Continuity

Objectives

1. Determine whether a function is continuous at a number
2. Determine the numbers for which a function is discontinuous

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Conditions for continuity at $x = a$:

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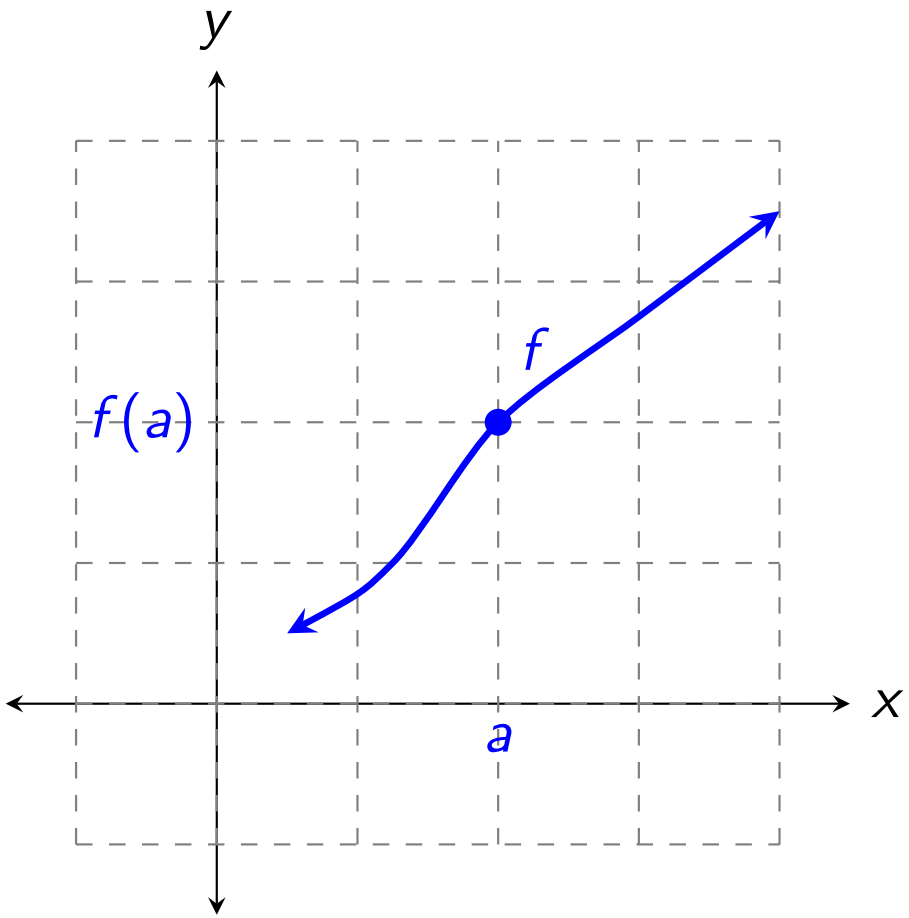
Conditions for continuity at $x = a$:

1. $f(a)$ exists
2. Left-hand and right-hand limits exist and are equal.
3. $\lim_{x \to a} f(x) = f(a)$

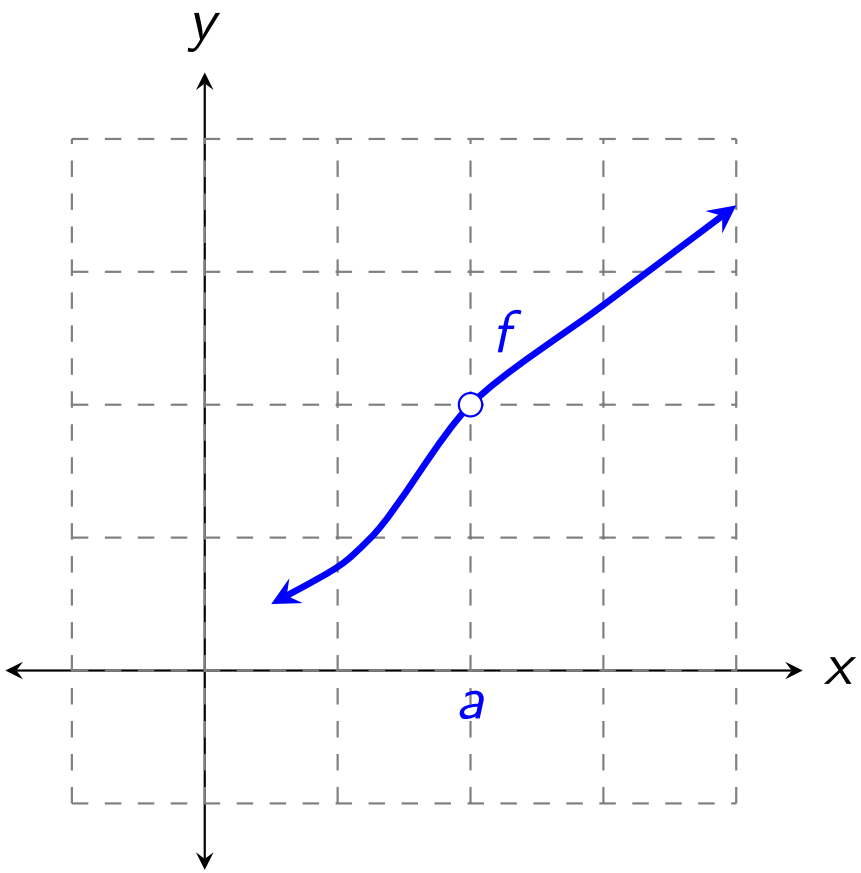
Graph of a function continuous at $x = a$



Example 1

Explain why each graph is *discontinuous* at $x = a$.

