Transforming Functions

Objectives

1. Determine vertical and horizontal shifts of the graph of a function.
2. Determine reflections across axes of the graph of a function.
3. Determine vertical and horizontal stretches and compressions of the graph of a function.
4. Perform multiple transformations of a function.

$f(x) \pm d$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x) \pm d$.

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Vertical shift

$f(x) \pm d$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x) \pm d$.

Vertical shift

- Up d units if d is positive.

$f(x) \pm d$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x) \pm d$.

Vertical shift

- Up d units if d is positive.
- Down d units if d is negative.

$f(x) \pm d$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x) \pm d$.

Vertical shift

- Up d units if d is positive.
- Down d units if d is negative.

Notice the number you are adding or subtracting is outside the function.

Example 1

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

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(a) $g(x) = \sqrt{x} + 11$

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Shift up 11 units

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Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = \sqrt{x} + 11$

Shift up 11 units

(b) $g(x) = \sqrt{x} - 15$

Example 1

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = \sqrt{x} + 11$

Shift up 11 units

(b) $g(x) = \sqrt{x} - 15$

Shift down 15 units

$f(x \pm c)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x \pm c)$

$f(x \pm c)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x \pm c)$

Horizontal shift

$f(x \pm c)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x \pm c)$

Horizontal shift

- Right c units for $f(x - c)$

$f(x \pm c)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x \pm c)$

Horizontal shift

- Right c units for $f(x - c)$
- Left c units for $f(x + c)$

$f(x \pm c)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(x \pm c)$

Horizontal shift

- Right c units for $f(x - c)$
- Left c units for $f(x + c)$

Notice the number you are adding or subtracting is inside the function.

Example 2

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

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Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = \sqrt{x + 11}$

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(a) $g(x) = \sqrt{x + 11}$

Shift left 11 units

Example 2

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = \sqrt{x + 11}$

Shift left 11 units

(b) $g(x) = \sqrt{x - 15}$

Example 2

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = \sqrt{x + 11}$

Shift left 11 units

(b) $g(x) = \sqrt{x - 15}$

Shift right 15 units

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$-f(x)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = -f(x)$

$-f(x)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = -f(x)$

Reflect across x-axis

$f(-x)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(-x)$

$f(-x)$

For the function $f(x) = \sqrt{x}$, determine the effects on the graph of $g(x) = f(-x)$

Reflect across y-axis

Example 3

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = -\sqrt{x} + 2$

Example 3

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = -\sqrt{x} + 2$

1. Reflect across x-axis

Example 3

Determine the effect on the graph of $f(x) = \sqrt{x}$ for each.

(a) $g(x) = -\sqrt{x} + 2$

1. Reflect across x-axis
2. Shift up 2 units

Example 3

(b) $g(x) = \sqrt{-x} - 3$

Example 3

(b) $g(x) = \sqrt{-x} - 3$

1. Reflect across y-axis

Example 3

(b) $g(x) = \sqrt{-x} - 3$

1. Reflect across y-axis
2. Shift down 3 units

Example 3

(c) $g(x) = 4 - \sqrt{x}$

Example 3

(c) $g(x) = 4 - \sqrt{x}$

$g(x) = -\sqrt{x} + 4$

Example 3

(c) $g(x) = 4 - \sqrt{x}$

$g(x) = -\sqrt{x} + 4$

1. Reflect across x-axis

Example 3

(c) $\quad g(x) = 4 - \sqrt{x}$

$g(x) = -\sqrt{x} + 4$

1. Reflect across x-axis
2. Shift up 4 units

Objectives

1. Determine vertical and horizontal shifts of the graph of a function.
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$a \cdot f(x)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = a \cdot f(x)$

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Vertical stretch if $a > 1$

$a \cdot f(x)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = a \cdot f(x)$

Vertical stretch if $a > 1$

- The factor is a

$a \cdot f(x)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = a \cdot f(x)$

Vertical stretch if $a > 1$

- The factor is a
- The y-coordinates are now a times further away from the x-axis

$a \cdot f(x)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = a \cdot f(x)$

$a \cdot f(x)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = a \cdot f(x)$

Vertical compression if $0 < a < 1$

$a \cdot f(x)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = a \cdot f(x)$

Vertical compression if $0 < a < 1$

- The factor is the reciprocal of a

$a \cdot f(x)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = a \cdot f(x)$

Vertical compression if $0 < a < 1$

- The factor is the reciprocal of a
- The y-coordinates are now a times closer to the x-axis

Example 4

Determine the effect on the graph of $f(x) = \sin x$ for each.

