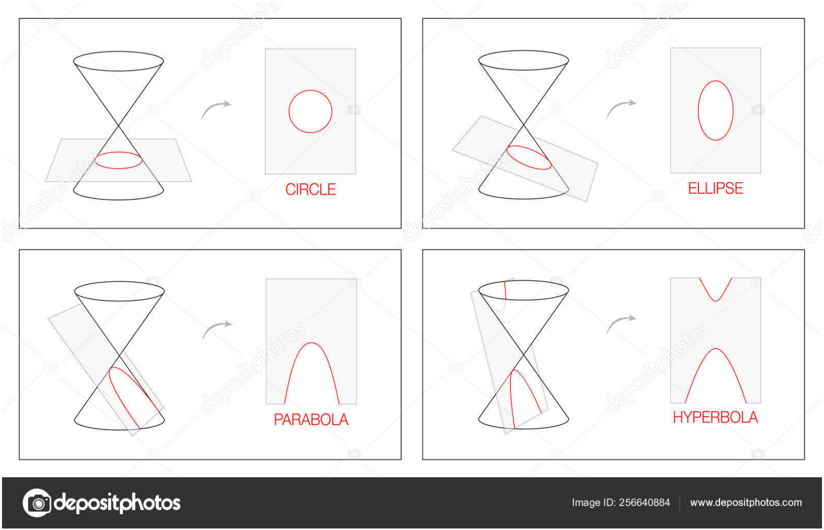
Ellipses


CIRCLE
ELLIPSE
PARABOLA
HYPERBOLA
depositphotos
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Objectives

1. Identify the center, vertices, and foci of an ellipse.
2. Write the equation of an ellipse in standard form.

Ellipses

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The set of points such that the **sum** of their distances from 2 fixed points (called **foci**) is constant.

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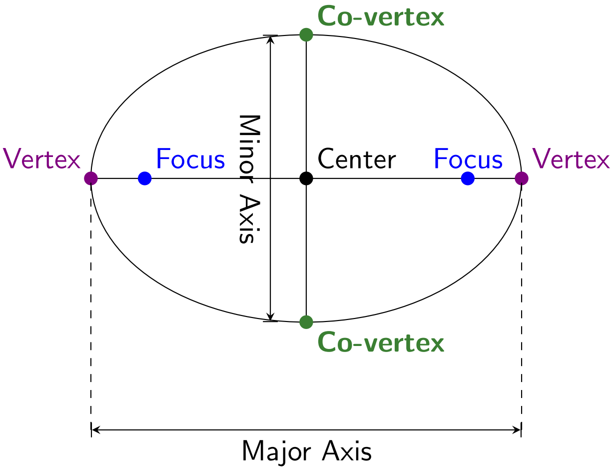
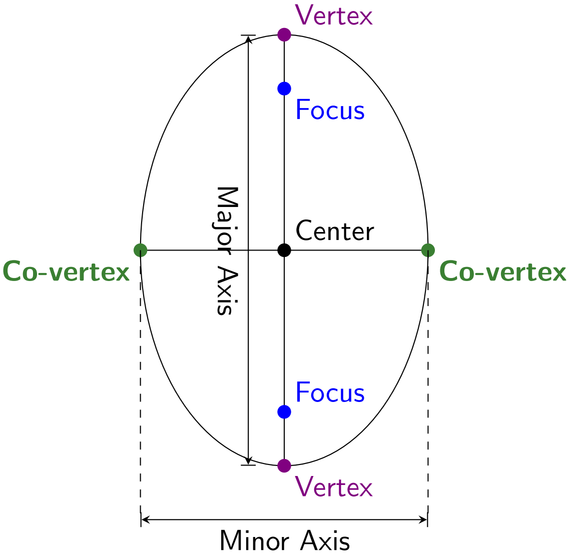
Appearance

Ellipses will typically either appear wider or taller based on their equation. Below are key parts of each type of ellipse:



Appearance


Vertex
Focus
Major Axis
Center
Co-vertex
Co-vertex
Focus
Vertex
Minor Axis

Vocab

The *center* of an ellipse is denoted by (h, k), just like with circles.

