Function Operations

Objectives

1. Perform arithmetic operations to functions
2. Find the domain of the sum, difference, product, or quotient of two functions
3. Find the difference quotient of a function

Notation

We can add, subtract, multiply, and divide functions much like we can with real numbers:

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- $(f-g)(x) = f(x) - g(x)$

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- $(f+g)(x) = f(x) + g(x)$
  - add corresponding y-coordinates
- $(f-g)(x) = f(x) - g(x)$
  - subtract corresponding y-coordinates

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- $(fg)(x) = f(x) \cdot g(x)$
  - multiply corresponding y-coordinates
- $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}, \quad g(x) \neq 0$
  - divide corresponding y-coordinates

Example 1

For $f(x) = 6x^2 - 2x$ and $g(x) = 3 - \dfrac{1}{x}$

(a) Simplify $(f + g)(x)$

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For $f(x) = 6x^2 - 2x$ and $g(x) = 3 - \frac{1}{x}$

(a) Simplify $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

$$= 6x^2 - 2x + 3 - \frac{1}{x}$$

Example 1 $f(x) = 6x^2 - 2x$ and $g(x) = 3 - \dfrac{1}{x}$

(b) Evaluate $(f+g)(-1)$

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$$= 8 + 4$$

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$$= 12$$

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(b) Evaluate $(f+g)(-1)$

$$(f+g)(-1) = f(-1) + g(-1)$$

$$= 8 + 4$$

$$= 12$$

$$(f+g)(-1) = 6(-1)^2 - 2(-1) + 3 - \left(\frac{1}{-1}\right)$$

Example 1 $\quad f(x) = 6x^2 - 2x$ and $g(x) = 3 - \dfrac{1}{x}$

(b) Evaluate $(f+g)(-1)$

$$\begin{aligned}(f+g)(-1) &= f(-1) + g(-1)\\ &= 8 + 4\\ &= 12\end{aligned}$$

$$\begin{aligned}(f+g)(-1) &= 6(-1)^2 - 2(-1) + 3 - \left(\frac{1}{-1}\right)\\ &= 12\end{aligned}$$

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(c) Simplify $(f - g)(x)$

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(c) Simplify $(f - g)(x)$

$$
\begin{aligned}
(f - g)(x) &= f(x) - g(x) \\
&= (6x^2 - 2x) - \left(3 - \frac{1}{x}\right)
\end{aligned}
$$

Example 1 $f(x) = 6x^2 - 2x$ and $g(x) = 3 - \frac{1}{x}$

(c) Simplify $(f - g)(x)$

$$\begin{aligned}
(f - g)(x) &= f(x) - g(x) \\
&= (6x^2 - 2x) - \left(3 - \frac{1}{x}\right) \\
&= 6x^2 - 2x - 3 + \frac{1}{x}
\end{aligned}$$

Example 1 $\quad f(x) = 6x^2 - 2x$ and $g(x) = 3 - \dfrac{1}{x}$

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$$(g - f)(x) = g(x) - f(x)$$

$$= 3 - \frac{1}{x} - (6x^2 - 2x)$$

Example 1 $f(x) = 6x^2 - 2x$ and $g(x) = 3 - \dfrac{1}{x}$

(d) Simplify $(g - f)(x)$

$$
\begin{aligned}
(g - f)(x) &= g(x) - f(x) \\
&= 3 - \frac{1}{x} - (6x^2 - 2x) \\
&= 3 - \frac{1}{x} - 6x^2 + 2x
\end{aligned}
$$

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$$= \left(6x^2 - 2x\right)\left(3 - \frac{1}{x}\right)$$

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$$
\begin{aligned}
(fg)(x) &= f(x) \cdot g(x) \\
&= \left(6x^2 - 2x\right)\left(3 - \frac{1}{x}\right) \\
&= 18x^2 - 6x - 6x + 2
\end{aligned}
$$

