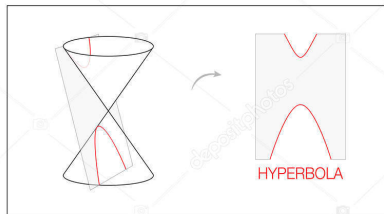
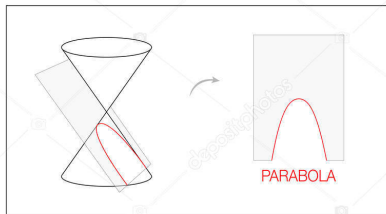
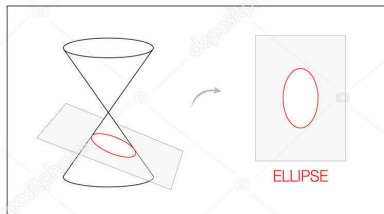
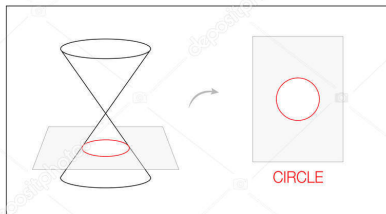


Hyperbolas



Objectives

- 1 Find the vertices and foci for a hyperbola in standard form.
- 2 Write the equation for a hyperbola in standard form.

Hyperbolas

Hyperbolas

The set of points such that the **difference** of their distances from 2 fixed points (called **foci**) is constant.

Comparing Hyperbolas and Ellipses

Just like an ellipse, the midpoint joining the foci is the **center**.

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Just like an ellipse, the midpoint joining the foci is the **center**.

Whereas ellipses could appear taller or wider, hyperbolas will open up and down, or left and right.

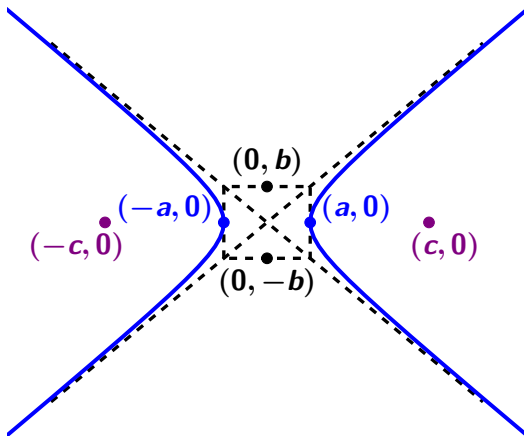
Comparing Hyperbolas and Ellipses

Just like an ellipse, the midpoint joining the foci is the **center**.

Whereas ellipses could appear taller or wider, hyperbolas will open up and down, or left and right.

A key difference, however, is that hyperbolas will open left/right if the sign in front of x is positive, and will open up/down if the sign in front of y is positive; regardless of the values of a and b .

Opening Left and Right



Properties

Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
-----------------	---

Center	(h, k)
---------------	----------

Vertices	$(h \pm a, 0)$
-----------------	----------------

Foci	$(h \pm c, 0)$
-------------	----------------

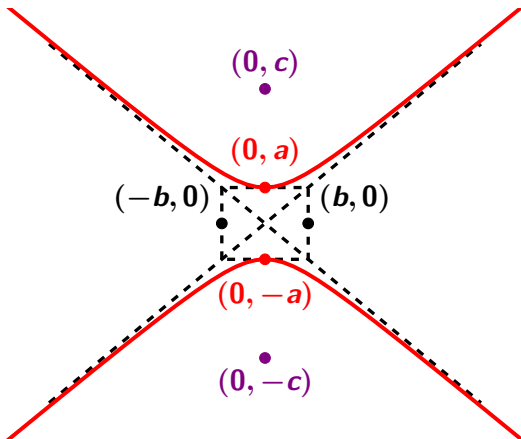
Co-vertices	$(h, k \pm b)$
--------------------	----------------

x-Axis	Transverse Axis
---------------	-----------------

y-Axis	Conjugate Axis
---------------	----------------

c^2	$a^2 + b^2$
-------	-------------

Opening Up and Down



Properties

Equation	$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$
-----------------	---

Center	(h, k)
---------------	----------

Vertices	$(h, k \pm a)$
-----------------	----------------

Foci	$(h, k \pm c)$
-------------	----------------

Co-vertices	$(h \pm a, k)$
--------------------	----------------

x-Axis	Conjugate Axis
---------------	----------------

y-Axis	Transverse Axis
---------------	-----------------

c^2	$a^2 + b^2$
-------	-------------

Example 1

Find the exact coordinates for the vertices and foci for each of the following.

