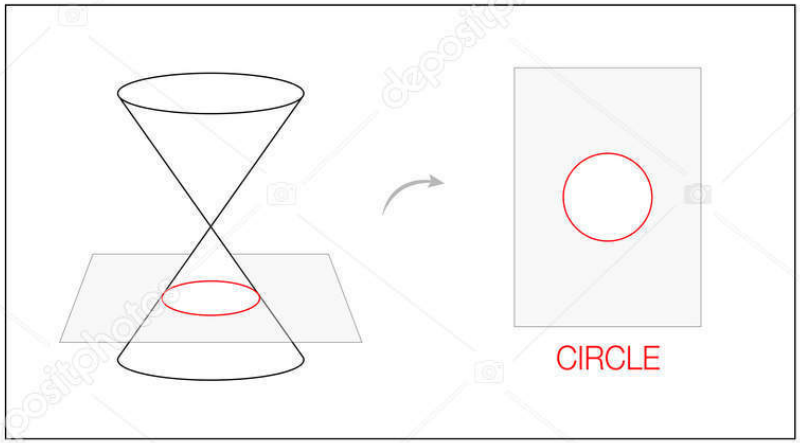
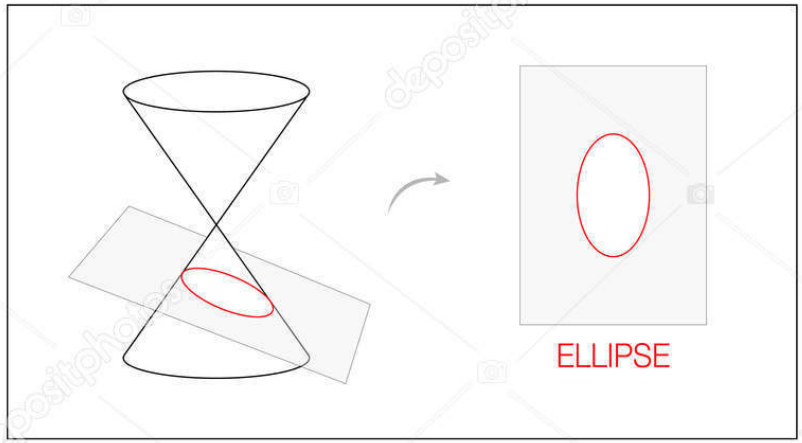
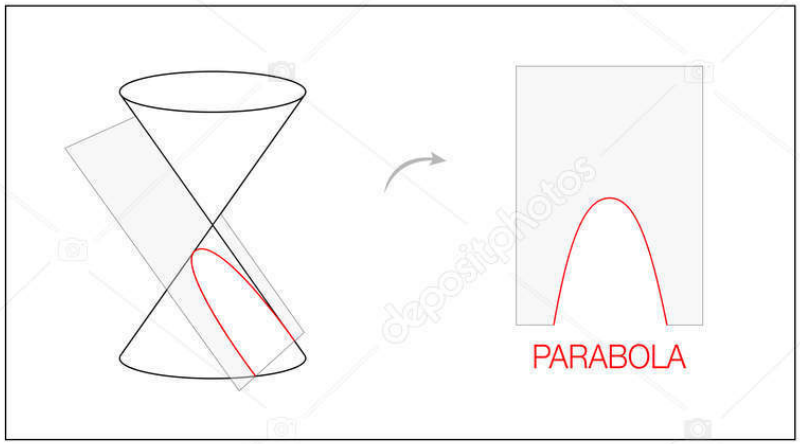
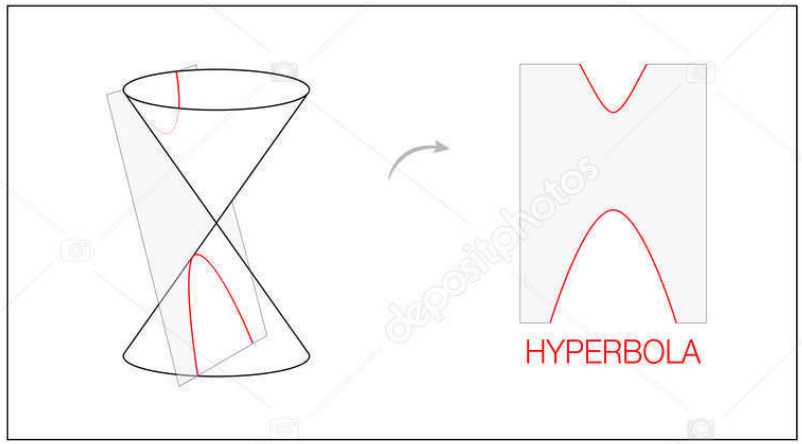
Hyperbolas