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$$= \left(6x^2 - 2x\right)\left(3 - \frac{1}{x}\right)$$

$$= 18x^2 - 6x - 6x + 2$$

$$= 18x^2 - 12x + 2$$

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$$= \frac{6x^2 - 2x}{3 - \frac{1}{x}}$$

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$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{6x^2 - 2x}{3 - \frac{1}{x}}\left(\frac{x}{x}\right)$$

$$= \frac{6x^3 - 2x^2}{3x - 1}$$

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$$= \frac{2x^2(3x - 1)}{3x - 1}$$

$$= 2x^2$$

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Finding Domain

When finding the domain of the sum, difference, product, or quotient of two functions, one method is to analyze the sum/difference/product/quotient <u>before</u> simplifying.

Example 2

(a) Find the domain of $(f+g)(x)$ if $f(x) = 6x^2 - 2x$ and $g(x) = 3 - \dfrac{1}{x}$

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$$x \neq 0$$

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$$(f+g)(x) = 6x^2 - 2x + 3 - \frac{1}{x}$$

$$x \neq 0$$

$$(-\infty, 0) \cup (0, \infty)$$

Example 2

(b) Find the domain of $\left(\frac{f}{g}\right)(x)$ if $f(x) = 6x^2 - 2x$ and $g(x) = 3 - \frac{1}{x}$

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For $g(x)$, $x \neq 0$

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$$3 - \frac{1}{x} \neq 0$$

$$3 \neq \frac{1}{x}$$

$$3x \neq 1$$

$$x \neq \frac{1}{3}$$

Example 2

$$x \neq 0, \frac{1}{3}$$

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$$(-\infty, 0) \cup \left(0, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$$

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Difference Quotient

The difference quotient is fundamental to the idea of the **derivative** in Calculus.

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We will break it down into 3 steps:

1. Evaluate $f(x+h)$

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1. Evaluate $f(x+h)$
2. Subtract original function from that
3. Divide that result by h

Example 3

Find and simplify the difference quotient for each of the following.

(a) $f(x) = x^2 - x - 2$

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(a) $f(x) = x^2 - x - 2$

$$\begin{aligned} f(x+h) &= (x+h)^2 - (x+h) - 2 \\ &= x^2 + 2hx + h^2 - x - h - 2 \end{aligned}$$

Example 3

Find and simplify the difference quotient for each of the following.

(a) $f(x) = x^2 - x - 2$

$$f(x+h) = (x+h)^2 - (x+h) - 2$$

$$= x^2 + 2hx + h^2 - x - h - 2$$

$$f(x+h) - f(x) = x^2 + 2hx + h^2 - x - h - 2 - (x^2 - x - 2)$$

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f(x+h) &= (x+h)^2 - (x+h) - 2 \\
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f(x+h) - f(x) &= x^2 + 2hx + h^2 - x - h - 2 - (x^2 - x - 2) \\
&= x^2 + 2hx + h^2 - x - h - 2 - x^2 + x + 2
\end{aligned}$$

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(a) $f(x) = x^2 - x - 2$

$$
\begin{aligned}
f(x+h) &= (x+h)^2 - (x+h) - 2 \\
&= x^2 + 2hx + h^2 - x - h - 2 \\
f(x+h) - f(x) &= x^2 + 2hx + h^2 - x - h - 2 - (x^2 - x - 2) \\
&= x^2 + 2hx + h^2 - x - h - 2 - x^2 + x + 2 \\
&= 2hx + h^2 - h
\end{aligned}
$$

Example 3

$$\frac{f(x+h)-f(x)}{h}=\frac{2hx+h^2-h}{h}$$

Example 3

$$\frac{f(x+h)-f(x)}{h} = \frac{2hx+h^2-h}{h}$$

$$= 2x+h-1$$

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(b) $g(x) = \dfrac{3}{2x+1}$

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$$\begin{aligned} g(x+h) &= \frac{3}{2(x+h)+1} \\ &= \frac{3}{2x+2h+1} \end{aligned}$$

