Zeros of Polynomial Functions

Objectives

1. Use the Rational Zeros Theorem to list out potential rational zeros of a polynomial

Example 4

Find all zeros for $f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$

Example 4

Find all zeros for $f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$

Potential Rational Zeros:

$$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12},$$

Example 4

Find all zeros for $f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$

Potential Rational Zeros:

$$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12},$$

Actual Rational Zeros:

$$\frac{1}{2}, -\frac{1}{3}$$

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

x	$12x^5$				
$-\frac{1}{2}$					

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

$$\begin{array}{c|c|c|c|c|c|}
 & 12x^4 & & & & \\ \hline
x & 12x^5 & & & & \\ \hline
-\frac{1}{2} & & & & & \\ \hline
\end{array}$$

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$				
x	$12x^5$				
$-\frac{1}{2}$	$-6x^4$				

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$				
x	$12x^5$	$-14x^4$			
$-\frac{1}{2}$	$-6x^4$				

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$			
x	$12x^5$	$-14x^4$			
$-\frac{1}{2}$	$-6x^4$				

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$			
x	$12x^5$	$-14x^4$			
$-\frac{1}{2}$	$-6x^4$	$7x^3$			

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$			
x	$12x^5$	$-14x^4$	$12x^3$		
$-\frac{1}{2}$	$-6x^4$	$7x^3$			

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$		
x	$12x^5$	$-14x^4$	$12x^3$		
$-\frac{1}{2}$	$-6x^4$	$7x^3$			

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$		
x	$12x^5$	$-14x^4$	$12x^3$		
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$		

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$		
x	$12x^5$	$-14x^4$	$12x^3$	$0x^2$	
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$		

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$	$0x$	
x	$12x^5$	$-14x^4$	$12x^3$	$0x^2$	
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$		

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$	$0x$	
x	$12x^5$	$-14x^4$	$12x^3$	$0x^2$	
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$	$0x$	

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$	$0x$	
x	$12x^5$	$-14x^4$	$12x^3$	$0x^2$	$-2x$
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$	$0x$	

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$	$0x$	-2
x	$12x^5$	$-14x^4$	$12x^3$	$0x^2$	$-2x$
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$	$0x$	

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$	$0x$	-2
x	$12x^5$	$-14x^4$	$12x^3$	$0x^2$	$-2x$
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$	$0x$	1

Example 4

$$f(x) = 12x^5 - 20x^4 + 19x^3 - 6x^2 - 2x + 1$$

	$12x^4$	$-14x^3$	$12x^2$	$0x$	-2
x	$12x^5$	$-14x^4$	$12x^3$	$0x^2$	$-2x$
$-\frac{1}{2}$	$-6x^4$	$7x^3$	$-6x^2$	$0x$	1

$$12\left(\tfrac{1}{2}\right)^4 - 14\left(\tfrac{1}{2}\right)^3 + 12\left(\tfrac{1}{2}\right)^2 - 2 = 0,$$

so $x = -\frac{1}{2}$ has another root.

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

x	$12x^4$			
$-\frac{1}{2}$				

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$			
x	$12x^4$			
$-\frac{1}{2}$				

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$			
x	$12x^4$			
$-\frac{1}{2}$	$-6x^3$			

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$\color{red}{12x^3}$			
x	$12x^4$	$-8x^3$		
$-\frac{1}{2}$	$-6x^3$			

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$		
x	$12x^4$	$-8x^3$		
$-\frac{1}{2}$	$-6x^3$			

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$		
x	$12x^4$	$-8x^3$		
$-\frac{1}{2}$	$-6x^3$	$4x^2$		

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$		
x	$12x^4$	$-8x^3$	$8x^2$	
$-\frac{1}{2}$	$-6x^3$	$4x^2$		

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$	$8x$	
x	$12x^4$	$-8x^3$	$8x^2$	
$-\frac{1}{2}$	$-6x^3$	$4x^2$		

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$	$8x$	
x	$12x^4$	$-8x^3$	$8x^2$	
$-\frac{1}{2}$	$-6x^3$	$4x^2$	$-4x$	

