

# Linear Functions and Slope

# Objectives

- 1 Calculate the slope of a line connecting two points
- 2 Write the point-slope form of a line
- 3 Write the equation of a line in slope-intercept form
- 4 Determine the average rate of change of a function
- 5 Find the least-squares regression line

# Slope

The slope of the line connect points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  is

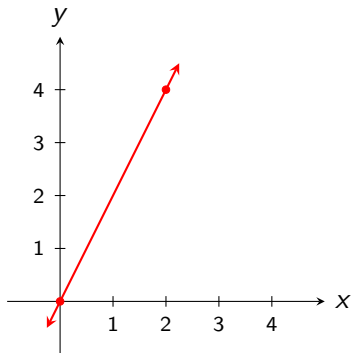
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

provided  $x_2 \neq x_1$

## Example 1

Find the slope of the line connecting each pair of points.

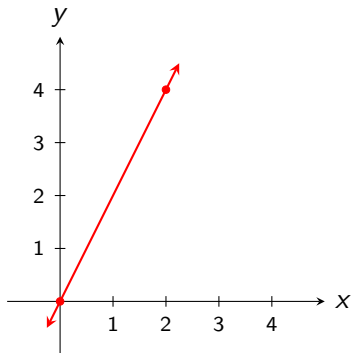
(a)  $(0, 0)$  and  $(2, 4)$



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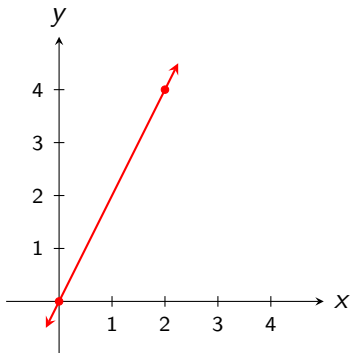


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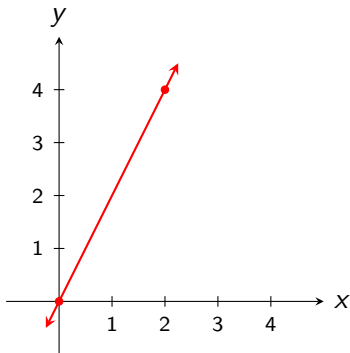


$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 0}{2 - 0} \end{aligned}$$

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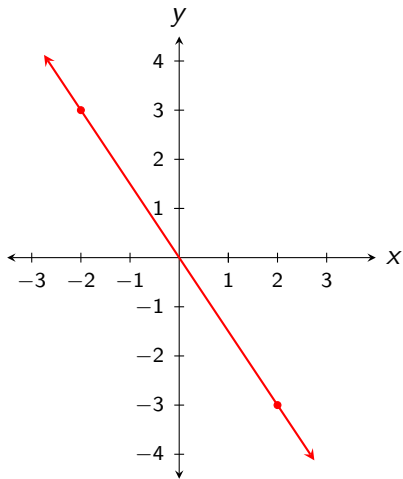
(a)  $(0, 0)$  and  $(2, 4)$



$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 0}{2 - 0} \\ &= 2 \end{aligned}$$

