Complex Fractions

Objectives

1. Simplify Complex Fractions

Complex Fraction.

A **complex fraction** is a rational expression which contains other rational expressions in the numerator and/or denominator.

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Some examples of complex fractions are given below:

$$\frac{\frac{2}{x} - 3}{\frac{5}{x} + \frac{7}{x}} \qquad \text{and} \qquad \frac{\frac{x}{x+1} + \frac{7}{x}}{\frac{3}{2x} + \frac{8}{x-4}}$$

Simplifying Complex Fractions

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Recall that when we add or subtract fractions with unlike denominators, we need to find a common denominator first.

Rather than take this approach with complex fractions (which we could, by the way), we are going to clear out our “tiny” fractions by multiplying every term by the **least common tiny denominator**, or **LCTD**.

Least Common Tiny Denominator

We find the least common tiny denominator by finding the least common denominator of all of the “tiny” fractions in the expression. We can then simplify, if possible.

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- To find the LCTD of numbers, find the least common multiple of those numbers.

Least Common Tiny Denominator

We find the least common tiny denominator by finding the least common denominator of all of the "tiny" fractions in the expression. We can then simplify, if possible.

- To find the LCTD of numbers, find the least common multiple of those numbers.
- To find the LCTD of variable terms, select the highest power of each term.

Example 1

Simplify each of the following as much as possible.

(a) $$\frac{\left(3+\dfrac{1}{x}\right)}{\left(\dfrac{2}{x}+4\right)}$$

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$$\frac{\left(3+\dfrac{1}{x}\right)}{\left(\dfrac{2}{x}+4\right)}\left(\frac{x}{x}\right)$$

Example 1

Simplify each of the following as much as possible.

(a) $$\frac{\left(3+\frac{1}{x}\right)}{\left(\frac{2}{x}+4\right)}$$ LCTD is x

$$\frac{\left(3+\frac{1}{x}\right)}{\left(\frac{2}{x}+4\right)}\left(\frac{x}{x}\right) \longrightarrow \frac{3x+1}{2+4x}$$

Example 1

(b) $$\frac{\left(1 - \dfrac{5}{x^2}\right)}{\left(\dfrac{2}{x^2} - 7\right)}$$

Example 1

(b) $$\frac{\left(1-\dfrac{5}{x^2}\right)}{\left(\dfrac{2}{x^2}-7\right)}$$ LCTD is x^2

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$$\frac{\left(1-\dfrac{5}{x^2}\right)}{\left(\dfrac{2}{x^2}-7\right)}\left(\frac{x^2}{x^2}\right)$$

Example 1

(b) $$\frac{\left(1-\dfrac{5}{x^2}\right)}{\left(\dfrac{2}{x^2}-7\right)} \quad \text{LCTD is } x^2$$

$$\frac{\left(1-\dfrac{5}{x^2}\right)}{\left(\dfrac{2}{x^2}-7\right)}\left(\frac{x^2}{x^2}\right) \quad \longrightarrow \quad \frac{x^2-5}{2-7x^2}$$

Example 1

(c) $$\frac{\left(\dfrac{1}{x}+\dfrac{y}{x^2}\right)}{\left(\dfrac{1}{y}+\dfrac{x}{y^2}\right)}$$

Example 1

(c) $$\frac{\left(\dfrac{1}{x}+\dfrac{y}{x^2}\right)}{\left(\dfrac{1}{y}+\dfrac{x}{y^2}\right)}$$ LCTD is x^2y^2

Example 1

(c) $$\frac{\left(\dfrac{1}{x} + \dfrac{y}{x^2}\right)}{\left(\dfrac{1}{y} + \dfrac{x}{y^2}\right)}$$ LCTD is x^2y^2

$$\frac{\left(\dfrac{1}{x} + \dfrac{y}{x^2}\right)}{\left(\dfrac{1}{y} + \dfrac{x}{y^2}\right)}\left(\frac{x^2y^2}{x^2y^2}\right)$$

Example 1

(c) $$\frac{\left(\frac{1}{x}+\frac{y}{x^2}\right)}{\left(\frac{1}{y}+\frac{x}{y^2}\right)}$$ LCTD is x^2y^2

$$\frac{\left(\frac{1}{x}+\frac{y}{x^2}\right)}{\left(\frac{1}{y}+\frac{x}{y^2}\right)}\left(\frac{x^2y^2}{x^2y^2}\right) \quad\longrightarrow\quad \frac{xy^2+y^3}{x^2y+x^3}$$

Example 1c

$$\frac{xy^2 + y^3}{x^2y + x^3}$$

Example 1c

$$\frac{xy^2 + y^3}{x^2y + x^3}$$

$$= \frac{y^2(x+y)}{x^2(y+x)}$$

Example 1c

$$\frac{xy^2 + y^3}{x^2y + x^3}$$

$$= \frac{y^2(x + y)}{x^2(y + x)}$$

$$= \frac{y^2}{x^2}$$

Example 1

(d) $$\frac{\left(\dfrac{x}{y} - 1\right)}{\left(\dfrac{x^2}{y^2} - 1\right)}$$

Example 1

(d) $$\frac{\left(\dfrac{x}{y}-1\right)}{\left(\dfrac{x^2}{y^2}-1\right)}$$ LCTD is y^2

$$\frac{\left(\dfrac{x}{y}-1\right)}{\left(\dfrac{x^2}{y^2}-1\right)}\left(\frac{y^2}{y^2}\right)$$