(a)



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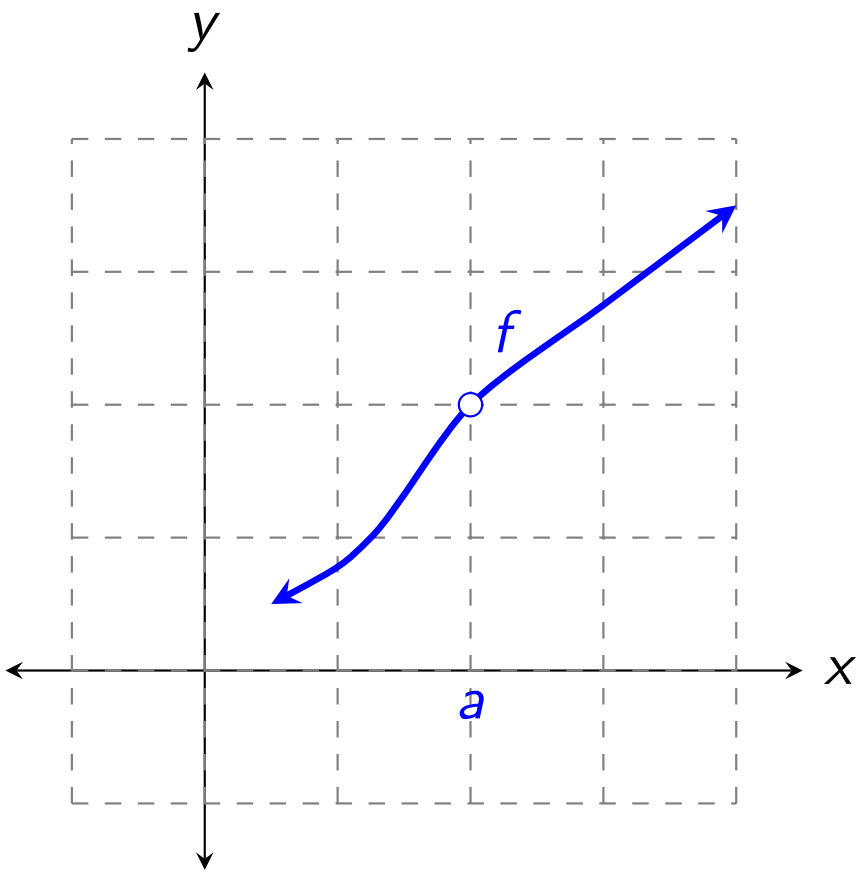

$f(a)$ is not defined.

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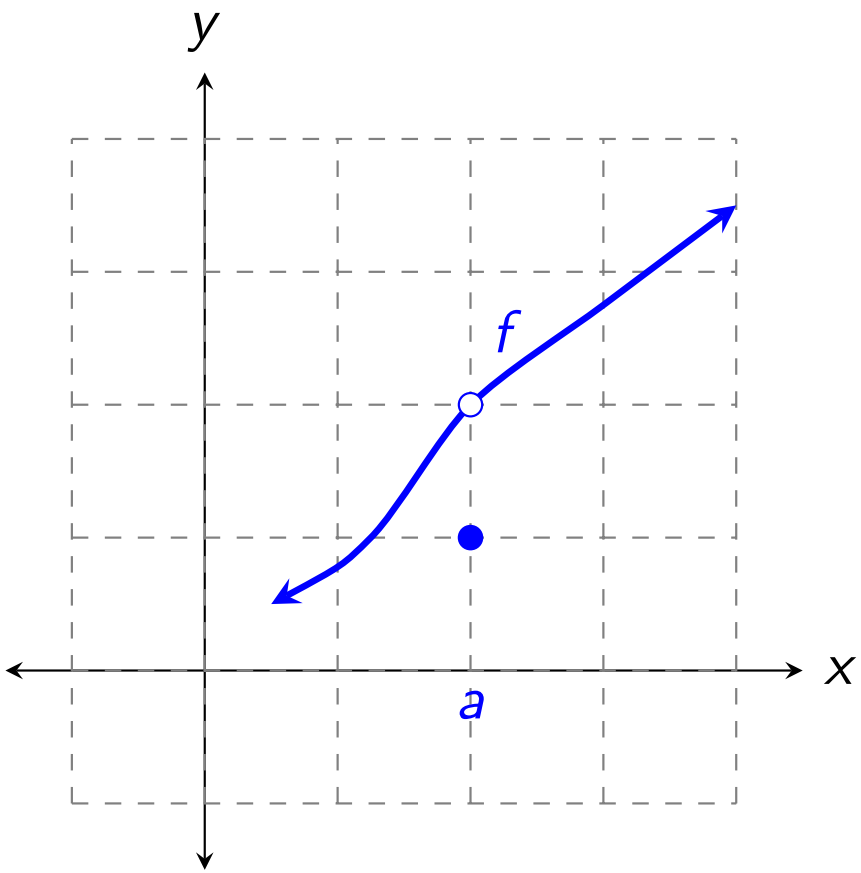
Explain why each graph is *discontinuous* at $x = a$.

