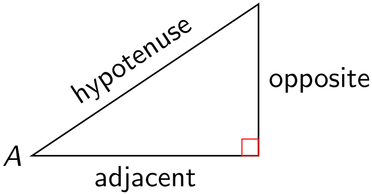
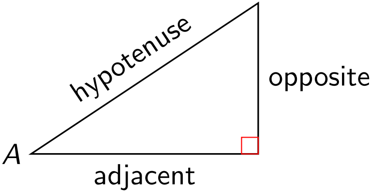
Right Triangle Trigonometry

Table of Contents

In geometry class, you learned about three trigonometric ratios: sine, cosine, and tangent.



In geometry class, you learned about three trigonometric ratios: sine, cosine, and tangent.



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan A = \frac{\text{opposite}}{\text{adjacent}}$$

SOH-CAH-TOA

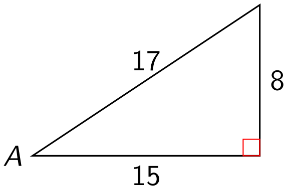
We usually remember this as SOH-CAH-TOA.

Sometimes you may need to use the Pythagorean Theorem, $a^2 + b^2 = c^2$, in order to find any missing sides.

Example 1a

Write the exact ratios for sine, cosine, and tangent of angle A for each of the following.

(a)



Example 1a

Write the exact ratios for sine, cosine, and tangent of angle A for each of the following.

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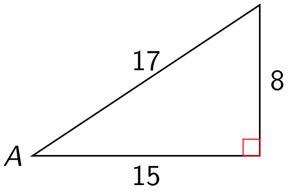
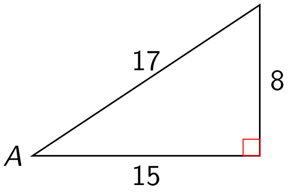


$$\sin A = \frac{8}{17}$$

Example 1a

Write the exact ratios for sine, cosine, and tangent of angle A for each of the following.

(a)



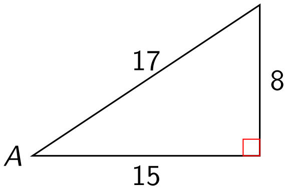
$$\sin A = \frac{8}{17}$$

$$\cos A = \frac{15}{17}$$

Example 1a

Write the exact ratios for sine, cosine, and tangent of angle A for each of the following.

(a)



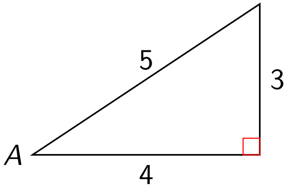
$$\sin A = \frac{8}{17}$$

$$\cos A = \frac{15}{17}$$

$$\tan A = \frac{8}{15}$$

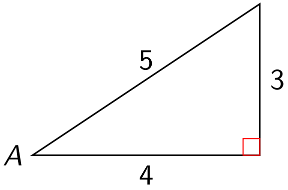
Example 1b

(b)



Example 1b

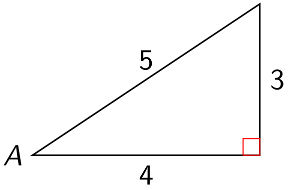
(b)



$$\sin A = \frac{3}{5}$$

Example 1b

(b)

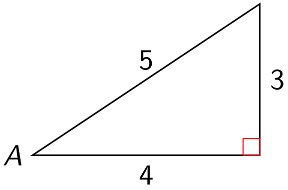


$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

Example 1b

(b)



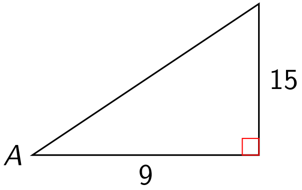
$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

$$\tan A = \frac{3}{4}$$

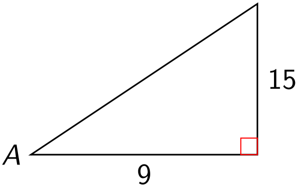
Example 1c

(c)



