Exponential Equations

Objectives

1. Solve exponential equations

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An exponential equation is an equation involving exponential functions.

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$$2^x = 128$$

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$$2^x = 129$$

$$\log_2(2^x) = \log_2(129)$$

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\begin{aligned}
2^x &= 129 \\
\log_2(2^x) &= \log_2(129) \\
x &= \log_2(129)
\end{aligned}
$$

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- Isolate the exponential function
    - If convenient, express both sides with a common base and equate the exponents.
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$$
\begin{aligned}
2^x &= 129 \\
\log_2(2^x) &= \log_2(129) \\
x &= \log_2(129) \\
&\approx 7.0112
\end{aligned}
$$

Example 1

Solve each. Round your answers to 4 decimal places.

(a) $2^{3x} = 16^{1-x}$

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$$\approx 0.5714$$

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$$3x \cdot \log_2(2) = (1-x) \cdot \log_2(16) \qquad \text{Power Prop.}$$

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$$3x = (1-x) \cdot 4 \qquad \log_2(16) = 4$$

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\begin{aligned}
2^{3x} &= 16^{1-x} \\
\log_2\left(2^{3x}\right) &= \log_2\left(16^{1-x}\right) \\
3x \cdot \log_2(2) &= (1-x) \cdot \log_2(16) && \text{Power Prop.} \\
3x &= (1-x) \cdot 4 && \log_2(16) = 4 \\
3x &= 4 - 4x \\
x &= \frac{4}{7}
\end{aligned}
$$

Example 1

(b) $2000 = 1000 \cdot 3^{-0.1t}$

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$\log_3(2) = -0.1t \cdot \log_3(3)$ Power Prop.

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(b) $2000 = 1000 \cdot 3^{-0.1t}$

$2 = 3^{-0.1t}$ Isolate expon. func.

$\log_3(2) = \log_3(3^{-0.1t})$

$\log_3(2) = -0.1t \cdot \log_3(3)$ Power Prop.

$\log_3(2) = -0.1t$ $\log_3(3) = 1$

Example 1

$$\begin{aligned}
\text{(b)} \qquad 2000 &= 1000 \cdot 3^{-0.1t} \\
2 &= 3^{-0.1t} && \text{Isolate expon. func.} \\
\log_3(2) &= \log_3(3^{-0.1t}) \\
\log_3(2) &= -0.1t \cdot \log_3(3) && \text{Power Prop.} \\
\log_3(2) &= -0.1t && \log_3(3) = 1 \\
t &= \frac{\log_3(2)}{-0.1}
\end{aligned}$$

Example 1

(b) $2000 = 1000 \cdot 3^{-0.1t}$

$$2 = 3^{-0.1t} \qquad \text{Isolate expon. func.}$$

$$\log_3(2) = \log_3(3^{-0.1t})$$

$$\log_3(2) = -0.1t \cdot \log_3(3) \qquad \text{Power Prop.}$$

$$\log_3(2) = -0.1t \qquad \log_3(3) = 1$$

$$t = \frac{\log_3(2)}{-0.1}$$

$$t \approx -6.3093$$

Example 1

(c) $9 \cdot 3^x = 7^{2x}$

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$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

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$$3^{2+x} = 7^{2x} \qquad \text{Product Prop.}$$

$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

$$(2+x) \cdot \log_3(3) = 2x \cdot \log_3(7) \qquad \text{Power Prop.}$$

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(c) $9 \cdot 3^x = 7^{2x}$

$$
\begin{aligned}
3^2 \cdot 3^x &= 7^{2x} && 9 = 3^2 \\
3^{2+x} &= 7^{2x} && \text{Product Prop.} \\
\log_3(3^{2+x}) &= \log_3(7^{2x}) && \\
(2+x) \cdot \log_3(3) &= 2x \cdot \log_3(7) && \text{Power Prop.} \\
2 + x &= 2x \cdot \log_3(7) && \log_3(3) = 1
\end{aligned}
$$

