

CEE6501 — Lecture 6.3

Fixed End Forces (FEFs)

Learning Objectives

By the end of this lecture, you will be able to:

- Distinguish between **joint loads** and **member loads**
- Define and interpret **fixed-end forces (FEFs)**
- Explain why $\mathbf{Q} = \mathbf{k}\mathbf{u}$ is insufficient for member loading
- Incorporate FEFs into the beam element formulation

Agenda

1. Part 1 — What are Fixed End Forces
2. Part 2 — Why Do We Need FEFs
3. Part 3 — Incorporating FEFs into DSM

Part 1 — What are Fixed End Forces

Definition

Fixed-end forces (FEFs) are:

The forces and moments that develop at the ends of a member due to external loading, assuming both ends are **fully fixed** (no translation or rotation).

Joint Loads vs Member Loads

- **Joint loads** → applied directly at nodes
- **Member loads** → applied between nodes
 - distributed loads
 - point loads along a span
 - applied moments

In deriving the beam stiffness matrix, we assumed:

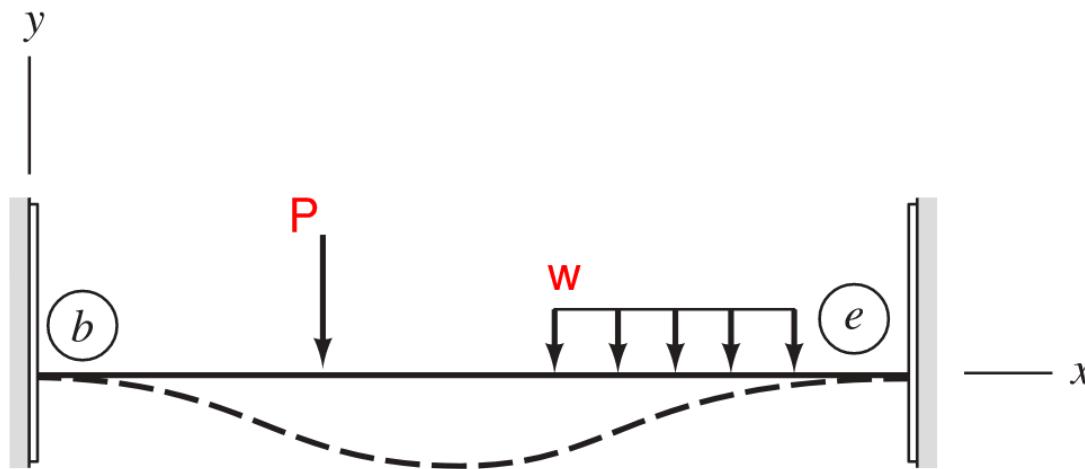
- loads act only at nodes
- no loading within the element

→ Now we relax this assumption.

Part 2 — Why Do We Need FEFs?

A Simple Case

Consider a **fully fixed beam** subjected to loading along its span.



All DOFs are restrained:

$$\mathbf{u} = \mathbf{0}$$

From typical local element force-displacement relationship:

$$\mathbf{Q} = \mathbf{k}\mathbf{u}$$

we obtain:

