# Operations with Functions

# Objectives

1. Write the sum, difference, product, and quotient of two functions
2. Evaluate the sum, difference, product, and quotient of two functions at a value

# Sum, Difference, Product, and Quotient of Functions

We can add, subtract, multiply, and divide functions just like we can with real numbers.

| | |
|---|---|
| **Sum** | $(f+g)(x) = f(x) + g(x)$ |
| **Difference** | $(f-g)(x) = f(x) - g(x)$ |
| **Product** | $(fg)(x) = f(x) \cdot g(x)$ |
| **Quotient** | $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$ |

# Sum, Difference, Product, and Quotient of Functions

So if $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

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$$(f + g)(x) = f(x) + g(x)$$

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So if $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= x + 2 + x^2 - 4\end{aligned}$$

# Sum, Difference, Product, and Quotient of Functions

So if $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$
\begin{aligned}
(f+g)(x) &= f(x) + g(x) \\
&= x + 2 + x^2 - 4 \\
&= x^2 + x - 2
\end{aligned}
$$

# Example 1

Find each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(x)$

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Find each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(x)$

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= x^2 - 3 + 4x + 5\end{aligned}$$

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Find each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f+g)(x)$

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= x^2 - 3 + 4x + 5\\ &= x^2 + 4x + 2\end{aligned}$$

# Example 1 $\quad f(x) = x^2 - 3 \quad g(x) = 4x + 5$

(b) $\quad (f - g)(x)$

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(b) $\quad (f - g)(x)$

$$\begin{aligned}(f - g)(x) &= f(x) - g(x) \\ &= x^2 - 3 - (4x + 5)\end{aligned}$$

# Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(b) $(f - g)(x)$

$$
\begin{aligned}
(f - g)(x) &= f(x) - g(x) \\
&= x^2 - 3 - (4x + 5) \\
&= x^2 - 3 - 4x - 5
\end{aligned}
$$

## Example 1 $\quad f(x) = x^2 - 3 \quad g(x) = 4x + 5$

(b) $\quad (f - g)(x)$

$$\begin{aligned} (f - g)(x) &= f(x) - g(x) \\ &= x^2 - 3 - (4x + 5) \\ &= x^2 - 3 - 4x - 5 \\ &= x^2 - 4x - 8 \end{aligned}$$

# Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

(c) $(g - f)(x)$

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$$\begin{aligned} (g - f)(x) &= g(x) - f(x) \\ &= 4x + 5 - (x^2 - 3) \end{aligned}$$

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$$
\begin{aligned}
(g - f)(x) &= g(x) - f(x) \\
&= 4x + 5 - (x^2 - 3) \\
&= 4x + 5 - x^2 + 3
\end{aligned}
$$

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(c) $\quad (g - f)(x)$

$$
\begin{aligned}
(g - f)(x) &= g(x) - f(x) \\
&= 4x + 5 - (x^2 - 3) \\
&= 4x + 5 - x^2 + 3 \\
&= -x^2 + 4x + 8
\end{aligned}
$$

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(d) $\quad (fg)(x)$

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(d) $\quad (fg)(x)$

$$
\begin{aligned}
(fg)(x) &= f(x) \cdot g(x) \\
&= (x^2 - 3)(4x + 5)
\end{aligned}
$$

# Example 1 $\quad f(x) = x^2 - 3 \quad g(x) = 4x + 5$

(d) $\quad (fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x^2 - 3)(4x + 5)$$

| | $4x$ | $5$ |
|---|---|---|
| $x^2$ | | |
| $-3$ | | |

# Example 1 $f(x) = x^2 - 3$ $g(x) = 4x + 5$

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|---|---|---|
| $x^2$ | $4x^3$ | |
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| | $4x$ | $5$ |
|---|---|---|
| $x^2$ | $4x^3$ | $5x^2$ |
| $-3$ | $-12x$ | |

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| | $4x$ | $5$ |
|---|---|---|
| $x^2$ | $4x^3$ | $5x^2$ |
| $-3$ | $-12x$ | $-15$ |