Objectives

1. Find the vertices and foci for a hyperbola in standard form.
2. Write the equation for a hyperbola in standard form.

Hyperbolas

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The set of points such that the **difference** of their distances from 2 fixed points (called **foci**) is constant.

Comparing Hyperbolas and Ellipses

Just like an ellipse, the midpoint joining the foci is the **center**.

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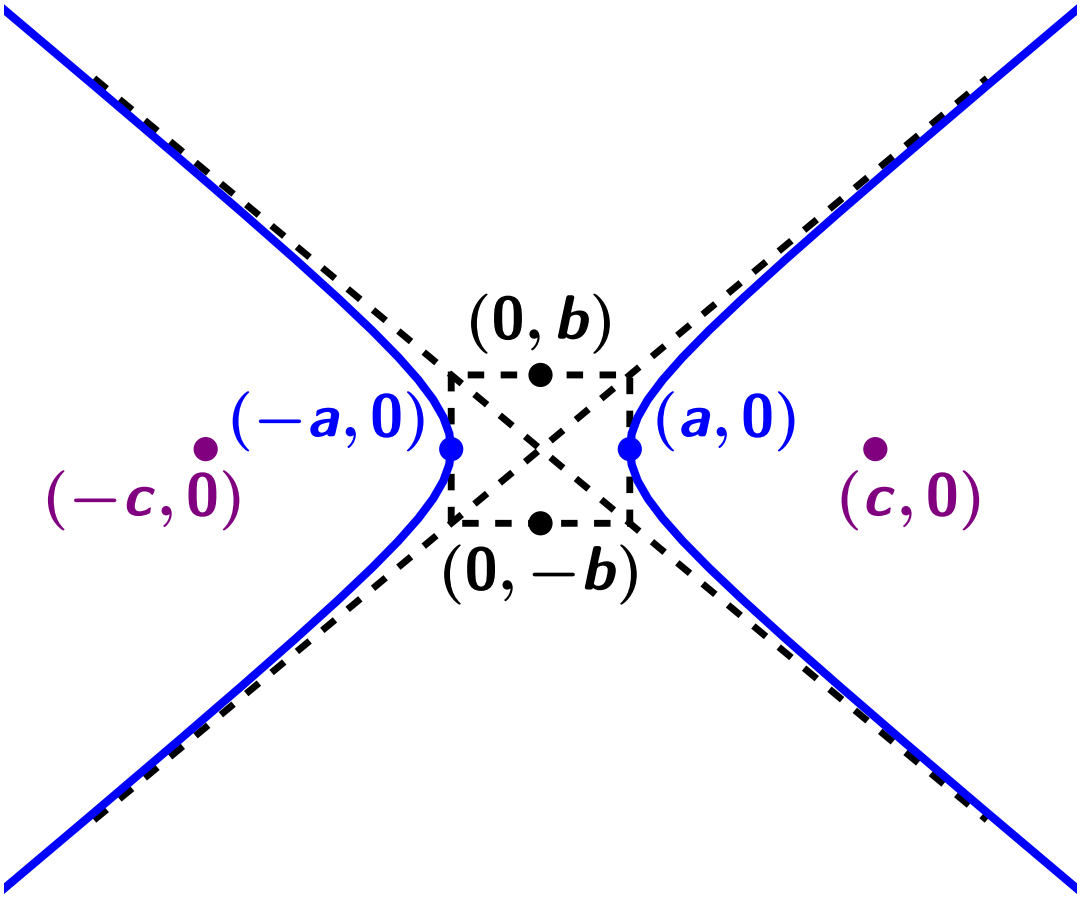
A key difference, however, is that hyperbolas will open left/right if the sign in front of x is positive, and will open up/down if the sign in front of y is positive; regardless of the values of a and b.

