# **Dividing Polynomials**

# Objectives

1 Divide polynomials without a remainder

2 Divide polynomials with a remainder

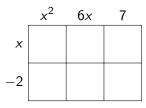
3 Use the Remainder Theorem and Factor Theorem

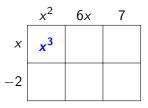
#### Division Basics

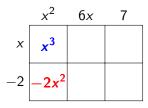
In the expression  $a \div b = c$ , a is the dividend, b is the divisor, and c is the quotient.

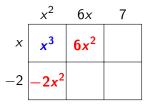
When dividing polynomials, it will help to write your terms in standard form (descending powers). You may also need to fill in any missing terms using 0 as a coefficient.

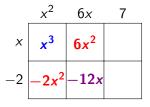
Before we get to division, let's review an organizational technique for multiplying polynomials.











$$\begin{array}{c|cccc}
x^2 & 6x & 7 \\
x & x^3 & 6x^2 & 7x \\
-2 & -2x^2 & -12x
\end{array}$$

$$\begin{array}{c|cccc}
x^2 & 6x & 7 \\
x & x^3 & 6x^2 & 7x \\
-2 & -2x^2 & -12x & -14
\end{array}$$

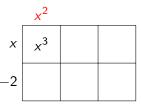
To find the product of  $(x-2)(x^2+6x+7)$ , we can use the following method:

When we combine all of terms inside the box, we get the product,  $x^3 + 4x^2 - 5x - 14$ .

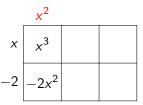
$$(x^3 + 4x^2 - 5x - 14) \div (x - 2)$$

X	<i>x</i> <sup>3</sup>	
-2		

$$(x^3 + 4x^2 - 5x - 14) \div (x - 2)$$



$$(x^3 + 4x^2 - 5x - 14) \div (x - 2)$$



$$(x^{3} + 4x^{2} - 5x - 14) \div (x - 2)$$

$$x^{2}$$

$$x \quad x^{3} \quad 6x^{2}$$

$$-2 \quad -2x^{2}$$

$$(x^{3} + 4x^{2} - 5x - 14) \div (x - 2)$$

$$x^{2} \quad 6x$$

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$$(x^{3} + 4x^{2} - 5x - 14) \div (x - 2)$$

$$x^{2} \quad 6x$$

$$x \quad x^{3} \quad 6x^{2}$$

$$-2 \quad -2x^{2} - 12x$$

$$(x^{3} + 4x^{2} - 5x - 14) \div (x - 2)$$

$$\begin{array}{c|cccc}
x^{2} & 6x \\
x & x^{3} & 6x^{2} & 7x \\
-2 & -2x^{2} & -12x
\end{array}$$

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$$x^{2} \quad 6x \quad 7$$

$$x \quad x^{3} \quad 6x^{2} \quad 7x$$

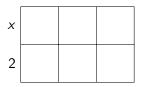
$$-2 \quad -2x^{2} - 12x \quad -14$$

$$x^{2} + 6x + 7$$

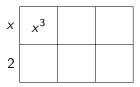
Divide 
$$(x^3 + 8) \div (x + 2)$$

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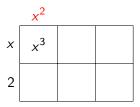
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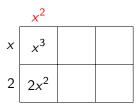
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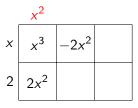
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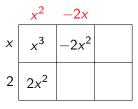
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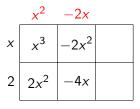
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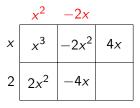
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$$\begin{array}{c|cccc}
x^2 & -2x & 4 \\
x & x^3 & -2x^2 & 4x \\
2 & 2x^2 & -4x & 8
\end{array}$$

$$x^2 - 2x + 4$$

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### Dividing Polynomials With a Remainder

In the previous examples, everything "balanced out" from within the grid.

In other words, after combining like terms, all terms in the dividend were accounted for.

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In the previous examples, everything "balanced out" from within the grid.

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In the next group of examples, we will need to figure out what to add to our quotient to "balance out the problem."

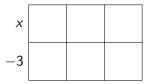
We will write our answers in the form

$$quotient + \frac{remainder}{divisor}$$

(a) 
$$(5x^3 - 2x^2 + 1) \div (x - 3)$$

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$$(5x^3 - 2x^2 + 1) \div (x - 3)$$
  
 $(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$ 

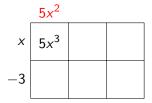
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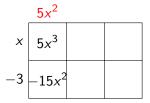
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$$(5x^3 - 2x^2 + 1) \div (x - 3)$$
  
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$$\begin{bmatrix} x & 5x^3 & & \\ & -3 & & & \end{bmatrix}$$

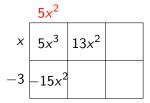
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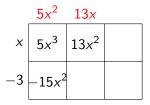
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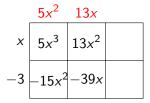
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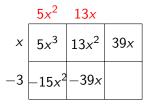
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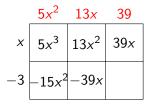
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 $5x^2 + 13x + 39$  remainder: 108

