Introduction to Cryptography

Michael Clear

4 Feb 2015

Cryptography

- = "Secret writing" throughout most of its history.
 - More precisely, "writing" with a hidden meaning as opposed to steganography where the existence of the "writing" itself is hidden.
- The idea is to make a message unintelligble except to the intended receiver.

- ▶ Up until the 1970's, the typical pattern was ([1])
 - Somebody creates a cipher.
 - ▶ They claim (or assume) the cipher is unbreakable.
 - Their enemy breaks the cipher using cryptanalysis.

Two Periods: BDH and ADH

- ▶ BDH Before Diffie-Hellman < 1976
- ► ADH After Diffie-Hellman > 1976

Two big changes in 1976

- Selection of Data Encryption Standard (DES) block cipher.
- Public-key cryptography Diffie-Hellman.

BDH: Symmetric Cryptography

- A symmetric cipher uses the same key for encryption and decryption.
- ► Two main types:
 - Stream cipher.
 - Block cipher.
- ▶ Prior to 1970's, most ciphers were stream ciphers.
- ► A symmetric cipher consists of three algorithms G, E and D:
 - G generates a secret key k.
 - ▶ E takes key k and plaintext m and ouputs a ciphertext c i.e. c = E(k, m).
 - ▶ D takes a key k and a ciphertext c and ouputs a plaintext m i.e. m = D(k, c).

▶ Measure of uncertainty in a given source of information.

- ▶ Measure of uncertainty in a given source of information.
- It measures the average amount of information contained in a given message, where a message is drawn from some distribution.

- ▶ Measure of uncertainty in a given source of information.
- It measures the average amount of information contained in a given message, where a message is drawn from some distribution.
- ► Shannon's definition of entropy *H*:

- ▶ Measure of uncertainty in a given source of information.
- It measures the average amount of information contained in a given message, where a message is drawn from some distribution.
- ► Shannon's definition of entropy H:
 - ▶ Let X be a discrete random variable.

- ▶ Measure of uncertainty in a given source of information.
- It measures the average amount of information contained in a given message, where a message is drawn from some distribution.
- ► Shannon's definition of entropy H:
 - ▶ Let X be a discrete random variable.
 - Let P(X) denote the probability mass function of X.

- Measure of uncertainty in a given source of information.
- It measures the average amount of information contained in a given message, where a message is drawn from some distribution.
- ► Shannon's definition of entropy H:
 - ▶ Let X be a discrete random variable.
 - Let P(X) denote the probability mass function of X.
 - ▶ Define H as $H(X) = -\sum_i P(x_i) \log_2 P(x_i)$.

- ▶ Measure of uncertainty in a given source of information.
- It measures the average amount of information contained in a given message, where a message is drawn from some distribution.
- Shannon's definition of entropy H:
 - ▶ Let X be a discrete random variable.
 - Let P(X) denote the probability mass function of X.
 - ▶ Define H as $H(X) = -\sum_i P(x_i) \log_2 P(x_i)$.
- H gives the average number of bits of information contained in some message, which we call the amount of entropy.

Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- ▶ In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- ▶ In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.
- ► The keys other than the correct key that yield a sensible plaintext on decryption are called *spurious keys*.

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- ▶ In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.
- ► The keys other than the correct key that yield a sensible plaintext on decryption are called *spurious keys*.
- ► The unicity distance is the needed length of ciphertext for the number of spurious keys to be zero.

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.
- ► The keys other than the correct key that yield a sensible plaintext on decryption are called *spurious keys*.
- ► The unicity distance is the needed length of ciphertext for the number of spurious keys to be zero.
- ▶ We calculate the unicity distance as:

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.
- ► The keys other than the correct key that yield a sensible plaintext on decryption are called *spurious keys*.
- ► The unicity distance is the needed length of ciphertext for the number of spurious keys to be zero.
- We calculate the unicity distance as:
 - Let k be the number of keys.

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.
- The keys other than the correct key that yield a sensible plaintext on decryption are called spurious keys.
- ► The unicity distance is the needed length of ciphertext for the number of spurious keys to be zero.
- ▶ We calculate the unicity distance as:
 - Let k be the number of keys.
 - ▶ Let s be the number of "sensible" (readable) plaintexts.

