Simplifying Radical Expressions

Objectives

1. Write radicals as rational exponents and vice versa.
2. Simplify square root and higher root expressions.
3. Perform operations with radical expressions.

Radicand

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For $\sqrt[n]{a}$, the **radicand** is $\boldsymbol{a}$.

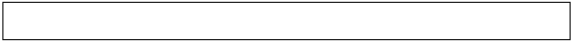
Radicals

What is $\left(\sqrt[3]{27}\right)^2$?

Well, if we evaluate $\sqrt[3]{27}$ first, we get 3. Then when we square that, we get the final answer of 9.

Radicals

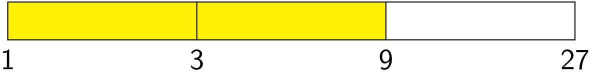
To put things visually, suppose the block below represents 27.



Since we are taking the cube root, we can divide the large block up into 3 equal blocks where we multiply by 3 to get each new number. (*Note*: We can divide it up into 2 equal blocks for square root, 3 for cube root, 4 for fourth root, etc.):

Radicals

Now let's look what happens as we shade in the block from the left, up to our answer of 9:



Notice that out of the three equal areas, two of them are shaded.

This is a visual approach to the idea that $\left(\sqrt[3]{27}\right)^2 = 27^{2/3}$

In general,

$$\sqrt[\text{root}]{x^{\text{power}}} = x^{\text{power}/\text{root}}$$

Example 1

Write each of the following using rational exponents.

(a) $\sqrt{6}$

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$$6^{1/2}$$

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Write each of the following using rational exponents.

(a) $\sqrt{6}$

$$6^{1/2}$$

(b) $\sqrt[3]{8}$

Example 1

Write each of the following using rational exponents.

(a) $\sqrt{6}$

$$6^{1/2}$$

(b) $\sqrt[3]{8}$

$$8^{1/3}$$

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Write each of the following using rational exponents.

(a) $\sqrt{6}$

$$6^{1/2}$$

(b) $\sqrt[3]{8}$

$$8^{1/3}$$

(c) $\sqrt[4]{x^3}$

Example 1

Write each of the following using rational exponents.

(a) $\sqrt{6}$

$$6^{1/2}$$

(b) $\sqrt[3]{8}$

$$8^{1/3}$$

(c) $\sqrt[4]{x^3}$

$$x^{3/4}$$

Example 2

Write each of the following in radical form.

(a) $5^{1/2}$

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Write each of the following in radical form.

(a) $5^{1/2}$

$$\sqrt{5}$$

Example 2

Write each of the following in radical form.

(a) $5^{1/2}$

$$\sqrt{5}$$

(b) $(-9)^{5/3}$

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(a) $5^{1/2}$

$$\sqrt{5}$$

(b) $(-9)^{5/3}$

$$\sqrt[3]{(-9)^5}$$

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Write each of the following in radical form.

(a) $5^{1/2}$

$$\sqrt{5}$$

(b) $(-9)^{5/3}$

$$\sqrt[3]{(-9)^5}$$

(c) $x^{-1/3}$

Example 2

Write each of the following in radical form.

(a) $5^{1/2}$

$$\sqrt{5}$$

(b) $(-9)^{5/3}$

$$\sqrt[3]{(-9)^5}$$

(c) $x^{-1/3}$

$$\sqrt[3]{x^{-1}} \quad \text{or} \quad \sqrt[3]{\frac{1}{x}}$$

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Even Roots and Exponents

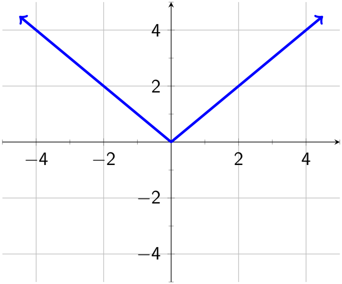
When dealing with **even** roots and exponents, keep in mind that the root and exponent don't "cancel each other out."

For instance, $\sqrt{5^2} = \sqrt{25} = 5$, but $\sqrt{(-5)^2} = \sqrt{25} = 5$.

