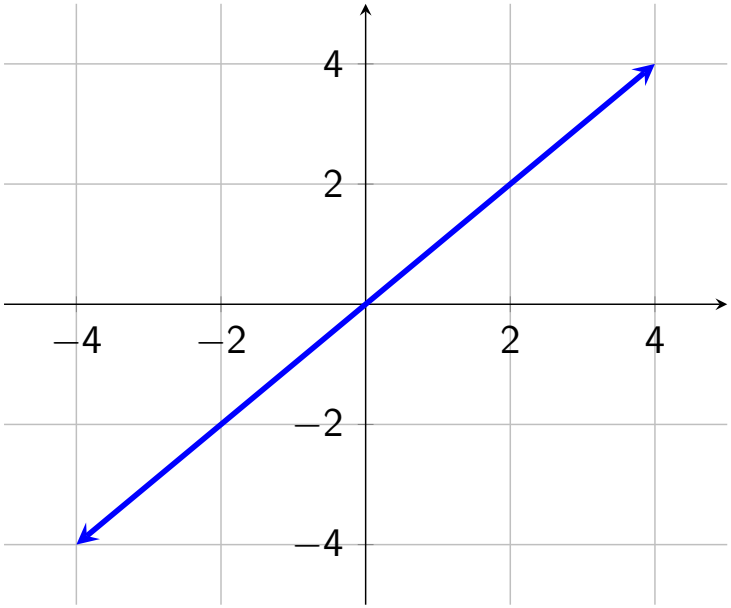
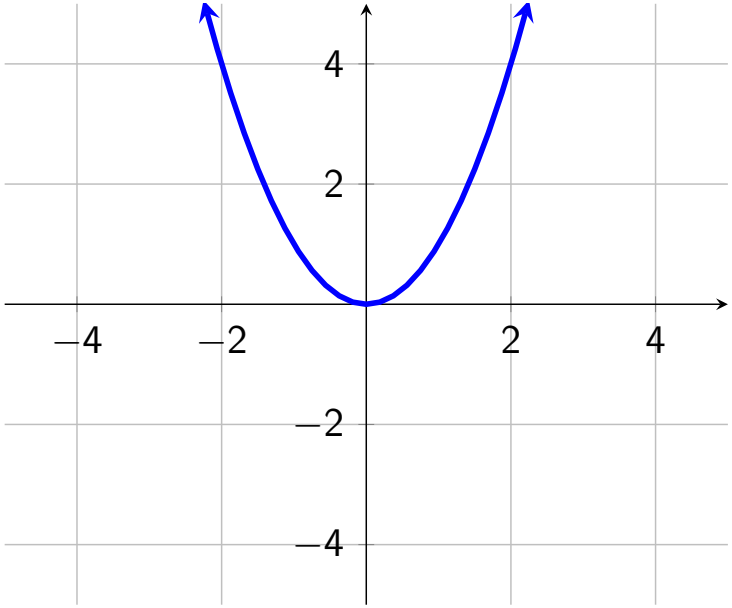
Transforming Functions