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.
- The keys other than the correct key that yield a sensible plaintext on decryption are called spurious keys.
- ► The unicity distance is the needed length of ciphertext for the number of spurious keys to be zero.
- ▶ We calculate the unicity distance as:
 - ▶ Let *k* be the number of keys.
 - ▶ Let s be the number of "sensible" (readable) plaintexts.
 - Let *p* be the total number of possible plaintexts.

- Minimum length of ciphertext needed for a computationally unbounded adversary to recover the (unique) encryption key.
- In a brute force attack where every key is tried, there should be just one key that decrypts to a sensible plaintext.
- The keys other than the correct key that yield a sensible plaintext on decryption are called spurious keys.
- The unicity distance is the needed length of ciphertext for the number of spurious keys to be zero.
- We calculate the unicity distance as:
 - Let k be the number of keys.
 - ▶ Let s be the number of "sensible" (readable) plaintexts.
 - ▶ Let *p* be the total number of possible plaintexts.
 - ► Then the unicity distance is the length of ciphertext U such that $k \cdot \frac{s}{p} = 1$.



▶ We can alternatively define the unicity distance:

- ▶ We can alternatively define the unicity distance:
 - ▶ Let K be the key space. The entropy of K is H(K).

- ▶ We can alternatively define the unicity distance:
 - ▶ Let K be the key space. The entropy of K is H(K).
 - Let H(M) be the entropy of a given "sensible" (e.g. English language) plaintext character.

- ▶ We can alternatively define the unicity distance:
 - ▶ Let K be the key space. The entropy of K is H(K).
 - Let H(M) be the entropy of a given "sensible" (e.g. English language) plaintext character.
 - Let *n* be the number of bits per plaintext character.

- ▶ We can alternatively define the unicity distance:
 - ▶ Let K be the key space. The entropy of K is H(K).
 - Let H(M) be the entropy of a given "sensible" (e.g. English language) plaintext character.
 - Let *n* be the number of bits per plaintext character.
 - ▶ Then we have U = H(K)/(n H(M)).

- ▶ We can alternatively define the unicity distance:
 - ▶ Let K be the key space. The entropy of K is H(K).
 - Let H(M) be the entropy of a given "sensible" (e.g. English language) plaintext character.
 - Let *n* be the number of bits per plaintext character.
 - ▶ Then we have U = H(K)/(n H(M)).
 - ▶ The value D = n H(M) is the redundancy of the plaintext.

A cipher is perfectly secure if the entropy of the plaintext given the ciphertext is equivalent to the entropy of the plaintext.

- ► A cipher is perfectly secure if the entropy of the plaintext given the ciphertext is equivalent to the entropy of the plaintext.
 - ▶ Let H(M) be the entropy of the plaintext.

- ► A cipher is perfectly secure if the entropy of the plaintext given the ciphertext is equivalent to the entropy of the plaintext.
 - ▶ Let H(M) be the entropy of the plaintext.
 - ▶ Let *C* be the probability distribution of the ciphertext.

- ► A cipher is perfectly secure if the entropy of the plaintext given the ciphertext is equivalent to the entropy of the plaintext.
 - ▶ Let *H*(*M*) be the entropy of the plaintext.
 - ▶ Let C be the probability distribution of the ciphertext.
 - ► Then

$$H(M) = H(M \mid C)$$

where H(M|C) is the *conditional entropy* of M given a ciphertext from C.

- ► A cipher is perfectly secure if the entropy of the plaintext given the ciphertext is equivalent to the entropy of the plaintext.
 - ▶ Let *H*(*M*) be the entropy of the plaintext.
 - ▶ Let C be the probability distribution of the ciphertext.
 - ► Then

$$H(M) = H(M \mid C)$$

where H(M|C) is the conditional entropy of M given a ciphertext from C.

▶ Put another way, for all plaintexts $m \in M$ and all ciphertexts $c \in C$, we have

$$\Pr(m) = \Pr(m \mid c)$$

Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- ► A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- ► A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ► The random key is called a pad.

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ▶ The random key is called a pad.
- ► The cipher is unbreakable as long as the key is

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ► The random key is called a pad.
- ▶ The cipher is unbreakable as long as the key is
 - ▶ truly random

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- ► A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ► The random key is called a pad.
- The cipher is unbreakable as long as the key is
 - ▶ truly random
 - kept secret

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- ► A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ► The random key is called a pad.
- The cipher is unbreakable as long as the key is
 - ▶ truly random
 - kept secret
 - used only once

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- ► A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ► The random key is called a pad.
- ► The cipher is unbreakable as long as the key is
 - ▶ truly random
 - kept secret
 - used only once
 - the same length as the plaintext.

