Graphs of Quadratic Expressions

Objectives

1. Determine the vertex and axis of symmetry of a quadratic function in standard form

2. Convert between standard and general form of quadratic expressions

Graphing Quadratic Expressions

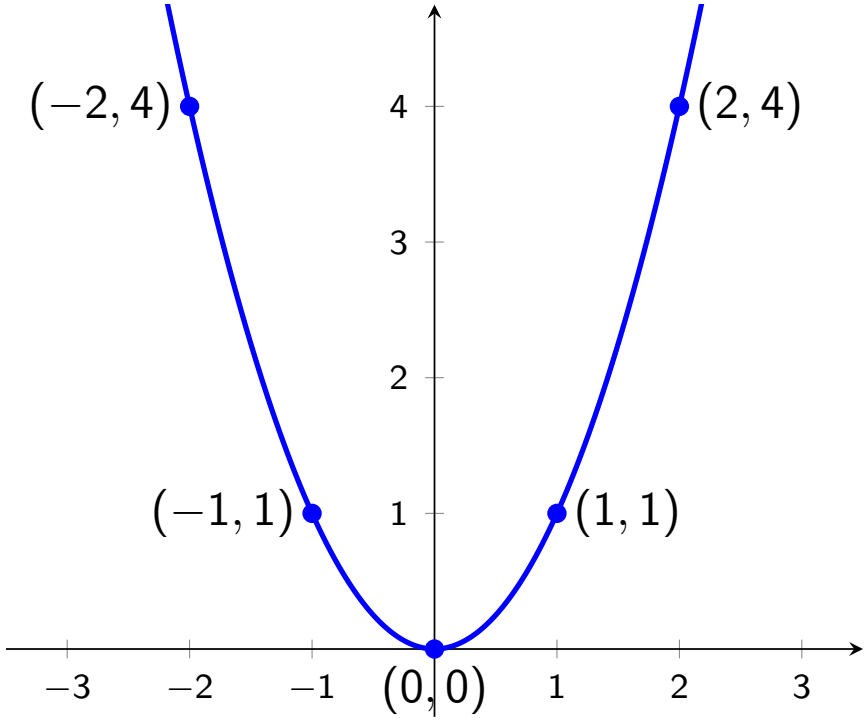
We will be looking at graphing equations in the form

$$y = ax^2 + bx + c$$

where a, b, and c are real numbers with $a \neq 0$.

Graph of a Quadratic Expression

For $y = x^2$, the graph below is a parabola.



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Through the vertex is a vertical line called the axis of symmetry that divides the parabola into 2 equal halves.

Example 1

Find the vertex, state whether the vertex is a maximum or minimum, and find the equation of the axis of symmetry for each.

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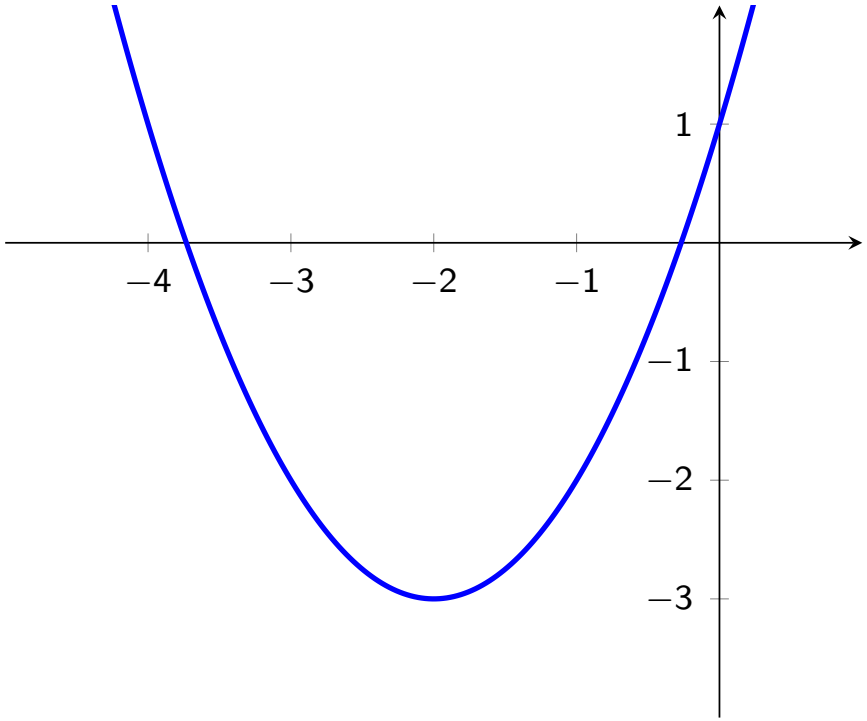

