Complex Numbers

Objectives

1. Write square roots of negative numbers as imaginary numbers
2. Add and subtract complex numbers
3. Multiply complex numbers
4. Divide complex numbers

Background

Imaginary numbers arose from trying to solve equations such as

$$x^2 = -1$$

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We define the imaginary unit i to be that value:

$$i = \sqrt{-1}$$

Writing Square Roots of Negative Numbers

When writing square roots of negative values as imaginary numbers, we can factor out

$$\sqrt{-1}$$

from the expression

as long as what remains is the square root of a positive number.

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from the expression

as long as what remains is the square root of a positive number.

$$\begin{aligned}\sqrt{-25} &= \sqrt{-1} \cdot \sqrt{25} \\ &= i \cdot 5 \\ &= 5i\end{aligned}$$

Example 1

Write each as an imaginary number.

(a) $\sqrt{-36}$

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$$= 6i$$

Example 1

(b) $\sqrt{-100}$

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$$\sqrt{-100} = \sqrt{-1} \cdot \sqrt{100}$$

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$$\sqrt{-100} = \sqrt{-1} \cdot \sqrt{100}$$
$$= i \cdot 10$$

Example 1

(b) $\sqrt{-100}$

$$\begin{aligned}\sqrt{-100} &= \sqrt{-1}\cdot\sqrt{100} \\ &= i\cdot 10 \\ &= 10i\end{aligned}$$

Example 1

(c) $\sqrt{-8}$

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$$\sqrt{-8} = \sqrt{-1} \cdot \sqrt{8}$$

Example 1

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$$\begin{aligned}\sqrt{-8} &= \sqrt{-1}\cdot\sqrt{8} \\ &= i\cdot 2\sqrt{2}\end{aligned}$$

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(c) $\sqrt{-8}$

$$\begin{aligned}\sqrt{-8} &= \sqrt{-1}\cdot\sqrt{8}\\ &= i\cdot 2\sqrt{2}\\ &= 2i\sqrt{2}\end{aligned}$$

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Complex Numbers

A complex number is a number written in the form

$$a + bi$$

where a (the real part) and b (the imaginary part) are real numbers.

Adding and Subtracting Complex Numbers

Complex numbers can be added and subtracted much like combining like terms in Algebra 1.

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The real parts are added (or subtracted) together, as are the imaginary parts.

Answers are then typically written in $a + bi$ form.

Example 2

Simplify each.

(a) $(3 + 2i) + (-1 + 8i)$

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(a) $(3 + 2i) + (-1 + 8i)$

$$(3 + 2i) + (-1 + 8i) = (3 + (-1)) + (2i + 8i)$$

Example 2

Simplify each.

(a) $(3+2i)+(-1+8i)$

$$\begin{aligned}(3+2i)+(-1+8i) &= (3+(-1))+(2i+8i)\\ &= 2+10i\end{aligned}$$

Example 2

(b) $(-5 - i) + (2 + 4i)$

Example 2

(b) $(-5 - i) + (2 + 4i)$

$$(-5 - i) + (2 + 4i) = (-5 + 2) + (-i + 4i)$$

Example 2

(b) $(-5 - i) + (2 + 4i)$

$$\begin{aligned}(-5 - i) + (2 + 4i) &= (-5 + 2) + (-i + 4i) \\ &= -3 + 3i\end{aligned}$$

Example 2

(c) $(7 - 9i) - (3 + 5i)$

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$$(7 - 9i) - (3 + 5i) = 7 - 9i - 3 - 5i$$

Example 2

(c) $(7 - 9i) - (3 + 5i)$

$$\begin{aligned}(7 - 9i) - (3 + 5i) &= 7 - 9i - 3 - 5i \\ &= 4 - 14i\end{aligned}$$

Example 2

(d) $(-1+2i)-(-8+10i)$

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$$(-1+2i)-(-8+10i) = -1+2i+8-10i$$

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(d) $(-1+2i)-(-8+10i)$

$$\begin{aligned}(-1+2i)-(-8+10i) &= -1+2i+8-10i \\ &= 7-8i\end{aligned}$$

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Multiplying Complex Numbers

Complex numbers can be multiplied in the same manner that binomials are multiplied in Algebra 1, such as $(x + 3)(x - 8)$.

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Complex numbers can be multiplied in the same manner that binomials are multiplied in Algebra 1, such as $(x+3)(x-8)$.

However, since $i = \sqrt{-1}$, if we square both sides we get

$$i^2 = -1$$

So when multiplying complex numbers, you will substitute a -1 whenever you see an i^2.

Example 3

Multiply each.

(a) $(2+3i)(5+6i)$

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(a) $(2 + 3i)(5 + 6i)$

	5	$6i$
2		
$3i$		

Example 3

Multiply each.

