

Objectives

- 1 Perform vertical and horizontal shifts of functions
- 2 Perform vertical stretches and compressions of functions
- 3 Perform reflections of functions across the x- and y-axes
- 4 Perform multiple transformations of functions

$$f(x) \pm d$$

INVESTIGATION:

For the function $f(x) = |x|$, examine the effects of

$$g(x) = |x| \pm d$$

where d is a real number.

Vertical shifts

A **vertical shift** (a.k.a. **vertical translation**) moves each point on the graph either up or down by a given number of spaces.

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| Equation | Shift Graph | Visually |
|------------|-------------|------------------------------------|
| $f(x) + d$ | Up | Add d to y -coordinates |
| $f(x) - d$ | Down | Subtract d from y -coordinates |

Example 1

For each, list the parent function and indicate the vertical translation. *Be specific.*

(a) $g(x) = x^2 + 3$

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Parent function: $f(x) = x^2$

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(a) $g(x) = x^2 + 3$

Parent function: $f(x) = x^2$

Transformation: Shift up 3 units

Example 1

$$(b) \quad g(x) = x^3 - 2$$

Example 1

$$(b) \quad g(x) = x^3 - 2$$

Parent function: $f(x) = x^3$

Example 1

$$(b) \quad g(x) = x^3 - 2$$

Parent function: $f(x) = x^3$

Transformation: Shift down 2 units

Example 1

$$(c) \quad g(x) = |x| + 4.7$$

Example 1

$$(c) \quad g(x) = |x| + 4.7$$

Parent function: $f(x) = |x|$

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Parent function: $f(x) = |x|$

Transformation: Shift up 4.7 units

$$f(x \pm c)$$

INVESTIGATION:

For the function $f(x) = |x|$, examine the effects of

$$g(x) = |x \pm c|$$

where c is a real number.

Horizontal shifts

A **horizontal shift** (a.k.a. **horizontal translation**) moves each point on the graph either left or right by a given number of spaces.

Horizontal shifts

A **horizontal shift** (a.k.a. **horizontal translation**) moves each point on the graph either left or right by a given number of spaces.

| Equation | Shift Graph |
|------------|-------------|
| $f(x + c)$ | Left |
| $f(x - c)$ | Right |

Horizontal shifts

A **horizontal shift** (a.k.a. **horizontal translation**) moves each point on the graph either left or right by a given number of spaces.

| Equation | Shift Graph |
|------------|-------------|
| $f(x + c)$ | Left |
| $f(x - c)$ | Right |

Note: With horizontal shifts, the addition or subtraction is done inside the function.

Horizontal shifts

A **horizontal shift** (a.k.a. **horizontal translation**) moves each point on the graph either left or right by a given number of spaces.

| Equation | Shift Graph |
|------------|-------------|
| $f(x + c)$ | Left |
| $f(x - c)$ | Right |

Note: With horizontal shifts, the addition or subtraction is done inside the function.

You will need to use () for functions like x^2 and x^3 .

Example 2

For each, list the parent function and indicate the horizontal translation. *Be specific.*

(a) $g(x) = (x - 2)^2$

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(a) $g(x) = (x - 2)^2$

Parent function: $f(x) = x^2$

Transformation: Shift right 2 units

Example 2

$$(b) \quad g(x) = |x + 3|$$

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$$(b) \quad g(x) = |x + 3|$$

Parent function: $f(x) = |x|$

Example 2

$$(b) \quad g(x) = |x + 3|$$

Parent function: $f(x) = |x|$

Transformation: Shift left 3 units

Example 2

$$(c) \quad g(x) = \sqrt{x - 6}$$

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$$\text{Parent function: } f(x) = \sqrt{x}$$

Example 2

$$(c) \quad g(x) = \sqrt{x - 6}$$

Parent function: $f(x) = \sqrt{x}$

Transformation: Shift right 6 units

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$$a \cdot f(x)$$

INVESTIGATION:

For the function $f(x) = \sin x$, examine the effects of

$$g(x) = a \cdot \sin x$$

where $a > 1$ and also where $0 < a < 1$.

Vertical Stretches and Compressions

A **vertical stretch** or **vertical compression** is obtained by vertically pulling on (or vertically pressing on) the graph.

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A vertical stretch pulls the points away from the x -axis, while a vertical compression pushes the points towards the x -axis.

Vertical Stretches and Compressions

A **vertical stretch** or **vertical compression** is obtained by vertically pulling on (or vertically pressing on) the graph.