Example 1c

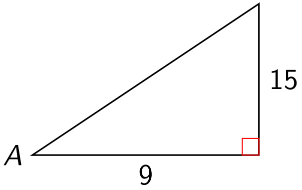
(c)



$$9^2 + 15^2 = c^2$$

Example 1c

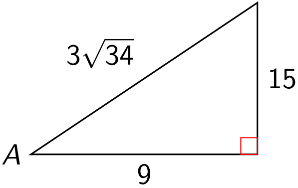
(c)



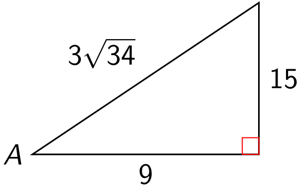
$$9^2 + 15^2 = c^2$$

$$c = \sqrt{306} = 3\sqrt{34}$$

Example 1c

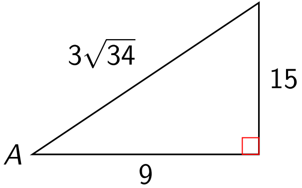


Example 1c



$$\sin A = \frac{15}{3\sqrt{34}}$$

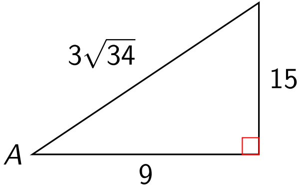
Example 1c



$$\sin A = \frac{15}{3\sqrt{34}}$$

$$= \frac{5}{\sqrt{34}}$$

Example 1c

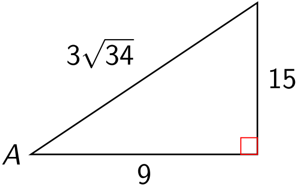


$$\sin A = \frac{15}{3\sqrt{34}}$$

$$= \frac{5}{\sqrt{34}}$$

$$= \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$$

Example 1c



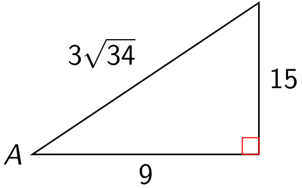
$$\sin A = \frac{15}{3\sqrt{34}}$$

$$= \frac{5}{\sqrt{34}}$$

$$= \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$$

$$= \frac{5\sqrt{34}}{34}$$

Example 1c



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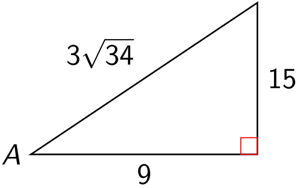
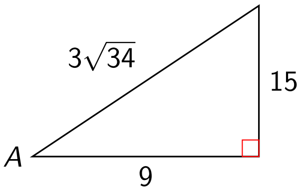


$$\cos A = \frac{9}{3\sqrt{34}}$$

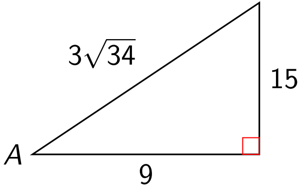
Example 1c



$$\cos A = \frac{9}{3\sqrt{34}}$$

$$= \frac{3}{\sqrt{34}}$$

Example 1c

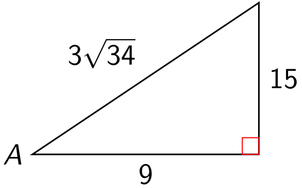


$$\cos A = \frac{9}{3\sqrt{34}}$$

$$= \frac{3}{\sqrt{34}}$$

$$= \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$$

Example 1c



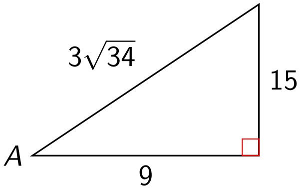
$$\cos A = \frac{9}{3\sqrt{34}}$$

$$= \frac{3}{\sqrt{34}}$$

$$= \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}}$$

$$= \frac{3\sqrt{34}}{34}$$

Example 1c



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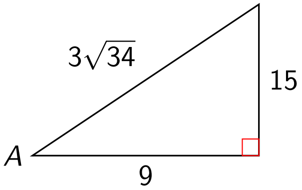
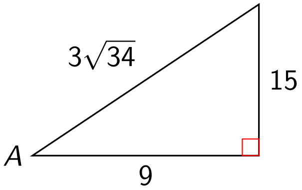


$$\tan A = \frac{15}{9}$$

Example 1c



$$\tan A = \frac{15}{9}$$

$$= \frac{5}{3}$$

Other Trig Ratios

In addition to the "Big-3": sine, cosine, and tangent, there are three additional trigonometric ratios.

