

Complex Fractions

Objectives

- 1 Add and subtract rational expressions with like denominators
- 2 Add and subtract rational expressions with unlike denominators

Fractions with like denominators

Recall that to add or subtract fractions with **like denominators**, you keep the denominators and add (or subtract) the numerators.

Fractions with like denominators

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Remember to **distribute the sign** to the numerator in the second fraction.

Example 1

Simplify each.

$$(a) \quad \frac{x+6}{9x^3+54x^2} + \frac{x+2}{9x^3+54x^2}$$

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$$= \frac{x+6+(x+2)}{9x^3+54x^2}$$

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$$= \frac{x+6+(x+2)}{9x^3+54x^2}$$

$$= \frac{2x+8}{9x^3+54x^2}$$

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Simplify each.

$$\begin{aligned} \text{(a)} \quad & \frac{x+6}{9x^3+54x^2} + \frac{x+2}{9x^3+54x^2} \\ & \frac{x+6}{9x^3+54x^2} + \frac{x+2}{9x^3+54x^2} \\ & = \frac{x+6+(x+2)}{9x^3+54x^2} \\ & = \frac{2x+8}{9x^3+54x^2} \\ & = \frac{2(x+4)}{9x^2(x+6)} \end{aligned}$$

Example 1

$$(b) \quad \frac{x-2}{2x^2-9x-18} + \frac{6x+1}{2x^2-9x-18}$$

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$$\frac{x-2}{2x^2-9x-18} + \frac{6x+1}{2x^2-9x-18}$$

$$= \frac{x-2+(6x+1)}{2x^2-9x-18}$$

$$= \frac{7x-1}{2x^2-9x-18}$$

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$$(b) \quad \frac{x-2}{2x^2-9x-18} + \frac{6x+1}{2x^2-9x-18}$$

$$\frac{x-2}{2x^2-9x-18} + \frac{6x+1}{2x^2-9x-18}$$

$$= \frac{x-2+(6x+1)}{2x^2-9x-18}$$

$$= \frac{7x-1}{2x^2-9x-18}$$

$$= \frac{7x-1}{(2x-3)(x+6)}$$

Example 1

$$(c) \quad \frac{x+5}{6x+4} - \frac{4x-1}{6x+4}$$

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$$= \frac{x+5-4x+1}{6x+4}$$

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$$= \frac{x+5-4x+1}{6x+4}$$

$$= \frac{-3x+6}{6x+4}$$

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$$\frac{x+5}{6x+4} - \frac{4x-1}{6x+4}$$

$$= \frac{x+5-(4x-1)}{6x+4}$$

$$= \frac{x+5-4x+1}{6x+4}$$

$$= \frac{-3x+6}{6x+4}$$

$$= \frac{-3(x-2)}{2(3x+2)}$$

Example 1

$$(d) \quad \frac{x-1}{3x^2-10x-8} - \frac{x+6}{3x^2-10x-8}$$

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$$= \frac{x-1-(x+6)}{3x^2-10x-8}$$

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$$= \frac{x-1-(x+6)}{3x^2-10x-8}$$

$$= \frac{x-1-x-6}{3x^2-10x-8}$$

$$= \frac{-7}{3x^2-10x-8}$$

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Adding and Subtracting Fractions with Unlike Denominators

Recall that before adding or subtracting fractions with unlike denominators, **you need to get a common denominator first.**

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For instance,

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$$\begin{aligned}\frac{2}{5} + \frac{1}{3} \\ = \frac{2(3)}{5(3)} + \frac{1(5)}{3(5)}\end{aligned}$$

Adding and Subtracting Fractions with Unlike Denominators

Recall that before adding or subtracting fractions with unlike denominators, **you need to get a common denominator first.**

For instance,

$$\begin{aligned}\frac{2}{5} + \frac{1}{3} \\&= \frac{2(3)}{5(3)} + \frac{1(5)}{3(5)} \\&= \frac{6}{15} + \frac{5}{15}\end{aligned}$$

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Recall that before adding or subtracting fractions with unlike denominators, **you need to get a common denominator first.**

For instance,

$$\begin{aligned}\frac{2}{5} + \frac{1}{3} \\&= \frac{2(3)}{5(3)} + \frac{1(5)}{3(5)} \\&= \frac{6}{15} + \frac{5}{15} \\&= \frac{11}{15}\end{aligned}$$

Ancient Secret to This Method

Notice you had to find the **least common multiple** of the denominators 3 and 5 (which ended up being 15).

