

Operations with Functions

Objectives

- 1 Write the sum, difference, product, and quotient of two functions
- 2 Evaluate the sum, difference, product, and quotient of two functions at a value

Sum, Difference, Product, and Quotient of Functions

We can add, subtract, multiply, and divide functions just like we can with real numbers.

Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(fg)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Sum, Difference, Product, and Quotient of Functions

So if $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

Sum, Difference, Product, and Quotient of Functions

So if $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$(f + g)(x) = f(x) + g(x)$$

Sum, Difference, Product, and Quotient of Functions

So if $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\ &= x + 2 + x^2 - 4\end{aligned}$$

Sum, Difference, Product, and Quotient of Functions

So if $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$(f + g)(x) = f(x) + g(x)$$

$$= x + 2 + x^2 - 4$$

$$= x^2 + x - 2$$

Example 1

Find each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(x)$

Example 1

Find each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

Example 1

Find each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

$$= x^2 - 3 + 4x + 5$$

Example 1

Find each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

$$= x^2 - 3 + 4x + 5$$

$$= x^2 + 4x + 2$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(b) $(f - g)(x)$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(b) $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(b) $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

$$= x^2 - 3 - (4x + 5)$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(b) $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

$$= x^2 - 3 - (4x + 5)$$

$$= x^2 - 3 - 4x - 5$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(b) $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

$$= x^2 - 3 - (4x + 5)$$

$$= x^2 - 3 - 4x - 5$$

$$= x^2 - 4x - 8$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(c) $(g - f)(x)$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(c) $(g - f)(x)$

$$(g - f)(x) = g(x) - f(x)$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(c) $(g - f)(x)$

$$(g - f)(x) = g(x) - f(x)$$

$$= 4x + 5 - (x^2 - 3)$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(c) $(g - f)(x)$

$$(g - f)(x) = g(x) - f(x)$$

$$= 4x + 5 - (x^2 - 3)$$

$$= 4x + 5 - x^2 + 3$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(c) $(g - f)(x)$

$$(g - f)(x) = g(x) - f(x)$$

$$= 4x + 5 - (x^2 - 3)$$

$$= 4x + 5 - x^2 + 3$$

$$= -x^2 + 4x + 8$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

	4x	5
x ²		
-3		

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

	4x	5
x ²	4x ³	
-3		

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

	4x	5
x ²	4x ³	5x ²
-3		

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

	4x	5
x ²	4x ³	5x ²
-3	-12x	

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

	4x	5
x ²	4x ³	5x ²
-3	-12x	-15

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(d) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

	4x	5
x ²	4x ³	5x ²
-3	-12x	-15

$$4x^3 + 5x^2 - 12x - 15$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(e) $\left(\frac{f}{g}\right)(x)$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(e) $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(e) $\left(\frac{f}{g}\right)(x)$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x^2 - 3}{4x + 5}\end{aligned}$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(x)$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(x)$

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$$

Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(x)$

$$\begin{aligned}\left(\frac{g}{f}\right)(x) &= \frac{g(x)}{f(x)} \\ &= \frac{4x + 5}{x^2 - 3}\end{aligned}$$

Objectives

- 1 Write the sum, difference, product, and quotient of two functions
- 2 Evaluate the sum, difference, product, and quotient of two functions at a value

Methods

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

Methods

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$(f + g)(3) = f(3) + g(3)$$

Methods

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}(f + g)(3) &= f(3) + g(3) \\ &= (3 + 2) + (3^2 - 4)\end{aligned}$$

Methods

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}(f + g)(3) &= f(3) + g(3) \\ &= (3 + 2) + (3^2 - 4) \\ &= 10\end{aligned}$$

Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$(f + g)(x) = f(x) + g(x)$$

Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\ &= x + 2 + x^2 - 4\end{aligned}$$

Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\&= x + 2 + x^2 - 4 \\&= x^2 + x - 2\end{aligned}$$

Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$(f + g)(x) = f(x) + g(x)$$

$$= x + 2 + x^2 - 4$$

$$= x^2 + x - 2$$

$$(f + g)(3) = 3^2 + 3 - 2$$

Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$(f + g)(x) = f(x) + g(x)$$

$$= x + 2 + x^2 - 4$$

$$= x^2 + x - 2$$

$$(f + g)(3) = 3^2 + 3 - 2$$

$$= 10$$

Example 2

Evaluate each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(3)$

Example 2

Evaluate each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(3)$

$$(f + g)(3) = f(3) + g(3)$$

Example 2

Evaluate each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(3)$

$$\begin{aligned}(f + g)(3) &= f(3) + g(3) \\ &= 6 + 17\end{aligned}$$

Example 2

Evaluate each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(3)$

$$(f + g)(3) = f(3) + g(3)$$

$$= 6 + 17$$

$$= 23$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(b) $(f - g)(0)$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(b) $(f - g)(0)$

$$(f - g)(0) = f(0) - g(0)$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(b) $(f - g)(0)$

$$(f - g)(0) = f(0) - g(0)$$

$$= -3 - 5$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(b) $(f - g)(0)$

$$(f - g)(0) = f(0) - g(0)$$

$$= -3 - 5$$

$$= -8$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(c) $(fg)(2)$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(c) $(fg)(2)$

$$(fg)(2) = f(2) \cdot g(2)$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(c) $(fg)(2)$

$$\begin{aligned}(fg)(2) &= f(2) \cdot g(2) \\ &= 1(13)\end{aligned}$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(c) $(fg)(2)$

$$(fg)(2) = f(2) \cdot g(2)$$

$$= 1(13)$$

$$= 13$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(d) $(gg)(1)$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(d) $(gg)(1)$

$$(gg)(1) = g(1) \cdot g(1)$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(d) $(gg)(1)$

$$\begin{aligned}(gg)(1) &= g(1) \cdot g(1) \\ &= 9(9)\end{aligned}$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(d) $(gg)(1)$

$$(gg)(1) = g(1) \cdot g(1)$$

$$= 9(9)$$

$$= 81$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(e) $\left(\frac{f}{g}\right)(1)$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(e) $\left(\frac{f}{g}\right)(1)$

$$\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)}$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(e) $\left(\frac{f}{g}\right)(1)$

$$\begin{aligned}\left(\frac{f}{g}\right)(1) &= \frac{f(1)}{g(1)} \\ &= \frac{-2}{9}\end{aligned}$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(8)$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(8)$

$$\left(\frac{g}{f}\right)(8) = \frac{g(8)}{f(8)}$$

Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(8)$

$$\begin{aligned}\left(\frac{g}{f}\right)(8) &= \frac{g(8)}{f(8)} \\ &= \frac{37}{61}\end{aligned}$$