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These ratios (*cosecant*, *secant*, and *cotangent*) are the reciprocals of sine, cosine, and tangent, respectively.

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In addition to the "Big-3": sine, cosine, and tangent, there are three additional trigonometric ratios.

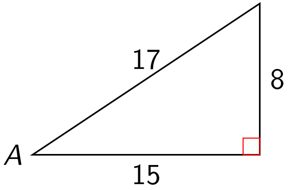
These ratios (*cosecant*, *secant*, and *cotangent*) are the reciprocals of sine, cosine, and tangent, respectively.

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec A = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot A = \frac{\text{adjacent}}{\text{opposite}}$$

Example 2a

Write the exact ratios for cosecant, secant, and cotangent of angle A for each of the following.

(a)



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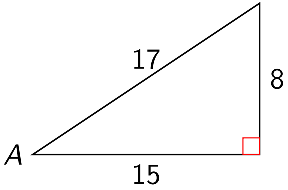
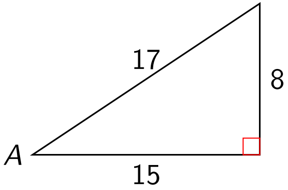


$$\csc A = \frac{17}{8}$$

Example 2a

Write the exact ratios for cosecant, secant, and cotangent of angle A for each of the following.

(a)



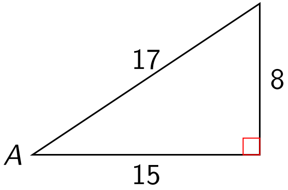
$$\csc A = \frac{17}{8}$$

$$\sec A = \frac{17}{15}$$

Example 2a

Write the exact ratios for cosecant, secant, and cotangent of angle A for each of the following.

(a)



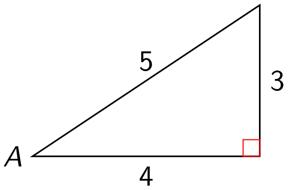
$$\csc A = \frac{17}{8}$$

$$\sec A = \frac{17}{15}$$

$$\cot A = \frac{15}{8}$$

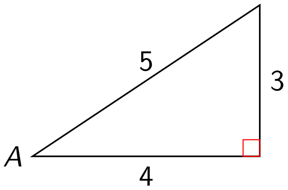
Example 2b

(b)



Example 2b

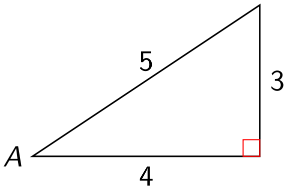
(b)



$$\csc A = \frac{5}{3}$$

Example 2b

(b)

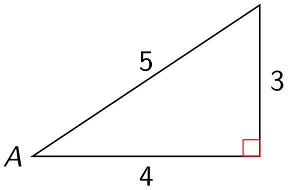


$$\csc A = \frac{5}{3}$$

$$\sec A = \frac{5}{4}$$

Example 2b

(b)



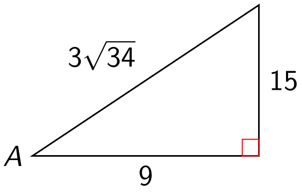
$$\csc A = \frac{5}{3}$$

$$\sec A = \frac{5}{4}$$

$$\cot A = \frac{4}{3}$$

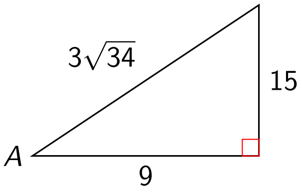
Example 2c

(c)



