Radical Equations and Inequalities

Objectives

1. Solve radical equations
2. Solve equations with rational exponents
3. Solve radical inequalities

Solving Radical Equations

When solving radical equations, we want to try our best to isolate the radical on one side of the equation (if possible).

Solving Radical Equations

When solving radical equations, we want to try our best to isolate the radical on one side of the equation (if possible).

Then we can raise both sides to the power that is the root of the radical.

Solving Radical Equations

When solving radical equations, we want to try our best to isolate the radical on one side of the equation (if possible).

Then we can raise both sides to the power that is the root of the radical.

However, sometimes you may end up with extraneous solutions.

Example 1

Solve each. Remember to check for extraneous solutions.

(a) $\sqrt{5x+1} = 4$

Example 1

Solve each. Remember to check for extraneous solutions.

(a) $\sqrt{5x+1} = 4$

$$\sqrt{5x+1} = 4$$

Example 1

Solve each. Remember to check for extraneous solutions.

(a) $\sqrt{5x+1} = 4$

$$\sqrt{5x+1} = 4$$

$$\left(\sqrt{5x+1}\right)^2 = 4^2$$

Example 1

Solve each. Remember to check for extraneous solutions.

(a) $\sqrt{5x+1} = 4$

$$\sqrt{5x+1} = 4$$

$$\left(\sqrt{5x+1}\right)^2 = 4^2$$

$$5x+1 = 16$$

Example 1

Solve each. Remember to check for extraneous solutions.

(a) $\sqrt{5x+1} = 4$

$$\begin{aligned} \sqrt{5x+1} &= 4 \\ \left(\sqrt{5x+1}\right)^2 &= 4^2 \\ 5x+1 &= 16 \\ x &= 3 \end{aligned}$$

Example 1

Solve each. Remember to check for extraneous solutions.

(a) $\sqrt{5x+1} = 4$

$$\begin{aligned} \sqrt{5x+1} &= 4 \\ \left(\sqrt{5x+1}\right)^2 &= 4^2 \\ 5x+1 &= 16 \\ x &= 3 \end{aligned}$$

Check: $\sqrt{5(3)+1} = 4?$

Example 1

Solve each. Remember to check for extraneous solutions.

(a) $\sqrt{5x+1} = 4$

$$\begin{aligned} \sqrt{5x+1} &= 4 \\ \left(\sqrt{5x+1}\right)^2 &= 4^2 \\ 5x+1 &= 16 \\ x &= 3 \end{aligned}$$

Check: $\sqrt{5(3)+1} = 4?$ Yes

Example 1

(b) $\sqrt{8-x}+7=10$

Example 1

(b) $\sqrt{8-x}+7=10$

$$\sqrt{8-x}+7=10$$

Example 1

(b) $\sqrt{8-x}+7=10$

$$\sqrt{8-x}+7=10$$

$$\sqrt{8-x}=3$$

Example 1

(b) $\sqrt{8-x}+7=10$

$$\sqrt{8-x}+7=10$$

$$\sqrt{8-x}=3$$

$$8-x=9$$

Example 1

(b) $\sqrt{8-x}+7=10$

$$\begin{aligned}\sqrt{8-x}+7&=10\\\sqrt{8-x}&=3\\8-x&=9\\x&=-1\end{aligned}$$

Example 1

(b) $\sqrt{8-x}+7=10$

$$\begin{aligned}\sqrt{8-x}+7&=10\\ \sqrt{8-x}&=3\\ 8-x&=9\\ x&=-1\end{aligned}$$

Check: $\sqrt{8-(-1)}+7=10?$

Example 1

(b) $\sqrt{8-x}+7=10$

$$\begin{aligned}\sqrt{8-x}+7&=10\\ \sqrt{8-x}&=3\\ 8-x&=9\\ x&=-1\end{aligned}$$

