Composition of Functions

Summary

1. With compositions of functions, the output of one function is used as input into the other.
2. You can even evaluate entire functions into other functions.

Compositions of functions involve **substituting** one function into the variable(s) of another.

The composition of a function f and g denoted

$$(f \circ g)(x)$$

is

$$(f \circ g)(x) = f(g(x))$$

where we plug $g(x)$ into the variable for $f(x)$.

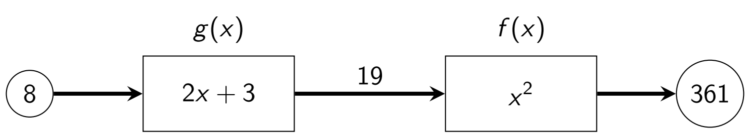
In other words, the output of $g(x)$ becomes the **input** of $f(x)$.

The following illustrates finding $(f \circ g)(8)$, or $f(g(8))$, in which

$$g(x) = 2x + 3 \text{ and } f(x) = x^2$$

:

1. Evaluate $g(8)$ to get $2(8) + 3$, or 19.
2. Evaluate $f(19)$ to get 19^2, or 361.



Example 1. Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

(b) $(g \circ f)(2)$

(c) $(f \circ f)(1)$

We can even substitute an entire function into another and simplify.

Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$\begin{aligned}(f \circ g)(x) &= f(2x+3) \\ &= (2x+3)^2 \\ &= (2x+3)(2x+3) \\ &= \boxed{4x^2 + 12x + 9}\end{aligned}$$

Example 2. Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $(f \circ f)(x)$

Tabular and Visual Methods

Remember, $f(x)$ tells you the output (or y-coordinate) of the function when plugging in a value for x.

For instance, $f(3) = -2$ means

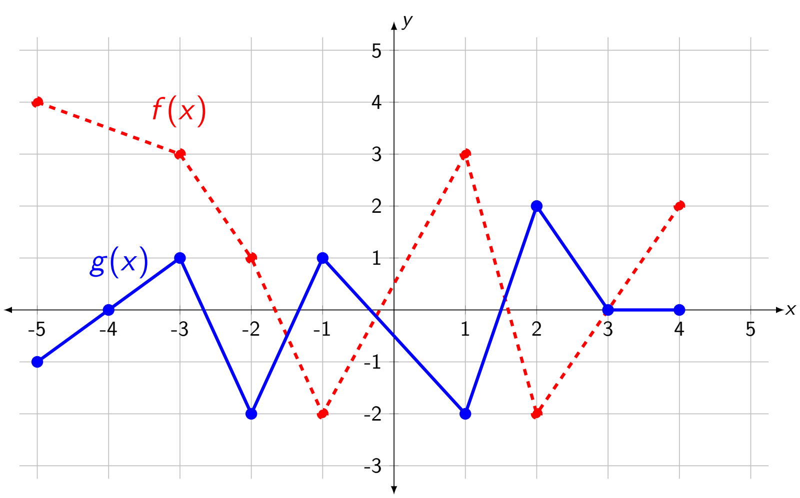
- When the input is 3, the output is -2
- The point $(3, -2)$ is on the graph of the function.

Example 3. Find each given the table below.

x	-3	-2	-1	0	1	2	3
$f(x)$	1	-2	-3	-1	3	2	0
$g(x)$	-2	2	-3	3	0	1	-1

(a) $(f \circ g)(-1)$ (b) $f(g(2))$ (c) $(g \circ f)(0)$ (d) $g(g(-3))$

Example 4. Find each given the graph below.



(a) $(f \circ g)(-5)$ (b) $g(f(-2))$ (c) $f(f(2))$ (d) $(g \circ g)(-4)$