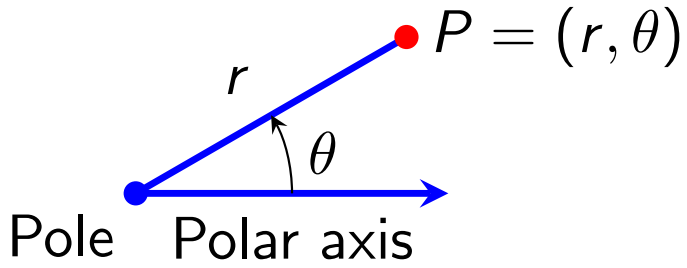
Polar Coordinates

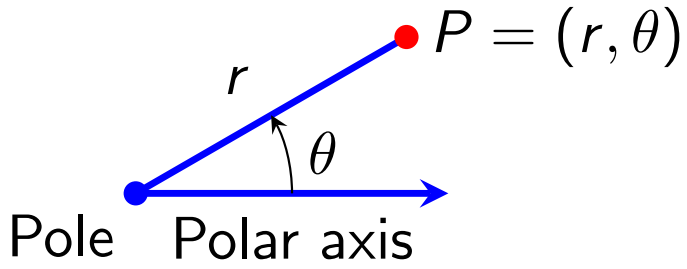
Objectives

1. Plot polar coordinates.
2. Convert from polar to rectangular coordinates.
3. Convert from rectangular to polar coordinates.
4. Convert rectangular and polar equations.

Polar Coordinates



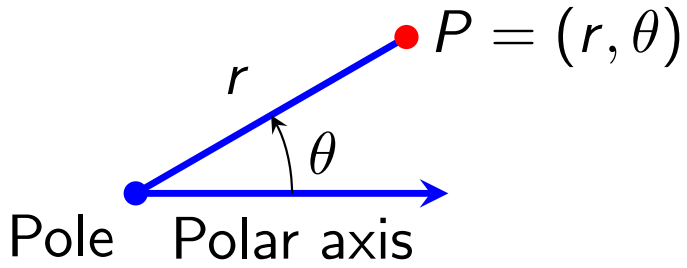
Polar Coordinates



For polar coordinates:

- Start at the origin (pole)

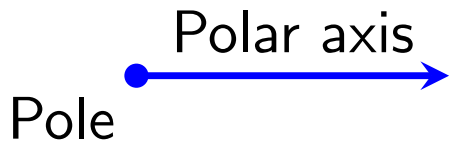
Polar Coordinates

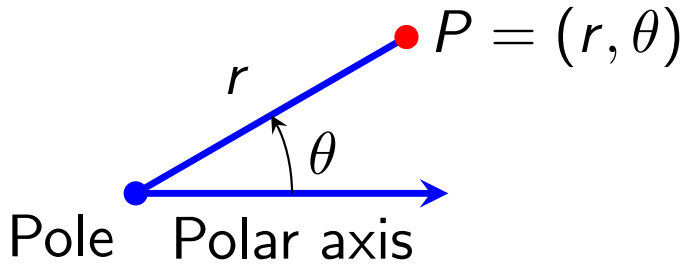


For polar coordinates:

- Start at the origin (pole)
- Go out r units right ($r > 0$) or left ($r < 0$)

Polar Coordinates

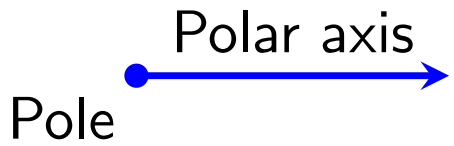


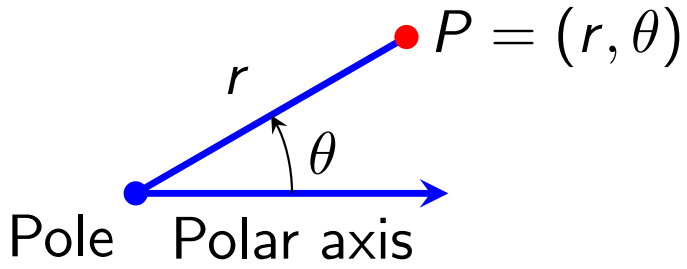


For polar coordinates:

- Start at the origin (pole)
- Go out r units right ($r > 0$) or left ($r < 0$)
- Rotate by the amount given (****direction****)

Polar Coordinates



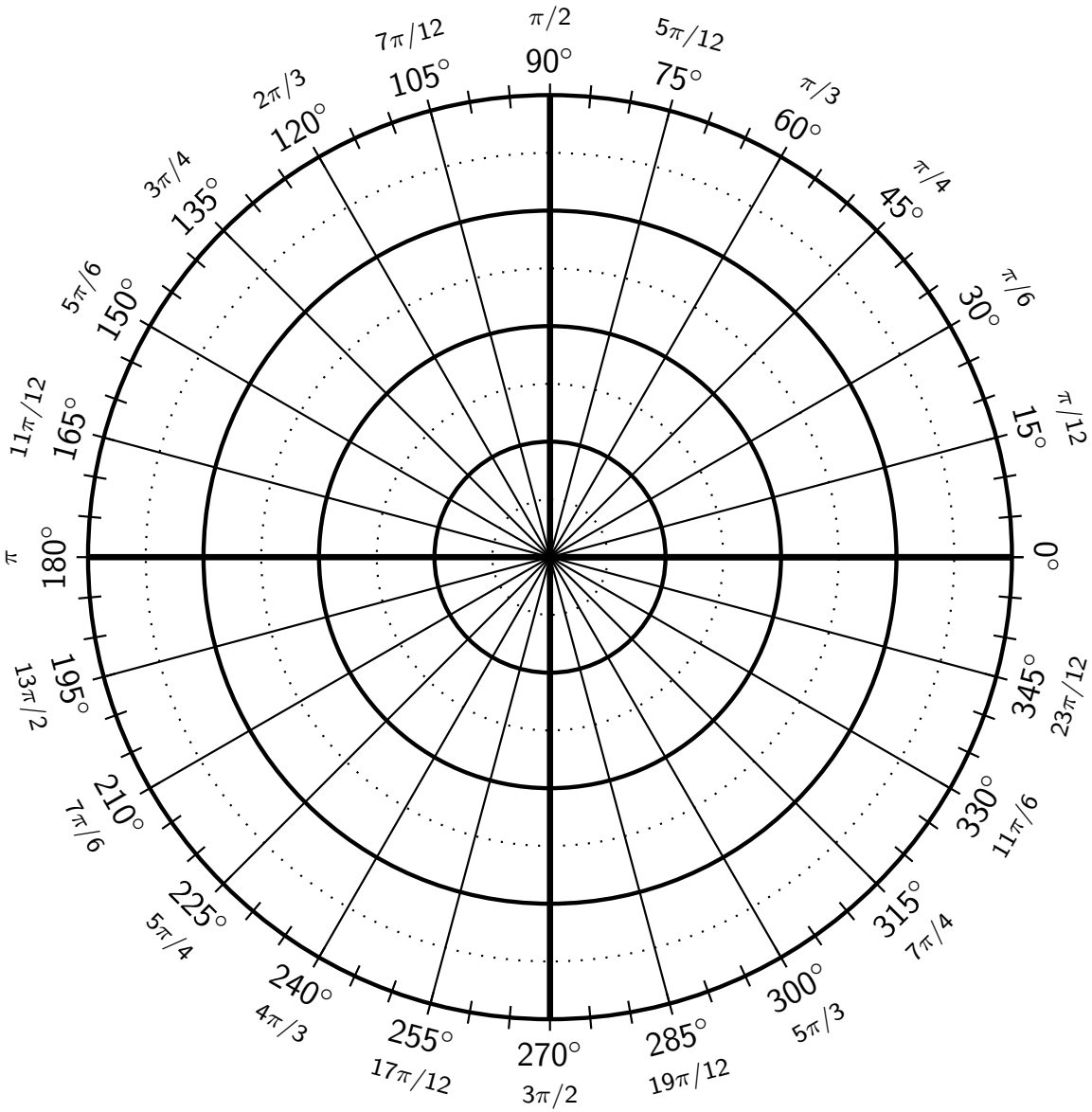


For polar coordinates:

- Start at the origin (pole)
- Go out r units right ($r > 0$) or left ($r < 0$)
- Rotate by the amount given (****direction****)

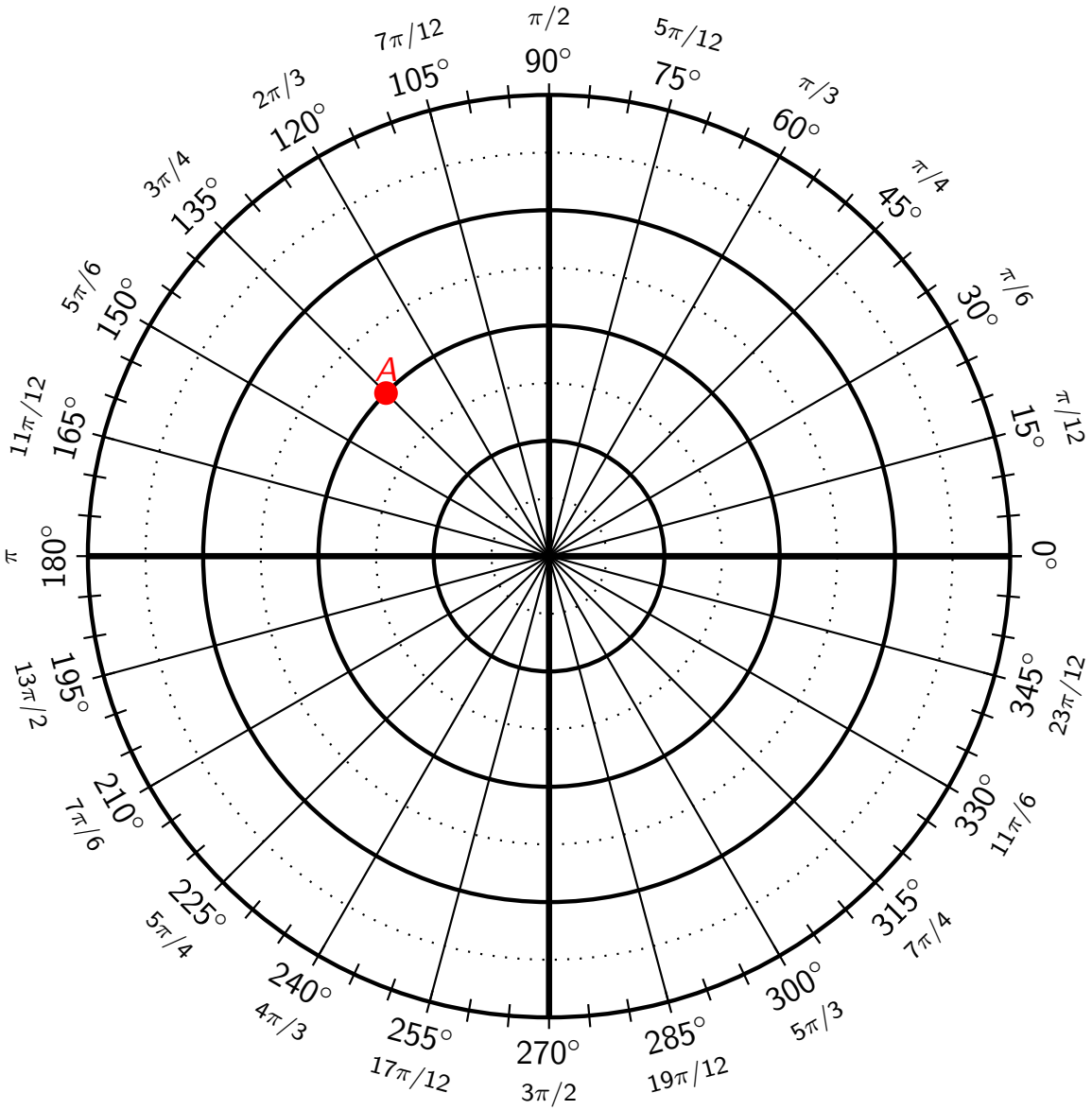
The polar coordinates of a point are (r, θ).