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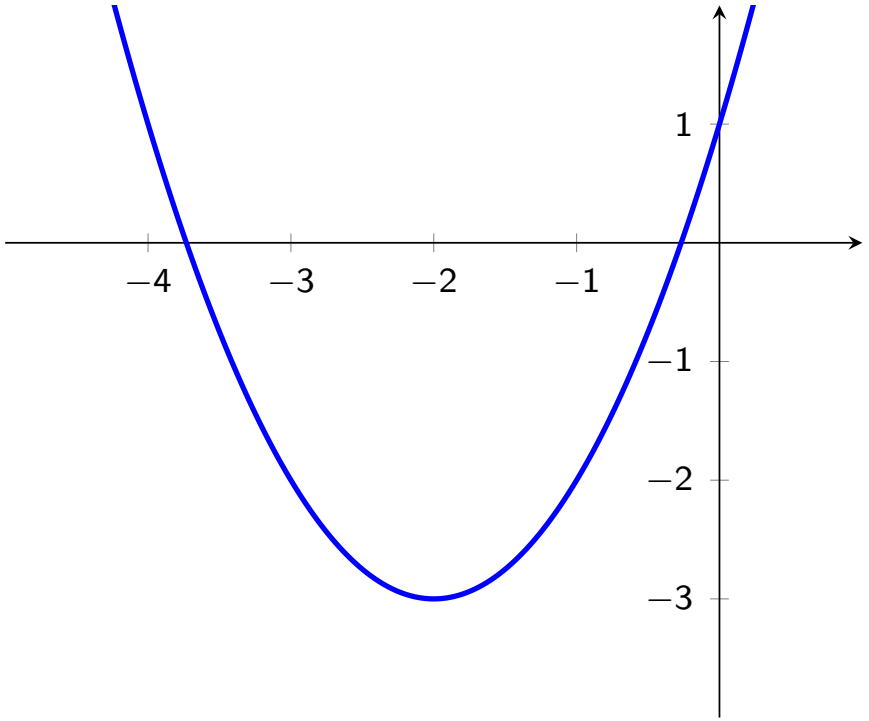

Vertex $(-2, -3)$

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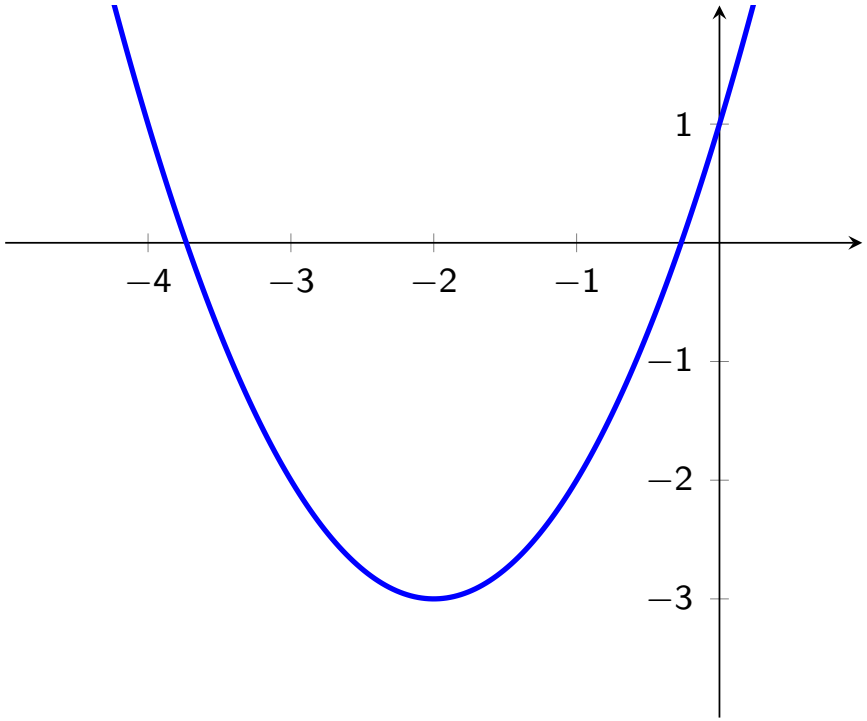