## Example 1

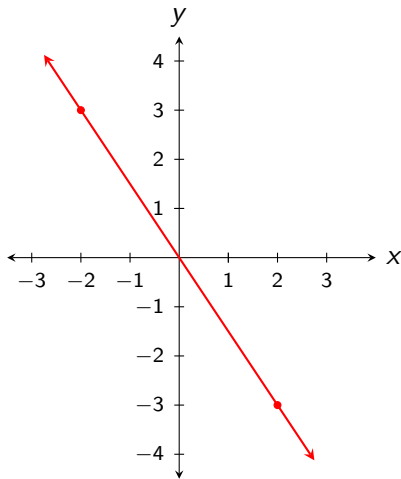
(b)  $(-2, 3)$  and  $(2, -3)$





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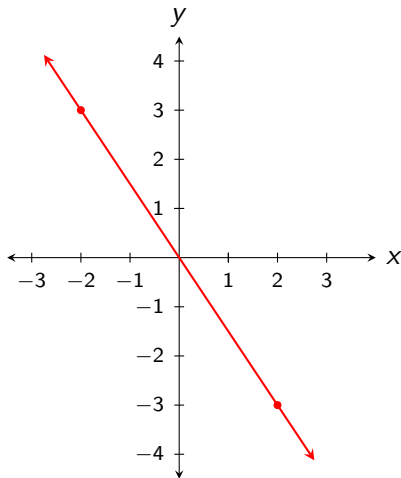
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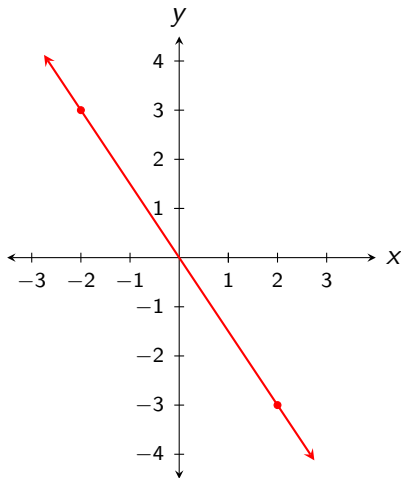
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$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 3}{2 - (-2)} \end{aligned}$$

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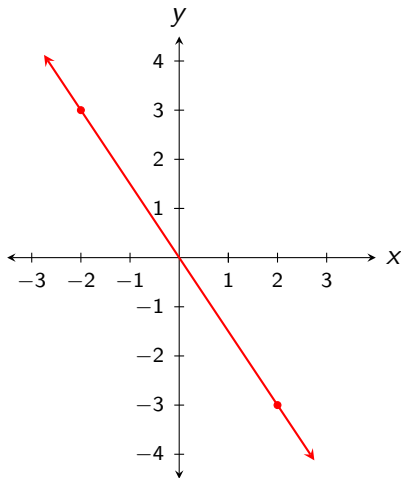
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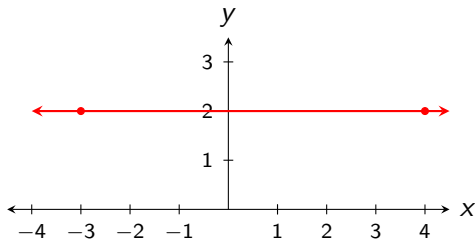
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$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 3}{2 - (-2)} \\ &= \frac{-6}{4} \\ &= -\frac{3}{2} \end{aligned}$$

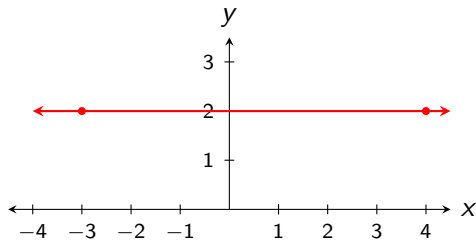
## Example 1

(c)  $(-3, 2)$  and  $(4, 2)$



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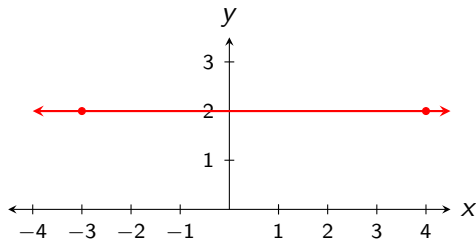
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## Example 1

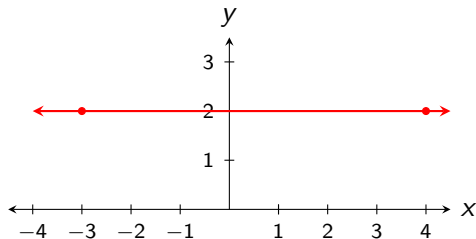
(c)  $(-3, 2)$  and  $(4, 2)$



$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 2}{4 - (-3)} \end{aligned}$$