Example 1. Plot each of the following



(a) $A\,(2, 135°)$

Example 1. Plot each of the following



(a) $A\,(2, 135°)$

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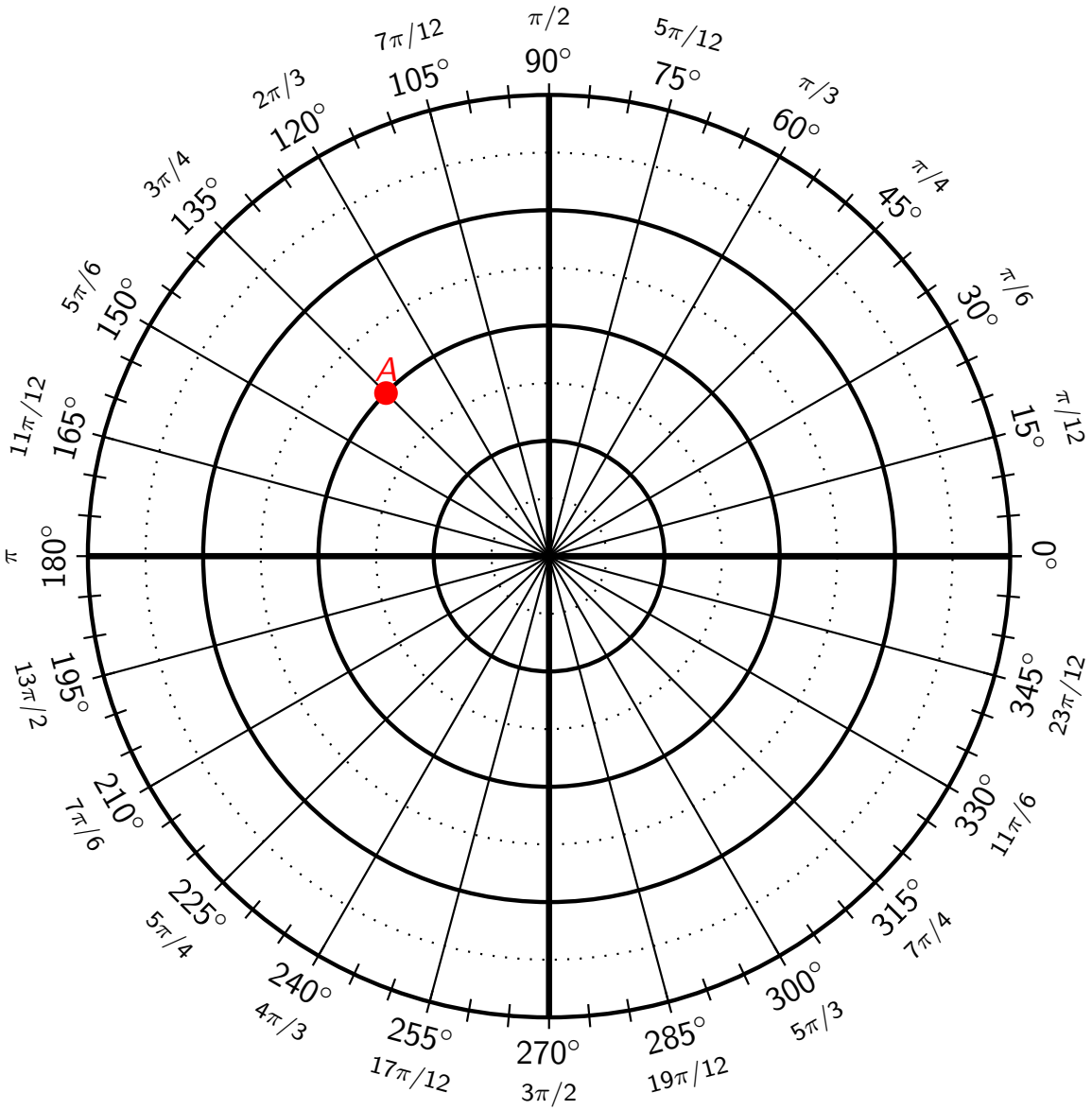
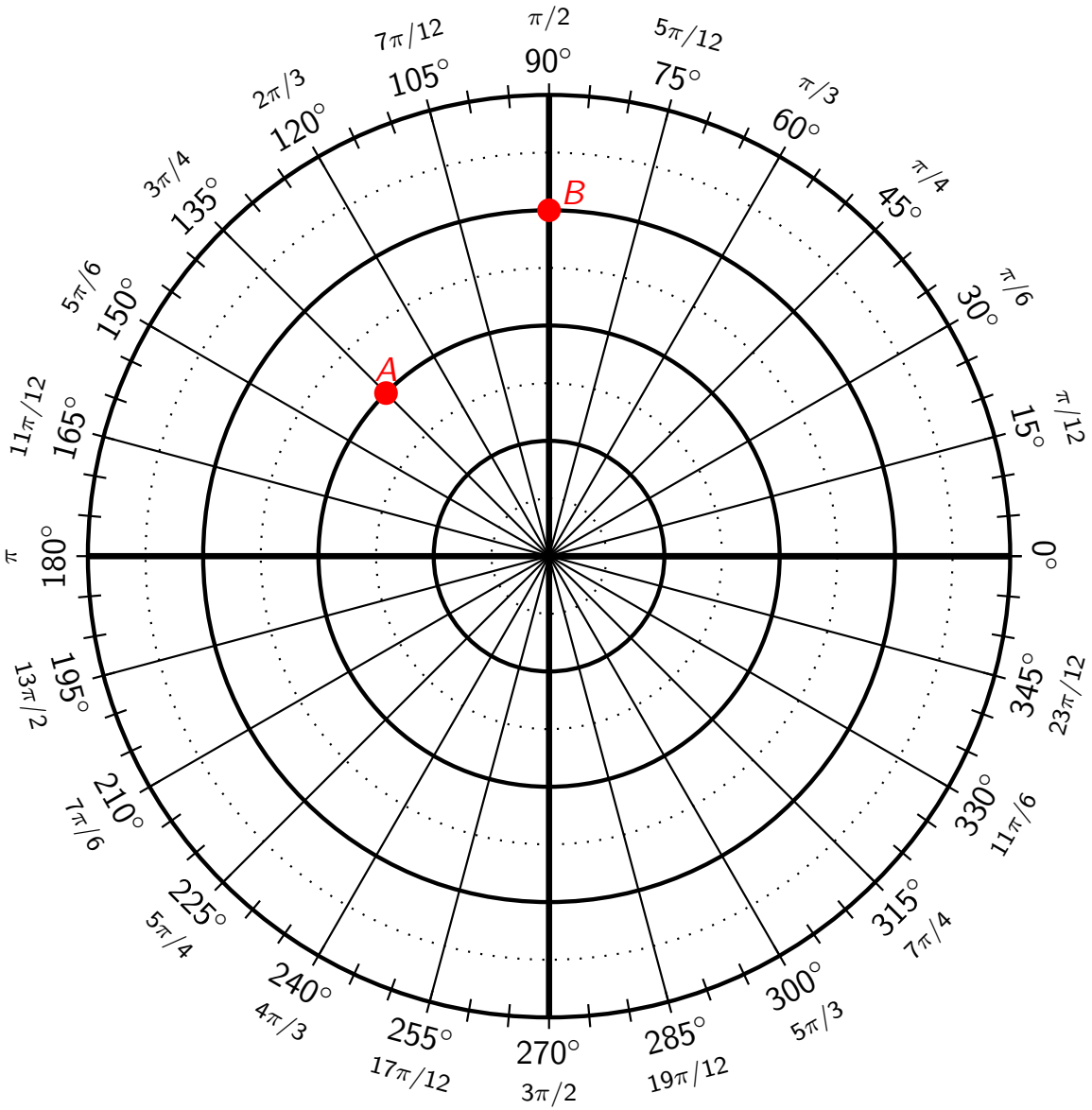


(a) $A\,(2, 135^\circ)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

Example 1. Plot each of the following



(a) $A\,(2, 135°)$

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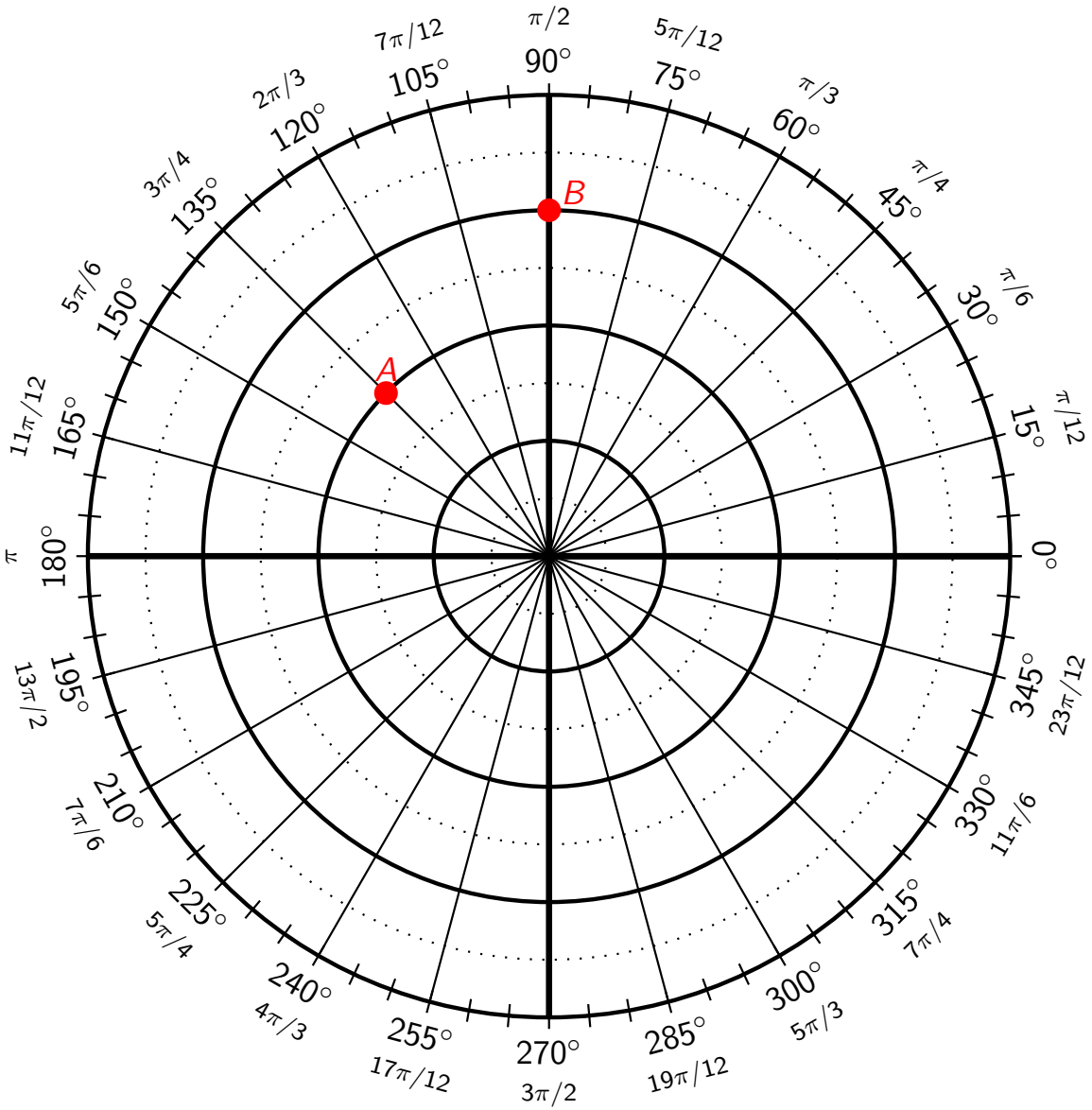
Example 1. Plot each of the following

