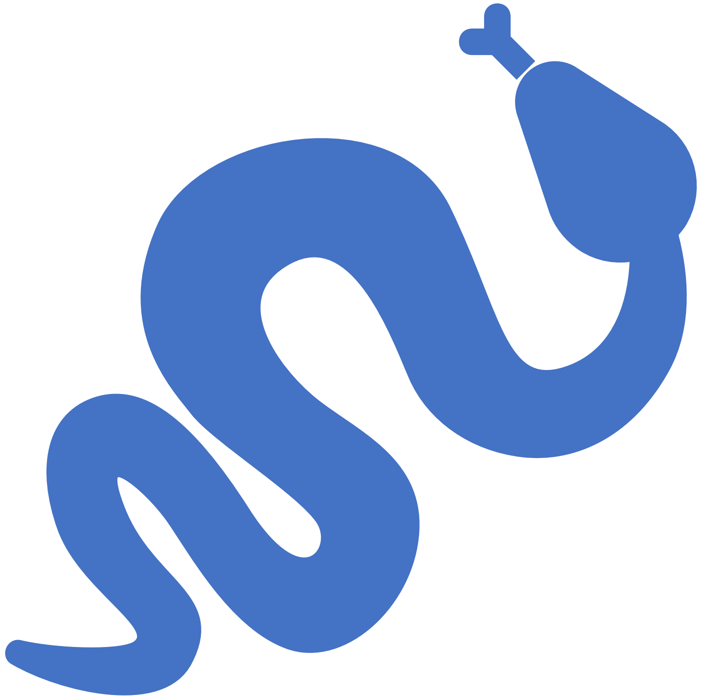

Reinforcement Learning
Snake

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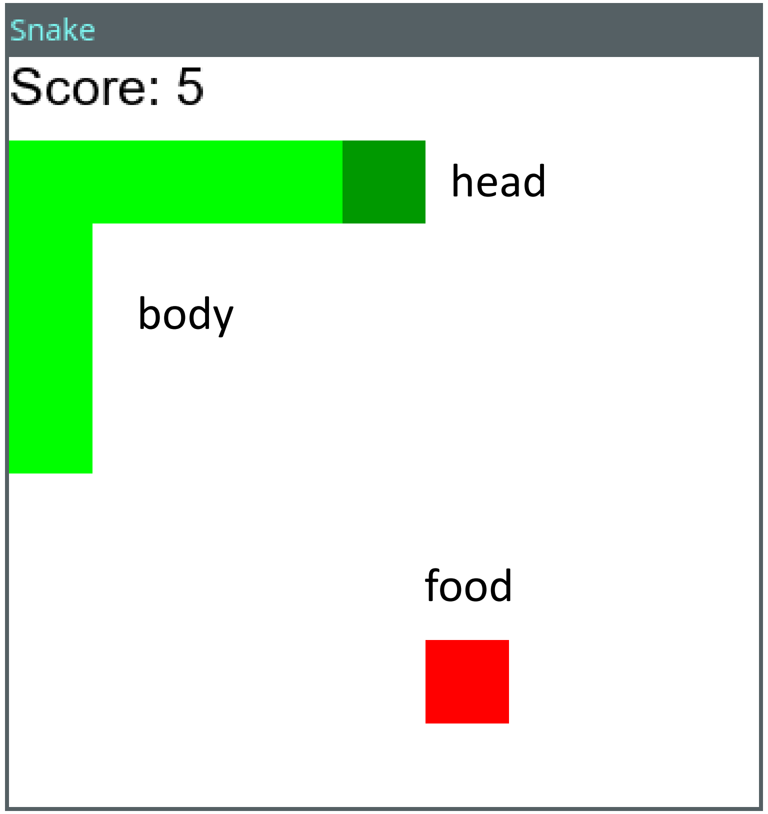
- 10 July 2023
- github.com/Brusa99/Snake_RL

What is Snake?

- Snake is a 1977 video game.
- The player maneuvers the end of a growing line, often themed as a snake.
- The objective is to grow as much as possible by eating food.
- While maneuvering the player has to avoid obstacles, such as the walls and the body of the snake. Hitting the obstacles results in game over.

How I represent it



- You can play the game by running: `python Game.py`
- You can move in 4 directions using the arrow keys. The head will move and the body will trail on.
- If the head reaches the food, the snake will become one block longer and you will increase your score.
- The objective of the game is to score as high as possible.

Model of the environment

- The playground is a matrix of size `self.w * self.h`
- Current direction, score, head and food position, and rest of the body position are class attributes.
- Walls are not modeled: head position outside of boundaries is considered as a collision.

```
# init game state
self.direction = Direction.DOWN
# snake
self.head = Point(self.w // 2, self.h // 2)  # start in the center
self.snake = [self.head,
              Point(self.head.x, self.head.y - 1),
              Point(self.head.x, self.head.y - 2),
              ]
self.score = 0
self.food = None
self._place_food()
```

Reinforcement Learning Framework

Action Space

4 absolute actions

- Up
- Down
- Left
- Right

3 relative actions

- Turn Left
- Straight
- Turn Right

All actions are deterministic.

Relative action

- In this way the action space has a lower cardinality, which reduces model complexity.
- Effectiveness of the model is equivalent: going backwards always results in game over.
- The agent must know current direction.

