Doors

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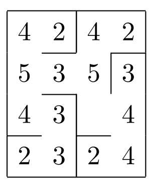
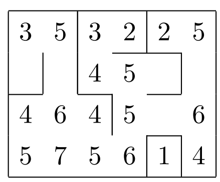
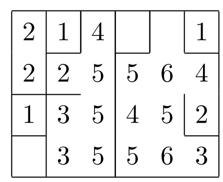
Abstract. In this paper we describe and explore a simple restraint logic programming solution for the logic problem *Doors*.

Keywords: doors · logic puzzle · rooms

1 Examples







2 Problem Description

The logic problem *Doors* is a puzzle on a rectangular board whose cells are either empty or contain natural numbers. The board is thought of like a *house*. Each cell is a *room*, and two adjacent cells are separated by a *wall* with one *door*. That door may be either open or closed. If it is open, then the cell can *see* its adjacent room through the doorway. A man standing in a room can look in all four directions — north, east, west, south — and count the number of *visible rooms*.

The puzzle consists in discovering an assignment of open and closed doors to the walls of the board such that the natural number in each non-empty cell is how many rooms are visible from that cell (including itself). There may be multiple solutions, or none at all.

3 Approach

A puzzle of size $n \times m$ is represented internally by three matrices (list of lists): *Board*, of size $n \times m$, holding the cell numbers; *Vertical*, of size $n \times (m-1)$, holding the vertical walls; and *Horizontal*, of size $(n-1) \times m$, holding the horizontal walls.

For each cell $(R, C), R = 1, 2, \cdots, n, C = 1, 2, \cdots, m$, indices in *Board*, the left wall has index $(R, C-1)$ in *Vertical*, the right wall has index (R, C) in *Vertical*, the top wall has index $(R-1, C)$ in *Horizontal*, the bottom wall has index (R, C) in *Horizontal*. Each wall in the board is assigned the number 0 for closed door and 1 for open door.

3.1 Restrictions

When solving a puzzle *Board* is fully instantiated, while *Vertical* and *Horizontal* contain domain variables (domain $\{0, 1\}$). Empty cells in the *Board* are represented by a 0, as it is never a valid visible room counter.

1	T	?	?	?	?	?	2
X	A	B	C	D	E	F	G
?	M	?	?	?	?	?	?
?	N	?	?	?	?	?	?

Consider the puzzle above. The horizontal range $A - G$ consists of 7 rooms and 6 vertical doors: let $\{b, c, d, e, f, g\}$ be these vertical doors, from left to right.

Focus on room A. If $b = 0$ then A sees no rooms to its right. If $b = 1$ and $c = 0$ then A sees only room B. A general formula can be deduced by noticing that closed doors behave as zero elements.

Let e_A be the total number of rooms A sees to its right (east), then

$$e_A = b + b(c + c(d + d(e + e(f + f(g + g \cdot 0))))) \tag{1}$$

Now, if we analogously define w_A for west, n_A for north and s_A for south, then we find that the number in cell A must be $e_A + w_A + n_A + s_A + 1$.

Implementing these restrictions in PROLOG is surprisingly simple. We start with a predicate to compute formula (1):

```
calculate_value([], 0).
calculate_value([H|T], V) :-
    calculate_value(T, V1),
    V #= H + H*V1.
```

Then, for each non-zero cell (R, C) on the *Board*, we retrieve as a list the four ranges of doors to the right, left, top and bottom of (R, C), apply the formula for each list, and finally the restriction:

```
restrict_cell(Board, _, _, [R,C]) :-
    matrixnth1([R,C], Board, 0), !. % empty cell
restrict_cell(Board, Vertical, Horizontal, [R,C]) :-
    matrixnth1([R,C], Board, Value),
    right_total(Vertical, [R,C], Right),
    left_total(Vertical, [R,C], Left),
    top_total(Horizontal, [R,C], Top),
    bot_total(Horizontal, [R,C], Bot),
    Right + Left + Top + Bot + 1 #= Value.
```

4 Solution presentation

We represent the board using Unicode box drawing characters. Cells are sufficiently sized for all board numbers with `max_width_number/3` and centered with `center_number/2`.

Predicate `write_connector/4` serves to select the appropriate unicode character to connect the cell corners inside and in the edges of the board, and `write_multiple/2` serves to write the same character multiple characters. Using these auxiliary predicates, the board is written in a straightforward fashion, with number rows and horizontal wall rows interleaved.

References

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