Check: $\sqrt{8-(-1)}+7=10?$ Yes

Example 1

(c) $\sqrt[3]{x-10} = 3$

Example 1

(c) $\sqrt[3]{x-10} = 3$

$$\sqrt[3]{x-10} = 3$$

Example 1

(c) $\sqrt[3]{x-10} = 3$

$$\sqrt[3]{x-10} = 3$$

$$\left(\sqrt[3]{x-10}\right)^3 = 3^3$$

Example 1

(c) $\sqrt[3]{x-10} = 3$

$$\sqrt[3]{x-10} = 3$$

$$\left(\sqrt[3]{x-10}\right)^3 = 3^3$$

$$x - 10 = 27$$

Example 1

(c) $\sqrt[3]{x-10}=3$

$$\begin{aligned}\sqrt[3]{x-10}&=3\\ \left(\sqrt[3]{x-10}\right)^3&=3^3\\ x-10&=27\\ x&=37\end{aligned}$$

Example 1

(d) $3\sqrt{x} + 12 = 9$

Example 1

(d) $3\sqrt{x} + 12 = 9$

$$3\sqrt{x} + 12 = 9$$

Example 1

(d) $3\sqrt{x} + 12 = 9$

$$3\sqrt{x} + 12 = 9$$

$$3\sqrt{x} = -3$$

Example 1

(d) $3\sqrt{x} + 12 = 9$

$$\begin{aligned} 3\sqrt{x} + 12 &= 9 \\ 3\sqrt{x} &= -3 \\ \sqrt{x} &= -1 \end{aligned}$$

Example 1

(d) $3\sqrt{x} + 12 = 9$

$$\begin{aligned} 3\sqrt{x} + 12 &= 9 \\ 3\sqrt{x} &= -3 \\ \sqrt{x} &= -1 \\ x &= 1 \end{aligned}$$

Example 1

(d) $3\sqrt{x} + 12 = 9$

$$\begin{aligned} 3\sqrt{x} + 12 &= 9 \\ 3\sqrt{x} &= -3 \\ \sqrt{x} &= -1 \\ x &= 1 \end{aligned}$$

Check: $3\sqrt{1} + 12 = 9$?

Example 1

(d) $3\sqrt{x} + 12 = 9$

$$\begin{aligned} 3\sqrt{x} + 12 &= 9 \\ 3\sqrt{x} &= -3 \\ \sqrt{x} &= -1 \\ x &= 1 \end{aligned}$$

Check: $3\sqrt{1} + 12 = 9?$ No

Example 1

(d) $3\sqrt{x} + 12 = 9$

$$\begin{aligned} 3\sqrt{x} + 12 &= 9 \\ 3\sqrt{x} &= -3 \\ \sqrt{x} &= -1 \\ x &= 1 \end{aligned}$$

