Functions and Their Graphs

Objectives

1. Determine if a relation is a function.
2. Write a function given a verbal description
3. Evaluate functions using function notation.
4. Find the domain of a function.
5. Find the intercepts of a function

Vocab

A relation is a set of ordered pairs.

The domain is the set of all input values (usually x).

The range is the set of all output values (usually y).

Relations and Functions

A relation is a function if each element in the domain has only 1 corresponding element in the range.

In other words, each x-coordinate has only 1 y-coordinate.

Example 1

Which of the following describes y as a function of x? For the ones that do, state the domain and range.

(a) $\{(-2, 1), (1, 3), (1, 4), (3, -1)\}$

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(a) $\{(-2, 1), (1, 3), (1, 4), (3, -1)\}$

$$\{(-2, 1), ({\color{red}1}, 3), ({\color{red}1}, 4), (3, -1)\}$$

Not a function ($x = 1$ has 2 different y-coordinates)

Example 1

(b) $\{(-2,1),\,(1,3),\,(2,3),\,(3,-1)\}$

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Since all x-coordinates are different, this IS a function.

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Since all x-coordinates are different, this IS a function.

Domain: -2, 1, 2, 3

Range: -1, 1, 3

Vertical Line Test

If you are given the equation, you can graph it and use the Vertical Line Test:

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1. Graph the equation.

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2. Draw vertical lines through the graph.

Vertical Line Test

If you are given the equation, you can graph it and use the Vertical Line Test:

1. Graph the equation.
2. Draw vertical lines through the graph.
3. If each vertical line hits the graph only once (or not at all), it is a function.

Example 2

Determine if each defines y as a function of x.

(a) $x^2 + y = 4$

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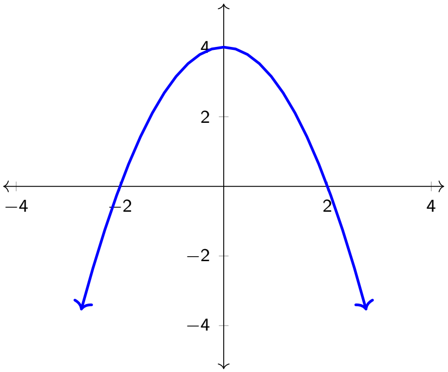
(a) $x^2 + y = 4$



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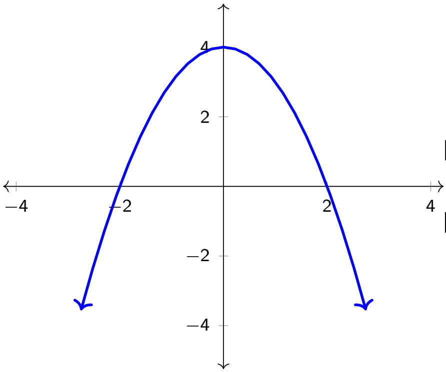
(a) $x^2 + y = 4$



Passes Vertical Line Test

Is a function.

Example 2

(b) $x^2 + y^2 = 4$

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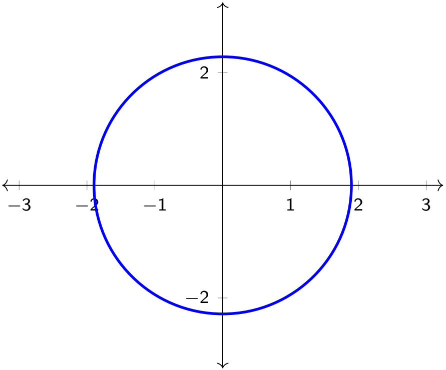
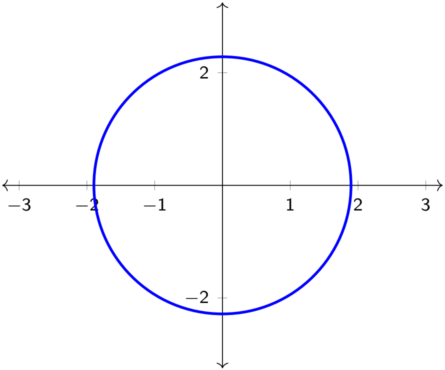


