Objectives

1. Find the inverse of a function

2. State the Domain and Range of an Inverse Function

Inverse of an Ordered Pair

The inverse of the ordered pair (x, y) is (y, x).

Example 1

Find the inverse of each.

(a) $(2, -7)$

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(a) $(2, -7)$ $(-7, 2)$

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Find the inverse of each.

(a) $(2, -7)$ $(-7, 2)$

(b) $(0, 3)$

Example 1

Find the inverse of each.

(a) $(2, -7)$ $(-7, 2)$

(b) $(0, 3)$ $(3, 0)$

Review of Functions

Recall that a function is nothing more than a machine that

1. Accepts an input, x

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Recall that a function is nothing more than a machine that

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2. Performs some operation(s)

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The inverse function is somewhat of an "undo" function.

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Recall that a function is nothing more than a machine that

1. Accepts an input, x
2. Performs some operation(s)
3. Gives an output, y

The inverse function is somewhat of an "undo" function.

It allows us to take the output of a function, put it into our inverse function, and get our original input value back.

Visualization

Suppose we put a value of 10 into the function

$$f(x) = x^2$$

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If we put the output (100) into the inverse, we get our 10 back.

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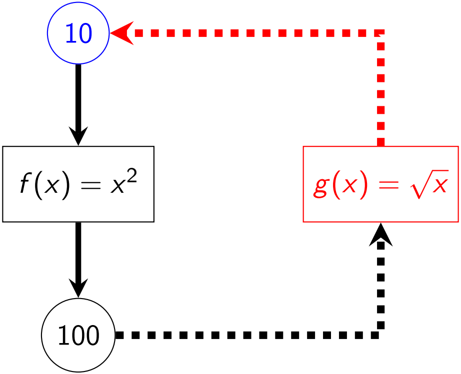

Inverse Notation

We use the notation

$$f^{-1}(x)$$

to denote the inverse of $f(x)$.

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$$f^{-1}(x)$$

to denote the inverse of $f(x)$.

Note: The notation **does not mean** raise the function to the -1 power.

Steps in Finding the Inverse of a Function

1. Rewrite $f(x) =$ as $y =$

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1. Rewrite $f(x) =$ as $y =$
2. Switch your x and y variables.

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1. Rewrite $f(x) =$ as $y =$
2. Switch your x and y variables.
3. Solve this result for y and rewrite using inverse notation.

Example 2

Find the inverse of each of the following.

(a) $f(x) = 5x$

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(a) $f(x) = 5x$

$$y = 5x$$

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Find the inverse of each of the following.

(a) $f(x) = 5x$

$$y = 5x$$

$$x = 5y$$

Example 2

Find the inverse of each of the following.

(a) $f(x) = 5x$

$$y = 5x$$

$$x = 5y$$

$$\frac{x}{5} = y$$

Example 2

Find the inverse of each of the following.

(a) $f(x) = 5x$

$$y = 5x$$

$$x = 5y$$

$$\frac{x}{5} = y$$

$$f^{-1}(x) = \frac{x}{5}$$

Example 2

(b) $f(x) = 3x + 2$

Example 2

(b) $f(x) = 3x + 2$

$$y = 3x + 2$$

Example 2

(b) $f(x) = 3x + 2$

$$y = 3x + 2$$

$$x = 3y + 2$$

Example 2

(b) $f(x) = 3x + 2$

$$\begin{aligned} y &= 3x + 2 \\ x &= 3y + 2 \\ x - 2 &= 3y \end{aligned}$$

Example 2

(b) $f(x) = 3x + 2$

$$\begin{aligned} y &= 3x + 2 \\ x &= 3y + 2 \\ x - 2 &= 3y \\ \frac{x-2}{3} &= y \end{aligned}$$

Example 2

(b) $f(x) = 3x + 2$

$$
\begin{aligned}
y &= 3x + 2 \\
x &= 3{\color{red}y} + 2 \\
x - 2 &= 3{\color{red}y} \\
\frac{x-2}{3} &= {\color{red}y} \\
f^{-1}(x) &= \frac{x-2}{3}
\end{aligned}
$$