(a) $g(x) = 12 \sin x$

Example 4

Determine the effect on the graph of $f(x) = \sin x$ for each.

(a) $g(x) = 12 \sin x$

Vertical stretch by factor of 12

Example 4

Determine the effect on the graph of $f(x) = \sin x$ for each.

(a) $g(x) = 12 \sin x$

Vertical stretch by factor of 12

(b) $g(x) = \dfrac{1}{3} \sin x$

Example 4

Determine the effect on the graph of $f(x) = \sin x$ for each.

(a) $g(x) = 12 \sin x$

Vertical stretch by factor of 12

(b) $g(x) = \dfrac{1}{3} \sin x$

Vertical compression by factor of 3

Example 4

(c) $g(x) = -80 \sin x$

Example 4

(c) $g(x) = -80 \sin x$

1. Reflect across x-axis

Example 4

(c) $g(x) = -80 \sin x$

1. Reflect across x-axis
2. Vertical stretch by factor of 80

Example 4

(d) $\quad g(x) = -\dfrac{2}{3}\sin x$

Example 4

(d) $\quad g(x) = -\dfrac{2}{3}\sin x$

1. Reflect across x-axis

Example 4

(d) $g(x) = -\frac{2}{3}\sin x$

1. Reflect across x-axis
2. Vertical compression by factor of $\frac{3}{2}$

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

Horizontal compression if $b > 1$

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

Horizontal compression if $b > 1$

- The factor is b

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

Horizontal compression if $b > 1$

- The factor is b
- The x-coordinates are now b times closer to the y-axis

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

Horizontal stretch if $0 < b < 1$

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

Horizontal stretch if $0 < b < 1$

- The factor is the reciprocal of b

$f(bx)$

For $f(x) = \sin x$, determine the effects on the graph of $g(x) = f(bx)$

Horizontal stretch if $0 < b < 1$

- The factor is the reciprocal of b
- The x-coordinates are now b times further away from the y-axis

Example 5

Determine the effect on the graph of $f(x) = \sin x$ for each.

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(a) $g(x) = \sin(12x)$

Example 5

Determine the effect on the graph of $f(x) = \sin x$ for each.

(a) $g(x) = \sin(12x)$

Horizontal compression by factor of 12

Example 5

Determine the effect on the graph of $f(x) = \sin x$ for each.

(a) $g(x) = \sin(12x)$

Horizontal compression by factor of 12

(b) $g(x) = \sin\left(\frac{1}{5}x\right)$

Example 5

Determine the effect on the graph of $f(x) = \sin x$ for each.

(a) $g(x) = \sin(12x)$

Horizontal compression by factor of 12

(b) $g(x) = \sin\left(\frac{1}{5}x\right)$

Horizontal stretch by factor of 5

Objectives

1. Determine vertical and horizontal shifts of the graph of a function.
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Example 6

Parent function: $f(x) = \sqrt{x}$

(a) $g(x) = -\sqrt{-x+1} - 5$

Example 6

Parent function: $f(x) = \sqrt{x}$

(a) $\quad g(x) = -\sqrt{-x+1} - 5$

1. Shift left 1 unit

Example 6

Parent function: $f(x) = \sqrt{x}$

(a) $g(x) = -\sqrt{-x+1} - 5$

1. Shift left 1 unit
2. Reflect across y-axis

Example 6

Parent function: $f(x) = \sqrt{x}$

(a) $g(x) = -\sqrt{-x+1} - 5$

1. Shift left 1 unit
2. Reflect across y-axis
3. Reflect across x-axis

Example 6

Parent function: $f(x) = \sqrt{x}$

(a) $g(x) = -\sqrt{-x+1} - 5$

1. Shift left 1 unit
2. Reflect across y-axis
3. Reflect across x-axis
4. Shift down 5 units

Example 6a Alternate answer

$$g(x) = -\sqrt{-x+1} - 5$$

Example 6a Alternate answer

$$g(x) = -\sqrt{-x+1} - 5$$

$$g(x) = -\sqrt{-(x-1)} - 5$$

Example 6a Alternate answer

$$g(x) = -\sqrt{-x+1} - 5$$

$$g(x) = -\sqrt{-(x-1)} - 5$$

1. Reflect across y-axis

Example 6a Alternate answer

$$g(x) = -\sqrt{-x+1} - 5$$

$$g(x) = -\sqrt{-(x-1)} - 5$$

1. Reflect across y-axis
2. Shift right 1 unit

Example 6a Alternate answer

$$g(x) = -\sqrt{-x+1} - 5$$

$$g(x) = -\sqrt{-(x-1)} - 5$$

1. Reflect across y-axis
2. Shift right 1 unit
3. Reflect across x-axis

Example 6a Alternate answer

$$g(x) = -\sqrt{-x+1} - 5$$

$$g(x) = -\sqrt{-(x-1)} - 5$$

1. Reflect across y-axis
2. Shift right 1 unit
3. Reflect across x-axis
4. Shift down 5 units

Preferred Order for Transformations

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1. Horizontal shifts

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1. Horizontal shifts
2. Horizontal stretches/compressions and/or y-axis reflection

