Graphs of Sine and Cosine

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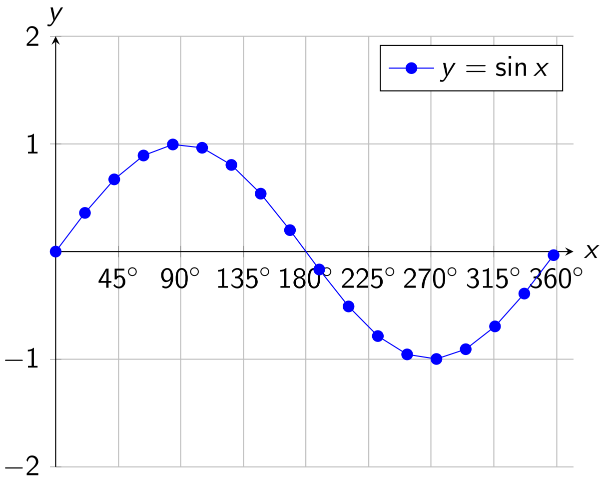
If we let the x-coordinates represent the measure of the angle and let the y-coordinates represent the value of the ratio, we can plot the graph of the sine function.



y
2
1
−1
−2
x
45° 90° 135° 180° 225° 270° 315° 360°
y = sin x

Table of Contents

Amplitude

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In the graph previously, the maximum of the graph is 1. The minimum of the graph is -1. Thus, the amplitude of $y = \sin x$ is

$$\frac{1}{2}\,(1 - (-1)) = 1$$

If we want to change the amplitude, we need to **stretch the graph vertically**. Recall that vertical stretches involve multiplying the function by a positive value (negative values reflect the graph across the x-axis).

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For functions in the form

$$y = A \sin x \qquad \text{or} \qquad y = B \cos x$$

the amplitude is $|A|$.

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Determine the amplitude of each of the following.

(a) $y = 3\sin x$

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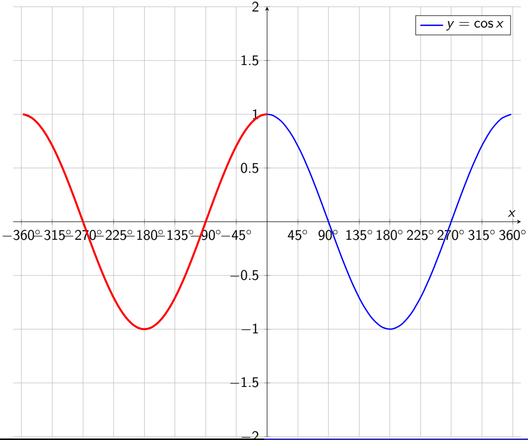
(d) $y = -2\cos x$

$$\text{Amplitude} = |-2| = 2$$

Table of Contents

Period

The graphs of sine and cosine are **periodic**, in that the pattern repeats itself infinitely in both directions. The graph of $y = \cos x$ is shown below.



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- How long until the graph starts to repeat the same values in the same order.

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Each of the following are ways in which you can think of the period of the graph of a function:

- How long until the graph starts to repeat the same values in the same order.
- What is the least amount of the graph you would need to copy to paste it before and after the copy?
- If the units along the x-axis were length, period would be the wavelength from science class.

Changing the Period

We can adjust the period of the sine and cosine functions by multiplying the input values, x, by a positive number other than 1.

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We can adjust the period of the sine and cosine functions by multiplying the input values, x, by a positive number other than 1.

Thus, the equation for adjusting the period of sine and cosine functions is

$$y = \sin(Bx) \quad \text{and} \quad y = \cos(Bx)$$

Example 2

Use a graphing utility to determine the period of each of the following.

(a) $y = \sin(2x)$

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(b) $y = \cos(3x)$

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Use a graphing utility to determine the period of each of the following.

(a) $y = \sin(2x)$

Period $= 180^\circ$

(b) $y = \cos(3x)$

Period $= 120^\circ$

Example 2

(c) $y = \sin\left(\frac{1}{2}x\right)$

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Period $= 720^\circ$

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(c) $y = \sin\left(\frac{1}{2}x\right)$

Period $= 720^\circ$

(d) $y = \cos\left(\frac{1}{4}x\right)$

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(c) $y = \sin\left(\frac{1}{2}x\right)$

Period $= 720^\circ$

(d) $y = \cos\left(\frac{1}{4}x\right)$

Period $= 1440^\circ$

Period

With $y = \sin(Bx)$ and $y = \cos(Bx)$, looking at the graphs, it would seem that the different values of B affect the graphs in different ways.

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Notice in each case, our answers are whatever $360^\circ/B$ equals.

Period

With $y = \sin(Bx)$ and $y = \cos(Bx)$, looking at the graphs, it would seem that the different values of B affect the graphs in different ways.

