Function Compositions

Objectives

1. Find compositions of functions and state their domain

Idea of Function Composition

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The **composition of a function f and g**, denoted $(f \circ g)(x)$ is

$$(f \circ g)(x) = f(g(x))$$

where we plug $g(x)$ into the variable for $f(x)$.

Illustrative Example

For instance the diagram below shows the result of $(f \circ g)(8)$, in which $g(x) = 2x + 3$ and $f(x) = x^2$.

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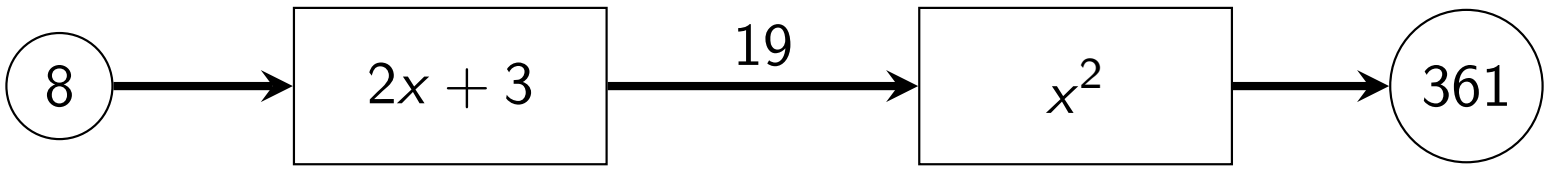

Domain of Composition of Functions

The domain of the compositions of two functions f and g is the domain of the result before simplifying.

Example 1

Given $f(x) = x^2 - 4x$, $g(x) = 2 - \sqrt{x+3}$, and $h(x) = \dfrac{2x}{x+1}$, simplify each and find the domain of the composition.

(a) $(g \circ f)(x)$

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$$[-3, \infty)$$

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Critical values at $x = -1$ and $x = -3/5$

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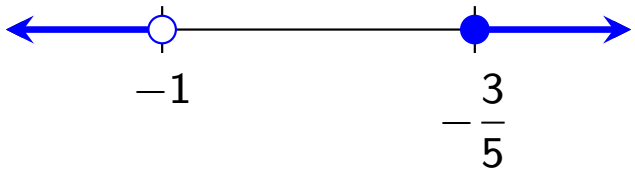

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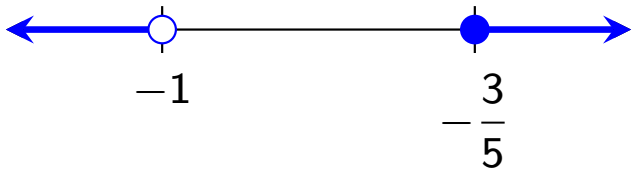

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