(b)



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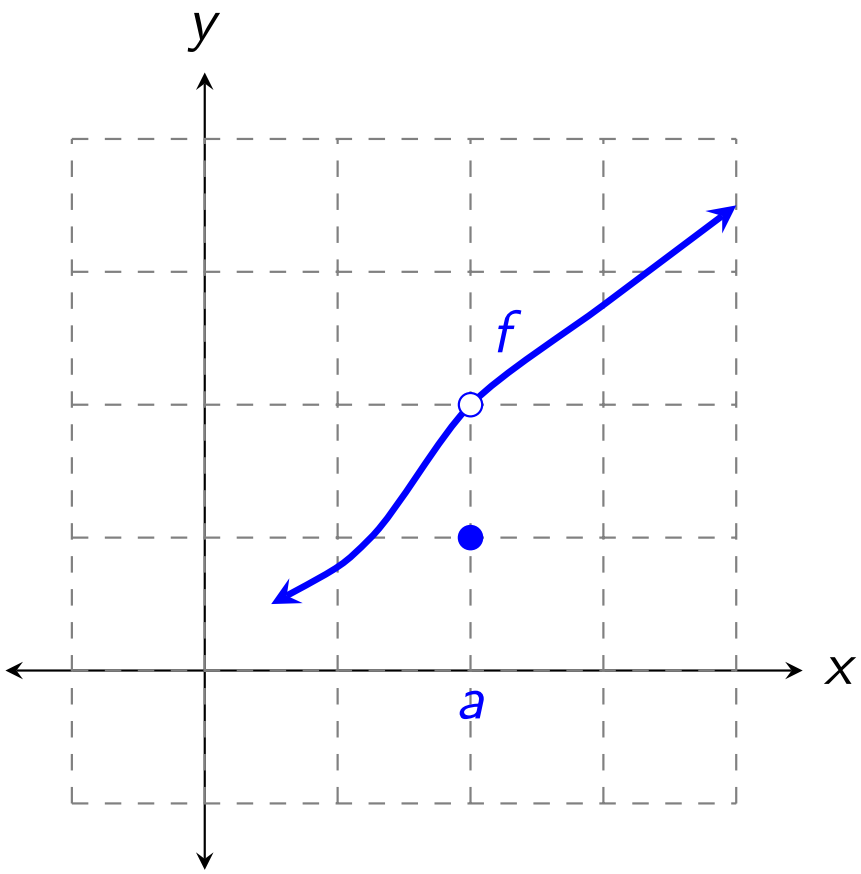

$$\lim_{x \to a} f(x) \neq f(a)$$

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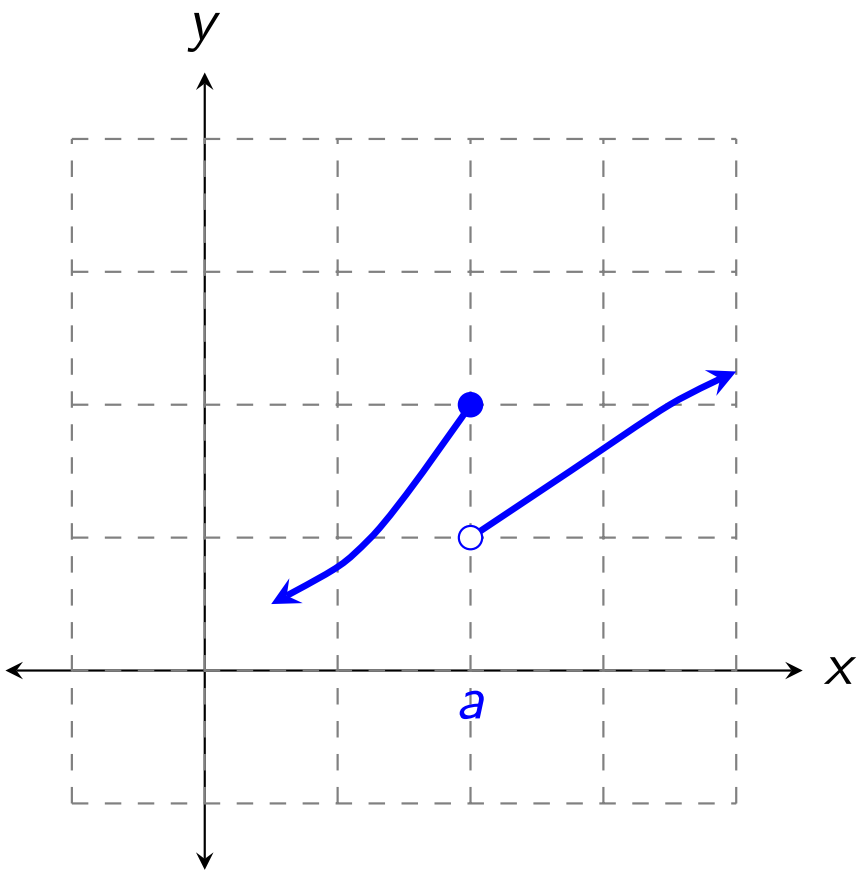