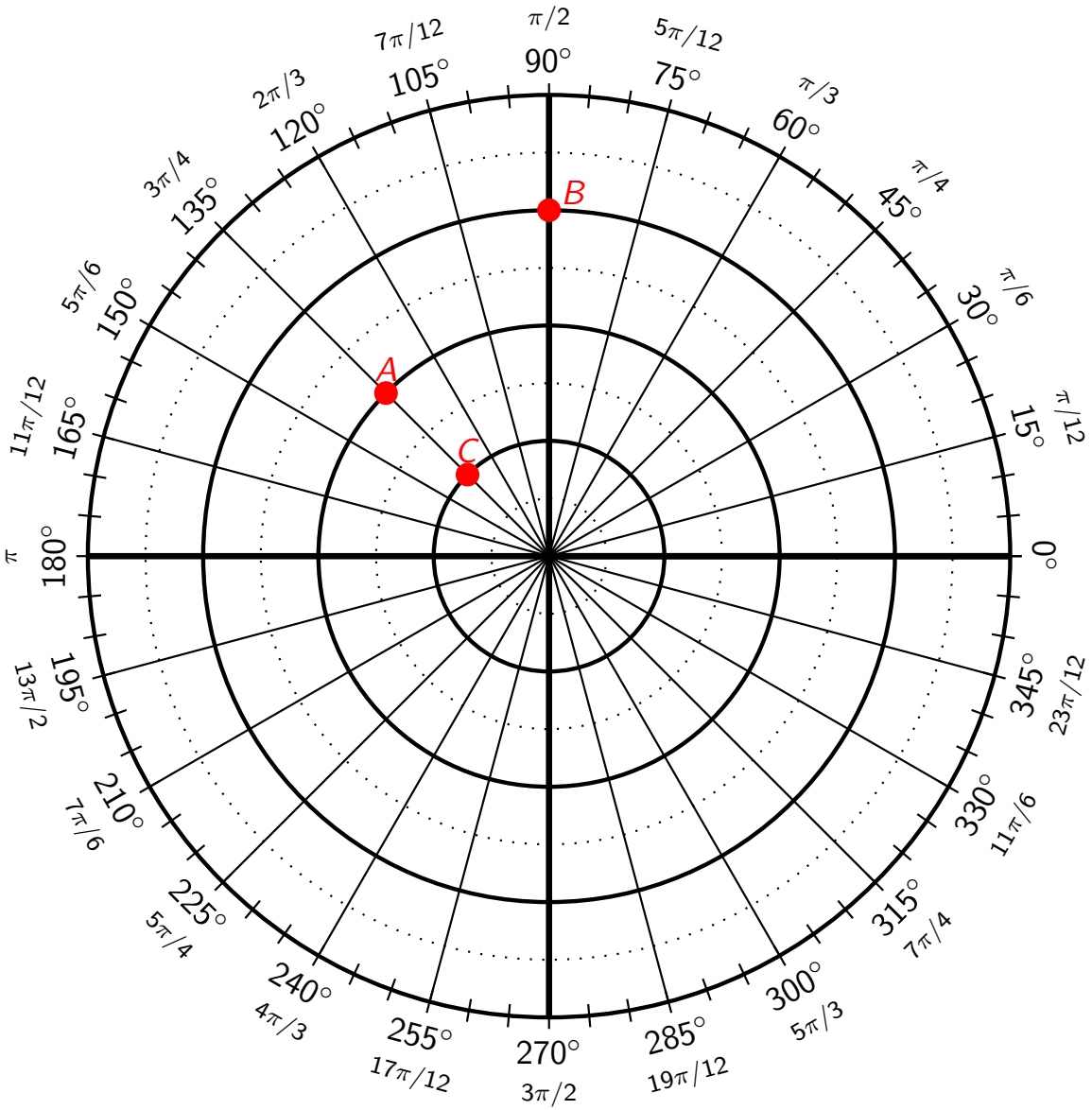


(a) $A\,(2, 135°)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

(c) $C\left(-1, -\frac{\pi}{4}\right)$

Example 1. Plot each of the following



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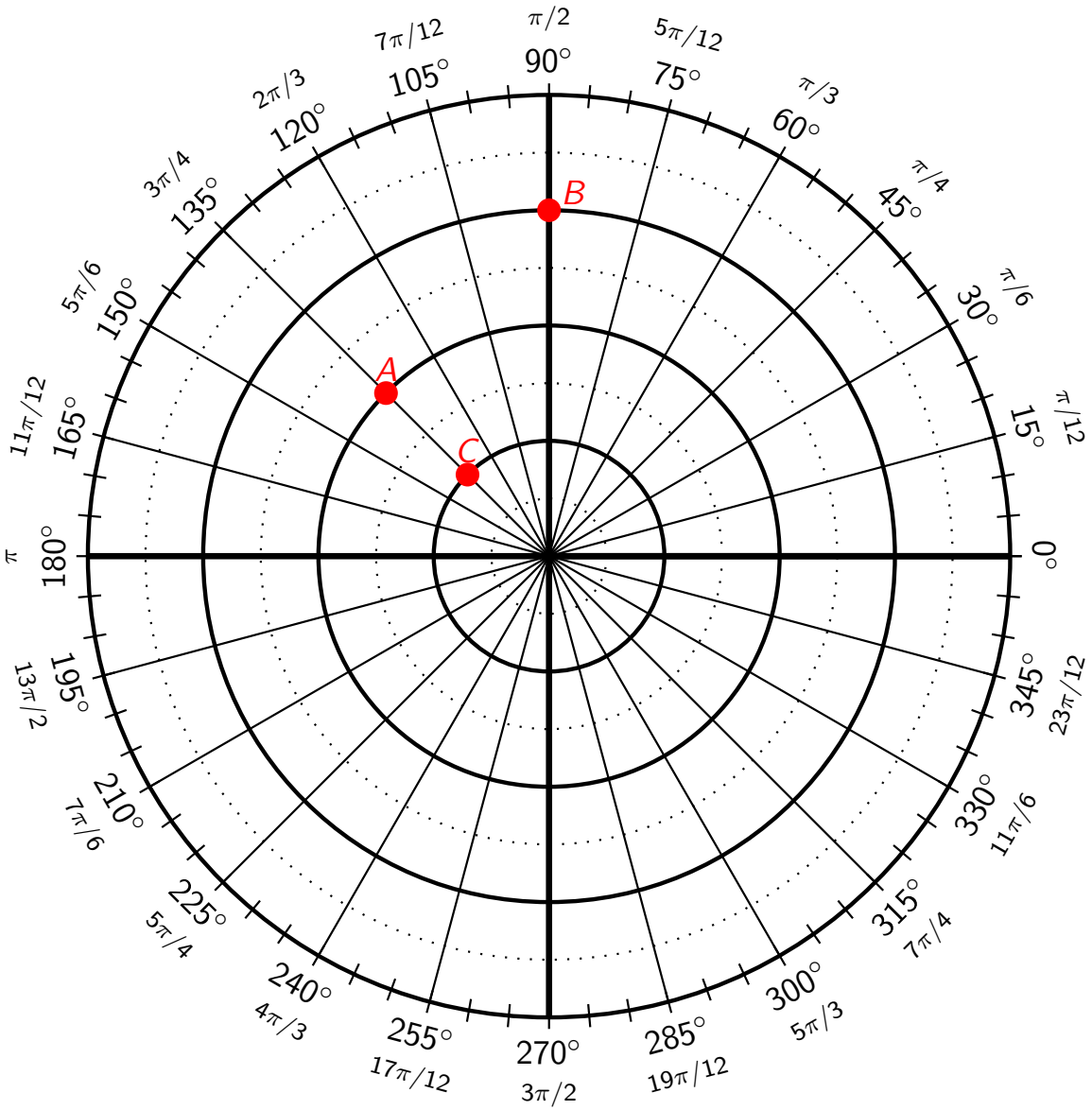


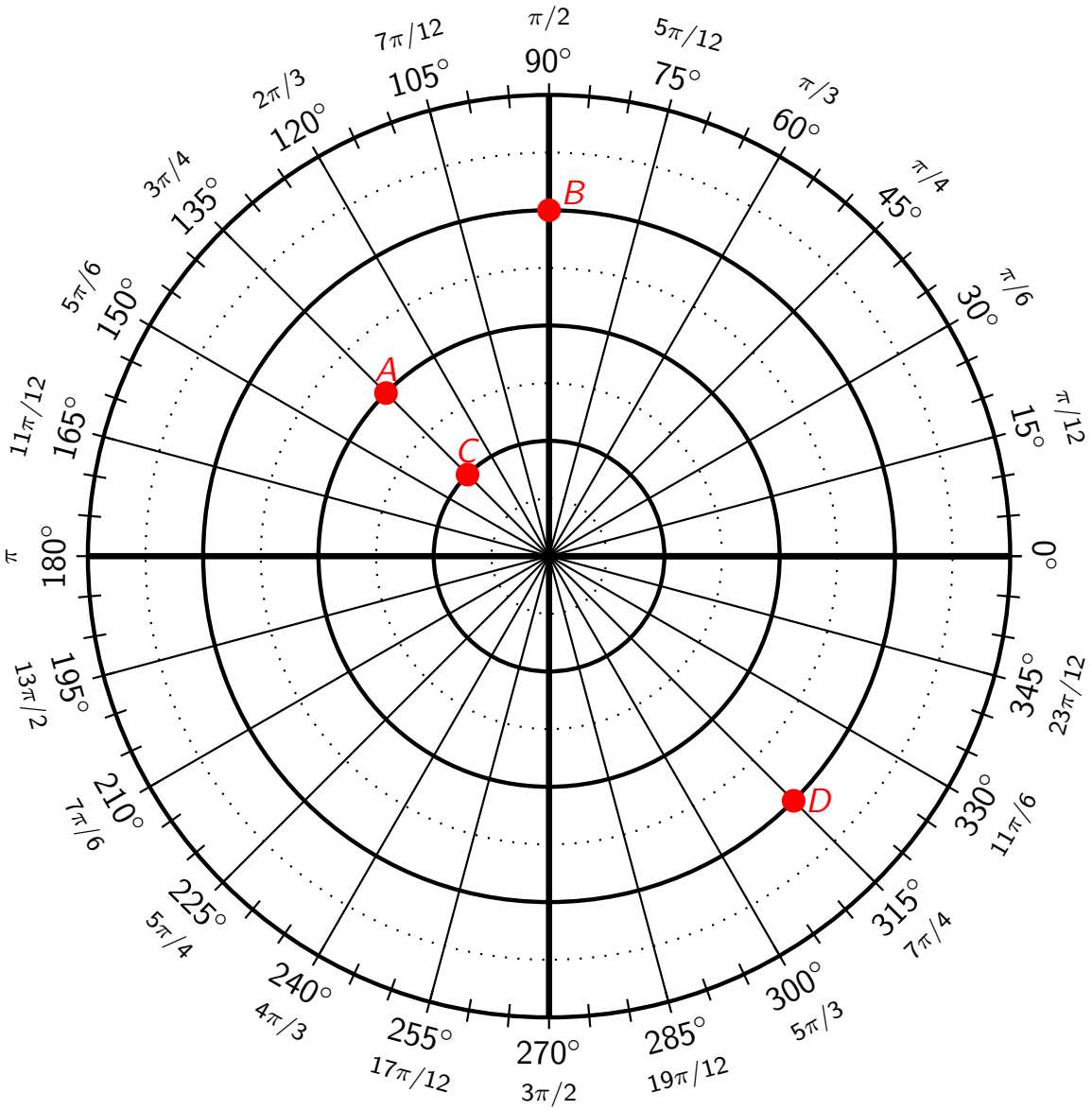
(a) $A\,(2, 135^\circ)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

(c) $C\left(-1, -\frac{\pi}{4}\right)$

(d) $D\,(3, 315^\circ)$

Example 1. Plot each of the following



(a) $A\,(2, 135^\circ)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

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(d) $D\,(3, 315^\circ)$

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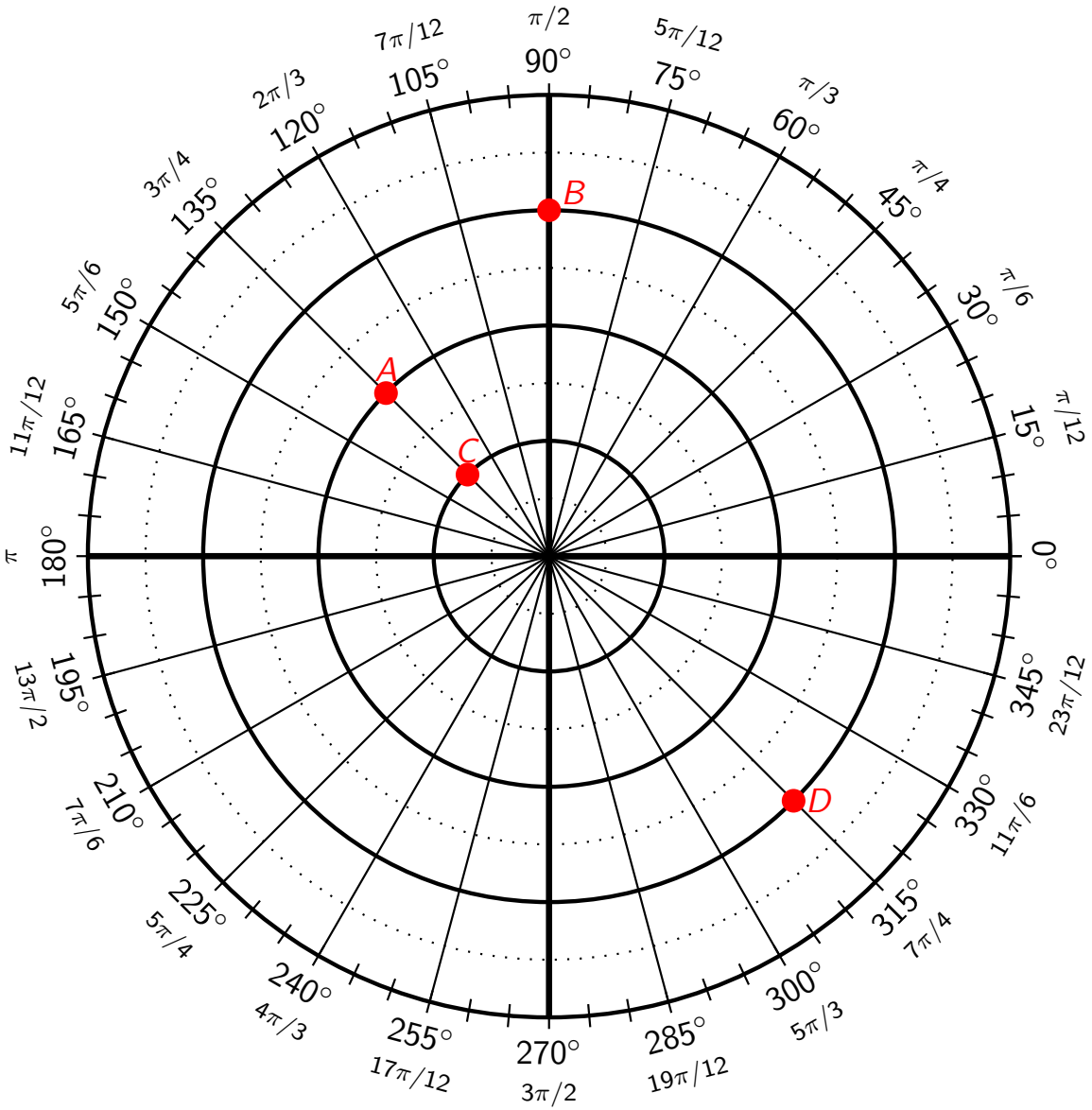