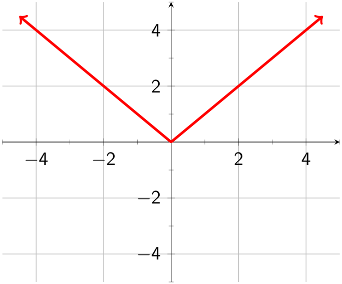
Even Roots and Exponents

The graphs of $y = \sqrt{x^2}$ and $y = |x|$ are shown. Notice they are identical.

$y = \sqrt{x^2}$



$y = |x|$



Thus, for any real number x, $\sqrt{x^2} = |x|$.

Odd Roots and Exponents

Odd roots, such as $\sqrt[3]{}$ do not follow the same rule as even roots.

So for any real number x, $\sqrt[3]{x^3} = x$.

And, in general, for any real number x,

1. If n is even, $\sqrt[n]{x^n} = |x|$.
2. If n is odd, $\sqrt[n]{x^n} = x$.

Example 3

Simplify each of the following. Exact answers only. Use absolute value bars when necessary.

(a) $\sqrt{72x^2}$

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(a) $\sqrt{72x^2}$

$$\sqrt{72x^2} = \sqrt{72} \cdot \sqrt{x^2}$$

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Simplify each of the following. Exact answers only. Use absolute value bars when necessary.

(a) $\sqrt{72x^2}$

$$\begin{aligned}\sqrt{72x^2} &= \sqrt{72} \cdot \sqrt{x^2} \\ &= 6\sqrt{2} \cdot |x|\end{aligned}$$

Example 3

Simplify each of the following. Exact answers only. Use absolute value bars when necessary.

(a) $\sqrt{72x^2}$

$$\begin{aligned}\sqrt{72x^2} &= \sqrt{72} \cdot \sqrt{x^2} \\ &= 6\sqrt{2} \cdot |x| \\ &= 6|x|\sqrt{2}\end{aligned}$$

Example 3

(b) $\sqrt{175x^3}$

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(b) $\sqrt{175x^3}$

$$\sqrt{175x^3} = \sqrt{175} \cdot \sqrt{x^3}$$

Example 3

(b) $\sqrt{175x^3}$

$$\begin{aligned}\sqrt{175x^3} &= \sqrt{175} \cdot \sqrt{x^3} \\ &= 5\sqrt{7} \cdot |x|\sqrt{x}\end{aligned}$$

Example 3

(b) $\sqrt{175x^3}$

$$\begin{aligned}\sqrt{175x^3} &= \sqrt{175}\cdot\sqrt{x^3}\\ &= 5\sqrt{7}\cdot|x|\sqrt{x}\\ &= 5|x|\sqrt{7x}\end{aligned}$$

Example 3

(c) $\sqrt{18x^4}$

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(c) $\sqrt{18x^4}$

$$\sqrt{18x^4} = \sqrt{18} \cdot \sqrt{x^4}$$

Example 3

(c) $\sqrt{18x^4}$

$$\begin{aligned}\sqrt{18x^4} &= \sqrt{18}\cdot\sqrt{x^4} \\ &= 3\sqrt{2}\cdot x^2\end{aligned}$$

Example 3

(c) $\sqrt{18x^4}$

$$\begin{aligned}\sqrt{18x^4} &= \sqrt{18}\cdot\sqrt{x^4}\\ &= 3\sqrt{2}\cdot x^2\\ &= 3x^2\cdot\sqrt{2}\end{aligned}$$

Example 3

(d) $\sqrt{65x^5}$

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(d) $\sqrt{65x^5}$

$$\sqrt{65x^5} = \sqrt{65} \cdot \sqrt{x^5}$$

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(d) $\sqrt{65x^5}$

$$\sqrt{65x^5} = \sqrt{65} \cdot \sqrt{x^5}$$

$$= \sqrt{65} \cdot x^2\sqrt{x}$$

Example 3

(d) $\sqrt{65x^5}$

$$\begin{aligned}\sqrt{65x^5} &= \sqrt{65}\cdot\sqrt{x^5}\\ &= \sqrt{65}\cdot x^2\sqrt{x}\\ &= x^2\cdot\sqrt{65x}\end{aligned}$$