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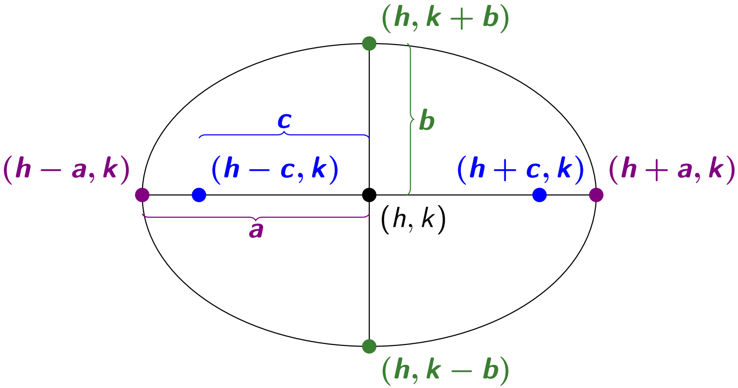
Each vertex (*pl: vertices*) also lies on the major axis, and is a units away from the center.

Each *co-vertex* (*pl: co-vertices*) lies on the *minor axis*, which is perpendicular to the major axis. The co-vertices are each b units away from the center.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



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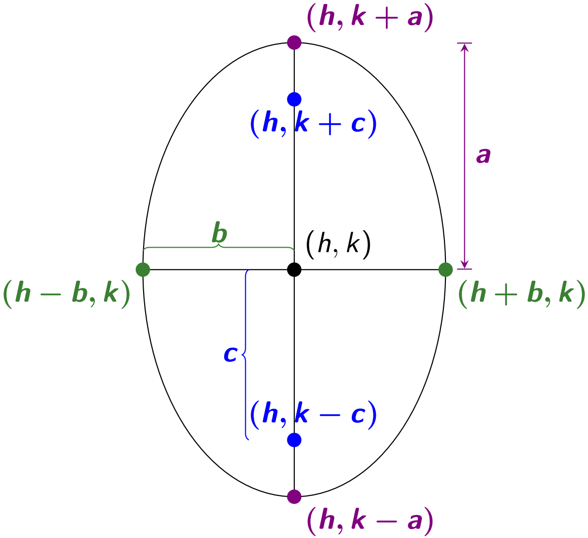

Foci and a vs. b

Note: In both cases, $c^2 = a^2 - b^2$, and $a > b$.

Example 1

Identify the coordinates of the center, vertices, and foci for each. Exact answers only.

(a) $$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

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Vertices: $a = 36 \longrightarrow a = \pm 6$

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Vertices at $(0 \pm 6, 0)$

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Vertices at $(0 \pm 6, 0)$

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Example 1a $\frac{x^2}{36} + \frac{y^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

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$$c^2 = 36 - 25$$

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Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 36 - 25$$

$$c^2 = 11$$

Example 1a $\frac{x^2}{36} + \frac{y^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 36 - 25$$

$$c^2 = 11$$

$$c = \pm\sqrt{11}$$

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Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 36 - 25$$

$$c^2 = 11$$

$$c = \pm\sqrt{11}$$

Foci at $(0 \pm \sqrt{11}, 0)$

Example 1a $\frac{x^2}{36} + \frac{y^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 36 - 25$$

$$c^2 = 11$$

$$c = \pm\sqrt{11}$$

Foci at $(0 \pm \sqrt{11}, 0) \longrightarrow (\pm\sqrt{11}, 0)$

Example 1

(b) $$\frac{y^2}{16} + \frac{x^2}{1} = 1$$

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Vertices: $a = 16$

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Center: $(0,0)$

Vertices: $a = 16 \longrightarrow a = \pm 4$

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Center: $(0, 0)$

Vertices: $a = 16 \longrightarrow a = \pm 4$

Vertices at $(0, 0 \pm 4)$

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(b) $\dfrac{y^2}{16} + \dfrac{x^2}{1} = 1$

