Absolute Value Equations and Inequalities

Objectives

1. Solve absolute value equations.

2. Solve and graph absolute value inequalities

Absolute Value Equations

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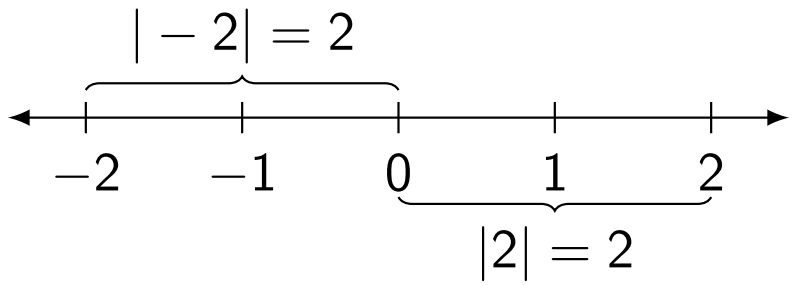

Solving Absolute Value Equations

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Make sure your absolute value bars are isolated on one side before separating your problem into 2 equations.

Example 1

Solve each.

(a) $|2x - 3| = 11$

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(b) $|3x - 1| = 5$

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$$x = 2 \qquad x = -\frac{4}{3}$$

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$$x = 2 \text{ or } x = -\frac{4}{3}$$

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(c) $|x + 1| = -2$

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No solution ($\varnothing$)

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(d) $|-x+2|-4=10$

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$$-x=12 \qquad -x=-16$$

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1. Solve absolute value equations.

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Absolute Value Inequalities: $<$ and $\leq$

Absolute value inequalities are similar to absolute value equations.

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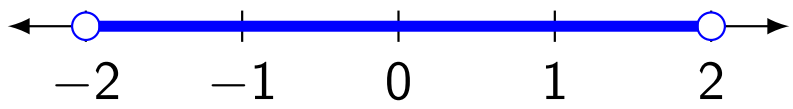

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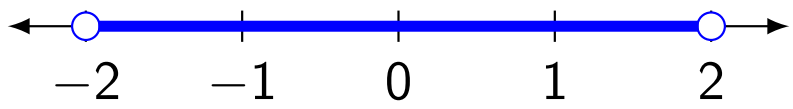

$|x| < 2$ means $-2 < x < 2$

Example 2

Solve and graph each.

(a) $|2x - 2| < 5$

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Alternate Solution Method for Example 2a

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$x = -\frac{3}{2}$ or $x = \frac{7}{2}$

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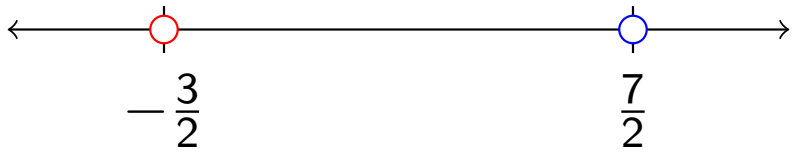

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Example 2

(c) $|x + 2| \leq -1$

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No solution, $\varnothing$, since $|x+2|$ is guaranteed to always be negative.

Alternate Approach to Example 2c

(c) $|x+2| \leq -1$ Treat as $|x+2| = -1$

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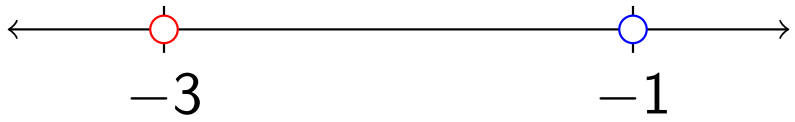

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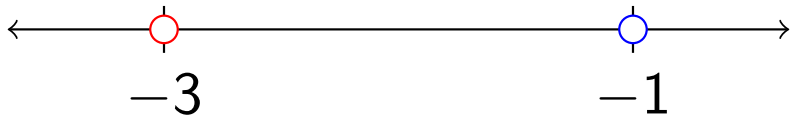

No test values work.

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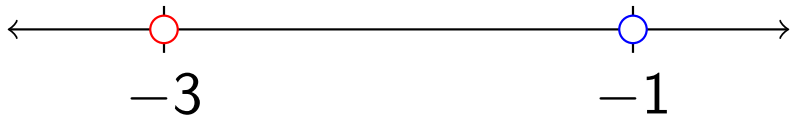

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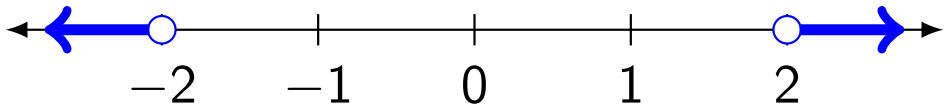

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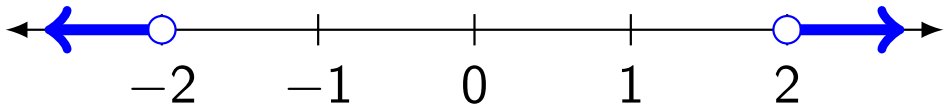

$|x| > 2$ means that $x < -2$ or $x > 2$

Example 3

Solve and graph each.

(a) $|2x + 3| \geq 5$

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$2x - 5 < -3$ $\qquad$ $2x - 5 > 3$

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$2x - 5 < -3$ $\qquad$ $2x - 5 > 3$

$2x < 2$ $\qquad$ $2x > 8$

Example 3

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$$2x - 5 < -3 \qquad 2x - 5 > 3$$

$$2x < 2 \qquad 2x > 8$$

$$x < 1 \qquad x > 4$$

Example 3

(b) $|2x - 5| > 3$

$2x - 5 < -3$ or $2x - 5 > 3$

$2x < 2$ or $2x > 8$

$x < 1$ or $x > 4$

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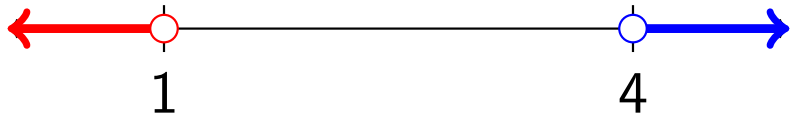

Example 3

(c) $|2x + 2| > -2$

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All real numbers, $\mathbb{R}$, since $|2x + 2|$ is guaranteed to always be greater than a negative number.

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All test values work.

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