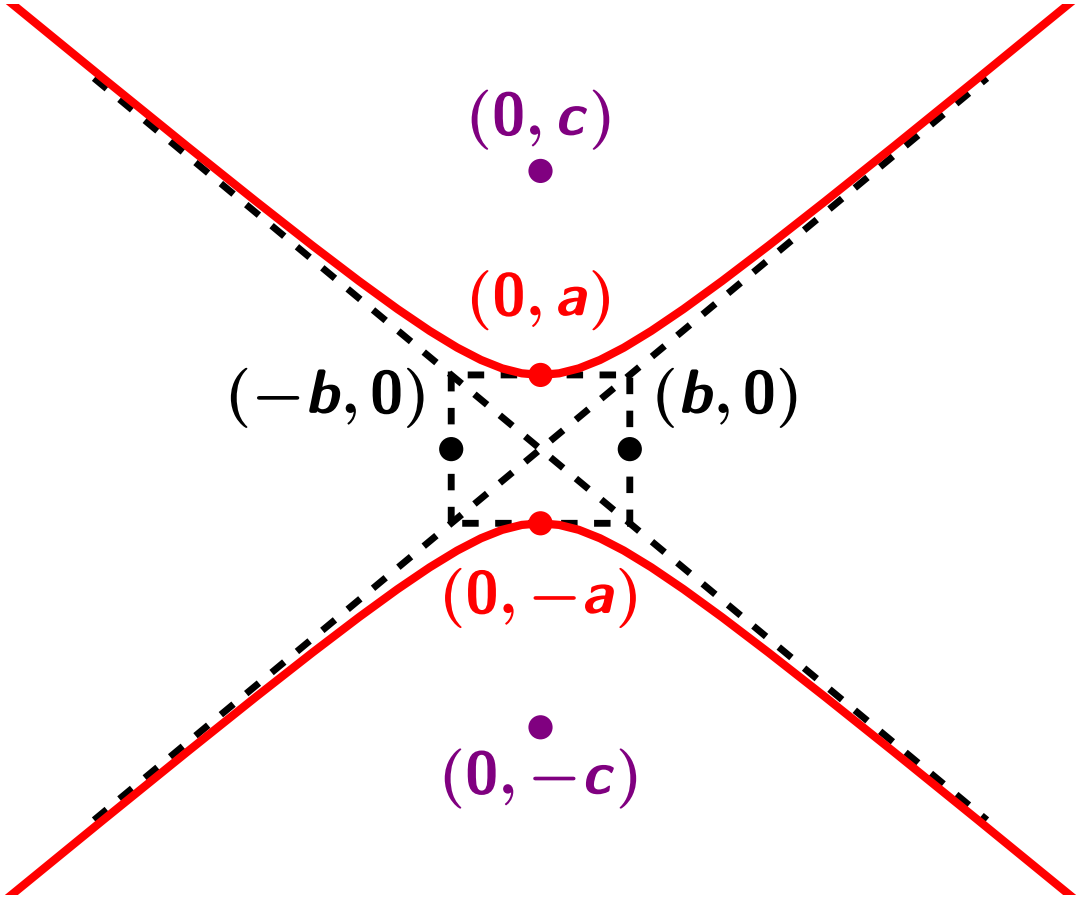
Opening Left and Right



Properties

Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
Center	(h, k)
Vertices	$(h \pm a, 0)$
Foci	$(h \pm c, 0)$
Co-vertices	$(h, k \pm b)$
x-Axis	Transverse Axis
y-Axis	Conjugate Axis
c^2	$a^2 + b^2$

Opening Up and Down



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Center	(h, k)
Vertices	$(h, k \pm a)$
Foci	$(h, k \pm c)$
Co-vertices	$(h \pm a, k)$
x-Axis	Conjugate Axis
y-Axis	Transverse Axis
c^2	$a^2 + b^2$

Example 1

Find the exact coordinates for the vertices and foci for each of the following.