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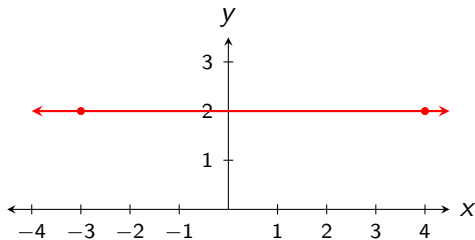


$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 2}{4 - (-3)} \\ &= \frac{0}{7} \end{aligned}$$



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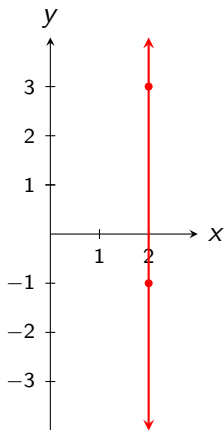
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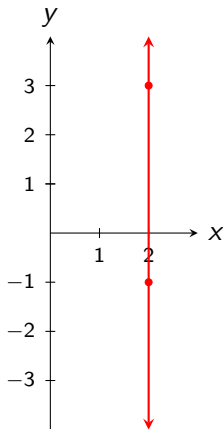
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(d)  $(2, 3)$  and  $(2, -1)$



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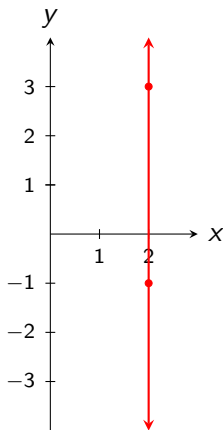
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$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

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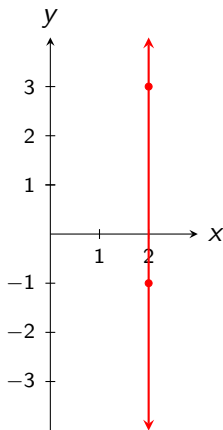
(d)  $(2, 3)$  and  $(2, -1)$



$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 3}{2 - 2} \end{aligned}$$

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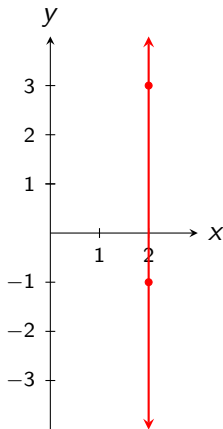
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$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 3}{2 - 2} \\ &= \frac{-4}{0} \\ &= \text{undefined} \end{aligned}$$

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# Point-Slope Form of a Line

The **point-slope form** of a line is a general equation involving the slope of the line and a point on the line.



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$$y - 3 = -\frac{2}{3}(x + 1)$$



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# Slope-Intercept Form

The **slope-intercept form** of the equation of a line is

$$y = mx + b$$

where  $b$  is the  $y$ -intercept.

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where  $b$  is the  $y$ -intercept.

To convert from point-slope form to slope-intercept form, solve the point-slope form for  $y$ .

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$$y - 3 = -\frac{2}{3}(x + 1)$$

$$y - 3 = -\frac{2}{3}x - \frac{2}{3}$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

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# Average Rate of Change

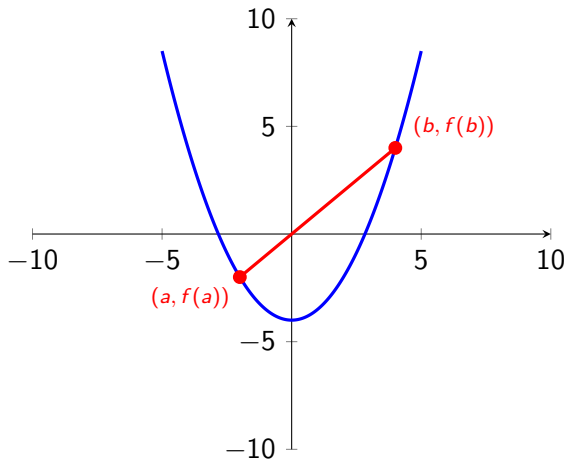
The **average rate of change** over an interval  $[a, b]$  of a function is the slope of the line connecting the endpoints of the interval.

