Logarithmic Functions

Objectives

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2. Find the domain of logarithmic functions

Logarithms as Inverse Exponents

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It is denoted

$$f^{-1}(x) = \log_b x$$

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The natural logarithm of a real number x is $\log_e x$ and is usually written

$$\ln x$$

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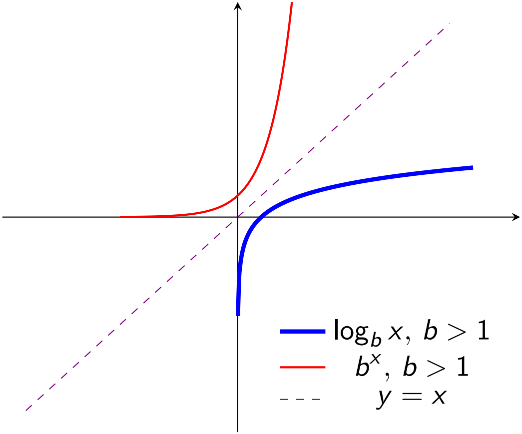
Graphs



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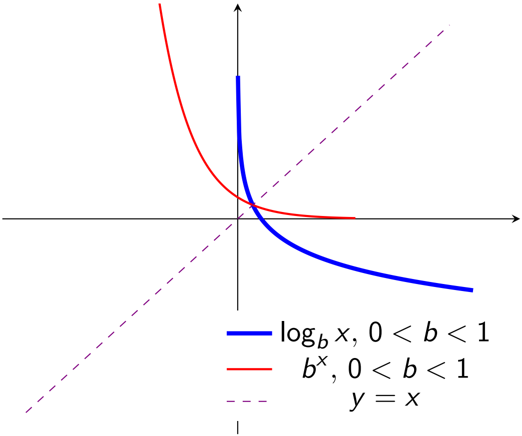

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- $\log_b b^x = x$ for all x and $b^{\log_b x} = x$ for all $x > 0$

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Example 1

Simplify each of the following.

(a) $\log_3 81$

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$$\log_3 81 = 4$$

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(b) $\log_2\left(\frac{1}{8}\right)$

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$$\log_2\left(\frac{1}{8}\right) = -3$$

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(c) $\log_{\sqrt{5}} 25$

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$$= \frac{1}{6}$$

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Up until now, the only domain restrictions we have had have been

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Example 2

Find the domain of each. Write your answer in interval notation.

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$$(-\infty, 3)$$

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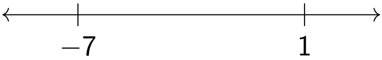

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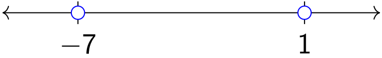

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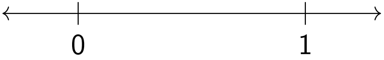

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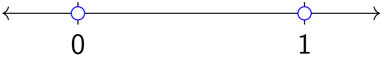

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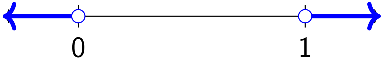

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