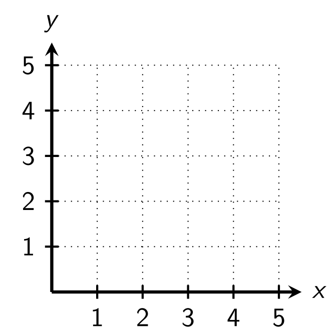
Vectors: Adding, Subtracting, and Scalar Multiplication

Summary

1. A vector is a directed line segment.
2. We can add or subtract vectors via corresponding elements if they have the same dimensions and we can multiply any vector by a scalar.
3. Any 2-dimensional vector can be written in terms of basis vectors $\hat{\imath}$ and $\hat{\jmath}$.

Any point in the coordinate plane can be represented by a directed line segment called a **vector**. For instance, we can draw a vector from the origin to the point $(3, 4)$ below:



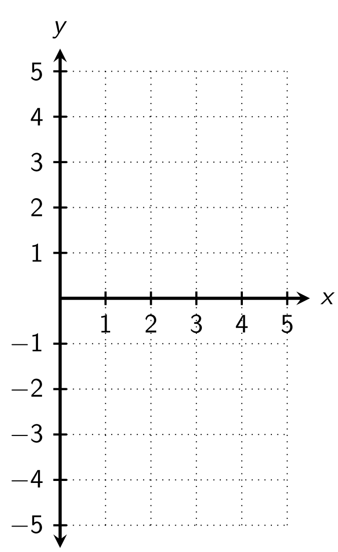
In the above picture, $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, where $\vec{v}$ is a **column vector** with the dimensions 2 rows by 1 column.

Note: the values in the vector, 3 and 4, are called **elements**.

Vector Addition

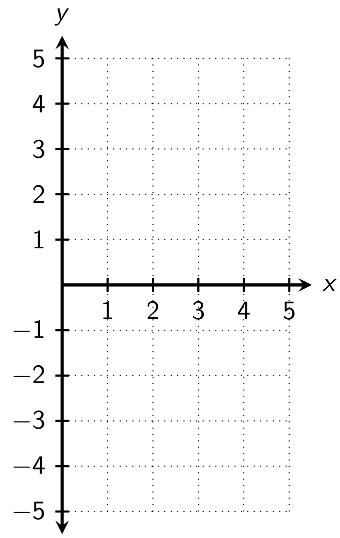
We can add two vectors together if they have the same dimensions by adding their corresponding elements.

For instance, suppose $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ then $\vec{v} + \vec{w}$ is



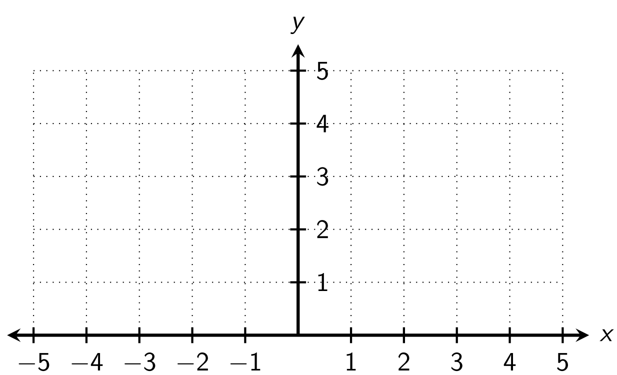
When we physically add vectors, we can move the second vector (without changing its length or direction) so that it starts where the first ends:

Visually, our previous problem of $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \vec{w} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ is



Example 1. Find and graph the sum of the vectors below.

$$\vec{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

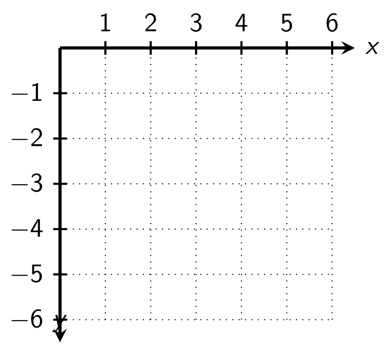


Scalar Multiplication

A **scalar** is another name for a real number.

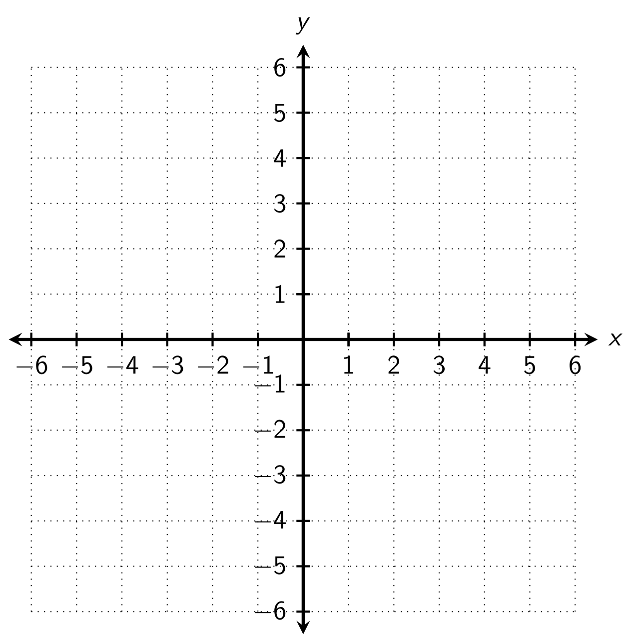
When we multiply a vector by a scale, it scales the vector's length accordingly.

If $\vec{a} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ what is $2\vec{a}$?



What happens when we multiply our vector by -1?