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$$\frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

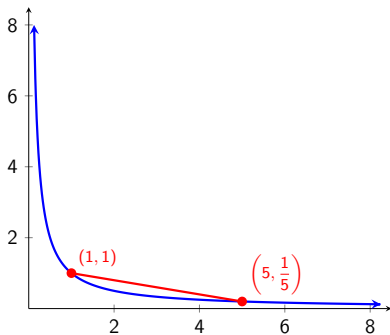
# Average Rate of Change



## Example 4

Find the average rate of change for each.

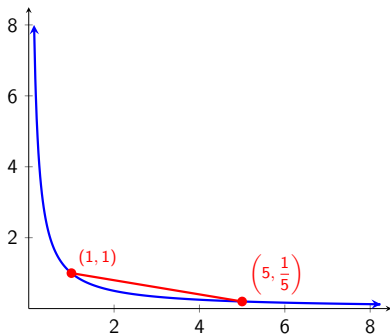
(a)  $f(x) = \frac{1}{x}$   $[1, 5]$



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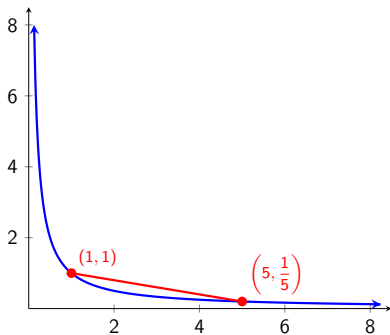
$$f(5) = \frac{1}{5}$$

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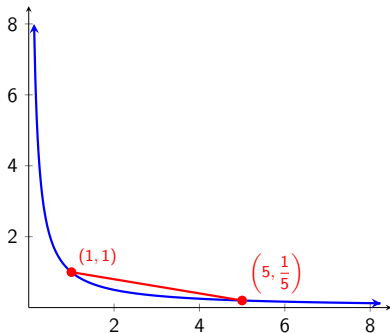
$$f(1) = 1$$

$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{5} - 1}{5 - 1} \left( \frac{5}{5} \right)$$

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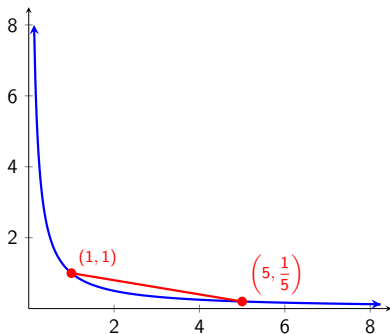
$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{5} - 1}{5 - 1} \left( \frac{5}{5} \right)$$

$$\frac{\Delta f}{\Delta x} = \frac{1 - 5}{25 - 5}$$

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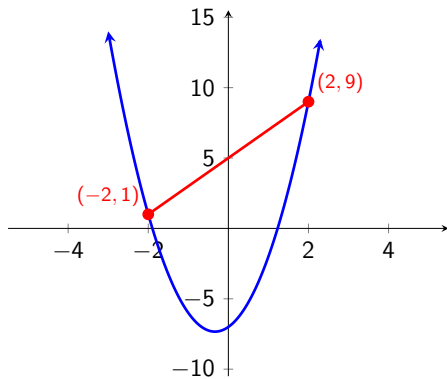
$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{5} - 1}{5 - 1} \left( \frac{5}{5} \right)$$

$$\begin{aligned} \frac{\Delta f}{\Delta x} &= \frac{1 - 5}{25 - 5} \\ &= -\frac{1}{5} \end{aligned}$$



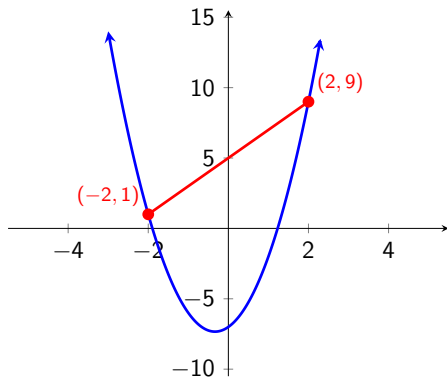
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(b)  $f(x) = 3x^2 + 2x - 7$   $[-2, 2]$



