

# Graphs of Quadratic Expressions

# Objectives

- 1 Determine the vertex and axis of symmetry of a quadratic function in standard form
- 2 Convert between standard and general form of quadratic expressions

# Graphing Quadratic Expressions

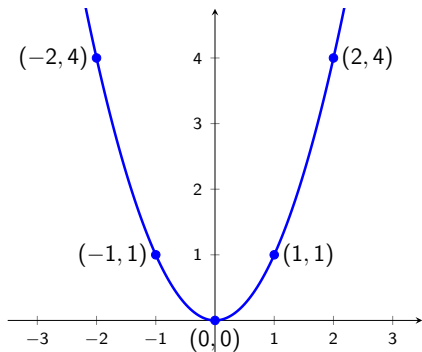
We will be looking at graphing equations in the form

$$y = ax^2 + bx + c$$

where  $a$ ,  $b$ , and  $c$  are real numbers with  $a \neq 0$ .

# Graph of a Quadratic Expression

For  $y = x^2$ , the graph below is a **parabola**.



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The point  $(0, 0)$  is called the **vertex** of the parabola and can be either a minimum (smile) or maximum point (frown).

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Through the vertex is a vertical line called the **axis of symmetry** that divides the parabola into 2 equal halves.

## Example 1

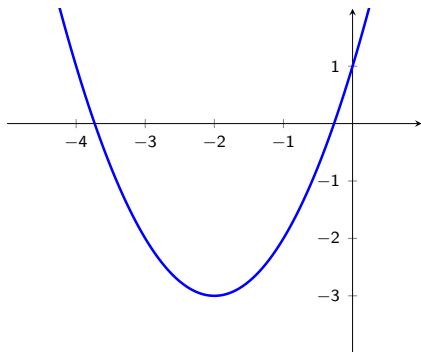
Find the vertex, state whether the vertex is a maximum or minimum, and find the equation of the axis of symmetry for each.

(a)  $y = x^2 + 4x + 1$

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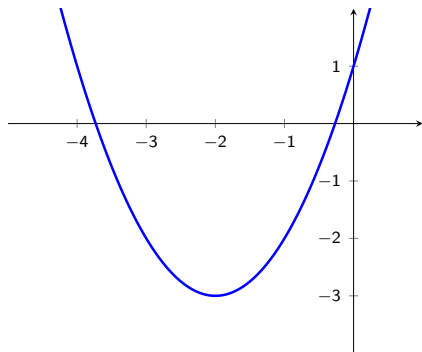




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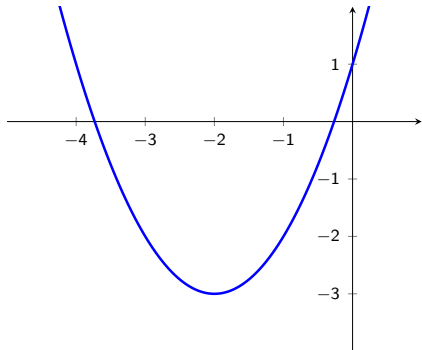


Vertex  $(-2, -3)$

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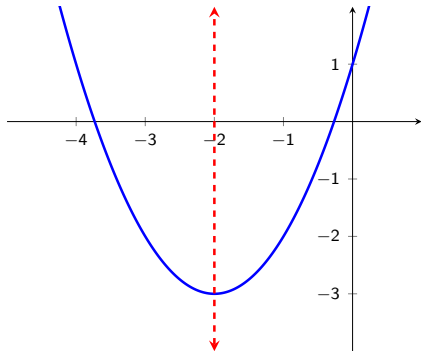
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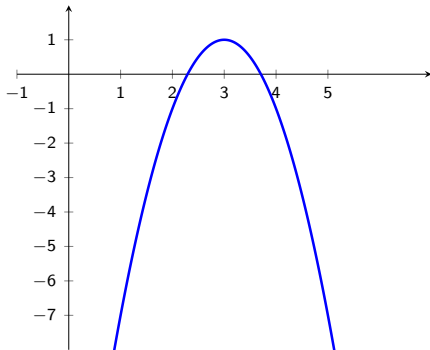
Axis of symmetry:  $x = -2$

