Quadratic Functions

Objectives

1. Determine the vertex, range, and intercepts of a quadratic function in standard form
2. Convert between standard and general form of quadratic functions

Quadratic Functions

A quadratic function is a function in the form

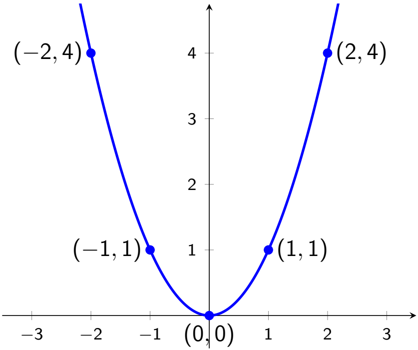
$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers with $a \neq 0$.

The domain of a quadratic function is $(-\infty, \infty)$.

Graph of a Quadratic Function

For $f(x) = x^2$, the graph below is a parabola.



Graph of a Quadratic Function

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We can graph parabolas using transformations to the parent function $f(x) = x^2$.

Example 1

Graph the following functions starting with the graph of $f(x) = x^2$ and using transformations. Find the vertex, state the range, and find the x- and y-intercepts, if any exist.

(a) $g(x) = (x+2)^2 - 3$

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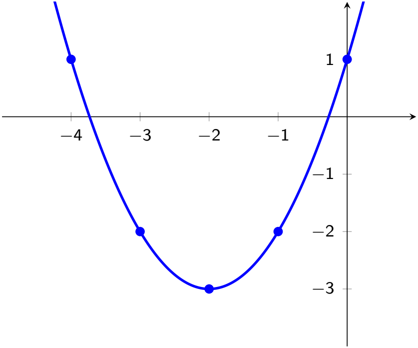

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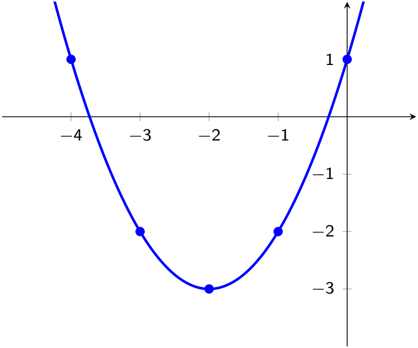

Shift left 2 units

Shift down 3 units

Example 1

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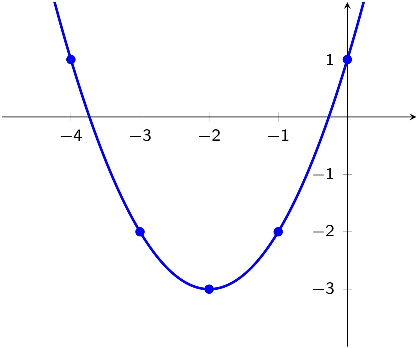

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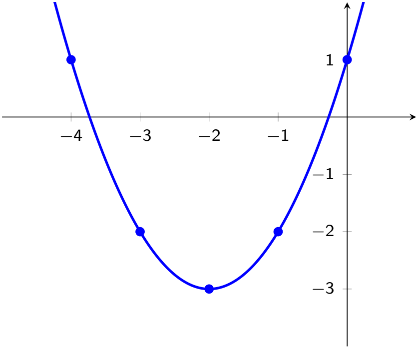

Vertex at $(-2, -3)$

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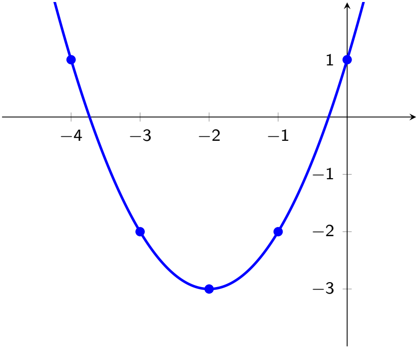

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Range $[-3, \infty)$

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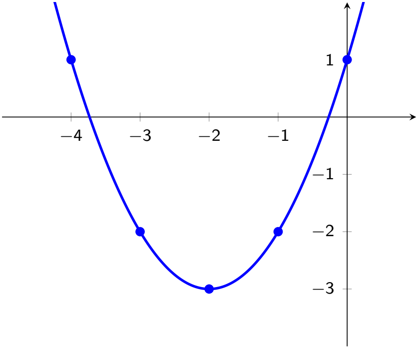

