Angles and Radian Measure
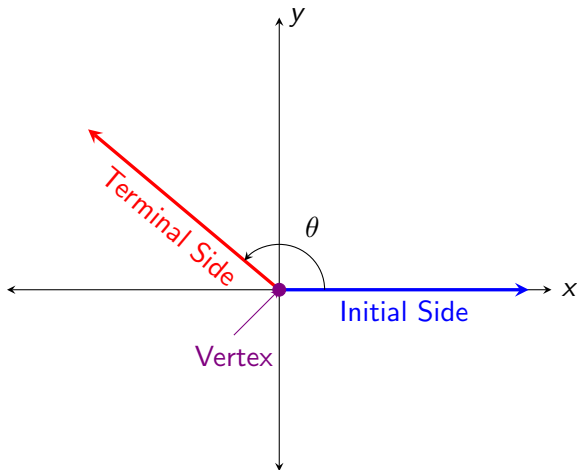
Angles in Standard Position

An angle is in standard position if

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- Vertex is at the origin.

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Positive angles open **counter-clockwise** and negative angles open **clockwise**.

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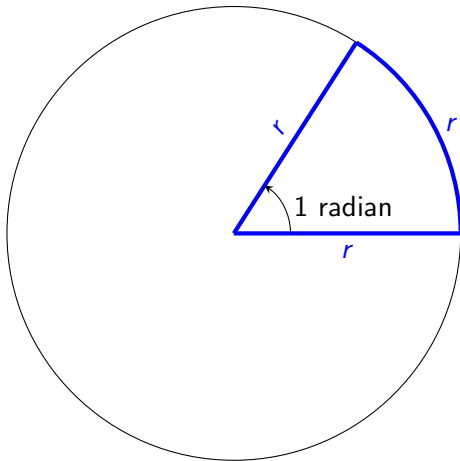
- Vertex is at the origin.

- Initial side runs along positive x-axis.

Positive angles open **counter-clockwise** and negative angles open **clockwise**.

A quadrantal angle is one whose terminal side lies on an axis. In other words, it's a multiple of $90°$.

Radians

One **radian** is the measure of the central angle of a circle in which the radius equals the length of the intercepted arc.

Radian Interpretation

In other words, its when the length of your slice of pizza's crust is equal to the radius of the pizza.

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Relationship Between Radians and Degrees

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$$360^\circ = 2\pi \text{ radians}$$

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$$360^\circ = 2\pi \text{ radians}$$

$$180^\circ = \pi \text{ radians}$$

Relationship Between Radians and Degrees

One rotation is the circumference of the circle ($2\pi r$) and also $360°$:

$$360° = 2\pi \text{ radians}$$

$$180° = \pi \text{ radians}$$

$$\frac{180°}{\pi} = 1 \text{ radian}$$

Convert Degrees to Radians and Radians to Degrees

- To convert degrees to radians, multiply by $\frac{\pi}{180°}$.

Convert Degrees to Radians and Radians to Degrees

- To convert degrees to radians, multiply by $\dfrac{\pi}{180^\circ}$.

- To convert radians to degrees, multiply by $\dfrac{180^\circ}{\pi}$.

Example 1

Convert each angle to radians.

(a) $30°$

Example 1

Convert each angle to radians.

(a) $30°$

$$30°$$

Example 1

Convert each angle to radians.

(a) $30°$

$$30° \to 30° \left(\frac{\pi}{180°} \right)$$

Example 1

Convert each angle to radians.

(a) $30°$

$$30° \to 30° \left(\frac{\pi}{180°} \right)$$

$$= \frac{30\pi}{180}$$

Example 1

Convert each angle to radians.

(a) 30°

$$30^\circ \rightarrow 30^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$= \frac{30\pi}{180}$$

$$= \frac{\pi}{6}$$

Example 1

Convert each angle to radians.

(b) $90°$

Example 1

Convert each angle to radians.

(b) $90°$

$$90°$$

Example 1

Convert each angle to radians.