(a) $A\,(2, 135^\circ)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

(c) $C\left(-1, -\frac{\pi}{4}\right)$

(d) $D\,(3, 315^\circ)$

(e) $E\,(2, \pi)$

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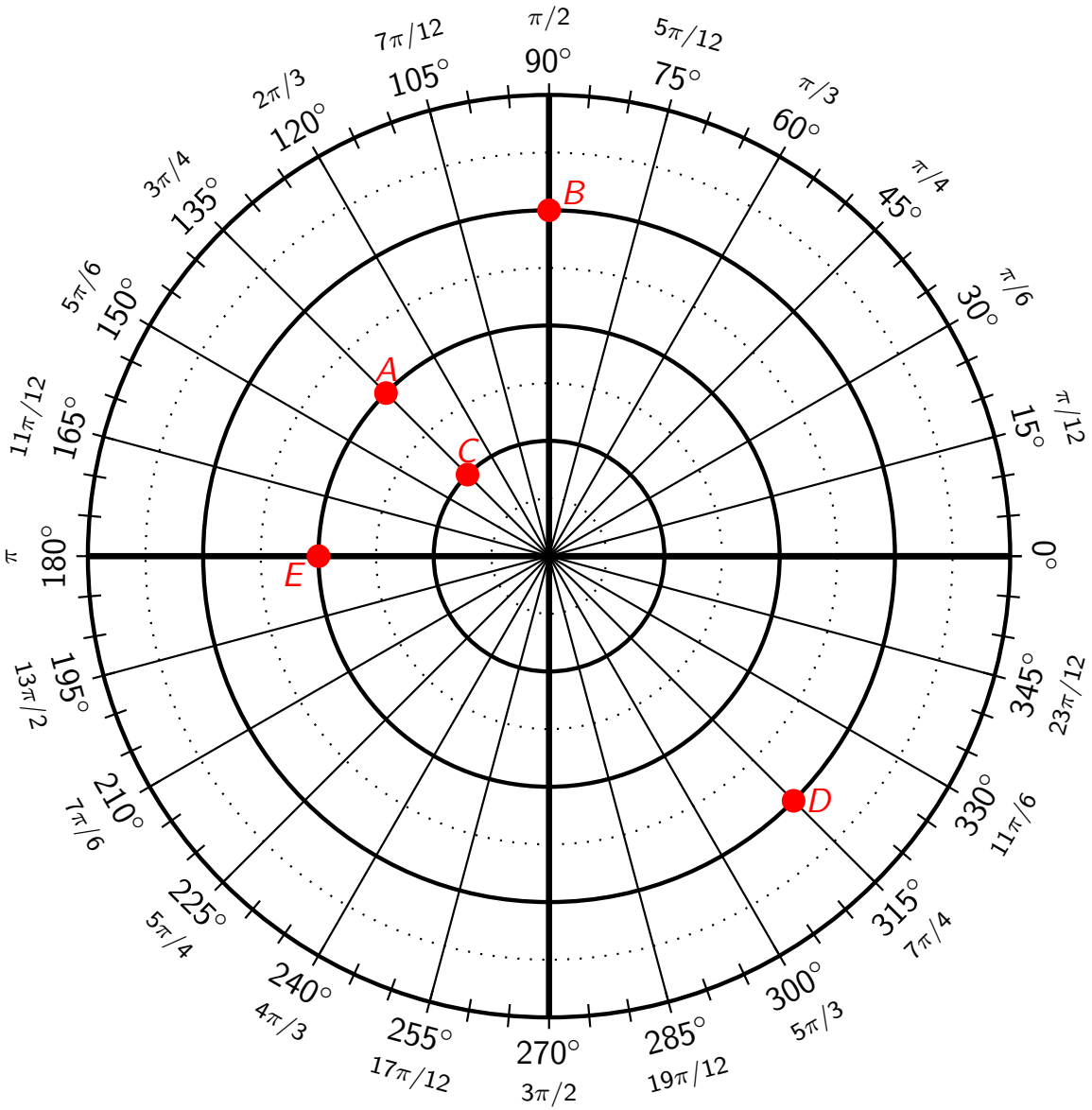

(a) $A\,(2, 135°)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

(c) $C\left(-1, -\frac{\pi}{4}\right)$

(d) $D\,(3, 315°)$

(e) $E\,(2, \pi)$

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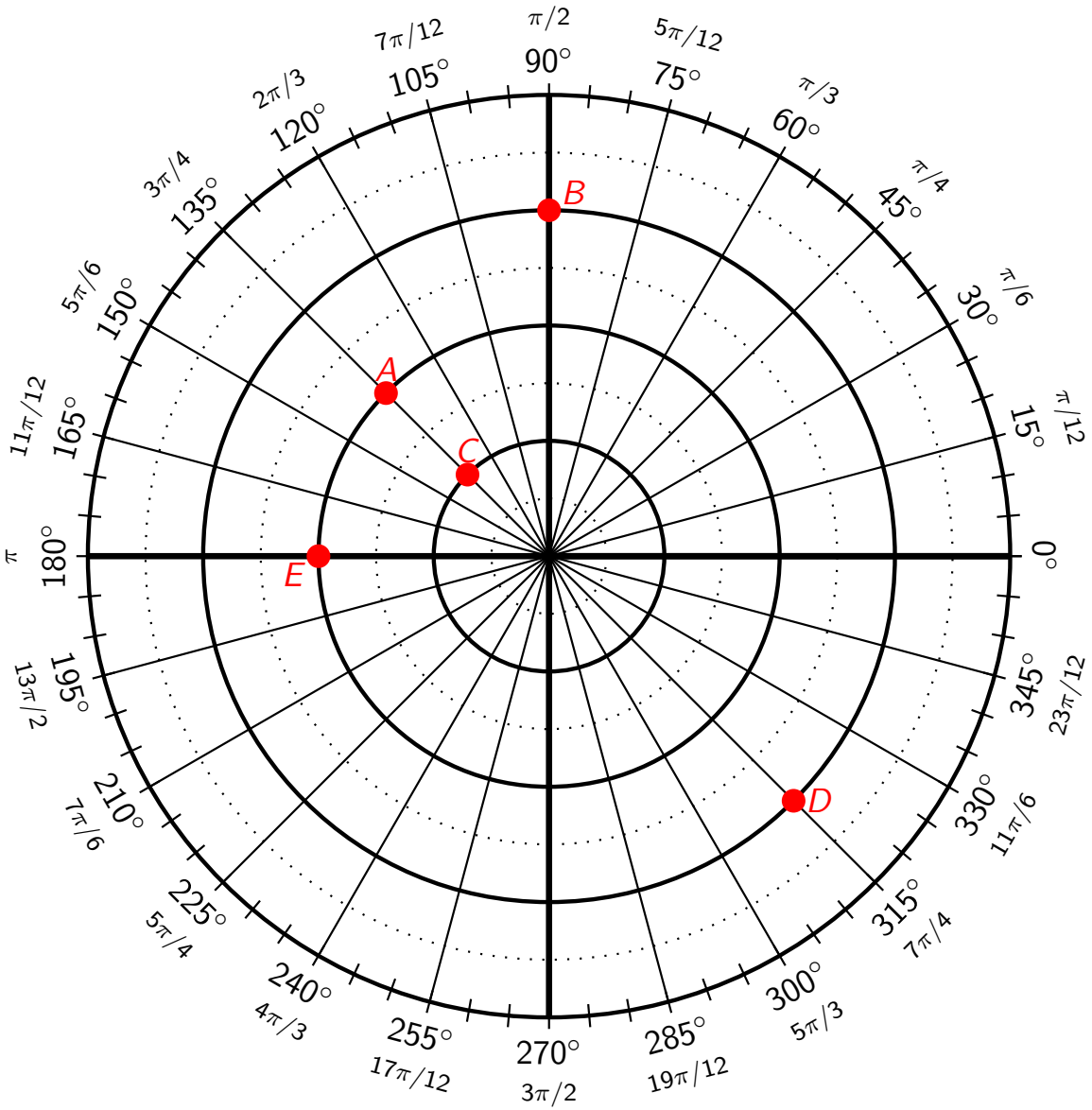


