Complex Numbers

Objectives

1 Write square roots of negative numbers as imaginary numbers

2 Add and subtract complex numbers

- Multiply complex numbers
- 4 Divide complex numbers

Background

Imaginary numbers arose from trying to solve equations such as

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We define the imaginary unit i to be that value:

$$i = \sqrt{-1}$$

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$$\sqrt{-1}$$

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$$\sqrt{-25} = \sqrt{-1} \cdot \sqrt{25}$$
$$= i \cdot 5$$
$$= 5i$$

(a)
$$\sqrt{-36}$$

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$$\sqrt{-36} = \sqrt{-1} \cdot \sqrt{36}$$

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$$= i \cdot 6$$

(a)
$$\sqrt{-36}$$

$$\sqrt{-36} = \sqrt{-1} \cdot \sqrt{36}$$
$$= i \cdot 6$$
$$= 6i$$

(b)
$$\sqrt{-100}$$

(b)
$$\sqrt{-100}$$

$$\sqrt{-100} = \sqrt{-1} \cdot \sqrt{100}$$

(b)
$$\sqrt{-100}$$

$$\sqrt{-100} = \sqrt{-1} \cdot \sqrt{100}$$
$$= i \cdot 10$$

(b)
$$\sqrt{-100}$$

$$\sqrt{-100} = \sqrt{-1} \cdot \sqrt{100}$$
$$= i \cdot 10$$
$$= 10i$$



(c)
$$\sqrt{-8}$$

$$\sqrt{-8} = \sqrt{-1} \cdot \sqrt{8}$$

(c)
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$$= i \cdot 2\sqrt{2}$$

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$$= 2i\sqrt{2}$$

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A complex number is a number written in the form

$$a + bi$$

where a (the real part) and b (the imaginary part) are real numbers.

Adding and Subtracting Complex Numbers

Complex numbers can be added and subtracted much like combining like terms in Algebra 1.

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Answers are then typically written in a + bi form.

Simplify each.

(a)
$$(3+2i)+(-1+8i)$$

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 $(3+2i)+(-1+8i)=(3+(-1))+(2i+8i)$

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(a)
$$(3+2i) + (-1+8i)$$

 $(3+2i) + (-1+8i) = (3+(-1)) + (2i+8i)$
 $= 2+10i$

(b)
$$(-5-i)+(2+4i)$$

(b)
$$(-5-i) + (2+4i)$$

 $(-5-i) + (2+4i) = (-5+2) + (-i+4i)$

(b)
$$(-5-i) + (2+4i)$$

 $(-5-i) + (2+4i) = (-5+2) + (-i+4i)$
 $= -3+3i$

(c)
$$(7-9i)-(3+5i)$$

(c)
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 $(7-9i)-(3+5i)=7-9i-3-5i$

(c)
$$(7-9i) - (3+5i)$$

 $(7-9i) - (3+5i) = 7-9i-3-5i$
 $= 4-14i$

(d)
$$(-1+2i)-(-8+10i)$$

(d)
$$(-1+2i) - (-8+10i)$$

 $(-1+2i) - (-8+10i) = -1+2i+8-10i$

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 $= 7-8i$

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Multiplying Complex Numbers

Complex numbers can be multiplied in the same manner that binomials are multiplied in Algebra 1, such as (x + 3)(x - 8).

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However, since $i = \sqrt{-1}$, if we square both sides we get

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So when multiplying complex numbers, you will substitute a -1 whenever you see an i^2 .

(a)
$$(2+3i)(5+6i)$$

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	5	6 <i>i</i>
2		
3i		

(a)
$$(2+3i)(5+6i)$$

	5	6 <i>i</i>
2	10	
3 <i>i</i>		

(a)
$$(2+3i)(5+6i)$$

	5	6 <i>i</i>
2	10	12 <i>i</i>
3 <i>i</i>		

(a)
$$(2+3i)(5+6i)$$

	5	6 <i>i</i>
2	10	12 <i>i</i>
3 <i>i</i>	15 <i>i</i>	

(a)
$$(2+3i)(5+6i)$$

	5	6 <i>i</i>
2	10	12 <i>i</i>
3 <i>i</i>	15 <i>i</i>	18 <i>i</i> ²

(a)
$$(2+3i)(5+6i)$$

	5	6 <i>i</i>
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3 <i>i</i>	15 <i>i</i>	

