Properties of Functions

Objectives

1. Determine increasing, decreasing, and constant intervals of a function
2. Determine relative (local) maximum and minimum coordinates
3. Determine if a function is even or odd
4. Evaluate piecewise-defined functions

Increasing Intervals

A function is increasing in an interval if the y-coordinates increase in value as the x-coordinates increase in value.

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- Mathematically, if $a < b$ then $f(a) < f(b)$.

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*****FOCUS ON THE X-COORDINATES*****

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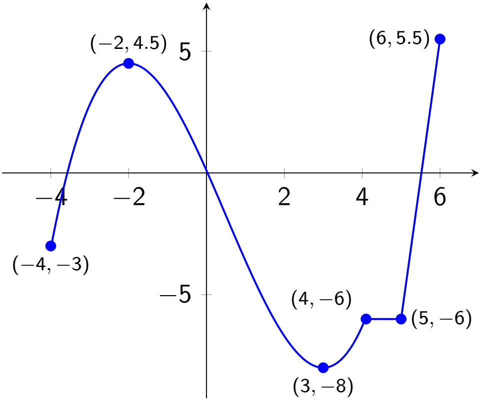
*****FOCUS ON THE X-COORDINATES*****

Example 1

Determine the intervals in which the function is increasing, decreasing, and constant.



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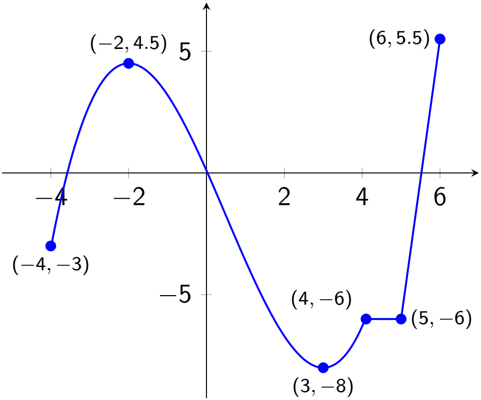

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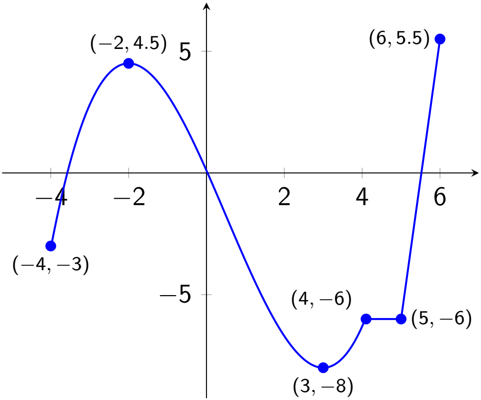

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$(-4,-2) \cup (3,4) \cup (5,6)$

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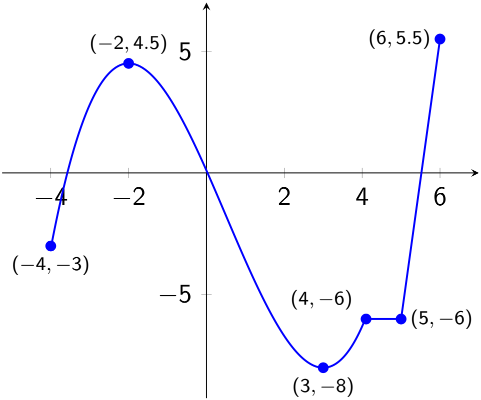

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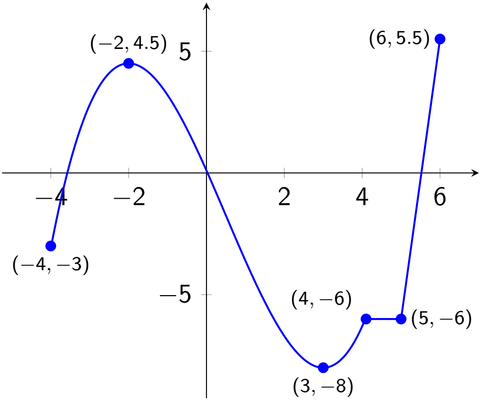

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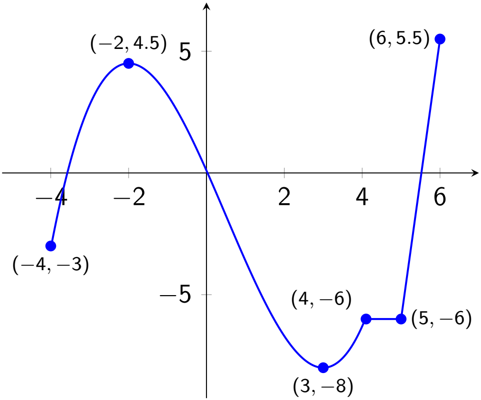

