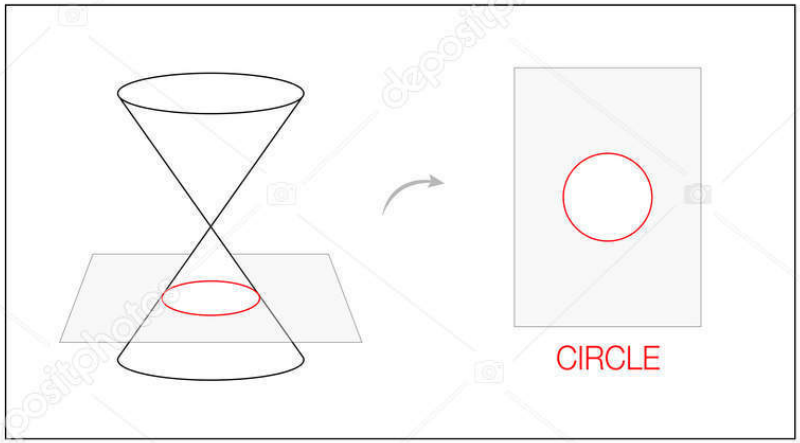
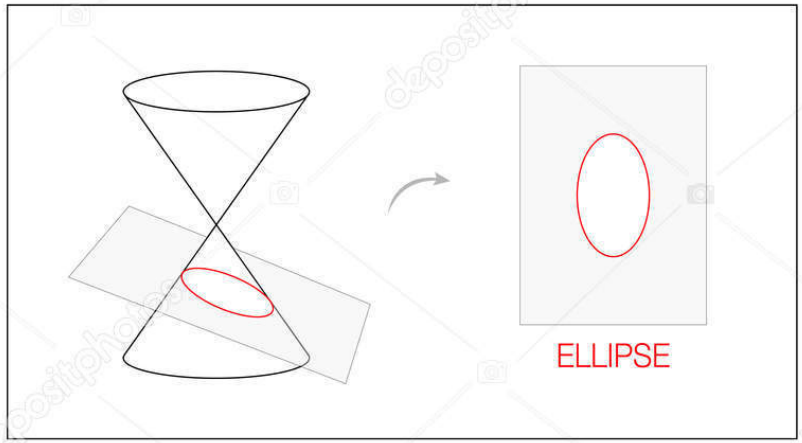
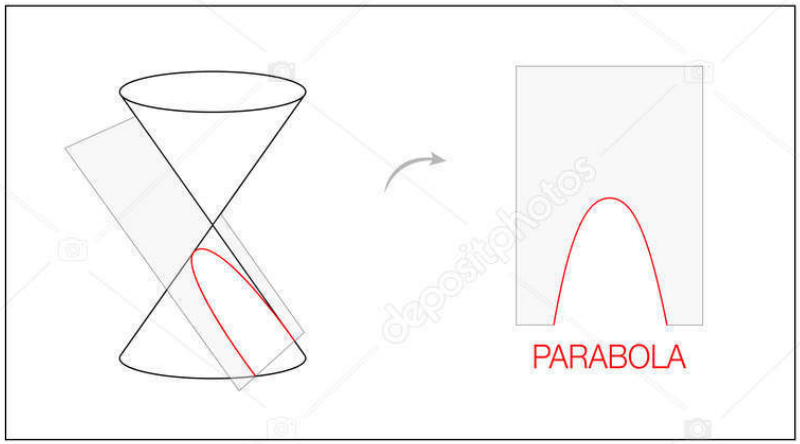
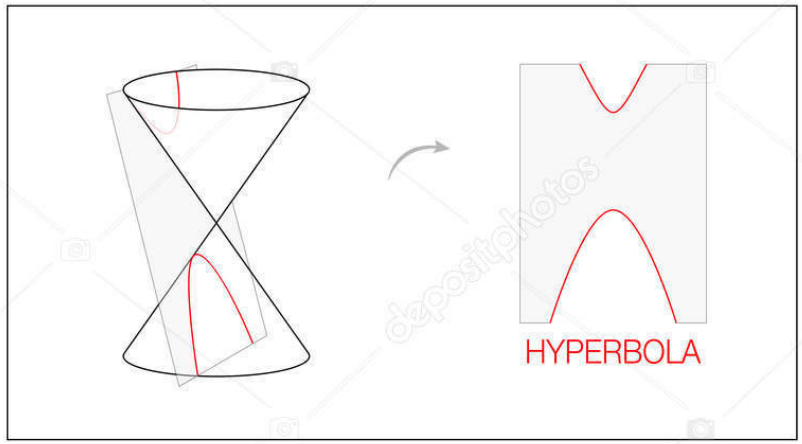
# Hyperbolas



