Complex Numbers

Summary

1. Complex numbers are in the form $a+bi$ where a is the real part, b is the imaginary part, and $i=\sqrt{-1}$.
2. When multiplying complex numbers, $i^2=-1$.

For $x^2=1$

$x=1$ or $x=-1$

However, for $x^2=-1$

- No real solutions exist
- Mathematicians came up with a solution:
  - $i=\sqrt{-1}$
  - $i^2=-1$
- For $x^2=-1$, $x=i$ or $x=-i$

Complex Number

A **complex number** is a number in the form

$$a+bi$$

where a is the **real part** and b is the **imaginary part**.

$$\begin{aligned}\sqrt{-16}&=\sqrt{16}\cdot\sqrt{-1}\\&=4i\end{aligned}$$

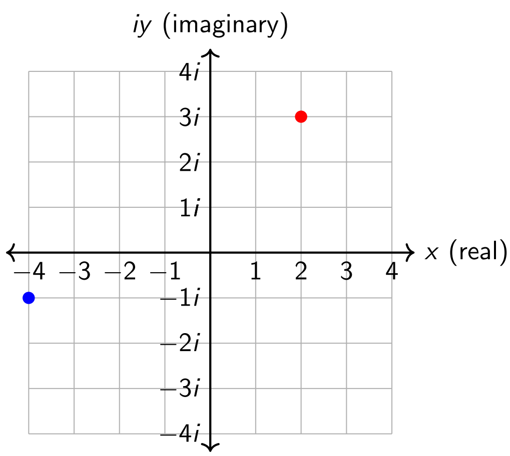
$$\begin{aligned}\sqrt{-81}&=\sqrt{81}\cdot\sqrt{-1}\\&=9i\end{aligned}$$

$$\begin{aligned}\sqrt{-72}&=\sqrt{72}\cdot\sqrt{-1}\\&=\sqrt{36}\cdot\sqrt{2}\cdot\sqrt{-1}\\&=6i\sqrt{2}\end{aligned}$$

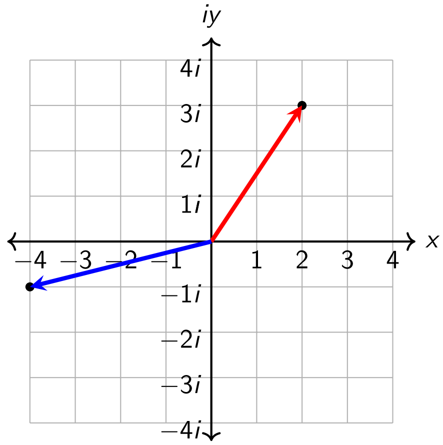
Plotting Complex Numbers

We can plot complex numbers using an *Argand diagram* (complex plane).

- Similar to plotting points normally
- x-coordinate is the real part
- y-coordinate is the imaginary part
- Plot below shows $\mathbf{2+3i}$ and $\mathbf{-4-i}$.



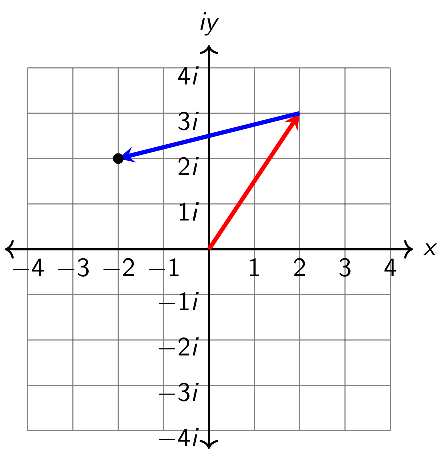
We can draw arrows (*vectors*) from the origin to each point.



Adding and Subtracting Complex Numbers

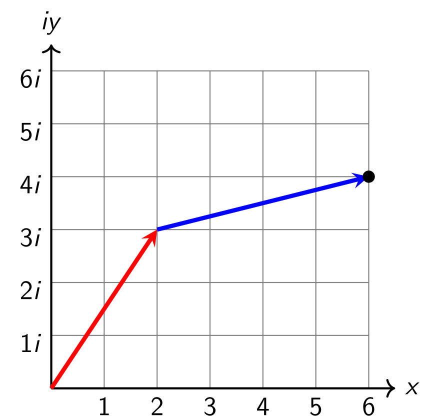
- Second arrow can start where first ends.
- Second arrow still points the same direction.
- You are just combining like terms.

For instance, $\mathbf{(2+3\boldsymbol{i})} + \mathbf{(-4-\boldsymbol{i})} = \mathbf{2+2\boldsymbol{i}}$ is shown below.



For subtraction,

1. Distribute the negative
2. Combine like terms
3. $\mathbf{(2+3\boldsymbol{i})} - \mathbf{(-4-\boldsymbol{i})}$



Example 1. Simplify each.

(a) $(1-2i)+(3+4i)$

(b) $(1-2i)-(3+4i)$

Multiplying Complex Numbers

- Multiply like normal
- Substitute -1 whenever you see an i^2
- Simplify and combine like terms

Example 2. Multiply and simplify each.

(a) $(4+3i)(2+i)$

(b) $(1-2i)(3+4i)$

Visual Intepretation of Multiplying Complex Numbers

Example 3.

(a) Can you find a relationship between the lengths of $4+3i$ and $2+i$ compared to the length of product?

(b) Can you find a relationship between the angles that $4+3i$ and $2+i$ make with the positive x-axis compared to the angle that the product makes?

Dividing Complex Numbers

Visually, what do you think happens with division?

Example 4. Find the quotient of $\frac{4+3i}{2+i}$.

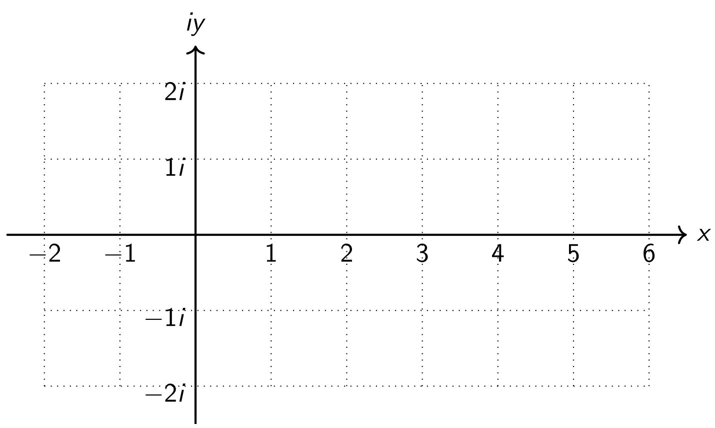
Complex Conjugate

The **complex conjugate** of a complex number in the form

$$z = a + bi$$

is $\overline{z} = a - bi$ and vice versa.

Example 5. Find and graph the product of $(2+i)(2-i)$.



To divide complex numbers, you multiply the numerator and denominator by the *conjugate* of the denominator.

Example 6. Divide each.

(a) $\dfrac{4+3i}{2+i}$

(b) $\dfrac{1-2i}{3+4i}$

We can even use complex numbers to solve quadratic equations when the graph of the equation does not intersect the x-axis.

Example 7. Solve each. None of these problems are factorable.

(a) $x^2 + 4x + 8 = 0$

(b) $12x^2 - 12x + 12 = 0$

(c) $9x^2 + 5x + 14 = 3$

(d) $-4x^2 - 4x + 18 = -10x^2 + 7x + 6$