Sequences

Objectives

1. List the terms of an explicit sequence.
2. List the terms of a recursive sequence.
3. Understand factorial notation
4. Find terms of an arithmetic sequence
5. Find terms of a geometric sequence

Sequences

An sequence $\{a_n\}$ is a function whose domain is the set of non-negative integers (*i.e.* 0, 1, 2, ...).

We will mostly stick with positive integers (1, 2, 3, ...).

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The function values (**terms**) are $a_1, a_2, a_3, a_4, \ldots, a_n$; or $a_0, a_1, a_2, a_3, \ldots, a_n$ if using a starting value of 0.

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The value $a(n)$ is often written as a_n and is called the n^{th} term of the sequence.

The sequence is denoted by a_n or $\{a_n\}_{n=1}^{\infty}$.

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Note: We can use any variable to name a sequence a_n, b_n, c_n, etc. We can also use any letter we want instead of n.

Example 1

Write the first 4 terms of the sequence whose explicit rule is given.

(a) $a_n = \dfrac{5^{n-1}}{3^n}$

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(b) $b_k = \dfrac{(-1)^k}{2k+1},\ k \geq 0$

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$$b(2) = \frac{(-1)^2}{2(2)+1} = \frac{1}{5} \qquad b(3) = \frac{(-1)^3}{2(3)+1} = -\frac{1}{7}$$

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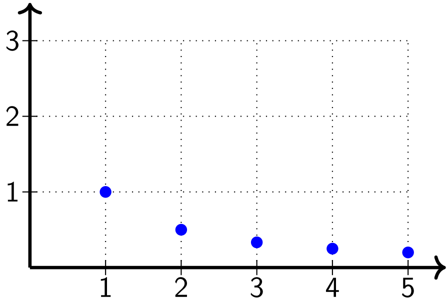
$$c(4) = \frac{1+(-1)^4}{4} = \frac{1}{2} \qquad c(5) = \frac{1+(-1)^5}{5} = 0$$

Graphs of Sequences

We can graph a sequence by using coordinates.

$$a_n = \frac{1}{n}$$



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Recursive Sequences

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You are given a starting term value and then a rule for generating additional terms.

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The notations for successive terms are either

$$a_n \text{ and } a_{n+1}$$

or

$$a_{n-1} \text{ and } a_n$$

Example 2

Write the first four terms of each sequence.

(a) $a_1 = 5$ and $a_n = 3a_{n-1} + 2$ for $n \geq 2$

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$a_3 = 3(17) + 2 = 53$

$a_4 = 3(53) + 2 = 161$

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$f_0 = 1$ $\qquad$ $f_1 = 1(1) = 1$

$f_2 = 2(1) = 2$ $\qquad$ $f_3 = 3(2) = 6$

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Factorial Notation

Products of consecutive positive integers can be expressed using factorial notation.

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If n is a positive integer, then

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

By definition, $0! = 1$.

Factorial Values

Factorial values grow very quickly.

$$
\begin{aligned}
1! &= 1 \\
2! &= 2 \cdot 1 = 2 \\
3! &= 3 \cdot 2 \cdot 1 = 6 \\
4! &= 4 \cdot 3 \cdot 2 \cdot 1 = 24 \\
5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \\
6! &= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720
\end{aligned}
$$

Grouping Symbols

Unless grouping symbols, like () are involved, factorials only affect the number or variable they follow.

$2 \cdot 3! = 2(3 \cdot 2 \cdot 1) = 12$, but $(2 \cdot 3)! = 6! = 720$.

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$$a_3 = \frac{2^3}{(3-1)!} = 4 \qquad a_4 = \frac{2^4}{(4-1)!} = \frac{8}{3}$$

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$$b_3 = \frac{20}{(3+1)!} = \frac{5}{6} \qquad b_4 = \frac{20}{(4+1)!} = \frac{1}{6}$$

Simplifying Factorial Expressions

When simplifying factorial expressions, it helps to remember that

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

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Simplify each.