(b) $90°$

$$90° \rightarrow 90° \left(\frac{\pi}{180°} \right)$$

Example 1

Convert each angle to radians.

(b) $90°$

$$90° \rightarrow 90° \left(\frac{\pi}{180°} \right)$$

$$= \frac{90\pi}{180}$$

Example 1

Convert each angle to radians.

(b) $90°$

$$90° \to 90° \left(\frac{\pi}{180°} \right)$$

$$= \frac{90\pi}{180}$$

$$= \frac{\pi}{2}$$

Example 1

Convert each angle to radians.

(c) $-135°$

Example 1

Convert each angle to radians.

(c) $-135°$

$$-135°$$

Example 1

Convert each angle to radians.

(c) $-135°$

$$-135° \rightarrow -135° \left(\frac{\pi}{180°} \right)$$

Example 1

Convert each angle to radians.

(c) $-135°$

$$-135° \rightarrow -135° \left(\frac{\pi}{180°}\right)$$

$$= \frac{-135\pi}{180}$$

Example 1

Convert each angle to radians.

(c) $-135°$

$$-135° \rightarrow -135° \left(\frac{\pi}{180°}\right)$$

$$= \frac{-135\pi}{180}$$

$$= \frac{-3\pi}{4}$$

Example 2

Convert each angle to degrees.

(a) $\dfrac{\pi}{3}$

Example 2

Convert each angle to degrees.

(a) $\dfrac{\pi}{3}$

$$\dfrac{\pi}{3}$$

Example 2

Convert each angle to degrees.

(a) $\dfrac{\pi}{3}$

$$\frac{\pi}{3} \to \frac{\pi}{3}\left(\frac{180^\circ}{\pi}\right)$$

Example 2

Convert each angle to degrees.

(a) $\dfrac{\pi}{3}$

$$\frac{\pi}{3} \to \frac{\pi}{3}\left(\frac{180^\circ}{\pi}\right)$$

$$= \frac{180^\circ}{3}$$

Example 2

Convert each angle to degrees.

(a) $\dfrac{\pi}{3}$

$$\frac{\pi}{3} \to \frac{\pi}{3}\left(\frac{180^\circ}{\pi}\right)$$

$$= \frac{180^\circ}{3}$$

$$= 60^\circ$$

Example 2
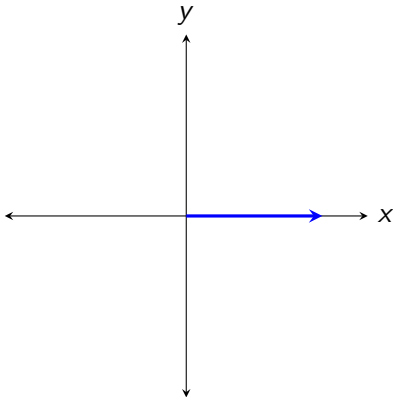
Convert each angle to degrees.

(b) $\quad -\dfrac{5\pi}{4}$

Example 2

Convert each angle to degrees.

(b) $-\dfrac{5\pi}{4}$

$$-\frac{5\pi}{4}$$

Example 2

Convert each angle to degrees.

(b) $\quad -\dfrac{5\pi}{4}$

$$-\frac{5\pi}{4} \rightarrow -\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right)$$

Example 2

Convert each angle to degrees.

(b) $-\dfrac{5\pi}{4}$

$$-\frac{5\pi}{4} \rightarrow -\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right)$$

$$= -\frac{900^\circ}{4}$$

Example 2

Convert each angle to degrees.

(b) $-\dfrac{5\pi}{4}$

$$-\frac{5\pi}{4} \rightarrow -\frac{5\pi}{4}\left(\frac{180^\circ}{\pi}\right)$$

$$= -\frac{900^\circ}{4}$$

$$= -225^\circ$$

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Example 3

Draw and label each angle in standard position.