Example 2

(b) $x^2 + y^2 = 4$

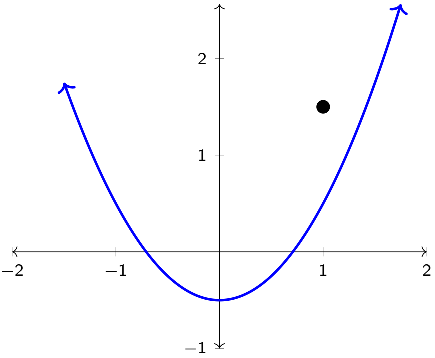


Fails Vertical Line Test

Not a function

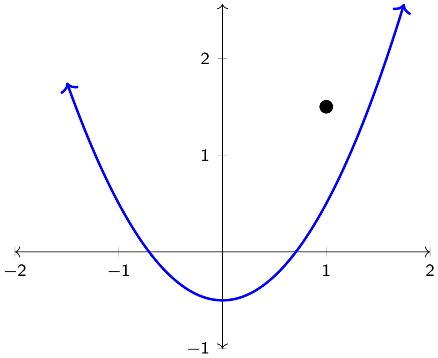
Example 2

(c)



Example 2

(c)

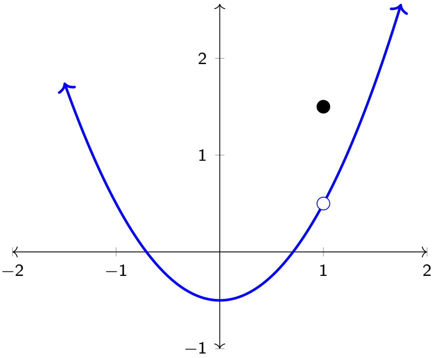


Fails Vertical Line Test

Not a Function

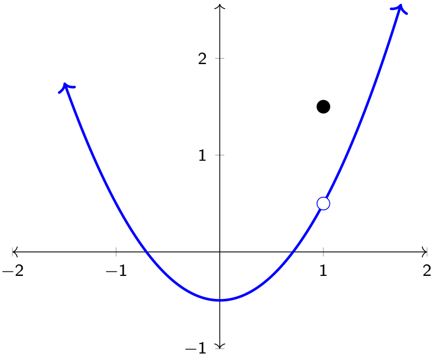
Example 2

(d)



Example 2

(d)



Passes Vertical Line Test

Is a Function

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Verbal Descriptions

Always keep in mind that functions act as a "machine" with a specific job to do.

They take input, such as x, perform their job, and then give you back output, such as y.

The next examples will give you practice in constructing a function based on the steps it performs.

Example 3

(a) Write the function f is described by the following sequential steps.

1. multiply by 3
2. add 4

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1. multiply by 3 $3x$
2. add 4

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(a) Write the function f is described by the following sequential steps.

1. multiply by 3 $3x$
2. add 4 $3x + 4$

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1. multiply by 3 $3x$
2. add 4 $3x + 4$

$$f(x) = 3x + 4$$

Example 3

(b) Write the function g that is described by the following sequential steps:

1. add 4
2. multiply by 3

Example 3

(b) Write the function g that is described by the following sequential steps:

1. add 4 $x + 4$
2. multiply by 3

Example 3

(b) Write the function g that is described by the following sequential steps:

1. add 4 $x + 4$
2. multiply by 3 $3(x + 4)$

Example 3

(b) Write the function g that is described by the following sequential steps:

1. add 4 $x + 4$
2. multiply by 3 $3(x + 4)$

$$g(x) = 3x + 12$$

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Function Notation

We typically use function notation such as $f(x)$ or $g(x)$ when writing functions rather than y.

To evaluate a function, substitute the value in parentheses into the function.

Example 4

For $f(x) = -x^2 + 3x + 4$, evaluate each.

(a) $f(-1)$

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(a) $f(-1)$

$$f(-1) = -(-1)^2 + 3(-1) + 4$$

Example 4

For $f(x) = -x^2 + 3x + 4$, evaluate each.

(a) $f(-1)$

$$\begin{aligned} f(-1) &= -(-1)^2 + 3(-1) + 4 \\ &= -1 - 3 + 4 \end{aligned}$$

Example 4

For $f(x) = -x^2 + 3x + 4$, evaluate each.

(a) $f(-1)$

$$\begin{aligned} f(-1) &= -(-1)^2 + 3(-1) + 4 \\ &= -1 - 3 + 4 \\ &= 0 \end{aligned}$$

Example 4

(b) $f(0)$

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$$f(0) = -(0)^2 + 3(0) + 4$$

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(b) $f(0)$

$$\begin{aligned} f(0) &= -(0)^2 + 3(0) + 4 \\ &= 0 + 0 + 4 \end{aligned}$$

Example 4

(b) $f(0)$

$$\begin{aligned} f(0) &= -(0)^2 + 3(0) + 4 \\ &= 0 + 0 + 4 \\ &= 4 \end{aligned}$$

Example 4

(c) $f(2)$

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$$f(2) = -(2)^2 + 3(2) + 4$$

