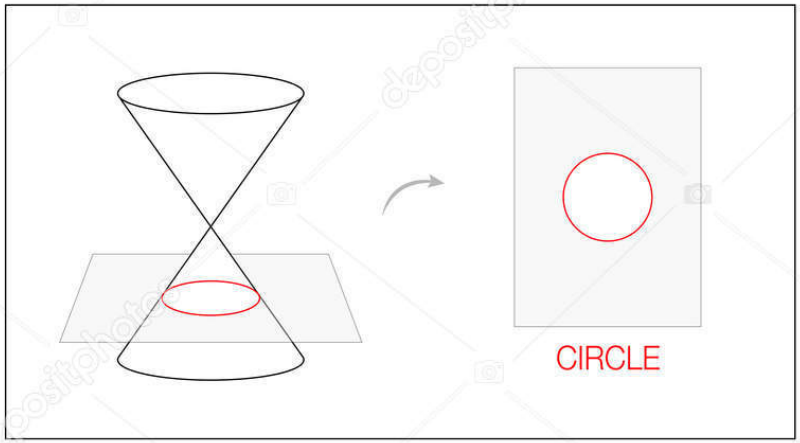
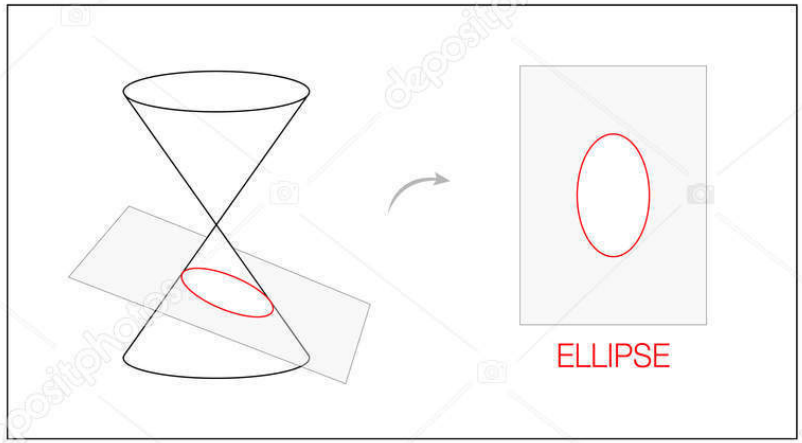
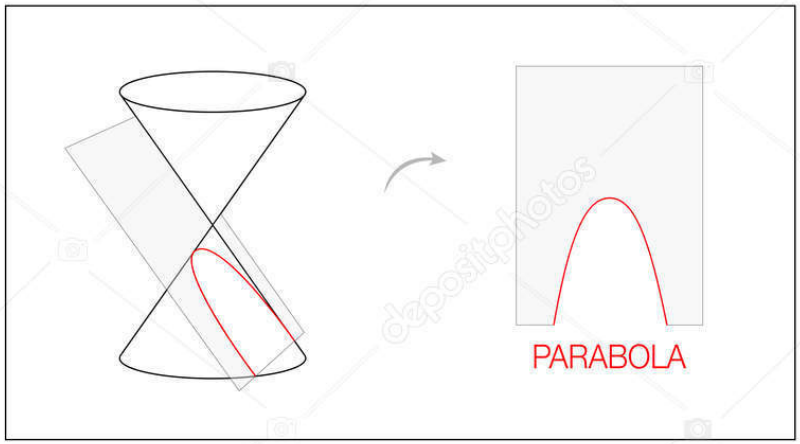
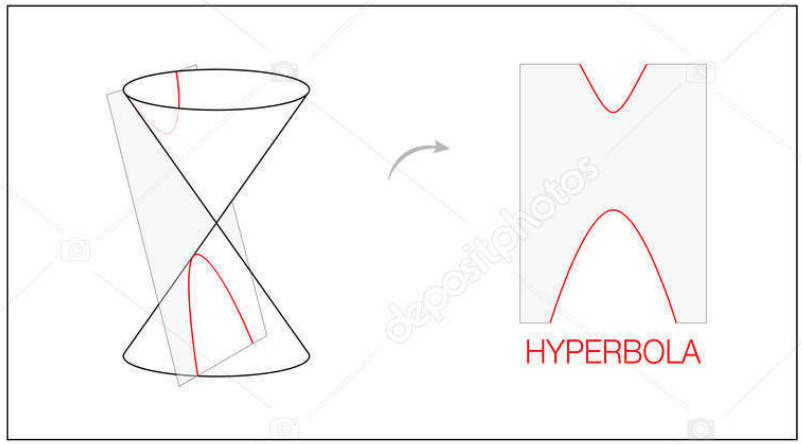
Circles