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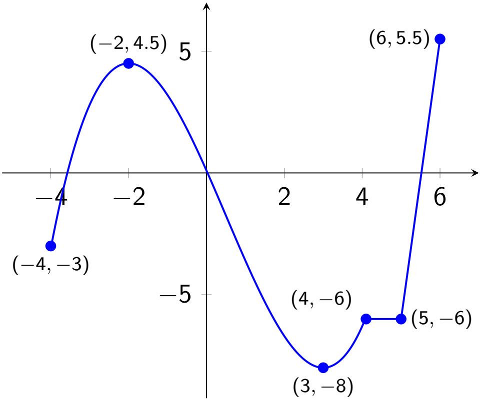

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Constant:

$(4,5)$

Objectives

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Relative Maximum

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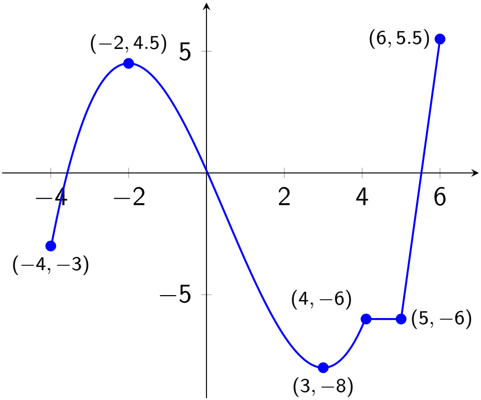
Example 2

Determine the relative minimum and relative maximum for each. Then determine the global minimum and global maximum.



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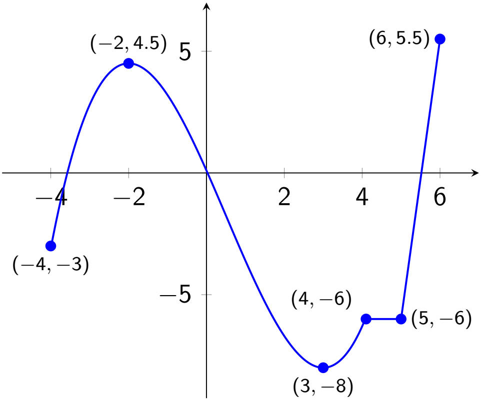

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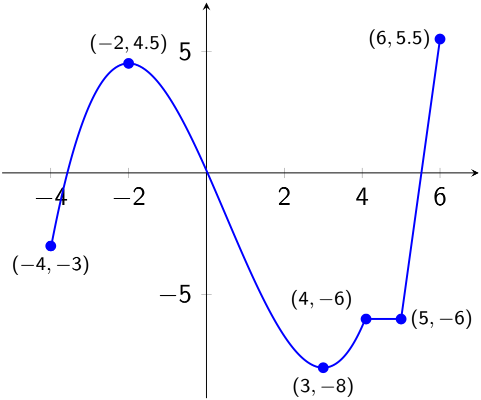

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$(3, -8)$

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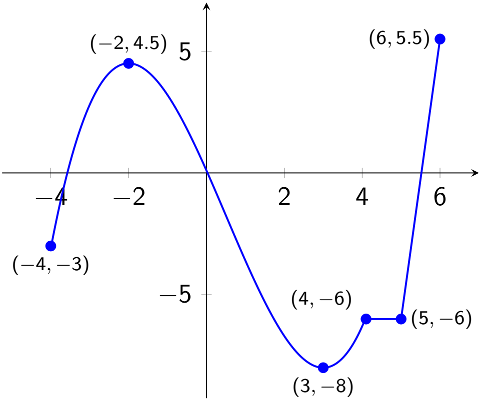

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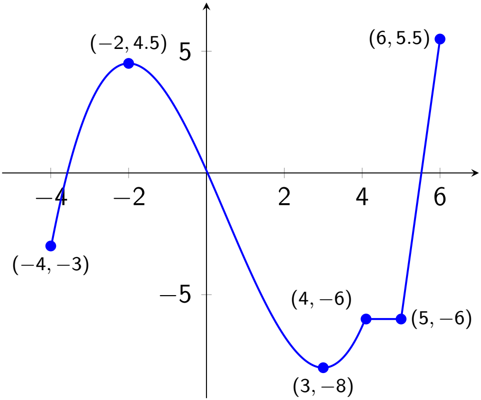

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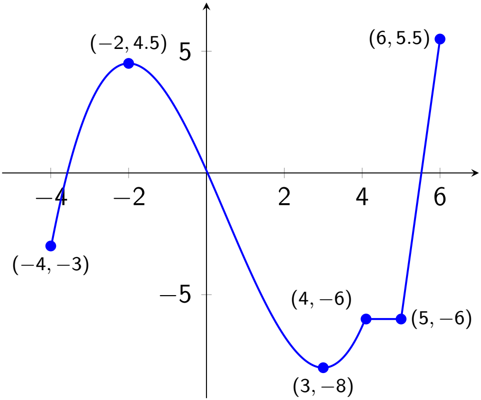