# Objectives

1. Find the vertices and foci for a hyperbola in standard form.
2. Write the equation for a hyperbola in standard form.

# Hyperbolas

## Hyperbolas

The set of points such that the **difference** of their distances from 2 fixed points (called **foci**) is constant.

# Comparing Hyperbolas and Ellipses

Just like an ellipse, the midpoint joining the foci is the **center**.

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Whereas ellipses could appear taller or wider, hyperbolas will open up and down, or left and right.

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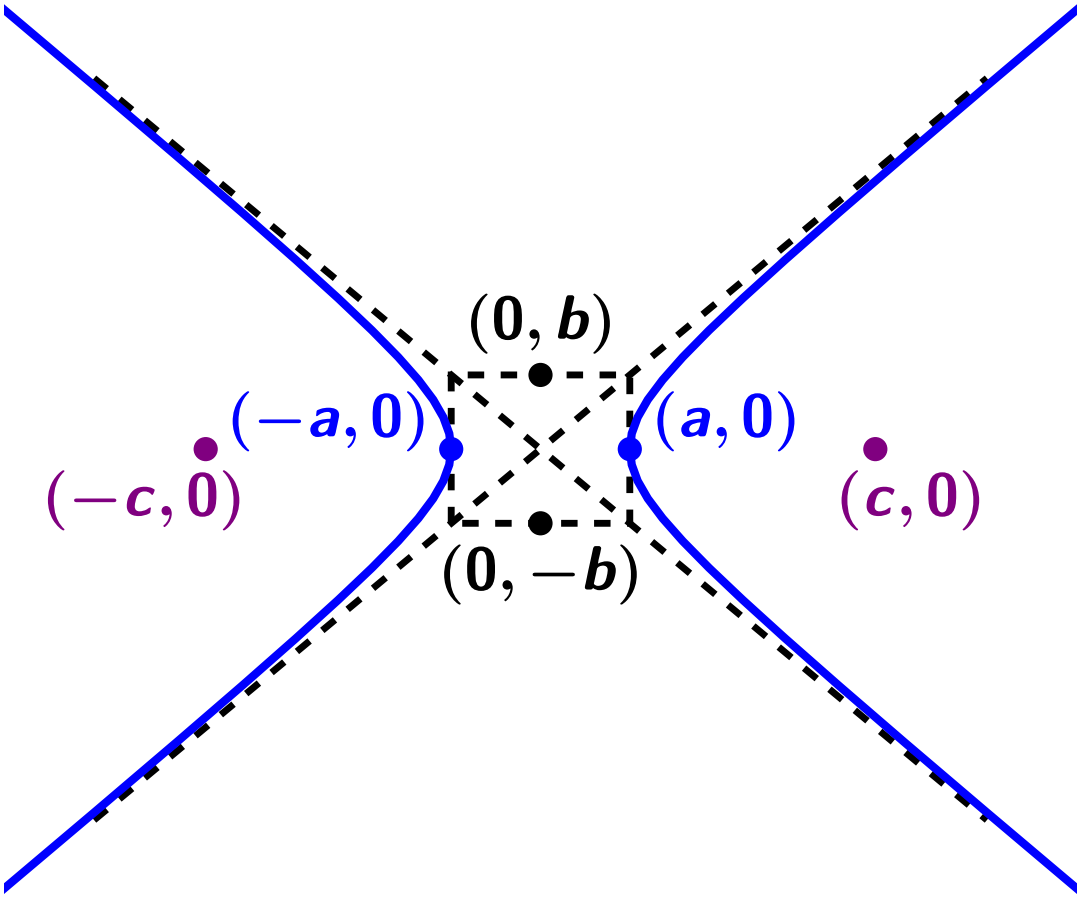
Just like an ellipse, the midpoint joining the foci is the **center**.