Parent Functions

$y = x$

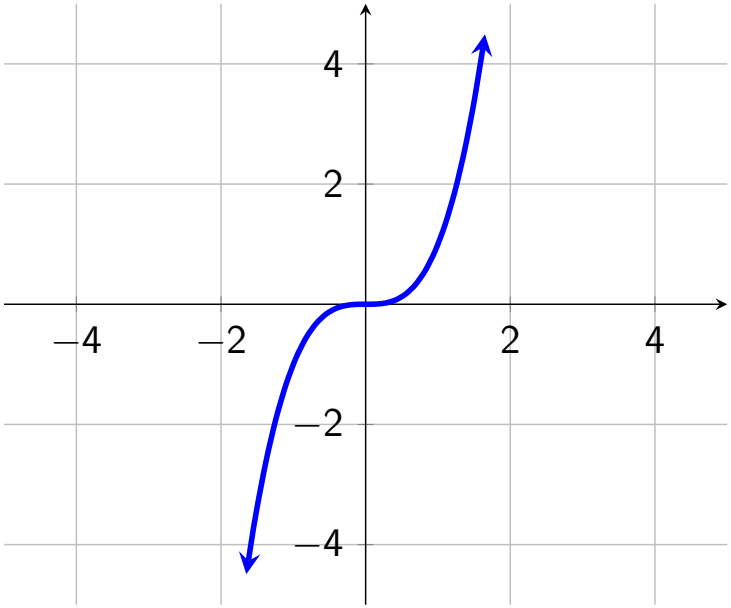


$y = x^2$

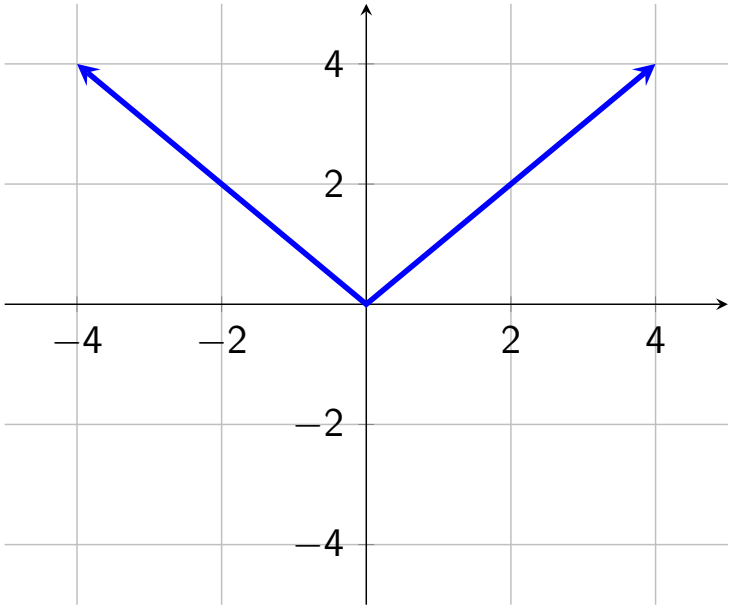


Parent Functions

$y = x^3$

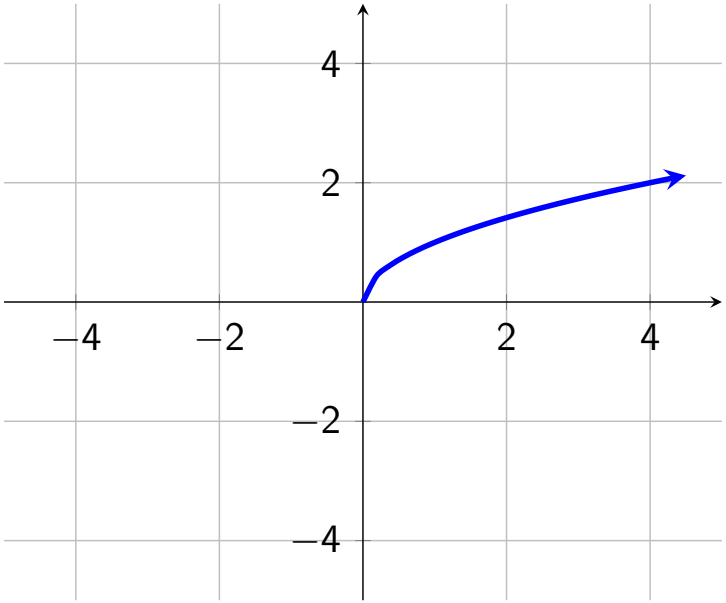


$y = |x|$

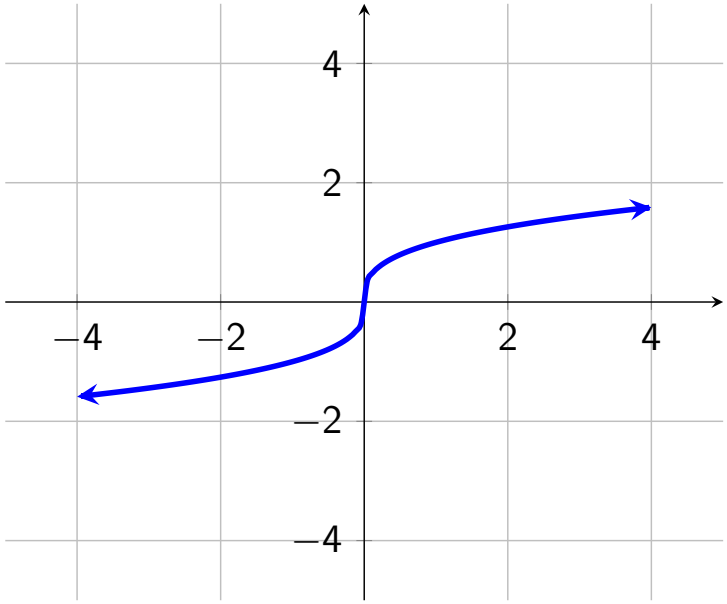


Parent Functions

$y = \sqrt{x}$



$y = \sqrt[3]{x}$



Objectives

1. Perform vertical and horizontal shifts of functions
2. Perform vertical stretches and compressions of functions
3. Perform reflections of functions across the x- and y-axes
4. Perform multiple transformations of functions

$f(x) \pm d$

Investigation:

For the function $f(x) = |x|$, examine the effects of

$$g(x) = |x| \pm d$$

where d is a real number.

