Series

Objectives

1. Expand the terms of a series
2. Write a series using sigma notation
3. Work with arithmetic series
4. Find the sum of a finite geometric series
5. Find the sum of an infinite geometric series

Series

A series is a sequence in which we add the terms.

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Series are written using the explicit rule for a sequence with the Greek letter sigma Σ indicating the terms of the sequence are to be added.

General Form

The general form of a series is given below:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

where

- i is the **index of summation**
- 1 is the **lower limit of summation**
- n is the **upper limit of summation**

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When you write out the terms of the series to add, that is called expanding the series.

Example 1

Expand each of the following. Then find the sum.

(a) $\displaystyle\sum_{k=1}^{4} \frac{2}{3^k}$

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$$k = 4: \quad \frac{2}{81}$$

Sum: $\dfrac{80}{81}$

Example 1

(b) $$\sum_{n=0}^{3} \frac{n!}{2}$$

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Sum:

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$$n = 0: \quad \frac{1}{2}$$

$$n = 1: \quad \frac{1}{2}$$

$$n = 2: \quad 1$$

$$n = 3: \quad 3$$

Sum: 5

Example 1

(c) $$\sum_{n=0}^{3} \frac{x^n}{n!}$$

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$$n = 0 : \quad 1$$

$$n = 1 : \quad x$$

$$n = 2 : \quad \frac{x^2}{2}$$

$$n = 3 : \quad \frac{x^3}{6}$$

Sum:

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

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Writing in Sigma Notation

It can be time-consuming to add a lot of terms of a series. One strategy is to find the sequence rule for the given series and evaluate it in a calculator.

Example 2

Express each sum in sigma notation. Then evaluate. Round to 4 decimal places.

(a) $1 + 3 + 5 + \cdots + 987$

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Lower limit:

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Arithmetic sequence: 1, 3, 5, 7, ..., 987

$$a_n = 2n - 1$$

Lower limit:

$$2n - 1 = 1$$

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Express each sum in sigma notation. Then evaluate. Round to 4 decimal places.

(a) $1 + 3 + 5 + \cdots + 987$

Arithmetic sequence: 1, 3, 5, 7, ..., 987

$$a_n = 2n - 1$$

Lower limit:

$$\begin{aligned} 2n - 1 &= 1 \\ n &= 1 \end{aligned}$$

Example 2

Upper limit:

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$$2n - 1 = 987$$

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$$\begin{aligned} 2n - 1 &= 987 \\ n &= 494 \end{aligned}$$

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$$\sum_{n=1}^{494} (2n - 1)$$

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Upper limit:

$$\begin{aligned} 2n - 1 &= 987 \\ n &= 494 \end{aligned}$$

$$\sum_{n=1}^{494} (2n-1) = 244,036$$

Example 2

(b) $3 + 9 + 27 + \cdots + 387,420,489$

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$$3^n = 3$$

Example 2

(b) $3 + 9 + 27 + \cdots + 387,420,489$

Geometric sequence: 3, 9, 27, 81, ..., 387,420,489

$$a_n = 3^n$$

Lower limit:

$$3^n = 3$$

$$n = 1$$

Example 2

Upper limit:

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$$n \cdot \log_3(3) = \log_3(387,420,489)$$

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$$\sum_{n=1}^{18} 3^n$$

Example 2

Upper limit:

$$3^n = 387,420,489$$

$$n \cdot \log_3(3) = \log_3(387,420,489)$$

$$n = 18$$

$$\sum_{n=1}^{18} 3^n = 581,130,732$$

Example 2

(c) $\quad 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots + \frac{1}{117}$

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Rule: $$\frac{(-1)^{n+1}}{n}$$

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Rule: $\dfrac{(-1)^{n+1}}{n}$

Lower Limit: $n = 1$

Example 2

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$$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots + \frac{1}{117}$$

Rule: $$\frac{(-1)^{n+1}}{n}$$

Lower Limit: $n = 1$ Upper Limit: $n = 117$

Example 2

$$\sum_{n=1}^{117} \frac{(-1)^{n+1}}{n}$$

Example 2

$$\sum_{n=1}^{117} \frac{(-1)^{n+1}}{n} \approx 0.6974$$

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Arithmetic Series

An arithmetic series can be found by adding the terms of the arithmetic sequence.

Example 3

Without using a calculator, find the sum of the first 100 natural numbers.