A vertical stretch pulls the points away from the x -axis, while a vertical compression pushes the points towards the x -axis.

Algebraically, we obtain vertical stretches and compressions by multiplying the entire function by a positive value.

Vertical Stretches and Compressions

| Value of a | Stretch or Compress? | Factor |
|--------------|----------------------|---------------|
| $a > 1$ | Stretch | a |
| $0 < a < 1$ | Compression | $\frac{1}{a}$ |

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Vertical stretches and compressions will multiply the y -coordinates of the function by whatever the value of a is.

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Vertical stretches and compressions will multiply the y -coordinates of the function by whatever the value of a is.

Note: The factor is expressed as a value greater than 1.

Example 3

List the parent function and indicate if there is a vertical stretch or vertical compression. Then list the factor.

(a) $g(x) = 2\sqrt{x}$

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(a) $g(x) = 2\sqrt{x}$

Parent function: $f(x) = \sqrt{x}$

Transformation: Vertical stretch by factor of 2

Example 3

$$(b) \quad g(x) = 3.5\sqrt[3]{x}$$

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Parent function: $f(x) = \sqrt[3]{x}$

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$$(b) \quad g(x) = 3.5\sqrt[3]{x}$$

Parent function: $f(x) = \sqrt[3]{x}$

Transformation: Vertical stretch by factor of 3.5

Example 3

$$(c) \quad g(x) = \frac{1}{3}x^2$$

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$$(c) \quad g(x) = \frac{1}{3}x^2$$

Parent function: $f(x) = x^2$

Example 3

$$(c) \quad g(x) = \frac{1}{3}x^2$$

Parent function: $f(x) = x^2$

Transformation: Vertical compression by factor of 3

Example 3

$$(d) \quad g(x) = \frac{2}{5}|x|$$

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$$(d) \quad g(x) = \frac{2}{5}|x|$$

Parent function: $f(x) = |x|$

Example 3

$$(d) \quad g(x) = \frac{2}{5}|x|$$

Parent function: $f(x) = |x|$

Transformation: Vertical compression by factor of $\frac{5}{2}$

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Reflections of Functions Across the x- and y-axes

INVESTIGATION:

For the function $f(x) = \sqrt{x}$, examine the effects of

$$g(x) = -\sqrt{x}$$

Reflections of Functions Across the x- and y-axes

INVESTIGATION:

For the function $f(x) = \sqrt{x}$, examine the effects of

$$g(x) = \sqrt{-x}$$

Reflections Across the x - and y -Axes

When we multiply by -1 , we reflect our graph across either the x -axis or the y -axis.

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When we multiply by -1 , we reflect our graph across either the x -axis or the y -axis.

| Function | Reflect Across |
|----------|----------------|
| $-f(x)$ | x -axis |
| $f(-x)$ | y -axis |

Example 4

List the parent function and indicate the axis it is reflected across.

(a) $g(x) = \sqrt[3]{-x}$

Example 4

List the parent function and indicate the axis it is reflected across.

(a) $g(x) = \sqrt[3]{-x}$

Parent function: $f(x) = \sqrt[3]{x}$

Example 4

List the parent function and indicate the axis it is reflected across.

(a) $g(x) = \sqrt[3]{-x}$

Parent function: $f(x) = \sqrt[3]{x}$

Transformation: Reflected across the y -axis

Example 4

$$(b) \quad g(x) = -|x|$$

Example 4

$$(b) \quad g(x) = -|x|$$

Parent function: $f(x) = |x|$

Example 4

$$(b) \quad g(x) = -|x|$$

Parent function: $f(x) = |x|$

Transformation: Reflected across the x -axis

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Order of Multiple Transformations

For multiple transformations, perform them in the following order:

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- 1 Horizontal translations (shift left or right)

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

- 1 Horizontal translations (shift left or right)
- 2 Reflections

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

- 1 Horizontal translations (shift left or right)
- 2 Reflections
- 3 Vertical stretches or compressions

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

- 1 Horizontal translations (shift left or right)
- 2 Reflections
- 3 Vertical stretches or compressions
- 4 Vertical translations (shift up or down)

Order of Multiple Transformations

For multiple transformations, perform them in the following order:

- 1 Horizontal translations (shift left or right)
- 2 Reflections
- 3 Vertical stretches or compressions
- 4 Vertical translations (shift up or down)

Note: Steps 2 and 3 may be switched without penalty.

