

Complex Fractions

Objectives

- 1 Add and subtract rational expressions with like denominators
- 2 Add and subtract rational expressions with unlike denominators

Fractions with like denominators

Recall that to add or subtract fractions with **like denominators**, you keep the denominators and add (or subtract) the numerators.

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Remember to **distribute the sign** to the numerator in the second fraction.

Example 1

Simplify each.

$$(a) \quad \frac{x+6}{9x^3+54x^2} + \frac{x+2}{9x^3+54x^2}$$

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$$= \frac{x+6+(x+2)}{9x^3+54x^2}$$

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$$= \frac{2x+8}{9x^3+54x^2}$$

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$$\begin{aligned} \text{(a)} \quad & \frac{x+6}{9x^3+54x^2} + \frac{x+2}{9x^3+54x^2} \\ & \frac{x+6}{9x^3+54x^2} + \frac{x+2}{9x^3+54x^2} \\ & = \frac{x+6+(x+2)}{9x^3+54x^2} \\ & = \frac{2x+8}{9x^3+54x^2} \\ & = \frac{2(x+4)}{9x^2(x+6)} \end{aligned}$$

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$$(b) \quad \frac{x-2}{2x^2-9x-18} + \frac{6x+1}{2x^2-9x-18}$$

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$$= \frac{x-2+(6x+1)}{2x^2-9x-18}$$

$$= \frac{7x-1}{2x^2-9x-18}$$

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$$= \frac{x-2+(6x+1)}{2x^2-9x-18}$$

$$= \frac{7x-1}{2x^2-9x-18}$$

$$= \frac{7x-1}{(2x-3)(x+6)}$$

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$$= \frac{x+5-(4x-1)}{6x+4}$$

$$= \frac{x+5-4x+1}{6x+4}$$

$$= \frac{-3x+6}{6x+4}$$

$$= \frac{-3(x-2)}{2(3x+2)}$$

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$$= \frac{x-1-(x+6)}{3x^2-10x-8}$$

$$= \frac{x-1-x-6}{3x^2-10x-8}$$

$$= \frac{-7}{3x^2-10x-8}$$

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Adding and Subtracting Fractions with Unlike Denominators

Recall that before adding or subtracting fractions with unlike denominators, **you need to get a common denominator first.**

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For instance,

$$\begin{aligned}\frac{2}{5} + \frac{1}{3} \\&= \frac{2(3)}{5(3)} + \frac{1(5)}{3(5)} \\&= \frac{6}{15} + \frac{5}{15}\end{aligned}$$

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Ancient Secret to This Method

Notice you had to find the **least common multiple** of the denominators 3 and 5 (which ended up being 15).

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Notice you had to find the **least common multiple** of the denominators 3 and 5 (which ended up being 15).

For adding and subtracting rational expressions with unlike denominators, **factor the denominators completely and multiply by factors that differ**. (Easier to see that process in action than it is to understand it written down like that).

Example 2

Simplify each.

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$$= \frac{6}{x+4} + \frac{2}{3(x+2)}$$

$$= \frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

Example 2a

$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

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$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

Example 2a

$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

$$= \frac{18x+36+2x+8}{3(x+2)(x+4)}$$

Example 2a

$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

$$= \frac{18x+36+2x+8}{3(x+2)(x+4)}$$

$$= \frac{20x+44}{3(x+2)(x+4)}$$

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$$\frac{6(3(x+2))}{(x+4)(3(x+2))} + \frac{2(x+4)}{3(x+2)(x+4)}$$

$$\frac{6(3x+6) + 2x+8}{3(x+2)(x+4)}$$

$$= \frac{18x+36+2x+8}{3(x+2)(x+4)}$$

$$= \frac{20x+44}{3(x+2)(x+4)}$$

$$= \frac{4(5x+11)}{3(x+2)(x+4)}$$