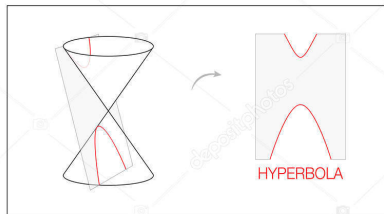
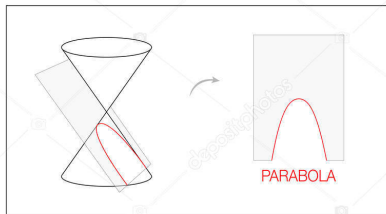
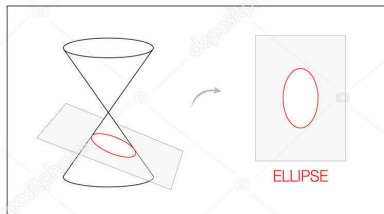
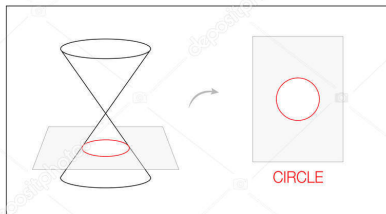


# Parabolas



# Objectives

- 1 Identify vertex, focus, and directrix from an equation.
- 2 Find the equation given vertex, focus, and/or directrix.
- 3 Convert parabolas from general to vertex form.

# Parabolas

If we look at the graph of the quadratic function  
 $f(x) = ax^2 + bx + c$ , we obtain what is known as a *parabola*.

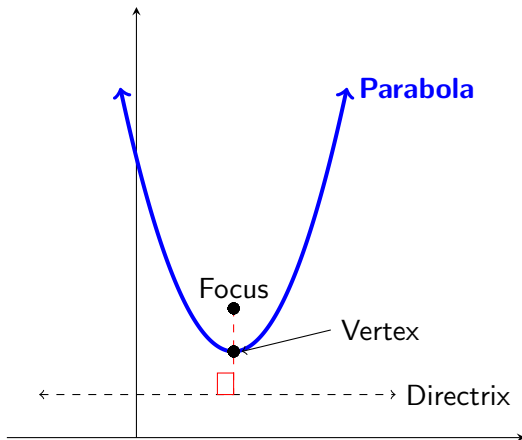
# Parabolas

If we look at the graph of the quadratic function  $f(x) = ax^2 + bx + c$ , we obtain what is known as a *parabola*.

## Parabolas

The set of all points in the plane that are the same distance from the focus and the directrix line.

# Parabolas



# General Form of a Parabola

The general form of a parabola is the familiar quadratic equation

$$f(x) = ax^2 + bx + c.$$

# General Form of a Parabola

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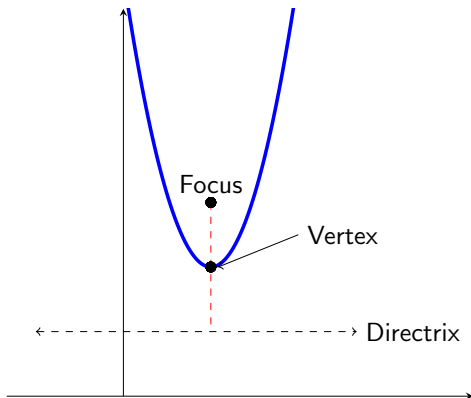
The **vertex forms** of a parabola are given below.

	<b>Opens Up or Down</b>	<b>Opens Left or Right</b>
	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Vertex	$(h, k)$	$(h, k)$
Focus	$(h, k + p)$ where $a = \frac{1}{4p}$	$(h + p, k)$ where $a = \frac{1}{4p}$
Directrix	$y = k - p$	$x = h - p$



# General Form of a Parabola

*Note:*  $p$  is the distance from the focus to the vertex, or the distance from the vertex to the directrix; either interpretation is correct.



## Example 1

Identify the vertex, focus, and directrix for each of the following.

(a)  $y = 2(x + 1)^2$

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Identify the vertex, focus, and directrix for each of the following.