Objectives

1. Identify the center and radius of a circle.
2. Write the general form of the equation of a circle in standard form

Circles

You may remember circles from geometry class. In this chapter, we will look at equations and properties of circles.

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The standard form of the equation of a circle is

$$(x-h)^2+(y-k)^2=r^2$$

with center (h, k) and radius r.

Example 1

Identify the center and exact radius of each.

(a) $(x-4)^2+(y+3)^2=49$

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Radius: $\sqrt{49}=7$

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(b) $(x+1)^2 + (y-7)^2 = 72$

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Center: $(-1, 7)$

Radius: $\sqrt{72} = 6\sqrt{2}$

Writing the Standard Form of the Equation of a Circle

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    - The x-coordinates of each vertex will represent h and k, respectively.
    - The absolute value y-coordinates will be added to the constant.

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Identify the center and exact radius of each.

(a) $x^2 - 4x + y^2 + 6y - 23 = 0$

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$$(x-2)^2 + (y-3)^2 = 36$$

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Center: $(2, 3)$ Radius: 6

Example 2

(b) $x^2 + 16x + y^2 - 8y - 1 = 0$

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$$\mathbf{x^2 + 16x} \qquad + \mathbf{y^2 - 8y} \qquad = 1$$

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$$\mathbf{(x+8)^2} + \mathbf{(y-4)^2} = 1 + |-64| + |-16|$$

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$$\mathbf{x^2 + 16x} \qquad \mathbf{+ y^2 - 8y} \qquad = 1$$

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$$\mathbf{(x+8)^2} + \mathbf{(y-4)^2} = 1 + |-64| + |-16|$$

$$(x+8)^2 + (y-4)^2 = 81$$

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Vertex: $\mathbf{(-8, -64)}$ **Vertex:** $\mathbf{(4, -16)}$

$$\mathbf{(x+8)^2} + \mathbf{(y-4)^2} = 1 + |-64| + |-16|$$

$$(x+8)^2 + (y-4)^2 = 81$$

Center: $(-8, 4)$ Radius: 9

Example 2

(c) $x^2 - 10x + y^2 + 2y + 14 = 0$

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$$\mathbf{(x-5)^2} + \mathbf{(y+1)^2} = 14 + |-25| + |-1|$$

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$$\boldsymbol{(x-5)^2} + \boldsymbol{(y+1)^2} = 14 + |-25| + |-1|$$

$$(x-5)^2 + (y+1)^2 = 40$$

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Vertex: $\boldsymbol{(5, -25)}$ **Vertex:** $\boldsymbol{(-1, -1)}$

$$\boldsymbol{(x-5)^2} + \boldsymbol{(y+1)^2} = 14 + |-25| + |-1|$$

$$(x-5)^2 + (y+1)^2 = 40$$

Center: $(5, -1)$ Radius: $2\sqrt{10}$

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General Form

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General form will have all terms on one side of the equation and 0 on the other.

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(a) $(x-3)^2 + y^2 = 6$

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$$x^2 + 14x + 49 + y^2 - 10y + 25 = 10$$

Example 3

(b) $(x+7)^2+(y-5)^2=18$

$$(x+7)^2+(y-5)^2=18$$

$$x^2+14x+49+y^2-10y+25=10$$

$$x^2+14x+y^2-10y+74=10$$

Example 3

(b) $(x+7)^2+(y-5)^2=18$

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$$x^2+14x+49+y^2-10y+25=10$$

$$x^2+14x+y^2-10y+74=10$$

$$x^2+14x+y^2-10y+64=0$$