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$$S = 1 + 2 + 3 + \cdots + 98 + 99 + 100$$

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$$1 \quad + \quad 2 \quad + \quad 3 \quad + \quad \cdots \quad + \quad 98 \quad + \quad 99 \quad + \quad 100$$

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$$S = 1 + 2 + 3 + \cdots + 98 + 99 + 100$$

$$\begin{array}{cccccccccccccc}
 & 1 & + & 2 & + & 3 & + & \cdots & + & 98 & + & 99 & + & 100 \\
+ & 100 & + & 99 & + & 98 & + & \cdots & + & 3 & + & 2 & + & 1 \\
\hline
\end{array}$$

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Without using a calculator, find the sum of the first 100 natural numbers.

$$S = 1 + 2 + 3 + \cdots + 98 + 99 + 100$$

$$\begin{array}{cccccccccccccc} & 1 & + & 2 & + & 3 & + & \cdots & + & 98 & + & 99 & + & 100 \\ + & 100 & + & 99 & + & 98 & + & \cdots & + & 3 & + & 2 & + & 1 \\ \hline & 101 & + & 101 & + & 101 & + & \cdots & + & 101 & + & 101 & + & 101 \end{array}$$

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$$2S = 100(101)$$

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+ & 100 & + & 99 & + & 98 & + & \cdots & + & 3 & + & 2 & + & 1 \\
\hline
 & 101 & + & 101 & + & 101 & + & \cdots & + & 101 & + & 101 & + & 101
\end{array}$$

$$2S = 100(101)$$

$$S = 50(101)$$

Example 3

Without using a calculator, find the sum of the first 100 natural numbers.

$$S = 1 + 2 + 3 + \cdots + 98 + 99 + 100$$

$$\begin{array}{cccccccccccccc}
 & 1 & + & 2 & + & 3 & + & \cdots & + & 98 & + & 99 & + & 100 \\
+ & 100 & + & 99 & + & 98 & + & \cdots & + & 3 & + & 2 & + & 1 \\
\hline
 & 101 & + & 101 & + & 101 & + & \cdots & + & 101 & + & 101 & + & 101
\end{array}$$

$$2S = 100(101)$$

$$S = 50(101) = 5,050$$

General Formula for Arithmetic Series

The method of solving the example above suggests that to find the sum of an arithmetic series, add the first and last terms then multiply that sum by half the number of terms:

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$$S_n = \frac{n}{2}\left(a_1 + a_n\right)$$

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In the next example, you will need to find the number of terms, n, that are being added.

Example 4

Find the value of n in each.

(a) $$\sum_{i=1}^{n}(5i-9)=1,400$$

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First term:

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First term:

$$5(1)-9=-4$$

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(a) $$\sum_{i=1}^{n}(5i-9) = 1,400$$

First term:

$$5(1) - 9 = -4$$

Last term:

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First term:

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Last term:

$$5n-9$$

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$$1400 = \frac{n}{2}\left(-4 + 5n - 9\right)$$

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$$1400 = 2.5n^2 - 6.5n$$

Example 4

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$$1400 = \frac{n}{2}\left(5n - 13\right)$$

$$1400 = 2.5n^2 - 6.5n$$

$$0 = 2.5n^2 - 6.5n - 1400$$

Example 4

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$$n = -22.4,\ 25$$

Example 4

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$$1400 = \frac{n}{2}\left(5n - 13\right)$$

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$$0 = 2.5n^2 - 6.5n - 1400$$

$$n = -22.4,\ 25$$

$$n = 25$$

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(b) $$\sum_{i=1}^{n}(6i - 11) = 2,460$$

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First term:

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First term:

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Last term:

Example 4

Find the value of n in each.

(b) $\displaystyle\sum_{i=1}^{n}(6i - 11) = 2,460$

First term:

$$6(1) - 11 = -5$$

Last term:

$$6n - 11$$

Example 4

$$2460 = \frac{n}{2}\left(-5 + 6n - 11\right)$$

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$$2460 = \frac{n}{2}(-5 + 6n - 11)$$

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$$2460 = 3n^2 - 8n$$

$$0 = 3n^2 - 8n - 2460$$

Example 4

$$2460 = \frac{n}{2}\left(-5 + 6n - 11\right)$$

$$2460 = \frac{n}{2}\left(6n - 16\right)$$

$$2460 = 3n^2 - 8n$$

$$0 = 3n^2 - 8n - 2460$$

$$n = -27.\overline{3},\ 30$$

Example 4

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Find the value of n in each.

(c) $\displaystyle\sum_{i=0}^{n}(1-2i) = -399$ **Be careful with this one

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First term:

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(c) $\displaystyle\sum_{i=0}^{n}(1-2i) = -399$ **Be careful with this one

First term:

$$1-2(0)=1$$

Last term:

Example 4

Find the value of n in each.