## Example 1

$$(b) \quad y = -2x^2 + 12x - 17$$

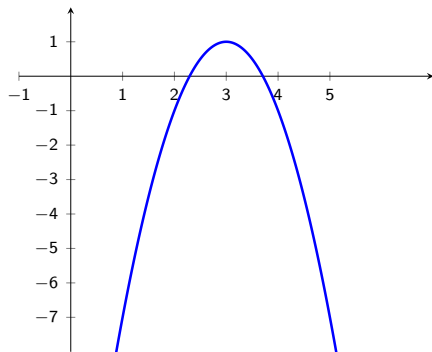
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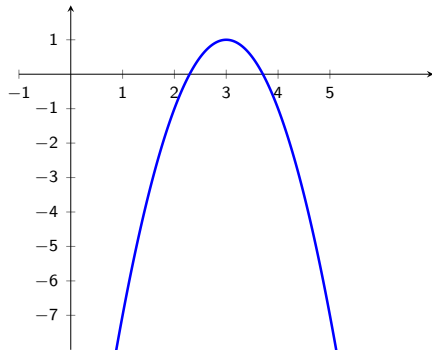
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Vertex (3, 1)

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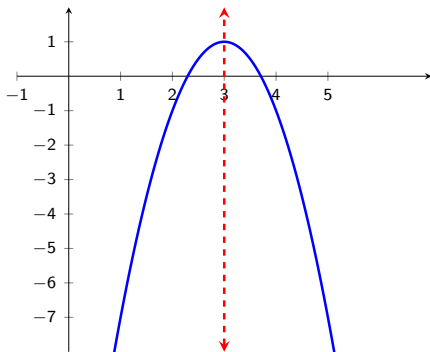


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Vertex (3, 1)

Vertex is a maximum

Axis of symmetry:  $x = 3$



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  - $a \neq 0$
  - $a$ ,  $h$ , and  $k$  are real numbers



# Converting From General to Standard Form

To convert from general form  $y = ax^2 + bx + c$  to standard form  $y = a(x - h)^2 + k$

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① Find the vertex:

- x-coordinate:  $\frac{-b}{2a}$
- y-coordinate: Evaluate expression at x-coordinate
- Or use graphing technology

② Use the same value of  $a$

## Example 2

Convert each to standard form.

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$$y = -1$$



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$$a = 1$$

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Vertex:  $(2, -1)$

$$a = 1$$

$$y = (x - 2)^2 - 1$$

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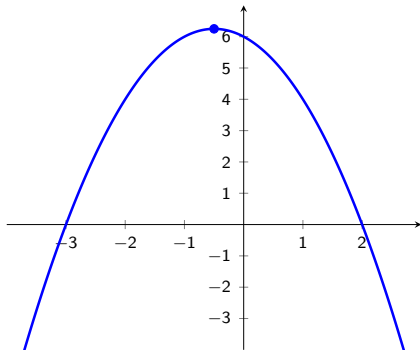
$$g(x) = -x^2 - x + 6$$

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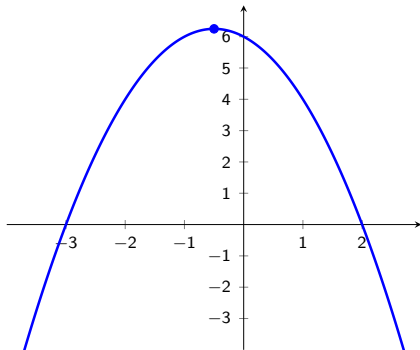


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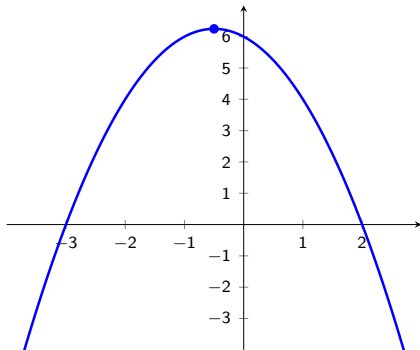
Vertex:  $\left(-\frac{1}{2}, \frac{25}{4}\right)$

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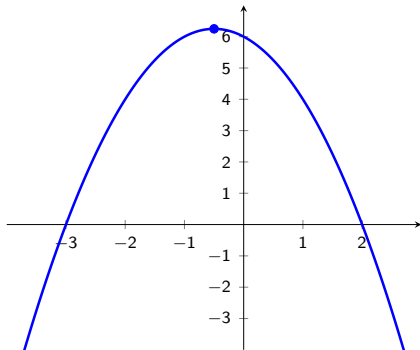


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$$\text{Vertex: } \left(-\frac{1}{2}, \frac{25}{4}\right)$$

$$a = -1$$

$$y = -\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$$

# Converting From Standard to General Form

To convert from

$$y = a(x - h)^2 + k$$

form to

$$y = ax^2 + bx + c$$

just **do the math** and remember your order of operations.

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$$y = -x^2 + 14x - 48$$