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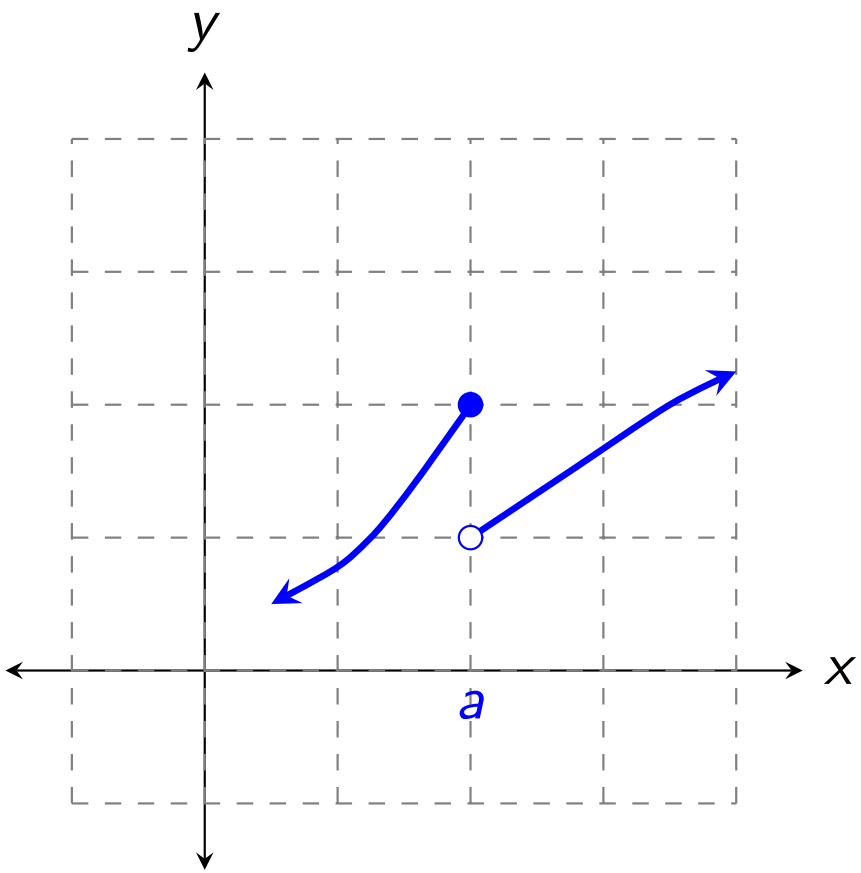

$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

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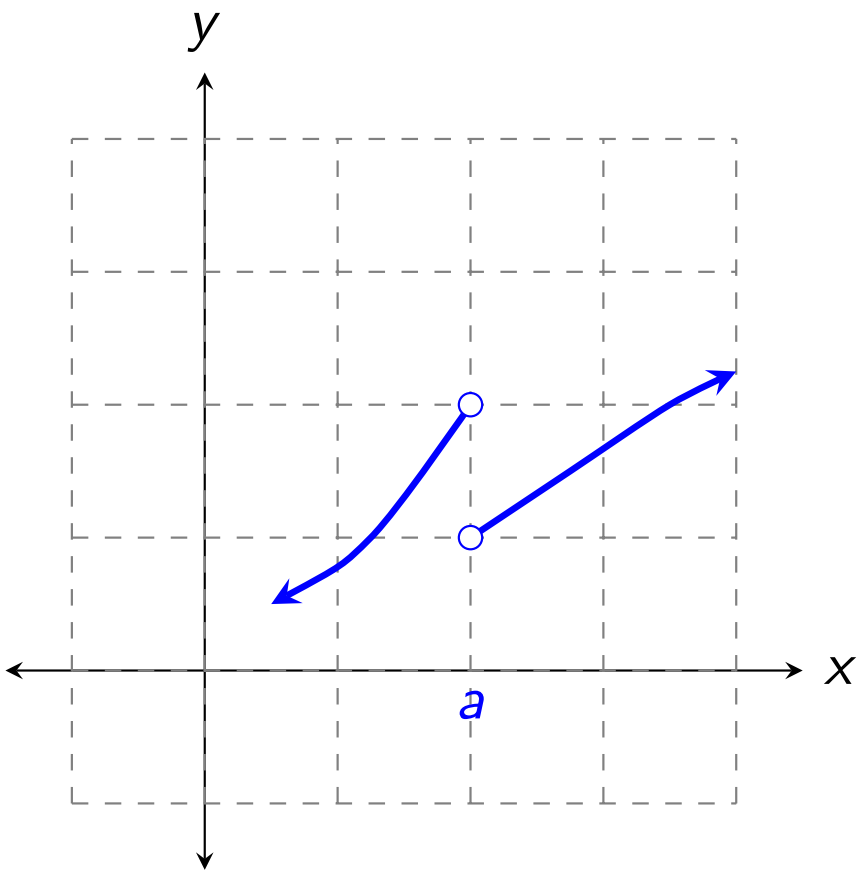