Example 2

(c) $f(x) = \dfrac{x+5}{7}$

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$$y = \frac{x+5}{7}$$

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(c) $\quad f(x) = \dfrac{x+5}{7}$

$$y = \frac{x+5}{7}$$

$$x = \frac{y+5}{7}$$

$$7x = y + 5$$

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(c) $f(x) = \dfrac{x+5}{7}$

$$y = \frac{x+5}{7}$$

$$x = \frac{y+5}{7}$$

$$7x = y + 5$$

$$7x - 5 = y$$

Example 2

(c) $f(x) = \dfrac{x+5}{7}$

$$y = \frac{x+5}{7}$$

$$x = \frac{y+5}{7}$$

$$7x = y + 5$$

$$7x - 5 = y$$

$$f^{-1}(x) = 7x - 5$$

Example 2

(d) $g(x) = x^3 + 1$

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(d) $g(x) = x^3 + 1$

$$y = x^3 + 1$$

Example 2

(d) $g(x) = x^3 + 1$

$$y = x^3 + 1$$

$$x = y^3 + 1$$

Example 2

(d) $g(x) = x^3 + 1$

$$y = x^3 + 1$$

$$x = y^3 + 1$$

$$x - 1 = y^3$$

Example 2

(d) $g(x) = x^3 + 1$

$$y = x^3 + 1$$

$$x = y^3 + 1$$

$$x - 1 = y^3$$

$$\sqrt[3]{x-1} = y$$

Example 2

(d) $g(x) = x^3 + 1$

$$
\begin{aligned}
y &= x^3 + 1 \\
x &= {\color{red}y}^3 + 1 \\
x - 1 &= {\color{red}y}^3 \\
\sqrt[3]{x-1} &= {\color{red}y} \\
g^{-1}(x) &= \sqrt[3]{x-1}
\end{aligned}
$$

Example 2

(e) $h(x) = 4x^5 - 1$

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(e) $h(x) = 4x^5 - 1$

$$y = 4x^5 - 1$$

Example 2

(e) $h(x) = 4x^5 - 1$

$$y = 4x^5 - 1$$

$$x = 4y^5 - 1$$

Example 2

(e) $h(x) = 4x^5 - 1$

$$\begin{aligned} y &= 4x^5 - 1 \\ x &= 4{\color{red}y}^5 - 1 \\ x + 1 &= 4{\color{red}y}^5 \end{aligned}$$

Example 2

(e) $h(x) = 4x^5 - 1$

$$
\begin{aligned}
y &= 4x^5 - 1 \\
x &= 4y^5 - 1 \\
x + 1 &= 4y^5 \\
\frac{x+1}{4} &= y^5
\end{aligned}
$$

Example 2

(e) $h(x) = 4x^5 - 1$

$$
\begin{aligned}
y &= 4x^5 - 1 \\
x &= 4y^5 - 1 \\
x + 1 &= 4y^5 \\
\frac{x+1}{4} &= y^5 \\
\sqrt[5]{\frac{x+1}{4}} &= y
\end{aligned}
$$

Example 2

(e) $h(x) = 4x^5 - 1$

$$
\begin{aligned}
y &= 4x^5 - 1 \\
x &= 4y^5 - 1 \\
x + 1 &= 4y^5 \\
\frac{x+1}{4} &= y^5 \\
\sqrt[5]{\frac{x+1}{4}} &= y \\
h^{-1}(x) &= \sqrt[5]{\frac{x+1}{4}}
\end{aligned}
$$

Example 2

(f) $f(x) = \sqrt{x+3}$

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$$y = \sqrt{x+3}$$

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Example 2

(f) $f(x) = \sqrt{x+3}$

$$y = \sqrt{x+3}$$

$$x = \sqrt{y+3}$$

$$x^2 = y+3$$

Example 2

(f) $f(x) = \sqrt{x+3}$

$$\begin{aligned} y &= \sqrt{x+3} \\ x &= \sqrt{y+3} \\ x^2 &= y+3 \\ x^2 - 3 &= y \end{aligned}$$

