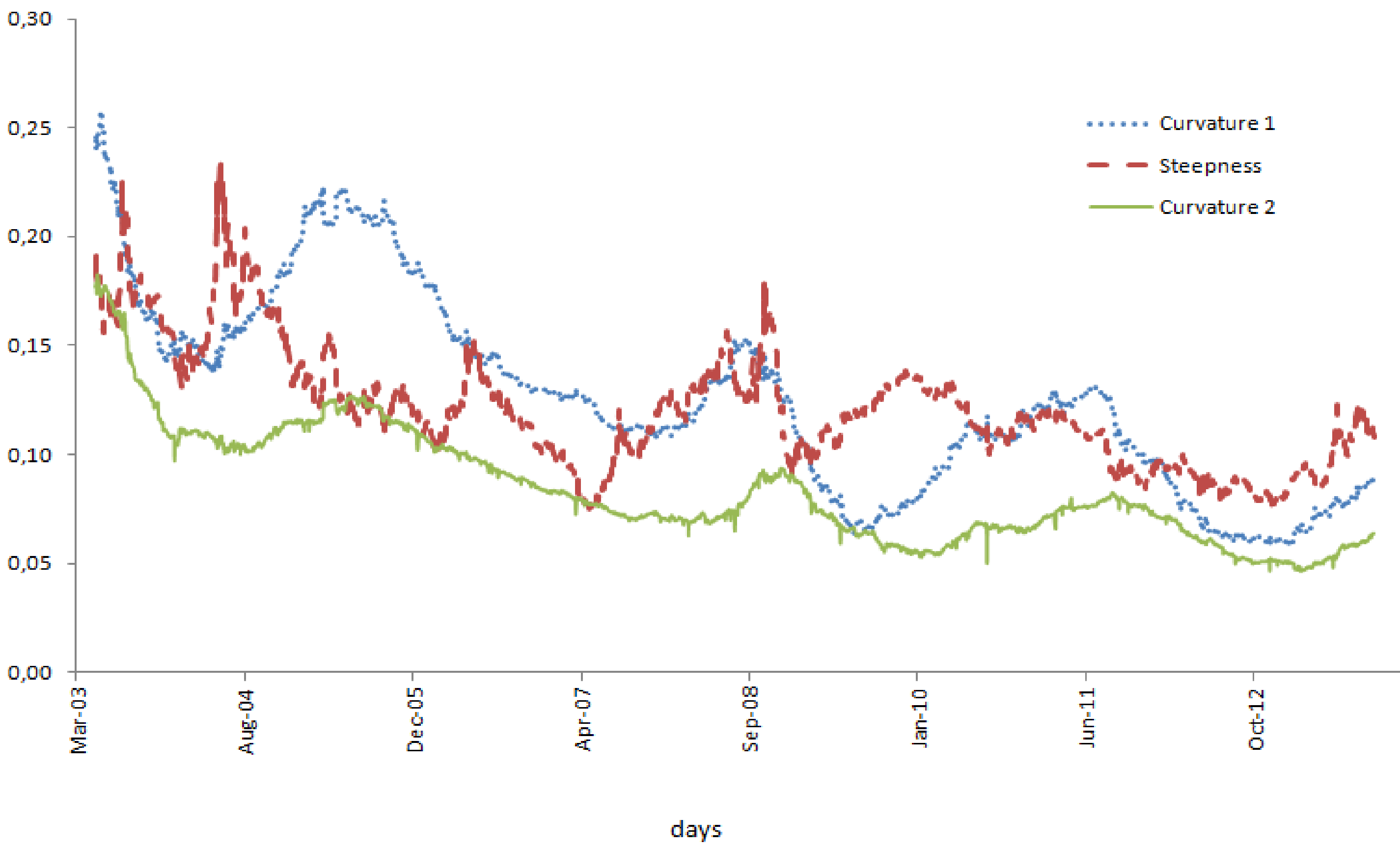
- **Easy interpretation…**



Using PCA.

Factors in fixed income?