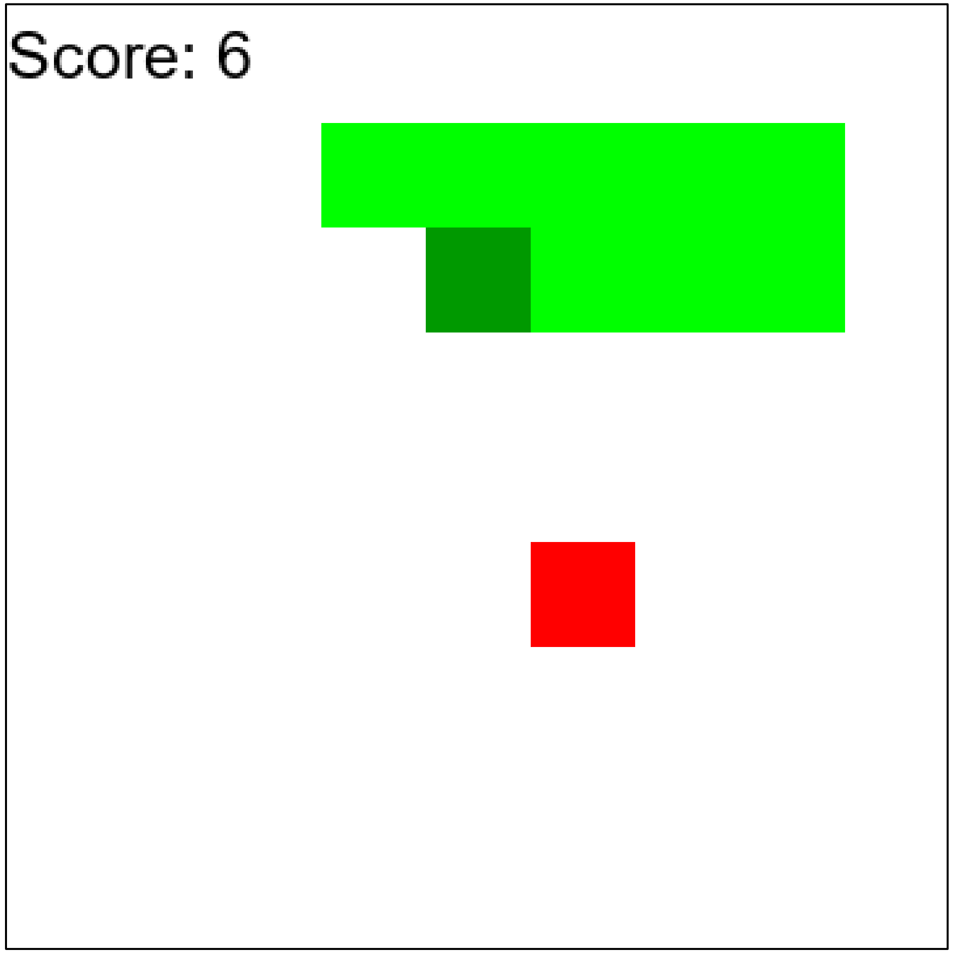
```
# determine direction based on action ~ maps action to direction
clock_wise = [Direction.RIGHT, Direction.DOWN, Direction.LEFT, Direction.UP]
idx = clock_wise.index(self.direction)
if np.array_equal(action, LEFT_TURN):
    new_dir = clock_wise[(idx - 1) % 4]
elif np.array_equal(action, STRAIGHT):
    new_dir = clock_wise[idx]
else:  # np.array_equal(action, RIGHT_TURN):
    new_dir = clock_wise[(idx + 1) % 4]

self.direction = new_dir
```

State Space

- 3 Boolean attributes that signal an immediate collision for each relative direction
- 4 Boolean attributes for each direction
- 4 Boolean attributes for food position:
  - `food.x < head.x`
  - `food.x > head.x`
  - `food.y < head.y`
  - `food.y > head.y`

***Note** that this isn't the only option. Different state spaces will be discussed later.

Example

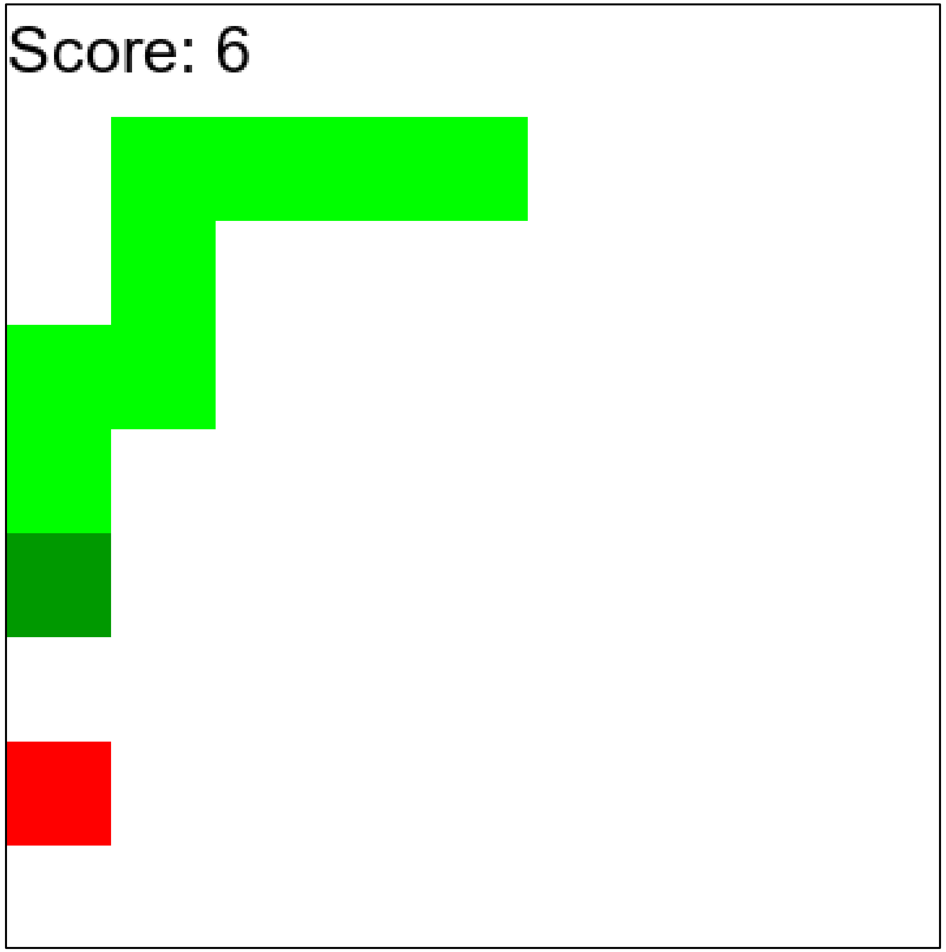


- 0 danger left
- 0 danger straight
- 1 danger right

- 1 direction left
- 0 direction right
- 0 direction up
- 0 direction down

- 0 food left
- 1 food right
- 0 food up
- 1 food down

Example



- 0 danger left
- 0 danger straight
- 1 danger right

- 0 direction left
- 0 direction right
- 0 direction up
- 1 direction down

- 0 food left
- 0 food right
- 0 food up
- 1 food down

State Space Dimension

- We have 11 variables.
- All variables are Boolean.
- We have a total of $2^{11} = 2048$ possible states.

$$S = \underbrace{D_l \times D_s \times D_r}_{\texttt{danger}} \times \underbrace{L \times R \times U \times D}_{\texttt{direction}} \times \underbrace{Fx_< \times Fx_> \times Fy_< \times Fy_>}_{\texttt{food position}}$$

But…

- The Boolean variables are sub-optimal: not all tuples are valid states.
- For example it is not possible that both `direction up` and `direction down` are `1`.
- A better state space would be:

$$S = D_l \times D_s \times D_r \times \underset{\left\{\begin{matrix}\texttt{up}\\ \texttt{down}\\ \texttt{left}\\ \texttt{right}\end{matrix}\right\}}{Dir} \times \underset{\left\{\begin{matrix}<\\ =\\ >\end{matrix}\right\}}{F_x} \times \underset{\left\{\begin{matrix}<\\ =\\ >\end{matrix}\right\}}{F_y}$$

- Which would give us a total of $2^4 * 4 * 3^2 = 576$ possible states.