Example 3

(e) $\sqrt[3]{27x^7}$

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(e) $\sqrt[3]{27x^7}$

$$\sqrt[3]{27x^7} = \sqrt[3]{27} \cdot \sqrt[3]{x^7}$$

Example 3

(e) $\sqrt[3]{27x^7}$

$$\sqrt[3]{27x^7} = \sqrt[3]{27} \cdot \sqrt[3]{x^7}$$

$$= 3 \cdot x^2 \sqrt[3]{x}$$

Example 3

(e) $\sqrt[3]{27x^7}$

$$\begin{aligned}\sqrt[3]{27x^7} &= \sqrt[3]{27} \cdot \sqrt[3]{x^7} \\ &= 3 \cdot x^2 \sqrt[3]{x} \\ &= 3x^2 \cdot \sqrt[3]{x}\end{aligned}$$

Example 3

(f) $\sqrt[3]{128x^6}$

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(f) $\sqrt[3]{128x^6}$

$$\sqrt[3]{128x^6} = \sqrt[3]{128} \cdot \sqrt[3]{x^6}$$

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(f) $\sqrt[3]{128x^6}$

$$\sqrt[3]{128x^6} = \sqrt[3]{128} \cdot \sqrt[3]{x^6}$$

$$= 4 \cdot \sqrt[3]{2} \cdot x^2$$

Example 3

(f) $\sqrt[3]{128x^6}$

$$\begin{aligned}\sqrt[3]{128x^6} &= \sqrt[3]{128} \cdot \sqrt[3]{x^6} \\ &= 4 \cdot \sqrt[3]{2} \cdot x^2 \\ &= 4x^2 \cdot \sqrt[3]{2}\end{aligned}$$

Objectives

1. Write radicals as rational exponents and vice versa.
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Performing Operations with Radical Expressions

Once we have simplified the radicals, we can add and subtract radical expressions with the same radicands and roots. This is essentially combining like terms.

Example 4

Simplify each of the following. Exact answers only.

(a) $\sqrt{98} + \sqrt{8}$

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Simplify each of the following. Exact answers only.

(a) $\sqrt{98}+\sqrt{8}$

$$\sqrt{98}+\sqrt{8}=7\sqrt{2}+2\sqrt{2}$$

Example 4

Simplify each of the following. Exact answers only.

(a) $\sqrt{98} + \sqrt{8}$

$$\sqrt{98} + \sqrt{8} = 7\sqrt{2} + 2\sqrt{2}$$

$$= 9\sqrt{2}$$

Example 4

(b) $\sqrt{108x} - \sqrt{300x}$

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(b) $\sqrt{108x} - \sqrt{300x}$

$$\sqrt{108x} - \sqrt{300x} = 6\sqrt{3x} - 10\sqrt{3x}$$

Example 4

(b) $\sqrt{108x} - \sqrt{300x}$

$$\begin{aligned}\sqrt{108x} - \sqrt{300x} &= 6\sqrt{3x} - 10\sqrt{3x} \\ &= -4\sqrt{3x}\end{aligned}$$

Example 4

(c) $4\sqrt{6} + 3\sqrt{54} - 5\sqrt{45}$

Example 4

(c) $4\sqrt{6} + 3\sqrt{54} - 5\sqrt{45}$

$$4\sqrt{6} + 3\sqrt{54} - 5\sqrt{45} = 4\sqrt{6} + 3(3\sqrt{6}) - 5(3\sqrt{5})$$

Example 4

(c) $4\sqrt{6} + 3\sqrt{54} - 5\sqrt{45}$

$$\begin{aligned} 4\sqrt{6} + 3\sqrt{54} - 5\sqrt{45} &= 4\sqrt{6} + 3(3\sqrt{6}) - 5(3\sqrt{5}) \\ &= 4\sqrt{6} + 9\sqrt{6} - 15\sqrt{5} \end{aligned}$$

