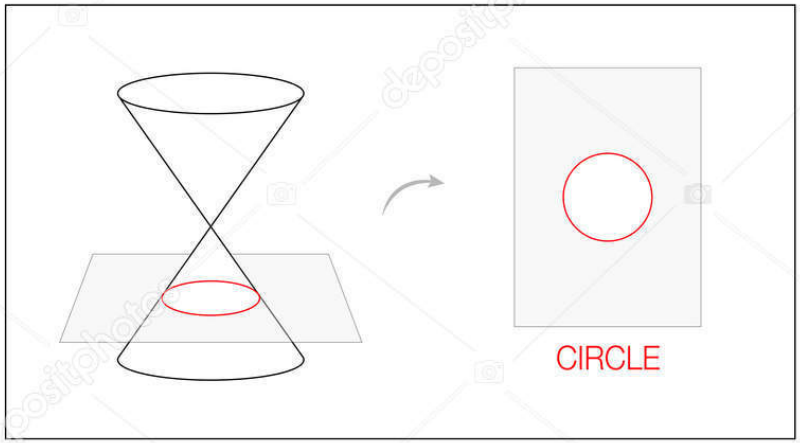
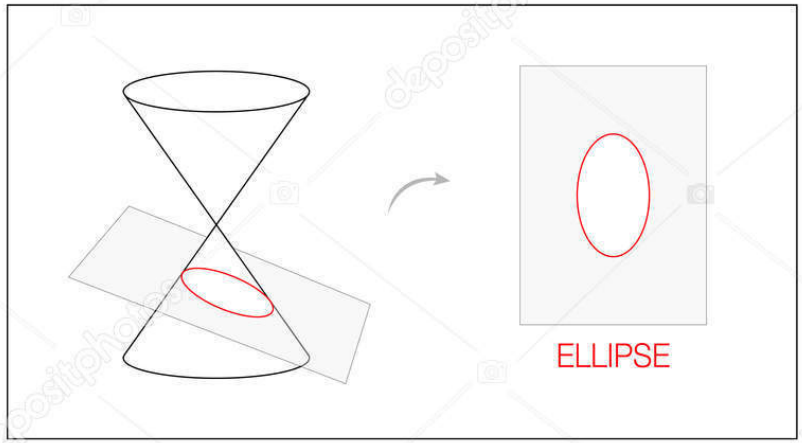
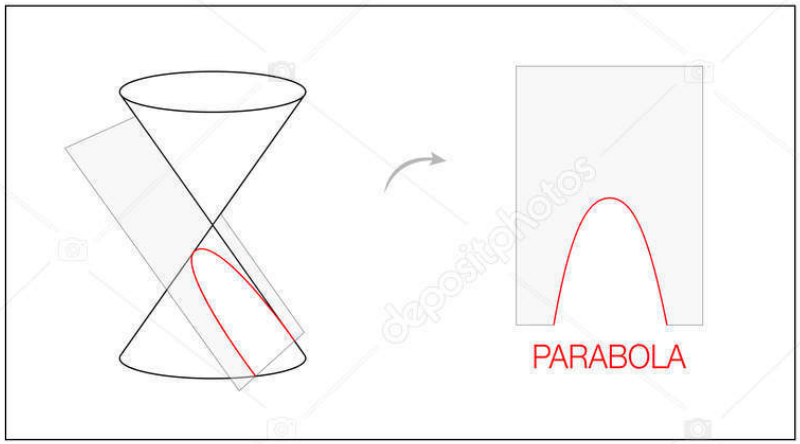
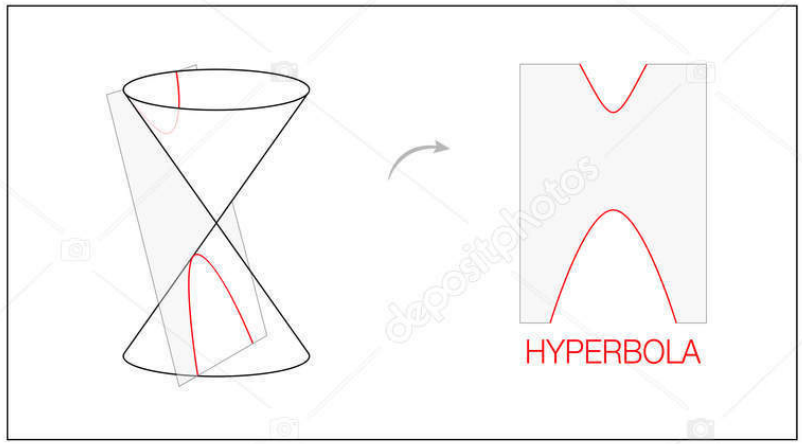
Parabolas









Objectives

1. Identify vertex, focus, and directrix from an equation.
2. Find the equation given vertex, focus, and/or directrix.
3. Convert parabolas from general to vertex form.