Rewards

- `+15` for eating the food
- `-10` for colliding
- `-10` if the game goes on for too long (`MAX_ITER * len(snake)`)
- `-0.1` else*

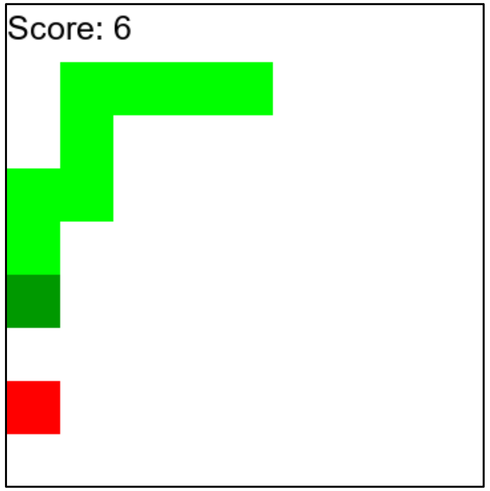
*this is designed to avoid the loop bottleneck: In a vast playground the chance of randomly stumble into the food is very low. So, the agent may adopt “don’t die” as a policy, which is guaranteed in a loop.

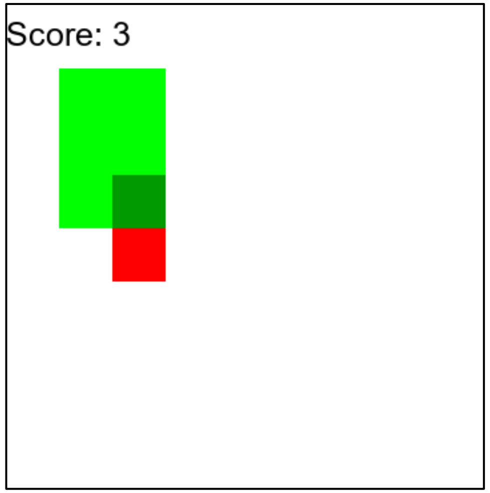
Solving the problem

Model based or model free?

- We have no prior knowledge on the transition probability $p(s'|s,a)$
- We have limited knowledge on the expected immediate reward $r(s,a)$.

Example: The figures on the right are interpreted as the same state.
Taking action `Straight` will lead to a reward of `0` in the left game configuration and of `+15` in the right configuration.
Taking action `Turn right` will lead to a reward of `-10` (and `game over`) in the first case, while in the second case it will produce a reward of `-0.01`.





We will opt for a model free approach

- In a model free context, we know N sequences obtained by the agent by interacting with the enviroment:

$$S_0^n, A_0^n, R_1^n, S_1^n, A_1^n, R_2^n, \ldots \text{ with } n = 1, \ldots, N$$

- We will use TD-Learning to estimate the state-action value:

$$Q^\pi(s, a) = \mathbb{E}_\pi \left[R_t | s_t = s, a_t = a \right]$$

Value of states

- We can get the value of the states from the action-value. Since

$$
\begin{aligned}
Q_\pi(s,a) &= \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_{t+1} | S_0 = s, A_0 = a\right] \\
&= \sum_{s'} P(s'|s,a)\left[r(s,a,s') + \gamma \sum_{a'} \pi(a'|s) Q_\pi(s',a')\right] \\
&= \sum_{s'} P(s'|s,a)\left[r(s,a,s') + \gamma V_\pi(s')\right]
\end{aligned}
$$

- We obtain:

$$V_\pi(s) = \sum_{a'} \pi(a'|s) Q_\pi(s',a')$$

TD-Learning

- To estimate the state action values, we will use the TD-Learning algorithm.
- Other methods, such as Monte Carlo, can be used.

Algorithm 2 TD-Learning

Input Learning Rate $\alpha \in (0; 1]$, small $\epsilon > 0$

Initialize $\hat{Q}(s, a)\ \forall\ s \in S,\ a \in A$

loop for each episode:

    Initialize s

    loop Derive π from $\hat{Q}$ (with $\epsilon - greedy$)

        Choose a from A

        Take A, observe R, S'

        Compute TD-error δ

        Compute eligibility e

        $Q \leftarrow Q + +\alpha\delta e$

        $S \leftarrow S'$

- For eligibility we will use $1(S_t = s,\ A_t = a)$
- To calculate the TD-error we will try different algorithms, for example SARSA:

$$\delta = R + \gamma\hat{Q}(S', A') - \hat{Q}(S, A)$$

- We will follow an ε-greedy policy, that means that we will take the action which we estimate has the most value with probability 1-ε otherwise we will take an action at random. This permits exploration.

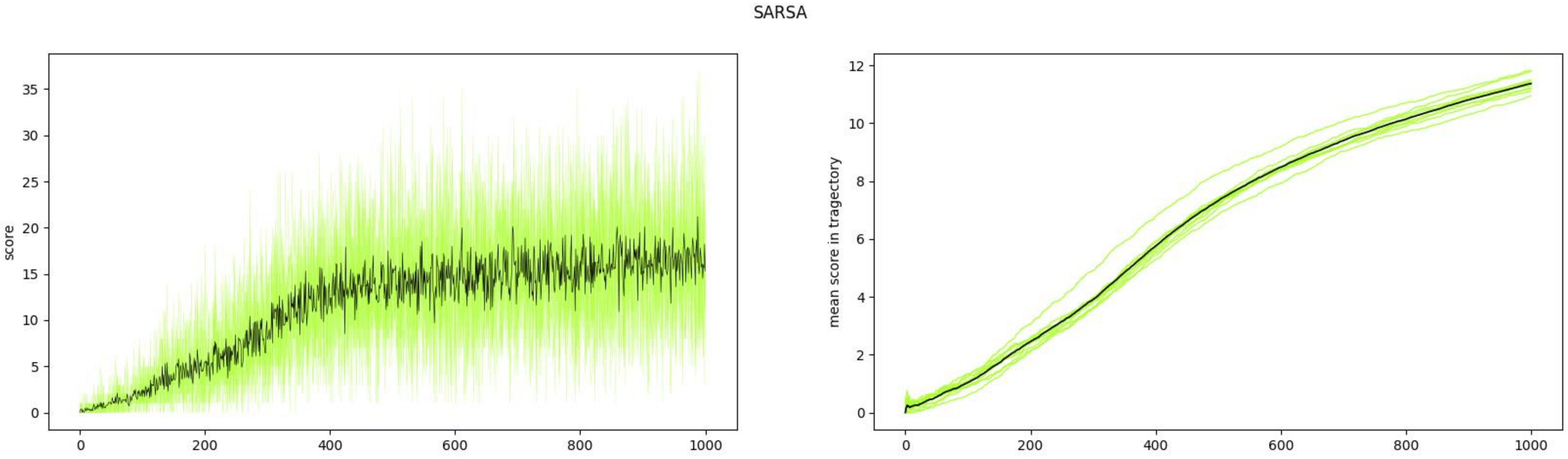
Algorithm 2: Epsilon-Greedy Action Selection