Vertical shifts

A vertical shift (a.k.a. vertical translation) moves each point on the graph either up or down by a given number of spaces.

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Equation	**Shift Graph**	**Visually**
$f(x) + d$	Up	Add d to y-coordinates
$f(x) - d$	Down	Subtract d from y-coordinates

Example 1

For each, list the parent function and indicate the vertical translation. *Be specific*.

(a) $g(x) = x^2 + 3$

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For each, list the parent function and indicate the vertical translation. *Be specific*.

(a) $g(x) = x^2 + 3$

Parent function: $f(x) = x^2$

Example 1

For each, list the parent function and indicate the vertical translation. *Be specific*.

(a) $g(x) = x^2 + 3$

Parent function: $f(x) = x^2$

Transformation: Shift up 3 units

Example 1

(b) $g(x) = x^3 - 2$

Example 1

(b) $g(x) = x^3 - 2$

Parent function: $f(x) = x^3$

Example 1

(b) $g(x) = x^3 - 2$

Parent function: $f(x) = x^3$

Transformation: Shift down 2 units

Example 1

(c) $g(x) = |x| + 4.7$

Example 1

(c) $g(x) = |x| + 4.7$

Parent function: $f(x) = |x|$

Example 1

(c) $g(x) = |x| + 4.7$

Parent function: $f(x) = |x|$

Transformation: Shift up 4.7 units

$f(x \pm c)$

Investigation:

For the function $f(x) = |x|$, examine the effects of

$$g(x) = |x \pm c|$$

where c is a real number.

Horizontal shifts

A horizontal shift (a.k.a. horizontal translation) moves each point on the graph either left or right by a given number of spaces.

Horizontal shifts

A horizontal shift (a.k.a. horizontal translation) moves each point on the graph either left or right by a given number of spaces.

Equation	**Shift Graph**
$f(x+c)$	Left
$f(x-c)$	Right

Horizontal shifts

A horizontal shift (a.k.a. horizontal translation) moves each point on the graph either left or right by a given number of spaces.

Equation	**Shift Graph**
$f(x+c)$	Left
$f(x-c)$	Right

Note: With horizontal shifts, the addition or subtraction is done inside the function.

Horizontal shifts

A horizontal shift (a.k.a. horizontal translation) moves each point on the graph either left or right by a given number of spaces.

Equation	**Shift Graph**
$f(x+c)$	Left
$f(x-c)$	Right

Note: With horizontal shifts, the addition or subtraction is done inside the function.

You will need to use ( ) for functions like x^2 and x^3.

Example 2

For each, list the parent function and indicate the horizontal translation. *Be specific.*

(a) $g(x) = (x - 2)^2$

Example 2

For each, list the parent function and indicate the horizontal translation. *Be specific.*

(a) $g(x) = (x - 2)^2$

Parent function: $f(x) = x^2$

Example 2

For each, list the parent function and indicate the horizontal translation. *Be specific.*

(a) $g(x) = (x - 2)^2$

Parent function: $f(x) = x^2$

Transformation: Shift right 2 units

Example 2

(b) $g(x) = |x + 3|$

Example 2

(b) $g(x) = |x + 3|$

Parent function: $f(x) = |x|$

Example 2

(b) $g(x) = |x + 3|$

Parent function: $f(x) = |x|$

Transformation: Shift left 3 units

Example 2

(c) $g(x) = \sqrt{x - 6}$

Example 2

(c) $g(x) = \sqrt{x-6}$

Parent function: $f(x) = \sqrt{x}$

Example 2

(c) $g(x) = \sqrt{x-6}$

Parent function: $f(x) = \sqrt{x}$

Transformation: Shift right 6 units

Objectives

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$a \cdot f(x)$

Investigation:

For the function $f(x) = \sin x$, examine the effects of

$$g(x) = a \cdot \sin x$$

where $a > 1$ and also where $0 < a < 1$.

Vertical Stretches and Compressions

A vertical stretch or vertical compression is obtained by vertically pulling on (or vertically pressing on) the graph.

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A vertical stretch or vertical compression is obtained by vertically pulling on (or vertically pressing on) the graph.

A vertical stretch pulls the points away from the x-axis, while a vertical compression pushes the points towards the x-axis.

Vertical Stretches and Compressions

A vertical stretch or vertical compression is obtained by vertically pulling on (or vertically pressing on) the graph.