Check: $3\sqrt{1} + 12 = 9?$ No

No Solution $\varnothing$

Example 1

(e) $\sqrt{2x-7} = \sqrt{3x-12}$

Example 1

(e) $\sqrt{2x-7} = \sqrt{3x-12}$

$$\sqrt{2x-7} = \sqrt{3x-12}$$

Example 1

(e) $\sqrt{2x-7} = \sqrt{3x-12}$

$$\sqrt{2x-7} = \sqrt{3x-12}$$

$$2x-7 = 3x-12$$

Example 1

(e) $\sqrt{2x-7} = \sqrt{3x-12}$

$$\sqrt{2x-7} = \sqrt{3x-12}$$

$$2x-7 = 3x-12$$

$$x = 5$$

Example 1

(f) $x - 3 = \sqrt{x - 1}$

Example 1

(f) $x - 3 = \sqrt{x - 1}$

$$x - 3 = \sqrt{x - 1}$$

Example 1

(f) $x - 3 = \sqrt{x-1}$

$$x - 3 = \sqrt{x-1}$$

$$(x-3)^2 = \left(\sqrt{x-1}\right)^2$$

Example 1

(f) $\quad x-3=\sqrt{x-1}$

$$x-3=\sqrt{x-1}$$

$$(x-3)^2=\left(\sqrt{x-1}\right)^2$$

$$x^2-6x+9=x-1$$

Example 1

(f) $x - 3 = \sqrt{x-1}$

$$x - 3 = \sqrt{x-1}$$

$$(x-3)^2 = \left(\sqrt{x-1}\right)^2$$

$$x^2 - 6x + 9 = x - 1$$

$$x^2 - 7x + 10 = 0$$

Example 1

(f) $x - 3 = \sqrt{x-1}$

$$x - 3 = \sqrt{x-1}$$

$$(x-3)^2 = \left(\sqrt{x-1}\right)^2$$

$$x^2 - 6x + 9 = x - 1$$

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 0$$

Example 1

(f) $x - 3 = \sqrt{x - 1}$

$$
\begin{aligned}
x - 3 &= \sqrt{x - 1} \\
(x - 3)^2 &= \left(\sqrt{x - 1}\right)^2 \\
x^2 - 6x + 9 &= x - 1 \\
x^2 - 7x + 10 &= 0 \\
(x - 2)(x - 5) &= 0 \\
x &= 2, 5
\end{aligned}
$$

Example 1 $x - 3 = \sqrt{x - 1}$

$$x = 2 \quad x = 5$$

Check:

Example 1 $x - 3 = \sqrt{x - 1}$

$$x = 2 \quad x = 5$$

Check:

$x = 2$ is extraneous

Example 1 $x - 3 = \sqrt{x - 1}$

$$x = 2 \quad x = 5$$

Check:

$x = 2$ is extraneous

$x = 5$ is valid

Example 1 $x - 3 = \sqrt{x - 1}$

$$x = 2 \quad x = 5$$

Check:

$x = 2$ is extraneous

$x = 5$ is valid

Final answer: $x = 5$

Objectives

1. Solve radical equations
2. Solve equations with rational exponents
3. Solve radical inequalities

Equations with Rational Exponents

When dealing with rational exponents, recall that raising a power to a power will result in multiplying the exponents together.

Equations with Rational Exponents

When dealing with rational exponents, recall that raising a power to a power will result in multiplying the exponents together.

Using this knowledge, we can isolate the radicand by raising both sides of the equation to the reciprocal of the exponent.

Example 2

Solve each of the following. Remember to check for extraneous solutions.

(a) $x^{2/3} = 3$

Example 2

Solve each of the following. Remember to check for extraneous solutions.

(a) $x^{2/3} = 3$

$$x^{2/3} = 3$$

Example 2

Solve each of the following. Remember to check for extraneous solutions.

(a) $x^{2/3} = 3$

$$x^{2/3} = 3$$

$$\left(x^{2/3}\right)^{3/2} = 3^{3/2}$$

Example 2

Solve each of the following. Remember to check for extraneous solutions.

(a) $x^{2/3} = 3$

$$\begin{aligned} x^{2/3} &= 3 \\ \left(x^{2/3}\right)^{3/2} &= 3^{3/2} \\ x &= 3^{3/2} \end{aligned}$$

Example 2

Solve each of the following. Remember to check for extraneous solutions.

(a) $x^{2/3} = 3$

$$
\begin{aligned}
x^{2/3} &= 3 \\
\left(x^{2/3}\right)^{3/2} &= 3^{3/2} \\
x &= 3^{3/2} \\
&= \sqrt{27}
\end{aligned}
$$

Example 2

Solve each of the following. Remember to check for extraneous solutions.

(a) $x^{2/3} = 3$

$$\begin{aligned} x^{2/3} &= 3 \\ \left(x^{2/3}\right)^{3/2} &= 3^{3/2} \\ x &= 3^{3/2} \\ &= \sqrt{27} \\ &= 3\sqrt{3} \end{aligned}$$