Data: Q: Q-table generated so far, : a small number, S: current state
Result: Selected action
Function *SELECT-ACTION(Q, S, ϵ)* **is**
  n $\leftarrow$ uniform random number between 0 and 1;
  if $n < \epsilon$ **then**
    A $\leftarrow$ random action from the action space;
  else
    A $\leftarrow$ maxQ(S,.);
  end
  return selected action A;
end

Results

Training the model

- We trained the model on a $9 * 9$ board.
- At around 400 games, the agent stops to learn
- We repeated the learning process 10 times.



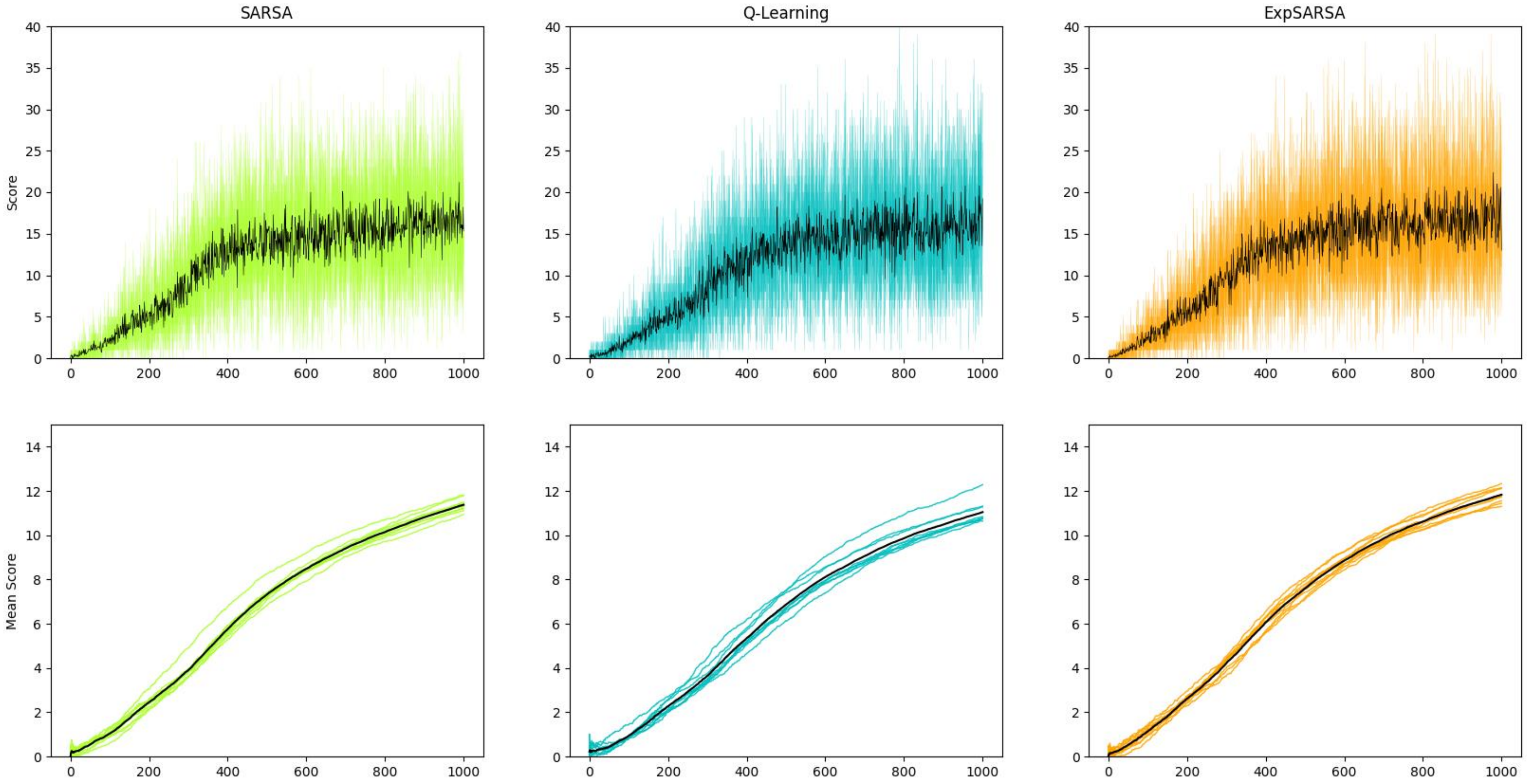
Other algorithms

- We can try to use different TD-errors for the algorithm, for example:
- Q-Learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \Big]$$