$$(a) \quad \frac{(y-3)^2}{4} - \frac{x^2}{16} = 1$$

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Center: $(0, 3)$

$$a^2 = 4$$

$$a = \pm 2$$

Example 1

Find the exact coordinates for the vertices and foci for each of the following.

$$(a) \quad \frac{(y-3)^2}{4} - \frac{x^2}{16} = 1$$

Center: $(0, 3)$

$$a^2 = 4$$

$$a = \pm 2$$

Vertices: $(0, 3 \pm 2)$

Example 1

Find the exact coordinates for the vertices and foci for each of the following.

$$(a) \quad \frac{(y-3)^2}{4} - \frac{x^2}{16} = 1$$

Center: $(0, 3)$

$$a^2 = 4$$

$$a = \pm 2$$

Vertices: $(0, 3 \pm 2) \rightarrow (0, 1)$ and $(0, 5)$

Example 1a

$$c^2 = a^2 + b^2$$

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$$c^2 = a^2 + b^2$$

$$c^2 = 4 + 16$$

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$$c^2 = 4 + 16$$

$$c^2 = 20$$

Example 1a

$$c^2 = a^2 + b^2$$

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$$c^2 = 20$$

$$c = \pm 2\sqrt{5}$$

Example 1a

$$c^2 = a^2 + b^2$$

$$c^2 = 4 + 16$$

$$c^2 = 20$$

$$c = \pm 2\sqrt{5}$$

Foci: $(0, 3 \pm 2\sqrt{5})$

Example 1

$$(b) \quad \frac{(y+1)^2}{16} - (x-1)^2 = 1$$

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$$(b) \quad \frac{(y+1)^2}{16} - (x-1)^2 = 1$$

Center: $(1, -1)$

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Vertices: $(1, -1 \pm 4)$

Example 1

$$(b) \quad \frac{(y+1)^2}{16} - (x-1)^2 = 1$$

Center: $(1, -1)$

$$a^2 = 16$$

$$a = \pm 4$$

Vertices: $(1, -1 \pm 4) \longrightarrow (1, -5) \text{ and } (1, 3)$

Example 1b

$$c^2 = a^2 + b^2$$

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$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 1$$

Example 1b

$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 1$$

$$c^2 = 17$$

Example 1b

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$$c^2 = 16 + 1$$

$$c^2 = 17$$

$$c = \pm\sqrt{17}$$

Example 1b

$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 1$$

$$c^2 = 17$$

$$c = \pm\sqrt{17}$$

Foci: $(1, -1 \pm \sqrt{17})$

Example 1

$$(c) \quad (x+4)^2 - \frac{(y+2)^2}{9} = 1$$

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Center: $(-4, -2)$

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Center: $(-4, -2)$

$$a^2 = 1$$

$$a = \pm 1$$

Example 1

$$(c) \quad (x+4)^2 - \frac{(y+2)^2}{9} = 1$$

Center: $(-4, -2)$

$$a^2 = 1$$

$$a = \pm 1$$

Vertices: $(-4 \pm 1, -2)$

Example 1

$$(c) \quad (x+4)^2 - \frac{(y+2)^2}{9} = 1$$

Center: $(-4, -2)$

$$a^2 = 1$$

$$a = \pm 1$$

Vertices: $(-4 \pm 1, -2) \longrightarrow (-5, -2) \text{ and } (-3, -2)$

Example 1c

$$c^2 = a^2 + b^2$$

Example 1c

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

Example 1c

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

$$c^2 = 10$$

Example 1c

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

$$c^2 = 10$$

$$c = \pm\sqrt{10}$$

Example 1c

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 9$$

$$c^2 = 10$$

$$c = \pm\sqrt{10}$$

Foci: $(-4 \pm \sqrt{10}, -2)$

Example 1

$$(d) \quad \frac{(x+2)^2}{9} - \frac{(y-2)^2}{4} = 1$$

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Center: $(-2, 2)$

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$$(d) \quad \frac{(x+2)^2}{9} - \frac{(y-2)^2}{4} = 1$$