Example 2

(b) $(x-1)^{-2/3} = 1$

Example 2

(b) $(x-1)^{-2/3} = 1$

$$(x-1)^{-2/3} = 1$$

Example 2

(b) $(x-1)^{-2/3} = 1$

$$(x-1)^{-2/3} = 1$$

$$\left((x-1)^{-2/3}\right)^{-3/2} = 1^{-3/2}$$

Example 2

(b) $(x-1)^{-2/3} = 1$

$$(x-1)^{-2/3} = 1$$

$$\left((x-1)^{-2/3}\right)^{-3/2} = 1^{-3/2}$$

$$x - 1 = 1$$

Example 2

(b) $(x-1)^{-2/3}=1$

$$(x-1)^{-2/3}=1$$

$$\left((x-1)^{-2/3}\right)^{-3/2}=1^{-3/2}$$

$$x-1=1$$

$$x=2$$

Example 2

(c) $(x+2)^{3/2} = -1$

Example 2

(c) $(x+2)^{3/2} = -1$

$$(x+2)^{3/2} = -1$$

Example 2

(c) $(x+2)^{3/2} = -1$

$$(x+2)^{3/2} = -1$$

$$\left((x+2)^{3/2}\right)^{2/3} = (-1)^{2/3}$$

Example 2

(c) $(x+2)^{3/2} = -1$

$$(x+2)^{3/2} = -1$$

$$\left((x+2)^{3/2}\right)^{2/3} = (-1)^{2/3}$$

$$x + 2 = 1$$

Example 2

(c) $(x+2)^{3/2} = -1$

$$\begin{aligned} (x+2)^{3/2} &= -1 \\ \left((x+2)^{3/2}\right)^{2/3} &= (-1)^{2/3} \\ x+2 &= 1 \\ x &= -1 \end{aligned}$$

Example 2

(c) $(x+2)^{3/2} = -1$

$$
\begin{aligned}
(x+2)^{3/2} &= -1 \\
\left((x+2)^{3/2}\right)^{2/3} &= (-1)^{2/3} \\
x+2 &= 1 \\
x &= -1
\end{aligned}
$$

No solution $\varnothing$

Objectives

1. Solve radical equations
2. Solve equations with rational exponents
3. Solve radical inequalities

Solving Radical Inequalities

When solving inequalities, use the same techniques as solving the equations, then use number lines and test values to solve inequalities.

Solving Radical Inequalities

When solving inequalities, use the same techniques as solving the equations, then use number lines and test values to solve inequalities.

This gives us the advantage of solving any inequality after learning how to solve its equation form.

Solving Radical Inequalities

When solving inequalities, use the same techniques as solving the equations, then use number lines and test values to solve inequalities.

This gives us the advantage of solving any inequality after learning how to solve its equation form.

Important: Remember, when dealing with **even roots**, the domain of the radicand is ≥ 0.

Example 3

Solve each of the following and graph your solution on a number line.

(a) $\sqrt{x-3} - 3 < 4$

Example 3

Solve each of the following and graph your solution on a number line.

(a) $\sqrt{x-3} - 3 < 4$

$$\sqrt{x-3} - 3 = 4$$

Example 3

Solve each of the following and graph your solution on a number line.

(a) $\sqrt{x-3} - 3 < 4$

$$\sqrt{x-3} - 3 = 4$$

$$\sqrt{x-3} = 7$$

Example 3

Solve each of the following and graph your solution on a number line.

(a) $\sqrt{x-3} - 3 < 4$

$$\sqrt{x-3} - 3 = 4$$

$$\sqrt{x-3} = 7$$

$$x - 3 = 49$$

Example 3

Solve each of the following and graph your solution on a number line.

(a) $\sqrt{x-3} - 3 < 4$

$$\begin{aligned} \sqrt{x-3} - 3 &= 4 \\ \sqrt{x-3} &= 7 \\ x - 3 &= 49 \\ x &= 52 \end{aligned}$$

Example 3 $\sqrt{x-3}-3<4$

Critical value of x is 52.

Example 3 $\sqrt{x-3} - 3 < 4$

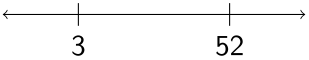
Critical value of x is 52.

For $\sqrt{x-3}$, $x - 3$ must be ≥ 0, so $x \geq 3$

Example 3 $\sqrt{x-3}-3<4$

Critical value of x is 52.