Example 2

(f) $f(x) = \sqrt{x+3}$

$$y = \sqrt{x+3}$$

$$x = \sqrt{y+3}$$

$$x^2 = y + 3$$

$$x^2 - 3 = y$$

$$f^{-1}(x) = x^2 - 3$$

Example 2

(g) $\quad g(x) = \dfrac{5}{x}$

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$$y = \frac{5}{x}$$

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(g) $\quad g(x) = \dfrac{5}{x}$

$$y = \frac{5}{x}$$

$$x = \frac{5}{y}$$

$$xy = 5$$

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(g) $g(x) = \dfrac{5}{x}$

$$y = \frac{5}{x}$$

$$x = \frac{5}{y}$$

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Example 2

(g) $\quad g(x) = \dfrac{5}{x}$

$$y = \frac{5}{x}$$

$$x = \frac{5}{y}$$

$$xy = 5$$

$$y = \frac{5}{x}$$

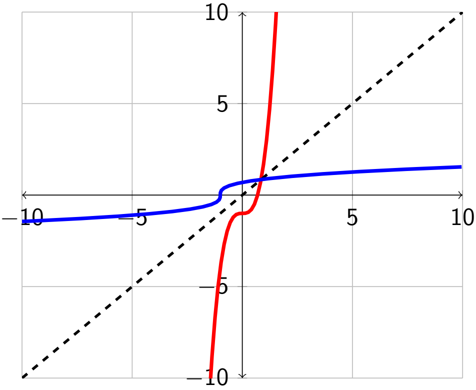
$$g^{-1}(x) = \frac{5}{x}$$

Visual Interpretation of Inverse Functions

Visually, when finding the inverse of a function, you are **reflecting that function across the line $y = x$**.

Visual Interpretation of Inverse Functions

Below are the graphs of $f(x) = 3x^3 - 1$ and $f^{-1}(x) = \sqrt[3]{\frac{x+1}{3}}$ as well as the line $y = x$:



Objectives

1. Find the inverse of a function
2. State the Domain and Range of an Inverse Function

Domain and Range of Inverse Functions

When switching the x and y in finding the inverse function, you also switch the domain and range of the function and its inverse.

Domain and Range of Inverse Functions

When switching the x and y in finding the inverse function, you also switch the domain and range of the function and its inverse.

When you graph a function and its inverse, **you'll want to make sure that they are reflections across the line $y = x$.** This is VERY IMPORTANT for a function such as $y = x^2$.

Domain and Range of Inverse Functions

When switching the x and y in finding the inverse function, you also switch the domain and range of the function and its inverse.

When you graph a function and its inverse, **you'll want to make sure that they are reflections across the line** $y = x$. This is VERY IMPORTANT for a function such as $y = x^2$.

This might mean we need to restrict the domain and/or range of our original function.

Relationship Between Domain and Range

$$\text{Domain of } f = \text{Range of } f^{-1}$$

and

$$\text{Range of } f = \text{Domain of } f^{-1}$$

Example 3

Find the domain and range of both the function and its inverse.

(a) $f(x) = 5x$

	Domain (x)	**Range (y)**
$f(x)$		
$f^{-1}(x)$		

Example 3

Find the domain and range of both the function and its inverse.

(a) $f(x) = 5x$ $f^{-1}(x) = \frac{x}{5}$

	Domain (x)	**Range (y)**
$f(x)$		
$f^{-1}(x)$		

Example 3

Find the domain and range of both the function and its inverse.

(a) $f(x) = 5x \quad f^{-1}(x) = \frac{x}{5}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	
$f^{-1}(x)$		

Example 3

Find the domain and range of both the function and its inverse.

(a) $f(x) = 5x$ $\quad f^{-1}(x) = \frac{x}{5}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	
$f^{-1}(x)$		$\mathbb{R}$

Example 3

Find the domain and range of both the function and its inverse.

(a) $f(x) = 5x \quad f^{-1}(x) = \frac{x}{5}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	
$f^{-1}(x)$	$\mathbb{R}$	$\mathbb{R}$

Example 3

Find the domain and range of both the function and its inverse.