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Vertex:  $(-1, 0)$

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$$\frac{1}{4p} = 2$$

$$1 = 8p$$

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Identify the vertex, focus, and directrix for each of the following.

(a)  $y = 2(x + 1)^2$

Vertex:  $(-1, 0)$

Focus:

$$\frac{1}{4p} = 2$$

$$1 = 8p$$

$$p = \frac{1}{8}$$

Example 1    Vertex:  $(-1, 0)$ ,  $p = \frac{1}{8}$

Focus:  $\left(-1, 0 + \frac{1}{8}\right)$

Example 1    Vertex:  $(-1, 0)$ ,  $p = \frac{1}{8}$

$$\text{Focus: } \left(-1, 0 + \frac{1}{8}\right) = \left(-1, \frac{1}{8}\right)$$

Example 1    Vertex:  $(-1, 0)$ ,  $p = \frac{1}{8}$

$$\text{Focus: } \left(-1, 0 + \frac{1}{8}\right) = \left(-1, \frac{1}{8}\right)$$

$$\text{Directrix: } y = -\frac{1}{8}$$



## Example 1

$$(b) \quad y = -0.25(x - 3)^2 + 4$$

## Example 1

$$(b) \quad y = -0.25(x - 3)^2 + 4$$

Vertex:  $(3, 4)$

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$$(b) \quad y = -0.25(x - 3)^2 + 4$$

Vertex:  $(3, 4)$

Focus:

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$$4p = 4$$

## Example 1

$$(b) \quad y = -0.25(x - 3)^2 + 4$$

Vertex:  $(3, 4)$

Focus:

$$\frac{1}{4p} = \frac{1}{4}$$

$$4p = 4$$

$$p = 1$$

Example 1    Vertex:  $(3, 4)$ ,  $p = 1$

Focus:  $(3, 4 - 1)$

Example 1    Vertex:  $(3, 4)$ ,  $p = 1$

Focus:  $(3, 4 - 1) = (3, 3)$

## Example 1    Vertex: $(3, 4)$ , $p = 1$

$$\text{Focus: } (3, 4 - 1) = (3, 3)$$

$$\text{Directrix: } y = 5$$



## Example 1

$$(c) \quad x = 2(y + 7)^2 - 9$$

## Example 1

$$(c) \quad x = 2(y + 7)^2 - 9$$

Vertex:  $(-9, -7)$

## Example 1

$$(c) \quad x = 2(y + 7)^2 - 9$$

Vertex:  $(-9, -7)$

Focus:

$$\frac{1}{4p} = 2$$

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$$(c) \quad x = 2(y + 7)^2 - 9$$

Vertex:  $(-9, -7)$

Focus:

$$\frac{1}{4p} = 2$$

$$1 = 8p$$

$$p = \frac{1}{8}$$

Example 1    Vertex:  $(-9, -7)$ ,  $p = \frac{1}{8}$

Focus:  $\left(-9 + \frac{1}{8}, -7\right)$

Example 1    Vertex:  $(-9, -7)$ ,  $p = \frac{1}{8}$

$$\text{Focus: } \left(-9 + \frac{1}{8}, -7\right) = \left(-\frac{71}{8}, -7\right)$$

Example 1    Vertex:  $(-9, -7)$ ,  $p = \frac{1}{8}$

$$\text{Focus: } \left(-9 + \frac{1}{8}, -7\right) = \left(-\frac{71}{8}, -7\right)$$

$$\text{Directrix: } x = -\frac{73}{8}$$



## Example 1

$$(d) \quad x = -3y^2$$

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Vertex:  $(0, 0)$

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$$(d) \quad x = -3y^2$$

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$$\frac{1}{4p} = 3$$

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Vertex:  $(0, 0)$

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## Example 1

$$(d) \quad x = -3y^2$$

Vertex:  $(0, 0)$

Focus:

$$\frac{1}{4p} = 3$$

$$1 = 12p$$

$$p = \frac{1}{12}$$

Example 1    Vertex:  $(0, 0)$ ,  $p = \frac{1}{12}$

Focus:  $\left(0 - \frac{1}{12}, 0\right)$

Example 1    Vertex:  $(0, 0)$ ,  $p = \frac{1}{12}$

$$\text{Focus: } \left(0 - \frac{1}{12}, 0\right) = \left(-\frac{1}{12}, 0\right)$$



