

# Exponential Equations

# Objectives

- 1 Solve exponential equations

# Exponential Equations

An **exponential equation** is an equation involving exponential functions.

# Exponential Equations

An **exponential equation** is an equation involving exponential functions.

$$2^x = 128$$

# Exponential Equations

An **exponential equation** is an equation involving exponential functions.

$$2^x = 128$$

$$x = 7$$

# Exponential Equations

An **exponential equation** is an equation involving exponential functions.

$$2^x = 128$$

$$x = 7$$

$$2^x = 129$$

# General Technique for Solving Exponential Equations

# General Technique for Solving Exponential Equations

- Isolate the exponential function



# General Technique for Solving Exponential Equations

- Isolate the exponential function
  - If convenient, express both sides with a common base and equate the exponents.

# General Technique for Solving Exponential Equations

- Isolate the exponential function
  - If convenient, express both sides with a common base and equate the exponents.
  - Else, take the logarithm of both sides; (*recommendation*: Use the same base for logarithm as exponent).

# General Technique for Solving Exponential Equations

- Isolate the exponential function
  - If convenient, express both sides with a common base and equate the exponents.
  - Else, take the logarithm of both sides; (*recommendation*: Use the same base for logarithm as exponent).

$$2^x = 129$$

# General Technique for Solving Exponential Equations

- Isolate the exponential function
  - If convenient, express both sides with a common base and equate the exponents.
  - Else, take the logarithm of both sides; (*recommendation*: Use the same base for logarithm as exponent).

$$2^x = 129$$

$$\log_2(2^x) = \log_2(129)$$

# General Technique for Solving Exponential Equations

- Isolate the exponential function
  - If convenient, express both sides with a common base and equate the exponents.
  - Else, take the logarithm of both sides; (*recommendation*: Use the same base for logarithm as exponent).

$$2^x = 129$$

$$\log_2(2^x) = \log_2(129)$$

$$x = \log_2(129)$$

# General Technique for Solving Exponential Equations

- Isolate the exponential function
  - If convenient, express both sides with a common base and equate the exponents.
  - Else, take the logarithm of both sides; (*recommendation*: Use the same base for logarithm as exponent).

$$2^x = 129$$

$$\log_2(2^x) = \log_2(129)$$

$$x = \log_2(129)$$

$$\approx 7.0112$$

## Example 1

Solve each. Round your answers to 4 decimal places.

(a)  $2^{3x} = 16^{1-x}$

## Example 1

Solve each. Round your answers to 4 decimal places.

(a)  $2^{3x} = 16^{1-x}$

$$2^{3x} = (2^4)^{1-x}$$

$$16 = 2^4$$



## Example 1

Solve each. Round your answers to 4 decimal places.

(a)  $2^{3x} = 16^{1-x}$

$$2^{3x} = (2^4)^{1-x}$$

$$16 = 2^4$$

$$2^{3x} = 2^{4-4x}$$

Power Prop.

## Example 1

Solve each. Round your answers to 4 decimal places.

(a)  $2^{3x} = 16^{1-x}$

$$2^{3x} = (2^4)^{1-x}$$

$$16 = 2^4$$

$$2^{3x} = 2^{4-4x}$$

Power Prop.

$$3x = 4 - 4x$$

Equality Prop.

## Example 1

Solve each. Round your answers to 4 decimal places.

(a)  $2^{3x} = 16^{1-x}$

$$2^{3x} = (2^4)^{1-x}$$

$$16 = 2^4$$

$$2^{3x} = 2^{4-4x}$$

Power Prop.

$$3x = 4 - 4x$$

Equality Prop.

$$7x = 4$$

## Example 1

Solve each. Round your answers to 4 decimal places.

(a)  $2^{3x} = 16^{1-x}$

$$2^{3x} = (2^4)^{1-x}$$

$$16 = 2^4$$

$$2^{3x} = 2^{4-4x}$$

Power Prop.

$$3x = 4 - 4x$$

Equality Prop.

$$7x = 4$$

$$x = \frac{4}{7}$$

## Example 1

Solve each. Round your answers to 4 decimal places.