Parabolas

If we look at the graph of the quadratic function $f(x) = ax^2 + bx + c$, we obtain what is known as a *parabola*.

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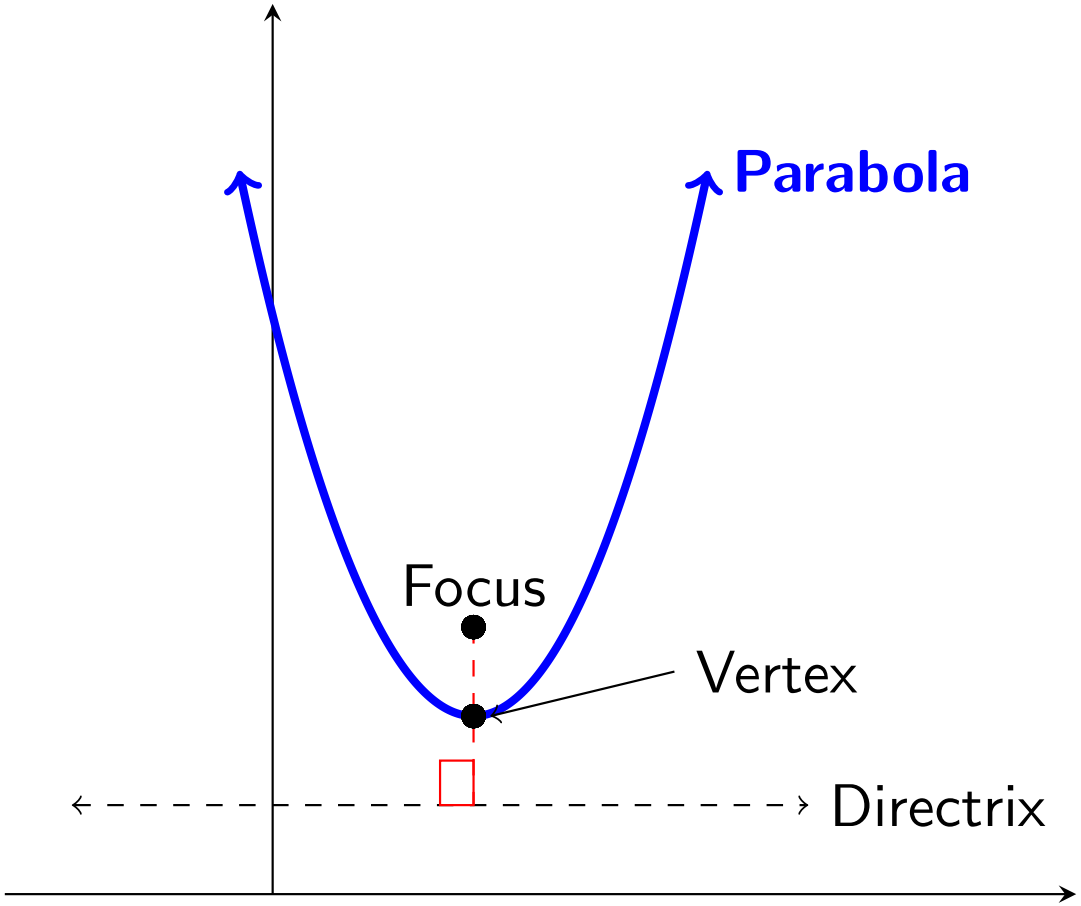
Parabolas

The set of all points in the plane that are the same distance from the focus and the directrix line.

Parabolas



General Form of a Parabola

The general form of a parabola is the familiar quadratic equation $f(x) = ax^2 + bx + c$.