Example 5

Determine the parent function and list all transformations done to it. Be specific.

(a) $g(x) = -2\sqrt{x-3}$

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Example 5

Determine the parent function and list all transformations done to it. Be specific.

$$(a) \quad g(x) = -2\sqrt{x-3}$$

Parent function: $f(x) = \sqrt{x}$

Shift right 3 units $\sqrt{x-3}$

Example 5

Determine the parent function and list all transformations done to it. Be specific.

$$(a) \quad g(x) = -2\sqrt{x-3}$$

Parent function: $f(x) = \sqrt{x}$

Shift right 3 units $\sqrt{x-3}$

Reflect across x -axis $-\sqrt{x-3}$

Example 5

Determine the parent function and list all transformations done to it. Be specific.

$$(a) \quad g(x) = -2\sqrt{x-3}$$

Parent function: $f(x) = \sqrt{x}$

Shift right 3 units $\sqrt{x-3}$

Reflect across x -axis $-\sqrt{x-3}$

Vertical stretch by factor of 2 $-2\sqrt{x-3}$

Example 5

$$(b) \quad g(x) = \frac{1}{2}(x + 3)^2$$

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Parent function: $f(x) = x^2$

Example 5

$$(b) \quad g(x) = \frac{1}{2}(x + 3)^2$$

Parent function: $f(x) = x^2$

Shift left 3 units $(x + 3)^2$

Example 5

$$(b) \quad g(x) = \frac{1}{2}(x + 3)^2$$

Parent function: $f(x) = x^2$

Shift left 3 units $(x + 3)^2$

Vertical compression by a factor of 2 $\frac{1}{2}(x + 3)^2$

Example 5

$$(c) \quad g(x) = |-x + 2| + 1$$

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Parent function: $f(x) = |x|$

Example 5

$$(c) \quad g(x) = |-x + 2| + 1$$

Parent function: $f(x) = |x|$

Shift left 2 units $|x + 2|$

Example 5

$$(c) \quad g(x) = |-x + 2| + 1$$

Parent function: $f(x) = |x|$

Shift left 2 units $|x + 2|$

Reflect across y -axis $|-x + 2|$

Example 5

$$(c) \quad g(x) = |-x + 2| + 1$$

Parent function: $f(x) = |x|$

Shift left 2 units $|x + 2|$

Reflect across y -axis $|-x + 2|$

Vertical shift up 1 unit $|-x + 2| + 1$

Example 5

$$(d) \quad g(x) = -\frac{1}{5}(-x - 7)^3 - 4$$

Example 5

$$(d) \quad g(x) = -\frac{1}{5}(-x - 7)^3 - 4$$

Parent function: $f(x) = x^3$

Example 5

$$(d) \quad g(x) = -\frac{1}{5}(-x - 7)^3 - 4$$

Parent function: $f(x) = x^3$

Shift right 7 units $(x - 7)^3$

Example 5

$$(d) \quad g(x) = -\frac{1}{5}(-x - 7)^3 - 4$$

Parent function: $f(x) = x^3$

Shift right 7 units $(x - 7)^3$

Reflect across y -axis $(-x - 7)^3$

Example 5

$$(d) \quad g(x) = -\frac{1}{5}(-x - 7)^3 - 4$$

Parent function: $f(x) = x^3$

Shift right 7 units $(x - 7)^3$

Reflect across y -axis $(-x - 7)^3$

Vertical compression by factor of 5 $\frac{1}{5}(-x - 7)^3$

Example 5

$$(d) \quad g(x) = -\frac{1}{5}(-x - 7)^3 - 4$$

Parent function: $f(x) = x^3$

Shift right 7 units $(x - 7)^3$

Reflect across y -axis $(-x - 7)^3$

Vertical compression by factor of 5 $\frac{1}{5}(-x - 7)^3$

Reflect across x -axis $-\frac{1}{5}(-x - 7)^3$

Example 5

$$(d) \quad g(x) = -\frac{1}{5}(-x - 7)^3 - 4$$

Parent function: $f(x) = x^3$

Shift right 7 units $(x - 7)^3$

Reflect across y -axis $(-x - 7)^3$

Vertical compression by factor of 5 $\frac{1}{5}(-x - 7)^3$

Reflect across x -axis $-\frac{1}{5}(-x - 7)^3$

Vertical shift down 4 units $-\frac{1}{5}(-x - 7)^3 - 4$