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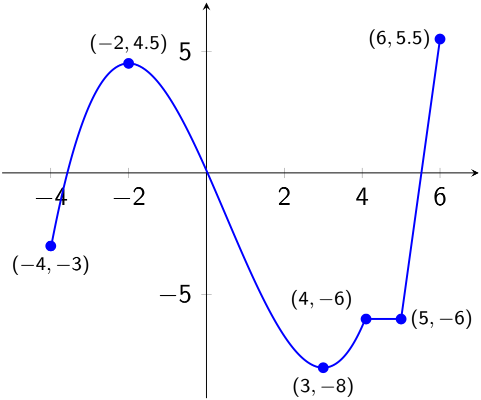

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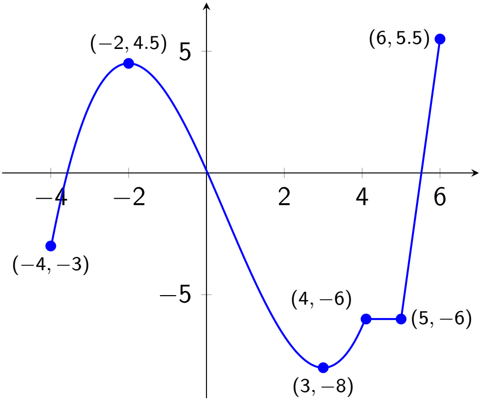

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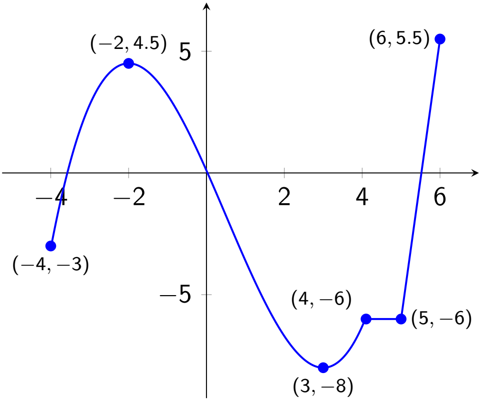

Relative Minimum
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Global Maximum:
$(6, 5.5)$

Objectives

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Even Functions

A function f is even if

$$f(-x) = f(x)$$

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Negative input values give you the same outputs as their positive opposites.

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Even functions are symmetric with respect to the y-axis.

Odd Functions

A function f is odd if

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Odd Functions

A function f is odd if

$$f(-x) = -f(x)$$

Negative input values give you the opposite outputs as their positive opposites.

Odd functions are symmetric with respect to the origin.

Example 3

Determine whether each of the following functions is even, odd, or neither.

(a) $$f(x) = \frac{5}{2 - x^2}$$

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Determine whether each of the following functions is even, odd, or neither.

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$$
\begin{aligned}
f(-x) &= \frac{5}{2-(-x)^2} \\
&= \frac{5}{2-x^2} \\
&= f(x)
\end{aligned}
$$

$f(x) = \dfrac{5}{2 - x^2}$ is even

Example 3

(b) $\quad g(x) = \dfrac{5x}{2 - x^2}$

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(b) $$g(x) = \frac{5x}{2 - x^2}$$

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$$\begin{aligned} g(-x) &= \frac{5(-x)}{2 - (-x)^2} \\ &= \frac{-5x}{2 - x^2} \end{aligned}$$

Example 3

(b) $g(x) = \dfrac{5x}{2 - x^2}$

$$\begin{aligned} g(-x) &= \frac{5(-x)}{2-(-x)^2} \\ &= \frac{-5x}{2-x^2} \\ &= -\left(\frac{5x}{2-x^2}\right) \end{aligned}$$

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(b) $g(x) = \dfrac{5x}{2 - x^2}$

$$\begin{aligned} g(-x) &= \frac{5(-x)}{2 - (-x)^2} \\ &= \frac{-5x}{2 - x^2} \\ &= -\left(\frac{5x}{2 - x^2}\right) \\ &= -g(x) \end{aligned}$$

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$g(x) = \dfrac{5x}{2 - x^2}$ is odd

Example 3

(c) $h(x) = \dfrac{5x}{2 - x^3}$

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(c) $h(x) = \dfrac{5x}{2 - x^3}$

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$$= \frac{-5x}{2 + x^3}$$

Example 3

(c) $\quad h(x) = \dfrac{5x}{2 - x^3}$

$$
\begin{aligned}
h(-x) &= \frac{5(-x)}{2 - (-x)^3} \\
&= \frac{-5x}{2 + x^3} \\
&= -\left(\frac{5x}{2 + x^3}\right)
\end{aligned}
$$

Example 3

(c) $h(x) = \dfrac{5x}{2 - x^3}$

$$h(-x) = \frac{5(-x)}{2 - (-x)^3}$$

$$= \frac{-5x}{2 + x^3}$$

$$= -\left(\frac{5x}{2 + x^3}\right)$$

$h(x) = \dfrac{5x}{2 - x^3}$ is neither odd nor even

Example 3

(d) $\quad i(x) = \dfrac{5x}{2x - x^3}$

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(d) $i(x) = \dfrac{5x}{2x - x^3}$

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(d) $i(x) = \dfrac{5x}{2x - x^3}$

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Example 3

(e) $j(x) = x^2 - \dfrac{x}{100} - 1$

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$$\begin{aligned} j(-x) &= (-x)^2 - \frac{-x}{100} - 1 \\ &= x^2 + \frac{x}{100} - 1 \end{aligned}$$

