# Absolute Value Inequalities

# Objectives

1. Solve absolute value equations
2. Solve absolute value inequalities

# Solving Absolute Value Equations

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Try to get the absolute value expression alone and break into 2 cases.

# Example 1

Solve $|3x - 1| = 6$

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$$x = \frac{7}{3} \qquad\qquad x = -\frac{5}{3}$$

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2. Solve absolute value inequalities

# General Method of Solving Inequalities

To solve inequalities, we can solve their equation equivalent.

Then we can use test values to determine which values make the original inequality true.

# Example 2

Solve each.

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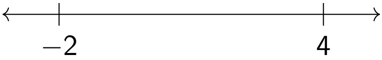

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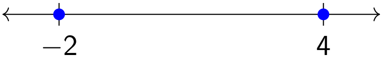

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$$(-\infty, -2] \cup [4, \infty)$$

# Graphical Interpretation of Example 2a

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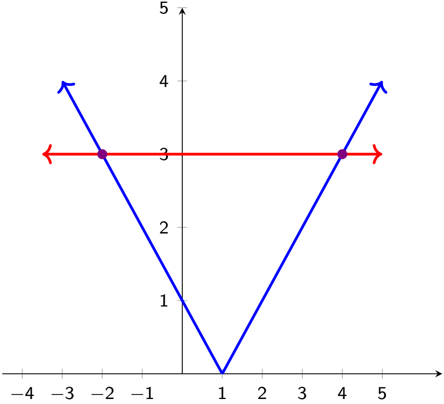

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$$|2x + 1| = 2$$

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(b) $4 - 3|2x + 1| > -2$

$$\begin{aligned} 4 - 3|2x + 1| &= -2 \\ -3|2x + 1| &= -6 \\ |2x + 1| &= 2 \end{aligned}$$

$$2x + 1 = 2 \qquad\qquad 2x + 1 = -2$$

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$$x = \frac{1}{2} \qquad\qquad x = -\frac{3}{2}$$

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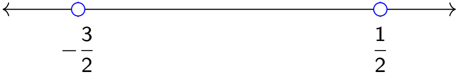

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$$\left(-\frac{3}{2}, \frac{1}{2}\right)$$

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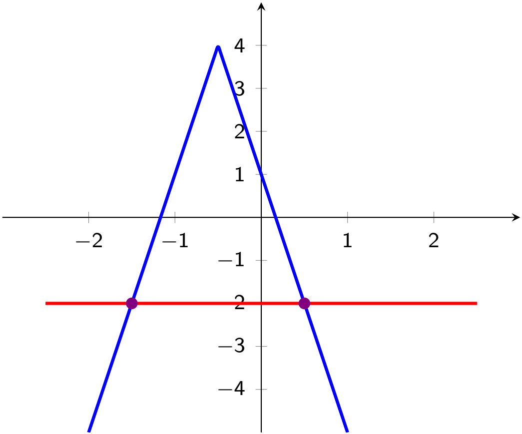

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(c) $\quad 2 < |x-1| \leq 5$

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$$\begin{aligned} x-1 &= -2 \\ x &= -1 \end{aligned} \qquad \begin{aligned} x-1 &= 2 \\ x &= 3 \end{aligned} \qquad \begin{aligned} x-1 &= 5 \\ x &= 6 \end{aligned} \qquad \begin{aligned} x-1 &= -5 \\ x &= -4 \end{aligned}$$

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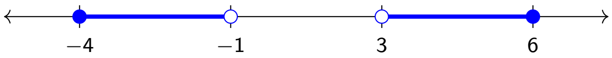
$$\begin{array}{cccc} x-1=-2 & x-1=2 & x-1=5 & x-1=-5 \\ x=-1 & x=3 & x=6 & x=-4 \end{array}$$



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$$[-4, -1) \cup (3, 6]$$

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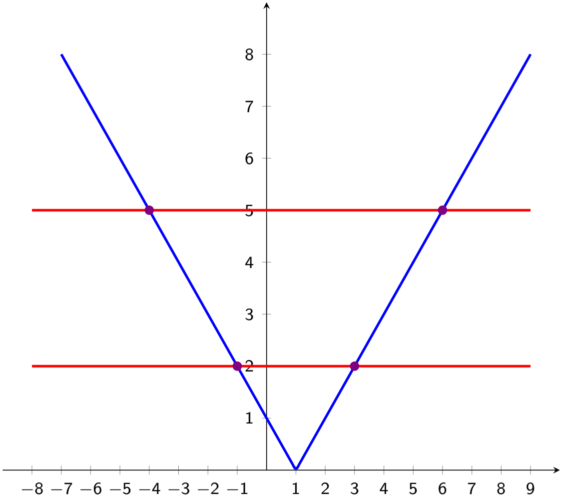

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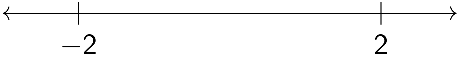

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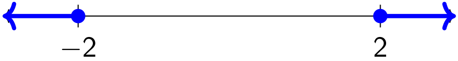

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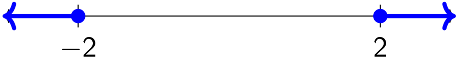


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