A vertical stretch pulls the points away from the x-axis, while a vertical compression pushes the points towards the x-axis.

Algebraically, we obtain vertical stretches and compressions by multiplying the entire function by a positive value.

Vertical Stretches and Compressions

Value of a	**Stretch or Compress?**	**Factor**
$a > 1$	Stretch	a
$0 < a < 1$	Compression	$\frac{1}{a}$

Vertical Stretches and Compressions

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Vertical stretches and compressions will multiply the y-coordinates of the function by whatever the value of a is.

Vertical Stretches and Compressions

Value of a	**Stretch or Compress?**	**Factor**
$a > 1$	Stretch	a
$0 < a < 1$	Compression	$\frac{1}{a}$

Vertical stretches and compressions will multiply the y-coordinates of the function by whatever the value of a is.

Note: The factor is expressed as a value greater than 1.

Example 3

List the parent function and indicate if there is a vertical stretch or vertical compression. Then list the factor.

(a) $g(x) = 2\sqrt{x}$

Example 3

List the parent function and indicate if there is a vertical stretch or vertical compression. Then list the factor.

(a) $g(x) = 2\sqrt{x}$

Parent function: $f(x) = \sqrt{x}$

Example 3

List the parent function and indicate if there is a vertical stretch or vertical compression. Then list the factor.

(a) $g(x) = 2\sqrt{x}$

Parent function: $f(x) = \sqrt{x}$

Transformation: Vertical stretch by factor of 2

Example 3

(b) $g(x) = 3.5\sqrt[3]{x}$

Example 3

(b) $g(x) = 3.5\sqrt[3]{x}$

Parent function: $f(x) = \sqrt[3]{x}$

Example 3

(b) $g(x) = 3.5\sqrt[3]{x}$

Parent function: $f(x) = \sqrt[3]{x}$

Transformation: Vertical stretch by factor of 3.5

Example 3

(c) $g(x) = \frac{1}{3}x^2$

Example 3

(c) $g(x) = \frac{1}{3}x^2$

Parent function: $f(x) = x^2$

Example 3

(c) $g(x) = \frac{1}{3}x^2$

Parent function: $f(x) = x^2$

Transformation: Vertical compression by factor of 3

Example 3

(d) $g(x) = \frac{2}{5}|x|$

Example 3

(d) $g(x) = \frac{2}{5}|x|$

Parent function: $f(x) = |x|$

Example 3

(d) $g(x) = \frac{2}{5}|x|$

Parent function: $f(x) = |x|$

Transformation: Vertical compression by factor of $\frac{5}{2}$

Objectives

1. Perform vertical and horizontal shifts of functions
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3. Perform reflections of functions across the x- and y-axes
4. Perform multiple transformations of functions

Reflections of Functions Across the x- and y-axes

Investigation:

For the function $f(x) = \sqrt{x}$, examine the effects of

$$g(x) = -\sqrt{x}$$

Reflections of Functions Across the x- and y-axes

Investigation:

For the function $f(x) = \sqrt{x}$, examine the effects of

$$g(x) = \sqrt{-x}$$

Reflections Across the x- and y-Axes

When we multiply by -1, we reflect our graph across either the x-axis or the y-axis.

Reflections Across the x- and y-Axes

When we multiply by -1, we reflect our graph across either the x-axis or the y-axis.

Function	**Reflect Across**
$-f(x)$	x-axis
$f(-x)$	y-axis

Example 4

List the parent function and indicate the axis it is reflected across.

(a) $g(x) = \sqrt[3]{-x}$

Example 4

List the parent function and indicate the axis it is reflected across.

(a) $g(x) = \sqrt[3]{-x}$

Parent function: $f(x) = \sqrt[3]{x}$

Example 4

List the parent function and indicate the axis it is reflected across.

(a) $g(x) = \sqrt[3]{-x}$

Parent function: $f(x) = \sqrt[3]{x}$

Transformation: Reflected across the y-axis

Example 4

(b) $g(x) = -|x|$

Example 4

(b) $g(x) = -|x|$

Parent function: $f(x) = |x|$

Example 4

(b) $g(x) = -|x|$

Parent function: $f(x) = |x|$

Transformation: Reflected across the x-axis

Objectives

1. Perform vertical and horizontal shifts of functions
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Order of Multiple Transformations

For multiple transformations, perform them in the following order:

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1. Horizontal translations (shift left or right)

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

1. Horizontal translations (shift left or right)
2. Reflections

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

1. Horizontal translations (shift left or right)
2. Reflections
3. Vertical stretches or compressions

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

1. Horizontal translations (shift left or right)
2. Reflections
3. Vertical stretches or compressions
4. Vertical translations (shift up or down)

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

1. Horizontal translations (shift left or right)
2. Reflections
3. Vertical stretches or compressions
4. Vertical translations (shift up or down)

Note: Steps 2 and 3 may be switched without penalty.