Example 3

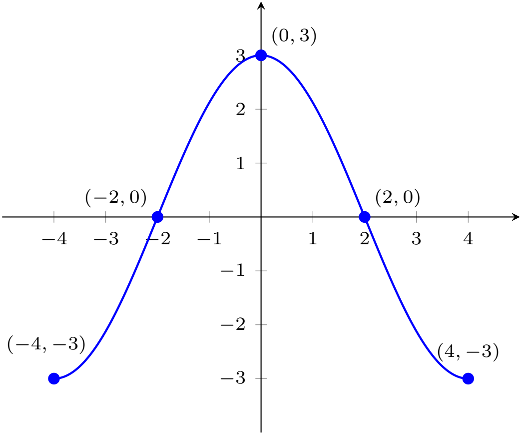
(e) $j(x) = x^2 - \dfrac{x}{100} - 1$

$$\begin{aligned} j(-x) &= (-x)^2 - \frac{-x}{100} - 1 \\ &= x^2 + \frac{x}{100} - 1 \end{aligned}$$

$j(x)$ is neither odd nor even.

Example 4

Given the graph of $y = f(x)$, find each.



(a) Domain of f

Example 4

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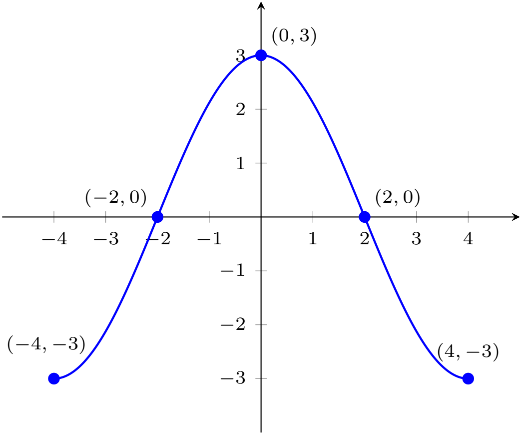

(a) Domain of f

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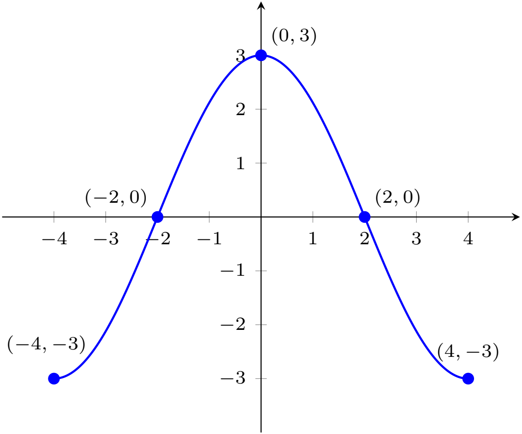

(a) Domain of f

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(b) Range of f

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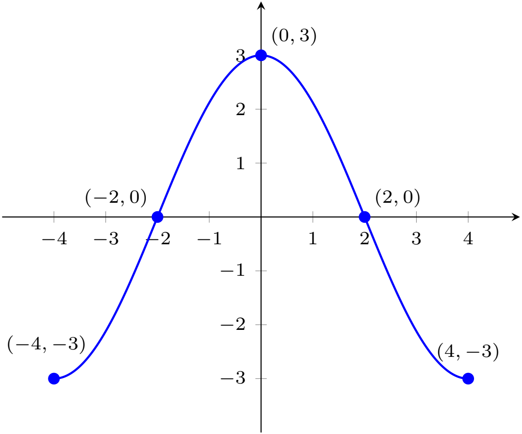

(a) Domain of f

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(b) Range of f

$[-3, 3]$

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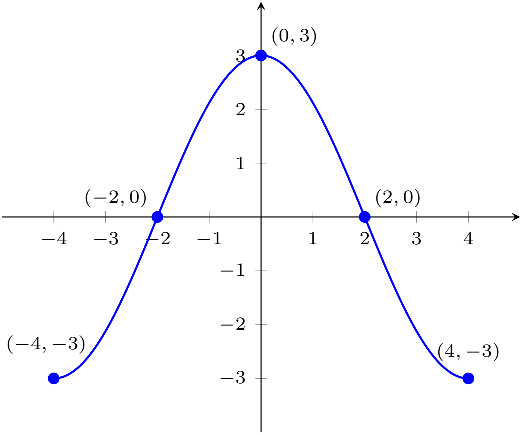

(c) x-intercepts

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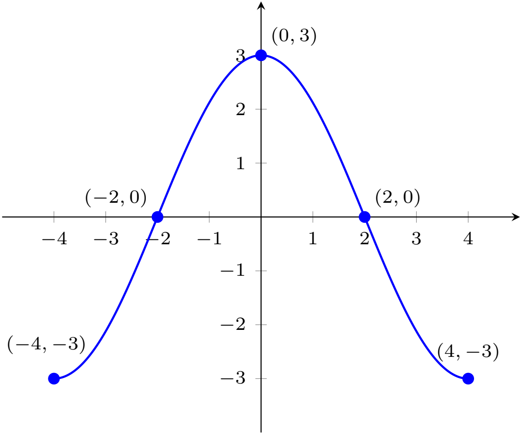