(a) $f(x) = 5x \quad f^{-1}(x) = \frac{x}{5}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	$\mathbb{R}$
$f^{-1}(x)$	$\mathbb{R}$	$\mathbb{R}$

Example 3

(b) $f(x) = 3x + 2$

	Domain (x)	**Range (y)**
$f(x)$		
$f^{-1}(x)$		

Example 3

(b) $f(x) = 3x + 2$ $f^{-1}(x) = \frac{x-2}{3}$

	Domain (x)	**Range (y)**
$f(x)$		
$f^{-1}(x)$		

Example 3

(b) $f(x) = 3x + 2 \quad f^{-1}(x) = \frac{x-2}{3}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	
$f^{-1}(x)$		

Example 3

(b) $f(x) = 3x + 2$ $\quad f^{-1}(x) = \frac{x-2}{3}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	
$f^{-1}(x)$		$\mathbb{R}$

Example 3

(b) $f(x) = 3x + 2 \quad f^{-1}(x) = \frac{x-2}{3}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	
$f^{-1}(x)$	$\mathbb{R}$	$\mathbb{R}$

Example 3

(b) $f(x) = 3x + 2 \qquad f^{-1}(x) = \frac{x-2}{3}$

	Domain (x)	**Range (y)**
$f(x)$	$\mathbb{R}$	$\mathbb{R}$
$f^{-1}(x)$	$\mathbb{R}$	$\mathbb{R}$

Example 3

(c) $g(x) = \sqrt{x+3}$

	Domain (x)	**Range (y)**
$g(x)$		
$g^{-1}(x)$		

Example 3

(c) $g(x) = \sqrt{x+3}$ $\quad g^{-1}(x) = x^2 - 3$

	Domain (x)	**Range (y)**
$g(x)$		
$g^{-1}(x)$		

Example 3

(c) $g(x) = \sqrt{x+3} \quad g^{-1}(x) = x^2 - 3$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq -3}$	
$g^{-1}(x)$		

Example 3

(c) $g(x) = \sqrt{x+3}$ $\quad g^{-1}(x) = x^2 - 3$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq -3}$	
$g^{-1}(x)$		$\boldsymbol{y \geq -3}$

Example 3

(c) $g(x) = \sqrt{x+3}$ $\quad g^{-1}(x) = x^2 - 3$

	Domain $(\boldsymbol{x})$	**Range** $(\boldsymbol{y})$
$g(x)$	$\boldsymbol{x \geq -3}$	$\boldsymbol{y \geq 0}$
$g^{-1}(x)$		$\boldsymbol{y \geq -3}$

Example 3

(c) $g(x) = \sqrt{x+3}$ $g^{-1}(x) = x^2 - 3$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq -3}$	$\boldsymbol{y \geq 0}$
$g^{-1}(x)$	$\boldsymbol{x \geq 0}$	$\boldsymbol{y \geq -3}$

Example 3

(d) $f(x) = -\sqrt{x-4} + 2$

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$$y = -\sqrt{x-4} + 2$$

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$$y = -\sqrt{x-4}+2$$

$$x = -\sqrt{y-4}+2$$

Example 3

(d) $f(x) = -\sqrt{x-4} + 2$

$$y = -\sqrt{x-4} + 2$$

$$x = -\sqrt{y-4} + 2$$

$$x - 2 = -\sqrt{y-4}$$

Example 3

(d) $f(x) = -\sqrt{x-4} + 2$

$$\begin{aligned} y &= -\sqrt{x-4} + 2 \\ x &= -\sqrt{y-4} + 2 \\ x - 2 &= -\sqrt{y-4} \\ -x + 2 &= \sqrt{y-4} \end{aligned}$$

Example 3

(d) $f(x) = -\sqrt{x-4}+2$

$$
\begin{aligned}
y &= -\sqrt{x-4}+2 \\
x &= -\sqrt{y-4}+2 \\
x-2 &= -\sqrt{y-4} \\
-x+2 &= \sqrt{y-4} \\
(-x+2)^2 &= y-4
\end{aligned}
$$

Example 3

(d) $f(x) = -\sqrt{x-4} + 2$

$$
\begin{aligned}
y &= -\sqrt{x-4} + 2 \\
x &= -\sqrt{y-4} + 2 \\
x - 2 &= -\sqrt{y-4} \\
-x + 2 &= \sqrt{y-4} \\
(-x+2)^2 &= y - 4 \\
(-x+2)^2 + 4 &= y
\end{aligned}
$$

Example 3

(d) $\quad f(x) = -\sqrt{x-4} + 2$

$$
\begin{aligned}
y &= -\sqrt{x-4} + 2 \\
x &= -\sqrt{y-4} + 2 \\
x - 2 &= -\sqrt{y-4} \\
-x + 2 &= \sqrt{y-4} \\
(-x+2)^2 &= y - 4 \\
(-x+2)^2 + 4 &= y \\
f^{-1}(x) &= (-x+2)^2 + 4
\end{aligned}
$$

