Table 1: Article Benchmarks - Generic problems with uncorrelated variables

LSF significance	**Case No.**	**Limit State function(s)**	**Stochastic variable(s)**	β	**Ref**
Iceland shape	(1)	$g(X) = 7 - 8\exp\big(-(X_1+1)^2 + (X_2+1)^2\big) + 2\exp\big(-(X_1-5)^2 - (X_2-4)^2\big) + 1 + \dfrac{X_1X_2}{10}$	$X_1 : N(2,1)$ $X_2 : N(2,1)$	4.0	[1]
With high-frequency, artificial noise term	(2)	$g(X) = X_1 + 2X_2 + 2X_3 + X_4 - 5X_5 - 5X_6 + 0.001\sum_{i=1}^{6}\sin(100X_i)$	$X_{1\ldots4} : LN(120,12)$ $X_5 : LN(50,15)$ $X_6 : LN(40,12)$	2.247	[2, 3]
Cubic performance function	(3)	$g(X) = 2.2257 - \dfrac{0.025\sqrt{2}}{27}(X_1 + X_2 - 20)^3 + \dfrac{33}{140}(X_1 - X_2)$	$X_1 : N(10,3)$ $X_2 : N(10,3)$	2.07	[4, 5]
Highly Nonlinear	(4)	$g(X) = X_1^3 + X_2^3 - 18$	$X_1 : N(10,5)$ $X_2 : N(9.9,5)$	2.52	[6, 5]
Highly Nonlinear	(5)	$g(X) = X_1^3 + X_2^3 - 67.5$	$X_1 : N(10,5)$ $X_2 : N(9.9,5)$	2.23	[6]
Highly Nonlinear	(6)	$g(X) = \dfrac{120}{X_1} - \dfrac{X_2}{X_1} - 1$	$X_1 : N(4.0,1.0)$ $X_2 : N(4.0,1.0)$	4.0	[7]
Quartic performance function	(7)	$g(X) = \dfrac{5}{2} + \dfrac{1}{216}(X_1 + X_2 - 20)^4 - \dfrac{33}{140}(X_1 + X_2)$	$X_1 : N(10.0,3.0)$ $X_2 : N(10.0,3.0)$	2.76	[8, 9]
Highly Nonlinear	(8)	$g(X) = X_1^4 + 2X_2^4 - 20$	$X_1 : N(10.0,5.0)$ $X_2 : N(10.0,5.0)$	2.90	[10, 9]
With multiple failure points	(9)	$g(X) = X_1X_2 - 146.14$	$X_1 : N(7.80644\times10^4,\ 1.17097\times10^4)$ $X_2 : N(0.0104,\ 0.00156)$	5.11	[7]
Non-Normal-Nonlinear	(10)	$g(X) = X_1X_2 - 1140$	$X_1 : LN(38.0,3.8)$ $X_2 : LN(54.0,2.7)$	5.21	[11]
Non-Normal-Nonlinear	(11)	$g(X) = X_1X_2 - 2000X_3$	$X_1 : N(0.32,0.032)$ $X_2 : N(1.4\times10^6,\ 7.0\times10^4)$ $X_3 : LN(100.0,40.0)$	2.184	[12]
Circle Shape	(12)	$g(X) = 9 - X_1^2 - X_2^2$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	2.28	[7]
Highly Nonlinear	(13)	$g(X) = 3 - X_2 + (4X_1)^4$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	3.57	[13]
Nonlinear	(14)	$g(X) = 2 + 0.015\sum_{i=1}^{10}X_i^2 - X_{10}$	$X_{1\ldots10} : N(0.0,1.0)$	2.13	[14]
Highly nonlinear with non-convex domains	(15)	$g(X) = 10 - \sum_{i=1}^{2}\left[X_i^2 - 5\cos(2\pi X_i)\right]$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	1.45	[15]
Nonlinear LS with saddle point	(16)	$g(X) = 2 - X_2 - 0.1X_1^2 + 0.06X_3^3$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$ $X_3 : N(0.0,1.0)$	1.819	[16]
Nonlinear Concave LS	(17)	$g(X) = -0.5(X_1 - X_2)^2 - \dfrac{X_1 + X_2}{\sqrt{2}} + 3$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	1.255	[17]
Nonlinear	(18)	$g(X) = 25 - 2(X_1 - X_2)^2 - 2(X_1^2 - X_2^2)$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	1.705	[8]
Nonlinear	(19)	$g(X) = -\dfrac{4}{25}(X_1 - 1)^2 - X_2 + 4$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	3.156	[18]
Highly Nonlinear	(20)	$g(X) = -0.16(X_1 - 1)^3 - X_2 + 4 - 0.04\cos(X_1X_2)$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	4.16	[19, 9]
Highly Nonlinear	(21)	$g(X) = \dfrac{1}{40}X_1^4 + 2X_3^3 + X_3 + 3$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$ $X_3 : N(0.0,1.0)$	3.43	[18]
Highly Nonlinear	(22)	$g(X) = \exp(0.4(X_1 + 2) + 6.2) - \exp(0.3X_2 + 5) - 200$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	2.709	[20, 9]
Highly Nonlinear	(23)	$g(X) = \exp(0.2X_1 + 1.4) - X_2$	$X_1 : N(0.0,1.0)$ $X_2 : N(0.0,1.0)$	3.385	[21, 20]

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Table Continuation 1

LSF significance	Case No.	Limit State function(s)	Stochastic variable(s)	β	Ref
Highly Nonlinear	(24)	$g(X) = X_3 + \left(\frac{X_1 - 1.1}{1.5}\right)^2 - \left(\frac{X_2 - 2}{3.0}\right)^2 + 3.6$	$X_1 : N(0.0, 1.0)$ $X_2 : N(0.0, 1.0)$ $X_3 : N(0.0, 1.0)$	3.705	[9]
Highly Nonlinear	(25)	$g(X) = \exp(0.2(1 + X_1 - X_2)) -$ $\exp(0.2(5 - 5X_1 - X_2) - 1$	$X_1 : N(0.0, 1.0)$ $X_2 : N(0.0, 1.0)$	4.44	[9]
Nonlinear	(26)	$g(X) = -0.5(X_1^2 + X_2^2 + X_3^2) - 2X_1X_2 - 2X_3X_2 -$ $2X_3X_1 - \frac{X_1 + X_2 + X_3}{\sqrt{3}} + 3$	$X_1 : N(0.0, 1.0)$ $X_2 : N(0.0, 1.0)$ $X_3 : N(0.0, 1.0)$	0.849	[8]
Nonlinear	(27)	$g(X) = \exp(0.2X_1 + 6.2) - \exp(0.47X_2 + 5.0)$	$X_1 : N(0.0, 1.0)$ $X_2 : N(0.0, 1.0)$	2.35	[8]

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