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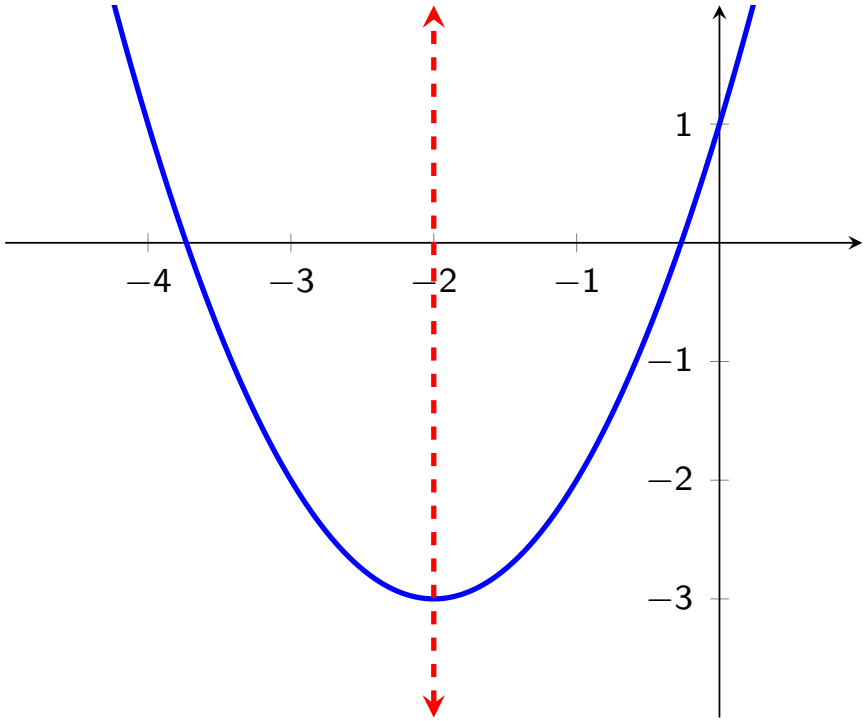

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Axis of symmetry: $x = -2$

Example 1

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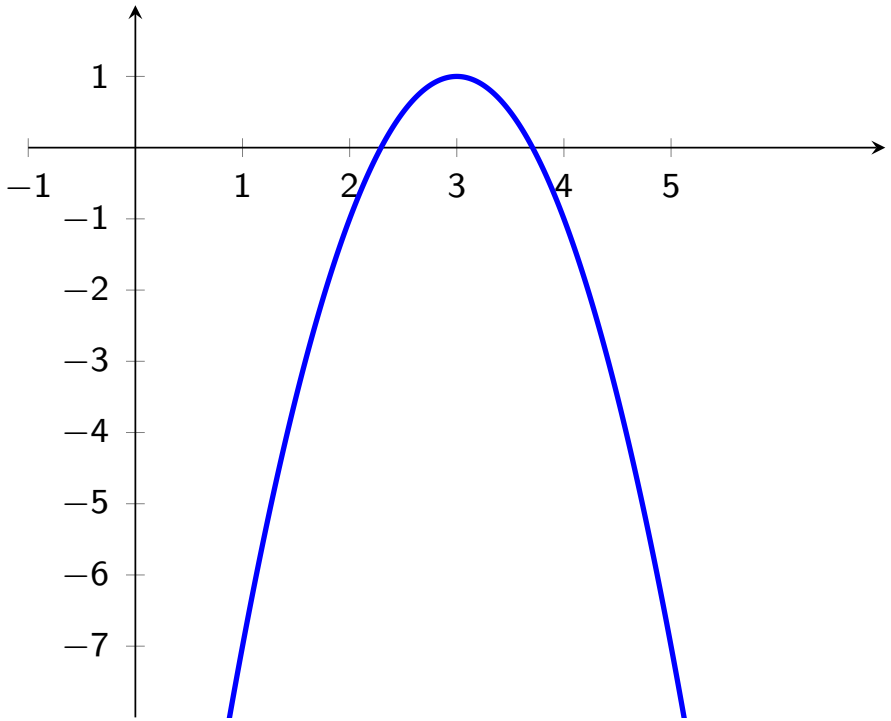

