Inverse Functions

Summary

1. Inverse functions act as an "undo" function for a given function.
2. The graph of the inverse function is a reflection of the original function across $y = x$.
3. $f^{-1}(\bullet)$ asks you to find the x-coordinate associated with the y-coordinate of $\bullet$.

The **inverse** of the ordered pair (x, y) is (y, x).

Example 1. Find the inverse of each.

(a) $(2, -7)$

(b) $(0, 3)$

Recall that a function is nothing more than a machine that

1. Accepts an input, x
2. Performs some operation(s)
3. Gives an output, y

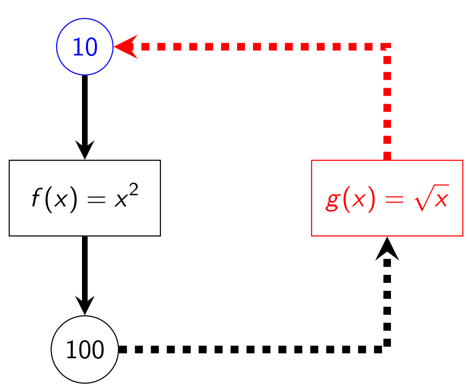
The inverse function is somewhat of an **"undo"** function.

Inverse functions allow us to take the output of a function, put it into our inverse function, and get our original input value back.

Suppose we put a value of 10 into the function

$$f(x) = x^2$$

If we put the output (100) into the inverse, we get our 10 back.



We use the notation $f^{-1}(x)$ to denote the inverse of $f(x)$.

Note: The notation **does not mean** raise the function to the -1 power.

STEPS IN FINDING THE INVERSE OF A FUNCTION

1. Rewrite $f(x) =$ as $y =$
2. Switch your x and y variables.
3. Solve this result for y and rewrite using inverse notation.

Example 2. Find the inverse of each of the following.

(a) $f(x) = 5x$

(b) $f(x) = 3x + 2$

(c) $f(x) = \dfrac{x+5}{7}$

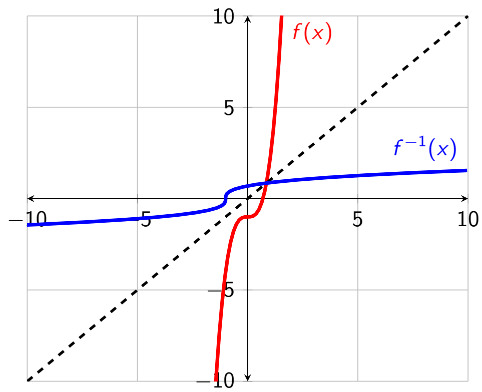
(d) $g(x) = x^3 + 1$

(e) $h(x) = 4x^5 - 1$

(f) $g(x) = \dfrac{5}{x}$

Visually, when finding the inverse of a function, you are **reflecting that function across the line $y = x$**.

Below are the graphs of $f(x) = 3x^3 - 1$ and $f^{-1}(x) = \sqrt[3]{\dfrac{x+1}{3}}$ as well as the line $y = x$:



Tabular and Visual Approaches to Inverse Functions

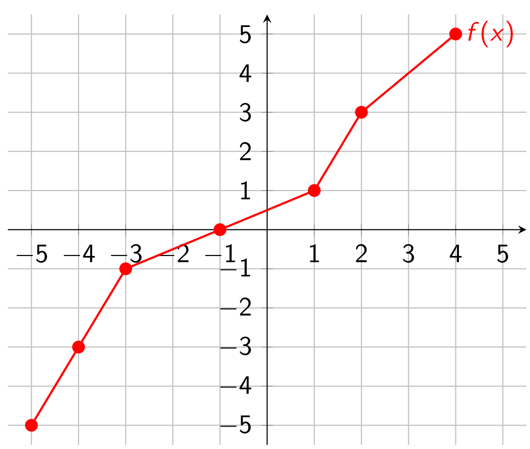
The notation $f^{-1}(\bullet)$ means what x-coordinate has a y-coordinate of $\bullet$?

Example 3. Find the value of each.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	2	4	1	-3	0	3	-4	-1	-2

(a) $f^{-1}(3)$ (b) $f^{-1}(-2)$ (c) $f^{-1}(2)$ (d) $f^{-1}(-1)$

Example 4. Find the value of each given the graph of $f(x)$.



(a) $f^{-1}(-3)$ (b) $f^{-1}(3)$ (c) $f^{-1}(5)$ (d) $f^{-1}(0)$