Factoring Techniques

Objectives

1. Find the greatest common factor of a quadratic expression.
2. Factor trinomials in the form $x^2 + bx + c$
3. Factor trinomials in the form $ax^2 + bx + c$
4. Use “reverse engineering” to factor quadratic expressions

Greatest Common Factor

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To find the greatest common factor of an expression:

- Find the greatest common factor of the coefficients.
- Select the lowest power of each unique variable.

Then divide each term by that GCF.

Example 1

Factor the GCF from each.

(a) $16x^5 + 72x^3 - 36x^2$

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GCF is $4x^2$

$$4x^2\left(4x^3 + 18x - 9\right)$$

Example 1

(b) $5a^4 + 70a^2 - 25a$

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$$5a\left(a^3 + 14a - 5\right)$$

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$x^2 + bx + c$

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We can then write the factorization in the form

$$(x + m)(x + n)$$

where $mn = c$ and $m + n = b$

Example 2

Factor each completely.

(a) $x^2 + 15x + 36$

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$$(x+3)(x+12)$$

Visual Approach to Factoring

$x^2 + 15x + 36$

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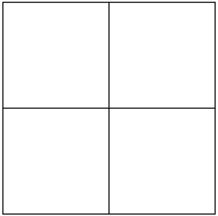

Visual Approach to Factoring

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x^2	

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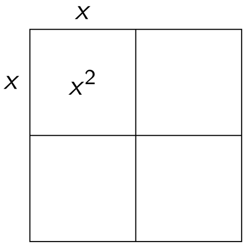
$x^2 + 15x + 36$



Visual Approach to Factoring

$x^2 + 15x + 36$

	x	
x	x^2	
		36

Visual Approach to Factoring

$x^2 + 15x + 36$

	x	3
x	x^2	
12		36

Visual Approach to Factoring

$x^2 + 15x + 36$

	x	3
x	x^2	$3x$
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Visual Approach to Factoring

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$$(x-7)(x-2)$$

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(c) $x^2 + 5x - 6$

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Numbers are -1 and 6

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Sum of the two numbers is 5.

Numbers are -1 and 6

$$(x-1)(x+6)$$

Example 2

(d) $x^2 - 7x - 8$

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Numbers are -8 and 1

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Sum of the two numbers is -7.

Numbers are -8 and 1

$$(x - 8)(x + 1)$$

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$ax^2 + bx + c$

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We can then factor like in the last example, but we will have to remember to **transform the expression back**; as we will see.

Example 3

Factor each completely.

(a) $3x^2 - 11x - 4$

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Multiply $a = 3$ and $c = -4$ to get $ac = -12$.

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We are not done yet!!

Example 3 $3x^2 - 11x - 4$

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$$\left(x - \frac{12}{3}\right)\left(x + \frac{1}{3}\right)$$

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$$(x-4)\left(x + \frac{1}{3}\right)$$

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Slide any remaining denominators in front of their variable.

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Slide any remaining denominators in front of their variable.

$$(x-4)(3x+1)$$

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(b) $8x^2 + 2x - 3$

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Multiply $a = 8$ and $c = -3$ to get $ac = -24$

Factor $x^2 + 2x - 24$

$$(x + 6)(x - 4)$$

Example 3

(b) $8x^2 + 2x - 3$

Multiply $a = 8$ and $c = -3$ to get $ac = -24$

Factor $x^2 + 2x - 24$

$$(x+6)(x-4)$$

$$\left(x + \frac{6}{8}\right)\left(x - \frac{4}{8}\right)$$

Example 3

(b) $8x^2 + 2x - 3$

Multiply $a = 8$ and $c = -3$ to get $ac = -24$

Factor $x^2 + 2x - 24$

$$(x+6)(x-4)$$

$$\left(x + \frac{6}{8}\right)\left(x - \frac{4}{8}\right)$$

$$\left(x + \frac{3}{4}\right)\left(x - \frac{1}{2}\right)$$

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(b) $8x^2 + 2x - 3$

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Factor $x^2 + 2x - 24$

$$(x+6)(x-4)$$

$$\left(x + \frac{6}{8}\right)\left(x - \frac{4}{8}\right)$$

$$\left(x + \frac{3}{4}\right)\left(x - \frac{1}{2}\right)$$

$$(4x+3)(2x-1)$$

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(c) $16x^2 + 46x + 15$

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Multiply $a = 16$ and $c = 15$ to get $ac = 240$

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Multiply $a = 16$ and $c = 15$ to get $ac = 240$

Factor $x^2 + 46x + 240$

$$(x + 40)(x + 6)$$

Example 3

(c) $16x^2 + 46x + 15$

Multiply $a = 16$ and $c = 15$ to get $ac = 240$

Factor $x^2 + 46x + 240$

$$(x + 40)(x + 6)$$

$$\left(x + \frac{40}{16}\right)\left(x + \frac{6}{16}\right)$$

$$\left(x + \frac{5}{2}\right)\left(x + \frac{3}{8}\right)$$

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(c) $16x^2 + 46x + 15$

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Factor $x^2 + 46x + 240$

$$(x+40)(x+6)$$

$$\left(x + \frac{40}{16}\right)\left(x + \frac{6}{16}\right)$$

$$\left(x + \frac{5}{2}\right)\left(x + \frac{3}{8}\right)$$

$$(2x+5)(8x+3)$$

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(d) $7x^2 - 55x + 42$

Multiply $a = 7$ and $c = 42$ to get $ac = 294$

Factor $x^2 - 55x + 294$

$$(x - 49)(x - 6)$$

Example 3

(d) $7x^2 - 55x + 42$

Multiply $a = 7$ and $c = 42$ to get $ac = 294$

Factor $x^2 - 55x + 294$

$$(x - 49)(x - 6)$$

$$\left(x - \frac{49}{7}\right)\left(x - \frac{6}{7}\right)$$

$$(x - 7)\left(x - \frac{6}{7}\right)$$

Example 3

(d) $7x^2 - 55x + 42$

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Factor $x^2 - 55x + 294$

$$(x - 49)(x - 6)$$

$$\left(x - \frac{49}{7}\right)\left(x - \frac{6}{7}\right)$$

$$(x - 7)\left(x - \frac{6}{7}\right)$$

$$(x - 7)(7x - 6)$$

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$$x + 6 = 0 \qquad\qquad x - 1 = 0$$

$$x^2 + 5x - 6 \text{ factors as } (x + 6)(x - 1)$$

Using the Graph for $ax^2 + bx + c$

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2. Find those x-intercepts
3. Transform back by dividing by the value of a and simplifying like in Example 3.

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Divide each of those by 3 to get $x = -\frac{1}{3}$ and $x = 4$

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Get each equal to 0:

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Get each equal to 0:

$$3x + 1 \quad \text{and} \quad x - 4$$

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Get each equal to 0:

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$$(3x + 1)(x - 4)$$