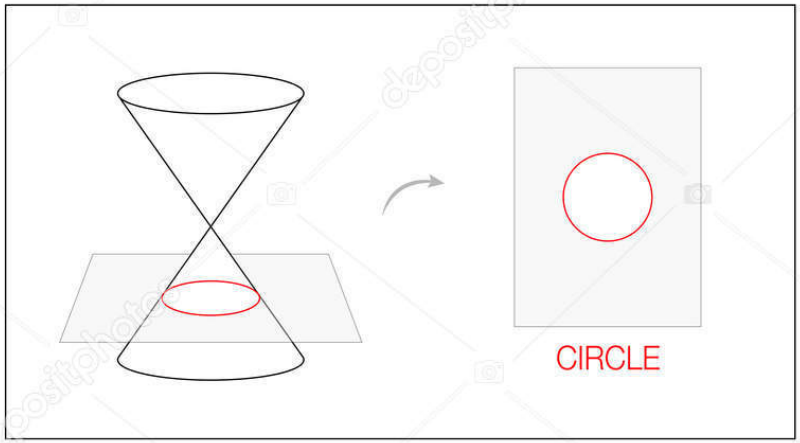
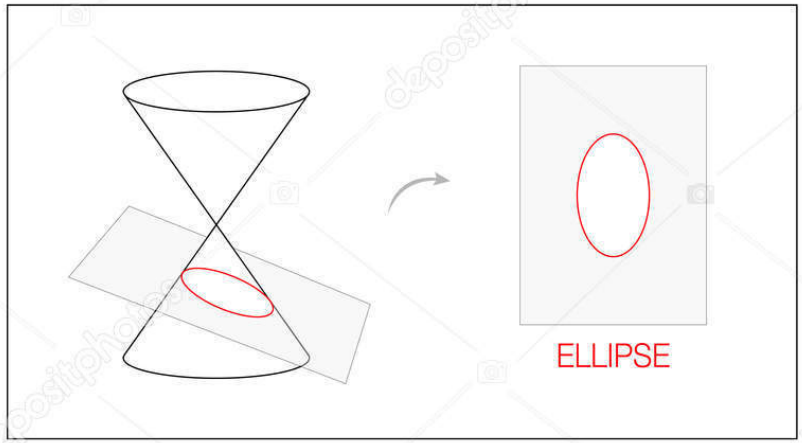
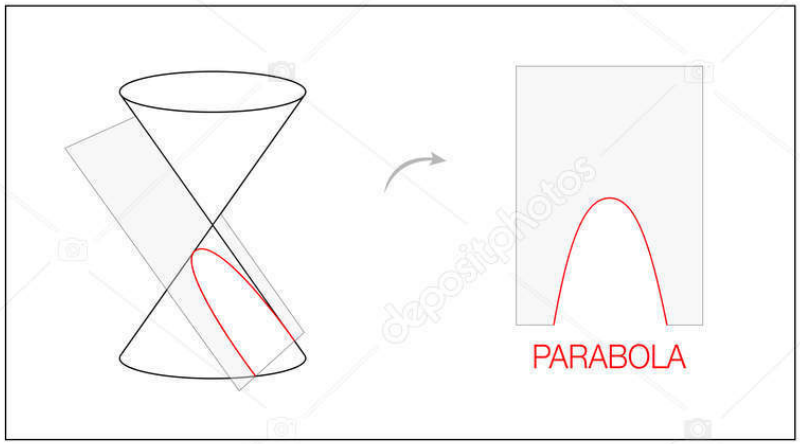
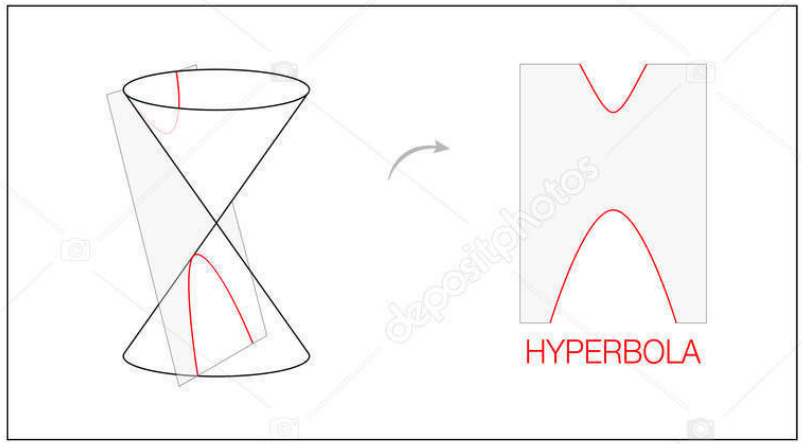
# Circles



# Objectives

1. Identify the center and radius of a circle.
2. Write the general form of the equation of a circle in standard form

# Circles

You may remember circles from geometry class. In this chapter, we will look at equations and properties of circles.