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Notice you had to find the **least common multiple** of the denominators 3 and 5 (which ended up being 15).

For adding and subtracting rational expressions with unlike denominators, **factor the denominators completely and multiply by factors that differ**. (Easier to see that process in action than it is to understand it written down like that).

Example 2

Simplify each.

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$$= \frac{6}{x+4} + \frac{2}{3(x+2)}$$

Example 2

Simplify each.

$$(a) \quad \frac{6}{x+4} + \frac{2}{3x+6}$$

$$\frac{6}{x+4} + \frac{2}{3x+6}$$

$$= \frac{6}{x+4} + \frac{2}{3(x+2)}$$

$$= \frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

Example 2a

$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

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$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

Example 2a

$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

$$= \frac{18x+36+2x+8}{3(x+2)(x+4)}$$

Example 2a

$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

$$= \frac{18x+36+2x+8}{3(x+2)(x+4)}$$

$$= \frac{20x+44}{3(x+2)(x+4)}$$

Example 2a

$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

$$= \frac{18x+36+2x+8}{3(x+2)(x+4)}$$

$$= \frac{20x+44}{3(x+2)(x+4)}$$

$$= \frac{4(5x+11)}{3(x+2)(x+4)}$$

Example 2

$$(b) \quad \frac{3x}{2x-1} + \frac{7}{5x+3}$$

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$$= \frac{3x(5x+3)}{(2x-1)(5x+3)} + \frac{7(2x-1)}{(5x+3)(2x-1)}$$

Example 2

$$(b) \quad \frac{3x}{2x-1} + \frac{7}{5x+3}$$

$$\frac{3x}{2x-1} + \frac{7}{5x+3}$$

$$= \frac{3x(5x+3)}{(2x-1)(5x+3)} + \frac{7(2x-1)}{(5x+3)(2x-1)}$$

$$= \frac{15x^2 + 9x + 14x - 7}{(2x-1)(5x+3)}$$

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$$= \frac{15x^2 + 9x + 14x - 7}{(2x-1)(5x+3)}$$

$$= \frac{15x^2 + 23x - 7}{(2x-1)(5x+3)} \longrightarrow \frac{15x^2 + 23x - 7}{10x^2 + x - 3}$$

Example 2

$$(c) \quad \frac{8x}{3x^2 + 8x - 3} + \frac{9}{x^2 - x - 12}$$

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$$\frac{8x}{(x+3)(3x-1)} + \frac{9}{(x+3)(x-4)}$$

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$$(c) \quad \frac{8x}{3x^2 + 8x - 3} + \frac{9}{x^2 - x - 12}$$

$$\frac{8x}{(x+3)(3x-1)} + \frac{9}{(x+3)(x-4)}$$

$$= \frac{8x(\textcolor{red}{x-4})}{(x+3)(3x-1)(\textcolor{red}{x-4})} + \frac{9(\textcolor{red}{3x-1})}{(x+3)(x-4)(\textcolor{red}{3x-1})}$$

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$$= \frac{8x^2 - 32x + 27x - 9}{(x+3)(3x-1)(x-4)}$$

Example 2

$$(c) \quad \frac{8x}{3x^2 + 8x - 3} + \frac{9}{x^2 - x - 12}$$

$$\frac{8x}{(x+3)(3x-1)} + \frac{9}{(x+3)(x-4)}$$

$$= \frac{8x(\textcolor{red}{x-4})}{(x+3)(3x-1)(\textcolor{red}{x-4})} + \frac{9(\textcolor{red}{3x-1})}{(x+3)(x-4)(\textcolor{red}{3x-1})}$$

$$= \frac{8x^2 - 32x + 27x - 9}{(x+3)(3x-1)(x-4)}$$

$$= \frac{8x^2 - 5x - 9}{(x+3)(3x-1)(x-4)}$$

Example 2

$$(d) \quad \frac{x}{x+3} - \frac{5}{x-1}$$

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$$= \frac{x(\textcolor{red}{x}-1)}{(x+3)(\textcolor{red}{x}-1)} - \frac{5(\textcolor{red}{x}+3)}{(x-1)(\textcolor{red}{x}+3)}$$