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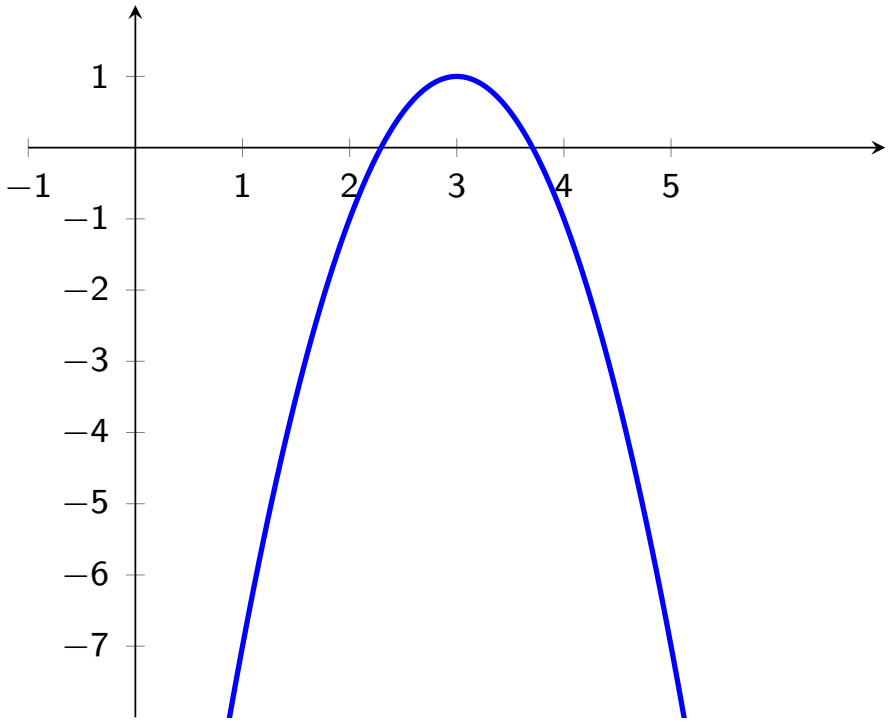

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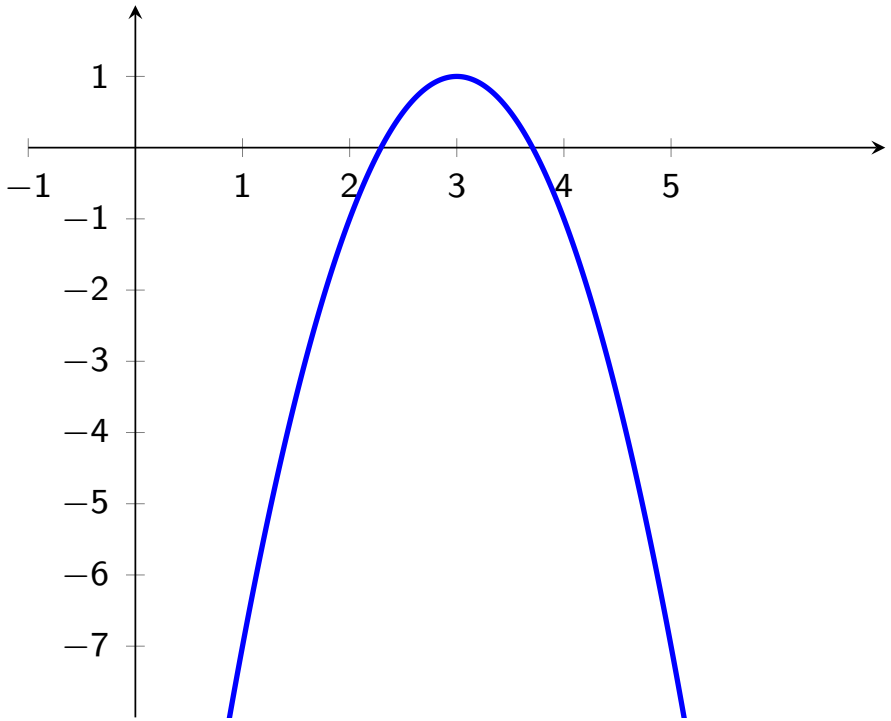