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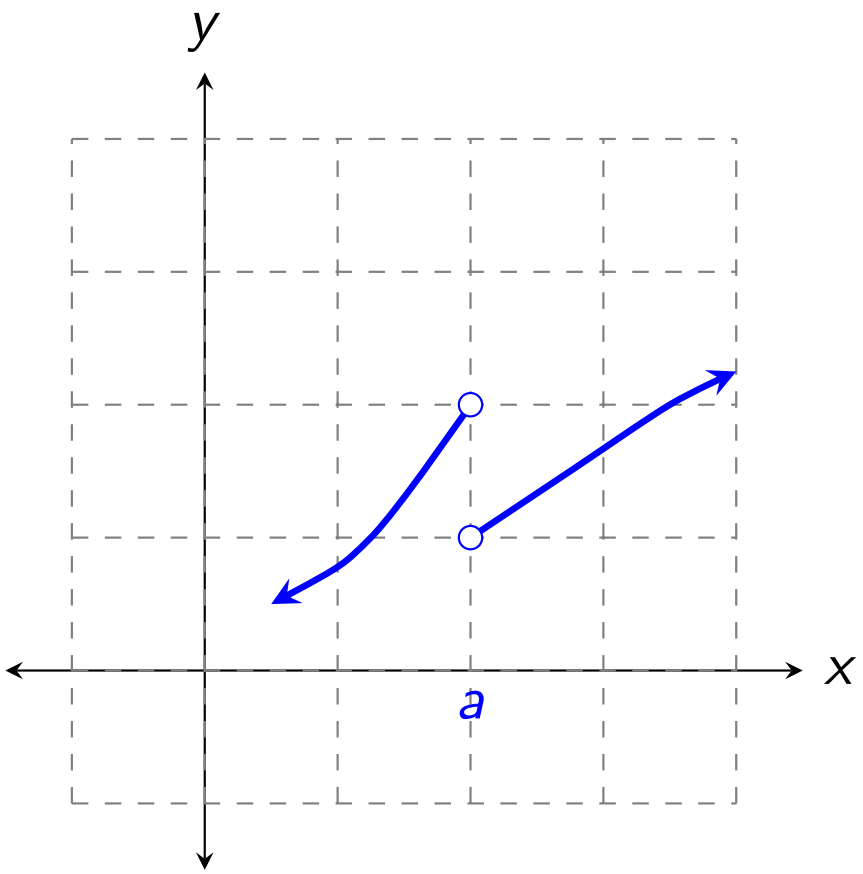

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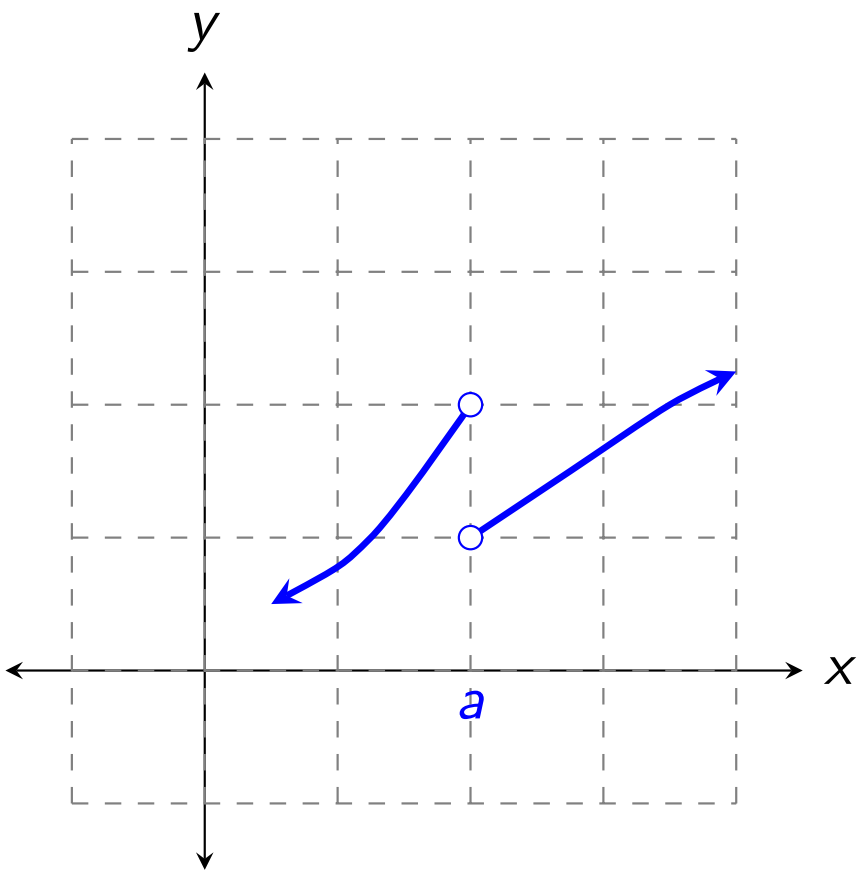

