Exponential Functions

Objectives

1. Use exponential functions to solve problems

Intro

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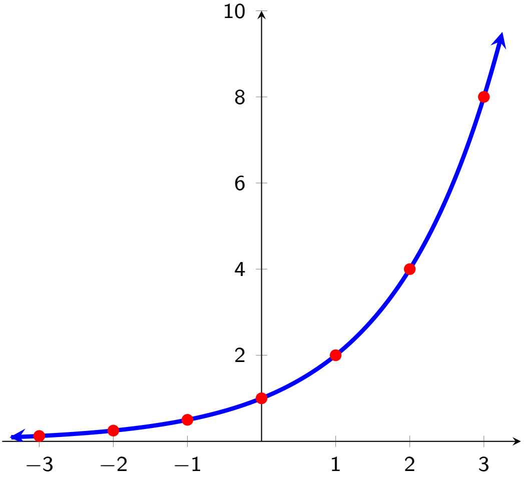
One example of an exponential function is the **doubling function**

$$f(x) = 2^x$$

Doubling Function



x	$f(x)$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

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For instance,

$$2^{50} = 1,125,899,906,842,624$$

Exponential Functions

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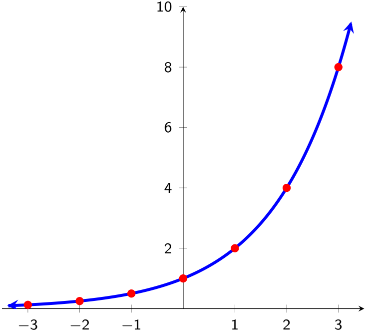
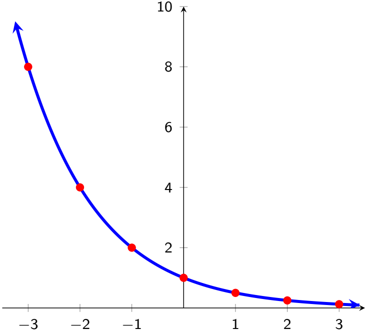
$f(x) = 2^x$



$g(x) = \left(\frac{1}{2}\right)^x$



Properties of Exponential Functions

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- f is one-to-one (has an inverse), continuous, and smooth.

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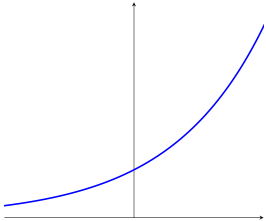

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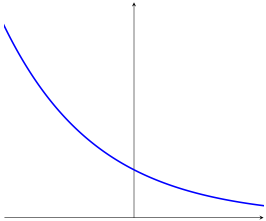

Special Bases

Of all possible bases for exponential functions, the 2 that occur most are base 10 (common base) and irrational base $e \approx 2.718$ (natural base).

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

Example 1

The value of a car can be modeled by $V(x) = 25\left(\frac{4}{5}\right)^x$, where $x \geq 0$ is the age of the car in years and $V(x)$ is the value in thousands of dollars.

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Brand new, the car is valued at \$25,000.

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$V(x) = 25\left(\frac{4}{5}\right)^x$ is a vertical stretch by a factor of 25.

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Over time, the value of the car will approach 0.

Example 2

According to Newton's Law of Cooling, the temperature of coffee T (in degrees Fahrenheit) t minutes after it is served can be modeled by

$$T(t) = 70 + 90e^{-0.1t}$$

(a) Find and interpret $T(0)$.

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The coffee was served at 160°F.

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- Reflection across y-axis (multiplying t by -1)
- Horizontal stretch by factor of 10 (multiplying t by 0.1)
- Vertical stretch by factor of 90
- Shift up 70 degrees Fahrenheit

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Over time, the coffee will cool to a temperature of 70°F.