(a) $\dfrac{(y-3)^2}{4} - \dfrac{x^2}{16} = 1$

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Vertices: $(0, 3 \pm 2)$

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(a) $\dfrac{(y-3)^2}{4} - \dfrac{x^2}{16} = 1$

Center: $(0, 3)$

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$a = \pm 2$

Vertices: $(0, 3 \pm 2) \longrightarrow (0, 1)$ and $(0, 5)$

Example 1a

$$c^2 = a^2 + b^2$$

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$$c = \pm 2\sqrt{5}$$

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$$c^2 = 4 + 16$$

$$c^2 = 20$$

$$c = \pm 2\sqrt{5}$$

Foci: $(0, 3 \pm 2\sqrt{5})$

Example 1

(b) $\dfrac{(y+1)^2}{16} - (x-1)^2 = 1$

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(b) $\dfrac{(y+1)^2}{16} - (x-1)^2 = 1$

Center: $(1, -1)$

$a^2 = 16$

$a = \pm 4$

Vertices: $(1, -1 \pm 4) \longrightarrow (1, -5)$ and $(1, 3)$

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$$c^2 = a^2 + b^2$$

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$$c = \pm\sqrt{17}$$

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$$c^2 = a^2 + b^2$$

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$$c^2 = 17$$

$$c = \pm\sqrt{17}$$

Foci: $(1, -1 \pm \sqrt{17})$

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(c) $(x+4)^2 - \dfrac{(y+2)^2}{9} = 1$

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Vertices: $(-4 \pm 1, -2)$

Example 1

(c) $(x+4)^2 - \dfrac{(y+2)^2}{9} = 1$

Center: $(-4, -2)$

$a^2 = 1$

$a = \pm 1$

Vertices: $(-4 \pm 1, -2) \longrightarrow (-5, -2)$ and $(-3, -2)$

Example 1c

$$c^2 = a^2 + b^2$$

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$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

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$$c^2 = a^2 + b^2$$

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$$c^2 = 10$$

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Foci: $(-4 \pm \sqrt{10}, -2)$

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(d) $\dfrac{(x+2)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

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Vertices: $(-2 \pm 3, 2)$

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(d) $\dfrac{(x+2)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

Center: $(-2, 2)$

$a^2 = 9$

$a = \pm 3$

Vertices: $(-2 \pm 3, 2) \longrightarrow (-5, 2)$ and $(1, 2)$

Example 1d

$$c^2 = a^2 + b^2$$

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$$c^2 = 9 + 4$$

$$c^2 = 13$$

$$c = \pm\sqrt{13}$$

Foci: $(-2 \pm \sqrt{13}, 2)$

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Similar to Ellipses, But Not the Same

We will use the technique that we used for ellipses **with one big difference.**

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We will use the technique that we used for ellipses **with one big difference.**

The absolute value of the y-coordinate after the minus sign gets subtracted from the right side.

Example 2

Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

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$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

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Write each of the following in standard form.

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Vertex: $(-6, 324)$

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Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

Vertex: $(-6, 324)$

Vertex: $(5, -25)$

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Vertex: $(-6, 324)$

Vertex: $(5, -25)$

$$-9(x+6)^2 + (y-5)^2 = 380 - |324| + |-25|$$

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$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

Vertex: $(-6, 324)$

Vertex: $(5, -25)$

$$-9(x+6)^2 + (y-5)^2 = 380 - |324| + |-25|$$

$$-9(x+6)^2 + (y-5)^2 = 81$$

Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

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$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

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$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

$$-\frac{(x+6)^2}{9} + \frac{(y-5)^2}{81} = 1$$

Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

$$-\frac{(x+6)^2}{9} + \frac{(y-5)^2}{81} = 1$$

$$\frac{(y-5)^2}{81} - \frac{(x+6)^2}{9} = 1$$