Complex Numbers

Objectives

1. Perform arithmetic operations with complex numbers
2. Solve quadratic equations with complex solutions

Intro

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$$x^2 + 1 = 0$$

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$\sqrt{-1}$ is the imaginary unit i.

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- If c is a real number with $c \geq 0$, then $\sqrt{-c} = i\sqrt{c}$

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Adding, subtracting, and multiplying complex numbers is a lot like that of real numbers.

However, keep in mind that $i^2 = -1$.

Example 1

Perform each indicated operation. Write your answers in $a + bi$ form.

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$$= -2 - 6i$$

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$$= x^2 - 2x + 5$$

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Complex conjugates are used to divide complex numbers and find complex solutions to equations.

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$$= \frac{11}{25} - \frac{2}{25}i$$

Properties of Complex Conjugates

Let z and w be complex numbers.

- $\overline{\overline{z}} = z$
- $\overline{z} + \overline{w} = \overline{z + w}$
- $\overline{z} \cdot \overline{w} = \overline{zw}$
- $(\overline{z})^n = \overline{z^n}$ for any natural number n
- z is a real number if and only if $\overline{z} = z$

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Quadratic Equations with Complex Solutions

In the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

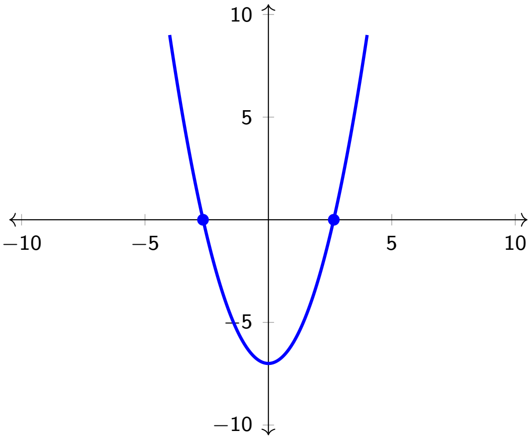
the discriminant

$$b^2 - 4ac$$

tells us what type of solutions we will have.

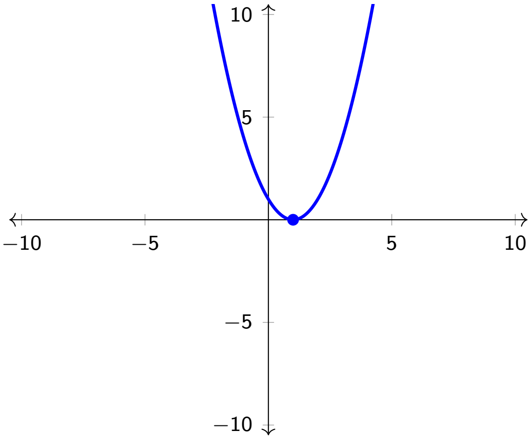
The Discriminant

$b^2 - 4ac > 0$ gives us 2 unique real solutions.



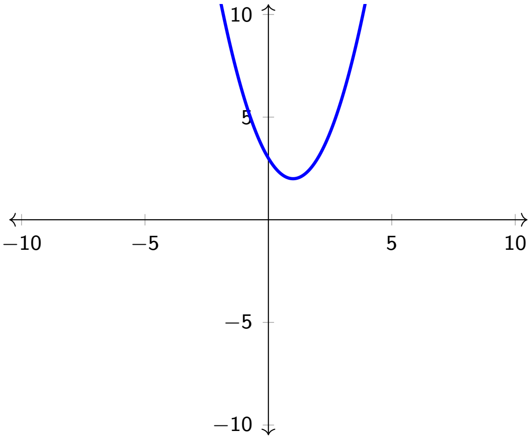
The Discriminant

$b^2 - 4ac = 0$ gives us 1 unique real solution (a double root).



The Discriminant

$b^2 - 4ac < 0$ gives us 2 complex solutions that are *conjugates*.



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$$x = \frac{2 \pm 4i}{2}$$

$$x = 1 \pm 2i$$