(a) $A\,(2, 135^\circ)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

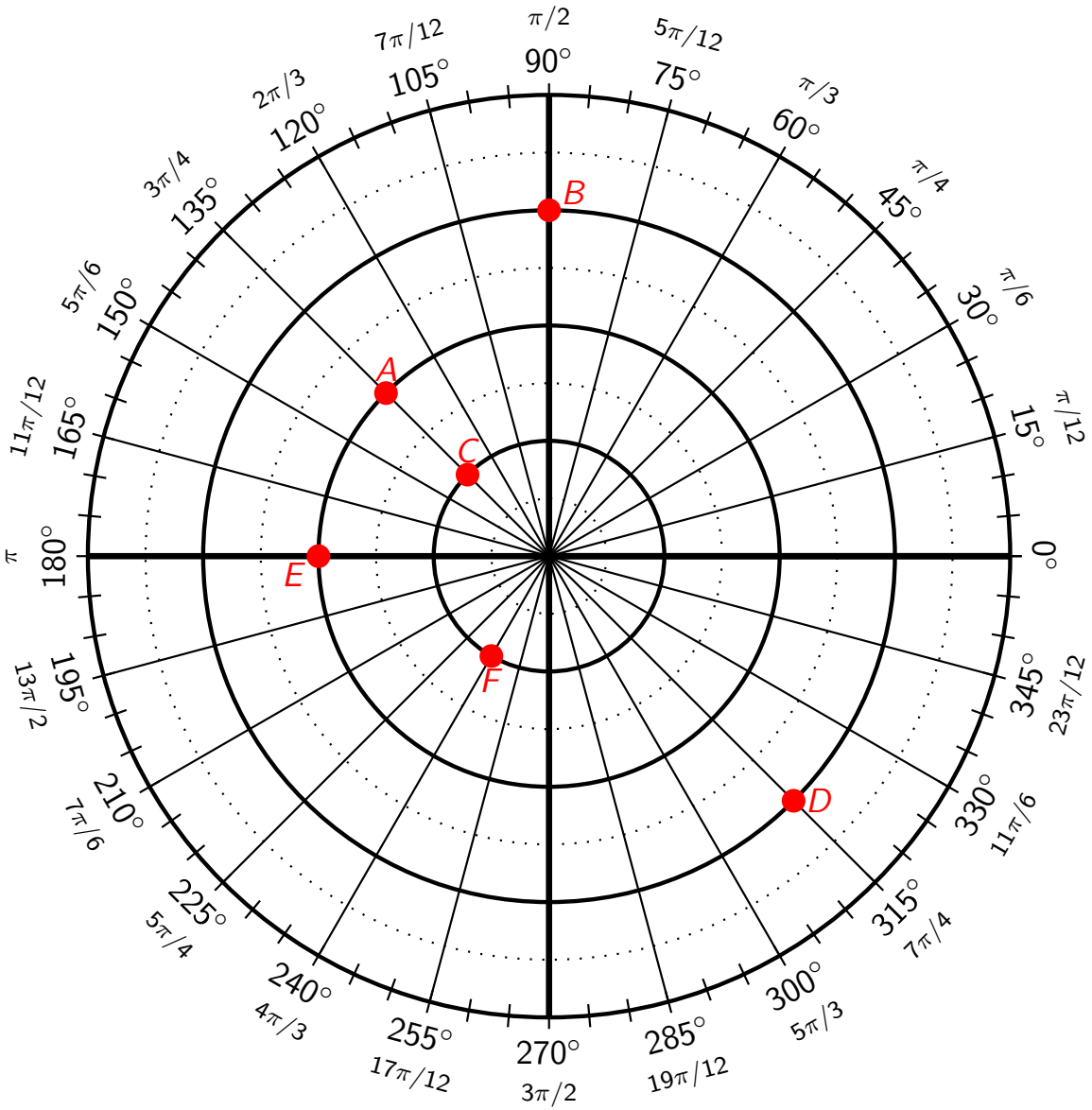
(c) $C\left(-1, -\frac{\pi}{4}\right)$

(d) $D\,(3, 315^\circ)$

(e) $E\,(2, \pi)$

(f) $F\left(-1, \frac{\pi}{3}\right)$

Example 1. Plot each of the following



(a) $A\,(2, 135^\circ)$

(b) $B\left(-3, \frac{3\pi}{2}\right)$

(c) $C\left(-1, -\frac{\pi}{4}\right)$

(d) $D\,(3, 315^\circ)$

(e) $E\,(2, \pi)$

(f) $F\left(-1, \frac{\pi}{3}\right)$

Objectives

1. Plot polar coordinates.
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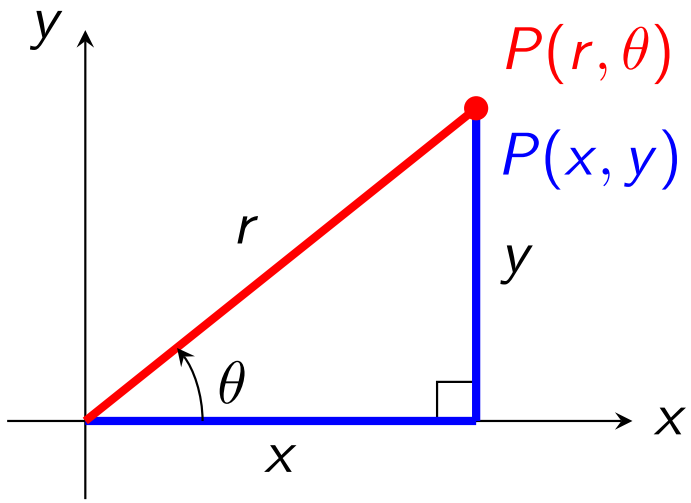
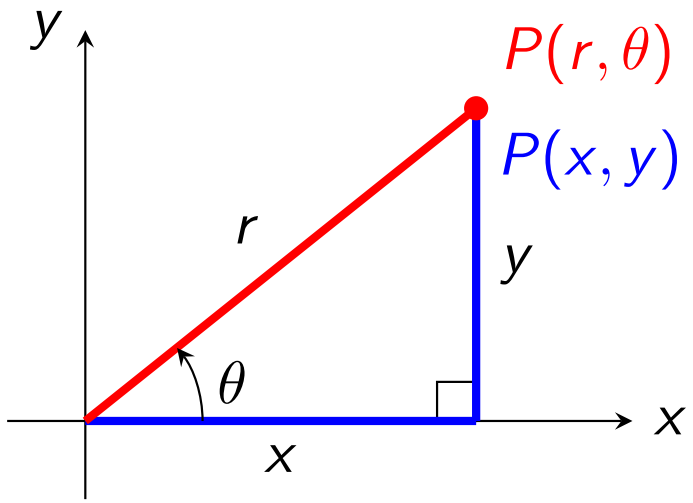
3. Convert from rectangular to polar coordinates.
4. Convert rectangular and polar equations.

Polar to Rectangular Coordinates

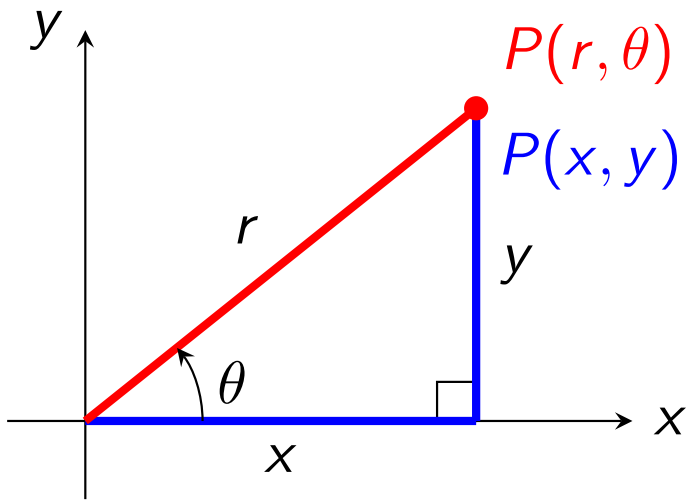


Polar to Rectangular Coordinates



$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$$

Polar to Rectangular Coordinates



$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

Example 2

Convert each to rectangular coordinates.

(a) $(2, 270^\circ)$

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(a) $(2, 270^\circ)$

$$x = 2\cos 270^\circ \qquad y = 2\sin 270^\circ$$

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(a) $(2, 270^\circ)$

$$x = 2\cos 270^\circ \qquad y = 2\sin 270^\circ$$

$$x = 2(0) \qquad y = 2(-1)$$

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Convert each to rectangular coordinates.

(a) $(2, 270^\circ)$

$$x = 2\cos 270^\circ \qquad y = 2\sin 270^\circ$$

$$x = 2(0) \qquad y = 2(-1)$$

$$x = 0 \quad y = -2$$

Example 2

Convert each to rectangular coordinates.

(a) $(2, 270^\circ)$

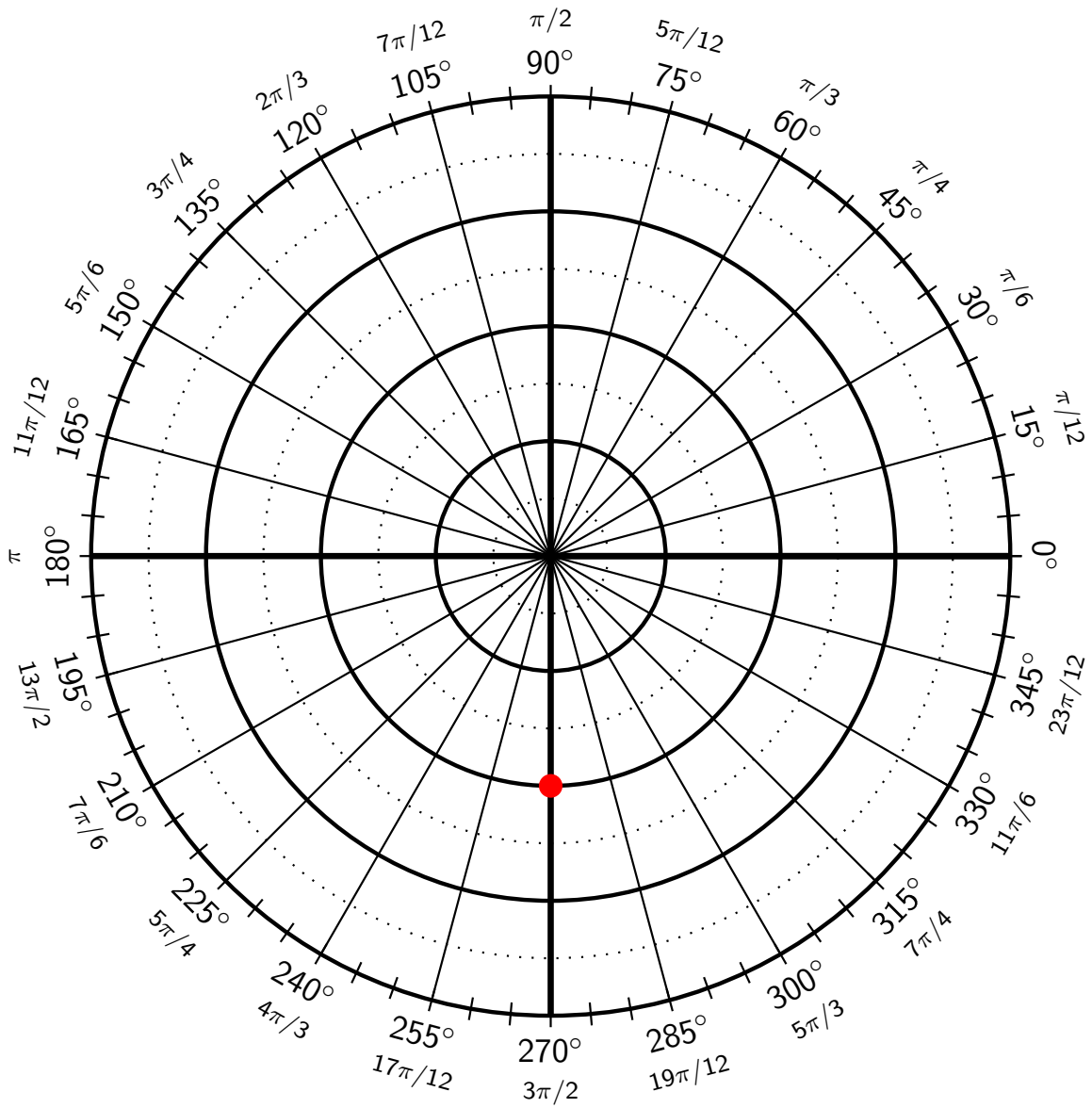
$$x = 2\cos 270^\circ \qquad y = 2\sin 270^\circ$$

$$x = 2(0) \qquad y = 2(-1)$$

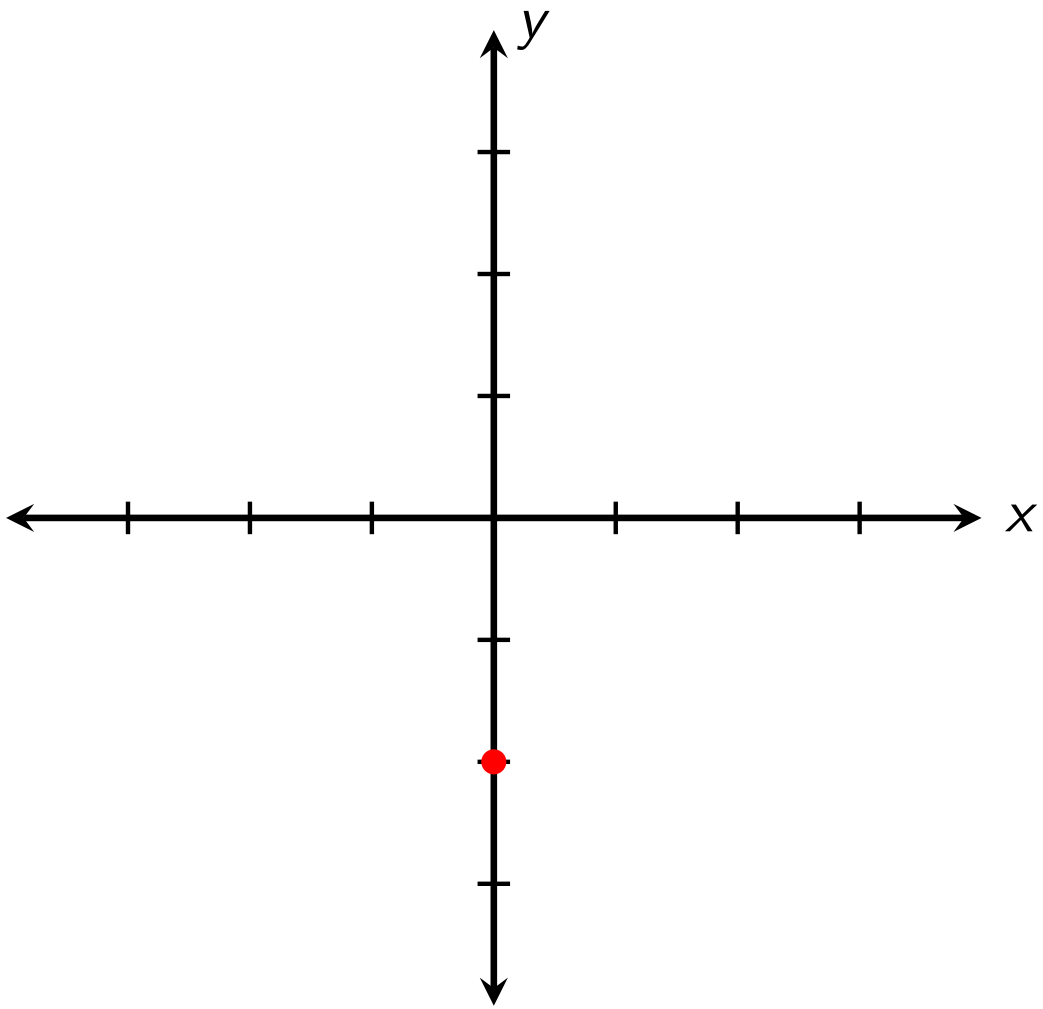
$$x = 0 \quad y = -2$$

$$(0, 2)$$

Example 2


π/2
90°
5π/12
75°
π/3
60°
π/4
45°
π/6
30°
π/12
15°
0°
23π/12
345°
11π/6
330°
7π/4
315°
5π/3
300°
19π/12
285°
3π/2
270°
17π/12
255°
4π/3
240°
5π/4
225°
7π/6
210°
13π/12
195°
π
180°
11π/12
165°
5π/6
150°
3π/4
135°
2π/3
120°
7π/12
105°

Example 2



Example 2

(b) $\left(-8, \frac{\pi}{3}\right)$

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$$x = -8\cos\left(\frac{\pi}{3}\right) \qquad y = -8\sin\left(\frac{\pi}{3}\right)$$

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$$x = -8\cos\left(\frac{\pi}{3}\right) \qquad y = -8\sin\left(\frac{\pi}{3}\right)$$

$$x = -8\left(\frac{1}{2}\right) \qquad y = -8\left(\frac{\sqrt{3}}{2}\right)$$

Example 2

(b) $\left(-8, \frac{\pi}{3}\right)$

$$x = -8\cos\left(\frac{\pi}{3}\right) \qquad y = -8\sin\left(\frac{\pi}{3}\right)$$

$$x = -8\left(\frac{1}{2}\right) \qquad y = -8\left(\frac{\sqrt{3}}{2}\right)$$

$$x = -4 \qquad y = -4\sqrt{3}$$

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(b) $\left(-8, \frac{\pi}{3}\right)$

$$x = -8\cos\left(\frac{\pi}{3}\right) \qquad y = -8\sin\left(\frac{\pi}{3}\right)$$