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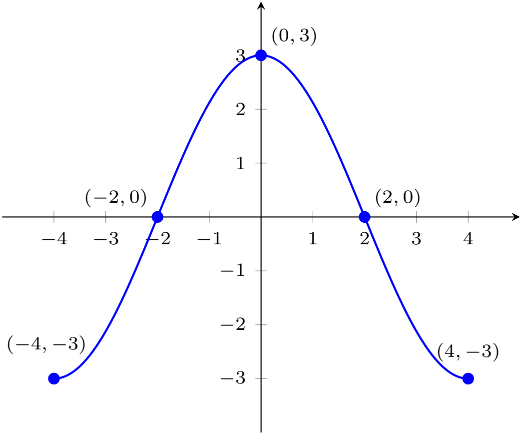

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(d) y-intercept

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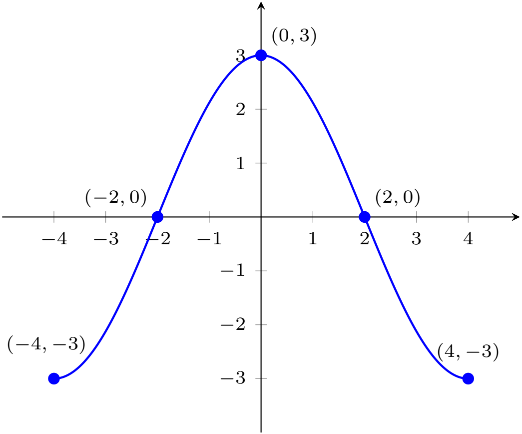

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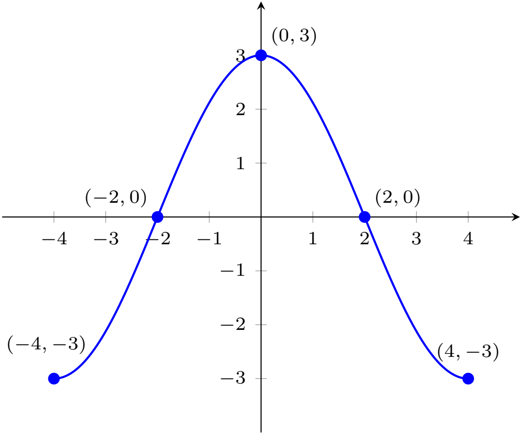

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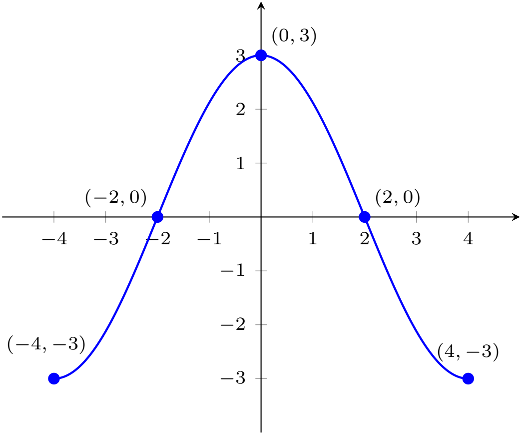

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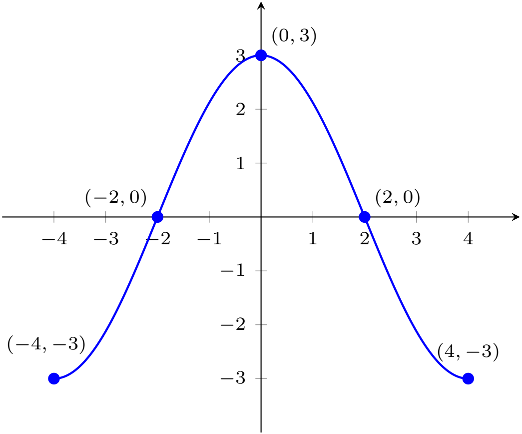

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(f) Solve $f(x) < 0$

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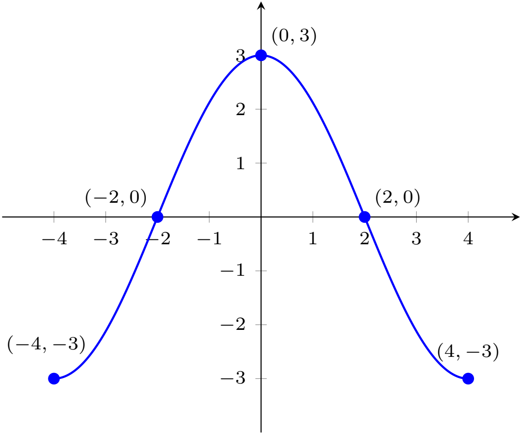


