Graphs of Quadratic Functions

Summary

1. Depending on how the graph is opening, the vertex is the minimum (or maximum) point on the graph.
2. The axis of symmetry runs through the vertex and creates a mirrored image.

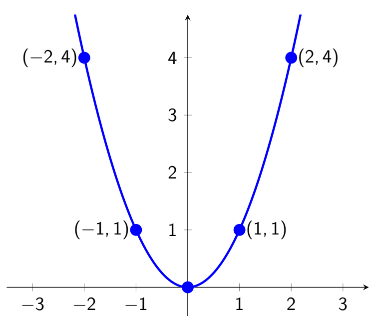
Vertex and Axis of Symmetry

We will be looking at graphing equations in the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers with $a \neq 0$.

For $f(x) = x^2$, the graph below is a **parabola**.



The point $(0, 0)$ is called the **vertex** of the parabola and can be either a minimum (smile) or maximum point (frown).

Through the vertex is a vertical line called the **axis of symmetry** that divides the parabola into 2 equal halves.

$$\text{Equation:} \quad x = x\text{-coordinate of the vertex}$$

The **zeros** (a.k.a. the *x-intercepts* or *roots*) of a function are the values of x where the graph crosses the x-axis (a.k.a. $y = 0$); <u>if it does at all</u>.

Example 1. For each of the following:

- Find the coordinates of the vertex (use a graphing utility).
- State whether the vertex is a maximum or minimum
- Find the equation of the axis of symmetry
- Find the zeros (if any)

(a) $y = x^2 + 8x + 15$

(b) $f(x) = -2x^2 + 11x - 12$

General and Vertex Forms

For a quadratic function:

- The **general form** is $y = ax^2 + bx + c$
  - a, b, and c are real numbers
  - $a \neq 0$

- The **vertex form** is $y = a(x-h)^2 + k$
  - Vertex is (h, k)
  - $a \neq 0$
  - a, h, and k are real numbers

To convert from general form $y = ax^2 + bx + c$ to standard form $y = a(x-h)^2 + k$

1. Find the vertex:
   - x-coordinate: $-\dfrac{b}{2a}$
   - y-coordinate: Evaluate expression at x-coordinate
   - **Or use graphing technology**
2. Use the value of a that is given.

Example 2. Write each of the following in vertex form.

(a) $y = x^2 - 4x + 3$ (b) $y = 6 - x - x^2$ (c) $y = 3x^2 - 8x + 7$

To convert from $y = a(x-h)^2 + k$ form to

$$y = ax^2 + bx + c$$

just **do the math** and remember your order of operations (PEMDAS).

Example 3. Convert each to general form.

(a) $y = (x+2)^2 - 3$ (b) $y = -(x-7)^2 + 10$ (c) $y = 4\,(x-1)^2 + 1$