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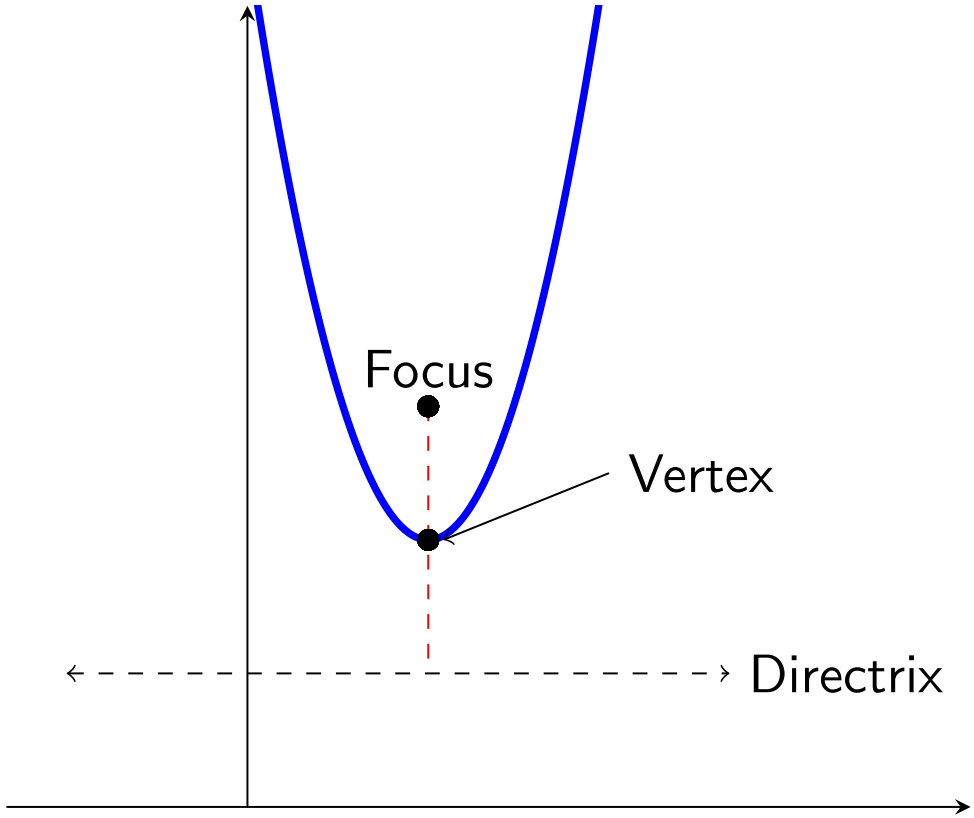
The **vertex forms** of a parabola are given below.

	Opens Up or Down	**Opens Left or Right**
	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
Vertex	(h, k)	(h, k)
Focus	$(h, k+p)$ where $a = \frac{1}{4p}$	$(h+p, k)$ where $a = \frac{1}{4p}$
Directrix	$y = k - p$	$x = h - p$

General Form of a Parabola

Note: p is the distance from the focus to the vertex, or the distance from the vertex to the directrix; either interpretation is correct.



Example 1

Identify the vertex, focus, and directrix for each of the following.

(a) $y = 2(x+1)^2$

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Vertex: $(-1, 0)$

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Focus:

$$\frac{1}{4p} = 2$$

$$1 = 8p$$

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(a) $y = 2(x+1)^2$

Vertex: $(-1, 0)$

Focus:

$$\frac{1}{4p} = 2$$

$$1 = 8p$$

$$p = \frac{1}{8}$$

Example 1 Vertex: $(-1, 0)$, $p = \frac{1}{8}$

Focus: $\left(-1, 0 + \frac{1}{8}\right)$

Example 1 Vertex: $(-1, 0)$, $p = \frac{1}{8}$

Focus: $\left(-1, 0 + \frac{1}{8}\right) = \left(-1, \frac{1}{8}\right)$

Example 1 Vertex: $(-1, 0)$, $p = \frac{1}{8}$

Focus: $\left(-1, 0 + \frac{1}{8}\right) = \left(-1, \frac{1}{8}\right)$

Directrix: $y = -\frac{1}{8}$

Example 1

(b) $y = -0.25(x-3)^2 + 4$

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Vertex: $(3, 4)$

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Focus:

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Focus:

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$$4p = 4$$

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(b) $y = -0.25(x-3)^2 + 4$

Vertex: $(3, 4)$

Focus:

$$\frac{1}{4p} = \frac{1}{4}$$

$$4p = 4$$

$$p = 1$$

Example 1 Vertex: $(3, 4)$, $p = 1$

Focus: $(3, 4 - 1)$

Example 1 Vertex: $(3, 4)$, $p = 1$

Focus: $(3, 4 - 1) = (3, 3)$

Example 1 Vertex: $(3, 4)$, $p = 1$

Focus: $(3, 4 - 1) \qquad = (3, 3)$

Directrix: $y = 5$

Example 1

(c) $x = 2(y + 7)^2 - 9$

Example 1

(c) $x = 2(y + 7)^2 - 9$

Vertex: $(-9, -7)$

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Vertex: $(-9, -7)$

Focus:

$$\frac{1}{4p} = 2$$

$$1 = 8p$$

$$p = \frac{1}{8}$$

Example 1 Vertex: $(-9, -7)$, $p = \frac{1}{8}$

Focus: $\left(-9 + \frac{1}{8}, -7\right)$

Example 1 Vertex: $(-9, -7)$, $p = \frac{1}{8}$

Focus: $\left(-9 + \frac{1}{8}, -7\right) = \left(-\frac{71}{8}, -7\right)$

Example 1 Vertex: $(-9, -7)$, $p = \frac{1}{8}$

Focus: $\left(-9 + \frac{1}{8}, -7\right) = \left(-\frac{71}{8}, -7\right)$

Directrix: $x = -\frac{73}{8}$

Example 1

(d) $x = -3y^2$

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Vertex: $(0, 0)$

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Focus:

$$\frac{1}{4p} = 3$$

$$1 = 12p$$

Example 1

(d) $x = -3y^2$

Vertex: $(0, 0)$

Focus:

$$\frac{1}{4p} = 3$$

$$1 = 12p$$

$$p = \frac{1}{12}$$

Example 1 Vertex: $(0, 0)$, $p = \frac{1}{12}$

Focus: $\left(0 - \frac{1}{12}, 0\right)$

Example 1 Vertex: $(0,0)$, $p = \frac{1}{12}$

Focus: $\left(0 - \frac{1}{12}, 0\right) = \left(-\frac{1}{12}, 0\right)$

Example 1 Vertex: $(0,0)$, $p = \frac{1}{12}$

Focus: $\left(0 - \frac{1}{12}, 0\right) = \left(-\frac{1}{12}, 0\right)$

Directrix: $x = \frac{1}{12}$

Objectives

1. Identify vertex, focus, and directrix from an equation.
2. Find the equation given vertex, focus, and/or directrix.
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How To

This is just working backwards from the previous section. Having any two of the three pieces of information is sufficient enough to create the equation for that parabola.

Example 2

Find the equation of each parabola given the following information.

(a) Focus $(0, -3)$, directrix $y = 3$

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(a) Focus $(0, -3)$, directrix $y = 3$

Vertex: $(0, 0)$

$p = 3$

$$a = \frac{1}{4p}$$

Example 2

Find the equation of each parabola given the following information.

(a) Focus $(0, -3)$, directrix $y = 3$

Vertex: $(0, 0)$

$p = 3$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(3)}$$

Example 2

Find the equation of each parabola given the following information.

(a) Focus $(0, -3)$, directrix $y = 3$

Vertex: $(0, 0)$

$p = 3$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(3)}$$

$$a = \frac{1}{12}$$

Example 2a Vertex: $(0, 0)$, $a = \frac{1}{12}$

Since parabola opens down, $a = -\frac{1}{12}$

Example 2a Vertex: $(0, 0)$, $a = \frac{1}{12}$

Since parabola opens down, $a = -\frac{1}{12}$

$$y = -\frac{1}{12}(x - 0)^2 + 0$$

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Since parabola opens down, $a = -\frac{1}{12}$