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- ► A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ► The random key is called a pad.
- The cipher is unbreakable as long as the key is
 - ▶ truly random
 - kept secret
 - used only once
 - the same length as the plaintext.
- The one-time pad has perfect secrecy.

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- The random key is called a pad.
- The cipher is unbreakable as long as the key is
 - ▶ truly random
 - kept secret
 - used only once
 - the same length as the plaintext.
- The one-time pad has perfect secrecy.
- ▶ The *n*-th character is enciphered by "adding" the *n*-the character of the key to the *n*-th character of the plaintext.

- Described by Gilbert Vernam in 1917. Known as the Vernam Cipher.
- ► A stream cipher in which the key is the same length as the plaintext, and is chosen to be truly random.
- ► The random key is called a pad.
- The cipher is unbreakable as long as the key is
 - ▶ truly random
 - kept secret
 - used only once
 - the same length as the plaintext.
- The one-time pad has perfect secrecy.
- ► The *n*-th character is enciphered by "adding" the *n*-the character of the key to the *n*-th character of the plaintext.
- Example: using addition modulo 2 (XOR) when encrypting a binary message.



► Encryption is based on the principles of confusion and diffusion.

- Encryption is based on the principles of confusion and diffusion.
- ► Confusion: Making the relationship between the ciphertext and the key complex. Each ciphertext character should depend on several different parts of the key.

- Encryption is based on the principles of confusion and diffusion.
- ► Confusion: Making the relationship between the ciphertext and the key complex. Each ciphertext character should depend on several different parts of the key.
 - Means that drastic changes are made from the input to the output.

- Encryption is based on the principles of confusion and diffusion.
- ► Confusion: Making the relationship between the ciphertext and the key complex. Each ciphertext character should depend on several different parts of the key.
 - Means that drastic changes are made from the input to the output.
 - Can be achieved via the technique of substitution.

- Encryption is based on the principles of confusion and diffusion.
- ► Confusion: Making the relationship between the ciphertext and the key complex. Each ciphertext character should depend on several different parts of the key.
 - Means that drastic changes are made from the input to the output.
 - ► Can be achieved via the technique of substitution.
- ▶ **Diffusion:** Changing a single character of the plaintext changes many characters of the ciphertext.

- Encryption is based on the principles of confusion and diffusion.
- ► Confusion: Making the relationship between the ciphertext and the key complex. Each ciphertext character should depend on several different parts of the key.
 - Means that drastic changes are made from the input to the output.
 - Can be achieved via the technique of substitution.
- ▶ **Diffusion**: Changing a single character of the plaintext changes many characters of the ciphertext.
 - ▶ Distributing the statistical structure of the plaintext across much larger structures in the ciphertext.

- Encryption is based on the principles of confusion and diffusion.
- ► Confusion: Making the relationship between the ciphertext and the key complex. Each ciphertext character should depend on several different parts of the key.
 - Means that drastic changes are made from the input to the output.
 - Can be achieved via the technique of substitution.
- ▶ **Diffusion:** Changing a single character of the plaintext changes many characters of the ciphertext.
 - Distributing the statistical structure of the plaintext across much larger structures in the ciphertext.
 - Can be achieved via the technique of permutation (aka transposition).

► Replace each character of the plaintext with a potentially different character determined by the key.

- Replace each character of the plaintext with a potentially different character determined by the key.
- ▶ In a monoalphabetic substitution cipher, the key decides the particular susbtitution table that is used.

- Replace each character of the plaintext with a potentially different character determined by the key.
- ▶ In a monoalphabetic substitution cipher, the key decides the particular susbtitution table that is used.
- ► Example Caesar Cipher: each character is shifted by *k* places in the alphabet where *k* is the key.

- Replace each character of the plaintext with a potentially different character determined by the key.
- ▶ In a monoalphabetic substitution cipher, the key decides the particular susbtitution table that is used.
- ► Example Caesar Cipher: each character is shifted by *k* places in the alphabet where *k* is the key.
 - ▶ For k = 3, the string "HELLO" is encrypted as "KHOOR".