Example 4

(c) $4\sqrt{6} + 3\sqrt{54} - 5\sqrt{45}$

$$\begin{aligned} 4\sqrt{6} + 3\sqrt{54} - 5\sqrt{45} &= 4\sqrt{6} + 3(3\sqrt{6}) - 5(3\sqrt{5}) \\ &= 4\sqrt{6} + 9\sqrt{6} - 15\sqrt{5} \\ &= 13\sqrt{6} - 15\sqrt{5} \end{aligned}$$

Multiplying and Dividing Radical Expressions

We can also multiply and divide radical expressions. It may be helpful to convert them to rational exponent form first, and then use your laws of exponents.

Example 5

Simplify each of the following. Leave no radical expressions in a denominator.

(a) $\sqrt{x^5} \cdot \sqrt{x^2}$

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Simplify each of the following. Leave no radical expressions in a denominator.

(a) $\sqrt{x^5} \cdot \sqrt{x^2}$

$$\sqrt{x^5} \cdot \sqrt{x^2} = x^{5/2} \cdot x^{2/2}$$

Example 5

Simplify each of the following. Leave no radical expressions in a denominator.

(a) $\sqrt{x^5} \cdot \sqrt{x^2}$

$$\sqrt{x^5} \cdot \sqrt{x^2} = x^{5/2} \cdot x^{2/2}$$

$$= x^{7/2}$$

Example 5

Simplify each of the following. Leave no radical expressions in a denominator.

(a) $\sqrt{x^5} \cdot \sqrt{x^2}$

$$\begin{aligned}\sqrt{x^5} \cdot \sqrt{x^2} &= x^{5/2} \cdot x^{2/2} \\ &= x^{7/2} \\ &= \sqrt{x^7}\end{aligned}$$

Example 5

Simplify each of the following. Leave no radical expressions in a denominator.

(a) $\sqrt{x^5} \cdot \sqrt{x^2}$

$$\begin{aligned}
\sqrt{x^5} \cdot \sqrt{x^2} &= x^{5/2} \cdot x^{2/2} \\
&= x^{7/2} \\
&= \sqrt{x^7} \\
&= |x^3|\sqrt{x}
\end{aligned}$$

Example 5

(b) $\sqrt{a^3} \cdot \sqrt[3]{a^2}$

Example 5

(b) $\sqrt{a^3} \cdot \sqrt[3]{a^2}$

$$\sqrt{a^3} \cdot \sqrt[3]{a^2} = a^{3/2} \cdot a^{2/3}$$

Example 5

(b) $\sqrt{a^3} \cdot \sqrt[3]{a^2}$

$$\begin{aligned}\sqrt{a^3} \cdot \sqrt[3]{a^2} &= a^{3/2} \cdot a^{2/3} \\ &= a^{13/6}\end{aligned}$$

Example 5

(b) $\sqrt{a^3} \cdot \sqrt[3]{a^2}$

$$\begin{aligned} \sqrt{a^3} \cdot \sqrt[3]{a^2} &= a^{3/2} \cdot a^{2/3} \\ &= a^{13/6} \\ &= \sqrt[6]{a^{13}} \end{aligned}$$

Example 5

(b) $\sqrt{a^3} \cdot \sqrt[3]{a^2}$

$$\begin{aligned}
\sqrt{a^3} \cdot \sqrt[3]{a^2} &= a^{3/2} \cdot a^{2/3} \\
&= a^{13/6} \\
&= \sqrt[6]{a^{13}} \\
&= a^2 \cdot \sqrt[6]{a}
\end{aligned}$$

Example 5

(c) $\sqrt[3]{y^6} \cdot \sqrt{y^3}$

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(c) $\sqrt[3]{y^6} \cdot \sqrt{y^3}$

$$\sqrt[3]{y^6} \cdot \sqrt{y^3} = y^{6/3} \cdot y^{3/2}$$

Example 5

(c) $\sqrt[3]{y^6} \cdot \sqrt{y^3}$

$$\begin{aligned} \sqrt[3]{y^6} \cdot \sqrt{y^3} &= y^{6/3} \cdot y^{3/2} \\ &= y^{7/2} \end{aligned}$$