Whereas ellipses could appear taller or wider, hyperbolas will open up and down, or left and right.

A key difference, however, is that hyperbolas will open left/right if the sign in front of $x$ is positive, and will open up/down if the sign in front of $y$ is positive; regardless of the values of $a$ and $b$.

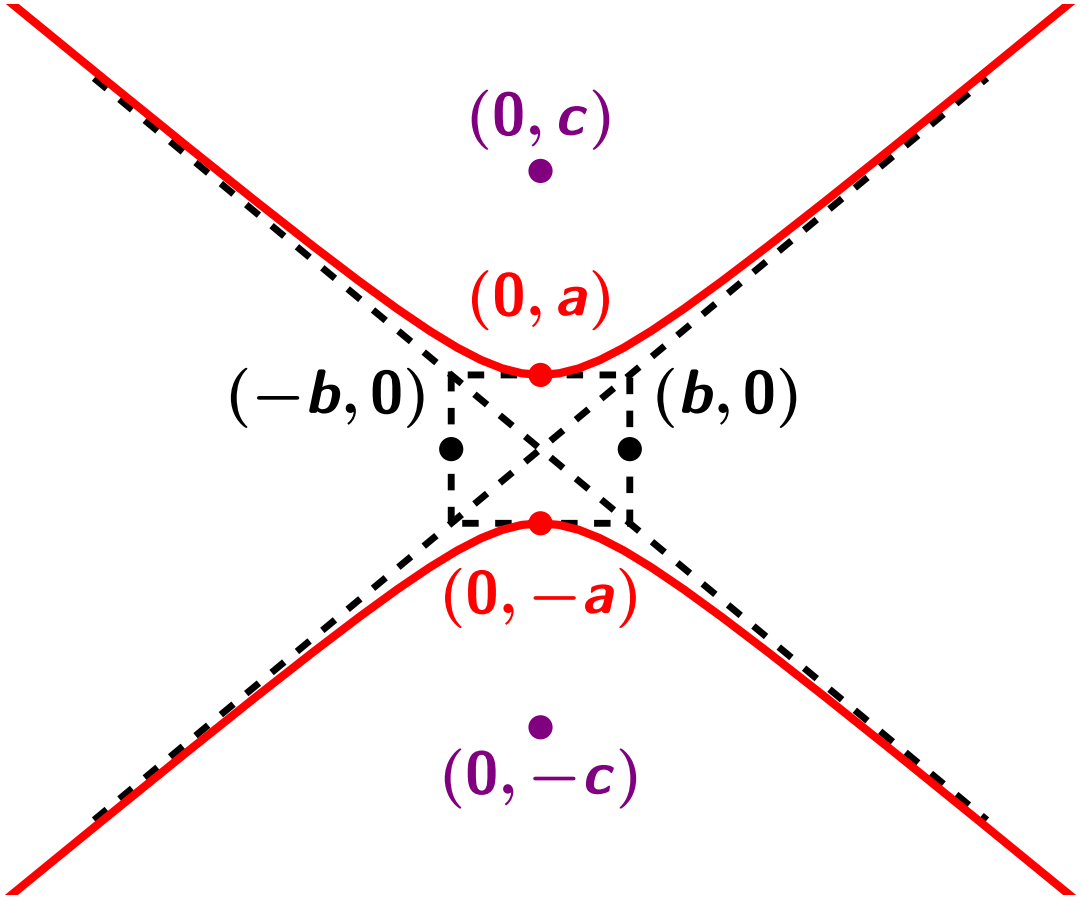
# Opening Left and Right



# Properties

| | |
|---|---|
| **Equation** | $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ |
| **Center** | $(h, k)$ |
| **Vertices** | $(h \pm a, k)$ |
| **Foci** | $(h \pm c, k)$ |
| **Co-vertices** | $(h, k \pm b)$ |
| **Left and Right** | Transverse Axis |
| **Up and Down** | Conjugate Axis |
| $c^2$ | $a^2 + b^2$ |

# Opening Up and Down



# Properties

| | |
|---|---|
| **Equation** | $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ |
| **Center** | $(h, k)$ |
| **Vertices** | $(h, k \pm a)$ |
| **Foci** | $(h, k \pm c)$ |
| **Co-vertices** | $(h \pm b, k)$ |
| **Left and Right** | Conjugate Axis |
| **Up and Down** | Transverse Axis |
| $c^2$ | $a^2 + b^2$ |

# Example 1

Find the exact coordinates for the vertices and foci for each of the following.