Center: $(0, 0)$

Vertices: $a = 16 \longrightarrow a = \pm 4$

Vertices at $(0, 0 \pm 4)$

Vertices $(0, \pm 4)$

Example 1b $\frac{y^2}{16} + \frac{x^2}{1} = 1$

Foci:

$$c^2 = a^2 - b^2$$

Example 1b $\frac{y^2}{16} + \frac{x^2}{1} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 1$$

Example 1b $\frac{y^2}{16} + \frac{x^2}{1} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 1$$

$$c^2 = 15$$

Example 1b $\frac{y^2}{16} + \frac{x^2}{1} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 1$$

$$c^2 = 15$$

$$c = \pm\sqrt{15}$$

Example 1b $\frac{y^2}{16} + \frac{x^2}{1} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 1$$

$$c^2 = 15$$

$$c = \pm\sqrt{15}$$

Foci at $(0, 0 \pm \sqrt{15})$

Example 1b $\frac{y^2}{16} + \frac{x^2}{1} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 1$$

$$c^2 = 15$$

$$c = \pm\sqrt{15}$$

Foci at $(0, 0 \pm \sqrt{15}) \longrightarrow (0, \pm\sqrt{15})$

Example 1

(c) $$\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$$

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Center: $(-5, 2)$

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Vertices at $(-5 \pm 7, 2)$

Example 1

(c) $$\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$$

Center: $(-5, 2)$

Vertices: $a = 49 \longrightarrow a = \pm 7$

Vertices at $(-5 \pm 7, 2)$

Vertices $(-12, 2)$ and $(2, 2)$

Example 1c $\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

Example 1c $\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 49 - 25$$

Example 1c $\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 49 - 25$$

$$c^2 = 24$$

Example 1c $\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 49 - 25$$

$$c^2 = 24$$

$$c = \pm 2\sqrt{6}$$

Example 1c $\frac{(x+5)^2}{49} + \frac{(y-2)^2}{25} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 49 - 25$$

$$c^2 = 24$$

$$c = \pm 2\sqrt{6}$$

Foci at $(-5 \pm 2\sqrt{6}, 2)$

Example 1

(d) $$(x-2)^2 + \frac{(y+1)^2}{36} = 1$$

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Center: $(2,-1)$

Example 1

(d) $\quad (x-2)^2 + \dfrac{(y+1)^2}{36} = 1$

Center: $(2, -1)$

Vertices: $a = 36$

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Vertices at $(2, -1 \pm 6)$

Example 1

(d) $\quad (x-2)^2 + \dfrac{(y+1)^2}{36} = 1$

Center: $(2, -1)$

Vertices: $a = 36 \longrightarrow a = \pm 6$

Vertices at $(2, -1 \pm 6)$

Vertices $(2, -7)$ and $(2, 5)$

Example 1d $\frac{(x-2)^2}{1}+\frac{(y+1)^2}{36}=1$

Foci:

$$c^2 = a^2 - b^2$$

Example 1d $\frac{(x-2)^2}{1} + \frac{(y+1)^2}{36} = 1$

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$$c^2 = 36 - 1$$

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Example 1d $\frac{(x-2)^2}{1} + \frac{(y+1)^2}{36} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 36 - 1$$

$$c^2 = 35$$

$$c = \pm\sqrt{35}$$

Example 1d $\frac{(x-2)^2}{1} + \frac{(y+1)^2}{36} = 1$

Foci:

$$c^2 = a^2 - b^2$$

$$c^2 = 36 - 1$$

$$c^2 = 35$$

$$c = \pm\sqrt{35}$$

Foci at $(2, -1 \pm \sqrt{35})$

Objectives

1. Identify the center, vertices, and foci of an ellipse.
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Converting From General to Standard Form

Just like circles may not be written in standard form, you may need to find the center and vertices to write an ellipse in standard form.

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Just like circles may not be written in standard form, you may need to find the center and vertices to write an ellipse in standard form.

Luckily, the process is similar to that for circles.

Example 2

Write the standard form for the equation of an ellipse for each of the following.