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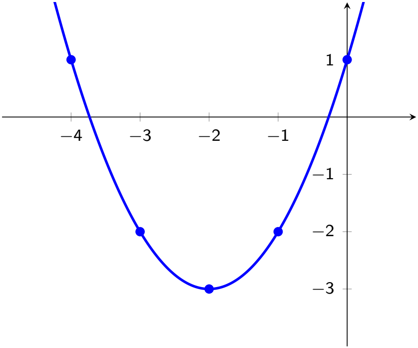

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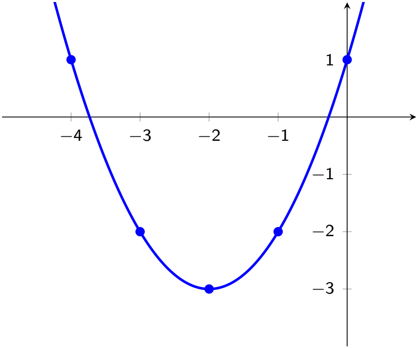

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$$(x+2)^2 - 3 = 0$$
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$$x + 2 = \pm\sqrt{3}$$

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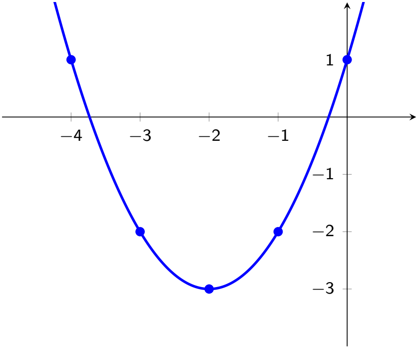

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x-intercepts:

$$\begin{aligned} (x+2)^2 - 3 &= 0 \\ (x+2)^2 &= 3 \\ x+2 &= \pm\sqrt{3} \\ x &= -2 \pm \sqrt{3} \end{aligned}$$

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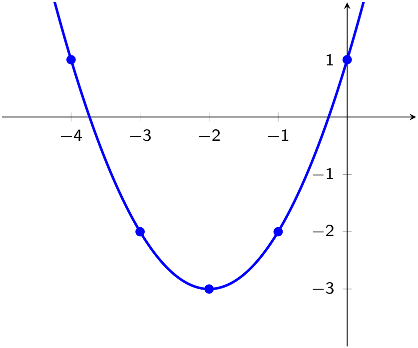

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$(-2 \pm \sqrt{3}, 0)$

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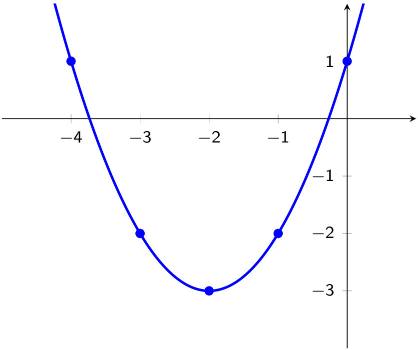

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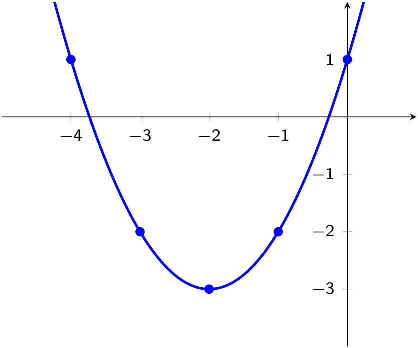

y-intercept at $(0, 1)$

Example 1

(b) $h(x) = -2(x-3)^2 + 1$

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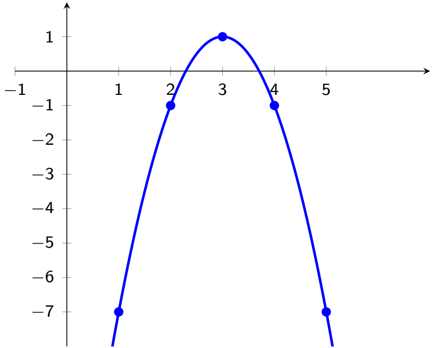