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$$= \frac{(n+1)\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cancel{\cdots}\cancel{(2)}\cancel{(1)}}{\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}\cancel{\cdots}\cancel{(2)}\cancel{(1)}}$$

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$$= n+1$$

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$$\frac{n!}{(n-1)!} = \frac{n(n-1)(n-2)\cdots(2)(1)}{(n-1)(n-2)\cdots(2)(1)}$$

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$$\frac{n!}{(n-1)!} = \frac{n(n-1)(n-2)\cdots(2)(1)}{(n-1)(n-2)\cdots(2)(1)}$$

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$$= n$$

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Arithmetic Sequences

When the difference between consecutive terms of a sequence is always the same, such as 8, 11, 14, 17, . . . , that sequence is an arithmetic sequence.

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To find the common difference of an arithmetic sequence, **subtract** any two consecutive terms.

Arithmetic Sequences and Linear Functions

Arithmetic sequences model linear functions.

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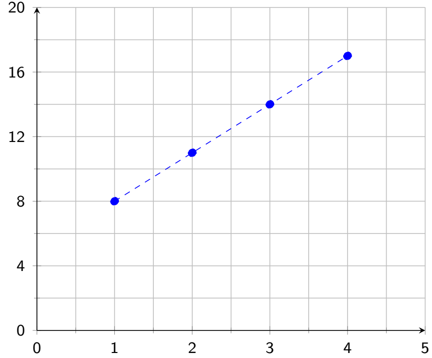
The following graph shows the first 4 terms of the sequence 8, 11, 14, 17:



Find the Rule for the Sequence

Notice that the common difference between terms (3 in this case) is the same as the slope of the line.

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If we subtract that 3 from the first term, we get 5; which is the y-intercept of the graph.

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Notice that the common difference between terms (3 in this case) is the same as the slope of the line.

If we subtract that 3 from the first term, we get 5; which is the y-intercept of the graph.

Thus, a sequence rule for this sequence is

$$y = 3x + 5 \qquad \text{or} \qquad a_n = 3n + 5 \text{ for } n \geq 1$$

Example 5

Find an explicit formula for the n^{th} term of each of the following sequences.

(a) $-31, -24, -17, -10, \ldots$

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y-intercept: $-31 - 7 = -38$

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(a) $-31, -24, -17, -10, \ldots$

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$$a_n = 7n - 38$$

Example 5

(b) $37, 67, 97, 127, \ldots$

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(b) $\quad 37, 67, 97, 127, \ldots$

Common Difference: $67 - 37 = 30$

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(b) 37, 67, 97, 127, $\ldots$

Common Difference: $67 - 37 = 30$

y-intercept: $37 - 30 = 7$

$$b_n = 30n + 7$$

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Geometric Sequences

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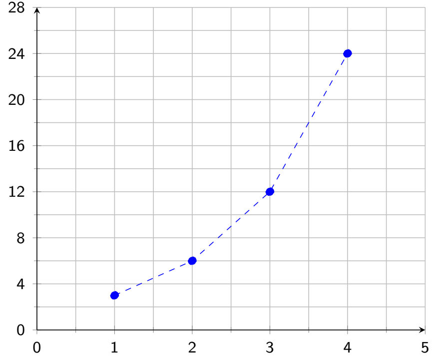
Geometric sequences model exponential growth or decay.

Model of a Geometric Sequence

The following graph shows the first 4 terms of the sequence 3, 6, 12, 18:



Finding Rule for Geometric Sequences

Notice each point's y-coordinate is twice the previous one. This indicates that the common ratio, r, is the base of the exponential function, or 2.

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Thus, the function can be modeled as

$$y = 1.5\left(2\right)^x \qquad \text{or} \qquad a_n = 1.5\left(2\right)^n \text{ for } n \geq 1$$

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Thus, the function can be modeled as

$$y = 1.5\,(2)^x \qquad \text{or} \qquad a_n = 1.5\,(2)^n \text{ for } n \geq 1$$

***Note:* Unlike exponential functions, geometric sequences can have negative common ratios.

Example 6

Find the 8th term of each geometric sequence.