(a) $(2 + 3i)(5 + 6i)$

	5	$6i$
2	10	
$3i$		

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	5	$6i$
2	10	$12i$
$3i$		

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	5	$6i$
2	10	$12i$
$3i$	$15i$	

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	5	$6i$
2	10	$12i$
$3i$	$15i$	$18i^2$

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	5	$6i$
2	10	$12i$
$3i$	$15i$	-18

$-8+27i$

Example 3

(b) $(-1+i)(4-3i)$

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	4	$-3i$
-1		
$1i$		

Example 3

(b) $(-1 + i)(4 - 3i)$

	4	$-3i$
-1	-4	
$1i$		

Example 3

(b) $(-1+i)(4-3i)$

	4	$-3i$
-1	-4	$3i$
$1i$		

Example 3

(b) $(-1+i)(4-3i)$

	4	$-3i$
-1	-4	$3i$
$1i$	$4i$	

Example 3

(b) $(-1+i)(4-3i)$

$$\begin{array}{r|c|c|}
 & 4 & -3i \\ \hline
-1 & -4 & 3i \\ \hline
1i & 4i & -3i^2 \\ \hline
\end{array}$$

Example 3

(b) $(-1 + i)(4 - 3i)$

	4	$-3i$
-1	-4	$3i$
$1i$	$4i$	

Example 3

(b) $(-1+i)(4-3i)$

$$\begin{array}{c|c|c|}
 & 4 & -3i \\
\hline
-1 & -4 & 3i \\
\hline
1i & 4i & 3 \\
\hline
\end{array}$$

Example 3

(b) $\quad (-1+i)(4-3i)$

$$\begin{array}{r|c|c|}
 & 4 & -3i \\ \hline
-1 & -4 & 3i \\ \hline
1i & 4i & 3 \\ \hline
\end{array}$$

$$-1+7i$$

Example 3

(c) $(7 - 7i)(7 + 7i)$

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	7	$7i$
7		
$-7i$		

Example 3

(c) $(7 - 7i)(7 + 7i)$

	7	$7i$
7	49	
$-7i$		

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	7	$7i$
7	49	$49i$
$-7i$		

Example 3

(c) $(7 - 7i)(7 + 7i)$

$$\begin{array}{c|c|c|} & 7 & 7i \\ \hline 7 & 49 & 49i \\ \hline -7i & -49i & \\ \hline \end{array}$$

Example 3

(c) $(7-7i)(7+7i)$

	7	$7i$
7	49	$49i$
$-7i$	$-49i$	$-49i^2$

Example 3

(c) $(7 - 7i)(7 + 7i)$

	7	$7i$
7	49	$49i$
$-7i$	$-49i$	

Example 3

(c) $(7 - 7i)(7 + 7i)$

	7	$7i$
7	49	$49i$
$-7i$	$-49i$	49

Example 3

(c) $(7 - 7i)(7 + 7i)$

	7	$7i$
7	49	$49i$
$-7i$	$-49i$	49

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$$\frac{3-i}{2+i} = \frac{3-\sqrt{-1}}{2+\sqrt{-1}}$$

Dividing Complex Numbers

Dividing complex numbers presents a bit of a challenge.

The reason is that the denominator will have a square root, which is a big no-no in math:

$$\frac{3-i}{2+i} = \frac{3-\sqrt{-1}}{2+\sqrt{-1}}$$

To remedy this, we need to find the conjugate of the **denominator**.

Complex Conjugates

The conjugate of a complex number

$$a + bi$$

is

$$a - bi$$

and vice versa

Conjugate Examples

Number	Conjugate

Conjugate Examples

Number	**Conjugate**
$7 + 2i$	

Conjugate Examples

Number	**Conjugate**
$7 + 2i$	$7 - 2i$

Conjugate Examples

Number	**Conjugate**
$7 + 2i$	$7 - 2i$
$-3 + i$	

Conjugate Examples

Number	**Conjugate**
$7+2i$	$7-2i$
$-3+i$	$-3-i$

Conjugate Examples

Number	**Conjugate**
$7 + 2i$	$7 - 2i$
$-3 + i$	$-3 - i$
$5 - 4i$	

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Number	**Conjugate**
$7 + 2i$	$7 - 2i$
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$5 - 4i$	$5 + 4i$

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Number	**Conjugate**
$7+2i$	$7-2i$
$-3+i$	$-3-i$
$5-4i$	$5+4i$

When you multiply complex conjugates, you will always get a real number, like in Example 3c.

Conjugate Examples

Number	Conjugate
$7+2i$	$7-2i$
$-3+i$	$-3-i$
$5-4i$	$5+4i$

When you multiply complex conjugates, you will always get a real number, like in Example 3c.

So, to divide complex numbers, multiply both numerator and denominator by the **conjugate of the denominator**.

Example 4

Divide $\dfrac{3-i}{2+i}$. Write your answer in $a+bi$ form.

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The conjugate of $2+i$ is $2-i$

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$$\frac{3-i}{2+i}$$

Example 4

Divide $\dfrac{3-i}{2+i}$. Write your answer in $a+bi$ form.

The conjugate of $2+i$ is $2-i$

$$\frac{3-i}{2+i}\left(\frac{2-i}{2-i}\right)$$

Example 4

Divide $\frac{3-i}{2+i}$. Write your answer in $a+bi$ form.

The conjugate of $2+i$ is $2-i$

$$\frac{3-i}{2+i}\left(\frac{2-i}{2-i}\right)$$

$$=\frac{5-5i}{5}$$

Example 4

Divide $\dfrac{3-i}{2+i}$. Write your answer in $a+bi$ form.

The conjugate of $2+i$ is $2-i$

$$\frac{3-i}{2+i}\left(\frac{2-i}{2-i}\right)$$

$$=\frac{5-5i}{5}$$

$$=1-i$$