- Replace each character of the plaintext with a potentially different character determined by the key.
- ▶ In a monoalphabetic substitution cipher, the key decides the particular susbtitution table that is used.
- ► Example Caesar Cipher: each character is shifted by *k* places in the alphabet where *k* is the key.
 - ▶ For k = 3, the string "HELLO" is encrypted as "KHOOR".
 - Encryption of m is $c = m + k \mod 26$.

- Replace each character of the plaintext with a potentially different character determined by the key.
- ▶ In a monoalphabetic substitution cipher, the key decides the particular susbtitution table that is used.
- ► Example Caesar Cipher: each character is shifted by *k* places in the alphabet where *k* is the key.
 - ▶ For k = 3, the string "HELLO" is encrypted as "KHOOR".
 - Encryption of m is $c = m + k \mod 26$.
- ▶ A monoalphabetic substitution cipher can have *P*! different keys where *P* is the number of plaintext characters.

- Replace each character of the plaintext with a potentially different character determined by the key.
- In a monoalphabetic substitution cipher, the key decides the particular susbtitution table that is used.
- ► Example Caesar Cipher: each character is shifted by *k* places in the alphabet where *k* is the key.
 - ▶ For k = 3, the string "HELLO" is encrypted as "KHOOR".
 - Encryption of m is $c = m + k \mod 26$.
- ► A monoalphabetic substitution cipher can have P! different keys where P is the number of plaintext characters.
 - ► So English language text with 26 characters results in 26! = 403291461126605635584000000 possible keys.

- Replace each character of the plaintext with a potentially different character determined by the key.
- ▶ In a monoalphabetic substitution cipher, the key decides the particular susbtitution table that is used.
- ► Example Caesar Cipher: each character is shifted by *k* places in the alphabet where *k* is the key.
 - ▶ For k = 3, the string "HELLO" is encrypted as "KHOOR".
 - Encryption of m is $c = m + k \mod 26$.
- ▶ A monoalphabetic substitution cipher can have P! different keys where P is the number of plaintext characters.
 - So English language text with 26 characters results in 26! = 403291461126605635584000000 possible keys.
 - ▶ The entropy per character of English is $H(M) \approx 1.5$.

- Replace each character of the plaintext with a potentially different character determined by the key.
- ▶ In a *monoalphabetic* substitution cipher, the key decides the particular susbtitution table that is used.
- ► Example Caesar Cipher: each character is shifted by *k* places in the alphabet where *k* is the key.
 - ▶ For k = 3, the string "HELLO" is encrypted as "KHOOR".
 - Encryption of m is $c = m + k \mod 26$.
- ▶ A monoalphabetic substitution cipher can have P! different keys where P is the number of plaintext characters.
 - So English language text with 26 characters results in 26! = 403291461126605635584000000 possible keys.
 - ▶ The entropy per character of English is $H(M) \approx 1.5$.
 - The unicity distance is $U = H(K)/(n H(M)) = \log_2 26!/(\log_2 26 1.5) \approx 28.$

► A monoalphabetic substitution cipher can be defeated with frequency analysis.

- ► A monoalphabetic substitution cipher can be defeated with frequency analysis.
- ▶ Invented by Al-Kindi in the 9th century.

- ► A monoalphabetic substitution cipher can be defeated with frequency analysis.
- Invented by Al-Kindi in the 9th century.
- ▶ Involves counting the number of occurences of each ciphertext character and matching against the frequency distribution of plaintext characters.

- ► A monoalphabetic substitution cipher can be defeated with frequency analysis.
- Invented by Al-Kindi in the 9th century.
- Involves counting the number of occurences of each ciphertext character and matching against the frequency distribution of plaintext characters.
- ▶ In English, the most frequently occurring letters are (in order) E, T, A, O, I, N, S, H, R, D, L, U...

- ► A monoalphabetic substitution cipher can be defeated with frequency analysis.
- Invented by Al-Kindi in the 9th century.
- Involves counting the number of occurences of each ciphertext character and matching against the frequency distribution of plaintext characters.
- ▶ In English, the most frequently occurring letters are (in order) E, T, A, O, I, N, S, H, R, D, L, U...
- So given a ciphertext generated from an English plaintext, the most frequently occurring character likely corresponds to E etc.

Polyalphabetic Substitution Cipher

Uses multiple substitution alphabets.

