Compositions of Functions

Objectives

1. Evaluate the composition of two functions at a value
2. Write the composition of two functions

Evaluating the Composition of Two Functions

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The composition of a function f and g denoted

$$(f \circ g)(x)$$

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where we plug $g(x)$ into the variable for $f(x)$.

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where we plug $g(x)$ into the variable for $f(x)$.

In other words, the output of $g(x)$ becomes the input of $f(x)$.

Functions as Machines

The following illustrates finding $(f \circ g)(8)$ in which

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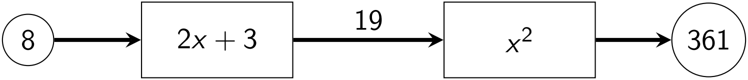

1. Evaluate $g(8)$ to get $2(8) + 3$, or 19.

2. Evaluate $f(19)$ to get 19^2, or 361.

Example 1

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

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$$= 26$$

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$$f(2) = 3(2) - 4$$

$$f(2) = 2$$

$$g(2) = 2^2 + 6$$

$$= 10$$

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$$f(-1) = 3(-1) - 4$$

Example 1 $f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(c) $(f \circ f)(1)$

$$
\begin{aligned}
(f \circ f)(1) &= f(f(1)) \\
f(1) &= 3(1) - 4 \\
f(1) &= -1 \\
f(-1) &= 3(-1) - 4 \\
&= -7
\end{aligned}
$$

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Writing the Composition of Two Functions

We can even substitute an entire function into another and simplify.

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Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$\begin{aligned} (f \circ g)(x) &= f(2x+3) \\ &= (2x+3)^2 \end{aligned}$$

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We can even substitute an entire function into another and simplify.

Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$\begin{aligned}(f \circ g)(x) &= f(2x+3) \\ &= (2x+3)^2 \\ &= (2x+3)(2x+3)\end{aligned}$$

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Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$
\begin{aligned}
(f \circ g)(x) &= f(2x + 3) \\
&= (2x + 3)^2 \\
&= (2x + 3)(2x + 3) \\
&= 4x^2 + 12x + 9
\end{aligned}
$$

Example 2

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

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$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= 3(x^2 + 6) - 4\end{aligned}$$

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$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= 3(x^2 + 6) - 4 \\ &= 3x^2 + 18 - 4\end{aligned}$$

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Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

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$$\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= 3(x^2 + 6) - 4 \\
&= 3x^2 + 18 - 4 \\
&= 3x^2 + 14
\end{aligned}$$

Example 2 $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

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$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= (3x-4)^2 + 6\end{aligned}$$

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(b) $\quad (g \circ f)(x)$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= (3x - 4)^2 + 6 \\
&= 9x^2 - 12x - 12x + 16 + 6
\end{aligned}
$$

(b) $(g \circ f)(x)$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= (3x - 4)^2 + 6 \\
&= 9x^2 - 12x - 12x + 16 + 6 \\
&= 9x^2 - 24x + 22
\end{aligned}
$$

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\begin{aligned}
(f \circ f)(x) &= f(f(x)) \\
&= 3(3x - 4) - 4 \\
&= 9x - 12 - 4 \\
&= 9x - 16
\end{aligned}
$$

Evaluating the Composition of Functions

In the previous video, we looked at things like

$$(f \circ g)(2)$$

We could also evaluate our answer for $(f \circ g)(x)$ at $x = 2$.