(a) $\dfrac{(y-3)^2}{4} - \dfrac{x^2}{16} = 1$

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Center: $(0, 3)$

$a^2 = 4$

$a = \pm 2$

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Find the exact coordinates for the vertices and foci for each of the following.

(a) $\dfrac{(y-3)^2}{4} - \dfrac{x^2}{16} = 1$

Center: $(0, 3)$

$a^2 = 4$

$a = \pm 2$

Vertices: $(0, 3 \pm 2)$

# Example 1

Find the exact coordinates for the vertices and foci for each of the following.

(a) $\dfrac{(y-3)^2}{4} - \dfrac{x^2}{16} = 1$

Center: $(0, 3)$

$a^2 = 4$

$a = \pm 2$

Vertices: $(0, 3 \pm 2) \longrightarrow (0, 1)$ and $(0, 5)$

# Example 1a

$$c^2 = a^2 + b^2$$

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$$c^2 = 4 + 16$$

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$$c^2 = 20$$

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$$c^2 = 4 + 16$$

$$c^2 = 20$$

$$c = \pm 2\sqrt{5}$$

# Example 1a

$$c^2 = a^2 + b^2$$

$$c^2 = 4 + 16$$

$$c^2 = 20$$

$$c = \pm 2\sqrt{5}$$

Foci: $(0, 3 \pm 2\sqrt{5})$

# Example 1

(b) $\dfrac{(y+1)^2}{16} - (x-1)^2 = 1$

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Center: $(1, -1)$

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# Example 1

(b) $\dfrac{(y+1)^2}{16} - (x-1)^2 = 1$

Center: $(1, -1)$

$a^2 = 16$

$a = \pm 4$

Vertices: $(1, -1 \pm 4)$

# Example 1

(b) $\dfrac{(y+1)^2}{16} - (x-1)^2 = 1$

Center: $(1, -1)$

$a^2 = 16$

$a = \pm 4$

Vertices: $(1, -1 \pm 4) \longrightarrow (1, -5)$ and $(1, 3)$

# Example 1b

$$c^2 = a^2 + b^2$$

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$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 1$$

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$$c^2 = a^2 + b^2$$

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$$c^2 = 16 + 1$$

$$c^2 = 17$$

$$c = \pm\sqrt{17}$$

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$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 1$$

$$c^2 = 17$$

$$c = \pm\sqrt{17}$$

Foci: $(1, -1 \pm \sqrt{17})$

# Example 1

(c) $(x+4)^2 - \dfrac{(y+2)^2}{9} = 1$

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Center: $(-4, -2)$

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$a^2 = 1$

$a = \pm 1$

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(c) $(x+4)^2 - \dfrac{(y+2)^2}{9} = 1$

Center: $(-4, -2)$

$a^2 = 1$

$a = \pm 1$

Vertices: $(-4 \pm 1, -2)$

# Example 1

(c) $(x+4)^2 - \dfrac{(y+2)^2}{9} = 1$

Center: $(-4, -2)$

$a^2 = 1$

$a = \pm 1$

Vertices: $(-4 \pm 1, -2) \longrightarrow (-5, -2)$ and $(-3, -2)$

# Example 1c

$$c^2 = a^2 + b^2$$

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$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

# Example 1c

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

$$c^2 = 10$$

# Example 1c

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

$$c^2 = 10$$

$$c = \pm\sqrt{10}$$

# Example 1c

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

$$c^2 = 10$$

$$c = \pm\sqrt{10}$$

Foci: $(-4 \pm \sqrt{10}, -2)$

# Example 1

(d) $\dfrac{(x+2)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

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Center: $(-2, 2)$

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Center: $(-2, 2)$

$a^2 = 9$

$a = \pm 3$

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(d) $\dfrac{(x+2)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