Notice in each case, our answers are whatever $360^\circ/B$ equals.

Therefore, the period of the graph of the sine and cosine functions is

$$\frac{360^\circ}{B} \quad \text{or} \quad \frac{2\pi}{B}$$

Table of Contents

Vertical Shifts

Recall from transforming functions that we shift functions vertically by adding or subtracting a value *from the function itself*.

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Recall from transforming functions that we shift functions vertically by adding or subtracting a value *from the function itself*.

For sine and cosine functions, the vertical shift becomes

$$y = \sin x + D \quad \text{and} \quad y = \cos x + D$$

Example 3

Determine the vertical shift for each of the following.

(a) $y = \sin x + 3$

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Vertical shift = Up 3 units

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(b) $y = \cos x - 1$

Example 3

Determine the vertical shift for each of the following.

(a) $y = \sin x + 3$

Vertical shift $=$ Up 3 units

(b) $y = \cos x - 1$

Vertical shift $=$ Down 1 unit

Example 3

(c) $y = 2\sin x - 4$

Example 3

(c) $y = 2\sin x - 4$

Vertical shift = Down 4 units

Example 3

(c) $y = 2\sin x - 4$

Vertical shift $=$ Down 4 units

(d) $y = -0.5\cos x$

Example 3

(c) $y = 2\sin x - 4$

Vertical shift = Down 4 units

(d) $y = -0.5\cos x$

Vertical shift = None (or 0 units)

Table of Contents

Phase Shifts

In trigonometry, phase shifts are the periodic functions' version of horizontal shifts.

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In trigonometry, phase shifts are the periodic functions' version of horizontal shifts.

For sine and cosine functions, phase shifts typically resemble the following:

$$y = \sin(Bx - C) \quad \text{and} \quad y = \cos(Bx - C)$$

Phase Shifts

To find the value of the phase shift, set $Bx - C = 0$ and solve.

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- Positive answer: shifts right

Phase Shifts

To find the value of the phase shift, set $Bx - C = 0$ and solve.

This value will be how far from the origin your graph shifts:

- Positive answer: shifts right
- Negative answer: shifts left

Example 4a

Determine the phase shift of each of the following. Don't forget to indicate direction.

(a) $y = \sin(x - 30^\circ)$

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Determine the phase shift of each of the following. Don't forget to indicate direction.

(a) $y = \sin(x - 30^\circ)$

$$x - 30 = 0$$

$$x = 30$$

Example 4a

Determine the phase shift of each of the following. Don't forget to indicate direction.

(a) $y = \sin(x - 30^\circ)$

$$x - 30 = 0$$

$$x = 30$$

The graph is shifted 30° to the right.

Example 4b

(b) $y = \cos(x + 135^\circ)$

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(b) $y = \cos(x + 135^\circ)$

$$x + 135 = 0$$

Example 4b

(b) $y = \cos(x + 135^\circ)$

$$x + 135 = 0$$

$$x = -135$$

Example 4b

(b) $y = \cos(x + 135°)$

$$x + 135 = 0$$

$$x = -135$$

The graph is shifted 135° to the left

Example 4c

(c) $y = \sin(2x + 90^\circ)$

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(c) $y = \sin(2x + 90^\circ)$

$$2x + 90 = 0$$

Example 4c

(c) $y = \sin(2x + 90^\circ)$

$$2x + 90 = 0$$

$$2x = -90$$

Example 4c

(c) $y = \sin(2x + 90^\circ)$

$$2x + 90 = 0$$

$$2x = -90$$

$$x = -45$$

Example 4c

(c) $y = \sin(2x + 90^\circ)$

$$2x + 90 = 0$$

$$2x = -90$$

$$x = -45$$

The graph is shifted 45° to the left

Example 4d

(d) $y = \cos(3x - 270^\circ)$

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(d) $y = \cos(3x - 270^\circ)$

$$3x - 270 = 0$$

Example 4d

(d) $y = \cos(3x - 270^\circ)$

$$3x - 270 = 0$$

$$3x = 270$$

Example 4d

(d) $y = \cos(3x - 270^\circ)$

$$3x - 270 = 0$$

$$3x = 270$$

$$x = 90$$

Example 4d

(d) $y = \cos(3x - 270^\circ)$

$$3x - 270 = 0$$

$$3x = 270$$

$$x = 90$$

The graph is shifted 90° to the right

Summary

For $y = A\sin(Bx - C) + D$ or $y = A\cos(Bx - C) + D$:

Amplitude	**Period**	**Phase Shift**	**Vertical Shift**
$\lvert A\rvert$	$\dfrac{360^\circ}{B}$ or $\dfrac{2\pi}{B}$	$\dfrac{C}{B}$	D