(a) $a_1 = -4 \quad r = -2$

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$$a_8 = -2\left(-2\right)^8$$

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y-intercept: -2

$$a_n = -2\left(-2\right)^n$$

$$a_8 = -2\left(-2\right)^8$$

$$a_8 = -512$$

Example 6

Find the 8th term of each geometric sequence.

(b) $a_1 = 80 \quad r = 0.5$

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(b) $a_1 = 80 \quad r = 0.5$

y-intercept: 160

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Find the 8th term of each geometric sequence.

(b) $a_1 = 80 \quad r = 0.5$

y-intercept: 160

$$a_n = 160\,(0.5)^n$$

Example 6

Find the 8th term of each geometric sequence.

(b) $a_1 = 80 \quad r = 0.5$

y-intercept: 160

$$a_n = 160\,(0.5)^n$$

$$a_8 = 160\,(0.5)^8$$

Example 6

Find the 8th term of each geometric sequence.

(b) $a_1 = 80 \quad r = 0.5$

y-intercept: 160

$$a_n = 160\,(0.5)^n$$

$$a_8 = 160\,(0.5)^8$$

$$a_8 = \frac{5}{8}$$

Example 6

(c) $\quad a_3 = -48 \quad a_5 = -768$

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$$\underline{\qquad\qquad} \qquad \underline{\qquad\qquad} \qquad \underline{\qquad\qquad} \qquad \underset{a_3}{\underline{\;-48\;}} \qquad \underline{\qquad\qquad} \qquad \underset{a_5}{\underline{\;-768\;}}$$

Example 6

(c) $a_3 = -48 \quad a_5 = -768$

$$\begin{tikzcd} \underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underset{a_3}{\underline{-48}} \arrow[r, bend right, "r"'] & \underline{\qquad} \arrow[r, bend right, "r"'] & \underset{a_5}{\underline{-768}} \end{tikzcd}$$

Example 6

(c) $a_3 = -48 \quad a_5 = -768$

$$\begin{tikzcd}
\underline{} & \underline{} & \underline{} & \underset{a_3}{\underline{-48}} \arrow[r, bend right, "r"'] & \underline{} \arrow[r, bend right, "r"'] & \underset{a_5}{\underline{-768}}
\end{tikzcd}$$

$$-48r^2 = -768$$

Example 6

(c) $\quad a_3 = -48 \quad a_5 = -768$

$$\underline{\qquad\quad} \qquad \underline{\qquad\quad} \qquad \underline{\qquad\quad} \qquad \underset{a_3}{\underline{\;-48\;}} \qquad \underline{\qquad\quad} \qquad \underset{a_5}{\underline{-768}}$$

$$r \qquad\qquad r$$

$$\begin{aligned} -48r^2 &= -768 \\ r^2 &= 16 \end{aligned}$$

Example 6

(c) $a_3 = -48 \quad a_5 = -768$

$$\begin{tikzcd}[column sep=small]
\underline{\qquad} & \underline{\qquad} & \underline{\qquad} & \underset{a_3}{\underline{-48}} \arrow[r, bend right, "r"'] & \underline{\qquad} \arrow[r, bend right, "r"'] & \underset{a_5}{\underline{-768}}
\end{tikzcd}$$

$$\begin{aligned} -48r^2 &= -768 \\ r^2 &= 16 \\ r &= \pm 4 \end{aligned}$$

Example 6

(c) $a_3 = -48 \quad a_5 = -768$

$$\begin{tikzcd} \underset{a_0}{\underline{\qquad}} & \underline{\qquad} & \underline{\qquad} & \underset{a_3}{\underline{-48}} \arrow[r, bend right, "r"'] & \underline{\qquad} \arrow[r, bend right, "r"'] & \underset{a_5}{\underline{-768}} \end{tikzcd}$$