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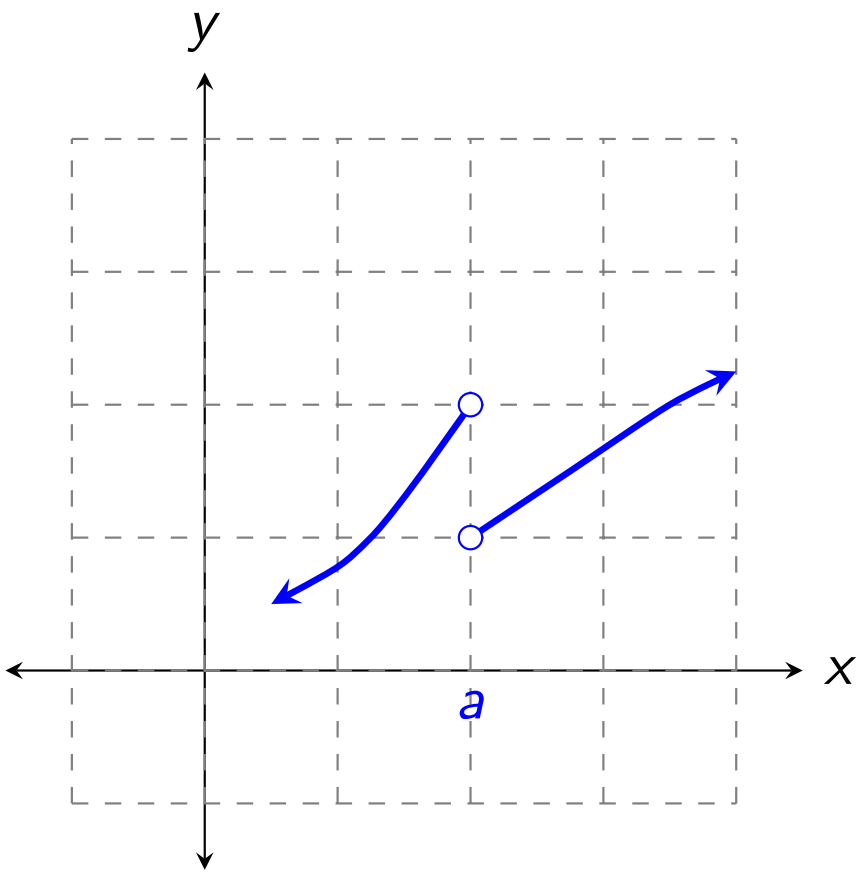

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Functions are discontinuous at points that involve

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Functions are discontinuous at points that involve

- Holes
- Vertical asymptotes
- Gaps in y-coordinates (left-hand limit $\neq$ right-hand limit)

Example 2

Identify all discontinuities for each.

(a) $f(x) = \dfrac{x^2 - 2x - 15}{x - 5}$

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The function is discontinuous at $x = 5$.

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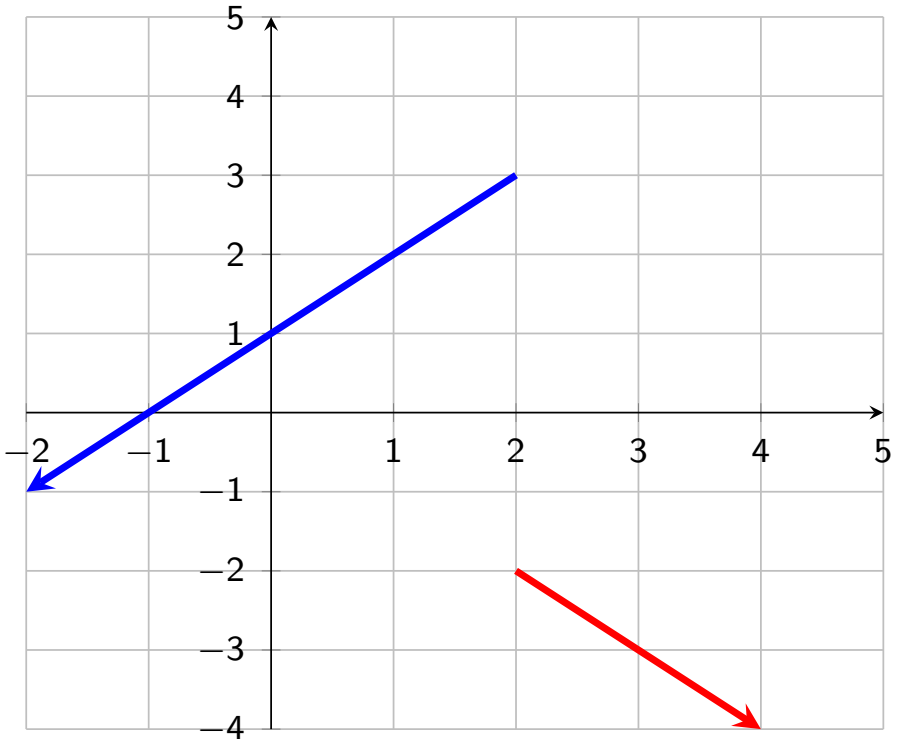
The function is discontinuous at $x = 6$