(a) $9x^2 + 16y^2 - 144 = 0$

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(a) $9x^2 + 16y^2 - 144 = 0$

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$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

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$$9x^2 + 16y^2 = 144$$

$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Example 2

(b) $25x^2 + y^2 - 25 = 0$

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Example 2

(b) $\quad 25x^2 + y^2 - 25 = 0$

$$25x^2 + y^2 - 25 = 0$$

$$25x^2 + y^2 = 25$$

$$\frac{25x^2}{25} + \frac{y^2}{25} = 1$$

Example 2

(b) $25x^2 + y^2 - 25 = 0$

$$\begin{aligned} 25x^2 + y^2 - 25 &= 0 \\ 25x^2 + y^2 &= 25 \\ \frac{25x^2}{25} + \frac{y^2}{25} &= 1 \\ x^2 + \frac{y^2}{25} &= 1 \end{aligned}$$

Example 2

(c) $9x^2 - 54x + 4y^2 - 8y + 49 = 0$

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Vertex: $(3, -81)$

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Vertex: $(3, -81)$

Vertex: $(1, -4)$

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$$\mathbf{9x^2 - 54x} + \mathbf{4y^2 - 8y} = -49$$

Vertex: $(3, -81)$

Vertex: $(1, -4)$

$$9(x-3)^2 + 4(y-1)^2 = -49 + |-81| + |-4|$$

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Vertex: $(1, -4)$

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$$9(x-3)^2 + 4(y-1)^2 = 36$$

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(c) $9x^2 - 54x + 4y^2 - 8y + 49 = 0$

$$\mathbf{9x^2 - 54x} + \mathbf{4y^2 - 8y} = -49$$

Vertex: $(3, -81)$

Vertex: $(1, -4)$

$$9(x-3)^2 + 4(y-1)^2 = -49 + |-81| + |-4|$$

$$9(x-3)^2 + 4(y-1)^2 = 36$$

$$\frac{9(x-3)^2}{36} + \frac{4(y-1)^2}{36} = 1$$

Example 2c

$$\frac{9(x-3)^2}{36}+\frac{4(y-1)^2}{36}=1$$

Example 2c

$$\frac{9(x-3)^2}{36} + \frac{4(y-1)^2}{36} = 1$$

$$\frac{(x-3)^2}{4} + \frac{(y-1)^2}{9} = 1$$

Example 2

(d) $x^2 + 6x + 9y^2 + 18y + 9 = 0$

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$$\color{blue}{\boldsymbol{x^2 + 6x}} + \color{red}{\boldsymbol{9y^2 + 18y}} = -9$$

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(d) $x^2 + 6x + 9y^2 + 18y + 9 = 0$

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Vertex: $(-3, -9)$

Example 2

(d) $x^2 + 6x + 9y^2 + 18y + 9 = 0$

$$\mathbf{x^2 + 6x} + \mathbf{9y^2 + 18y} = -9$$

Vertex: $(-3, -9)$

Vertex: $(-1, -9)$

Example 2

(d) $x^2 + 6x + 9y^2 + 18y + 9 = 0$

$$\mathbf{x^2 + 6x} + \mathbf{9y^2 + 18y} = -9$$

Vertex: $(-3, -9)$

Vertex: $(-1, -9)$

$$(x+3)^2 + 9(y+1)^2 = -9 + |-9| + |-9|$$

Example 2

(d) $x^2 + 6x + 9y^2 + 18y + 9 = 0$

$$\boldsymbol{x^2 + 6x} + \boldsymbol{9y^2 + 18y} = -9$$

Vertex: $(-3, -9)$

Vertex: $(-1, -9)$

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(d) $x^2 + 6x + 9y^2 + 18y + 9 = 0$

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Vertex: $(-3, -9)$

Vertex: $(-1, -9)$

$$(x+3)^2 + 9(y+1)^2 = -9 + |-9| + |-9|$$

$$(x+3)^2 + 9(y+1)^2 = 9$$

$$\frac{(x+3)^2}{9} + \frac{9(y-1)^2}{9} = 1$$

Example 2d

$$\frac{(x+3)^2}{9} + \frac{9(y-1)^2}{9} = 1$$

Example 2d

$$\frac{(x+3)^2}{9} + \frac{9(y-1)^2}{9} = 1$$

$$\frac{(x+3)^2}{9} + (y-1)^2 = 1$$