Center: $(-2, 2)$

$$a^2 = 9$$

$$a = \pm 3$$

Example 1

$$(d) \quad \frac{(x+2)^2}{9} - \frac{(y-2)^2}{4} = 1$$

Center: $(-2, 2)$

$$a^2 = 9$$

$$a = \pm 3$$

Vertices: $(-2 \pm 3, 2)$

Example 1

$$(d) \quad \frac{(x+2)^2}{9} - \frac{(y-2)^2}{4} = 1$$

Center: $(-2, 2)$

$$a^2 = 9$$

$$a = \pm 3$$

Vertices: $(-2 \pm 3, 2) \rightarrow (-5, 2)$ and $(1, 2)$

Example 1d

$$c^2 = a^2 + b^2$$

Example 1d

$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

Example 1d

$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

$$c^2 = 13$$

Example 1d

$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

$$c^2 = 13$$

$$c = \pm\sqrt{13}$$

Example 1d

$$c^2 = a^2 + b^2$$

$$c^2 = 9 + 4$$

$$c^2 = 13$$

$$c = \pm\sqrt{13}$$

Foci: $(-2 \pm \sqrt{13}, 2)$

Objectives

- 1 Find the vertices and foci for a hyperbola in standard form.
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Similar to Ellipses, But Not the Same

We will use the technique that we used for ellipses **with one big difference.**

Similar to Ellipses, But Not the Same

We will use the technique that we used for ellipses **with one big difference.**

The absolute value of the y -coordinate after the minus sign gets **subtracted** from the right side.

Example 2

Write each of the following in standard form.

(a) $-9x^2 - 108x + y^2 - 10y - 380 = 0$

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Write each of the following in standard form.

$$(a) \quad -9x^2 - 108x + y^2 - 10y - 380 = 0$$

$$-9x^2 - 108x + y^2 - 10y = 380$$

Example 2

Write each of the following in standard form.

$$(a) \quad -9x^2 - 108x + y^2 - 10y - 380 = 0$$

$$-9x^2 - 108x + y^2 - 10y = 380$$

Vertex: $(-6, 324)$

Example 2

Write each of the following in standard form.

$$(a) \quad -9x^2 - 108x + y^2 - 10y - 380 = 0$$

$$-9x^2 - 108x + y^2 - 10y = 380$$

Vertex: $(-6, 324)$

Vertex: $(5, -25)$

Example 2

Write each of the following in standard form.

$$(a) \quad -9x^2 - 108x + y^2 - 10y - 380 = 0$$

$$-9x^2 - 108x + y^2 - 10y = 380$$

Vertex: $(-6, 324)$

Vertex: $(5, -25)$

$$-9(x + 6)^2 + (y - 5)^2 = 380 - |324| + |-25|$$

Example 2

Write each of the following in standard form.

$$(a) \quad -9x^2 - 108x + y^2 - 10y - 380 = 0$$

$$-9x^2 - 108x + y^2 - 10y = 380$$

Vertex: $(-6, 324)$

Vertex: $(5, -25)$

$$-9(x+6)^2 + (y-5)^2 = 380 - |324| + |-25|$$

$$-9(x+6)^2 + (y-5)^2 = 81$$

Example 2a

$$-9(x + 6)^2 + (y - 5)^2 = 81$$

Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

$$-\frac{(x+6)^2}{9} + \frac{(y-5)^2}{81} = 1$$

Example 2a

$$-9(x+6)^2 + (y-5)^2 = 81$$

$$\frac{-9(x+6)^2}{81} + \frac{(y-5)^2}{81} = 1$$

$$-\frac{(x+6)^2}{9} + \frac{(y-5)^2}{81} = 1$$

$$\frac{(y-5)^2}{81} - \frac{(x+6)^2}{9} = 1$$

Example 2

$$(b) \quad -x^2 + 12x + 4y^2 - 24y - 144 = 0$$

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$$-x^2 + 12x + 4y^2 - 24y = 144$$

Example 2

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$$-x^2 + 12x + 4y^2 - 24y = 144$$

Vertex: (6, 36)

Example 2

$$(b) \quad -x^2 + 12x + 4y^2 - 24y - 144 = 0$$

$$-x^2 + 12x + 4y^2 - 24y = 144$$

Vertex: (6, 36)

Vertex: (3, -36)

Example 2

$$(b) \quad -x^2 + 12x + 4y^2 - 24y - 144 = 0$$

$$-x^2 + 12x + 4y^2 - 24y = 144$$

Vertex: (6, 36)

Vertex: (3, -36)

$$-(x - 6)^2 + 4(y - 3)^2 = 144 - |36| + |-36|$$

Example 2

$$(b) \quad -x^2 + 12x + 4y^2 - 24y - 144 = 0$$

$$-x^2 + 12x + 4y^2 - 24y = 144$$

Vertex: (6, 36)