Example 3

(b) $g(x) = \dfrac{3}{2x+1}$

$$g(x+h) = \frac{3}{2(x+h)+1}$$

$$= \frac{3}{2x+2h+1}$$

$$g(x+h) - g(x) = \frac{3}{2x+2h+1} - \frac{3}{2x+1}$$

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$$g(x+h) - g(x) = \frac{3}{2x+2h+1} - \frac{3}{2x+1}$$

$$\frac{g(x+h)-g(x)}{h} = \frac{\frac{3}{2x+2h+1} - \frac{3}{2x+1}}{h}$$

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$$\frac{g(x+h)-g(x)}{h} = \frac{\dfrac{3}{2x+2h+1} - \dfrac{3}{2x+1}}{h} \left(\frac{(2x+2h+1)(2x+1)}{(2x+2h+1)(2x+1)} \right)$$

Example 3

$$\frac{g(x+h)-g(x)}{h} = \frac{\dfrac{3}{2x+2h+1} - \dfrac{3}{2x+1}}{h} \left(\frac{(2x+2h+1)(2x+1)}{(2x+2h+1)(2x+1)}\right)$$

$$= \frac{3(2x+1)-3(2x+2h+1)}{h(2x+2h+1)(2x+1)}$$

Example 3

$$\frac{g(x+h)-g(x)}{h} = \frac{\dfrac{3}{2x+2h+1} - \dfrac{3}{2x+1}}{h} \left(\frac{(2x+2h+1)(2x+1)}{(2x+2h+1)(2x+1)} \right)$$

$$= \frac{3(2x+1) - 3(2x+2h+1)}{h(2x+2h+1)(2x+1)}$$

$$= \frac{6x+3-6x-6h-3}{h(2x+2h+1)(2x+1)}$$

Example 3

$$\frac{g(x+h)-g(x)}{h} = \frac{\dfrac{3}{2x+2h+1} - \dfrac{3}{2x+1}}{h}\left(\frac{(2x+2h+1)(2x+1)}{(2x+2h+1)(2x+1)}\right)$$

$$= \frac{3(2x+1)-3(2x+2h+1)}{h(2x+2h+1)(2x+1)}$$

$$= \frac{6x+3-6x-6h-3}{h(2x+2h+1)(2x+1)}$$

$$= \frac{-6h}{h(2x+2h+1)(2x+1)}$$

Example 3

$$\frac{g(x+h)-g(x)}{h} = \frac{-6h}{h(2x+2h+1)(2x+1)}$$

Example 3

$$\frac{g(x+h)-g(x)}{h} = \frac{-6h}{h(2x+2h+1)(2x+1)}$$

$$= \frac{-6}{(2x+2h+1)(2x+1)}$$

Example 3

(c) $r(x) = \sqrt{x}$

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$$r(x+h) = \sqrt{x+h}$$

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$$r(x+h) - r(x) = \sqrt{x+h} - \sqrt{x}$$

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$$r(x+h) - r(x) = \sqrt{x+h} - \sqrt{x}$$

$$\frac{r(x+h) - r(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

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$$\frac{r(x+h) - r(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

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$$r(x+h) - r(x) = \sqrt{x+h} - \sqrt{x}$$

$$\frac{r(x+h) - r(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

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(c) $r(x) = \sqrt{x}$

$$r(x+h) = \sqrt{x+h}$$

$$r(x+h) - r(x) = \sqrt{x+h} - \sqrt{x}$$

$$\frac{r(x+h) - r(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{x+h-x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

Example 3

$$\frac{r(x+h)-r(x)}{h} = \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

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$$\frac{r(x+h)-r(x)}{h} = \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

Example 3

$$\begin{aligned}\frac{r(x+h)-r(x)}{h} &= \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \\ &= \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \\ &= \frac{1}{\sqrt{x+h}+\sqrt{x}}\end{aligned}$$