Example 2c

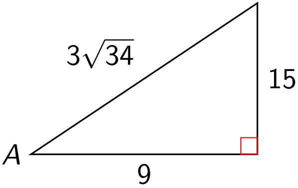
(c)



$$\csc A = \frac{3\sqrt{34}}{15} = \frac{\sqrt{34}}{5}$$

Example 2c

(c)

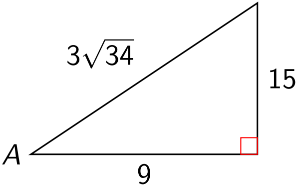


$$\csc A = \frac{3\sqrt{34}}{15} = \frac{\sqrt{34}}{5}$$

$$\sec A = \frac{3\sqrt{34}}{9} = \frac{\sqrt{34}}{3}$$

Example 2c

(c)



$$\csc A = \frac{3\sqrt{34}}{15} = \frac{\sqrt{34}}{5}$$

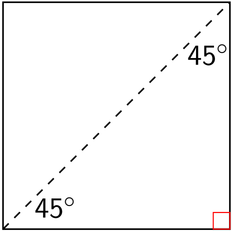
$$\sec A = \frac{3\sqrt{34}}{9} = \frac{\sqrt{34}}{3}$$

$$\cot A = \frac{9}{15} = \frac{3}{5}$$

Table of Contents

45-45-90 Triangles

45-45-90 triangles (also known as *isosceles right triangles*) can be created by drawing a diagonal across a square:



45-45-90 Triangles

Since each side of a square is the same length, we can use whatever length we want. For simplicity, we will use a length of 1.

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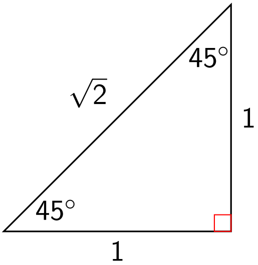
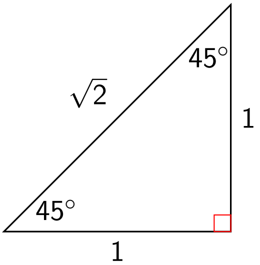


Example 3

Find the exact values of the six trig ratios for 45°.



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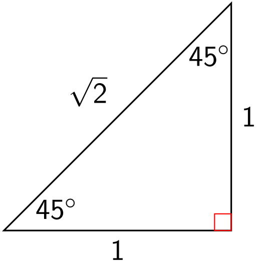
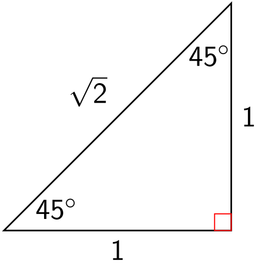


$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

Example 3

Find the exact values of the six trig ratios for 45°.

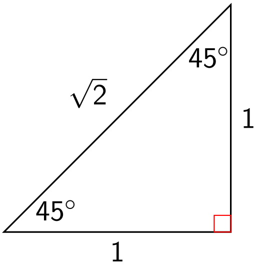


$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Example 3

Find the exact values of the six trig ratios for 45°.



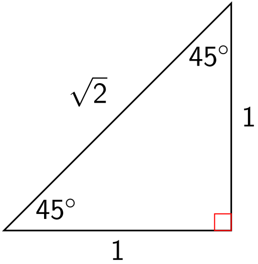
$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Example 3

Find the exact values of the six trig ratios for 45°.