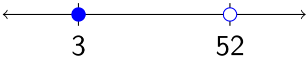
For $\sqrt{x-3}$, $x-3$ must be ≥ 0, so $x \geq 3$



Example 3 $\sqrt{x-3} - 3 < 4$

Critical value of x is 52.

For $\sqrt{x-3}$, $x - 3$ must be ≥ 0, so $x \geq 3$



Example 3 $\sqrt{x-3} - 3 < 4$

Critical value of x is 52.

For $\sqrt{x-3}$, $x-3$ must be ≥ 0, so $x \geq 3$

Example 3 $\sqrt{x-3} - 3 < 4$

Critical value of x is 52.

For $\sqrt{x-3}$, $x - 3$ must be ≥ 0, so $x \geq 3$

$$3 \leq x < 52$$

Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

$$\sqrt[3]{2x+3} = \sqrt[3]{x+12}$$

Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

$$\sqrt[3]{2x+3} = \sqrt[3]{x+12}$$

$$2x+3 = x+12$$

Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

$$\sqrt[3]{2x+3} = \sqrt[3]{x+12}$$

$$2x+3 = x+12$$

$$x = 9$$

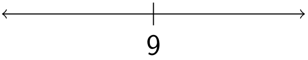
Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

$$\sqrt[3]{2x+3} = \sqrt[3]{x+12}$$

$$2x+3 = x+12$$

$$x = 9$$



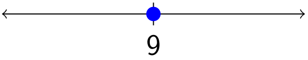
Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

$$\sqrt[3]{2x+3} = \sqrt[3]{x+12}$$

$$2x+3 = x+12$$

$$x = 9$$



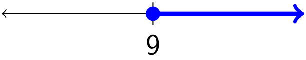
Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

$$\sqrt[3]{2x+3} = \sqrt[3]{x+12}$$

$$2x+3 = x+12$$

$$x = 9$$



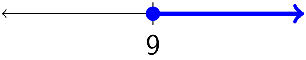
Example 3

(b) $\sqrt[3]{2x+3} \geq \sqrt[3]{x+12}$

$$\sqrt[3]{2x+3} = \sqrt[3]{x+12}$$

$$2x+3 = x+12$$

$$x = 9$$



$$x \geq 9$$

Example 3

(c) $\sqrt{2x-1} \leq 2x-1$

Example 3

(c) $\sqrt{2x-1} \leq 2x-1$

$$\sqrt{2x-1} = 2x-1$$

Example 3

(c) $\sqrt{2x-1} \leq 2x-1$

$$\sqrt{2x-1} = 2x-1$$

$$\left(\sqrt{2x-1}\right)^2 = (2x-1)^2$$

Example 3

(c) $\sqrt{2x-1} \leq 2x-1$

$$\sqrt{2x-1} = 2x-1$$

$$\left(\sqrt{2x-1}\right)^2 = (2x-1)^2$$

$$2x-1 = 4x^2-4x+1$$

Example 3

(c) $\sqrt{2x-1} \leq 2x-1$

$$\sqrt{2x-1} = 2x-1$$

$$\left(\sqrt{2x-1}\right)^2 = (2x-1)^2$$

$$2x-1 = 4x^2-4x+1$$

$$4x^2-6x+2=0$$

Example 3

(c) $\sqrt{2x-1} \le 2x-1$

$$\sqrt{2x-1} = 2x-1$$

$$\left(\sqrt{2x-1}\right)^2 = (2x-1)^2$$

$$2x-1 = 4x^2-4x+1$$

$$4x^2-6x+2=0$$

$$x = \frac{1}{2}, \quad 1$$

Example 3 $\sqrt{2x-1} \leq 2x-1$

Example 3 $\sqrt{2x-1} \le 2x-1$

Example 3 $\sqrt{2x-1} \le 2x-1$

Example 3 $\sqrt{2x-1} \le 2x-1$

$$x = \frac{1}{2} \quad \text{or} \quad x \ge 1$$