(a) $\alpha = \dfrac{\pi}{6}$

Example 3

Draw and label each angle in standard position.
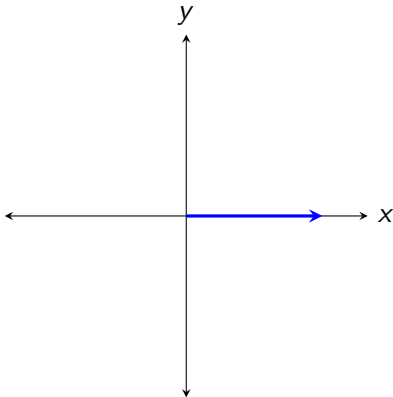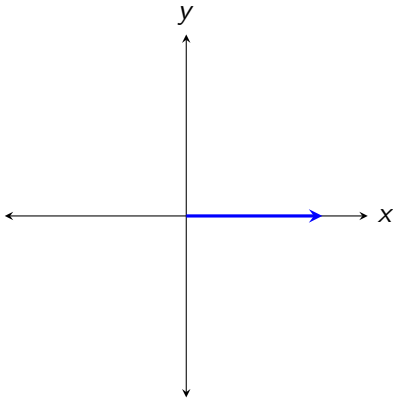
(a) $\alpha = \dfrac{\pi}{6}$ $\alpha = 30^\circ$

Example 3

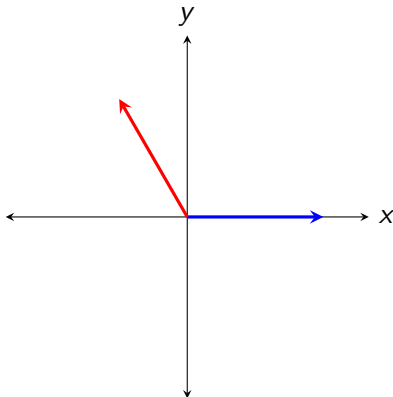Draw and label each angle in standard position.
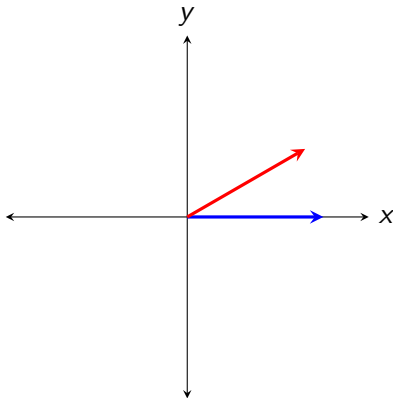
(a) $\alpha = \dfrac{\pi}{6}$ $\alpha = 30°$

Example 3

Draw and label each angle in standard position.

(a) $\alpha = \dfrac{\pi}{6}$ $\alpha = 30°$
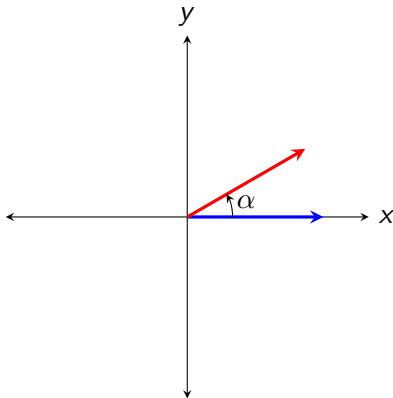
Example 3

(b) $\beta = -\dfrac{4\pi}{3}$

Example 3
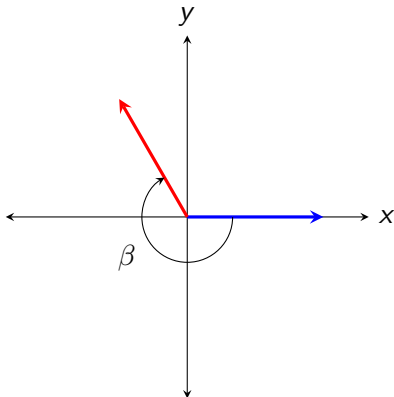
(b) $\quad \beta = -\dfrac{4\pi}{3} \qquad \beta = -240°$