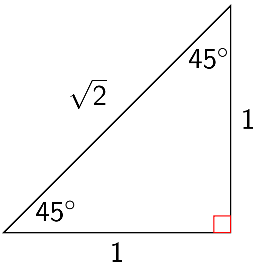
$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$

Example 3



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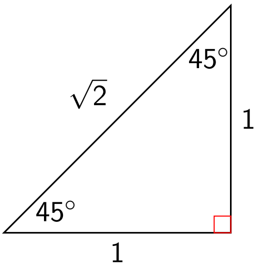
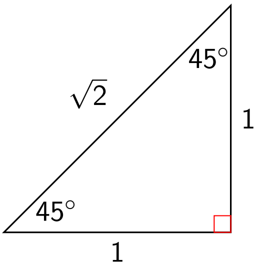


$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

Example 3



$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

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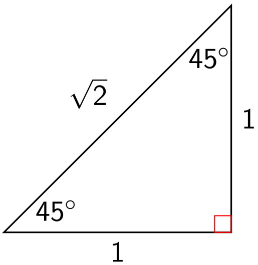
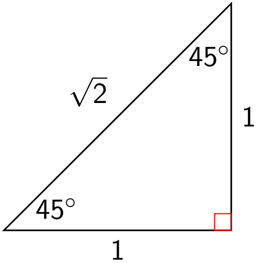


$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45^\circ = \frac{1}{1} = 1$$

Example 3



$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

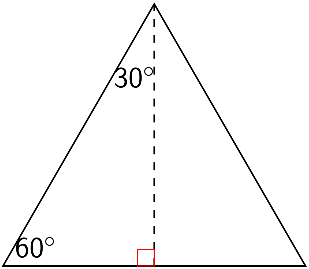
$$\sec 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45^\circ = \frac{1}{1} = 1$$

Note: Your answers from the above example will be the same if you replace 45° with $\frac{\pi}{4}$.

30-60-90 Triangles

We can create a 30-60-90 triangle by drawing an altitude in an equilateral triangle.



30-60-90 Triangles

Recall that the altitude of an equilateral triangle bisects one of the sides.

30-60-90 Triangles

Recall that the altitude of an equilateral triangle bisects one of the sides.

Rather than use a length of 1 for the sides of the equilateral triangle, we will use a length of 2 (if only to avoid using fractions).

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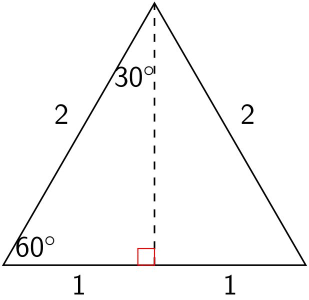
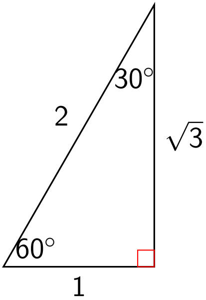
Recall that the altitude of an equilateral triangle bisects one of the sides.

Rather than use a length of 1 for the sides of the equilateral triangle, we will use a length of 2 (if only to avoid using fractions).



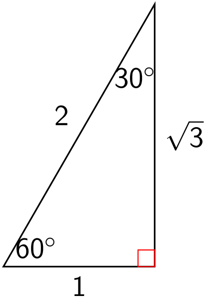
30-60-90 Triangles

We can use the Pythagorean Theorem to find the length of the altitude, $\sqrt{3}$:



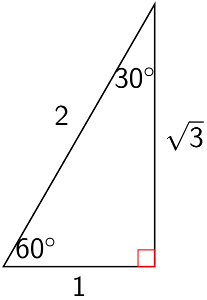
Example 4

Find the exact values of the six trig ratios for $60°$.



Example 4

Find the exact values of the six trig ratios for 60°.



$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

Example 4

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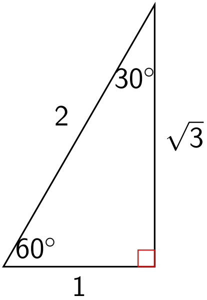

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

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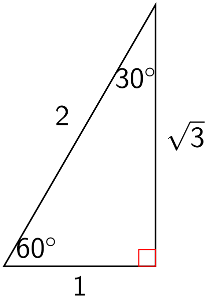
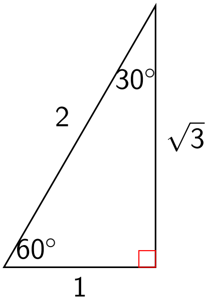