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**Circles**

The set of points, each of whose distance from a fixed point (the center) is the same.

The standard form of the equation of a circle is

$$(x-h)^2+(y-k)^2=r^2$$

with center $(h, k)$ and radius $r$.

# Example 1

Identify the center and exact radius of each.

(a) $(x-4)^2+(y+3)^2=49$

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Radius: $\sqrt{49}=7$

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(b) $(x+1)^2 + (y-7)^2 = 72$

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Center: $(-1, 7)$

Radius: $\sqrt{72} = 6\sqrt{2}$

# Writing the Standard Form of the Equation of a Circle

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2. Find the vertex of the $x$-terms and $y$-terms.
    - The $x$-coordinates of each vertex will represent $h$ and $k$, respectively.
    - The absolute value $y$-coordinates will be added to the constant.

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(a) $x^2 - 4x + y^2 + 6y - 23 = 0$

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**Vertex:** $\mathbf{(3, -9)}$

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$$(x-2)^2 + (y-3)^2 = 36$$

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$$(x-2)^2 + (y-3)^2 = 23 + |-4| + |-9|$$

$$(x-2)^2 + (y-3)^2 = 36$$

Center: $(2, 3)$ Radius: 6

# Example 2

(b) $x^2 + 16x + y^2 - 8y - 1 = 0$

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**Vertex: $(-8, -64)$**

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**Vertex:** $\mathbf{(4, -16)}$

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$$(x+8)^2 + (y-4)^2 = 1 + |-64| + |-16|$$

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(b) $x^2 + 16x + y^2 - 8y - 1 = 0$

$$\mathbf{x^2 + 16x} \qquad \mathbf{+ y^2 - 8y} \qquad = 1$$

**Vertex:** $\mathbf{(-8, -64)}$

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$$(x+8)^2 + (y-4)^2 = 1 + |-64| + |-16|$$

$$(x+8)^2 + (y-4)^2 = 81$$

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**Vertex: $(-8, -64)$**

**Vertex: $(4, -16)$**

$$(x+8)^2 + (y-4)^2 = 1 + |-64| + |-16|$$

$$(x+8)^2 + (y-4)^2 = 81$$

Center: $(-8, 4)$ Radius: 9

# Example 2

(c) $x^2 - 10x + y^2 + 2y + 14 = 0$

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**Vertex: $(5, -25)$**

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$x^2 - 10x + y^2 + 2y = -14$

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**Vertex:** $\mathbf{(5, -25)}$

**Vertex:** $\mathbf{(-1, -1)}$

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**Vertex:** $\boldsymbol{(5, -25)}$

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$$(x-5)^2 + (y+1)^2 = 14 + |-25| + |-1|$$

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$$(x-5)^2 + (y+1)^2 = 40$$

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$x^2 - 10x + y^2 + 2y = -14$

$$\boldsymbol{x^2 - 10x} \quad + \boldsymbol{y^2 + 2y} \quad = -14$$

**Vertex:** $\boldsymbol{(5, -25)}$

**Vertex:** $\boldsymbol{(-1, -1)}$

$$(x-5)^2 + (y+1)^2 = 14 + |-25| + |-1|$$

$$(x-5)^2 + (y+1)^2 = 40$$

Center: $(5, -1)$ Radius: $2\sqrt{10}$

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# General Form

Circles can also be written in general form. General form is standard form multiplied out and simplified.

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General form will have all terms on one side of the equation and 0 on the other.

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Write the general form of each of the following.

(a) $(x-3)^2 + y^2 = 6$

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Write the general form of each of the following.

(a) $(x-3)^2+y^2=6$

$$(x-3)^2+y^2=6$$

$$x^2-6x+9+y^2=6$$

$$x^2-6x+y^2+3=0$$

# Example 3

(b) $(x+7)^2 + (y-5)^2 = 10$

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(b) $(x+7)^2+(y-5)^2=10$

$$(x+7)^2+(y-5)^2=10$$

$$x^2+14x+49+y^2-10y+25=10$$

# Example 3

(b) $(x+7)^2+(y-5)^2=10$

$$(x+7)^2+(y-5)^2=10$$

$$x^2+14x+49+y^2-10y+25=10$$

$$x^2+14x+y^2-10y+74=10$$

# Example 3

(b) $(x+7)^2+(y-5)^2=10$

$$(x+7)^2+(y-5)^2=10$$

$$x^2+14x+49+y^2-10y+25=10$$

$$x^2+14x+y^2-10y+74=10$$

$$x^2+14x+y^2-10y+64=0$$