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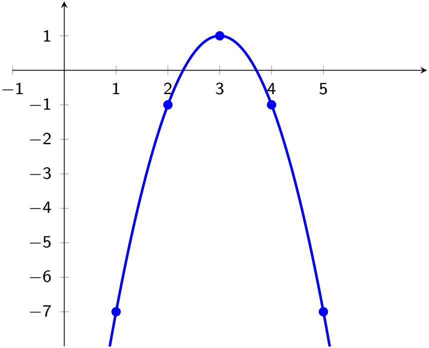

Reflect across x-axis

Vertical stretch by factor of 2

Shift right 3 units

Shift up 1 unit

Example 1

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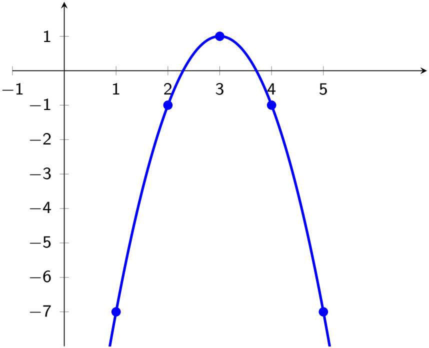

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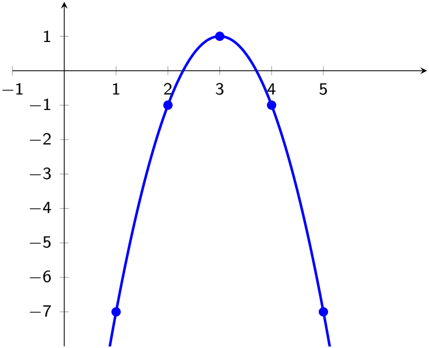

Vertex at $(3, 1)$

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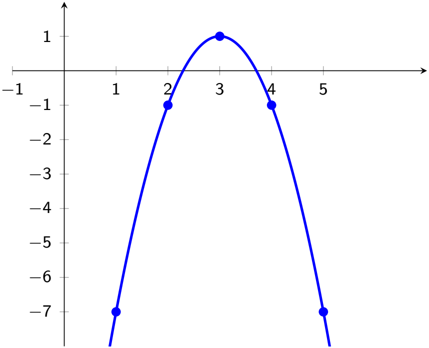

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Range $(-\infty, 1]$

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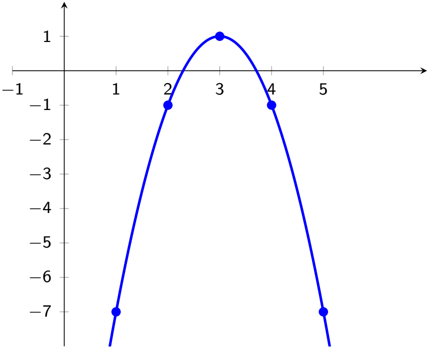

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x-intercepts:

$$-2(x-3)^2 + 1 = 0$$

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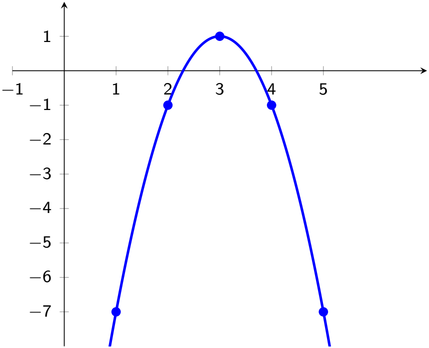

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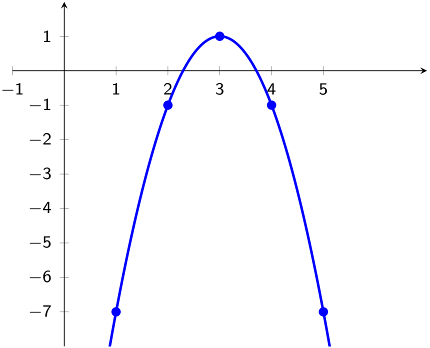