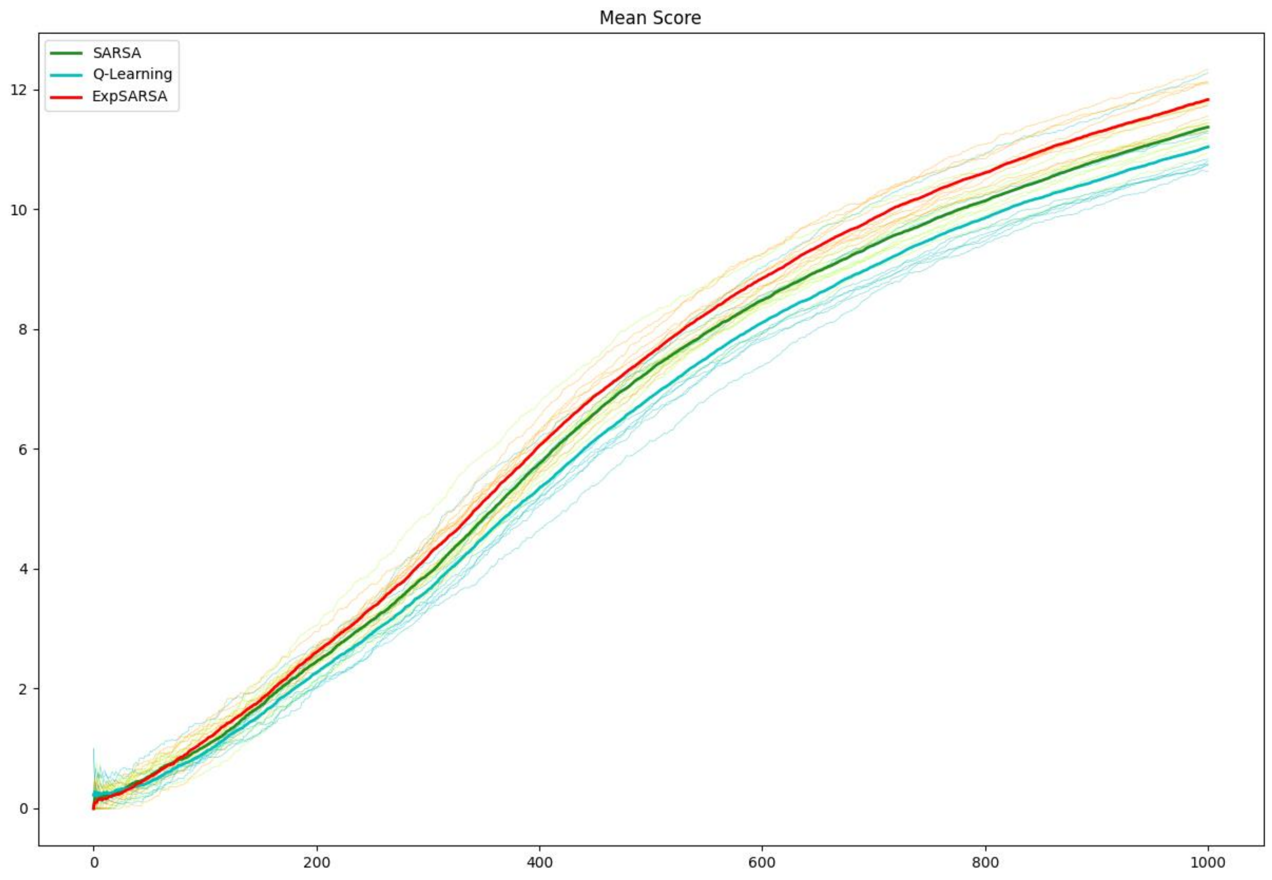
- Expected SARSA:

$$\begin{aligned} Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \mathbb{E}_\pi [Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t) \Big] \\ &\leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \sum_a \pi(a|S_{t+1}) Q(S_{t+1}, a) - Q(S_t, A_t) \Big] \end{aligned}$$


SARSA
Q-Learning
ExpSARSA
Score
Mean Score

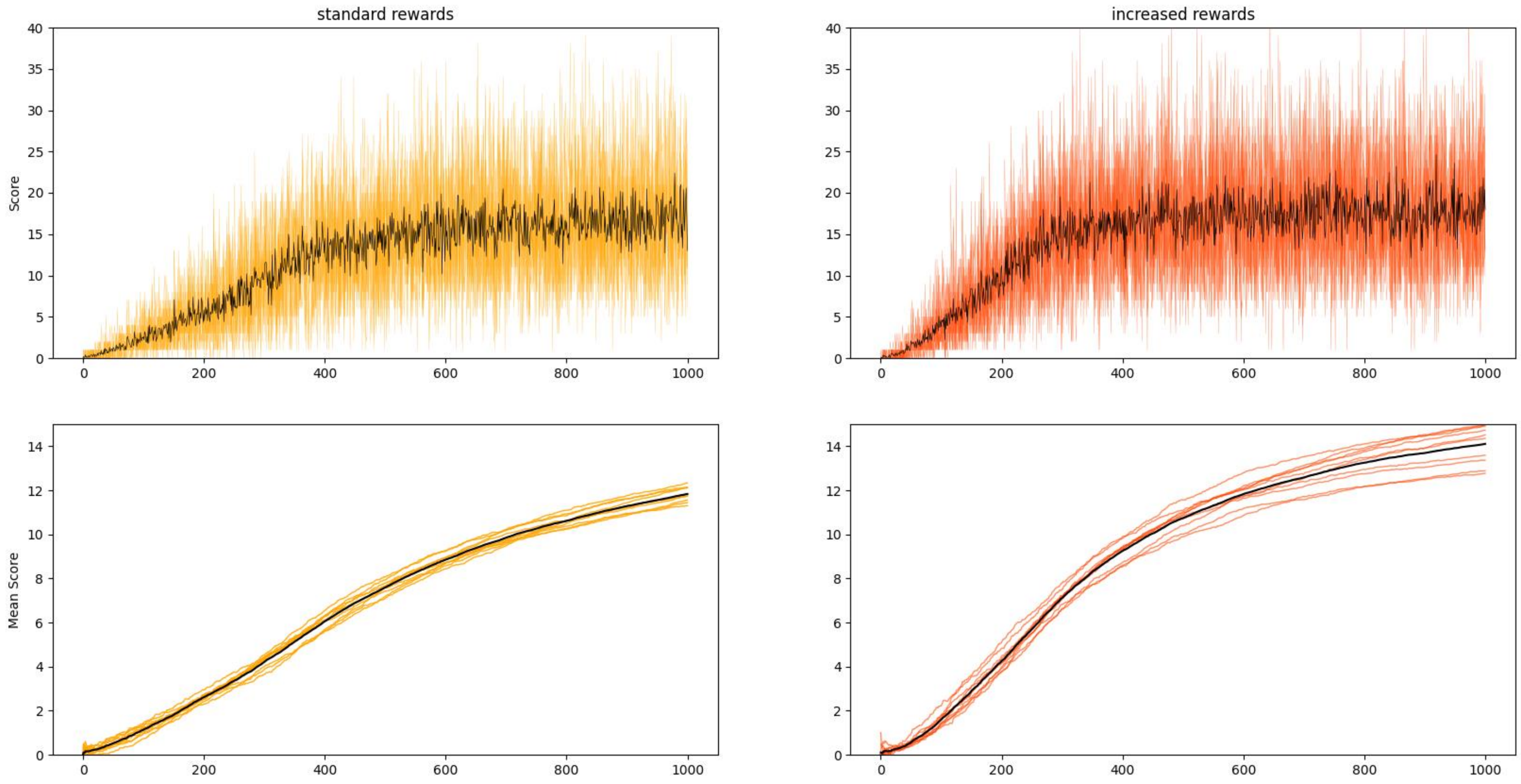
Comparing the algorithms

- We are looking at the average score of the models.
- The idea is that the faster it grows, the sooner the model has learned the optimal policy.
- The fastest model uses the expected SARSA algorithm.