Example 4

(c) $f(2)$

$$\begin{aligned} f(2) &= -(2)^2 + 3(2) + 4 \\ &= -4 + 6 + 4 \end{aligned}$$

Example 4

(c) $f(2)$

$$\begin{aligned} f(2) &= -(2)^2 + 3(2) + 4 \\ &= -4 + 6 + 4 \\ &= 6 \end{aligned}$$

Example 4

(d) $f(2x)$

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$$f(2x) = -(2x)^2 + 3(2x) + 4$$

Example 4

(d) $f(2x)$

$$\begin{aligned} f(2x) &= -(2x)^2 + 3(2x) + 4 \\ &= -4x^2 + 6x + 4 \end{aligned}$$

Example 4

(e) $f(x+2)$

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$$f(x+2) = -(x+2)^2 + 3(x+2) + 4$$

Example 4

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$$\begin{aligned} f(x+2) &= -(x+2)^2 + 3(x+2) + 4 \\ &= -(x^2 + 4x + 4) + 3x + 6 + 4 \end{aligned}$$

Example 4

(e) $f(x+2)$

$$\begin{aligned} f(x+2) &= -(x+2)^2 + 3(x+2) + 4 \\ &= -(x^2 + 4x + 4) + 3x + 6 + 4 \\ &= -x^2 - 4x - 4 + 3x + 6 + 4 \end{aligned}$$

Example 4

(e) $f(x+2)$

$$\begin{aligned} f(x+2) &= -(x+2)^2 + 3(x+2) + 4 \\ &= -(x^2 + 4x + 4) + 3x + 6 + 4 \\ &= -x^2 - 4x - 4 + 3x + 6 + 4 \\ &= -x^2 - x + 6 \end{aligned}$$

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Domains

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Restricted Domains:

- Taking an even ($\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$, etc.) of a negative number.

Domains

As we progress in the course, we will look at various functions; many having restrictions on what input values they will allow.

Restricted Domains:

- Taking an even ($\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$, etc.) of a negative number.
- Dividing by 0.

Example 5

Find the domain of each.

(a) $g(x) = \sqrt{4 - 3x}$

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$$4 - 3x \geq 0$$

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$$\begin{aligned} 4 - 3x &\geq 0 \\ -3x &\geq -4 \end{aligned}$$

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$$\begin{aligned} 4 - 3x &\geq 0 \\ -3x &\geq -4 \\ x &\leq \frac{4}{3} \end{aligned}$$

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(a) $g(x) = \sqrt{4 - 3x}$

$$\begin{aligned} 4 - 3x &\geq 0 \\ -3x &\geq -4 \\ x &\leq \frac{4}{3} \end{aligned}$$

$$\left(-\infty, \frac{4}{3}\right]$$

Example 5

(b) $h(x) = \sqrt[5]{4 - 3x}$

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$$(-\infty, \infty)$$

Example 5

(c) $$f(x) = \frac{2}{1 - \dfrac{4x}{x-3}}$$

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$$x - 3 \neq 0$$

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$$\begin{aligned} x - 3 &\neq 0 \\ x &\neq 3 \end{aligned}$$

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(c) $$f(x) = \frac{2}{1 - \dfrac{4x}{x-3}} \left(\frac{x-3}{x-3} \right)$$

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$$= \frac{2(x-3)}{x-3-4x}$$

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$$-3x - 3 \neq 0$$

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$$= \frac{2(x-3)}{x - 3 - 4x}$$

$$\begin{aligned} x - 3 - 4x &\neq 0 \\ -3x - 3 &\neq 0 \\ x &\neq -1 \end{aligned}$$

Example 5

(c)

$$x \neq -1, 3$$

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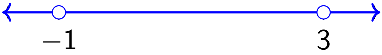
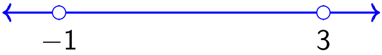
$$x \neq -1, 3$$



Example 5

(c)

$$x \neq -1, 3$$



$$(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$$

Example 5

(d) $$F(x) = \frac{\sqrt[4]{2x+1}}{x^2-1}$$

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$$2x \geq -1$$

$$x \geq -\frac{1}{2}$$

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$$x \neq -1, 1$$

Example 5

$$x \geq -\frac{1}{2}, \quad x \neq -1, 1$$

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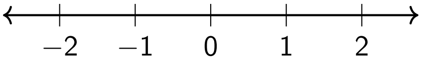

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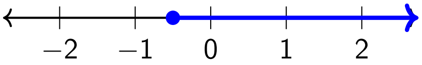
$$x \geq -\frac{1}{2}, \quad x \neq -1, 1$$