(c) $\sum_{i=0}^{n}(1-2i) = -399$ **Be careful with this one

First term:

$$1 - 2(0) = 1$$

Last term:

$$1 - 2n$$

Example 4

$$-399 = \frac{n+1}{2}\left(1+1-2n\right)$$

Example 4

$$-399 = \frac{n+1}{2}\left(1+1-2n\right)$$

$$-399 = \frac{n+1}{2}\left(2-2n\right)$$

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$$-399 = (n+1)(1-n)$$

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$$-399 = (n+1)(1-n)$$

$$-399 = -n^2+1$$

Example 4

$$-399 = \frac{n+1}{2}\left(1+1-2n\right)$$

$$-399 = \frac{n+1}{2}\left(2-2n\right)$$

$$-399 = (n+1)(1-n)$$

$$-399 = -n^2 + 1$$

$$-400 = -n^2$$

Example 4

$$-399 = \frac{n+1}{2}(1+1-2n)$$

$$-399 = \frac{n+1}{2}(2-2n)$$

$$-399 = (n+1)(1-n)$$

$$-399 = -n^2+1$$

$$-400 = -n^2$$

$$400 = n^2$$

Example 4

$$
\begin{aligned}
-399 &= \frac{n+1}{2}\,(1+1-2n) \\
-399 &= \frac{n+1}{2}\,(2-2n) \\
-399 &= (n+1)(1-n) \\
-399 &= -n^2+1 \\
-400 &= -n^2 \\
400 &= n^2 \quad \longrightarrow n=-20,\ 20
\end{aligned}
$$

Example 4

$$-399 = \frac{n+1}{2}(1+1-2n)$$

$$-399 = \frac{n+1}{2}(2-2n)$$

$$-399 = (n+1)(1-n)$$

$$-399 = -n^2+1$$

$$-400 = -n^2$$

$$400 = n^2 \quad \longrightarrow n = -20,\ 20$$

$$n = 20$$

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Geometric Series

The sum, S_n, of the first n terms of a geometric sequence is given by

$$S_n = \frac{(\text{first term})(1 - r^n)}{1 - r}$$

Example 5

(a) $\quad 0.5 + 2.5 + 12.5 + \cdots + 39062.5$

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Common Ratio:

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Common Ratio: $\quad 2.5/0.5 = 5$

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Common Ratio: $\quad 2.5/0.5 = 5$

y-intercept: $0.5/5 = 0.1$

Example 5

(a) $0.5 + 2.5 + 12.5 + \cdots + 39062.5$

Common Ratio: $2.5/0.5 = 5$

y-intercept: $0.5/5 = 0.1$

Rule:

$$0.1(5)^n$$

Example 5

Solving for n:

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$$0.1(5)^n = 39062.5$$

Example 5

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$$0.1(5)^n = 39062.5$$

$$5^n = 390,625$$

Example 5

Solving for n:

$$0.1(5)^n = 39062.5$$

$$5^n = 390,625$$

$$n = \log_5(390,625)$$

Example 5

Solving for n:

$$\begin{aligned} 0.1(5)^n &= 39062.5 \\ 5^n &= 390,625 \\ n &= \log_5(390,625) \\ n &= 8 \end{aligned}$$

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$$\sum_{i=1}^{8} 0.1(5)^i$$

Example 5

Solving for n:

$$\begin{aligned} 0.1(5)^n &= 39062.5 \\ 5^n &= 390,625 \\ n &= \log_5(390,625) \\ n &= 8 \end{aligned}$$

$$\sum_{i=1}^{8} 0.1(5)^i = 48,828$$

Example 5

Write each in sigma notation and find the sum.

(b) $2 + (-8) + 32 + (-128) + \cdots + 33,554,432$

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y-intercept: $-1/2$

Rule:

$$-\frac{1}{2}\left(-4\right)^{n}$$

Example 5

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(-4)^n &= -67,108,864 \\
n &= \log_4(67,108,864)
\end{aligned}
$$

Example 5

Solving for n:

$$-\frac{1}{2}(-4)^n = 33,554,432$$

$$(-4)^n = -67,108,864$$

$$n = \log_4(67,108,864)$$

$$n = 13$$

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$$\sum_{i=1}^{13} \left(-\frac{1}{2}\right)(-4)^i$$

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Solving for n:

$$\begin{aligned} -\frac{1}{2}(-4)^n &= 33{,}554{,}432 \\ (-4)^n &= -67{,}108{,}864 \\ n &= \log_4(67{,}108{,}864) \\ n &= 13 \end{aligned}$$

$$\sum_{i=1}^{13} \left(-\frac{1}{2}\right)(-4)^i = 26{,}843{,}546$$

Objectives

1. Expand the terms of a series
2. Write a series using sigma notation
3. Work with arithmetic series
4. Find the sum of a finite geometric series
5. Find the sum of an infinite geometric series

Formula Issues

The formula

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

works for a finite, or limited, number of terms.