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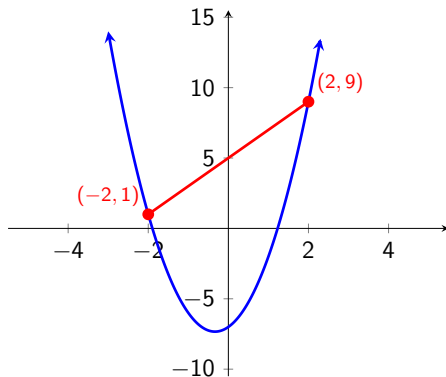


$$f(2) = 9$$

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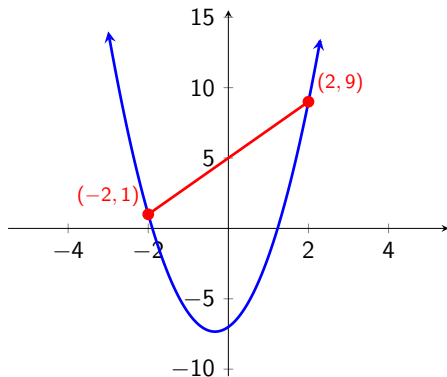


$$f(2) = 9 \qquad f(-2) = 1$$

$$\frac{\Delta f}{\Delta x} = \frac{9 - 1}{2 - (-2)}$$

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$$f(2) = 9$$

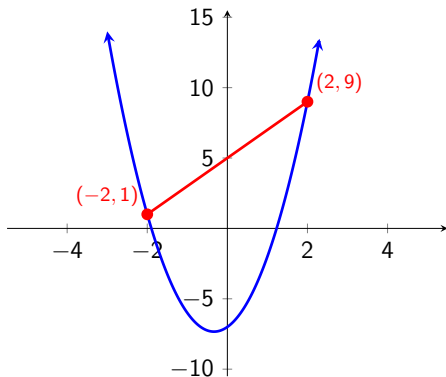
$$f(-2) = 1$$

$$\frac{\Delta f}{\Delta x} = \frac{9 - 1}{2 - (-2)}$$

$$\frac{\Delta f}{\Delta x} = \frac{8}{4}$$

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$$f(2) = 9$$

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$$\frac{\Delta f}{\Delta x} = \frac{8}{4}$$

$$= 2$$

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# Least-Squares Regression Line

When we plot data points, we can sometimes create a least squares regression line to describe and predict the data.

The value of  $r$  (the correlation coefficient) determines how well the data “falls into line.”

The closer  $r$  is to 1 (or  $-1$ ) the better the linear fit.

## Example 5

The census data for Lake County, Ohio is shown:

<b>Year</b>	1970	1980	1990	2000	2010
<b>Pop</b>	197200	212801	215499	227511	230041

(a) Find the least-squares regression line.



## Example 5

The census data for Lake County, Ohio is shown:

<b>Year</b>	1970	1980	1990	2000	2010
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(a) Find the least-squares regression line.

$$y = 803.92x + 200532$$

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(a) Find the least-squares regression line.

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(b) Interpret the slope.

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<b>Year</b>	1970	1980	1990	2000	2010
<b>Pop</b>	197200	212801	215499	227511	230041

(a) Find the least-squares regression line.

$$y = 803.92x + 200532$$

(b) Interpret the slope.

Population increases by about 804 people per year.

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$$803.92x + 200532 > 250000$$

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$$803.92x + 200532 > 250000$$

$$x > 61.5$$



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(c) Predict the population of Lake County in 2070.

$$803.92(100) + 200532 = 280924$$

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$$803.92x + 200532 > 250000$$

$$x > 61.5$$

Predicted to exceed 250,000 in the year 2031.