Example 5

(c) $\sqrt[3]{y^6} \cdot \sqrt{y^3}$

$$\begin{aligned} \sqrt[3]{y^6} \cdot \sqrt{y^3} &= y^{6/3} \cdot y^{3/2} \\ &= y^{7/2} \\ &= \sqrt{y^7} \end{aligned}$$

Example 5

(c) $\sqrt[3]{y^6} \cdot \sqrt{y^3}$

$$\begin{aligned}
\sqrt[3]{y^6} \cdot \sqrt{y^3} &= y^{6/3} \cdot y^{3/2} \\
&= y^{7/2} \\
&= \sqrt{y^7} \\
&= y^3 \cdot \sqrt{y}
\end{aligned}$$

Example 5

(d) $\dfrac{\sqrt{x^5}}{\sqrt{x}}$

Example 5

(d) $\dfrac{\sqrt{x^5}}{\sqrt{x}}$

$$\frac{\sqrt{x^5}}{\sqrt{x}} = \frac{x^{5/2}}{x^{1/2}}$$

Example 5

(d) $\dfrac{\sqrt{x^5}}{\sqrt{x}}$

$$\frac{\sqrt{x^5}}{\sqrt{x}} = \frac{x^{5/2}}{x^{1/2}}$$

$$= x^{4/2}$$

Example 5

(d) $\dfrac{\sqrt{x^5}}{\sqrt{x}}$

$$\frac{\sqrt{x^5}}{\sqrt{x}} = \frac{x^{5/2}}{x^{1/2}}$$

$$= x^{4/2}$$

$$= x^2$$

Example 5

(e) $\dfrac{7}{\sqrt{2}}$

Example 5

(e) $\dfrac{7}{\sqrt{2}}$

$$\frac{7}{\sqrt{2}} = \frac{7}{2^{1/2}}$$

Example 5

(e) $\dfrac{7}{\sqrt{2}}$

$$\frac{7}{\sqrt{2}} = \frac{7}{2^{1/2}}$$

$$= \frac{7}{2^{1/2}} \left(\frac{2^{1/2}}{2^{1/2}} \right)$$

Example 5

(e) $\dfrac{7}{\sqrt{2}}$

$$\begin{aligned}\frac{7}{\sqrt{2}} &= \frac{7}{2^{1/2}} \\ &= \frac{7}{2^{1/2}}\left(\frac{2^{1/2}}{2^{1/2}}\right) \\ &= \frac{7 \cdot 2^{1/2}}{2}\end{aligned}$$

Example 5

(e) $\dfrac{7}{\sqrt{2}}$

$$\begin{aligned}
\frac{7}{\sqrt{2}} &= \frac{7}{2^{1/2}} \\
&= \frac{7}{2^{1/2}}\left(\frac{2^{1/2}}{2^{1/2}}\right) \\
&= \frac{7 \cdot 2^{1/2}}{2} \\
&= \frac{7\sqrt{2}}{2}
\end{aligned}$$

Example 5

(f) $$\frac{\sqrt[3]{y^6}}{\sqrt{y^3}}$$

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(f) $\dfrac{\sqrt[3]{y^6}}{\sqrt{y^3}}$

$$\frac{\sqrt[3]{y^6}}{\sqrt{y^3}} = \frac{y^{6/3}}{y^{3/2}}$$

Example 5

(f) $\dfrac{\sqrt[3]{y^6}}{\sqrt{y^3}}$

$$\frac{\sqrt[3]{y^6}}{\sqrt{y^3}} = \frac{y^{6/3}}{y^{3/2}}$$

$$= y^{1/2}$$

Example 5

(f) $\dfrac{\sqrt[3]{y^6}}{\sqrt{y^3}}$

$$\frac{\sqrt[3]{y^6}}{\sqrt{y^3}} = \frac{y^{6/3}}{y^{3/2}}$$

$$= y^{1/2}$$

$$= \sqrt{y}$$