Factoring $x^2 + bx + c$

Summary

- 1. Factoring is "reverse multiplication."
- 2. For $x^2 + bx + c$, find 2 numbers that **multiply** to make c and also **add** to make b.
- 3. For numbers, find the greatest common factor (a.k.a. greatest common divisor).
- 4. For variables, pick the lowest power of each variable expression.

In the past, you learned about multiplying binomials.

Example 1. Simplify each.

(a)
$$(x-3)(x+5)$$

(b)
$$(x-4)(x-7)$$

Factoring is the *reverse* of the above process.

You start with the "answer" and build the "question."

Your goal is to find 2 numbers that

- Multiply to make the last value
- Add to make the middle value

Example 2. For each of the following find two numbers that meet each requirement.

- (a) Multiply to make -12; Add to make 1
- (b) Multiply to make -20; Add to make 8

- (c) Multiply to make 18; Add to make 7
- (d) Multiply to make 45; Add to make 14

- (e) Multiply to make 8; Add to make -9
- (f) Multiply to make -20; Add to make -1

Now we will put the last 2 examples together.

Example 3. Factor each completely.

(a)
$$x^2 + 7x - 60$$

(b)
$$x^2 + 19x - 66$$

(c)
$$x^2 - 4x - 21$$

(d)
$$x^2 + 5x + 6$$

(e)
$$x^2 + 6x + 8$$

(f)
$$x^2 - 14x + 24$$

Finding the Greatest Common Factor

Typically, when the first term is negative, include that negative when factoring out the GCF.

Example 4. Factor the greatest common factor (GCF) from each.

(a)
$$21x^2 + 28x$$

(b)
$$20x^2 + 30x$$

(c)
$$-3x^3 + 12x^2$$

(d)
$$-2x^3 + 10x^2$$

When factoring trinomials, sometimes you can still factor what remains after factoring out the GCF.

Example 5. Factor each *completely*.

(a)
$$3x^2 + 6x + 48$$

(b)
$$2x^2 - 20x - 120$$

(c)
$$5x^2 - 45x + 90$$

(d)
$$4x^2 + 12x - 40$$