Example 3d

$$f(x) = -\sqrt{x-4} + 2 \qquad f^{-1}(x) = (-x+2)^2 + 4$$

	Domain (x)	**Range (y)**
$g(x)$		
$g^{-1}(x)$		

Example 3d

$$f(x) = -\sqrt{x-4} + 2 \qquad f^{-1}(x) = (-x+2)^2 + 4$$

	Domain (x)	**Range (y)**
$g(x)$	$x \geq 4$	
$g^{-1}(x)$		

Example 3d

$$f(x) = -\sqrt{x-4} + 2 \qquad f^{-1}(x) = (-x+2)^2 + 4$$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq 4}$	
$g^{-1}(x)$		$\boldsymbol{y \geq 4}$

Example 3d

$$f(x) = -\sqrt{x-4} + 2 \qquad f^{-1}(x) = (-x+2)^2 + 4$$

	Domain (x)	**Range (y)**
$g(x)$	$x \geq 4$	$y \leq 2$
$g^{-1}(x)$		$y \geq 4$

Example 3d

$$f(x) = -\sqrt{x-4} + 2 \qquad f^{-1}(x) = (-x+2)^2 + 4$$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq 4}$	$\boldsymbol{y \leq 2}$
$g^{-1}(x)$	$\boldsymbol{x \leq 2}$	$\boldsymbol{y \geq 4}$

Example 3

(e) $f(x) = x^2$ with $x \geq 0$

Example 3

(e) $f(x) = x^2$ with $x \geq 0$

$$y = x^2$$

Example 3

(e) $f(x) = x^2$ with $x \geq 0$

$$y = x^2$$

$$x = y^2$$

Example 3

(e) $f(x) = x^2$ with $x \geq 0$

$$y = x^2$$

$$x = y^2$$

$$\pm\sqrt{x} = y$$

Example 3

(e) $f(x) = x^2$ with $x \geq 0$

$$y = x^2$$

$$x = y^2$$

$$\pm\sqrt{x} = y$$

$$f^{-1}(x) = \pm\sqrt{x}$$

Example 3

(e) $f(x) = x^2$ with $x \geq 0$

$$y = x^2$$

$$x = y^2$$

$$\pm\sqrt{x} = y$$

$$f^{-1}(x) = \pm\sqrt{x}$$

Note: It can NOT be both $\sqrt{x}$ and $-\sqrt{x}$ because it would fail the **vertical line test**.

Example 3

$f(x) = x^2$ with $x \geq 0 \quad f^{-1}(x) = \pm\sqrt{x}$

	Domain (x)	**Range (y)**
$g(x)$		
$g^{-1}(x)$		

Example 3

$f(x) = x^2$ with $x \geq 0$ $\quad f^{-1}(x) = \pm\sqrt{x}$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq 0}$	
$g^{-1}(x)$		

Example 3

$f(x) = x^2$ with $x \geq 0$ $\quad f^{-1}(x) = \pm\sqrt{x}$

	Domain (x)	**Range (y)**
$g(x)$	$x \geq 0$	
$g^{-1}(x)$		$y \geq 0$

Example 3

$f(x) = x^2$ with $x \geq 0$ $\quad f^{-1}(x) = \pm\sqrt{x}$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq 0}$	$\boldsymbol{y \geq 0}$
$g^{-1}(x)$		$\boldsymbol{y \geq 0}$

Example 3

$f(x) = x^2$ with $x \geq 0$ $\quad f^{-1}(x) = \pm\sqrt{x}$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq 0}$	$\boldsymbol{y \geq 0}$
$g^{-1}(x)$	$\boldsymbol{x \geq 0}$	$\boldsymbol{y \geq 0}$

Example 3

$f(x) = x^2$ with $x \geq 0$ $\quad f^{-1}(x) = \pm\sqrt{x}$

	Domain (x)	**Range (y)**
$g(x)$	$\boldsymbol{x \geq 0}$	$\boldsymbol{y \geq 0}$
$g^{-1}(x)$	$\boldsymbol{x \geq 0}$	$\boldsymbol{y \geq 0}$

$$\boldsymbol{f^{-1}(x) = \sqrt{x}}$$