

Logarithmic Equations

Objectives

- 1 Solve logarithmic equations

Logarithmic Equations

A **logarithmic equation** is one that involves logarithmic functions.

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General Techniques for Solving Logarithmic Equations

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 - If convenient, express both sides as logs with the same base and equate the arguments of the log functions.
 - Else, rewrite the log equation as an exponential equation.

For Instance

$$\log_2(x) = 3$$

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$$2^3 = x$$

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$$\log_2(x) = 3$$

$$2^3 = x$$

$$x = 8$$

*****Important*****

The domain of $\log_b(x)$ is $x > 0$

Check your answers!!!

Example 1

Solve each. Round your answers to 4 decimal places.

(a) $\log_{117}(1 - 3x) = \log_{117}(x^2 - 3)$

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$$(a) \quad \log_{117}(1 - 3x) = \log_{117}(x^2 - 3)$$

$$1 - 3x = x^2 - 3 \qquad \text{Equality Prop.}$$

$$x^2 + 3x - 4 = 0$$

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Solve each. Round your answers to 4 decimal places.

$$(a) \quad \log_{117}(1 - 3x) = \log_{117}(x^2 - 3)$$

$$1 - 3x = x^2 - 3 \qquad \text{Equality Prop.}$$

$$x^2 + 3x - 4 = 0$$

$$x = -4, 1$$

Example 1

Checking $x = -4$:

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$$\ln(x - 3) = 1$$

$$e^1 = x - 3$$

Write in expon. form

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Write in expon. form

$$x = e + 3$$

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Write in expon. form

$$x = e + 3$$

$$x \approx 5.7183$$

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$$(c) \quad \log_6(x + 4) + \log_6(3 - x) = 1$$

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$$\log_6((x+4)(3-x)) = 1$$

Prod. Property

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Prod. Property

$$\log_6(-x^2 - x + 12) = 1$$

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Prod. Property

$$\log_6(-x^2 - x + 12) = 1$$

$$-x^2 - x + 12 = 6$$

Write in expon. form

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Write in expon. form

$$-x^2 - x + 6 = 0$$

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Write in expon. form

$$-x^2 - x + 6 = 0$$

$$x^2 + x - 6 = 0$$

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Prod. Property

$$\log_6(-x^2 - x + 12) = 1$$

$$-x^2 - x + 12 = 6$$

Write in expon. form

$$-x^2 - x + 6 = 0$$

$$x^2 + x - 6 = 0$$

$$x = -3, 2$$

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Example 1

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Prod. Prop.

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Prod. Prop.

$$\log_7(2x^2 - 7x + 3) = 1$$

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Prod. Prop.

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$$2x^2 - 7x + 3 = 7$$

Write in expon. form

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Prod. Prop.

$$\log_7(2x^2 - 7x + 3) = 1$$

$$2x^2 - 7x + 3 = 7$$

Write in expon. form

$$2x^2 - 7x - 4 = 0$$

Example 1

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$$\log_7(1 - 2x) + \log_7(3 - x) = 1$$

$$\log_7((1 - 2x)(3 - x)) = 1 \quad \text{Prod. Prop.}$$

$$\log_7(2x^2 - 7x + 3) = 1$$

$$2x^2 - 7x + 3 = 7 \quad \text{Write in expon. form}$$

$$2x^2 - 7x - 4 = 0$$

$$x = -\frac{1}{2}, 4$$

Example 1 $\log_7(1 - 2x) = 1 - \log_7(3 - x)$

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$$\log_2(x+3) - \log_2(6-x) = 3$$

$$\log_2 \left(\frac{x+3}{6-x} \right) = 3$$

Quotient Prop.

Example 1

$$(e) \quad \log_2(x+3) = \log_2(6-x) + 3$$

$$\log_2(x+3) - \log_2(6-x) = 3$$

$$\log_2 \left(\frac{x+3}{6-x} \right) = 3$$

Quotient Prop.

$$\frac{x+3}{6-x} = 2^3$$

Write in expon. form

Example 1

$$(e) \quad \log_2(x+3) = \log_2(6-x) + 3$$

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Quotient Prop.

$$\frac{x+3}{6-x} = 2^3$$

Write in expon. form

$$\frac{x+3}{6-x} = 8$$

$$2^3 = 8$$

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Quotient Prop.

$$\frac{x+3}{6-x} = 2^3$$

Write in expon. form

$$\frac{x+3}{6-x} = 8$$

$$2^3 = 8$$

$$x+3 = 8(6-x) \quad \text{Eliminate the fraction}$$

Example 1 $\log_2(x + 3) = \log_2(6 - x) + 3$

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$$\log_4(x + 1) = \frac{\log_2(x + 1)}{\log_2(4)}$$

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$$\begin{aligned} \log_4(x + 1) &= \frac{\log_2(x + 1)}{\log_2(4)} \\ &= \frac{\log_2(x + 1)}{2} \end{aligned}$$

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$$1 + \log_2(x + 1) = \log_2(x^2)$$

$$1 = \log_2(x^2) - \log_2(x + 1)$$

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$$2x + 2 = x^2$$

$$x^2 - 2x - 2 = 0$$

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$$2^1 = \frac{x^2}{x + 1}$$

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$$2x + 2 = x^2$$

$$x^2 - 2x - 2 = 0$$

$$x = 1 \pm \sqrt{3}$$

Example 1 $1 + 2 \log_4(x + 1) = 2 \log_2(x)$

$$x = 1 + \sqrt{3}$$

$$x = 1 - \sqrt{3}$$

Example 1 $1 + 2 \log_4(x + 1) = 2 \log_2(x)$

$$x = 1 + \sqrt{3}$$

$$x = 1 - \sqrt{3}$$

$$x \approx 2.7321$$

$$x \approx -0.7321$$

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