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$$(d) \quad \frac{x}{x+3} - \frac{5}{x-1}$$

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$$= \frac{x^2 - x - (5x + 15)}{(x+3)(x-1)}$$

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$$= \frac{x(\textcolor{red}{x}-1)}{(x+3)(\textcolor{red}{x}-1)} - \frac{5(\textcolor{red}{x}+3)}{(x-1)(\textcolor{red}{x}+3)}$$

$$= \frac{x^2 - x - (5x + 15)}{(x+3)(x-1)}$$

$$= \frac{x^2 - 6x - 15}{(x+3)(x-1)}$$

Example 2

$$(d) \quad \frac{x}{x+3} - \frac{5}{x-1}$$

$$\frac{x}{x+3} - \frac{5}{x-1}$$

$$= \frac{x(\textcolor{red}{x}-1)}{(x+3)(\textcolor{red}{x}-1)} - \frac{5(\textcolor{red}{x}+3)}{(x-1)(\textcolor{red}{x}+3)}$$

$$= \frac{x^2 - x - (5x + 15)}{(x+3)(x-1)}$$

$$= \frac{x^2 - 6x - 15}{(x+3)(x-1)} \longrightarrow \frac{x^2 - 6x - 15}{x^2 + 2x - 3}$$

Example 2

$$(e) \quad \frac{x}{2x^2 + 5x - 12} - \frac{3}{2x^2 - 19x + 24}$$

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$$\frac{x}{2x^2 + 5x - 12} - \frac{3}{2x^2 - 19x + 24}$$

$$= \frac{x}{(2x - 3)(x + 4)} - \frac{3}{(2x - 3)(x - 8)}$$

Example 2

$$(e) \quad \frac{x}{2x^2 + 5x - 12} - \frac{3}{2x^2 - 19x + 24}$$

$$\frac{x}{2x^2 + 5x - 12} - \frac{3}{2x^2 - 19x + 24}$$

$$= \frac{x}{(2x - 3)(x + 4)} - \frac{3}{(2x - 3)(x - 8)}$$

$$= \frac{x(\textcolor{red}{x} - 8)}{(2x - 3)(x + 4)(\textcolor{red}{x} - 8)} - \frac{3(\textcolor{red}{x} + 4)}{(2x - 3)(x - 8)(\textcolor{red}{x} + 4)}$$

Example 2

$$(e) \quad \frac{x}{2x^2 + 5x - 12} - \frac{3}{2x^2 - 19x + 24}$$

$$\frac{x}{2x^2 + 5x - 12} - \frac{3}{2x^2 - 19x + 24}$$

$$= \frac{x}{(2x - 3)(x + 4)} - \frac{3}{(2x - 3)(x - 8)}$$

$$= \frac{x(\textcolor{red}{x} - \textcolor{red}{8})}{(2x - 3)(x + 4)(\textcolor{red}{x} - \textcolor{red}{8})} - \frac{3(\textcolor{red}{x} + \textcolor{red}{4})}{(2x - 3)(x - 8)(\textcolor{red}{x} + \textcolor{red}{4})}$$

$$= \frac{x^2 - 8x - (3x + 12)}{(2x - 3)(x + 4)(x - 8)}$$

Example 2e

$$\frac{x^2 - 8x - (3x + 12)}{(2x - 3)(x + 4)(x - 8)}$$

Example 2e

$$\frac{x^2 - 8x - (3x + 12)}{(2x - 3)(x + 4)(x - 8)}$$

$$\frac{x^2 - 11x - 12}{(2x - 3)(x + 4)(x - 8)}$$

Example 2e

$$\frac{x^2 - 8x - (3x + 12)}{(2x - 3)(x + 4)(x - 8)}$$

$$\frac{x^2 - 11x - 12}{(2x - 3)(x + 4)(x - 8)}$$

$$= \frac{(x - 12)(x + 1)}{(2x - 3)(x + 4)(x - 8)}$$