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$$\left(-4, -4\sqrt{3}\right)$$

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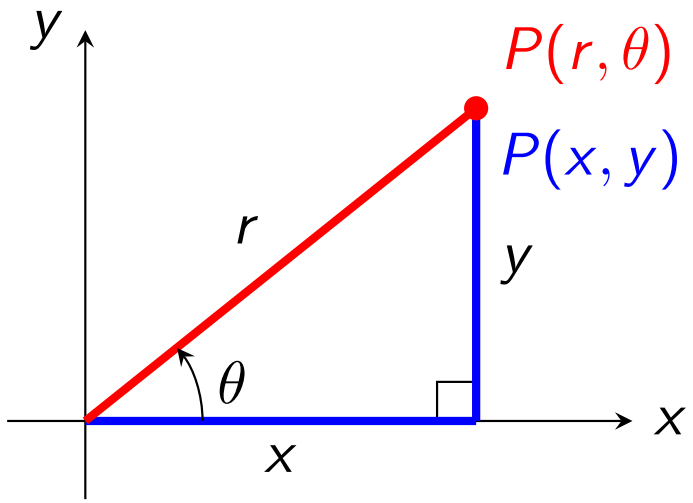
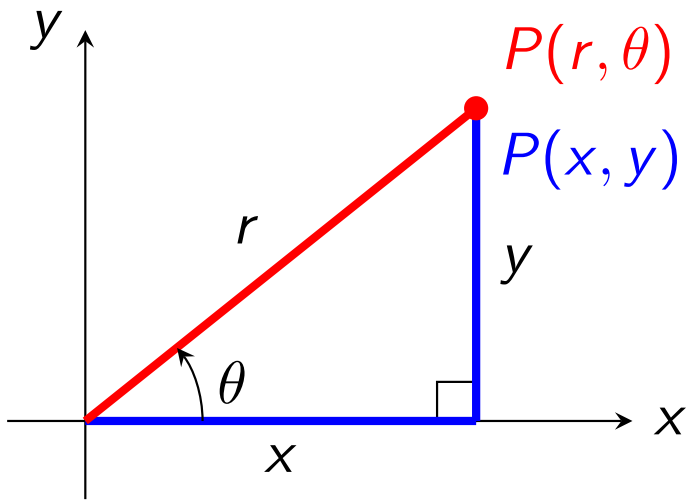
Rectangular to Polar

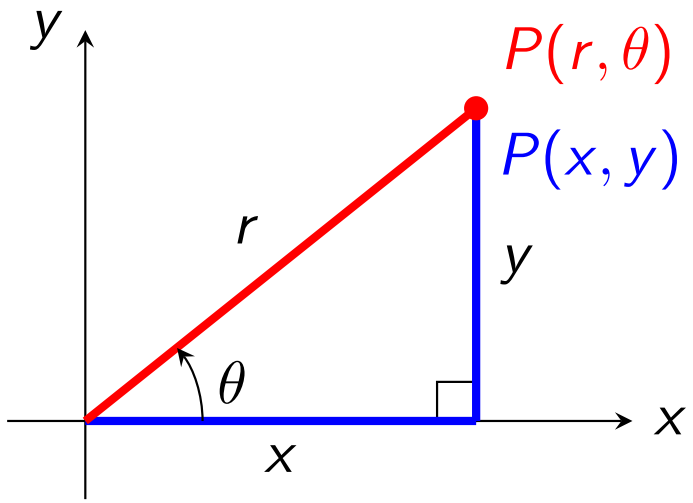


Rectangular to Polar



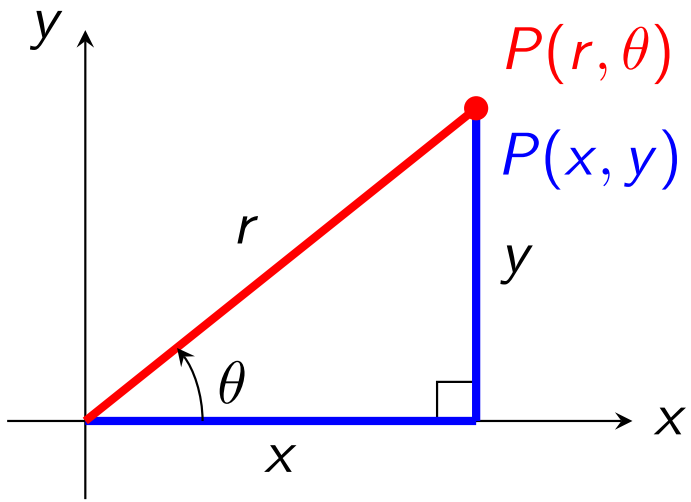
$$r = \sqrt{x^2 + y^2}$$

Rectangular to Polar



$$r = \sqrt{x^2 + y^2} \qquad \theta' = \tan^{-1}\left|\frac{y}{x}\right|$$

Rectangular to Polar



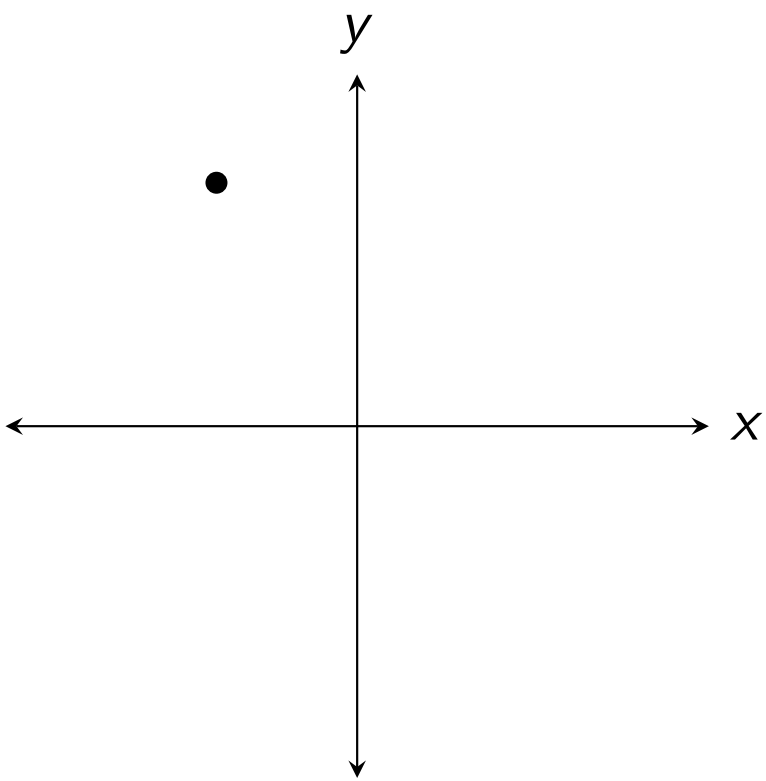
$$r = \sqrt{x^2 + y^2} \qquad \theta' = \tan^{-1}\left|\frac{y}{x}\right|$$

where θ' is the reference angle used to find the total angle rotated, θ.

Example 3

Convert each of the following to polar coordinates.

(a) $\left(-1, \sqrt{3}\right)$



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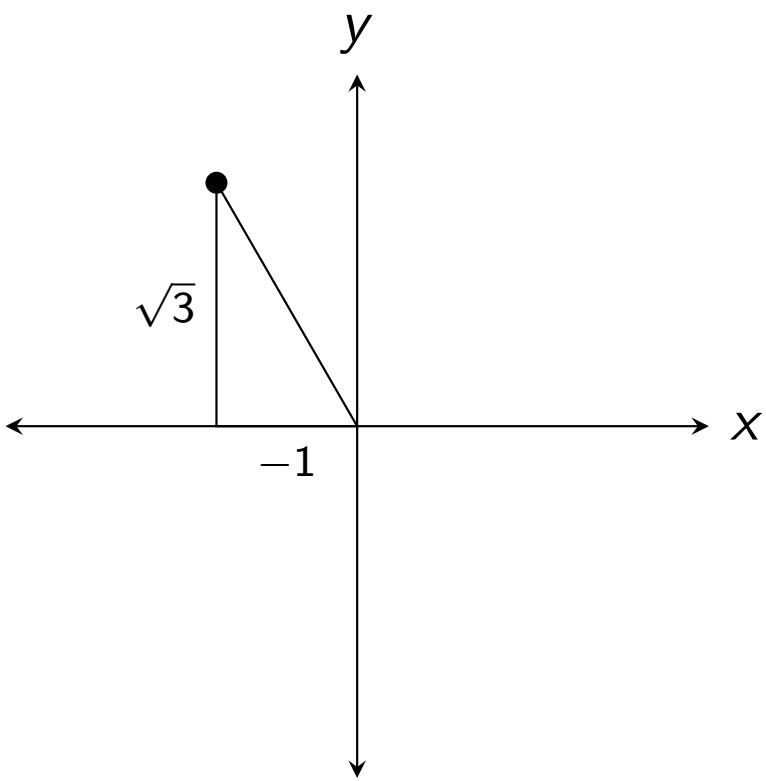

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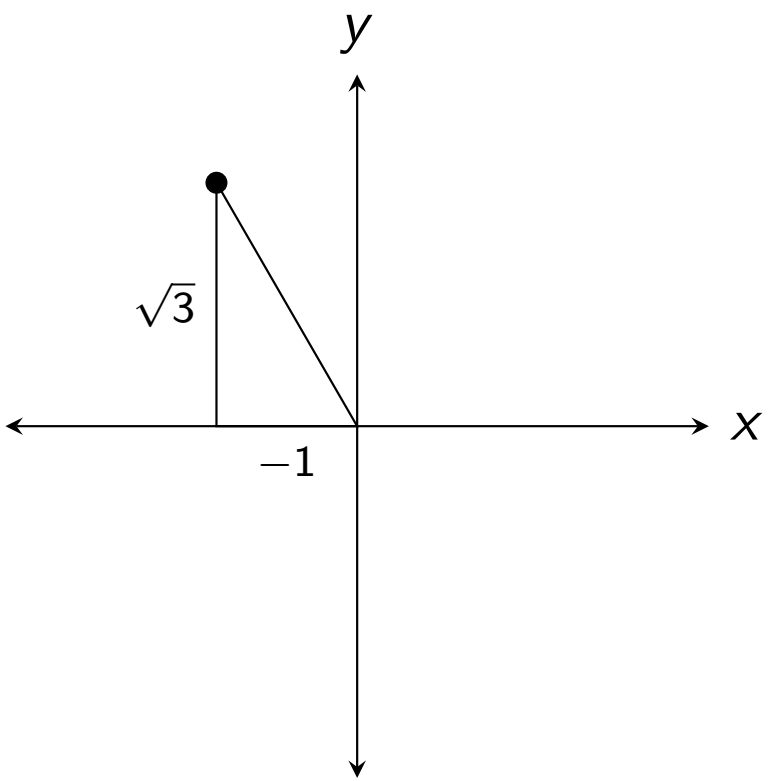
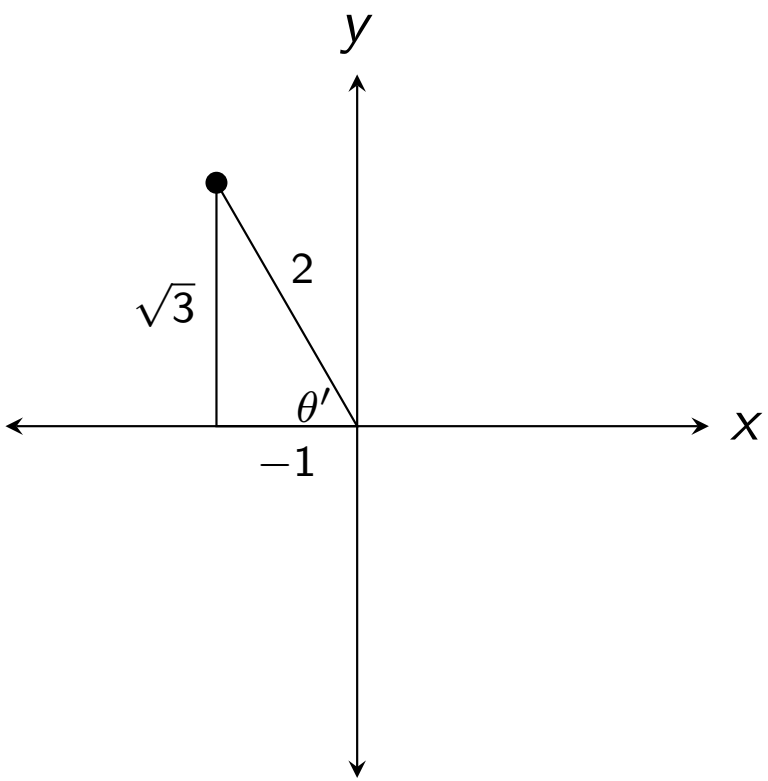


$$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

Example 3

Convert each of the following to polar coordinates.