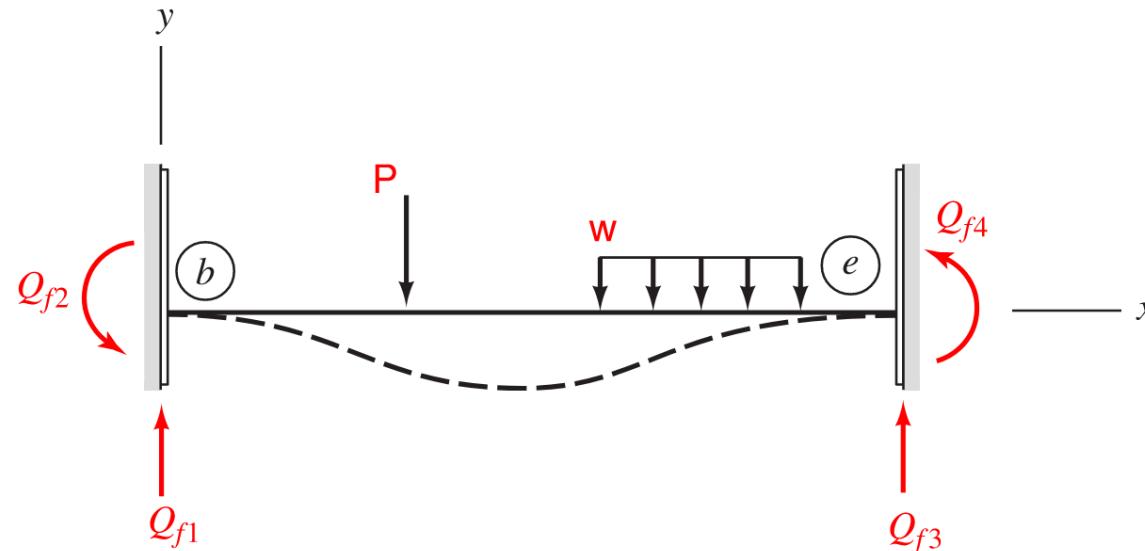
$$\mathbf{Q} = \mathbf{0}$$

✗ Clearly incorrect — reactions exist even with zero displacement.

Corrected Local Element Force-Displacement Relationship

$$\mathbf{Q} = \mathbf{k}\mathbf{u} + \mathbf{Q}_f$$

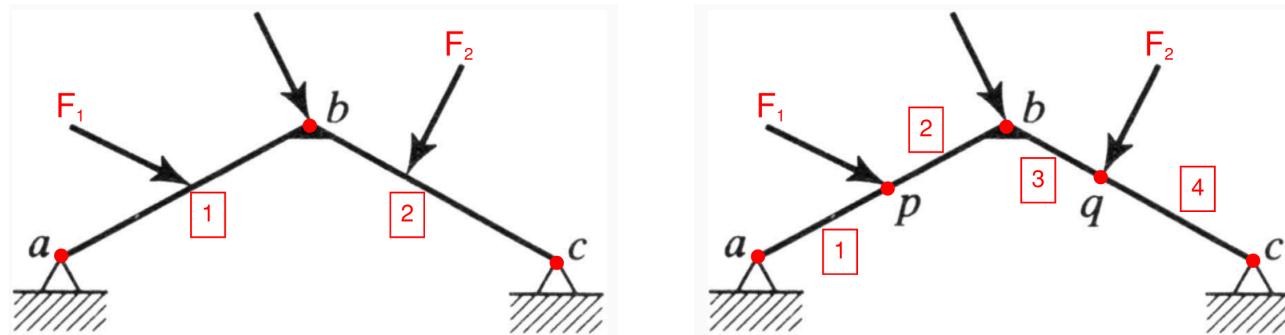
- \mathbf{Q}_f = **fixed-end force vector**
- Captures effects of **loads between nodes**



These forces are required whenever member loads are present

Avoiding FEFs with Artificial Nodes

One approach is to insert **artificial nodes** at load locations.



- Convert member loads → joint loads
- Enforce equilibrium at new nodes
- Maintain compatibility between segments

Artificial Nodes for Point Loads and Distributed Loads

This works for **point loads**, but:

- increases number of elements and DOFs in the solution
- not computationally efficient for general use

For **distributed loads**:

- Infinite number of points carry load
- Cannot represent exactly with finite nodes

Alternative: **lump distributed loads into point forces**

- preserves total force (equilibrium)
- but does not accurately represent local **internal shear and moment distributions**

Key Idea

We need a method that:

- accounts for **loads within elements**
- preserves **exact equilibrium**
- does not increase DOFs

Part 3 — Incorporating FEFs into DSM

Interpretation

$$\mathbf{Q} = \mathbf{k}\mathbf{u} + \mathbf{Q}_f$$

- $\mathbf{k}\mathbf{u}$ → response due to deformation
- \mathbf{Q}_f → response due to member loading

Total response = deformation + load effects

Role in Global System

- FEFs act as **equivalent nodal forces**
- Added to global force vector during assembly

This formulation allows standard DSM procedure, but with an additional FEF vector

List of Typical FEFs

Wrap-Up

Next Lecture