$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

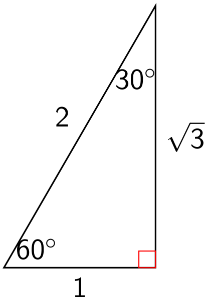
$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Example 4

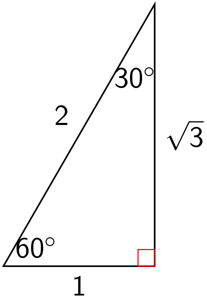


Example 4



$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

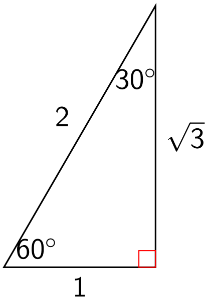
Example 4



$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Example 4

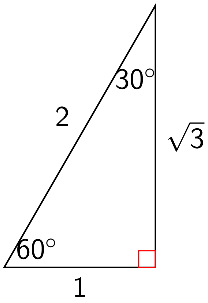


$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec 60^\circ = \frac{2}{1} = 2$$

Example 4



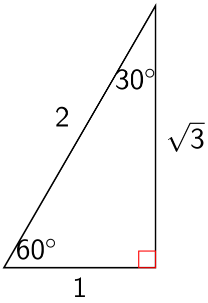
$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec 60^\circ = \frac{2}{1} = 2$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}}$$

Example 4



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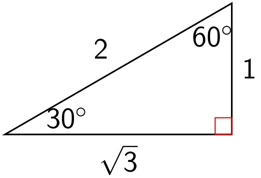
$$= \frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

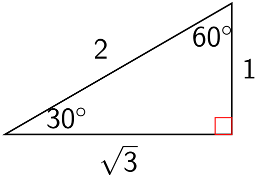
Example 5

Find the exact values of the six trig ratios for 30°.



Example 5

Find the exact values of the six trig ratios for 30°.



$$\sin 30^\circ = \frac{1}{2}$$

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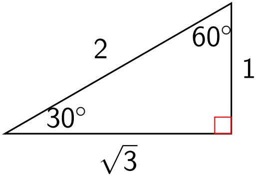
Find the exact values of the six trig ratios for 30°.



$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

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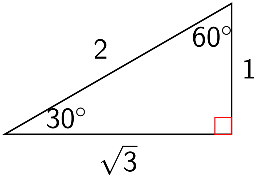
Find the exact values of the six trig ratios for 30°.

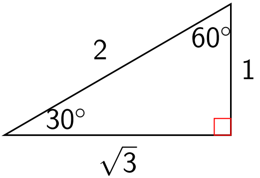


$$\sin 30^\circ = \frac{1}{2}$$

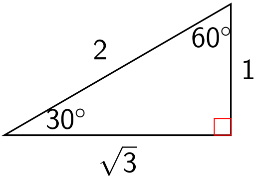
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Example 5



Example 5



$$\csc 30^\circ = \frac{2}{1} = 2$$

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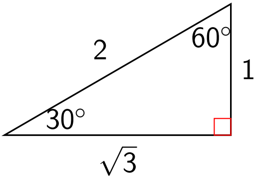

$$\csc 30^\circ = \frac{2}{1} = 2$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

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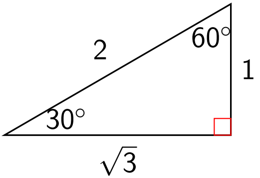

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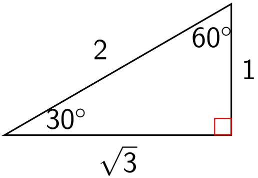

$$\csc 30^\circ = \frac{2}{1} = 2$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot 30^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Notice how $\sin 30^\circ = \cos 60^\circ$, $\tan 30^\circ = \cot 60^\circ$, etc. This is because these ratios are cofunctions.