Zeros of Polynomial Functions

Objectives

Use the Rational Zeros Theorem to list out potential rational zeros of a polynomial

Find all zeros for $f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$

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Potential Rational Zeros:

$$\pm 1,\,\pm \frac{1}{2},\,\pm \frac{1}{3},\,\pm \frac{1}{4},\,\pm \frac{1}{6},\,\pm \frac{1}{12},$$

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Potential Rational Zeros:

$$\pm 1,\,\pm \frac{1}{2},\,\pm \frac{1}{3},\,\pm \frac{1}{4},\,\pm \frac{1}{6},\,\pm \frac{1}{12},$$

Actual Rational Zeros:

$$\frac{1}{2},\,-\frac{1}{3}$$

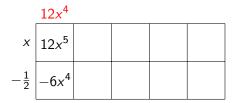
$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

x	12 <i>x</i> ⁵		
$-\frac{1}{2}$			

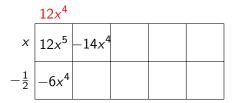
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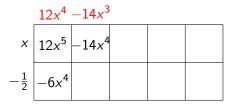
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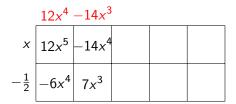
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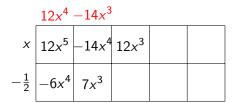
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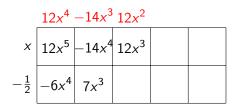
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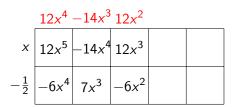
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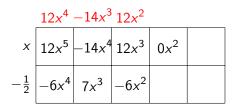
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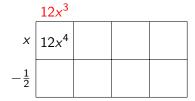
$$12\left(\frac{1}{2}\right)^4 - 14\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 2 = 0,$$

so $x = -\frac{1}{2}$ has another root.

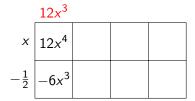
$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

X	12x ⁴		
$-\frac{1}{2}$			

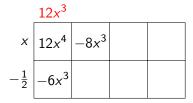
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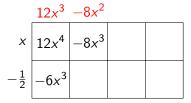
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$$12\left(\frac{1}{2}\right)^3 - 8\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) + 4 \neq 0$$

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

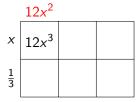
$$12\left(\frac{1}{2}\right)^3 - 8\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) + 4 \neq 0$$

Now we can use the $x = -\frac{1}{3}$ rational zero.

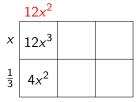
$$12x^3 - 8x^2 + 8x + 4$$

X	$12x^{3}$	
<u>1</u>		

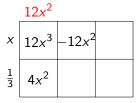
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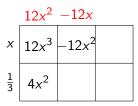
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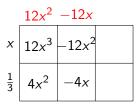
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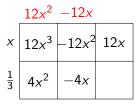
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$$12x^2 - 12x + 12 = 0$$

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$$x^2 - x + 1 = 0$$

$$12x^{2} - 12x + 12 = 0$$

$$x^{2} - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^{2} - 4(1)(1)}}{2(1)}$$

$$12x^{2} - 12x + 12 = 0$$

$$x^{2} - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^{2} - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$12x^{2} - 12x + 12 = 0$$

$$x^{2} - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^{2} - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

$$12x^{2} - 12x + 12 = 0$$

$$x^{2} - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^{2} - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

 $x = \frac{1}{2}$ (double root), $-\frac{1}{3}$, $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$