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x-intercepts:

$$\begin{aligned} -2(x-3)^2 + 1 &= 0 \\ -2(x-3)^2 &= -1 \\ (x-3)^2 &= \frac{1}{2} \end{aligned}$$

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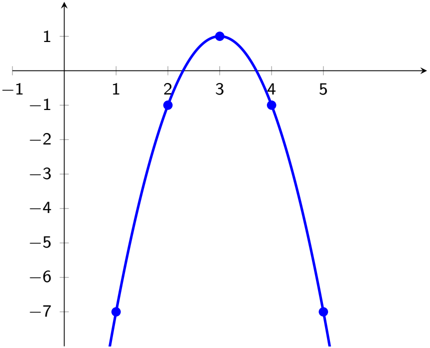

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$$\begin{aligned} -2(x-3)^2 + 1 &= 0 \\ -2(x-3)^2 &= -1 \\ (x-3)^2 &= \frac{1}{2} \\ x - 3 &= \pm\sqrt{\frac{1}{2}} \end{aligned}$$

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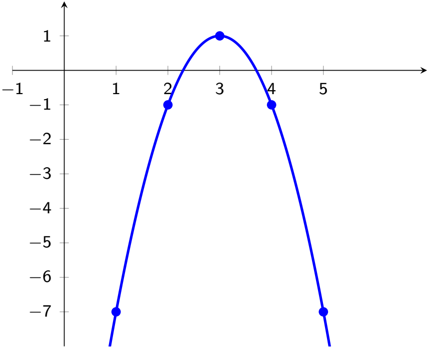

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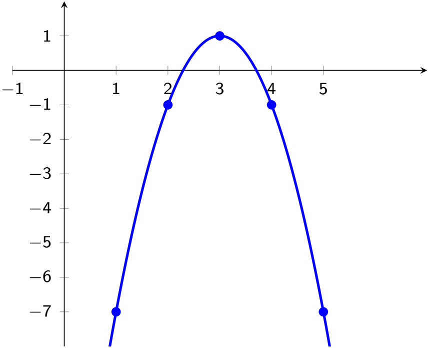

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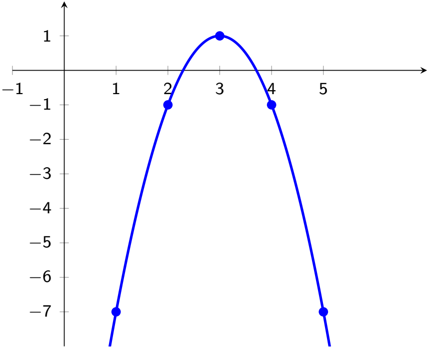

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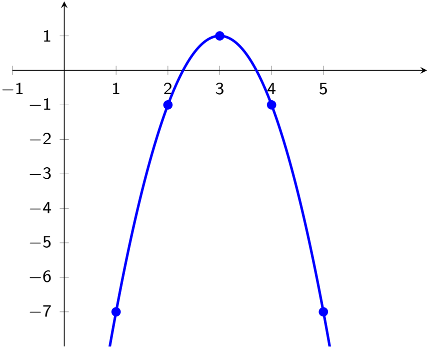

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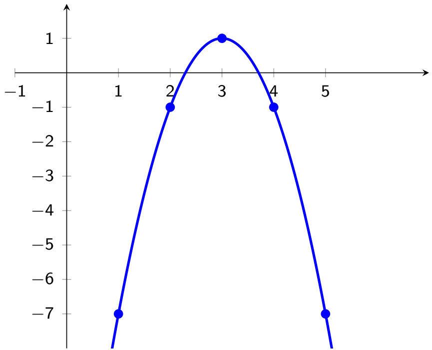

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y-intercept: $(0, -17)$

Objectives

1. Determine the vertex, range, and intercepts of a quadratic function in standard form
2. Convert between standard and general form of quadratic functions

Standard and General Form of Quadratic Functions

If f is a quadratic function:

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Converting From General to Standard Form

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To convert from general form $f(x) = ax^2 + bx + c$ to standard form $f(x) = a(x-h)^2 + k$

1. Find the vertex:
   - x-coordinate: $\dfrac{-b}{2a}$
   - y-coordinate: Evaluate function at x-coordinate
   - Or use graphing technology
2. Use the same value of a

Example 2

Convert each to standard form.