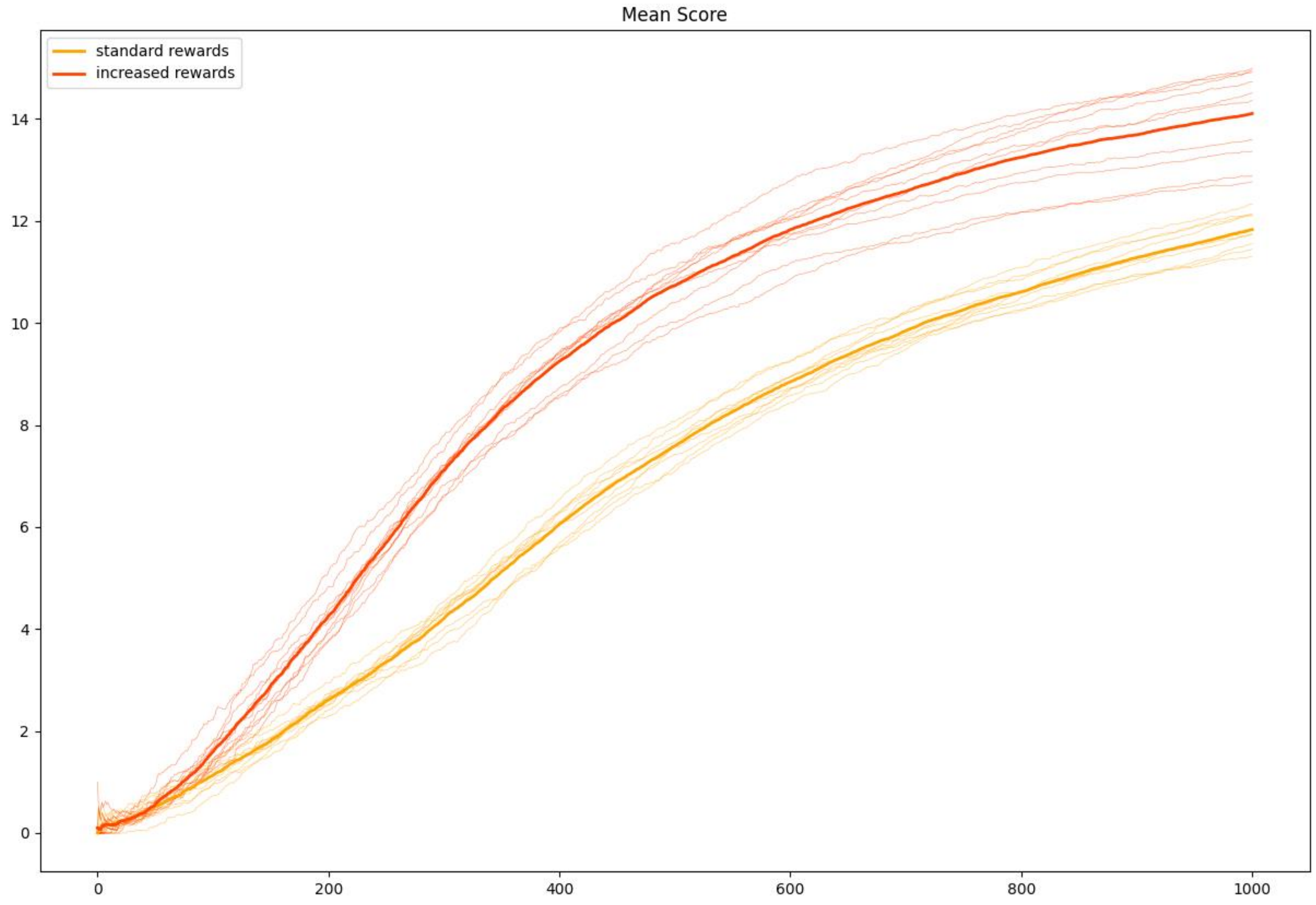


Changing parameters

Increased rewards

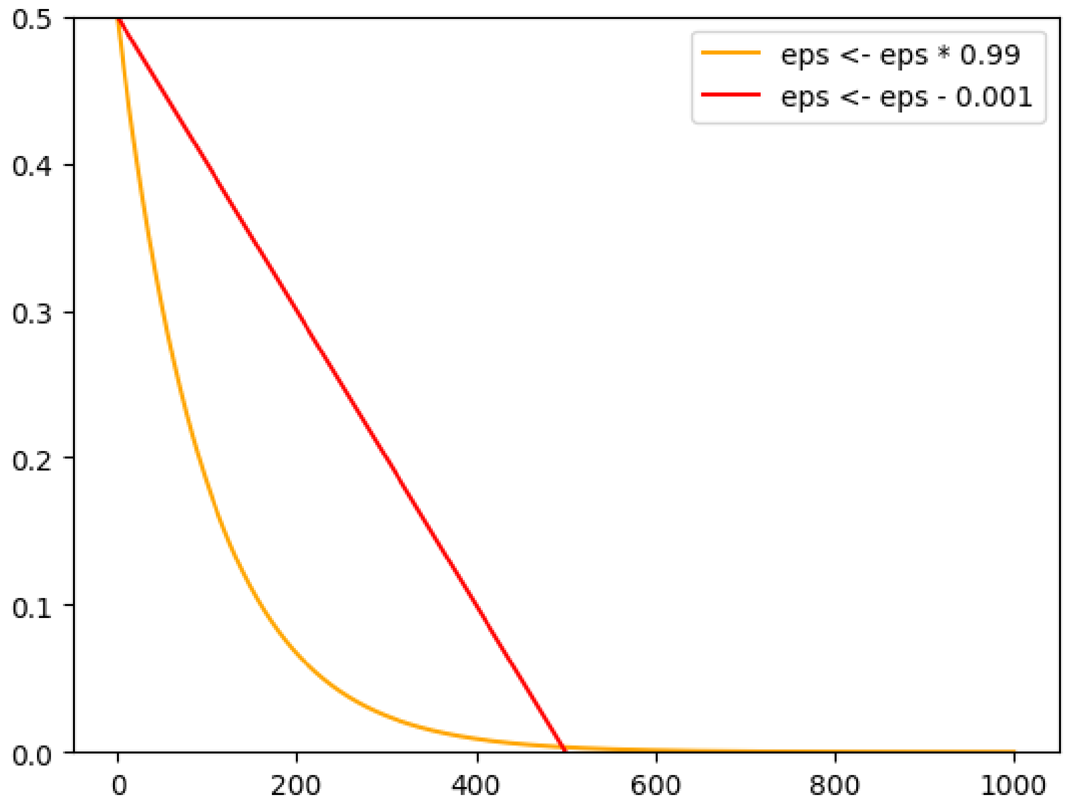


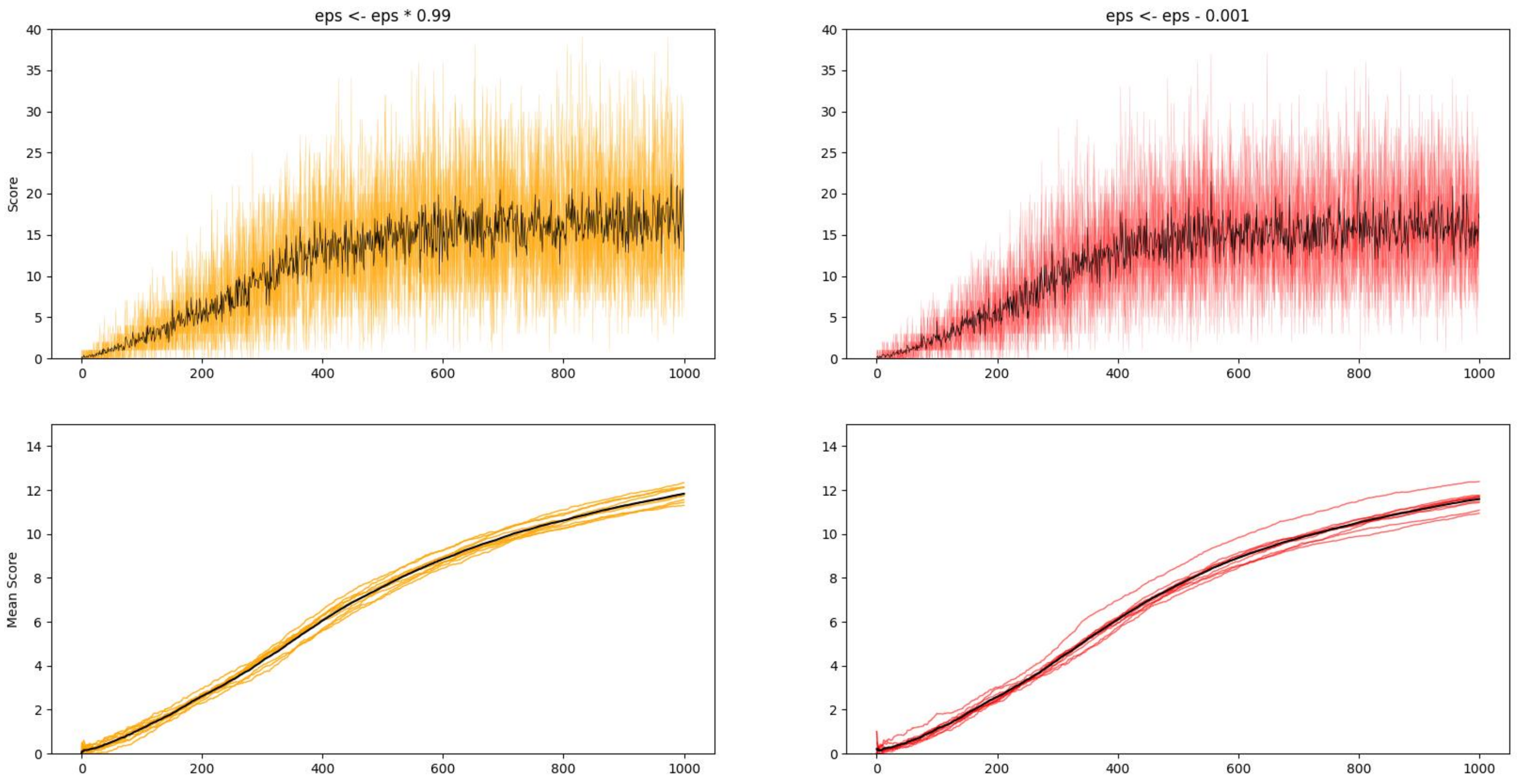
- We increased the positive of eating food from `15` to `150`.
- The algorithm converges faster but it has more variance.
- This is due to the rewards being sparse.



Change how ε decreases.

- In the previous models, to update ε, we used the rule:
$$\epsilon_t = 0.99\ \epsilon_{t-1}$$
- We tried instead a linear decrease:
$$\epsilon_t = \epsilon_{t-1} - 0.01$$