Center: $(-2, 2)$

$a^2 = 9$

$a = \pm 3$

Vertices: $(-2 \pm 3, 2)$

# Example 1

(d) $\dfrac{(x+2)^2}{9} - \dfrac{(y-2)^2}{4} = 1$

Center: $(-2, 2)$

$a^2 = 9$

$a = \pm 3$

Vertices: $(-2 \pm 3, 2) \longrightarrow (-5, 2)$ and $(1, 2)$

# Example 1d

$$c^2 = a^2 + b^2$$

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$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

# Example 1d

$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

$$c^2 = 13$$

# Example 1d

$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

$$c^2 = 13$$

$$c = \pm\sqrt{13}$$

# Example 1d

$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

$$c^2 = 13$$

$$c = \pm\sqrt{13}$$

Foci: $(-2 \pm \sqrt{13}, 2)$

# Objectives

1. Find the vertices and foci for a hyperbola in standard form.

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# Similar to Ellipses, But Not the Same

We will use the technique that we used for ellipses **with one big difference.**

# Similar to Ellipses, But Not the Same

We will use the technique that we used for ellipses **with one big difference.**

The absolute value of the $y$-coordinate after the minus sign gets subtracted from the right side.

# Example 2

Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

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Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

# Example 2

Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

**Vertex: $(-6, 324)$**

# Example 2

Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

**Vertex: $(-6, 324)$**

**Vertex: $(5, -25)$**

# Example 2

Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

**Vertex: $(-6, 324)$**

**Vertex: $(5, -25)$**

$$-9(x+6)^2 + (y-5)^2 = 380 - |324| + |-25|$$

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Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

$$\mathbf{-9x^2 - 108x} + \mathbf{y^2 - 10y} = 380$$

**Vertex: $(-6, 324)$**

**Vertex: $(5, -25)$**

$$-9(x+6)^2 + (y-5)^2 = 380 - |324| + |-25|$$

$$-9(x+6)^2 + (y-5)^2 = 81$$

# Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

# Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

# Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

$$-\frac{(x+6)^2}{9} + \frac{(y-5)^2}{81} = 1$$

# Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

$$-\frac{(x+6)^2}{9} + \frac{(y-5)^2}{81} = 1$$

$$\frac{(y-5)^2}{81} - \frac{(x+6)^2}{9} = 1$$

# Example 2

(b) $-x^2 + 12x + 4y^2 - 24y - 144 = 0$

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**Vertex: $(6, 36)$**

# Example 2

(b) $-x^2 + 12x + 4y^2 - 24y - 144 = 0$

$$-x^2 + 12x + 4y^2 - 24y = 144$$

**Vertex: $(6, 36)$**

**Vertex: $(3, -36)$**

# Example 2

(b) $-x^2 + 12x + 4y^2 - 24y - 144 = 0$

$$\mathbf{-x^2 + 12x} + \mathbf{4y^2 - 24y} = 144$$

**Vertex: $(6, 36)$**

**Vertex: $(3, -36)$**

$$-(x-6)^2 + 4(y-3)^2 = 144 - |36| + |-36|$$

# Example 2

(b) $-x^2 + 12x + 4y^2 - 24y - 144 = 0$

$$\mathbf{-x^2 + 12x} + \mathbf{4y^2 - 24y} = 144$$

**Vertex: $(6, 36)$**

**Vertex: $(3, -36)$**

$$-(x-6)^2 + 4(y-3)^2 = 144 - |36| + |-36|$$

$$-(x-6)^2 + 4(y-3)^2 = 144$$

# Example 2b

$$-(x-6)^2 + 4(y-3)^2 = 144$$

# Example 2b

$$-(x-6)^2 + 4(y-3)^2 = 144$$

$$-\frac{(x-6)^2}{144} + \frac{4(y-3)^2}{144} = 1$$

# Example 2b

$$-(x-6)^2 + 4(y-3)^2 = 144$$

$$-\frac{(x-6)^2}{144} + \frac{4(y-3)^2}{144} = 1$$

$$-\frac{(x-6)^2}{144} + \frac{(y-3)^2}{36} = 1$$

# Example 2b

$$-(x-6)^2 + 4(y-3)^2 = 144$$

$$-\frac{(x-6)^2}{144} + \frac{4(y-3)^2}{144} = 1$$

$$-\frac{(x-6)^2}{144} + \frac{(y-3)^2}{36} = 1$$

$$\frac{(y-3)^2}{36} - \frac{(x-6)^2}{144} = 1$$

# Example 2

(c) $x^2 + 20x - y^2 + 10y + 39 = 0$

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$$\mathbf{x^2 + 20x - y^2 + 10y} = -39$$