(a) $(-1, \sqrt{3})$



$$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

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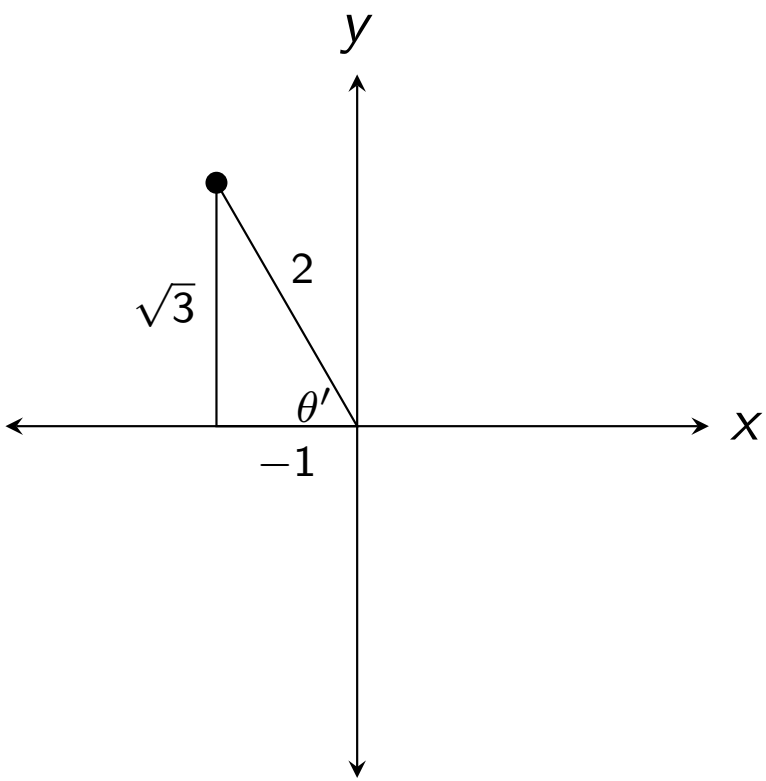
Convert each of the following to polar coordinates.

(a) $(-1, \sqrt{3})$



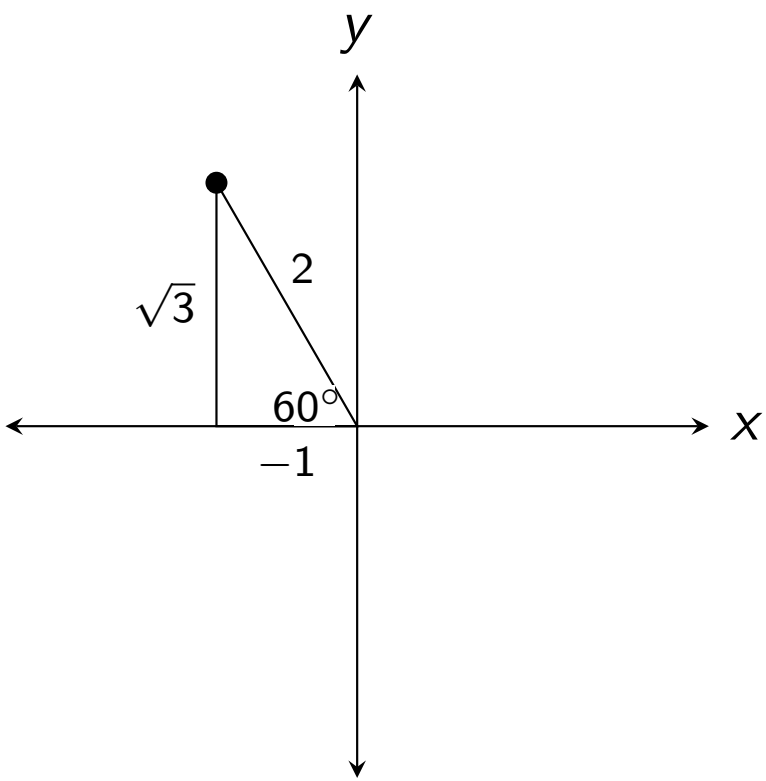
$$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\theta' = \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right| = 60^\circ$$

Example 3

Convert each of the following to polar coordinates.

(a) $(-1, \sqrt{3})$



$$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

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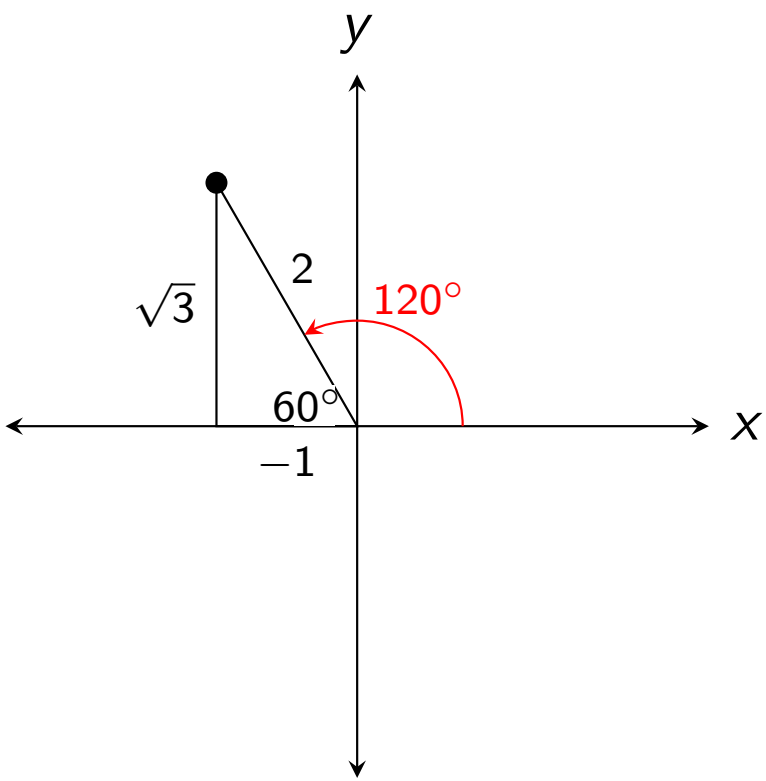

$$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\theta' = \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right| = 60^\circ$$

$$\theta = 120^\circ$$

Example 3

Convert each of the following to polar coordinates.

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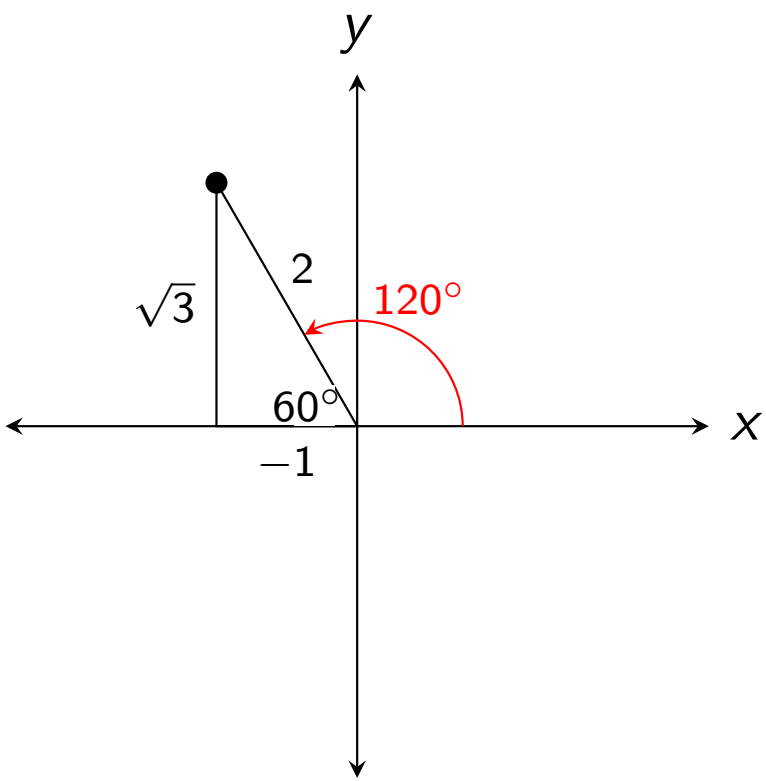


$$r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

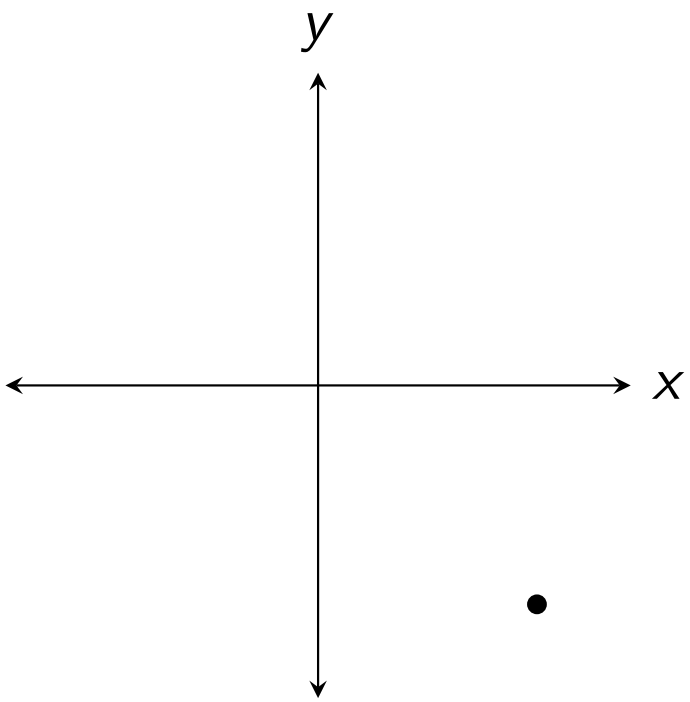
$$\theta' = \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right| = 60^\circ$$

