Objectives

1. Evaluate the composition of two functions at a value
2. Write the composition of two functions

Evaluating the Composition of Two Functions

Compositions of functions involve **substituting** one function into the variable(s) of another.

Evaluating the Composition of Two Functions

Compositions of functions involve **substituting** one function into the variable(s) of another.

The composition of a function f and g denoted

$$(f \circ g)(x)$$

is

$$(f \circ g)(x) = f(g(x))$$

where we plug $g(x)$ into the variable for $f(x)$.

Evaluating the Composition of Two Functions

Compositions of functions involve **substituting** one function into the variable(s) of another.

The composition of a function f and g denoted

$$(f \circ g)(x)$$

is

$$(f \circ g)(x) = f(g(x))$$

where we plug $g(x)$ into the variable for $f(x)$.

In other words, the output of $g(x)$ becomes the input of $f(x)$.

Functions as Machines

The following illustrates finding $(f \circ g)(8)$ in which

$$g(x) = 2x + 3 \text{ and } f(x) = x^2$$

:

Functions as Machines

The following illustrates finding $(f \circ g)(8)$ in which

$$g(x) = 2x + 3 \text{ and } f(x) = x^2$$

:

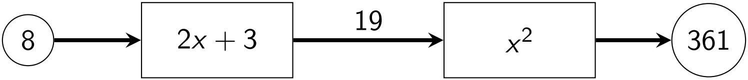
$$\textcircled{8} \longrightarrow \boxed{2x + 3} \xrightarrow{19} \boxed{x^2} \longrightarrow \textcircled{361}$$

Functions as Machines

The following illustrates finding $(f \circ g)(8)$ in which

$$g(x) = 2x + 3 \text{ and } f(x) = x^2$$

:



1. Evaluate $g(8)$ to get $2(8) + 3$, or 19.

Functions as Machines

The following illustrates finding $(f \circ g)(8)$ in which

$$g(x) = 2x + 3 \text{ and } f(x) = x^2$$

:

$$8 \longrightarrow \boxed{2x+3} \xrightarrow{19} \boxed{x^2} \longrightarrow 361$$

1. Evaluate $g(8)$ to get $2(8) + 3$, or 19.

2. Evaluate $f(19)$ to get 19^2, or 361.

Example 1

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

Example 1

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

$$(f \circ g)(2) = f(g(2))$$

Example 1

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

$$(f \circ g)(2) = f(g(2))$$

$$g(2) = 2^2 + 6$$

Example 1

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

$$(f \circ g)(2) = f(g(2))$$

$$g(2) = 2^2 + 6$$

$$g(2) = 10$$

Example 1

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

$$(f \circ g)(2) = f(g(2))$$

$$g(2) = 2^2 + 6$$

$$g(2) = 10$$

$$f(10) = 3(10) - 4$$

Example 1

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(2)$

$$(f \circ g)(2) = f(g(2))$$

$$g(2) = 2^2 + 6$$

$$g(2) = 10$$

$$f(10) = 3(10) - 4$$

$$= 26$$

Example 1 $f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(b) $(g \circ f)(2)$

Example 1 $\quad f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(b) $\quad (g \circ f)(2)$

$$(g \circ f)(2) = g(f(2))$$

Example 1 $f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(b) $(g \circ f)(2)$

$$(g \circ f)(2) = g(f(2))$$

$$f(2) = 3(2) - 4$$

Example 1 $\quad f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(b) $\quad (g \circ f)(2)$

$$(g \circ f)(2) = g(f(2))$$

$$f(2) = 3(2) - 4$$

$$f(2) = 2$$

Example 1 $f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(b) $(g \circ f)(2)$

$$(g \circ f)(2) = g(f(2))$$

$$f(2) = 3(2) - 4$$

$$f(2) = 2$$

$$g(2) = 2^2 + 6$$

Example 1 $\quad f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(b) $\quad (g \circ f)(2)$

$$\begin{aligned}(g \circ f)(2) &= g(f(2)) \\ f(2) &= 3(2) - 4 \\ f(2) &= 2 \\ g(2) &= 2^2 + 6 \\ &= 10\end{aligned}$$

Example 1 $f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(c) $(f \circ f)(1)$