Preferred Order for Transformations

1. Horizontal shifts
2. Horizontal stretches/compressions and/or y-axis reflection
3. Vertical stretches/compressions and/or x-axis reflection

Preferred Order for Transformations

1. Horizontal shifts
2. Horizontal stretches/compressions and/or y-axis reflection
3. Vertical stretches/compressions and/or x-axis reflection
4. Vertical shifts

Example 6

Parent function: $f(x) = \sqrt{x}$

(b) $\quad g(x) = 1 - \sqrt{\dfrac{x+3}{2}}$

Example 6

Parent function: $f(x) = \sqrt{x}$

(b) $g(x) = 1 - \sqrt{\dfrac{x+3}{2}}$

$$g(x) = -\sqrt{\frac{1}{2}x + \frac{3}{2}} + 1$$

Example 6

Parent function: $f(x) = \sqrt{x}$

(b) $\quad g(x) = 1 - \sqrt{\dfrac{x+3}{2}}$

$$g(x) = -\sqrt{\frac{1}{2}x + \frac{3}{2}} + 1$$

1. Shift left $\dfrac{3}{2}$ units

Example 6

Parent function: $f(x) = \sqrt{x}$

(b) $g(x) = 1 - \sqrt{\dfrac{x+3}{2}}$

$$g(x) = -\sqrt{\frac{1}{2}x + \frac{3}{2}} + 1$$

1. Shift left $\dfrac{3}{2}$ units
2. Horizontal stretch by factor of 2

Example 6

Parent function: $f(x) = \sqrt{x}$

(b) $g(x) = 1 - \sqrt{\dfrac{x+3}{2}}$

$$g(x) = -\sqrt{\frac{1}{2}x + \frac{3}{2}} + 1$$

1. Shift left $\dfrac{3}{2}$ units
2. Horizontal stretch by factor of 2
3. Reflect across x-axis

Example 6

Parent function: $f(x) = \sqrt{x}$

(b) $\quad g(x) = 1 - \sqrt{\dfrac{x+3}{2}}$

$$g(x) = -\sqrt{\frac{1}{2}x + \frac{3}{2}} + 1$$

1. Shift left $\dfrac{3}{2}$ units
2. Horizontal stretch by factor of 2
3. Reflect across x-axis
4. Shift up 1 unit

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

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Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $\qquad g(x) = x^2 + 2$

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $\quad g(x) = x^2 + 2$

2. Reflect across x-axis

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $$g(x) = x^2 + 2$$
2. Reflect across x-axis $$g(x) = -(x^2 + 2)$$

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $$g(x) = x^2 + 2$$
2. Reflect across x-axis $$g(x) = -(x^2 + 2)$$ $$g(x) = -x^2 - 2$$

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $$g(x) = x^2 + 2$$

2. Reflect across x-axis $$g(x) = -(x^2 + 2)$$ $$g(x) = -x^2 - 2$$

3. Shift right 1 unit

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $$g(x) = x^2 + 2$$
2. Reflect across x-axis $$g(x) = -(x^2 + 2)$$ $$g(x) = -x^2 - 2$$
3. Shift right 1 unit $$g(x) = -(x - 1)^2 - 2$$

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $\quad g(x) = x^2 + 2$
2. Reflect across x-axis $\quad g(x) = -(x^2 + 2)$

   $$g(x) = -x^2 - 2$$
3. Shift right 1 unit $\quad g(x) = -(x-1)^2 - 2$
4. Horizontal stretch by factor of 2

Example 7

Given parent function $f(x) = x^2$, write the child function $g(x)$ after the following sequence of transformations.

1. Shift up 2 units $$g(x) = x^2 + 2$$
2. Reflect across x-axis $$g(x) = -(x^2 + 2)$$ $$g(x) = -x^2 - 2$$
3. Shift right 1 unit $$g(x) = -(x-1)^2 - 2$$
4. Horizontal stretch by factor of 2 $$g(x) = -\left(\frac{1}{2}x - 1\right)^2 - 2$$