Example 3

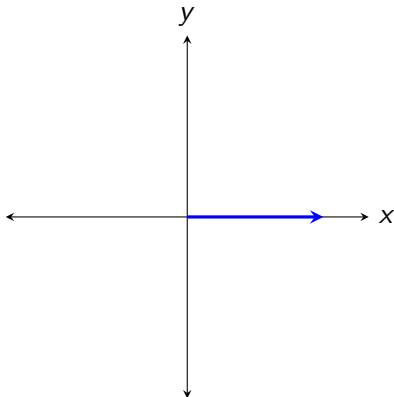(b) $\beta = -\dfrac{4\pi}{3}$ $\beta = -240°$

Example 3

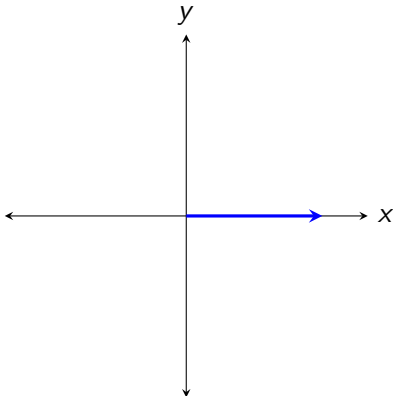(b) $\beta = -\dfrac{4\pi}{3}$ $\beta = -240^\circ$

Example 3

(c) $\gamma = -\dfrac{9\pi}{4}$

Example 3

(c) $\gamma = -\dfrac{9\pi}{4}$ $\gamma = -405°$
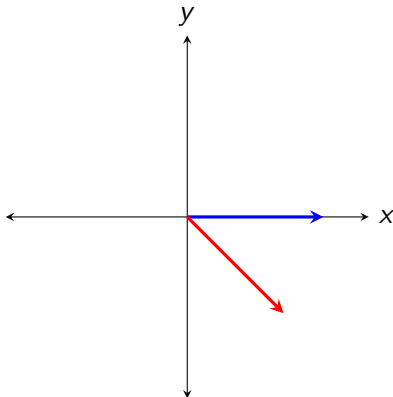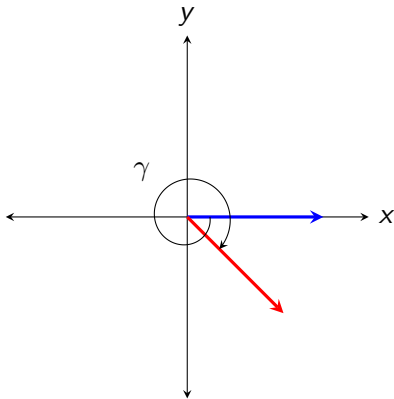
Example 3

(c) $\quad \gamma = -\dfrac{9\pi}{4} \qquad \gamma = -405^{\circ}$

Example 3

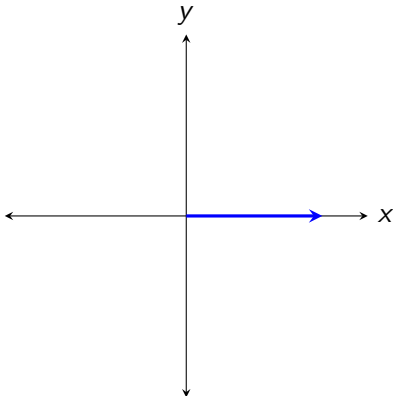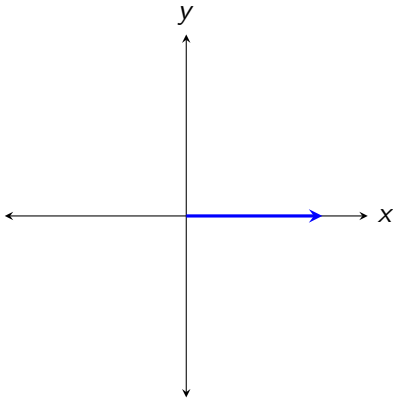(c) $\quad \gamma = -\dfrac{9\pi}{4} \qquad \gamma = -405^\circ$