Example 1 $\quad f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(c) $\quad (f \circ f)(1)$

$$(f \circ f)(1) = f(f(1))$$

Example 1 $\quad f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(c) $\quad (f \circ f)(1)$

$$(f \circ f)(1) = f(f(1))$$

$$f(1) = 3(1) - 4$$

Example 1 $f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(c) $(f \circ f)(1)$

$$(f \circ f)(1) = f(f(1))$$

$$f(1) = 3(1) - 4$$

$$f(1) = -1$$

Example 1 $\quad f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(c) $\quad (f \circ f)(1)$

$$(f \circ f)(1) = f(f(1))$$

$$f(1) = 3(1) - 4$$

$$f(1) = -1$$

$$f(-1) = 3(-1) - 4$$

Example 1 $\quad f(x) = 3x - 4 \quad g(x) = x^2 + 6$

(c) $\quad (f \circ f)(1)$

$$
\begin{aligned}
(f \circ f)(1) &= f(f(1)) \\
f(1) &= 3(1) - 4 \\
f(1) &= -1 \\
f(-1) &= 3(-1) - 4 \\
&= 7
\end{aligned}
$$

Objectives

1. Evaluate the composition of two functions at a value
2. Write the composition of two functions

Writing the Composition of Two Functions

We can even substitute an entire function into another and simplify.

Writing the Composition of Two Functions

We can even substitute an entire function into another and simplify.

Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

Writing the Composition of Two Functions

We can even substitute an entire function into another and simplify.

Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$(f \circ g)(x) = f(2x + 3)$$

Writing the Composition of Two Functions

We can even substitute an entire function into another and simplify.

Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$\begin{aligned}(f \circ g)(x) &= f(2x + 3) \\ &= (2x + 3)^2\end{aligned}$$

Writing the Composition of Two Functions

We can even substitute an entire function into another and simplify.

Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$\begin{aligned}(f \circ g)(x) &= f(2x + 3) \\ &= (2x + 3)^2 \\ &= (2x + 3)(2x + 3)\end{aligned}$$

Writing the Composition of Two Functions

We can even substitute an entire function into another and simplify.

Using $g(x) = 2x + 3$ and $f(x) = x^2$, the composition $(f \circ g)(x)$ becomes

$$\begin{aligned}(f \circ g)(x) &= f(2x+3) \\ &= (2x+3)^2 \\ &= (2x+3)(2x+3) \\ &= 4x^2 + 12x + 9\end{aligned}$$

Example 2

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(x)$

Example 2

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x))$$

Example 2

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(x)$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= 3(x^2 + 6) - 4\end{aligned}$$

Example 2

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(x)$

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= 3(x^2 + 6) - 4 \\
&= 3x^2 + 18 - 4
\end{aligned}
$$

Example 2

Find each of the following if $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(a) $(f \circ g)(x)$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= 3(x^2 + 6) - 4 \\ &= 3x^2 + 18 - 4 \\ &= 3x^2 + 14\end{aligned}$$

Example 2 $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(b) $(g \circ f)(x)$

Example 2 $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(b) $(g \circ f)(x)$

$$(g \circ f)(x) = g(f(x))$$

Example 2 $\quad f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(b) $\quad (g \circ f)(x)$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= (3x - 4)^2 + 6
\end{aligned}
$$

Example 2 $\quad f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(b) $\quad (g \circ f)(x)$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= (3x - 4)^2 + 6 \\
&= 9x^2 - 12x - 12x + 16 + 6
\end{aligned}
$$

Example 2 $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(b) $(g \circ f)(x)$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= (3x - 4)^2 + 6 \\
&= 9x^2 - 12x - 12x + 16 + 6 \\
&= 9x^2 - 24x + 22
\end{aligned}
$$

Example 2 $\quad f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(c) $\quad (f \circ f)(x)$

Example 2 $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(c) $(f \circ f)(x)$

$$(f \circ f)(x) = f(f(x))$$

Example 2 $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(c) $(f \circ f)(x)$

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= 3(3x - 4) - 4\end{aligned}$$

Example 2 $\quad f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(c) $\quad (f \circ f)(x)$

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= 3(3x - 4) - 4 \\ &= 9x - 12 - 4\end{aligned}$$

Example 2 $f(x) = 3x - 4$ and $g(x) = x^2 + 6$

(c) $(f \circ f)(x)$

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= 3(3x - 4) - 4 \\ &= 9x - 12 - 4 \\ &= 9x - 16\end{aligned}$$

Evaluating the Composition of Functions

In the previous video, we looked at things like

$$(f \circ g)(2)$$

We could also evaluate our answer for $(f \circ g)(x)$ at $x = 2$.