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**Vertex:** $\mathbf{(-10, -100)}$

# Example 2

(c) $x^2 + 20x - y^2 + 10y + 39 = 0$

$$\mathbf{x^2 + 20x - y^2 + 10y} = -39$$

**Vertex:** $\mathbf{(-10, -100)}$

**Vertex:** $\mathbf{(5, 25)}$

# Example 2

(c) $x^2 + 20x - y^2 + 10y + 39 = 0$

$$\mathbf{x^2 + 20x - y^2 + 10y} = -39$$

**Vertex:** $\mathbf{(-10, -100)}$

**Vertex:** $\mathbf{(5, 25)}$

$$(x + 10)^2 - (y - 5)^2 = -39 + |-100| - |25|$$

# Example 2

(c) $x^2 + 20x - y^2 + 10y + 39 = 0$

$$\mathbf{x^2 + 20x - y^2 + 10y} = -39$$

**Vertex:** $\mathbf{(-10, -100)}$

**Vertex:** $\mathbf{(5, 25)}$

$$(x + 10)^2 - (y - 5)^2 = -39 + |-100| - |25|$$

$$(x + 10)^2 - (y - 5)^2 = 36$$

# Example 2

(c) $x^2 + 20x - y^2 + 10y + 39 = 0$

$$\mathbf{x^2 + 20x - y^2 + 10y} = -39$$

**Vertex:** $\mathbf{(-10, -100)}$

**Vertex:** $\mathbf{(5, 25)}$

$$(x + 10)^2 - (y - 5)^2 = -39 + |-100| - |25|$$

$$(x + 10)^2 - (y - 5)^2 = 36$$

$$\frac{(x + 10)^2}{36} - \frac{(y - 5)^2}{36} = 1$$

# Example 2

(d) $x^2 + 18x - y^2 - 20y - 188 = 0$

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(d) $x^2 + 18x - y^2 - 20y - 188 = 0$

$$x^2 + 18x - y^2 - 20y = 188$$

# Example 2

(d) $x^2 + 18x - y^2 - 20y - 188 = 0$

$$\mathbf{x^2 + 18x - y^2 - 20y} = 188$$

**Vertex:** $\mathbf{(-9, -81)}$

# Example 2

(d) $x^2 + 18x - y^2 - 20y - 188 = 0$

$$\mathbf{x^2 + 18x - y^2 - 20y} = 188$$

**Vertex:** $\mathbf{(-9, -81)}$

**Vertex:** $\mathbf{(-10, 100)}$

# Example 2

(d) $x^2 + 18x - y^2 - 20y - 188 = 0$

$$\mathbf{x^2 + 18x - y^2 - 20y} = 188$$

**Vertex:** $\mathbf{(-9, -81)}$

**Vertex:** $\mathbf{(-10, 100)}$

$$(x + 9)^2 - (y + 10)^2 = 188 + |-81| - |100|$$

# Example 2

(d) $x^2 + 18x - y^2 - 20y - 188 = 0$

$$\mathbf{x^2 + 18x - y^2 - 20y} = 188$$

**Vertex:** $\mathbf{(-9, -81)}$

**Vertex:** $\mathbf{(-10, 100)}$

$$(x+9)^2 - (y+10)^2 = 188 + |-81| - |100|$$

$$(x+9)^2 - (y+10)^2 = 169$$

# Example 2

(d) $x^2 + 18x - y^2 - 20y - 188 = 0$

$$\mathbf{x^2 + 18x - y^2 - 20y} = 188$$

**Vertex:** $\mathbf{(-9, -81)}$

**Vertex:** $\mathbf{(-10, 100)}$

$$(x+9)^2 - (y+10)^2 = 188 + |-81| - |100|$$

$$(x+9)^2 - (y+10)^2 = 169$$

$$\frac{(x+9)^2}{169} - \frac{(y+10)^2}{169} = 1$$