Example 1    Vertex:  $(0, 0)$ ,  $p = \frac{1}{12}$

$$\text{Focus: } \left(0 - \frac{1}{12}, 0\right) = \left(-\frac{1}{12}, 0\right)$$

$$\text{Directrix: } x = \frac{1}{12}$$

# Objectives

- 1 Identify vertex, focus, and directrix from an equation.
- 2 Find the equation given vertex, focus, and/or directrix.
- 3 Convert parabolas from general to vertex form.

# How To

This is just working backwards from the previous section. Having any two of the three pieces of information is sufficient enough to create the equation for that parabola.

## Example 2

Find the equation of each parabola given the following information.

(a) Focus  $(0, -3)$ , directrix  $y = 3$

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## Example 2

Find the equation of each parabola given the following information.

(a) Focus  $(0, -3)$ , directrix  $y = 3$

Vertex:  $(0, 0)$

$$p = 3$$

$$a = \frac{1}{4p}$$

## Example 2

Find the equation of each parabola given the following information.

(a) Focus  $(0, -3)$ , directrix  $y = 3$

Vertex:  $(0, 0)$

$$p = 3$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(3)}$$



## Example 2

Find the equation of each parabola given the following information.

(a) Focus  $(0, -3)$ , directrix  $y = 3$

Vertex:  $(0, 0)$

$$p = 3$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(3)}$$

$$a = \frac{1}{12}$$

Example 2a    Vertex:  $(0, 0)$ ,  $a = \frac{1}{12}$

Since parabola opens down,  $a = -\frac{1}{12}$

Example 2a    Vertex:  $(0, 0)$ ,  $a = \frac{1}{12}$

Since parabola opens down,  $a = -\frac{1}{12}$

$$y = -\frac{1}{12}(x - 0)^2 + 0$$

Example 2a    Vertex:  $(0, 0)$ ,  $a = \frac{1}{12}$

Since parabola opens down,  $a = -\frac{1}{12}$

$$y = -\frac{1}{12}(x - 0)^2 + 0$$

$$y = -\frac{1}{12}x^2$$

## Example 2

(b) Focus  $(-1, 1)$ , directrix  $x = -3$

## Example 2

(b) Focus  $(-1, 1)$ , directrix  $x = -3$

Vertex:  $(-2, 1)$

## Example 2

(b) Focus  $(-1, 1)$ , directrix  $x = -3$

Vertex:  $(-2, 1)$

$$p = 1$$

## Example 2

(b) Focus  $(-1, 1)$ , directrix  $x = -3$

Vertex:  $(-2, 1)$

$$p = 1$$

$$a = \frac{1}{4p}$$



## Example 2

(b) Focus  $(-1, 1)$ , directrix  $x = -3$

Vertex:  $(-2, 1)$

$$p = 1$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

## Example 2

(b) Focus  $(-1, 1)$ , directrix  $x = -3$

Vertex:  $(-2, 1)$

$$p = 1$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

$$a = \frac{1}{4}$$

Example 2b    Vertex:  $(-2, 1)$ ,  $a = \frac{1}{4}$

Since parabola opens right,  $a = \frac{1}{4}$

Example 2b    Vertex:  $(-2, 1)$ ,  $a = \frac{1}{4}$

Since parabola opens right,  $a = \frac{1}{4}$

$$x = \frac{1}{4}(y - 1)^2 - 2$$

## Example 2

(c) Vertex  $(2, 1)$ , focus  $(2, 3)$

## Example 2

(c) Vertex  $(2, 1)$ , focus  $(2, 3)$

$$p = 2$$

## Example 2

(c) Vertex  $(2, 1)$ , focus  $(2, 3)$

$$p = 2$$

$$a = \frac{1}{4p}$$

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(c) Vertex  $(2, 1)$ , focus  $(2, 3)$

