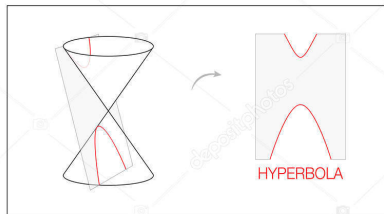
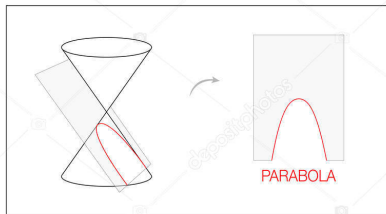
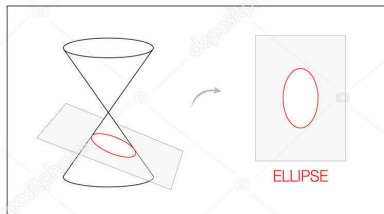
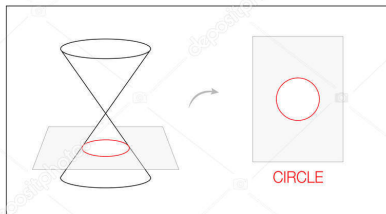


Circles



Objectives

- 1 Identify the center and radius of a circle.
- 2 Write the general form of the equation of a circle in standard form

Circles

You may remember circles from geometry class. In this chapter, we will look at equations and properties of circles.

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The set of points, each of whose distance from a fixed point (the center) is the same.

The standard form of the equation of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

with center (h, k) and radius r .

Example 1

Identify the center and exact radius of each.

(a) $(x - 4)^2 + (y + 3)^2 = 49$

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Center: $(4, -3)$

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Radius: $\sqrt{49}$

Example 1

Identify the center and exact radius of each.

$$(a) \quad (x - 4)^2 + (y + 3)^2 = 49$$

Center: $(4, -3)$

Radius: $\sqrt{49} = 7$

Example 1

$$(b) \quad (x + 1)^2 + (y - 7)^2 = 72$$

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Center: $(-1, 7)$

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Radius: $\sqrt{72}$

Example 1

$$(b) \quad (x + 1)^2 + (y - 7)^2 = 72$$

Center: $(-1, 7)$

Radius: $\sqrt{72} = 6\sqrt{2}$

Writing the Standard Form of the Equation of a Circle

To get the standard form, perform the following steps:

- 1 Bring the constant over to the other side of the equation.

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 - The x -coordinates of each vertex will represent h and k , respectively.

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To get the standard form, perform the following steps:

- 1 Bring the constant over to the other side of the equation.
- 2 Find the vertex of the x -terms and y -terms.
 - The x -coordinates of each vertex will represent h and k , respectively.
 - The absolute value y -coordinates will be added to the constant.

Example 2

Identify the center and exact radius of each.

(a) $x^2 - 4x + y^2 + 6y - 23 = 0$

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$$x^2 - 4x \quad + \quad y^2 - 6y \quad = 23$$

Vertex: $(2, -4)$

Example 2

Identify the center and exact radius of each.

$$(a) \quad x^2 - 4x + y^2 + 6y - 23 = 0$$

$$x^2 - 4x + y^2 + 6y = 23$$

Vertex: $(2, -4)$

Vertex: $(3, -9)$

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Identify the center and exact radius of each.

$$(a) \quad x^2 - 4x + y^2 + 6y - 23 = 0$$

$$x^2 - 4x + y^2 + 6y = 23$$

Vertex: $(2, -4)$

Vertex: $(3, -9)$

$$(x - 2)^2 + (y - 3)^2 = 23 + |-4| + |-9|$$

Example 2

Identify the center and exact radius of each.

$$(a) \quad x^2 - 4x + y^2 + 6y - 23 = 0$$

$$x^2 - 4x \quad + \quad y^2 + 6y \quad = 23$$

Vertex: $(2, -4)$

Vertex: $(3, -9)$

$$(x - 2)^2 + (y - 3)^2 = 23 + |-4| + |-9|$$

$$(x - 2)^2 + (y - 3)^2 = 36$$

Example 2

Identify the center and exact radius of each.

$$(a) \quad x^2 - 4x + y^2 + 6y - 23 = 0$$

$$x^2 - 4x \quad + \quad y^2 + 6y \quad = 23$$

Vertex: $(2, -4)$

Vertex: $(3, -9)$

$$(x - 2)^2 + (y - 3)^2 = 23 + |-4| + |-9|$$

$$(x - 2)^2 + (y - 3)^2 = 36$$

Center: $(2, 3)$ Radius: 6

Example 2

$$(b) \quad x^2 + 16x + y^2 - 8y - 1 = 0$$

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Vertex: $(-8, -64)$

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$$(b) \quad x^2 + 16x + y^2 - 8y - 1 = 0$$