(a) $f(x) = x^2 - 4x + 3$

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$$f(x) = (x-2)^2 - 1$$

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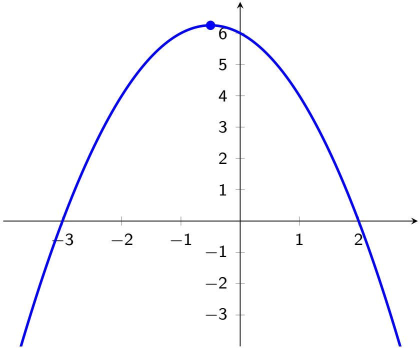

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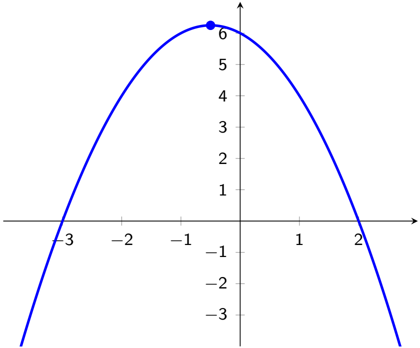

Vertex: $\left(-\frac{1}{2}, \frac{25}{4}\right)$

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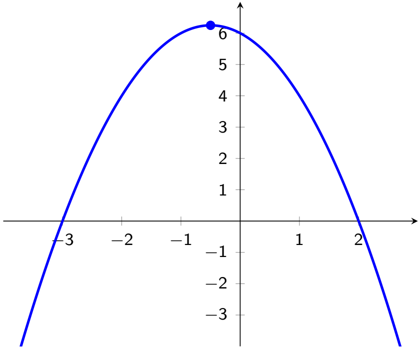

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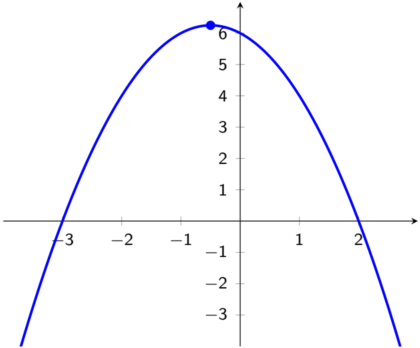


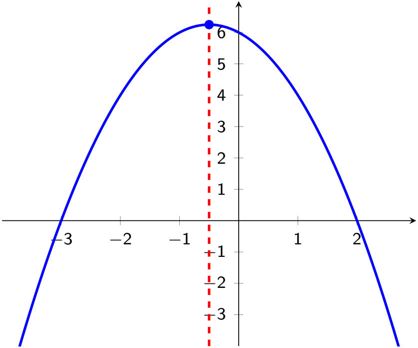
Vertex: $\left(-\frac{1}{2}, \frac{25}{4}\right)$

$a = -1$

$$g(x) = -\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$$

Axis of Symmetry

The graphs of the parabolas have a line of symmetry called the axis of symmetry that is a vertical line through the x-coordinate of the vertex:



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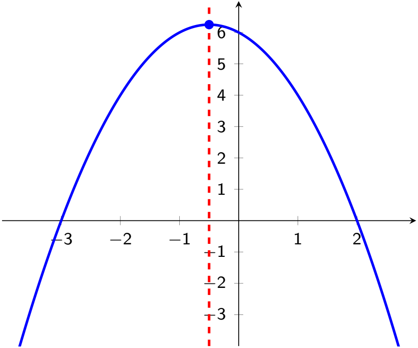

$$x = -\frac{1}{2}$$

Converting From Standard to General Form

To convert from

$$f(x) = a(x-h)^2 + k$$

form to

$$f(x) = ax^2 + bx + c$$

form, just **do the math**.

Example 3

Convert $g(x) = (x + 2)^2 - 3$ to general form.

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$$(x+2)^2 - 3 = x^2 + 4x + 4 - 3$$

Example 3

Convert $g(x) = (x+2)^2 - 3$ to general form.

$$\begin{aligned}(x+2)^2 - 3 &= x^2 + 4x + 4 - 3 \\ &= x^2 + 4x + 1\end{aligned}$$