Vertex: (3, -36)

$$-(x-6)^2 + 4(y-3)^2 = 144 - |36| + |-36|$$

$$-(x-6)^2 + 4(y-3)^2 = 144$$

Example 2b

$$-(x - 6)^2 + 4(y - 3)^2 = 144$$

Example 2b

$$-(x-6)^2 + 4(y-3)^2 = 144$$

$$-\frac{(x-6)^2}{144} + \frac{4(y-3)^2}{144} = 1$$

Example 2b

$$-(x-6)^2 + 4(y-3)^2 = 144$$

$$-\frac{(x-6)^2}{144} + \frac{4(y-3)^2}{144} = 1$$

$$-\frac{(x-6)^2}{144} + \frac{(y-3)^2}{36} = 1$$

Example 2b

$$-(x-6)^2 + 4(y-3)^2 = 144$$

$$-\frac{(x-6)^2}{144} + \frac{4(y-3)^2}{144} = 1$$

$$-\frac{(x-6)^2}{144} + \frac{(y-3)^2}{36} = 1$$

$$\frac{(y-3)^2}{36} - \frac{(x-6)^2}{144} = 1$$

Example 2

$$(c) \quad x^2 + 20x - y^2 + 10y + 39 = 0$$

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$$(c) \quad x^2 + 20x - y^2 + 10y + 39 = 0$$

$$x^2 + 20x - y^2 + 10y = -39$$

Example 2

$$(c) \quad x^2 + 20x - y^2 + 10y + 39 = 0$$

$$x^2 + 20x - y^2 + 10y = -39$$

Vertex: $(-10, -100)$

Example 2

$$(c) \quad x^2 + 20x - y^2 + 10y + 39 = 0$$

$$x^2 + 20x - y^2 + 10y = -39$$

Vertex: $(-10, -100)$

Vertex: $(5, 25)$

Example 2

$$(c) \quad x^2 + 20x - y^2 + 10y + 39 = 0$$

$$x^2 + 20x - y^2 + 10y = -39$$

Vertex: $(-10, -100)$

Vertex: $(5, 25)$

$$(x + 10)^2 - (y - 5)^2 = -39 + |-100| - |25|$$

Example 2

$$(c) \quad x^2 + 20x - y^2 + 10y + 39 = 0$$

$$x^2 + 20x - y^2 + 10y = -39$$

Vertex: $(-10, -100)$

Vertex: $(5, 25)$

$$(x + 10)^2 - (y - 5)^2 = -39 + |-100| - |25|$$

$$(x + 10)^2 - (y - 5)^2 = 36$$

Example 2

$$(c) \quad x^2 + 20x - y^2 + 10y + 39 = 0$$

$$x^2 + 20x - y^2 + 10y = -39$$

Vertex: $(-10, -100)$

Vertex: $(5, 25)$

$$(x + 10)^2 - (y - 5)^2 = -39 + |-100| - |25|$$

$$(x + 10)^2 - (y - 5)^2 = 36$$

$$\frac{(x + 10)^2}{36} - \frac{(y - 5)^2}{36} = 1$$

Example 2

$$(d) \quad x^2 + 18x - y^2 - 20y - 188 = 0$$

Example 2

$$(d) \quad x^2 + 18x - y^2 - 20y - 188 = 0$$

$$x^2 + 18x - y^2 - 20y = 188$$

Example 2

$$(d) \quad x^2 + 18x - y^2 - 20y - 188 = 0$$

$$x^2 + 18x - y^2 - 20y = 188$$

Vertex: $(-9, -81)$

Example 2

$$(d) \quad x^2 + 18x - y^2 - 20y - 188 = 0$$

$$x^2 + 18x - y^2 - 20y = 188$$

Vertex: $(-9, -81)$

Vertex: $(-10, 100)$

Example 2

$$(d) \quad x^2 + 18x - y^2 - 20y - 188 = 0$$

$$x^2 + 18x - y^2 - 20y = 188$$

Vertex: $(-9, -81)$

Vertex: $(-10, 100)$

$$(x + 9)^2 - (y + 10)^2 = 188 + |-81| - |100|$$

Example 2

$$(d) \quad x^2 + 18x - y^2 - 20y - 188 = 0$$

$$x^2 + 18x - y^2 - 20y = 188$$

Vertex: $(-9, -81)$

Vertex: $(-10, 100)$

$$(x + 9)^2 - (y + 10)^2 = 188 + |-81| - |100|$$

$$(x + 9)^2 - (y + 10)^2 = 169$$

Example 2

$$(d) \quad x^2 + 18x - y^2 - 20y - 188 = 0$$

$$x^2 + 18x - y^2 - 20y = 188$$

Vertex: $(-9, -81)$

Vertex: $(-10, 100)$

$$(x + 9)^2 - (y + 10)^2 = 188 + |-81| - |100|$$

$$(x + 9)^2 - (y + 10)^2 = 169$$

$$\frac{(x + 9)^2}{169} - \frac{(y + 10)^2}{169} = 1$$