$$\begin{aligned} -48r^2 &= -768 \\ r^2 &= 16 \\ r &= \pm 4 \end{aligned}$$

Example 6

(c) $\quad a_3 = -48 \quad a_5 = -768$

$$
\begin{tikzcd}
\underset{a_0}{\underline{\qquad\quad}} \arrow[r, bend right, "\times 4"'] & \underline{\qquad\quad} \arrow[r, bend right, "\times 4"'] & \underline{\qquad\quad} \arrow[r, bend right, "\times 4"'] & \underset{a_3}{\underline{\ -48\ }} \arrow[r, bend right, "r"'] & \underline{\qquad\quad} \arrow[r, bend right, "r"'] & \underset{a_5}{\underline{\ -768\ }}
\end{tikzcd}
$$

$$
\begin{aligned}
-48r^2 &= -768 \\
r^2 &= 16 \\
r &= \pm 4
\end{aligned}
$$

Example 6 $a_3 = -48$ $a_5 = -768$

$$64a_0 = -48$$

Example 6 $a_3 = -48 \quad a_5 = -768$

$$64a_0 = -48$$

$$a_0 = -0.75$$

Example 6 $a_3 = -48 \quad a_5 = -768$

$$64a_0 = -48$$

$$a_0 = -0.75$$

$$a_n = -0.75(4)^n$$

Example 6 $\quad a_3 = -48 \quad a_5 = -768$

$$
\begin{aligned}
64a_0 &= -48 \\
a_0 &= -0.75 \\
a_n &= -0.75(4)^n \\
a_8 &= -0.75(4)^8
\end{aligned}
$$

Example 6 $a_3 = -48$ $a_5 = -768$

$$\begin{aligned} 64a_0 &= -48 \\ a_0 &= -0.75 \\ a_n &= -0.75(4)^n \\ a_8 &= -0.75(4)^8 \\ &= -49,152 \end{aligned}$$

Example 6

(d) $a_1 = 5 \quad a_4 = 16.875$

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$$r^3 = 3.375$$

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$$5r^3 = 16.875$$

$$r^3 = 3.375$$

$$r = 1.5$$

Example 6

(d) $a_1 = 5 \quad a_4 = 16.875$

$$5r^3 = 16.875$$

$$r^3 = 3.375$$

$$r = 1.5$$

$$a_0 = 5/1.5$$

Example 6

(d) $a_1 = 5 \quad a_4 = 16.875$

$$5r^3 = 16.875$$

$$r^3 = 3.375$$

$$r = 1.5$$

$$a_0 = 5/1.5$$

$$a_0 = \frac{10}{3}$$

Example 6

$$a_n = \frac{10}{3}(1.5)^n$$

Example 6

$$a_n = \frac{10}{3}(1.5)^n$$

$$a_8 = \frac{10}{3}(1.5)^8$$

Example 6

$$a_n = \frac{10}{3}(1.5)^n$$

$$a_8 = \frac{10}{3}(1.5)^8$$

$$= \frac{10,935}{128}$$

Example 7

Write an explicit rule for each geometric sequence.

(a) $3, 6, 12, 24, 48, \ldots$

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Common ratio $= 6/3 = 2$

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Common ratio $= 6/3 = 2$

y-intercept $= 1.5$

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Write an explicit rule for each geometric sequence.

(a) $3, 6, 12, 24, 48, \ldots$

Common ratio $= 6/3 = 2$

y-intercept $= 1.5$

$$a_n = 1.5(2)^n$$

Example 7

(b) $2, -10, 50, -250, 1250, \ldots$

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(b) $2, -10, 50, -250, 1250, \ldots$

Common ratio = -10/2 = -5

Example 7

(b) $\quad 2, -10, 50, -250, 1250, \ldots$

Common ratio = -10/2 = -5

y-intercept $= -\frac{2}{5}$

Example 7

(b) $2, -10, 50, -250, 1250, \ldots$

Common ratio = -10/2 = -5

y-intercept $= -\frac{2}{5}$

$$b_n = -\frac{2}{5}\left(-5\right)^n$$