$$p = 2$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(2)}$$



## Example 2

(c) Vertex  $(2, 1)$ , focus  $(2, 3)$

$$p = 2$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(2)}$$

$$a = \frac{1}{8}$$

Example 2c    Vertex:  $(2, 1)$ ,  $a = \frac{1}{8}$

Since parabola opens up,  $a = \frac{1}{8}$

Example 2c    Vertex:  $(2, 1)$ ,  $a = \frac{1}{8}$

Since parabola opens up,  $a = \frac{1}{8}$

$$y = \frac{1}{8}(x - 2)^2 + 1$$

## Example 2

(d) Vertex  $(3, -1)$ , focus  $(2, -1)$

## Example 2

(d) Vertex  $(3, -1)$ , focus  $(2, -1)$

$$p = 1$$

## Example 2

(d) Vertex  $(3, -1)$ , focus  $(2, -1)$

$$p = 1$$

$$a = \frac{1}{4p}$$

## Example 2

(d) Vertex  $(3, -1)$ , focus  $(2, -1)$

$$p = 1$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

## Example 2

(d) Vertex  $(3, -1)$ , focus  $(2, -1)$

$$p = 1$$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

$$a = \frac{1}{4}$$



Example 2d    Vertex:  $(3, -1)$ ,  $a = \frac{1}{4}$

Since parabola opens to the right,  $a = -\frac{1}{4}$

Example 2d    Vertex:  $(3, -1)$ ,  $a = \frac{1}{4}$

Since parabola opens to the right,  $a = -\frac{1}{4}$

$$x = -\frac{1}{4}(y + 1)^2 + 3$$

# Objectives

- 1 Identify vertex, focus, and directrix from an equation.
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# Quadratic Expressions

This is revisiting writing quadratic expressions written in general form to standard form.

# Quadratic Expressions

This is revisiting writing quadratic expressions written in general form to standard form.

This is also known as **completing the square**

## Example 3

Write each in vertex form.

(a)  $y = -\frac{1}{4}x^2 + 5x - 24$

## Example 3

Write each in vertex form.

(a)  $y = -\frac{1}{4}x^2 + 5x - 24$

Vertex:  $(10, 1)$

## Example 3

Write each in vertex form.

$$(a) \quad y = -\frac{1}{4}x^2 + 5x - 24$$

Vertex:  $(10, 1)$

$$a = -\frac{1}{4}$$



## Example 3

Write each in vertex form.

$$(a) \quad y = -\frac{1}{4}x^2 + 5x - 24$$

Vertex:  $(10, 1)$

$$a = -\frac{1}{4}$$

$$y = -\frac{1}{4}(x - 10)^2 + 1$$

## Example 3

$$(b) \quad y = -x^2 - 4x - 14$$

## Example 3

$$(b) \quad y = -x^2 - 4x - 14$$

Vertex:  $(-2, -10)$

## Example 3

$$(b) \quad y = -x^2 - 4x - 14$$

Vertex:  $(-2, -10)$

$$a = -1$$

## Example 3

$$(b) \quad y = -x^2 - 4x - 14$$

Vertex:  $(-2, -10)$

$$a = -1$$

$$y = -(x + 2)^2 - 10$$

## Example 3

$$(c) \quad x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$$

## Example 3

$$(c) \quad x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$$

Vertex:  $(-4, -7)$

## Example 3

$$(c) \quad x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$$

Vertex:  $(-4, -7)$

$$a = \frac{1}{2}$$



## Example 3

$$(c) \quad x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$$

Vertex:  $(-4, -7)$

$$a = \frac{1}{2}$$

$$x = \frac{1}{2}(y + 7)^2 - 4$$

## Example 3

$$(d) \quad x = -y^2 - 2y + 9$$

## Example 3

$$(d) \quad x = -y^2 - 2y + 9$$

Vertex:  $(10, -1)$

## Example 3

$$(d) \quad x = -y^2 - 2y + 9$$

Vertex:  $(10, -1)$

$$a = -1$$

## Example 3

$$(d) \quad x = -y^2 - 2y + 9$$

Vertex:  $(10, -1)$

$$a = -1$$

$$x = -(y + 1)^2 + 10$$