Example 2

(c) $g(x) = \begin{cases} x+1, & x < 2 \\ -x, & x \geq 2 \end{cases}$



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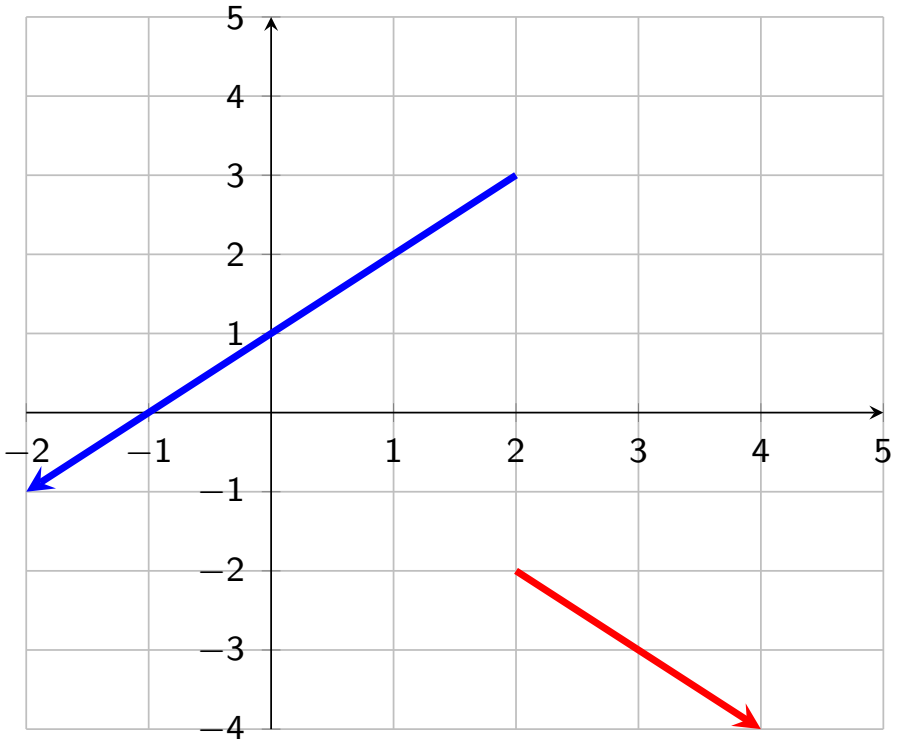
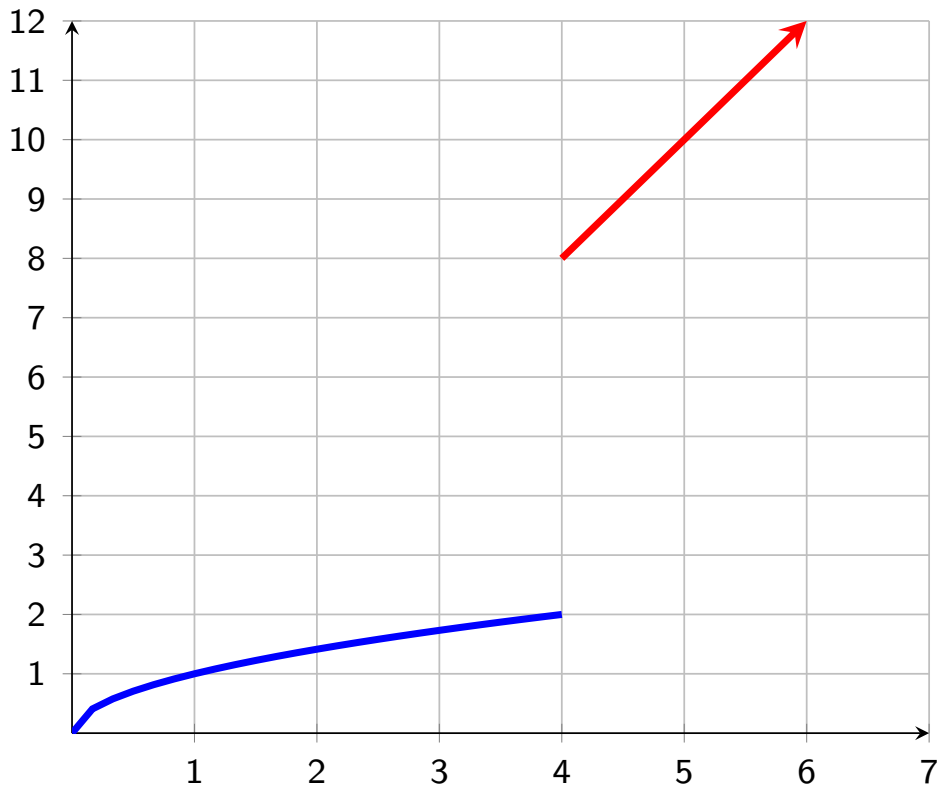


Discontinuous at $x = 2$

Example 2

(d) $$g(x) = \begin{cases} \sqrt{x}, & 0 \leq x < 4 \\ 2x, & x \geq 4 \end{cases}$$



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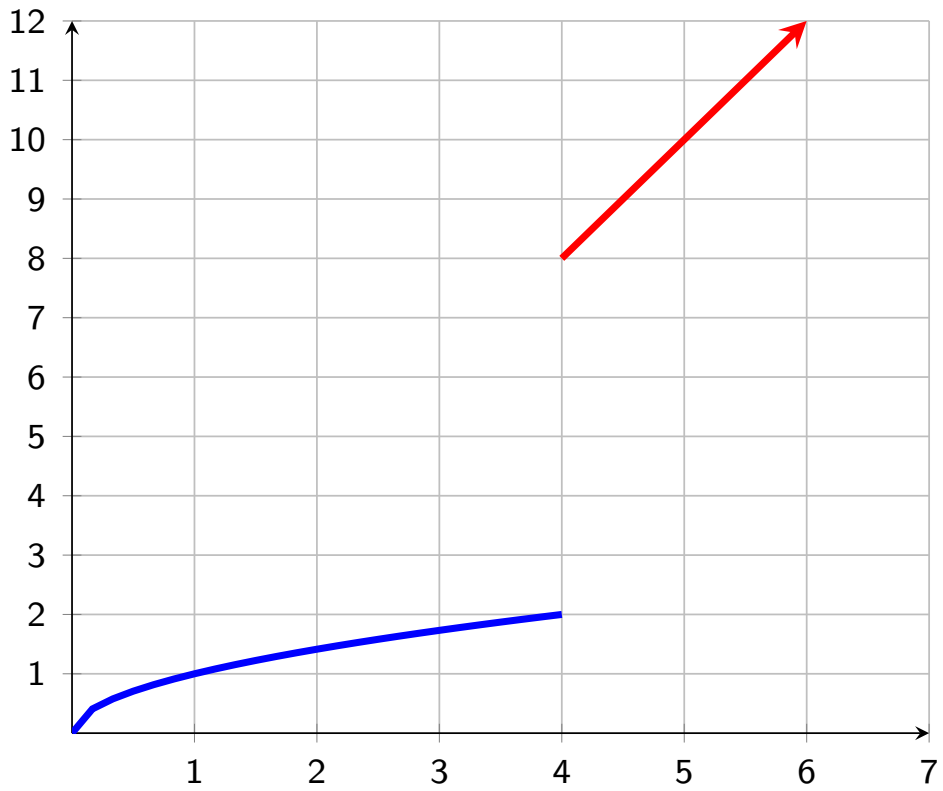