$$x^2 + 16x + y^2 - 8y = 1$$

Vertex: $(-8, -64)$

Vertex: $(4, -16)$

Example 2

$$(b) \quad x^2 + 16x + y^2 - 8y - 1 = 0$$

$$x^2 + 16x + y^2 - 8y = 1$$

Vertex: $(-8, -64)$

Vertex: $(4, -16)$

$$(x + 8)^2 + (y - 4)^2 = 1 + |-64| + |-16|$$

Example 2

$$(b) \quad x^2 + 16x + y^2 - 8y - 1 = 0$$

$$x^2 + 16x + y^2 - 8y = 1$$

Vertex: $(-8, -64)$

Vertex: $(4, -16)$

$$(x + 8)^2 + (y - 4)^2 = 1 + |-64| + |-16|$$

$$(x + 8)^2 + (y - 4)^2 = 81$$

Example 2

$$(b) \quad x^2 + 16x + y^2 - 8y - 1 = 0$$

$$x^2 + 16x + y^2 - 8y = 1$$

Vertex: $(-8, -64)$

Vertex: $(4, -16)$

$$(x + 8)^2 + (y - 4)^2 = 1 + |-64| + |-16|$$

$$(x + 8)^2 + (y - 4)^2 = 81$$

Center: $(-8, 4)$ Radius: 9

Example 2

$$(c) \quad x^2 - 10x + y^2 + 2y + 14 = 0$$

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$$(c) \quad x^2 - 10x + y^2 + 2y + 14 = 0$$

$$x^2 - 10x + y^2 + 2y = -14$$

$$x^2 - 10x + y^2 + 2y = 1$$

Vertex: (5, -25)

Example 2

$$(c) \quad x^2 - 10x + y^2 + 2y + 14 = 0$$

$$x^2 - 10x + y^2 + 2y = -14$$

$$x^2 - 10x + y^2 + 2y = 1$$

Vertex: $(5, -25)$

Vertex: $(-1, -1)$

Example 2

$$(c) \quad x^2 - 10x + y^2 + 2y + 14 = 0$$

$$x^2 - 10x + y^2 + 2y = -14$$

$$x^2 - 10x + y^2 + 2y = 1$$

Vertex: $(5, -25)$

Vertex: $(-1, -1)$

$$(x - 5)^2 + (y + 1)^2 = 14 + | - 25 | + | - 1 |$$

Example 2

$$(c) \quad x^2 - 10x + y^2 + 2y + 14 = 0$$

$$x^2 - 10x + y^2 + 2y = -14$$

$$x^2 - 10x + y^2 + 2y = 1$$

Vertex: $(5, -25)$

Vertex: $(-1, -1)$

$$(x - 5)^2 + (y + 1)^2 = 14 + |-25| + |-1|$$

$$(x - 5)^2 + (y + 1)^2 = 40$$

Example 2

$$(c) \quad x^2 - 10x + y^2 + 2y + 14 = 0$$

$$x^2 - 10x + y^2 + 2y = -14$$

$$x^2 - 10x + y^2 + 2y = 1$$

Vertex: $(5, -25)$

Vertex: $(-1, -1)$

$$(x - 5)^2 + (y + 1)^2 = 14 + |-25| + |-1|$$

$$(x - 5)^2 + (y + 1)^2 = 40$$

Center: $(5, -1)$ Radius: $2\sqrt{10}$

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- 1 Identify the center and radius of a circle.
- 2 Write the general form of the equation of a circle in standard form

General Form

Circles can also be written in general form. General form is standard form multiplied out and simplified.

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Circles can also be written in general form. General form is standard form multiplied out and simplified.

General form will have all terms on one side of the equation and 0 on the other.

Example 3

Write the general form of each of the following.

(a) $(x - 3)^2 + y^2 = 6$

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$$(x - 3)^2 + y^2 = 6$$

$$x^2 - 6x + 9 + y^2 = 6$$

Example 3

Write the general form of each of the following.

(a) $(x - 3)^2 + y^2 = 6$

$$(x - 3)^2 + y^2 = 6$$

$$x^2 - 6x + 9 + y^2 = 6$$

$$x^2 - 6x + y^2 + 3 = 0$$

Example 3

$$(b) \quad (x + 7)^2 + (y - 5)^2 = 18$$

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Example 3

$$(b) \quad (x + 7)^2 + (y - 5)^2 = 18$$

$$(x + 7)^2 + (y - 5)^2 = 18$$

$$x^2 + 14x + 49 + y^2 - 10y + 25 = 18$$

Example 3

$$(b) \quad (x + 7)^2 + (y - 5)^2 = 18$$

$$(x + 7)^2 + (y - 5)^2 = 18$$

$$x^2 + 14x + 49 + y^2 - 10y + 25 = 18$$

$$x^2 + 14x + y^2 - 10y + 74 = 18$$

Example 3

$$(b) \quad (x + 7)^2 + (y - 5)^2 = 18$$

$$(x + 7)^2 + (y - 5)^2 = 18$$

$$x^2 + 14x + 49 + y^2 - 10y + 25 = 18$$

$$x^2 + 14x + y^2 - 10y + 74 = 18$$

$$x^2 + 14x + y^2 - 10y + 64 = 0$$