Absolute Value Equations and Inequalities

Objectives

1. Solve absolute value equations.

2. Solve and graph absolute value inequalities

Absolute Value Equations

The absolute value of a number, b, denoted $|b|$, is the distance b is from 0 on a number line.

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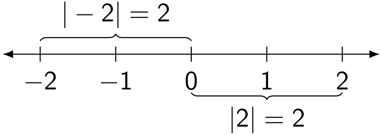

Solving Absolute Value Equations

When solving absolute value equations:

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Make sure your absolute value bars are isolated on one side before separating your problem into 2 equations.

Example 1

Solve each.

(a) $|2x - 3| = 11$

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$$x = 7 \qquad\qquad x = -4$$

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(a) $|2x-3| = 11$

$$2x - 3 = 11 \qquad\qquad 2x - 3 = -11$$

$$2x = 14 \qquad\qquad 2x = -8$$

$$x = 7 \qquad\qquad x = -4$$

$$x = 7 \text{ or } x = -4$$

Example 1

(b) $|3x - 1| = 5$

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$$x = 2 \qquad\qquad x = -\frac{4}{3}$$

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$$3x - 1 = 5 \qquad\qquad 3x - 1 = -5$$

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$$x = 2 \qquad\qquad x = -\frac{4}{3}$$

$$x = 2 \text{ or } x = -\frac{4}{3}$$

Example 1

(c) $|x+1| = -2$

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$$x + 1 = -2 \qquad\qquad x + 1 = 2$$

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No solution ($\varnothing$)

Example 1

(d) $|-x+2|-4=10$

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$$|-x+2| = 14 \qquad \text{add } 4$$

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(d) $|-x+2|-4=10$

$$|-x+2|=14 \qquad \text{add 4}$$

$$-x+2=14 \qquad\qquad -x+2=-14$$

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(d) $\quad |-x+2|-4=10$

$$|-x+2| = 14 \qquad \text{add } 4$$

$$-x+2=14 \qquad\qquad -x+2=-14$$

$$-x=12 \qquad\qquad -x=-16$$

Example 1

(d) $|-x+2|-4=10$

$$|-x+2|=14 \qquad \text{add 4}$$

$$-x+2=14 \qquad -x+2=-14$$

$$-x=12 \qquad -x=-16$$

$$x=-12 \qquad x=16$$

Example 1

(d) $|-x+2|-4=10$

$$|-x+2| = 14 \qquad \text{add 4}$$

$$-x+2 = 14 \qquad -x+2=-14$$

$$-x = 12 \qquad -x=-16$$

$$x=-12 \qquad x=16$$

$$x=-12 \text{ or } x=16$$

Example 1

(e) $|3x - 1| = |x + 5|$

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Example 1

(e) $|3x - 1| = |x + 5|$

$$
\begin{aligned}
3x - 1 &= x + 5 & \qquad 3x - 1 &= -(x + 5) \\
3x - 1 &= x + 5 & \qquad 3x - 1 &= -x - 5 \\
2x - 1 &= 5 & \qquad 4x - 1 &= -5 \\
2x &= 6 & \qquad 4x &= -4 \\
x &= 3 & \qquad x &= -1
\end{aligned}
$$

Example 1

(e) $|3x-1| = |x+5|$

$$3x-1 = x+5 \qquad\qquad 3x-1 = -(x+5)$$

$$3x-1 = x+5 \qquad\qquad 3x-1 = -x-5$$

$$2x-1 = 5 \qquad\qquad 4x-1 = -5$$

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1. Solve absolute value equations.
2. Solve and graph absolute value inequalities

Absolute Value Inequalities: $<$ and $\leq$

Absolute value inequalities are similar to absolute value equations.

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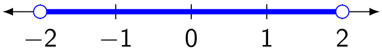

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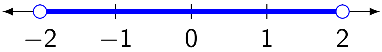

$$|x| < 2 \text{ means } -2 < x < 2$$

Example 2

Solve and graph each.

(a) $|2x - 2| < 5$

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$$-5 < 2x - 2 \qquad\qquad 2x - 2 < 5$$

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Alternate Solution Method for Example 2a

(a) $|2x - 2| < 5$

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Treat like an absolute value equation: $|2x - 2| = 5$ and then use **test values** in the *original inequality*.