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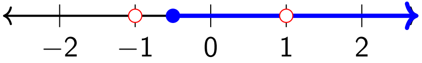
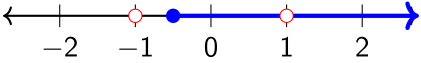


Example 5

$$x \geq -\frac{1}{2}, \quad x \neq -1, 1$$



$$\left[-\frac{1}{2}, 1\right) \cup (1, \infty)$$

Example 5

(e) $$r(t) = \frac{4}{6 - \sqrt{t+3}}$$

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$$6 \neq \sqrt{t+3}$$

$$6^2 \neq (\sqrt{t+3})^2$$

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$$\begin{aligned} 6 - \sqrt{t+3} &\neq 0 \\ 6 &\neq \sqrt{t+3} \\ 6^2 &\neq (\sqrt{t+3})^2 \\ 36 &\neq t+3 \end{aligned}$$

Example 5

(e) $r(t) = \dfrac{4}{6 - \sqrt{t+3}}$

$$
\begin{aligned}
6 - \sqrt{t+3} &\neq 0 \\
6 &\neq \sqrt{t+3} \\
6^2 &\neq (\sqrt{t+3})^2 \\
36 &\neq t+3 \\
t &\neq 33
\end{aligned}
$$

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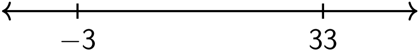

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$$[-3, 33) \cup (33, \infty)$$

Example 5

(f) $\quad I(x) = \dfrac{3x^2}{x}$

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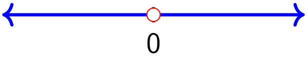

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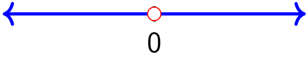
(f) $\quad I(x) = \dfrac{3x^2}{x}$

$$x \neq 0$$



$$(-\infty, 0) \cup (0, \infty)$$

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Intercepts

The x-intercept of a function is where it crosses the x-axis (i.e. the y-coordinate is 0).

The y-intercept of a function is where it crosses the y-axis (i.e. the x-coordinate is 0).

Example 6

Find the intercepts of $f(x) = x^2 + 4x + 3$.

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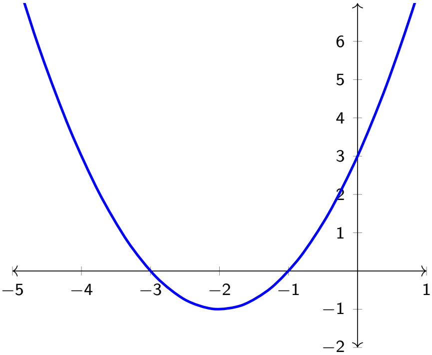
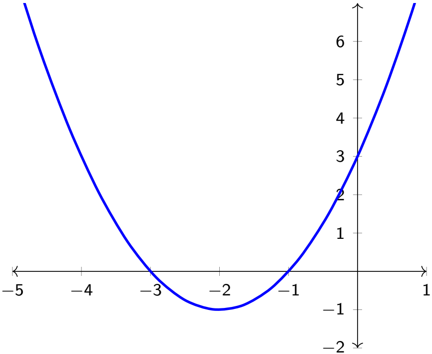


Example 6

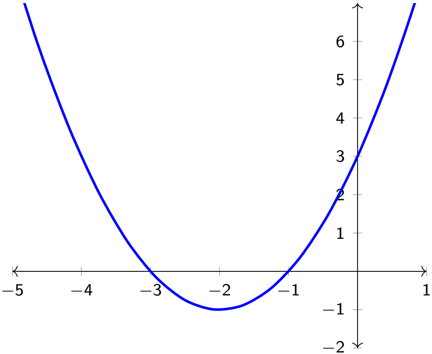
Find the intercepts of $f(x) = x^2 + 4x + 3$.



x-intercepts:

Example 6

Find the intercepts of $f(x) = x^2 + 4x + 3$.

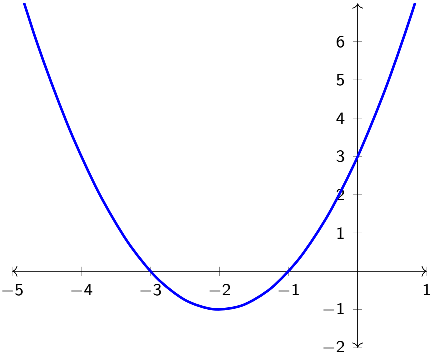


x-intercepts:

$(-3, 0)$ and $(-1, 0)$

Example 6

Find the intercepts of $f(x) = x^2 + 4x + 3$.



x-intercepts:

$(-3, 0)$ and $(-1, 0)$

y-intercept: $(0, 3)$