$$y = -\frac{1}{12}(x - 0)^2 + 0$$

$$y = -\frac{1}{12}x^2$$

Example 2

(b) Focus $(-1, 1)$, directrix $x = -3$

Example 2

(b) Focus $(-1, 1)$, directrix $x = -3$

Vertex: $(-2, 1)$

Example 2

(b) Focus $(-1, 1)$, directrix $x = -3$

Vertex: $(-2, 1)$

$p = 1$

Example 2

(b) Focus $(-1, 1)$, directrix $x = -3$

Vertex: $(-2, 1)$

$p = 1$

$$a = \frac{1}{4p}$$

Example 2

(b) Focus $(-1, 1)$, directrix $x = -3$

Vertex: $(-2, 1)$

$p = 1$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

Example 2

(b) Focus $(-1, 1)$, directrix $x = -3$

Vertex: $(-2, 1)$

$p = 1$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

$$a = \frac{1}{4}$$

Example 2b Vertex: $(-2, 1)$, $a = \frac{1}{4}$

Since parabola opens right, $a = \dfrac{1}{4}$

Example 2b Vertex: $(-2, 1)$, $a = \frac{1}{4}$

Since parabola opens right, $a = \dfrac{1}{4}$

$$x = \frac{1}{4}(y-1)^2 - 2$$

Example 2

(c) Vertex $(2, 1)$, focus $(2, 3)$

Example 2

(c) Vertex $(2, 1)$, focus $(2, 3)$

$p = 2$

Example 2

(c) Vertex $(2, 1)$, focus $(2, 3)$

$p = 2$

$$a = \frac{1}{4p}$$

Example 2

(c) Vertex $(2, 1)$, focus $(2, 3)$

$p = 2$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(2)}$$

Example 2

(c) Vertex $(2, 1)$, focus $(2, 3)$

$p = 2$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(2)}$$

$$a = \frac{1}{8}$$

Example 2c Vertex: $(2, 1)$, $a = \frac{1}{8}$

Since parabola opens up, $a = \dfrac{1}{8}$

Example 2c Vertex: $(2, 1)$, $a = \frac{1}{8}$

Since parabola opens up, $a = \dfrac{1}{8}$

$$y = \frac{1}{8}(x - 2)^2 + 1$$

Example 2

(d) Vertex $(3, -1)$, focus $(2, -1)$

Example 2

(d) Vertex $(3, -1)$, focus $(2, -1)$

$p = 1$

Example 2

(d) Vertex $(3, -1)$, focus $(2, -1)$

$p = 1$

$$a = \frac{1}{4p}$$

Example 2

(d) Vertex $(3, -1)$, focus $(2, -1)$

$p = 1$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

Example 2

(d) Vertex $(3, -1)$, focus $(2, -1)$

$p = 1$

$$a = \frac{1}{4p}$$

$$a = \frac{1}{4(1)}$$

$$a = \frac{1}{4}$$

Example 2d Vertex: $(3, -1)$, $a = \frac{1}{4}$

Since parabola opens to the right, $a = -\frac{1}{4}$

Example 2d Vertex: $(3, -1)$, $a = \frac{1}{4}$

Since parabola opens to the right, $a = -\frac{1}{4}$

$$x = -\frac{1}{4}(y+1)^2 + 3$$

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Quadratic Expressions

This is revisiting writing quadratic expressions written in general form to standard form.

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This is revisiting writing quadratic expressions written in general form to standard form.

This is also known as completing the square

Example 3

Write each in vertex form.

(a) $y = -\frac{1}{4}x^2 + 5x - 24$

Example 3

Write each in vertex form.

(a) $y = -\frac{1}{4}x^2 + 5x - 24$

Vertex: $(10, 1)$

Example 3

Write each in vertex form.

(a) $y = -\frac{1}{4}x^2 + 5x - 24$

Vertex: $(10, 1)$

$a = -\frac{1}{4}$

Example 3

Write each in vertex form.

(a) $y = -\frac{1}{4}x^2 + 5x - 24$

Vertex: $(10, 1)$

$a = -\frac{1}{4}$

$$y = -\frac{1}{4}(x - 10)^2 + 1$$

Example 3

(b) $y = -x^2 - 4x - 14$

Example 3

(b) $y = -x^2 - 4x - 14$

Vertex: $(-2, -10)$

Example 3

(b) $y = -x^2 - 4x - 14$

Vertex: $(-2, -10)$

$a = -1$

Example 3

(b) $y = -x^2 - 4x - 14$

Vertex: $(-2, -10)$

$a = -1$

$$y = -(x + 2)^2 - 10$$

Example 3

(c) $x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$

Example 3

(c) $x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$

Vertex: $(-4, -7)$

Example 3

(c) $x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$

Vertex: $(-4, -7)$

$a = \frac{1}{2}$

Example 3

(c) $x = \frac{1}{2}y^2 + 7y + \frac{41}{2}$

Vertex: $(-4, -7)$

$a = \frac{1}{2}$

$$x = \frac{1}{2}(y + 7)^2 - 4$$

Example 3

(d) $x = -y^2 - 2y + 9$

Example 3

(d) $x = -y^2 - 2y + 9$

Vertex: $(10, -1)$

Example 3

(d) $x = -y^2 - 2y + 9$

Vertex: $(10, -1)$

$a = -1$

Example 3

(d) $x = -y^2 - 2y + 9$

Vertex: $(10, -1)$

$a = -1$

$$x = -(y + 1)^2 + 10$$