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$	$8x$	
x	$12x^4$	$-8x^3$	$8x^2$	$4x$
$-\frac{1}{2}$	$-6x^3$	$4x^2$	$-4x$	

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$	$8x$	4
x	$12x^4$	$-8x^3$	$8x^2$	$4x$
$-\frac{1}{2}$	$-6x^3$	$4x^2$	$-4x$	

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$	$8x$	4
x	$12x^4$	$-8x^3$	$8x^2$	$4x$
$-\frac{1}{2}$	$-6x^3$	$4x^2$	$-4x$	-2

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$	$8x$	4
x	$12x^4$	$-8x^3$	$8x^2$	$4x$
$-\frac{1}{2}$	$-6x^3$	$4x^2$	$-4x$	-2

$$12\left(\frac{1}{2}\right)^3 - 8\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) + 4 \neq 0$$

Example 4

$$12x^4 - 14x^3 + 12x^2 + 0x - 2$$

	$12x^3$	$-8x^2$	$8x$	4
x	$12x^4$	$-8x^3$	$8x^2$	$4x$
$-\frac{1}{2}$	$-6x^3$	$4x^2$	$-4x$	-2

$$12\left(\frac{1}{2}\right)^3 - 8\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) + 4 \neq 0$$

Now we can use the $x = -\frac{1}{3}$ rational zero.

Example 4

$$12x^3 - 8x^2 + 8x + 4$$

x	$12x^3$		
$\frac{1}{3}$			

Example 4

$12x^3 - 8x^2 + 8x + 4$

$$\begin{array}{c|c|c|c|} & \color{red}{12x^2} & & \\ \hline x & 12x^3 & & \\ \hline \frac{1}{3} & & & \\ \hline \end{array}$$

Example 4

$$12x^3 - 8x^2 + 8x + 4$$

	$12x^2$		
x	$12x^3$		
$\frac{1}{3}$	$4x^2$		

Example 4

$$12x^3 - 8x^2 + 8x + 4$$

	$12x^2$		
x	$12x^3$	$-12x^2$	
$\frac{1}{3}$	$4x^2$		

Example 4

$$12x^3 - 8x^2 + 8x + 4$$

	$12x^2$	$-12x$	
x	$12x^3$	$-12x^2$	
$\frac{1}{3}$	$4x^2$		

Example 4

$12x^3 - 8x^2 + 8x + 4$

	$12x^2$	$-12x$	
x	$12x^3$	$-12x^2$	
$\frac{1}{3}$	$4x^2$	$-4x$	

Example 4

$$12x^3 - 8x^2 + 8x + 4$$

	$12x^2$	$-12x$	
x	$12x^3$	$-12x^2$	$12x$
$\frac{1}{3}$	$4x^2$	$-4x$	

Example 4

$$12x^3 - 8x^2 + 8x + 4$$

$$\begin{array}{c|c|c|c|}
 & 12x^2 & -12x & 12 \\
\hline
x & 12x^3 & -12x^2 & 12x \\
\hline
\frac{1}{3} & 4x^2 & -4x & \\
\hline
\end{array}$$

Example 4

$$12x^3 - 8x^2 + 8x + 4$$

	$12x^2$	$-12x$	12
x	$12x^3$	$-12x^2$	$12x$
$\frac{1}{3}$	$4x^2$	$-4x$	4

Example 4

$$12x^2 - 12x + 12 = 0$$

Example 4

$$
\begin{aligned}
12x^2 - 12x + 12 &= 0 \\
x^2 - x + 1 &= 0
\end{aligned}
$$

Example 4

$$12x^2 - 12x + 12 = 0$$
$$x^2 - x + 1 = 0$$
$$x = \frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

Example 4

$$12x^2 - 12x + 12 = 0$$
$$x^2 - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

Example 4

$$12x^2 - 12x + 12 = 0$$
$$x^2 - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

Example 4

$$12x^2 - 12x + 12 = 0$$
$$x^2 - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

$$x = \frac{1}{2} \text{ (double root)}, -\frac{1}{3}, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$