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	5	6 <i>i</i>
2	10	12 <i>i</i>
3 <i>i</i>	15 <i>i</i>	-18

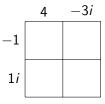
(a)
$$(2+3i)(5+6i)$$

$$\begin{array}{c|cccc}
5 & 6i \\
2 & 10 & 12i \\
3i & 15i & -18
\end{array}$$

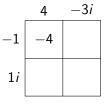
$$-8 + 27i$$

(b)
$$(-1+i)(4-3i)$$

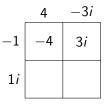
(b)
$$(-1+i)(4-3i)$$



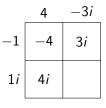
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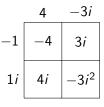
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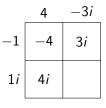
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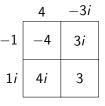
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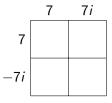
(b)
$$(-1+i)(4-3i)$$

$$\begin{array}{c|cc}
4 & -3i \\
-1 & -4 & 3i \\
1i & 4i & 3
\end{array}$$

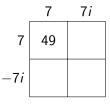
$$-1 + 7i$$

(c)
$$(7-7i)(7+7i)$$

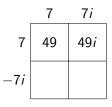
(c)
$$(7-7i)(7+7i)$$



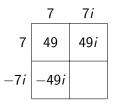
(c)
$$(7-7i)(7+7i)$$



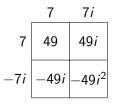
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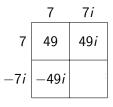
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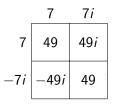
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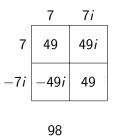
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Dividing Complex Numbers

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The reason is that the denominator will have a square root, which is a big no-no in math:

$$\frac{3-i}{2+i} = \frac{3-\sqrt{-1}}{2+\sqrt{-1}}$$

To remedy this, we need to find the conjugate of the denominator.

Complex Conjugates

The conjugate of a complex number

$$a + bi$$

is

and vice versa

Number Conjugate

Number	Conjugate
7 + 2i	_

Number	Conjugate
7 + 2i	7 – 2 <i>i</i>

Number	Conjugate
7 + 2i	7 – 2i
-3 + i	

Number	Conjugate
7 + 2 <i>i</i>	7 – 2i
-3 + i	-3 - i

Number	Conjugate
7 + 2i	7 – 2 <i>i</i>
-3 + i	-3 - i
5 – 4 <i>i</i>	

Number	Conjugate
7 + 2 <i>i</i>	7 – 2 <i>i</i>
-3 + i	-3 - i
5 – 4 <i>i</i>	5 + 4i

Number	Conjugate
7 + 2 <i>i</i>	7 – 2i
-3 + i	-3 - i
5 – 4 <i>i</i>	5 + 4i

When you multiply complex conjugates, you will always get a real number, like in Example 3c.

Number	Conjugate
7 + 2 <i>i</i>	7 – 2i
-3 + i	-3 - i
5 – 4 <i>i</i>	5 + 4 <i>i</i>

When you multiply complex conjugates, you will always get a real number, like in Example 3c.

So, to divide complex numbers, multiply both numerator and denominator by the **conjugate of the denominator**.

Divide $\frac{3-i}{2+i}$. Write your answer in a+bi form.

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$$\frac{3-i}{2+i}$$

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$$\frac{3-i}{2+i}\left(\frac{2-i}{2-i}\right)$$

Divide $\frac{3-i}{2+i}$. Write your answer in a+bi form.

$$\frac{3-i}{2+i}\left(\frac{2-i}{2-i}\right)$$

$$=\frac{5-5i}{5}$$

Divide $\frac{3-i}{2+i}$. Write your answer in a+bi form.

$$\frac{3-i}{2+i} \left(\frac{2-i}{2-i}\right)$$

$$= \frac{5-5i}{5}$$

$$= 1-i$$