(e) Zeros of f

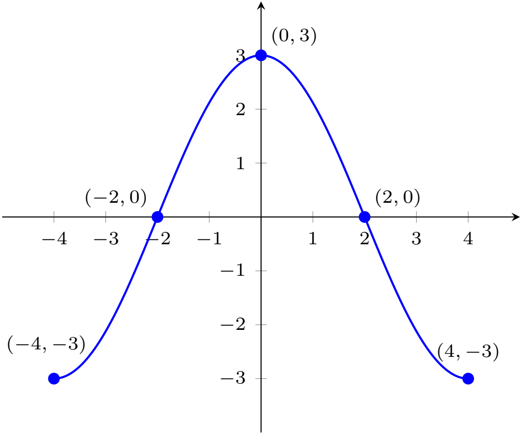
$(-2, 0)$ and $(2, 0)$

(f) Solve $f(x) < 0$

$[-4, -2) \cup (2, 4]$

Example 4

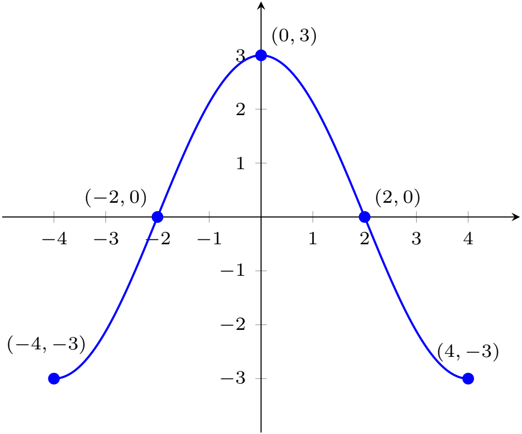
Given the graph of $y = f(x)$, find each.



(g) Determine $f(2)$

Example 4

Given the graph of $y = f(x)$, find each.

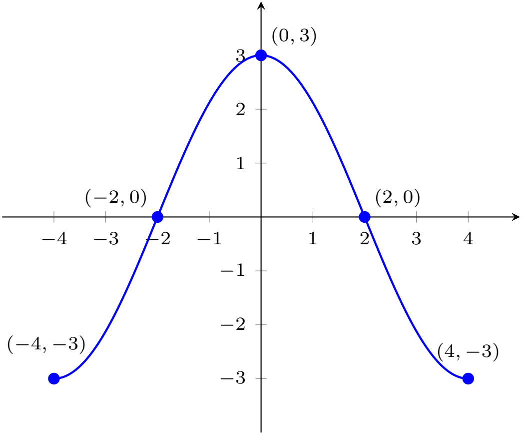


(g) Determine $f(2)$

0

Example 4

Given the graph of $y = f(x)$, find each.



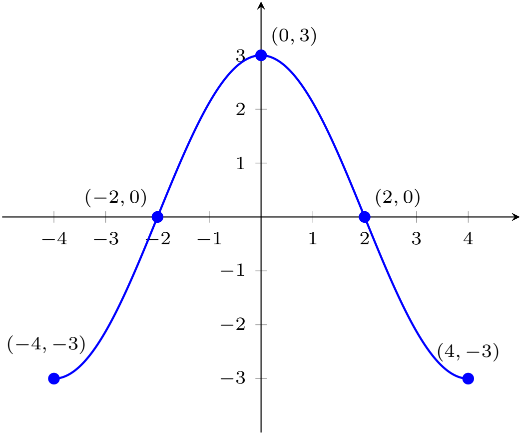
(g) Determine $f(2)$

0

(h) Solve $f(x) = -3$

Example 4

Given the graph of $y = f(x)$, find each.



(g) Determine $f(2)$

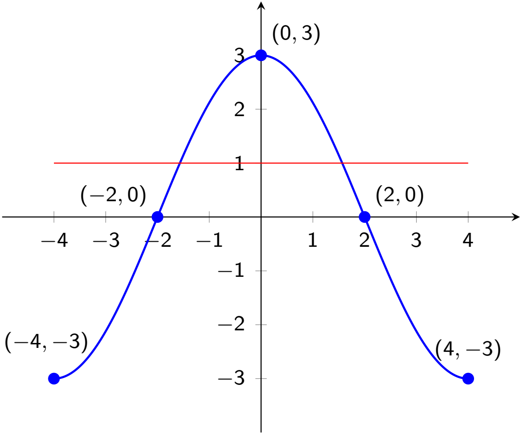
0

(h) Solve $f(x) = -3$

$x = -4$ and $x = 4$

Example 4

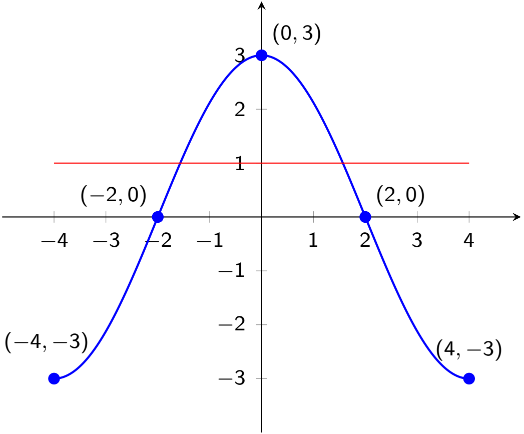
Given the graph of $y = f(x)$, find each.