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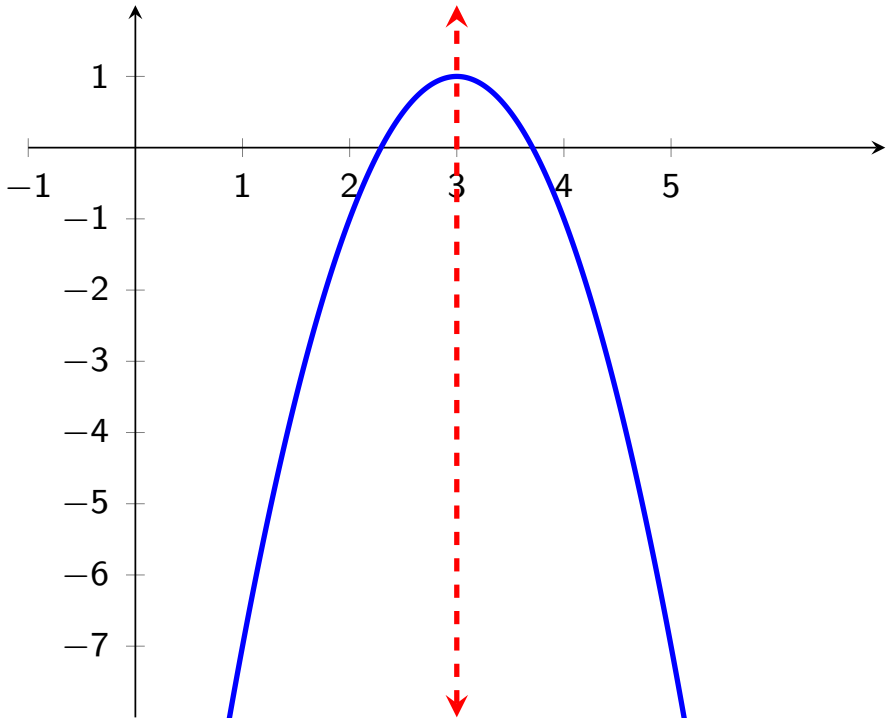

Vertex $(3, 1)$

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Axis of symmetry: $x = 3$

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Standard and General Form of Quadratic Expressions

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Converting From General to Standard Form

To convert from general form $y = ax^2 + bx + c$ to standard form $y = a(x - h)^2 + k$

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1. Find the vertex:
   - x-coordinate: $\dfrac{-b}{2a}$
   - y-coordinate: Evaluate expression at x-coordinate
   - Or use graphing technology
2. Use the same value of a