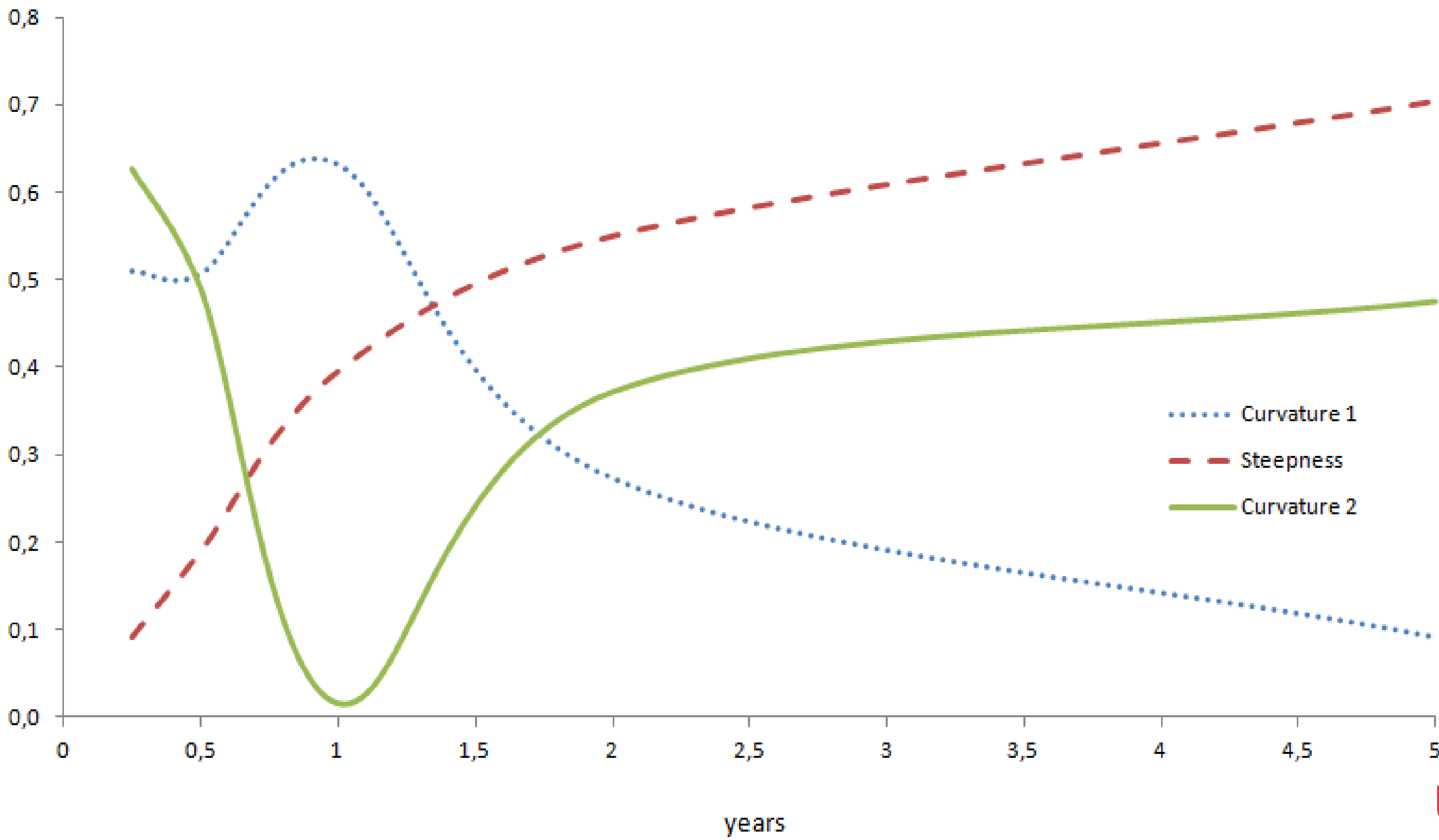
- **An alternative…**



Using NNMF.

Factors in fixed income?

- **Interesting…**



Using NNMF.

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Thank you!

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