(i) Number of solutions to $f(x) = 1$

Example 4

Given the graph of $y = f(x)$, find each.



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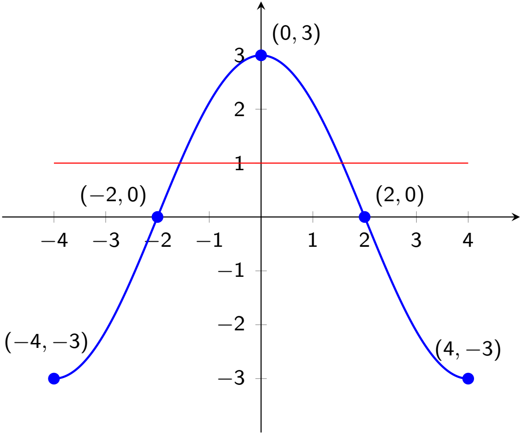
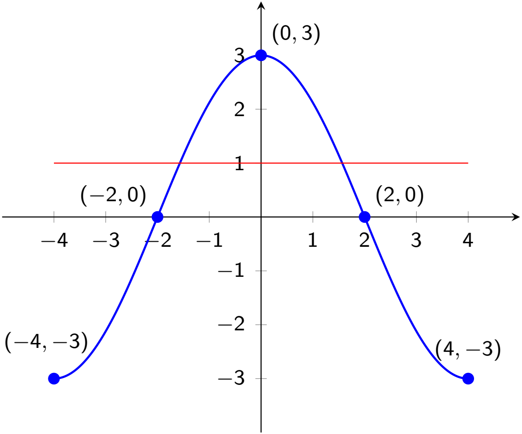


(i) Number of solutions to $f(x) = 1$

2

Example 4

Given the graph of $y = f(x)$, find each.



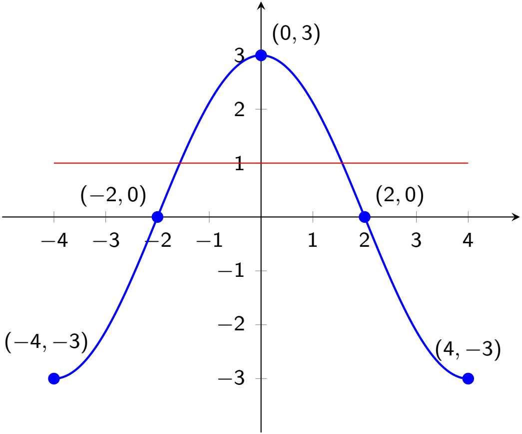
(i) Number of solutions to $f(x) = 1$

2

(j) Does f appear even, odd, or neither?

Example 4

Given the graph of $y = f(x)$, find each.



(i) Number of solutions to $f(x) = 1$

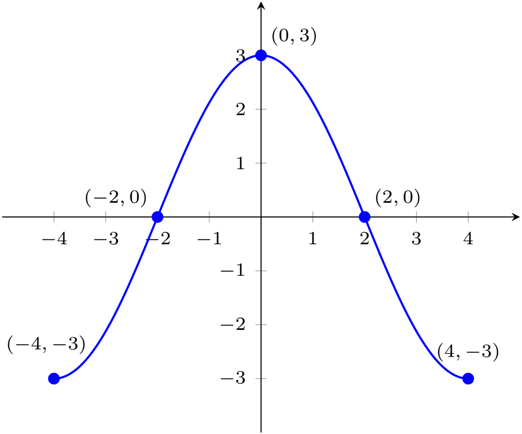
2

(j) Does f appear even, odd, or neither?

even

Example 4

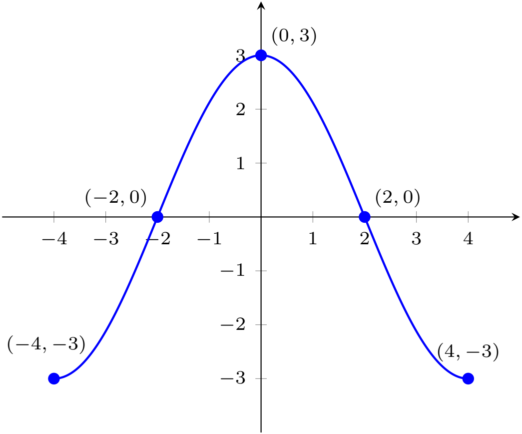
Given the graph of $y = f(x)$, find each.



(k) List intervals of increasing and decreasing.

Example 4

Given the graph of $y = f(x)$, find each.