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$a = 1$

$$y = (x - 2)^2 - 1$$

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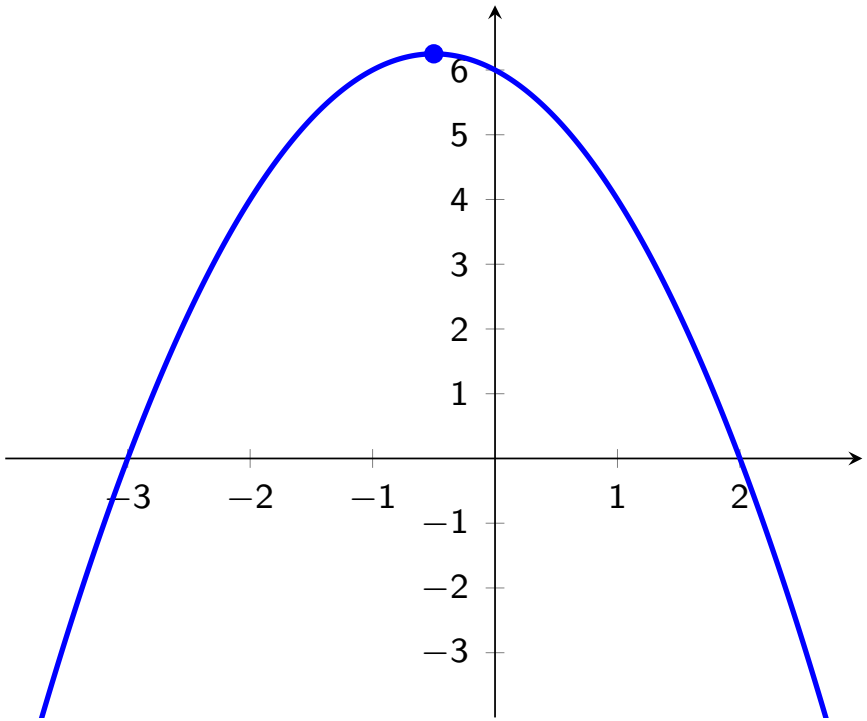

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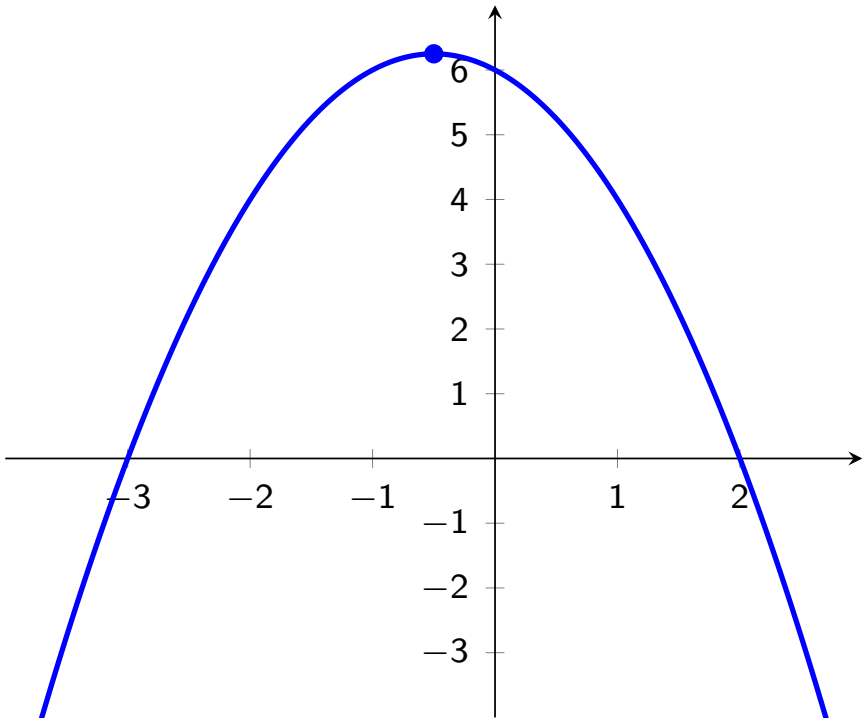