Example 3
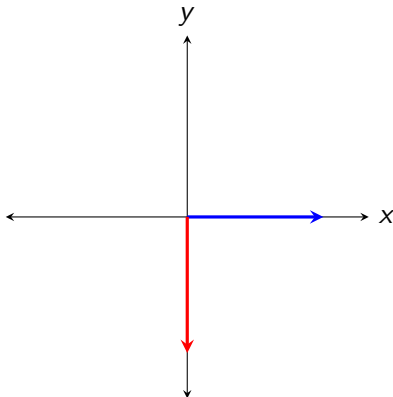
(d) $\quad \delta = -\dfrac{5\pi}{2}$

Example 3

(d) $\delta = -\dfrac{5\pi}{2}$ $\delta = -450^\circ$

Example 3

(d) $\delta = -\dfrac{5\pi}{2}$ $\delta = -450°$

Example 3

(d) $\quad \delta = -\dfrac{5\pi}{2} \qquad \delta = -450^\circ$

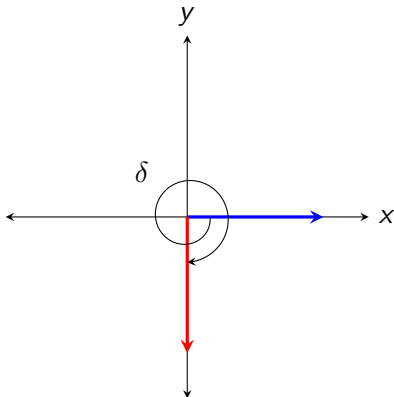# Table of Contents

Coterminal Angles

Two angles that have the same initial and terminal side are
coterminal angles.

Coterminal Angles

Two angles that have the same initial and terminal side are
coterminal angles.

To find coterminal angles, add (or subtract) multiples of $360°$ (or
2π radians).

Example 4

Find a coterminal angle between $0°$ and $360°$ (or 0 and 2π radians) for each.

(a) $400°$

Example 4

Find a coterminal angle between $0°$ and $360°$ (or 0 and 2π radians) for each.

(a) $400°$

$$400°$$

Example 4

Find a coterminal angle between $0°$ and $360°$ (or 0 and 2π radians) for each.

(a) $400°$

$$400° \rightarrow 400° - 360°$$

Example 4

Find a coterminal angle between $0°$ and $360°$ (or 0 and 2π radians) for each.

(a) $400°$

$$400° \rightarrow 400° - 360°$$

$$= 40°$$

Example 4

(b) $-\dfrac{4\pi}{3}$

Example 4

(b) $\quad -\dfrac{4\pi}{3}$

$$-\dfrac{4\pi}{3}$$

Example 4

(b) $\quad -\dfrac{4\pi}{3}$

$$-\frac{4\pi}{3} \rightarrow -\frac{4\pi}{3} + 2\pi$$

Example 4

(b) $-\dfrac{4\pi}{3}$

$$-\frac{4\pi}{3} \to -\frac{4\pi}{3} + 2\pi$$

$$= \frac{2\pi}{3}$$

Example 4

(c) $\dfrac{9\pi}{4}$

Example 4

(c) $\dfrac{9\pi}{4}$

$$\dfrac{9\pi}{4}$$

Example 4

(c) $\dfrac{9\pi}{4}$

$$\frac{9\pi}{4} \to \frac{9\pi}{4} - 2\pi$$

Example 4

(c) $\dfrac{9\pi}{4}$

$$\frac{9\pi}{4} \rightarrow \frac{9\pi}{4} - 2\pi$$

$$= \frac{\pi}{4}$$

Example 4

(d) $-785°$

Example 4

(d) $-785°$

$-785°$

Example 4

(d) -785°

$$-785^\circ \rightarrow -785^\circ + 1080^\circ$$

Example 4

(d) $-785°$

$$-785° \rightarrow -785° + 1080°$$

$$= 295°$$