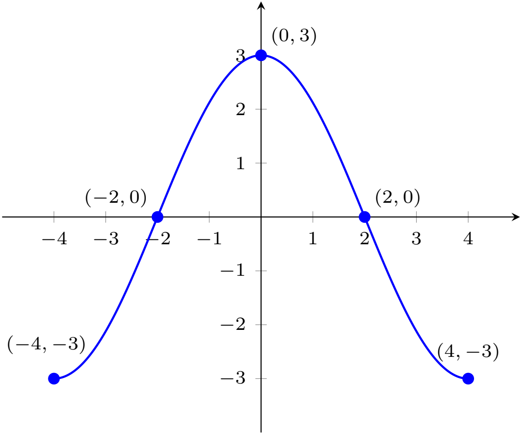


(k) List intervals of increasing and decreasing.

Increasing:

Example 4

Given the graph of $y = f(x)$, find each.



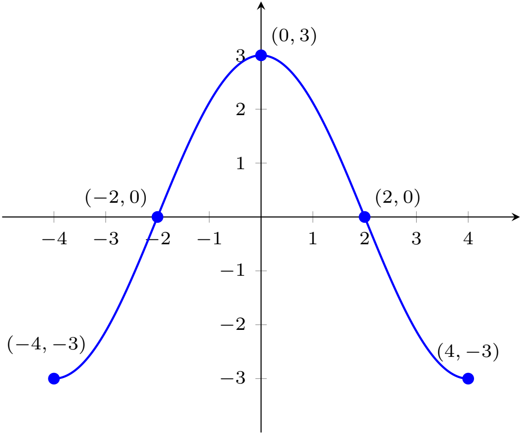
(k) List intervals of increasing and decreasing.

Increasing:

$(-4, 0)$

Example 4

Given the graph of $y = f(x)$, find each.



(k) List intervals of increasing and decreasing.

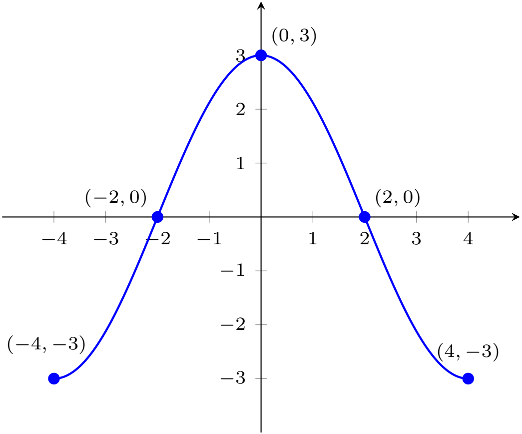
Increasing:

$(-4, 0)$

Decreasing:

Example 4

Given the graph of $y = f(x)$, find each.



(k) List intervals of increasing and decreasing.

Increasing:

$(-4, 0)$

Decreasing:

$(0, 4)$

Objectives

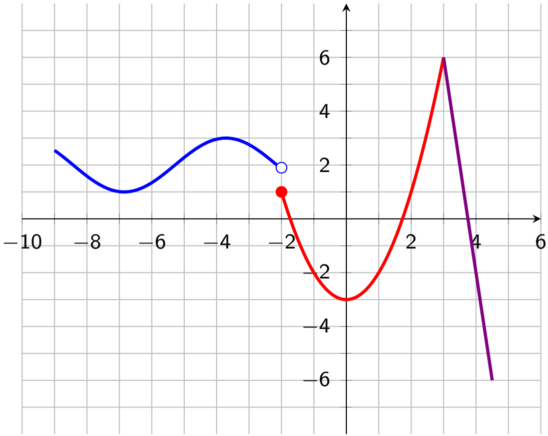
1. Determine increasing, decreasing, and constant intervals of a function
2. Determine relative (local) maximum and minimum coordinates
3. Determine if a function is even or odd
4. Evaluate piecewise-defined functions

Piecewise-Defined Functions

Piecewise-defined functions take pieces of other functions and put them together (a la Frankenstein).

$$f(x) = \begin{cases} \sin(x-1)+2 & \text{if } -9 \leq x < -2 \\ x^2 - 3 & \text{if } -2 \leq x \leq 3 \\ -8x + 30 & \text{if } 3 < x < 4.5 \end{cases}$$

Piecewise-Defined Functions



Piecewise-Defined Functions

When evaluating piecewise-defined functions, pay attention to the domain of each piece.

Example 5

Evaulate each for

(a) $f(-3)$

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

Example 5

Evaulate each for

(a) $f(-3) = -5$

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

Example 5

Evaulate each for

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

(a) $f(-3) = -5$

(b) $f(0)$

Example 5

Evaulate each for

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

(a) $f(-3) = -5$

(b) $f(0) = 4$

Example 5

Evaulate each for

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

(a) $f(-3) = -5$

(b) $f(0) = 4$

(c) $f(2)$

Example 5

Evaulate each for

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

(a) $f(-3) = -5$

(b) $f(0) = 4$

(c) $f(2) = -1$

Example 5

Evaulate each for

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

(a) $f(-3) = -5$

(b) $f(0) = 4$

(c) $f(2) = -1$

(d) $f\left(\dfrac{3}{2}\right)$

Example 5

Evaulate each for

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$$

(a) $f(-3) = -5$

(b) $f(0) = 4$

(c) $f(2) = -1$

(d) $f\left(\dfrac{3}{2}\right) = -\dfrac{3}{2}$