(a)  $2^{3x} = 16^{1-x}$

$$2^{3x} = (2^4)^{1-x}$$

$$16 = 2^4$$

$$2^{3x} = 2^{4-4x}$$

Power Prop.

$$3x = 4 - 4x$$

Equality Prop.

$$7x = 4$$

$$x = \frac{4}{7}$$

$$\approx 0.5714$$

## Example 1 Alternate Method

$$2^{3x} = 16^{1-x}$$

## Example 1 Alternate Method

$$2^{3x} = 16^{1-x}$$

$$\log_2 (2^{3x}) = \log_2 (16^{1-x})$$

## Example 1 Alternate Method

$$2^{3x} = 16^{1-x}$$

$$\log_2(2^{3x}) = \log_2(16^{1-x})$$

$$3x \cdot \log_2(2) = (1-x) \cdot \log_2(16) \quad \text{Power Prop.}$$



## Example 1 Alternate Method

$$2^{3x} = 16^{1-x}$$

$$\log_2(2^{3x}) = \log_2(16^{1-x})$$

$$3x \cdot \log_2(2) = (1-x) \cdot \log_2(16) \quad \text{Power Prop.}$$

$$3x = (1-x) \cdot 4 \quad \log_2(16) = 4$$

## Example 1 Alternate Method

$$2^{3x} = 16^{1-x}$$

$$\log_2(2^{3x}) = \log_2(16^{1-x})$$

$$3x \cdot \log_2(2) = (1-x) \cdot \log_2(16) \quad \text{Power Prop.}$$

$$3x = (1-x) \cdot 4 \quad \log_2(16) = 4$$

$$3x = 4 - 4x$$

## Example 1 Alternate Method

$$2^{3x} = 16^{1-x}$$

$$\log_2(2^{3x}) = \log_2(16^{1-x})$$

$$3x \cdot \log_2(2) = (1-x) \cdot \log_2(16) \quad \text{Power Prop.}$$

$$3x = (1-x) \cdot 4 \quad \log_2(16) = 4$$

$$3x = 4 - 4x$$

$$x = \frac{4}{7}$$

## Example 1

$$(b) \quad 2000 = 1000 \cdot 3^{-0.1t}$$

## Example 1

$$(b) \quad 2000 = 1000 \cdot 3^{-0.1t}$$

$$2 = 3^{-0.1t}$$

Isolate expon. func.

## Example 1

$$(b) \quad 2000 = 1000 \cdot 3^{-0.1t}$$

$$2 = 3^{-0.1t}$$

Isolate expon. func.

$$\log_3(2) = \log_3(3^{-0.1t})$$

## Example 1

$$(b) \quad 2000 = 1000 \cdot 3^{-0.1t}$$

$$2 = 3^{-0.1t}$$

Isolate expon. func.

$$\log_3(2) = \log_3(3^{-0.1t})$$

$$\log_3(2) = -0.1t \cdot \log_3(3)$$

Power Prop.

## Example 1

$$(b) \quad 2000 = 1000 \cdot 3^{-0.1t}$$

$$2 = 3^{-0.1t}$$

Isolate expon. func.

$$\log_3(2) = \log_3(3^{-0.1t})$$

$$\log_3(2) = -0.1t \cdot \log_3(3)$$

Power Prop.

$$\log_3(2) = -0.1t$$

$$\log_3(3) = 1$$



## Example 1

$$(b) \quad 2000 = 1000 \cdot 3^{-0.1t}$$

$$2 = 3^{-0.1t}$$

Isolate expon. func.

$$\log_3(2) = \log_3(3^{-0.1t})$$

$$\log_3(2) = -0.1t \cdot \log_3(3)$$

Power Prop.

$$\log_3(2) = -0.1t$$

$$\log_3(3) = 1$$

$$t = \frac{\log_3(2)}{-0.1}$$

## Example 1

$$(b) \quad 2000 = 1000 \cdot 3^{-0.1t}$$

$$2 = 3^{-0.1t}$$

Isolate expon. func.

$$\log_3(2) = \log_3(3^{-0.1t})$$

$$\log_3(2) = -0.1t \cdot \log_3(3)$$

Power Prop.

$$\log_3(2) = -0.1t$$

$$\log_3(3) = 1$$

$$t = \frac{\log_3(2)}{-0.1}$$

$$t \approx -6.3093$$

## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

$$3^2 \cdot 3^x = 7^{2x}$$

$$9 = 3^2$$

## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

$$3^2 \cdot 3^x = 7^{2x}$$

$$9 = 3^2$$

$$3^{2+x} = 7^{2x}$$

Product Prop.

## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

$$3^2 \cdot 3^x = 7^{2x}$$

$$9 = 3^2$$

$$3^{2+x} = 7^{2x}$$

Product Prop.

$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

$$3^2 \cdot 3^x = 7^{2x}$$

$$9 = 3^2$$

$$3^{2+x} = 7^{2x}$$

Product Prop.

$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

$$(2+x) \cdot \log_3(3) = 2x \cdot \log_3(7)$$

Power Prop.

## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

$$3^2 \cdot 3^x = 7^{2x}$$

$$9 = 3^2$$

$$3^{2+x} = 7^{2x}$$

Product Prop.

$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

$$(2+x) \cdot \log_3(3) = 2x \cdot \log_3(7)$$

Power Prop.

$$2+x = 2x \cdot \log_3(7)$$

$$\log_3(3) = 1$$



## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

$$3^2 \cdot 3^x = 7^{2x}$$

$$9 = 3^2$$

$$3^{2+x} = 7^{2x}$$

Product Prop.

$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

$$(2+x) \cdot \log_3(3) = 2x \cdot \log_3(7)$$

Power Prop.

$$2+x = 2x \cdot \log_3(7)$$

$$\log_3(3) = 1$$

$$2 = -x + 2x \cdot \log_3(7)$$

Subtract  $x$

## Example 1

$$(c) \quad 9 \cdot 3^x = 7^{2x}$$

$$3^2 \cdot 3^x = 7^{2x}$$

$$9 = 3^2$$

$$3^{2+x} = 7^{2x}$$

Product Prop.

$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

$$(2+x) \cdot \log_3(3) = 2x \cdot \log_3(7)$$

Power Prop.

$$2+x = 2x \cdot \log_3(7)$$

$$\log_3(3) = 1$$

$$2 = -x + 2x \cdot \log_3(7)$$

Subtract  $x$

$$2 = x(-1 + 2 \log_3(7))$$

Factor out  $x$

## Example 1

$$2 = x(-1 + 2\log_3(7))$$

Factor out  $x$

## Example 1

$$2 = x(-1 + 2\log_3(7))$$

Factor out  $x$

$$x = \frac{2}{-1 + 2\log_3(7)}$$

## Example 1

$$2 = x(-1 + 2\log_3(7))$$

Factor out  $x$

$$x = \frac{2}{-1 + 2\log_3(7)}$$

$$x \approx 0.7866$$

## Example 1

$$(d) \quad 75 = \frac{100}{1 + 3e^{-2t}}$$

## Example 1

$$(d) \quad 75 = \frac{100}{1 + 3e^{-2t}}$$

$$75(1 + 3e^{-2t}) = 100$$

Eliminate fraction

## Example 1

$$(d) \quad 75 = \frac{100}{1 + 3e^{-2t}}$$

$$75(1 + 3e^{-2t}) = 100$$

Eliminate fraction

$$1 + 3e^{-2t} = \frac{4}{3}$$

Divide by 75



## Example 1

$$(d) \quad 75 = \frac{100}{1 + 3e^{-2t}}$$

$$75(1 + 3e^{-2t}) = 100$$

Eliminate fraction

$$1 + 3e^{-2t} = \frac{4}{3}$$

Divide by 75

$$3e^{-2t} = \frac{1}{3}$$

Subtract 1

## Example 1

$$(d) \quad 75 = \frac{100}{1 + 3e^{-2t}}$$

$$75(1 + 3e^{-2t}) = 100$$

Eliminate fraction

$$1 + 3e^{-2t} = \frac{4}{3}$$

Divide by 75

$$3e^{-2t} = \frac{1}{3}$$

Subtract 1

$$e^{-2t} = \frac{1}{9}$$

Divide by 3

## Example 1

$$(d) \quad 75 = \frac{100}{1 + 3e^{-2t}}$$

$$75(1 + 3e^{-2t}) = 100$$

Eliminate fraction

$$1 + 3e^{-2t} = \frac{4}{3}$$

Divide by 75

$$3e^{-2t} = \frac{1}{3}$$

Subtract 1

$$e^{-2t} = \frac{1}{9}$$

Divide by 3

$$\ln(e^{-2t}) = \ln\left(\frac{1}{9}\right)$$

## Example 1

$$\ln(e^{-2t}) = \ln\left(\frac{1}{9}\right)$$

## Example 1

$$\ln(e^{-2t}) = \ln\left(\frac{1}{9}\right)$$

$$-2t \cdot \ln e = \ln\left(\frac{1}{9}\right)$$

Power Prop.