Example 5

Determine the parent function and list all transformations done to it. Be specific.

(a) $g(x) = -2\sqrt{x-3}$

Example 5

Determine the parent function and list all transformations done to it. Be specific.

(a) $g(x) = -2\sqrt{x-3}$

Parent function: $f(x) = \sqrt{x}$

Example 5

Determine the parent function and list all transformations done to it. Be specific.

(a) $g(x) = -2\sqrt{x-3}$

Parent function: $f(x) = \sqrt{x}$

Shift right 3 units $\sqrt{x-3}$

Example 5

Determine the parent function and list all transformations done to it. Be specific.

(a) $g(x) = -2\sqrt{x-3}$

Parent function: $f(x) = \sqrt{x}$

Shift right 3 units $\sqrt{x-3}$

Reflect across x-axis $-\sqrt{x-3}$

Example 5

Determine the parent function and list all transformations done to it. Be specific.

(a) $g(x) = -2\sqrt{x-3}$

Parent function: $f(x) = \sqrt{x}$

Shift right 3 units $\sqrt{x-3}$

Reflect across x-axis $-\sqrt{x-3}$

Vertical stretch by factor of 2 $-2\sqrt{x-3}$

Example 5

(b) $g(x) = \frac{1}{2}(x+3)^2$

Example 5

(b) $g(x) = \frac{1}{2}(x+3)^2$

Parent function: $f(x) = x^2$

Example 5

(b) $g(x) = \frac{1}{2}(x+3)^2$

Parent function: $f(x) = x^2$

Shift left 3 units $(x+3)^2$

Example 5

(b) $g(x) = \frac{1}{2}(x+3)^2$

Parent function: $f(x) = x^2$

Shift left 3 units $(x+3)^2$

Vertical compression by a factor of 2 $\frac{1}{2}(x+3)^2$

Example 5

(c) $g(x) = |-x+2|+1$

Example 5

(c) $g(x) = |-x + 2| + 1$

Parent function: $f(x) = |x|$

Example 5

(c) $g(x) = |-x+2|+1$

Parent function: $f(x) = |x|$

Shift left 2 units $|x+2|$

Example 5

(c) $g(x) = |-x+2|+1$

Parent function: $f(x) = |x|$

Shift left 2 units $|x+2|$

Reflect across y-axis $|-x+2|$

Example 5

(c) $g(x) = |-x+2|+1$

Parent function: $f(x) = |x|$

Shift left 2 units $|x+2|$

Reflect across y-axis $|-x+2|$

Vertical shift up 1 unit $|-x+2|+1$

Example 5

(d) $g(x) = -\frac{1}{5}(-x-7)^3 - 4$

Example 5

(d) $g(x) = -\frac{1}{5}(-x-7)^3 - 4$

Parent function: $f(x) = x^3$

Example 5

(d) $g(x) = -\frac{1}{5}(-x-7)^3 - 4$

Parent function: $f(x) = x^3$

Shift right 7 units $(x-7)^3$

Example 5

(d) $g(x) = -\frac{1}{5}(-x-7)^3 - 4$

Parent function: $f(x) = x^3$

Shift right 7 units $(x-7)^3$

Reflect across y-axis $(-x-7)^3$

Example 5

(d) $g(x) = -\frac{1}{5}(-x-7)^3 - 4$

Parent function: $f(x) = x^3$

Shift right 7 units $(x-7)^3$

Reflect across y-axis $(-x-7)^3$

Vertical compression by factor of 5 $\frac{1}{5}(-x-7)^3$

Example 5

(d) $g(x) = -\frac{1}{5}(-x-7)^3 - 4$

Parent function: $f(x) = x^3$

Shift right 7 units $(x-7)^3$

Reflect across y-axis $(-x-7)^3$

Vertical compression by factor of 5 $\frac{1}{5}(-x-7)^3$

Reflect across x-axis $-\frac{1}{5}(-x-7)^3$

Example 5

(d) $g(x) = -\frac{1}{5}(-x-7)^3 - 4$

Parent function: $f(x) = x^3$

Shift right 7 units $(x-7)^3$

Reflect across y-axis $(-x-7)^3$

Vertical compression by factor of 5 $\frac{1}{5}(-x-7)^3$

Reflect across x-axis $-\frac{1}{5}(-x-7)^3$

Vertical shift down 4 units $-\frac{1}{5}(-x-7)^3 - 4$