$$\theta = 120^\circ$$

$$(2, 120^\circ)$$

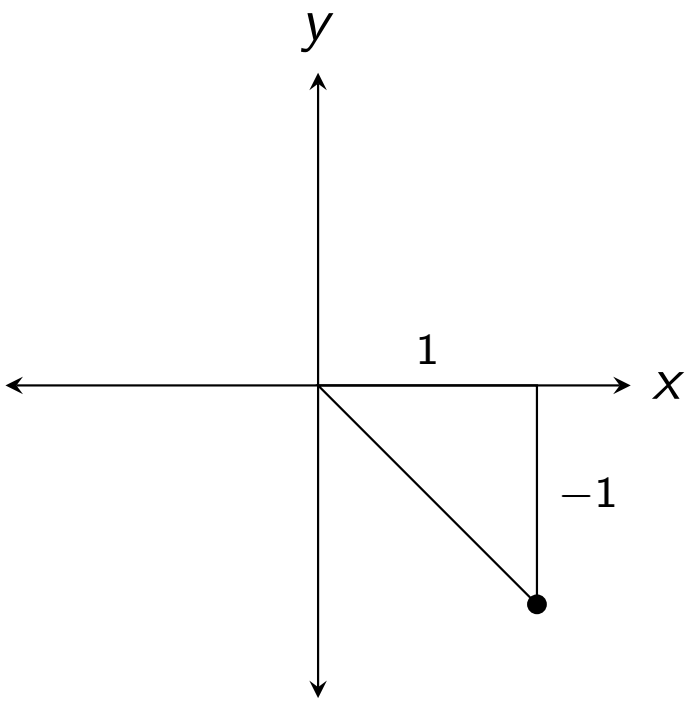
Example 3

(b) $(1, -1)$



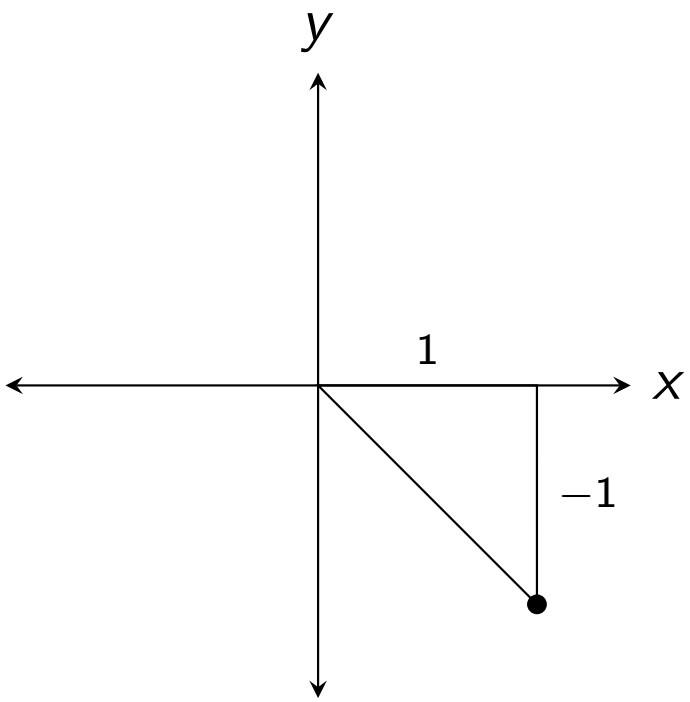
Example 3

(b) $(1, -1)$



Example 3

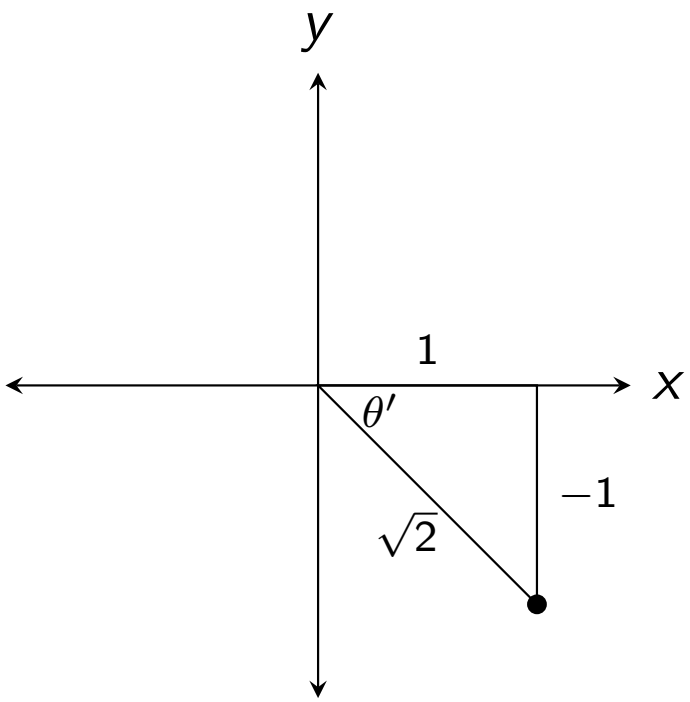
(b) $(1, -1)$



$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Example 3

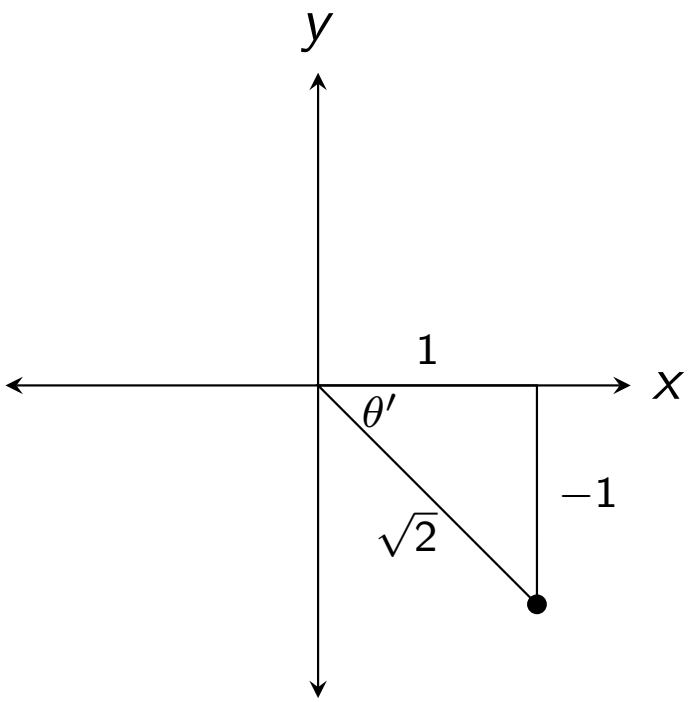
(b) $(1, -1)$



$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Example 3

(b) $(1, -1)$



$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta' = \tan^{-1}\left|\frac{-1}{1}\right| = 45^\circ$$

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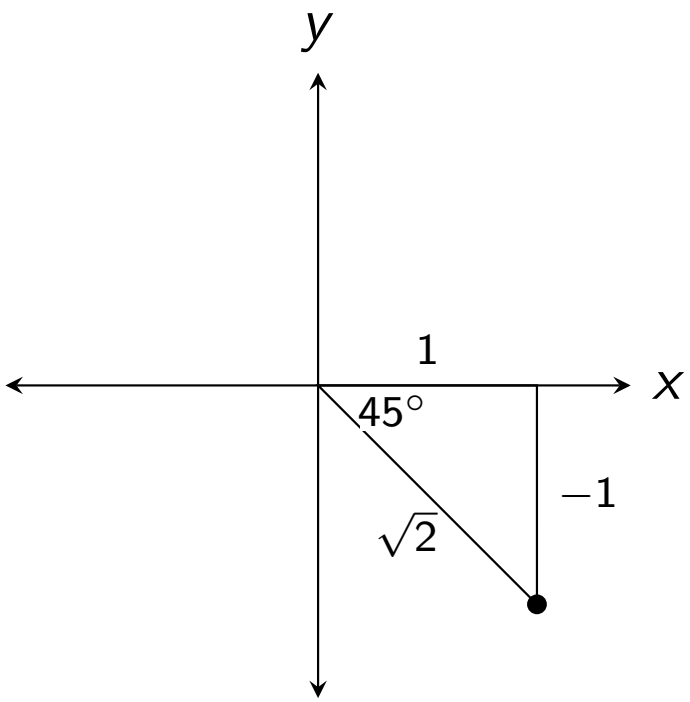

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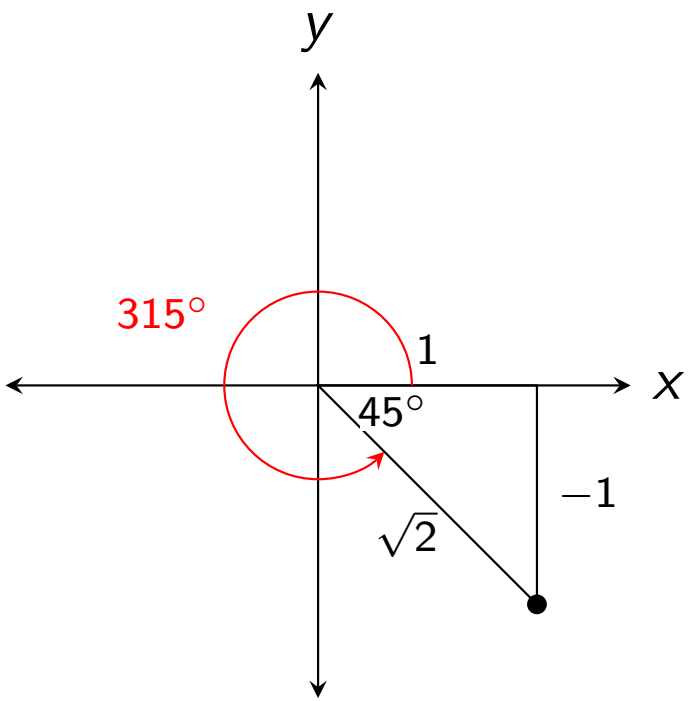


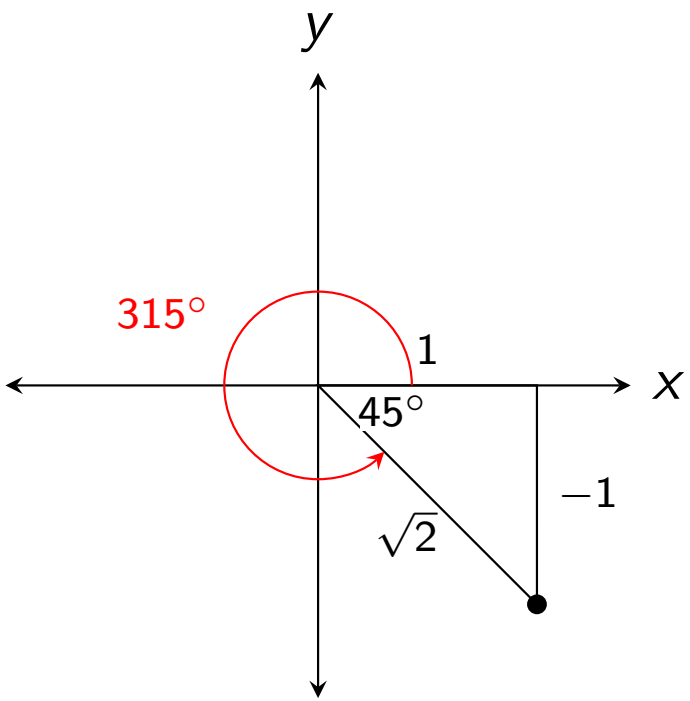
$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta' = \tan^{-1}\left|\frac{-1}{1}\right| = 45^\circ$$

$$\theta = 315^\circ$$

Example 3

(b) $(1, -1)$



$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta' = \tan^{-1}\left|\frac{-1}{1}\right| = 45^\circ$$

$$\theta = 315^\circ$$

$$\left(\sqrt{2}, 315^\circ\right)$$

Objectives

1. Plot polar coordinates.
2. Convert from polar to rectangular coordinates.
3. Convert from rectangular to polar coordinates.
4. Convert rectangular and polar equations.

Rectangular and Polar Equations

We can use the relationship between rectangular and polar coordinates to convert equations of one form to the other.

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$$x = r\cos\theta$$

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Rectangular and Polar Equations

We can use the relationship between rectangular and polar coordinates to convert equations of one form to the other.

$$x = r\cos\theta \qquad x^2 + y^2 = r^2$$

$$y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$$

Example 4

Convert each to polar equations.

(a) $x + y = 5$

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(a) $x + y = 5$

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(a) $x + y = 5$

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$$r\cos\theta + r\sin\theta = 5$$

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Convert each to polar equations.

(a) $x + y = 5$

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$$r\cos\theta + r\sin\theta = 5$$

$$r\left(\cos\theta + \sin\theta\right) = 5$$

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Convert each to polar equations.

(a) $x + y = 5$

$$x + y = 5$$

$$r\cos\theta + r\sin\theta = 5$$

$$r\left(\cos\theta + \sin\theta\right) = 5$$

$$r = \frac{5}{\cos\theta + \sin\theta}$$

Example 4

(b) $3x - y = 6$

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$$3x - y = 6$$

$$3r\cos\theta - r\sin\theta = 6$$

Example 4

(b) $3x - y = 6$

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$$3r\cos\theta - r\sin\theta = 6$$

$$r(3\cos\theta - \sin\theta) = 6$$

Example 4

(b) $3x - y = 6$

$$3x - y = 6$$

$$3r\cos\theta - r\sin\theta = 6$$

$$r(3\cos\theta - \sin\theta) = 6$$

$$r = \frac{6}{3\cos\theta - \sin\theta}$$

Example 5

Convert each of the following to rectangular equations.

(a) $r = 5$

Example 5

Convert each of the following to rectangular equations.

(a) $r = 5$

$$r = 5$$

Example 5

Convert each of the following to rectangular equations.

(a) $r = 5$

$$r = 5$$

$$r^2 = 25$$

Example 5

Convert each of the following to rectangular equations.

(a) $r = 5$

$$r = 5$$

$$r^2 = 25$$

$$x^2 + y^2 = 25$$

Example 5

(b) $\theta = \frac{\pi}{4}$

Example 5

(b) $\theta = \dfrac{\pi}{4}$

$$\theta = \frac{\pi}{4}$$

Example 5

(b) $\theta = \dfrac{\pi}{4}$

$$\theta = \frac{\pi}{4}$$

$$\tan\theta = \tan\left(\frac{\pi}{4}\right)$$

Example 5

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$$\tan\theta = \tan\left(\frac{\pi}{4}\right)$$

$$\frac{y}{x} = 1$$

Example 5

(b) $\theta = \dfrac{\pi}{4}$

$$\theta = \frac{\pi}{4}$$

$$\tan\theta = \tan\left(\frac{\pi}{4}\right)$$

$$\frac{y}{x} = 1$$

$$y = x$$

Example 5

(c) $r = 3 \csc \theta$

Example 5

(c) $r = 3\csc\theta$

$$r = 3\csc\theta$$

Example 5

(c) $r = 3\csc\theta$

$$r = 3\csc\theta$$

$$r = 3\left(\frac{1}{\sin\theta}\right)$$

Example 5

(c) $r = 3\csc\theta$

$$r = 3\csc\theta$$

$$r = 3\left(\frac{1}{\sin\theta}\right)$$

$$r = \frac{3}{\sin\theta}$$

Example 5

(c) $r = 3\csc\theta$

$$r = 3\csc\theta$$

$$r = 3\left(\frac{1}{\sin\theta}\right)$$

$$r = \frac{3}{\sin\theta}$$

$$r\sin\theta = 3$$

Example 5

(c) $r = 3\csc\theta$

$$r = 3\csc\theta$$

$$r = 3\left(\frac{1}{\sin\theta}\right)$$

$$r = \frac{3}{\sin\theta}$$

$$r\sin\theta = 3$$

$$y = 3$$

Example 5

(d) $r = -6\cos\theta$

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Example 5

(d) $r = -6\cos\theta$

$$r = -6\cos\theta$$

$$r^2 = -6r\cos\theta$$

Example 5

(d) $r = -6\cos\theta$

$$r = -6\cos\theta$$

$$r^2 = -6r\cos\theta$$

$$x^2 + y^2 = -6x$$

Example 5

(d) $r = -6\cos\theta$

$$r = -6\cos\theta$$

$$r^2 = -6r\cos\theta$$

$$x^2 + y^2 = -6x$$

$$x^2 + 6x + y^2 = 0$$