## Example 1

$$\ln(e^{-2t}) = \ln\left(\frac{1}{9}\right)$$

$$-2t \cdot \ln e = \ln\left(\frac{1}{9}\right)$$

Power Prop.

$$-2t = \ln\left(\frac{1}{9}\right)$$

$$\ln e = 1$$

## Example 1

$$\ln(e^{-2t}) = \ln\left(\frac{1}{9}\right)$$

$$-2t \cdot \ln e = \ln\left(\frac{1}{9}\right)$$

Power Prop.

$$-2t = \ln\left(\frac{1}{9}\right)$$

$$\ln e = 1$$

$$t = \frac{\ln\left(\frac{1}{9}\right)}{-2}$$

## Example 1

$$\ln(e^{-2t}) = \ln\left(\frac{1}{9}\right)$$

$$-2t \cdot \ln e = \ln\left(\frac{1}{9}\right)$$

Power Prop.

$$-2t = \ln\left(\frac{1}{9}\right)$$

$$\ln e = 1$$

$$t = \frac{\ln\left(\frac{1}{9}\right)}{-2}$$

$$t \approx 1.099$$



## Example 1

$$(e) \quad 25^x = 5^x + 6$$

## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

$$5^{2x} = 5^x + 6$$

Power Prop.

## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

$$5^{2x} = 5^x + 6$$

Power Prop.

$$(5^x)^2 = 5^x + 6$$

$$5^{2x} = (5^x)^2$$

## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

$$5^{2x} = 5^x + 6$$

Power Prop.

$$(5^x)^2 = 5^x + 6$$

$$5^{2x} = (5^x)^2$$

Let  $u = 5^x$

## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

$$5^{2x} = 5^x + 6$$

Power Prop.

$$(5^x)^2 = 5^x + 6$$

$$5^{2x} = (5^x)^2$$

Let  $u = 5^x$

$$u^2 = u + 6$$

Using substitution

## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

$$5^{2x} = 5^x + 6$$

Power Prop.

$$(5^x)^2 = 5^x + 6$$

$$5^{2x} = (5^x)^2$$

Let  $u = 5^x$

$$u^2 = u + 6$$

Using substitution

$$u^2 - u - 6 = 0$$

## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

$$5^{2x} = 5^x + 6$$

Power Prop.

$$(5^x)^2 = 5^x + 6$$

$$5^{2x} = (5^x)^2$$

Let  $u = 5^x$

$$u^2 = u + 6$$

Using substitution

$$u^2 - u - 6 = 0$$

$$u = -2, 3$$



## Example 1

$$(e) \quad 25^x = 5^x + 6$$

$$(5^2)^x = 5^x + 6$$

$$25 = 5^2$$

$$5^{2x} = 5^x + 6$$

Power Prop.