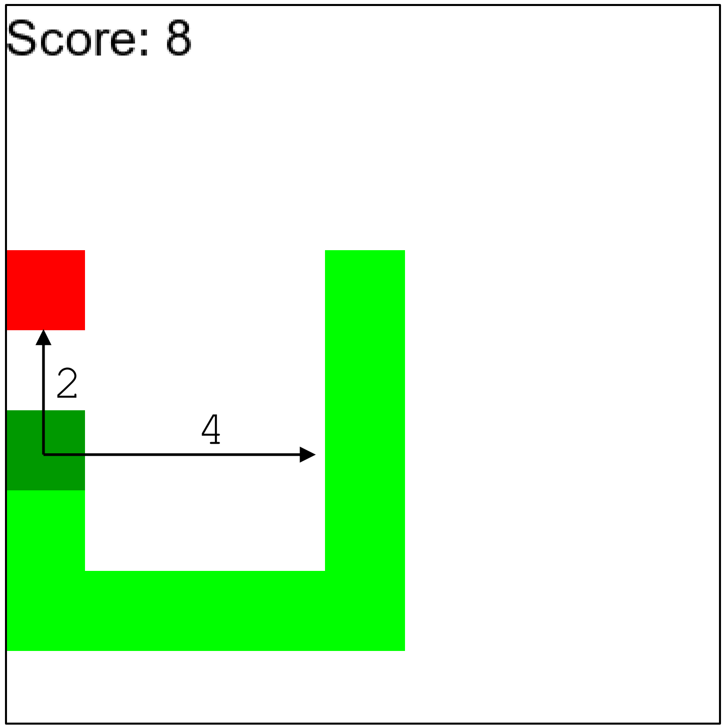


- It doesn’t have any effect

How could it improve

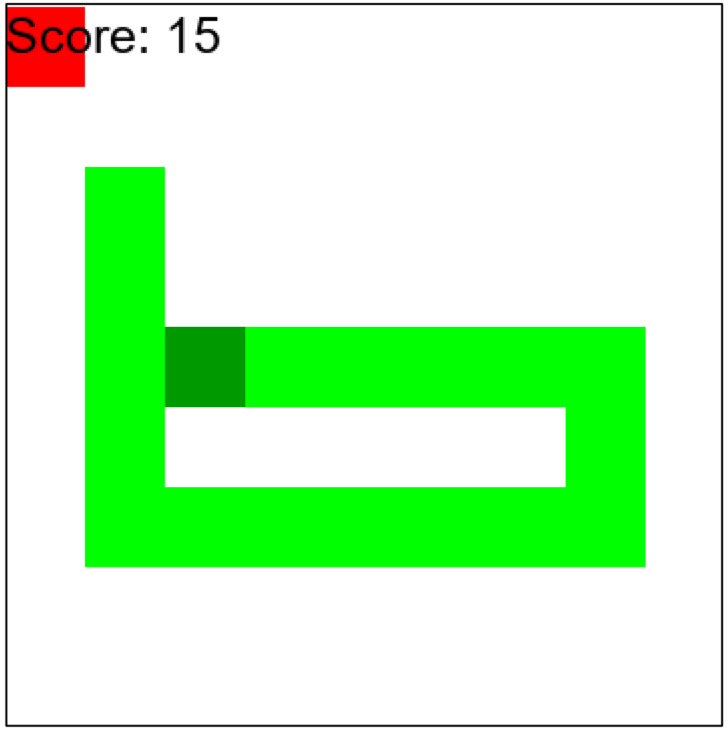
Different state space configurations

- Instead of signaling danger only in three directions, we can also signal the corners.
- Instead of a Boolean we could use distances.
- We could use binning to limit computational complexity increase.



The problem

- From the picture we can see that if the snakes turns left he will become stuck by his own body.
- The snake isn't aware of his body movement, due to how the game is represented through states.
- Getting stuck is the only way to lose if playing with the optimal policy.

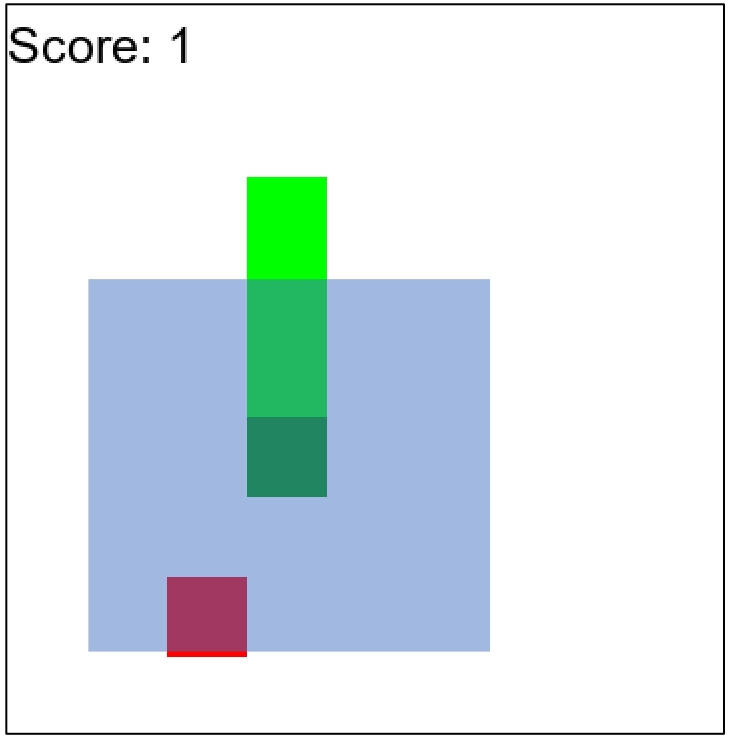


Solution?

- We could change our state space.
- We can make the state space a vector representing each cell of the playground, with different values representing `empty`, `blocked`, `head`, `food`.
- This will lead to a total of 4^{w*h} possible states. Which is computationally unfeasable.
- For example, for a $9 * 9$ board, there will be in the order of 10^{48}states.
- Moreover, the learned policy won’t work on different sized playgrounds.

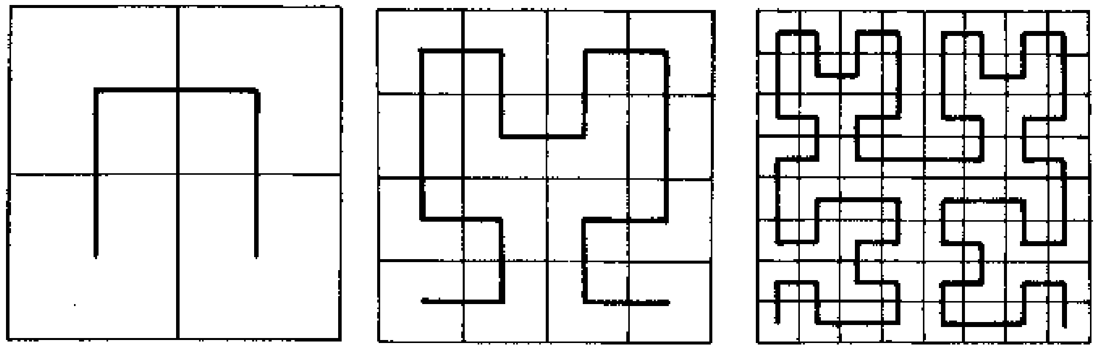
Slight improvements

- Instead of giving the whole board we can only give a limited area around the snake head.
- Still computationally expensive but better.
- State no longer depend on playground size.
- Doesn't solve the problem at its core.



Trivial Solution: Space filling curves

- Compute a curve that fills the square.
- Make the snake follow the curve in a loop.
- No actual learning.



(a) (b) (c)

Fig. 1. Hilbert's geometric construction of a space-filling curve.

The End