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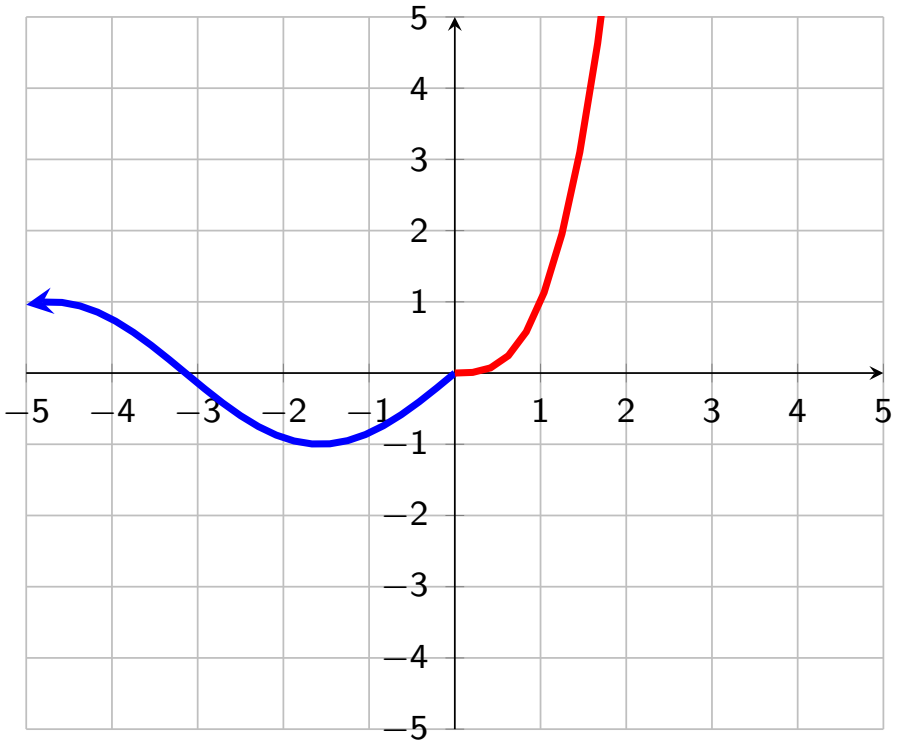

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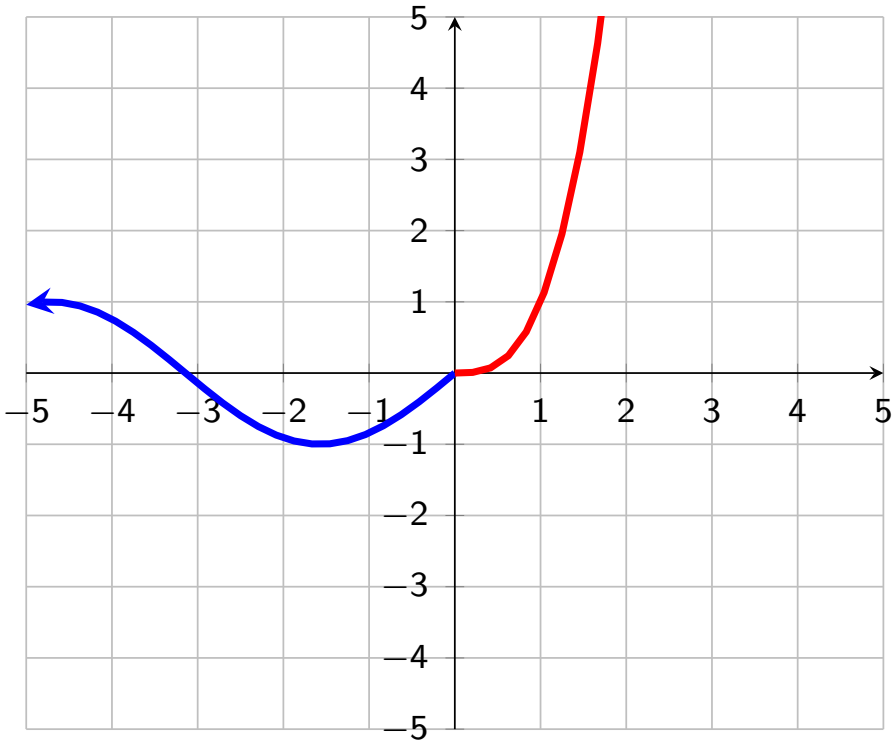

Discontinuous at $x = 0$