## Example 1 $\quad f(x) = x^2 - 3 \quad g(x) = 4x + 5$

(d) $\quad (fg)(x)$

$$
\begin{aligned}
(fg)(x) &= f(x) \cdot g(x) \\
&= (x^2 - 3)(4x + 5)
\end{aligned}
$$

$$
\begin{array}{c|c|c|}
 & 4x & 5 \\ \hline
x^2 & 4x^3 & 5x^2 \\ \hline
-3 & -12x & -15 \\ \hline
\end{array}
$$

$$4x^3 + 5x^2 - 12x - 15$$

# Example 1 $f(x) = x^2 - 3 \quad g(x) = 4x + 5$

(e) $\left(\dfrac{f}{g}\right)(x)$

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(e) $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{x^2 - 3}{4x + 5}$$

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(f) $\left(\frac{g}{f}\right)(x)$

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(f) $\left(\frac{g}{f}\right)(x)$

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$$

$$= \frac{4x+5}{x^2-3}$$

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1. Write the sum, difference, product, and quotient of two functions
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# Methods

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If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}(f+g)(3) &= f(3) + g(3) \\ &= (3+2) + (3^2 - 4)\end{aligned}$$

# Methods

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned} (f+g)(3) &= f(3) + g(3) \\ &= (3+2) + (3^2 - 4) \\ &= 10 \end{aligned}$$

# Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

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If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

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# Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$
\begin{aligned}
(f+g)(x) &= f(x) + g(x) \\
&= x + 2 + x^2 - 4 \\
&= x^2 + x - 2 \\
(f+g)(3) &= 3^2 + 3 - 2
\end{aligned}
$$

# Alternate Method

If $f(x) = x + 2$ and $g(x) = x^2 - 4$, then

$$\begin{aligned}
(f+g)(x) &= f(x) + g(x) \\
&= x + 2 + x^2 - 4 \\
&= x^2 + x - 2 \\
(f+g)(3) &= 3^2 + 3 - 2 \\
&= 10
\end{aligned}$$

# Example 2

Evaluate each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(3)$

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# Example 2

Evaluate each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(3)$

$$\begin{aligned}(f+g)(3) &= f(3) + g(3) \\ &= 6 + 17\end{aligned}$$

# Example 2

Evaluate each of the following if $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(a) $(f + g)(3)$

$$\begin{aligned}(f + g)(3) &= f(3) + g(3) \\ &= 6 + 17 \\ &= 23\end{aligned}$$

## Example 2 $\quad f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(b) $\quad (f - g)(0)$

# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

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(b) $(f - g)(0)$

$$\begin{aligned}(f-g)(0) &= f(0) - g(0) \\ &= -3 - 5\end{aligned}$$

# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(b) $(f - g)(0)$

$$\begin{aligned} (f-g)(0) &= f(0) - g(0) \\ &= -3 - 5 \\ &= -8 \end{aligned}$$

# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(c) $(fg)(2)$

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# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(c) $(fg)(2)$

$$\begin{aligned}(fg)(2) &= f(2) \cdot g(2) \\ &= 1(13)\end{aligned}$$

## Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(c) $(fg)(2)$

$$
\begin{aligned}
(fg)(2) &= f(2) \cdot g(2) \\
&= 1(13) \\
&= 13
\end{aligned}
$$

# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(d) $(gg)(1)$

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## Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(d) $(gg)(1)$

$$\begin{aligned}(gg)(1) &= g(1) \cdot g(1)\\ &= 9(9)\end{aligned}$$

# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(d) $(gg)(1)$

$$\begin{aligned}(gg)(1) &= g(1) \cdot g(1) \\ &= 9(9) \\ &= 81\end{aligned}$$

## Example 2 $\quad f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(e) $\quad \left(\dfrac{f}{g}\right)(1)$

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# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(e) $\left(\frac{f}{g}\right)(1)$

$$\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)}$$

$$= \frac{-2}{9}$$

# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(8)$

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(f) $\left(\frac{g}{f}\right)(8)$

$$\left(\frac{g}{f}\right)(8) = \frac{g(8)}{f(8)}$$

# Example 2 $f(x) = x^2 - 3$ and $g(x) = 4x + 5$

(f) $\left(\frac{g}{f}\right)(8)$

$$\left(\frac{g}{f}\right)(8) = \frac{g(8)}{f(8)}$$

$$= \frac{37}{61}$$