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$$(5x^3 - 2x^2 + 1) \div (x - 3)$$
  
 $5x^2 + 13x + 39$  remainder: 108

$$5x^2 + 13x + 39 + \frac{108}{x - 3}$$

(b) 
$$(4-8x-12x^2) \div (2x-3)$$

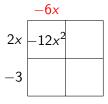
(b) 
$$(4-8x-12x^2) \div (2x-3)$$
  
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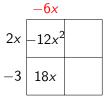
(b) 
$$(4-8x-12x^2) \div (2x-3)$$
  
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$$\begin{array}{c|c}
2x & -12x^2 \\
-3 & & & \\
\end{array}$$

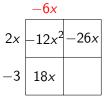
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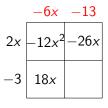
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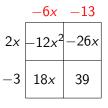
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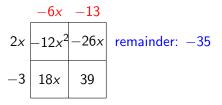
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(b) 
$$(4 - 8x - 12x^2) \div (2x - 3)$$
  
 $-6x - 13$ remainder  $-35$ 

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$$(4 - 8x - 12x^2) \div (2x - 3)$$
  
 $-6x - 13$  remainder  $-35$   
 $-6x - 13 - \frac{35}{2x - 3}$ 

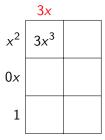
(c) 
$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$$

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$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$$
  
 $(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$ 

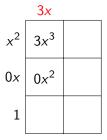
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$x^2$	$3x^3$	
0 <i>x</i>		
1		

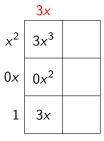
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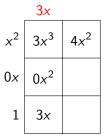
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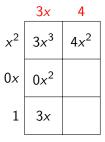
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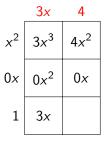
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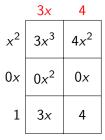
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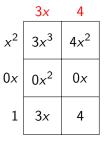
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remainder: -2x + 3

(c) 
$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$$
  
 $3x + 4$ remainder  $-2x + 3$ 

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$$3x + 4 + \frac{-2x + 3}{x^2 + 1}$$

# Objectives

1 Divide polynomials without a remainder

2 Divide polynomials with a remainder

3 Use the Remainder Theorem and Factor Theorem

#### The Remainder Theorem

There is a quicker way to determine if some polynomial division problems result in a remainder.

#### The Remainder Theorem

If p is a polynomial of degree at least 1, and c is a real number, then when p(x) is divided by x - c, the remainder is p(c).

What is the remainder when  $2x^3 - 5x + 3$  is divided by x + 2?

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$$p(-2) = 2(-2)^3 - 5(-2) + 3$$

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$$p(-2) = 2(-2)^3 - 5(-2) + 3$$
$$p(-2) = -3$$

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$$p(-2) = 2(-2)^3 - 5(-2) + 3$$
  
 $p(-2) = -3$ 

The remainder is -3

#### The Factor Theorem

The Factor Theorem states that if p is a nonzero polynomial, then the real number c is a zero of p if (x - c) is a factor of p(x).

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The Factor Theorem states that if p is a nonzero polynomial, then the real number c is a zero of p if (x - c) is a factor of p(x).

In other words, p(c) = 0.

Use the fact that x = 1 is a zero of  $p(x) = 2x^3 - 5x + 3$  to factor p(x) and find all of the real zeros of p.

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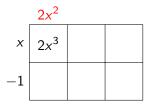
Use the fact that x = 1 is a zero of  $p(x) = 2x^3 - 5x + 3$  to factor p(x) and find all of the real zeros of p.

$$(2x^3 + 0x^2 - 5x + 3) \div (x - 1)$$
 will have a remainder of 0.

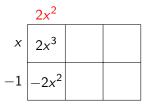
X	$2x^3$	
-1		

Use the fact that x = 1 is a zero of  $p(x) = 2x^3 - 5x + 3$  to factor p(x) and find all of the real zeros of p.

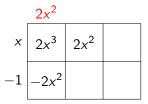
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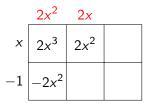
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2x^2 & 2x \\
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-1 & -2x^2 & -2x
\end{array}$$

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$$\begin{array}{c|cccc}
2x^2 & 2x \\
x & 2x^3 & 2x^2 & -3x \\
-1 & -2x^2 & -2x & \\
\end{array}$$

Use the fact that x = 1 is a zero of  $p(x) = 2x^3 - 5x + 3$  to factor p(x) and find all of the real zeros of p.

$$\begin{array}{c|cccc}
2x^2 & 2x & -3 \\
x & 2x^3 & 2x^2 & -3x \\
-1 & -2x^2 & -2x & \\
\end{array}$$

Use the fact that x = 1 is a zero of  $p(x) = 2x^3 - 5x + 3$  to factor p(x) and find all of the real zeros of p.

$$2x^2 + 2x - 3 = 0$$

$$2x^2 + 2x - 3 = 0$$

using the Quadratic Formula, we get

$$x = \frac{-1 \pm \sqrt{7}}{2}$$