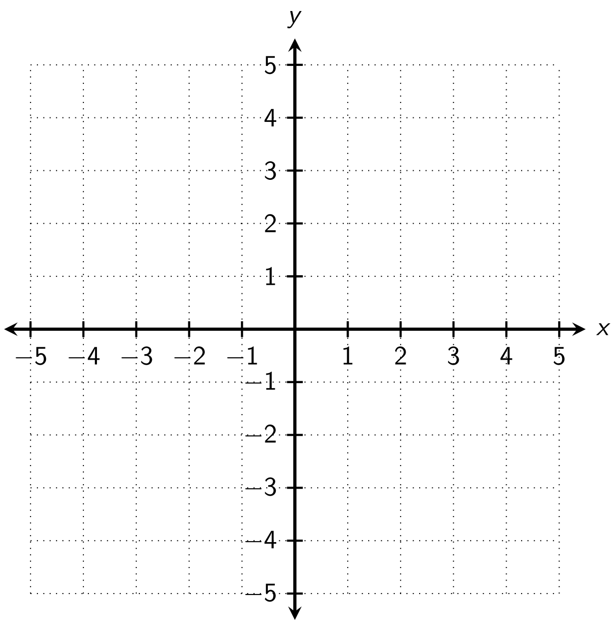
If $\vec{a} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, what is $-1\vec{a}$?



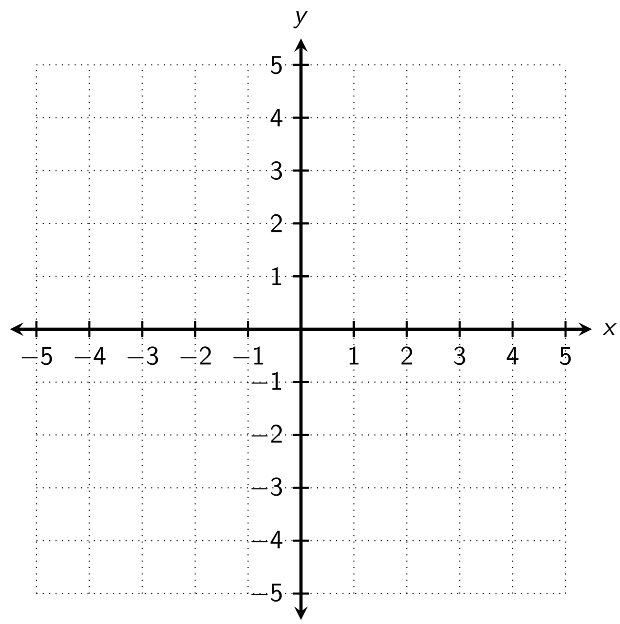
We can combine the scalar multiples with vector addition.

Example 2. Given $\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ find and graph each.

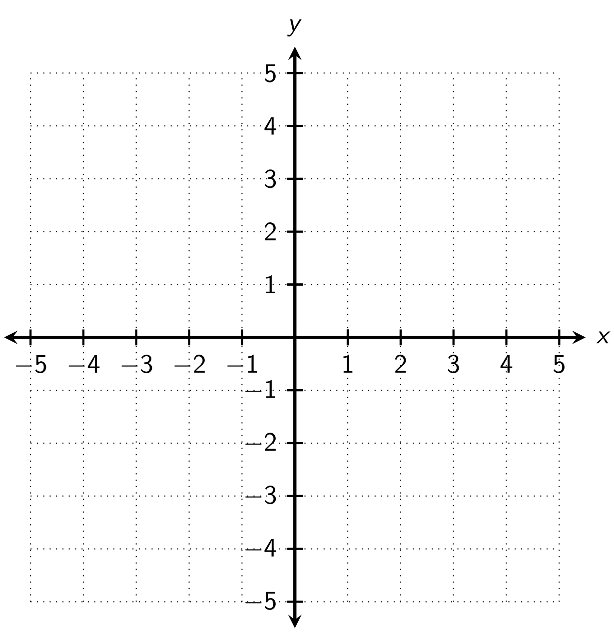
(a) $\vec{v} - \vec{w}$



(b) $3\vec{v} + \vec{w}$

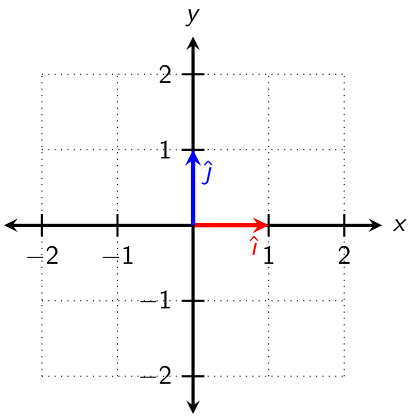


(c) $2\vec{v} - 1.5\vec{w}$



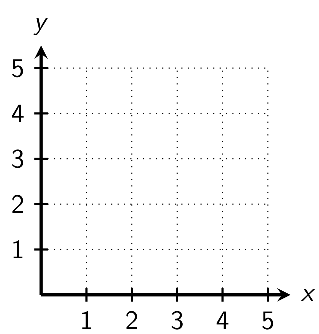
Basis Vectors $\hat{\imath}$ and $\hat{\jmath}$

The basis vectors $\hat{\imath} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\hat{\jmath} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ will serve as the foundation for matrix multiplication.



Any vector can be written as a combination of a scalar and the basis vectors $\hat{\imath}$ and $\hat{\jmath}$.

For instance, $\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ can be written as follows:



Example 3. Write each in terms of basis vectors $\hat{\imath}$ and $\hat{\jmath}$.

(a) $\vec{w} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(b) $\vec{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

(c)