Equations and Inequalities

Objectives

1. Solve linear equations and check solutions.
2. Solve and graph inequalities on a number line.

Solving Equations

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To do this, we use reverse order of operations:

1. Undo any addition or subtraction.
2. Undo any multiplication or division.
3. Undo any exponents.
4. Get rid of parentheses.

Checking Your Answer

You can make sure your answer is correct by **plugging it in** to the original problem and seeing if the left side and right side are equal.

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$2x - 20 = 4x - 20 - 2x$ distribute

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True statement, so $x =$ all real numbers, or $\mathbb{R}$

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False statement, so $x =$ No solution, or $\varnothing$

Visual Way to Check Answers

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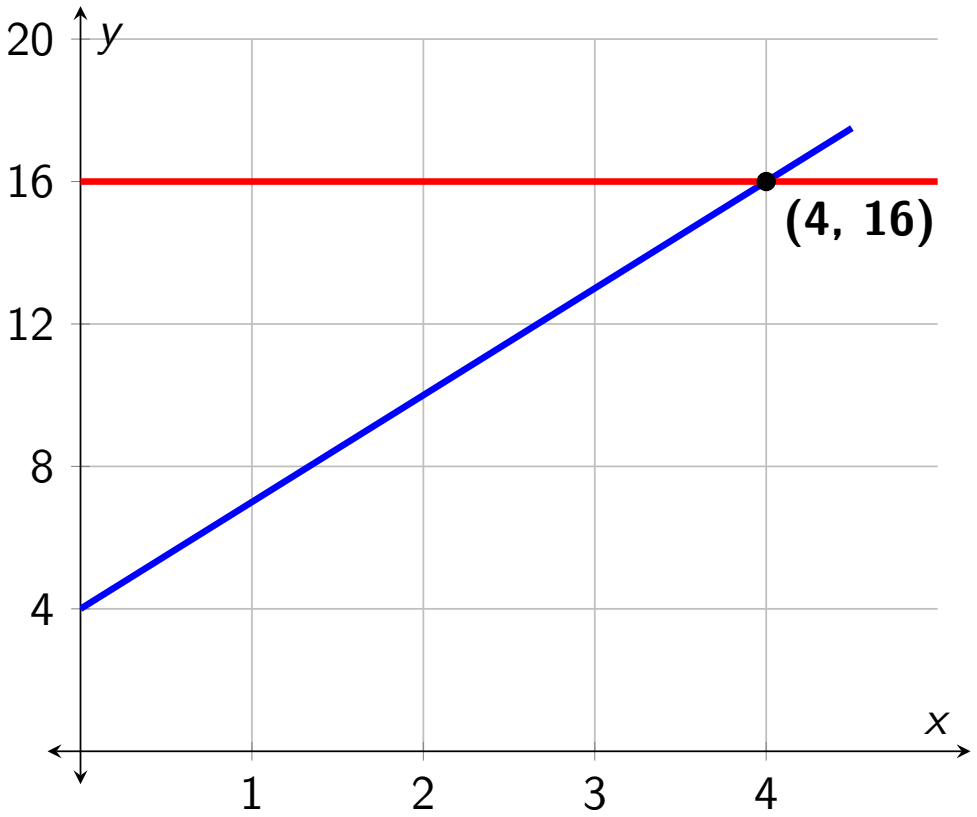
For "all real numbers" answers, the graphs will be **one in the same**.

Visual Way to Check Answers

The graphs of $y = 3x + 4$ and $y = 16$ are shown below:



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1. Solve linear equations and check solutions.

2. Solve and graph inequalities on a number line.

Solving Inequalities

Solving inequalities is a lot like solving equations.

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For instance, there are an infinite number of values that you can substitute into x for $x > -4$ to make it true.

Visual Solutions to Inequalities

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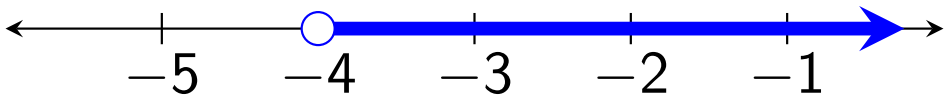

Summary of Graphing Inequalities

If your variable is on the left side when graphing an inequality, you can use the following table to help you graph:

Expression	**Circle**	**Shade**
$x <$	Open	Left
$x >$	Open	Right
$x \leq$	Closed	Left
$x \geq$	Closed	Right

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Graph each of the following on a number line.

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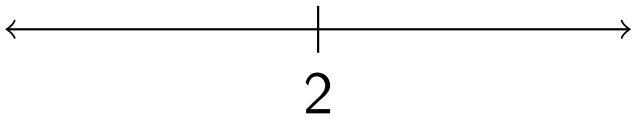

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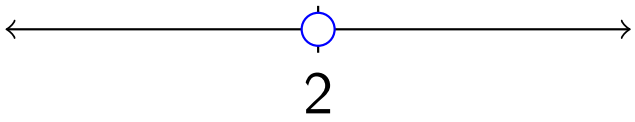

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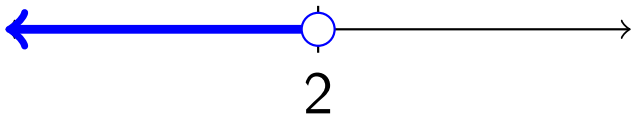

Solving Inequalities

With solving inequalities, you must remember to flip the inequality sign if you multiply or divide **both sides** by a negative number.

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Solve and graph each.

(a) $3x - 5 > -17$

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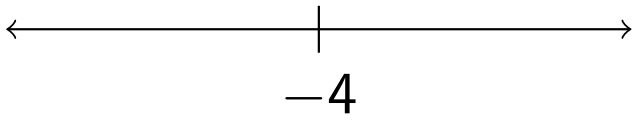

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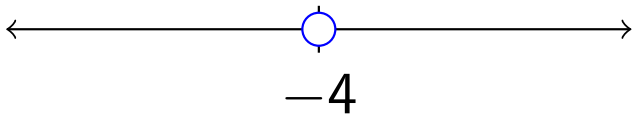

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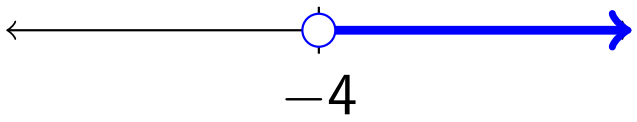

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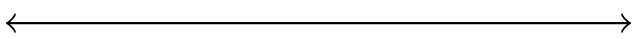

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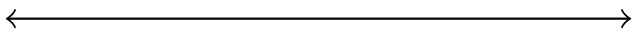

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