Polyalphabetic Substitution Cipher

- Uses multiple substitution alphabets.
- ▶ The main idea is to change the substitution alphabet with each plaintext character, so the first letter is encrypted according to one alphabet, the second according to a different alphabet and so on (note the alphabets may repeat after a certain period).

Example of a polyalphabetic substitution cipher.

- Example of a polyalphabetic substitution cipher.
- ► Works as follows:

- Example of a polyalphabetic substitution cipher.
- Works as follows:
 - ▶ Let k be a keyword such as "BLAZE", with n = 5 letters.

- Example of a polyalphabetic substitution cipher.
- Works as follows:
 - ▶ Let k be a keyword such as "BLAZE", with n = 5 letters.
 - Let k_i denote the numeric value (modulo 26) of the i-th letter of the keyword k.

- Example of a polyalphabetic substitution cipher.
- ► Works as follows:
 - ▶ Let k be a keyword such as "BLAZE", with n = 5 letters.
 - Let k_i denote the numeric value (modulo 26) of the i-th letter of the keyword k.
 - ► The *i*-th letter of plaintext m_i is encrypted as $c_i = m_i + k_i \pmod{n} \pmod{26}$ (when represented modulo 26)

- Example of a polyalphabetic substitution cipher.
- ► Works as follows:
 - ▶ Let k be a keyword such as "BLAZE", with n = 5 letters.
 - Let k_i denote the numeric value (modulo 26) of the i-th letter of the keyword k.
 - ▶ The *i*-th letter of plaintext m_i is encrypted as $c_i = m_i + k_i \pmod{n} \pmod{26}$ (when represented modulo 26)
- Encrypting the text "HELLO" with keyword "BLAZE" yields the ciphertext "IPLKS".

Viginère Cipher - Tabula Recta

ABCDEFGHIJKLMNOPQRSTUVWXYZ F G H I J K L M N O P Q R S T U V W X Y Z B B C D E F G H I J K L M N O P Q R S T U V W X Y Z A CCDEFGHIJKLMNOPQRSTUVW MNOPQRSTUVW LMNOPQRSTUVWXYZA F F G H I J K L M N O P Q R S T U V W X Y Z A B C D E I K L M N O P Q R S T U V W X Y Z A B C D E F LMNOPQRSTUVWXYZABC KLMNOPQRSTUVWXYZABCD LMNOPQRSTUVWXYZABCDEF K K L M N O P O R S T U V W X Y Z A B C D E F LLMNOPQRSTUVWXYZABCDE MMNOPORSTUVWXYZABCDEFGH NNOPQRSTUVWXYZABCDEFGHI OOPQRSTUVWXYZABCDE PPQRSTUVWXYZABCDEFGHI QQRSTUVWXYZABCDEFGHIJK RRSTUVWXYZABCDEFGHIJK SSTUVWXYZABCDEFGHI TTUVWXYZABCDEFGHIIKLMNO UUVWXYZABCDEFGH V V W X Y Z A B C D E F G H I J K L M N O P Q R S T U WWXYZABCDEFGHIJKLMNOPQRS XXYZABCDEFGHIJKLMNOPQRSTUVW Y Y Z A B C D E F G H I I K L M N O P Q R S T U V W X ZZABCDEFGHI IKLMNOPORSTUVWXY

Transposition

- Idea: rearrange the plaintext (change the order) to produce the ciphertext.
- ► The positions of the plaintext characters are shifted.
- Also known as permutation.
- Examples:
 - Rail Fence
 - Route Cipher
 - Columnar Transposition

Example: Columnar Transposition

- Write plaintext along rows whose length is determined by the key
- Example (here X denotes a null character):

```
T H E R E M U S T B E S O M E K I N D O F W A Y O U T O F H E R E X X
```

- ▶ Suppose the key specifies the row length as 5 and the order of columns to write out as 4, 2, 5, 1, 3.
- Then we get the ciphertext by writing out the columns in the specified order:
 - we obtain: RTMDYFXHUSTWTREBEOOHXTMEKFUEESONAO



Example: Columnar Transposition (Cont'd)

- The key could be alternatively given as a keyword such as TOWER
 - ▶ the length of the keyword represents the row length.
 - the alphabetical order of the letters in the keyword gives the order of the columns to be written out.

References



Barak, B.:

Cos 433: Cryptography.

www.cs.princeton.edu/courses/archive/fall07/cos433/.../lec1-intro.ppt (2007)