But what happens if we add up an unlimited, or infinite, number of terms?

Infinite Series Formula

We are able to find a sum on the condition that $|r| < 1$, or to put it another way, $-1 < r < 1$.

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The sum of the infinite geometric series becomes

$$S_\infty = \frac{\text{first term}}{1 - r}$$

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An infinite series that has a sum that can be found is said to converge.

A series that does not converge is said to diverge.

Example 6

Find the sum of each infinite series

(a) $\frac{3}{8} - \frac{3}{16} + \frac{3}{32} - \frac{3}{64} + \cdots$

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$$\frac{\frac{-3}{16}}{\frac{3}{8}}$$

Example 6

Find the sum of each infinite series

(a) $\frac{3}{8} - \frac{3}{16} + \frac{3}{32} - \frac{3}{64} + \cdots$

Common ratio:

$$\frac{\frac{-3}{16}}{\frac{3}{8}} = -\frac{1}{2}$$

$$-1 < -\frac{1}{2} < 1$$

Example 6

$$S_\infty = \frac{\text{first term}}{1 - r}$$

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$$= \frac{3/8}{1 - (-1/2)}$$

Example 6

$$S_\infty = \frac{\text{first term}}{1 - r}$$

$$= \frac{3/8}{1 - (-1/2)}$$

$$= \frac{1}{4}$$

Example 6

Find the sum of each infinite series

(b) $\quad 3+2+\frac{4}{3}+\frac{8}{9}+\cdots$

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$$= \frac{3}{1 - (2/3)}$$

$$= 9$$

Example 6

(c) $\displaystyle\sum_{i=1}^{\infty} -3.6(0.6)^{i-1}$

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First Term:

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Common Ratio:

$$0.6$$

$$-1 < 0.6 < 1$$

First Term:

$$-3.6(0.6)^{1-1}$$

Example 6

(c) $\displaystyle\sum_{i=1}^{\infty} -3.6(0.6)^{i-1}$

Common Ratio:

$$0.6$$

$$-1 < 0.6 < 1$$

First Term:

$$-3.6(0.6)^{1-1} = -3.6$$

Example 6

$$S_\infty = \frac{\text{first term}}{1 - r}$$

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$$S_\infty = \frac{\text{first term}}{1 - r}$$

$$= \frac{-3.6}{1 - 0.6}$$

Example 6

$$S_\infty = \frac{\text{first term}}{1 - r}$$

$$= \frac{-3.6}{1 - 0.6}$$

$$= -9$$

Example 6

(d) $$\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$$

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First Term:

$$2\left(\frac{1}{3}\right)^{1-1}$$

Example 6

(d) $\displaystyle\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$

Common Ratio:

$$\frac{1}{3}$$

$$-1 < \frac{1}{3} < 1$$

First Term:

$$2\left(\frac{1}{3}\right)^{1-1} = 2$$

Example 6

$$S_\infty = \frac{\text{first term}}{1 - r}$$

Example 6

$$S_\infty = \frac{\text{first term}}{1 - r}$$

$$= \frac{2}{1 - (1/3)}$$

Example 6

$$S_\infty = \frac{\text{first term}}{1 - r}$$

$$= \frac{2}{1 - (1/3)}$$

$$= 3$$

Example 6

(e) $\displaystyle\sum_{i=1}^{\infty}\left(\frac{4}{3}\right)^{i}$

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Common Ratio:

Example 6

(e) $\displaystyle\sum_{i=1}^{\infty}\left(\frac{4}{3}\right)^{i}$

Common Ratio:

$$\frac{4}{3}$$

Example 6

(e) $\sum_{i=1}^{\infty} \left(\frac{4}{3}\right)^{i}$

Common Ratio:

$$\frac{4}{3}$$

$$\frac{4}{3} > 1$$

Example 6

(e) $\displaystyle\sum_{i=1}^{\infty}\left(\frac{4}{3}\right)^{i}$

Common Ratio:

$$\frac{4}{3}$$

$$\frac{4}{3} > 1$$

Diverges

Example 6

(f) $\displaystyle\sum_{i=1}^{\infty} 0.1(-2.5)^i$

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Common Ratio:

Example 6

(f) $$\sum_{i=1}^{\infty} 0.1(-2.5)^i$$

Common Ratio:

$$-2.5$$

Example 6

(f) $\sum_{i=1}^{\infty} 0.1(-2.5)^{i}$

Common Ratio:

$$-2.5$$

$$-2.5 < -1$$

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Diverges