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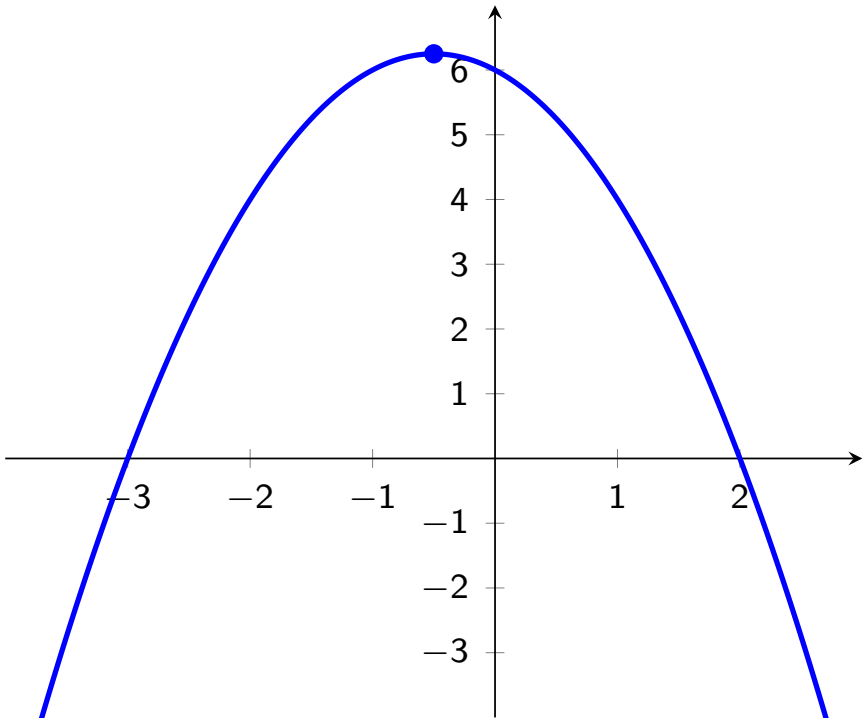

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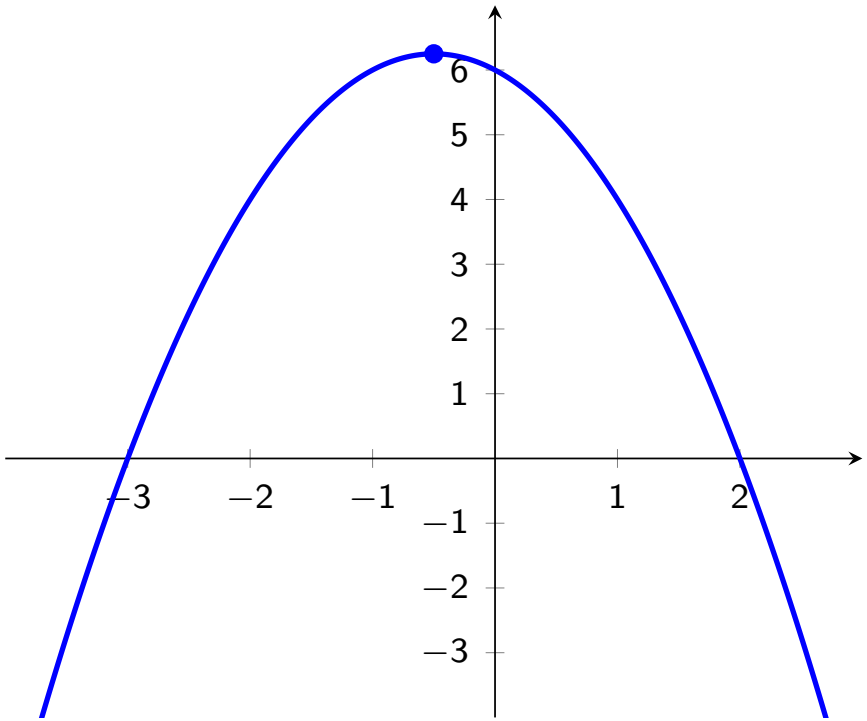

Vertex: $\left(-\frac{1}{2}, \frac{25}{4}\right)$

$a = -1$

$$y = -\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$$

Converting From Standard to General Form

To convert from

$$y = a(x - h)^2 + k$$

form to

$$y = ax^2 + bx + c$$

just **do the math** and remember your order of operations.

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$$y = -x^2 + 14x - 39$$