Example 1

(d) $$\frac{\left(\frac{x}{y}-1\right)}{\left(\frac{x^2}{y^2}-1\right)}$$ LCTD is y^2

$$\frac{\left(\frac{x}{y}-1\right)}{\left(\frac{x^2}{y^2}-1\right)}\left(\frac{y^2}{y^2}\right) \quad \longrightarrow \quad \frac{xy-y^2}{x^2-y^2}$$

Example 1d

$$\frac{xy - y^2}{x^2 - y^2}$$

Example 1d

$$\frac{xy - y^2}{x^2 - y^2}$$

$$= \frac{y(x - y)}{(x + y)(x - y)}$$

Example 1d

$$\frac{xy - y^2}{x^2 - y^2}$$

$$= \frac{y(x-y)}{(x+y)(x-y)}$$

$$= \frac{y}{x+y}$$

Example 1

(e) $$\frac{\left(\dfrac{1}{x+7} - \dfrac{1}{x}\right)}{7}$$

Example 1

(e) $$\frac{\left(\frac{1}{x+7} - \frac{1}{x}\right)}{7}$$ LCTD is $x(x+7)$

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(e) $$\frac{\left(\dfrac{1}{x+7}-\dfrac{1}{x}\right)}{7}$$ LCTD is $x(x+7)$

$$\frac{\left(\dfrac{1}{x+7}-\dfrac{1}{x}\right)}{7}\left(\frac{x(x+7)}{x(x+7)}\right)$$

Example 1

(e) $$\frac{\left(\dfrac{1}{x+7}-\dfrac{1}{x}\right)}{7}$$ LCTD is $x(x+7)$

$$\frac{\left(\dfrac{1}{x+7}-\dfrac{1}{x}\right)}{7}\left(\frac{x(x+7)}{x(x+7)}\right)$$

$$=\frac{x-(x+7)}{7x(x+7)}$$

Example 1

(e) $\dfrac{\left(\dfrac{1}{x+7} - \dfrac{1}{x}\right)}{7}$ LCTD is $x(x+7)$

$$\frac{\left(\dfrac{1}{x+7} - \dfrac{1}{x}\right)}{7}\left(\frac{x(x+7)}{x(x+7)}\right)$$

$$= \frac{x-(x+7)}{7x(x+7)}$$

$$= \frac{x-x-7}{7x(x+7)}$$

Example 1e

$$\frac{x - x - 7}{7x(x + 7)}$$

Example 1e

$$\frac{x - x - 7}{7x(x+7)}$$

$$= \frac{-7}{7x(x+7)}$$

Example 1e

$$\frac{x - x - 7}{7x(x+7)}$$

$$= \frac{-7}{7x(x+7)}$$

$$= \frac{-1}{x(x+7)}$$

Example 1

(f) $$\frac{\left(\dfrac{x+1}{x}+\dfrac{x+1}{x-1}\right)}{\left(\dfrac{x+2}{x}-\dfrac{2}{x-1}\right)}$$

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(f) $$\frac{\left(\dfrac{x+1}{x}+\dfrac{x+1}{x-1}\right)}{\left(\dfrac{x+2}{x}-\dfrac{2}{x-1}\right)}$$ LCTD is $x(x-1)$

Example 1

(f) $$\frac{\left(\dfrac{x+1}{x}+\dfrac{x+1}{x-1}\right)}{\left(\dfrac{x+2}{x}-\dfrac{2}{x-1}\right)}$$ LCTD is $x(x-1)$

$$\frac{\left(\dfrac{x+1}{x}+\dfrac{x+1}{x-1}\right)}{\left(\dfrac{x+2}{x}-\dfrac{2}{x-1}\right)}\left(\frac{x(x-1)}{x(x-1)}\right)$$

Example 1

(f) $$\frac{\left(\dfrac{x+1}{x}+\dfrac{x+1}{x-1}\right)}{\left(\dfrac{x+2}{x}-\dfrac{2}{x-1}\right)}$$ LCTD is $x(x-1)$

$$\frac{\left(\dfrac{x+1}{x}+\dfrac{x+1}{x-1}\right)}{\left(\dfrac{x+2}{x}-\dfrac{2}{x-1}\right)}\left(\frac{x(x-1)}{x(x-1)}\right)$$

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$= \frac{\qquad\qquad}{}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$= \frac{x^2-1}{}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$= \frac{x^2-1+x^2+x}{}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$= \frac{x^2-1+x^2+x}{x^2+x-2}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$= \frac{x^2-1+x^2+x}{x^2+x-2-2x}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$=\frac{x^2-1+x^2+x}{x^2+x-2-2x}$$

$$=\frac{2x^2+x-1}{x^2-x-2}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$=\frac{x^2-1+x^2+x}{x^2+x-2-2x}$$

$$=\frac{2x^2+x-1}{x^2-x-2}$$

$$=\frac{(2x-1)(x+1)}{(x-2)(x+1)}$$

Example 1f

$$\frac{(x+1)(x-1)+x(x+1)}{(x+2)(x-1)-2x}$$

$$=\frac{x^2-1+x^2+x}{x^2+x-2-2x}$$

$$=\frac{2x^2+x-1}{x^2-x-2}$$

$$=\frac{(2x-1)(x+1)}{(x-2)(x+1)} \quad \longrightarrow \quad =\frac{2x-1}{x-2}$$