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$x = -\frac{3}{2}$ or $x = \frac{7}{2}$

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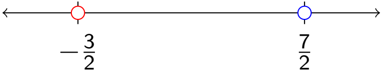

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$$x \geq 6.1 \qquad\qquad x \leq 11.9$$

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(b) $|x - 9| \leq 2.9$

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$$x \geq 6.1 \qquad\qquad x \leq 11.9$$

Example 2

(c) $|x+2| \leq -1$

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No solution, $\varnothing$, since $|x+2|$ is guaranteed to always be non-negative.

Alternate Approach to Example 2c

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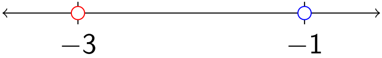

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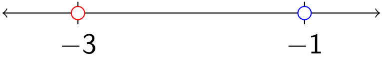

No test values work.

Alternate Approach to Example 2c

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$$x + 2 = -1 \qquad\qquad x + 2 = 1$$

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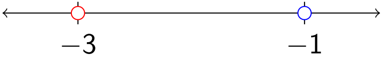

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Absolute Value Inequalities: $>$ and $\geq$

For $|x| > 2$, we want all the values of x that are greater than 2 units from 0 on a number line.

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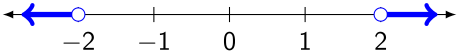

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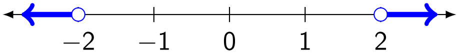

$|x| > 2$ means that $x < -2$ or $x > 2$

Example 3

Solve and graph each.

(a) $|2x+3| \geq 5$

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Solve and graph each.

(a) $|2x + 3| \geq 5$

$$2x + 3 \leq -5 \qquad\qquad 2x + 3 \geq 5$$

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Solve and graph each.

(a) $|2x + 3| \geq 5$

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$$2x \leq -8 \qquad\qquad 2x \geq 2$$

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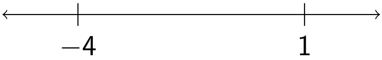

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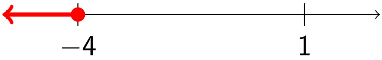

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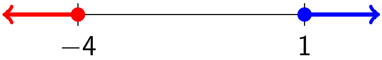

Alternate Solution Method for Example 3a

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Treat like an absolute value equation: $|2x + 3| = 5$ and then use **test values** in the *original inequality*.

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$x = -4$ or $x = 1$

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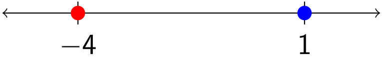

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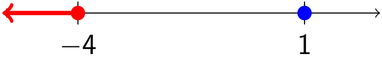

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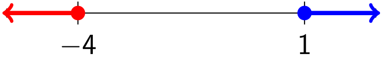

Example 3

(b) $|2x - 5| > 3$

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$$2x - 5 < -3 \qquad\qquad 2x - 5 > 3$$

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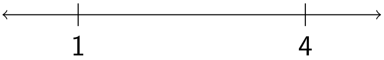


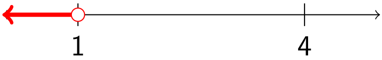
Example 3

(b) $|2x-5|>3$

$$2x-5<-3 \qquad\qquad 2x-5>3$$

$$2x<2 \qquad\qquad 2x>8$$

$$x<1 \qquad\qquad x>4$$



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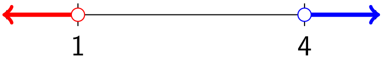

Example 3

(c) $|2x+2| > -2$

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All real numbers, $\mathbb{R}$, since $|2x + 2|$ is guaranteed to always be greater than a negative number.

Alternate Approach to Example 3c

(c) $|2x + 2| > -2$ Treat as $|2x + 2| = -2$

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$$2x+2=-2 \qquad\qquad 2x+2=2$$

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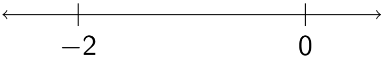

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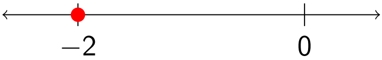

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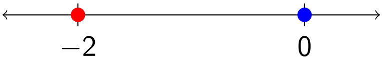

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All test values work.

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All test values work. $\mathbb{R}$