

# Polynomial and Rational Inequalities

# Objectives

1 Solve polynomial inequalities

2 Solve rational inequalities

# Polynomial Inequalities

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Then, set up a number line and **use test values**.

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$$\text{Zeros at } x = -2, -\frac{1}{4}, \frac{5}{2}$$



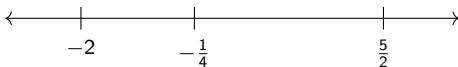
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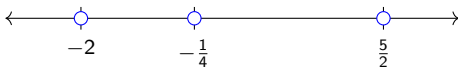
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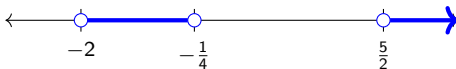
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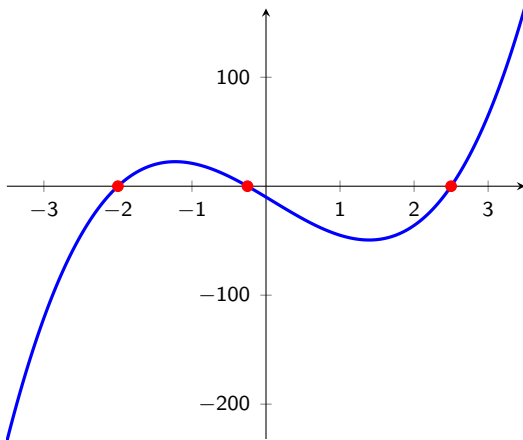
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Zeros at  $x = -2, -\frac{1}{4}, \frac{5}{2}$



## Example 1

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When solving rational inequalities, first be sure that there is a 0 on one side of your inequality. You'll want to simplify the other side (i.e. find a common denominator to combine the fractions.)

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Your **critical values** will be where the **numerator = 0** and the **denominator = 0**

*Note:* The critical values of the **denominator** will always be open circles on the number line.



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Solve each. Write your answers using interval notation.

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$$x = \frac{5}{2}$$

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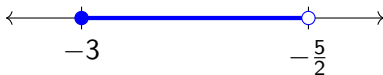
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$$\left[-3, -\frac{5}{2}\right)$$

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$$\frac{-3x+2}{x+1} > 0$$

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$$-3x + 2 = 0$$

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$$-3x + 2 = 0$$

$$x = \frac{2}{3}$$

$$x + 1 = 0$$

$$x = -1$$



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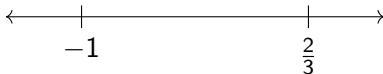
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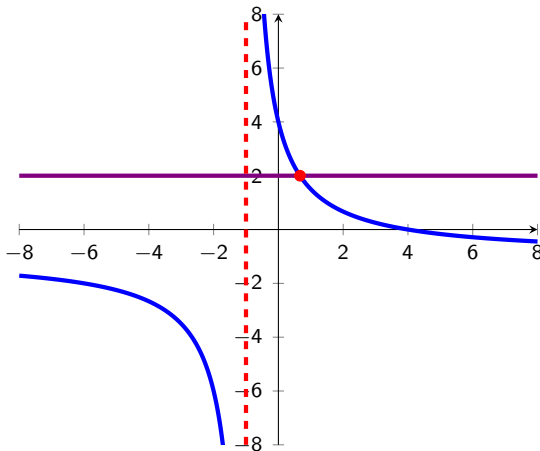
$$x = -1$$



$$\left(-1, \frac{2}{3}\right)$$

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$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$



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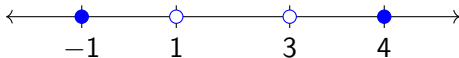
$$x - 1 = 0$$

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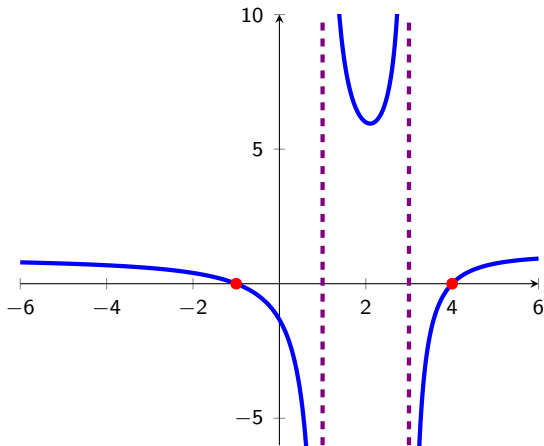
$$x = 1$$



$$(-\infty, -1] \cup (1, 3) \cup [4, \infty)$$

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$$x = \frac{1}{2}, 4$$

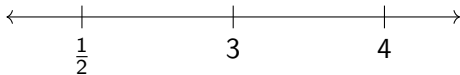
Example 2  $\frac{x+2}{x-3} < 2x - 2$

Critical values:  $x = \frac{1}{2}, 3, 4$



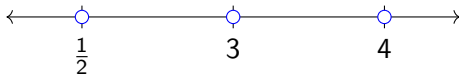
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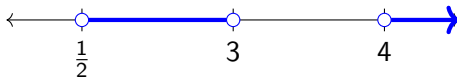
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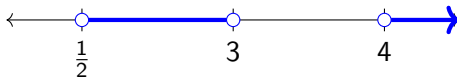
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$$\left(\frac{1}{2}, 3\right) \cup (4, \infty)$$