$$(5^x)^2 = 5^x + 6$$

$$5^{2x} = (5^x)^2$$

Let  $u = 5^x$

$$u^2 = u + 6$$

Using substitution

$$u^2 - u - 6 = 0$$

$$u = -2, 3$$

$$5^x = -2 \quad \text{and} \quad 5^x = 3$$

# Example 1

$$5^x = -2$$

$$5^x = 3$$

## Example 1

$$5^x = -2$$

$$x \cdot \log_5(5) = \log_5(-2)$$

$$5^x = 3$$

$$x \cdot \log_5(5) = \log_5(3)$$

## Example 1

$$5^x = -2$$

$$x \cdot \log_5(5) = \log_5(-2)$$

$$x = \log_5(-2)$$

$$5^x = 3$$

$$x \cdot \log_5(5) = \log_5(3)$$

$$x = \log_5(3)$$

## Example 1

$$5^x = -2$$

$$x \cdot \log_5(5) = \log_5(-2)$$

$$x = \log_5(-2)$$

$$x = \emptyset$$

$$5^x = 3$$

$$x \cdot \log_5(5) = \log_5(3)$$

$$x = \log_5(3)$$

$$x \approx 0.6826$$

## Example 1

$$5^x = -2$$

$$x \cdot \log_5(5) = \log_5(-2)$$

$$x = \log_5(-2)$$

$$x = \emptyset$$

$$5^x = 3$$

$$x \cdot \log_5(5) = \log_5(3)$$

$$x = \log_5(3)$$

$$x \approx 0.6826$$

$$x \approx 0.6826$$

## Example 1

$$(f) \quad \frac{e^x - e^{-x}}{2} = 5$$

## Example 1

$$(f) \quad \frac{e^x - e^{-x}}{2} = 5$$

$$e^x - e^{-x} = 10$$

Eliminate fraction



## Example 1

$$(f) \quad \frac{e^x - e^{-x}}{2} = 5$$

$$e^x - e^{-x} = 10$$

Eliminate fraction

$$e^x - \frac{1}{e^x} = 10$$

$$e^{-x} = \frac{1}{e^x}$$

## Example 1

$$(f) \quad \frac{e^x - e^{-x}}{2} = 5$$

$$e^x - e^{-x} = 10$$

Eliminate fraction

$$e^x - \frac{1}{e^x} = 10$$

$$e^{-x} = \frac{1}{e^x}$$

$$e^{2x} - 1 = 10e^x$$

Multiply by  $e^x$

## Example 1

$$(f) \quad \frac{e^x - e^{-x}}{2} = 5$$

$$e^x - e^{-x} = 10$$

Eliminate fraction

$$e^x - \frac{1}{e^x} = 10$$

$$e^{-x} = \frac{1}{e^x}$$

$$e^{2x} - 1 = 10e^x$$

Multiply by  $e^x$

$$e^{2x} - 10e^x - 1 = 0$$

## Example 1

$$(f) \quad \frac{e^x - e^{-x}}{2} = 5$$

$$e^x - e^{-x} = 10$$

Eliminate fraction

$$e^x - \frac{1}{e^x} = 10$$

$$e^{-x} = \frac{1}{e^x}$$

$$e^{2x} - 1 = 10e^x$$

Multiply by  $e^x$

$$e^{2x} - 10e^x - 1 = 0$$

$$\text{Let } u = e^x$$

## Example 1

$$(f) \quad \frac{e^x - e^{-x}}{2} = 5$$

$$e^x - e^{-x} = 10$$

Eliminate fraction

$$e^x - \frac{1}{e^x} = 10$$

$$e^{-x} = \frac{1}{e^x}$$

$$e^{2x} - 1 = 10e^x$$

Multiply by  $e^x$

$$e^{2x} - 10e^x - 1 = 0$$

$$\text{Let } u = e^x$$

$$u^2 - 10u - 1 = 0$$

Example 1    letting  $u = e^x$

$$u^2 - 10u - 1 = 0$$

Example 1    letting  $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

Example 1    letting  $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$



Example 1    letting  $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$

$$u = \frac{10 \pm 2\sqrt{26}}{2}$$

Example 1    letting  $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$

$$u = \frac{10 \pm 2\sqrt{26}}{2}$$

$$u = 5 \pm \sqrt{26}$$

Example 1    letting  $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$

$$u = \frac{10 \pm 2\sqrt{26}}{2}$$

$$u = 5 \pm \sqrt{26}$$

$$e^x = 5 \pm \sqrt{26}$$

## Example 1

$$e^x = 5 + \sqrt{26}$$

$$e^x = 5 - \sqrt{26}$$

## Example 1

$$e^x = 5 + \sqrt{26}$$

$$\ln(e^x) = \ln(5 + \sqrt{26})$$

$$e^x = 5 - \sqrt{26}$$

$$\ln(e^x) = \ln(5 - \sqrt{26})$$

## Example 1

$$e^x = 5 + \sqrt{26}$$

$$e^x = 5 - \sqrt{26}$$

$$\ln(e^x) = \ln(5 + \sqrt{26})$$

$$\ln(e^x) = \ln(5 - \sqrt{26})$$

$$x \ln e = \ln(5 + \sqrt{26})$$

$$x \ln e = \ln(5 - \sqrt{26})$$

## Example 1

$$e^x = 5 + \sqrt{26}$$

$$e^x = 5 - \sqrt{26}$$

$$\ln(e^x) = \ln(5 + \sqrt{26})$$

$$\ln(e^x) = \ln(5 - \sqrt{26})$$

$$x \ln e = \ln(5 + \sqrt{26})$$

$$x \ln e = \ln(5 - \sqrt{26})$$

$$x = \ln(5 + \sqrt{26})$$

$$x = \ln(5 - \sqrt{26})$$

## Example 1

$$e^x = 5 + \sqrt{26}$$

$$\ln(e^x) = \ln(5 + \sqrt{26})$$

$$x \ln e = \ln(5 + \sqrt{26})$$

$$x = \ln(5 + \sqrt{26})$$

$$x \approx 2.312$$

$$e^x = 5 - \sqrt{26}$$

$$\ln(e^x) = \ln(5 - \sqrt{26})$$

$$x \ln e = \ln(5 - \sqrt{26})$$

$$x = \ln(5 - \sqrt{26})$$

$$x = \emptyset$$



## Example 1

$$e^x = 5 + \sqrt{26}$$

$$e^x = 5 - \sqrt{26}$$

$$\ln(e^x) = \ln(5 + \sqrt{26})$$

$$\ln(e^x) = \ln(5 - \sqrt{26})$$

$$x \ln e = \ln(5 + \sqrt{26})$$

$$x \ln e = \ln(5 - \sqrt{26})$$

$$x = \ln(5 + \sqrt{26})$$

$$x = \ln(5 - \sqrt{26})$$

$$x \approx 2.312$$

$$x = \emptyset$$

$$x \approx 2.312$$