

# Rational Equations and Inequalities

# Objectives

- 1 Solve rational equations
- 2 Solve rational inequalities

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Similar to the technique of simplifying complex fractions, we will eliminate our fractions by multiplying everything on both sides by the least common denominator.

However, because our fractions contain variables in the denominator, we must remember that the denominator can never equal zero.

Thus, we must always check for extraneous solutions when solving rational equations and inequalities.

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Solve each of the following. Remember to check for extraneous solutions.

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Since  $x \neq -1$  from the domain, our final answer is  $x = 2$ .

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$$x \neq 5, -3 \quad \text{LCD is } 3(x-5)(x+3)$$

$$(3(x-5)(x+3)) \left( \frac{1}{3(x-5)} \right)$$

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$$\begin{aligned} \left( \frac{1}{(x-5)(x+3)} + \frac{x^2}{3(x-5)(x+3)} \right) (3(x-5)(x+3)) \\ = 3 + x^2 \end{aligned}$$

Example 1f  $x \neq 5, -3$

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## Example 1f $x \neq 5, -3$

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$$x = 0, 1$$

# Objectives

- 1 Solve rational equations
- 2 Solve rational inequalities

# Rational Inequalities

We will continue the theme of solving inequalities like equations, setting up a number line, and using test values.

However, in addition to the answers we get from treating the inequality like an equation, we must also use the values outside the domain (i.e. where the denominator equals zero) on our number line.

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Solve each of the following and graph your solution on a number line.

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From Example 1a, we got  $x = -4$  and  $x \neq 0$

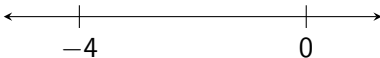
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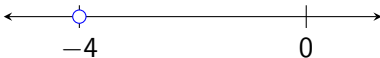
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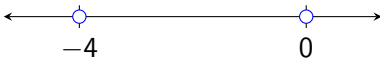
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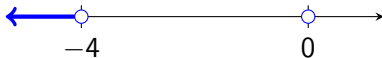
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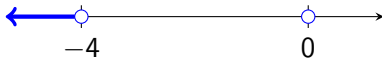
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$$x < -4$$

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$$x < 0 \text{ or } x \geq \frac{4}{5}$$