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$$\log_3(3^{2+x}) = \log_3(7^{2x})$$

$$(2+x) \cdot \log_3(3) = 2x \cdot \log_3(7) \qquad \text{Power Prop.}$$

$$2 + x = 2x \cdot \log_3(7) \qquad \log_3(3) = 1$$

$$2 = -x + 2x \cdot \log_3(7) \qquad \text{Subtract } x$$

Example 1

(c) $9 \cdot 3^x = 7^{2x}$

$$\begin{aligned}
3^2 \cdot 3^x &= 7^{2x} && 9 = 3^2 \\
3^{2+x} &= 7^{2x} && \text{Product Prop.} \\
\log_3(3^{2+x}) &= \log_3(7^{2x}) && \\
(2+x) \cdot \log_3(3) &= 2x \cdot \log_3(7) && \text{Power Prop.} \\
2 + x &= 2x \cdot \log_3(7) && \log_3(3) = 1 \\
2 &= -x + 2x \cdot \log_3(7) && \text{Subtract } x \\
2 &= x(-1 + 2\log_3(7)) && \text{Factor out } x
\end{aligned}$$

Example 1

$$2 = x(-1 + 2\log_3(7))$$ Factor out x

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$$x = \frac{2}{-1 + 2\log_3(7)}$$

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$$x = \frac{2}{-1 + 2\log_3(7)}$$

$$x \approx 0.7866$$

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(d) $$75 = \frac{100}{1 + 3e^{-2t}}$$

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$75(1 + 3e^{-2t}) = 100$ Eliminate fraction

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$$1 + 3e^{-2t} = \frac{4}{3}$$ Divide by 75

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$$75(1 + 3e^{-2t}) = 100$$ Eliminate fraction

$$1 + 3e^{-2t} = \frac{4}{3}$$ Divide by 75

$$3e^{-2t} = \frac{1}{3}$$ Subtract 1

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$$75(1 + 3e^{-2t}) = 100$$ Eliminate fraction

$$1 + 3e^{-2t} = \frac{4}{3}$$ Divide by 75

$$3e^{-2t} = \frac{1}{3}$$ Subtract 1

$$e^{-2t} = \frac{1}{9}$$ Divide by 3

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(d) $$75 = \frac{100}{1+3e^{-2t}}$$

$$75(1+3e^{-2t}) = 100$$ Eliminate fraction

$$1+3e^{-2t} = \frac{4}{3}$$ Divide by 75

$$3e^{-2t} = \frac{1}{3}$$ Subtract 1

$$e^{-2t} = \frac{1}{9}$$ Divide by 3

$$\ln\left(e^{-2t}\right) = \ln\left(\frac{1}{9}\right)$$

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$$-2t \cdot \ln e = \ln\left(\frac{1}{9}\right) \qquad \text{Power Prop.}$$

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$$-2t = \ln\left(\frac{1}{9}\right) \qquad \ln e = 1$$

$$t = \frac{\ln\left(\frac{1}{9}\right)}{-2}$$

$$t \approx 1.099$$

Example 1

(e) $25^x = 5^x + 6$

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$$\left(5^2\right)^x = 5^x + 6 \qquad\qquad 25 = 5^2$$

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Let $u = 5^x$

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$$(5^x)^2 = 5^x + 6 \qquad 5^{2x} = (5^x)^2$$

Let $u = 5^x$

$$u^2 = u + 6 \qquad \text{Using substitution}$$

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$$u^2 - u - 6 = 0$$

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$$u = -2,\ 3$$

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$$5^{2x} = 5^x + 6 \qquad \text{Power Prop.}$$

$$(5^x)^2 = 5^x + 6 \qquad 5^{2x} = (5^x)^2$$

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$$u^2 = u + 6 \qquad \text{Using substitution}$$

$$u^2 - u - 6 = 0$$

$$u = -2,\ 3$$

$$5^x = -2 \qquad \text{and} \qquad 5^x = 3$$

Example 1

$5^x = -2$ $\qquad$ $5^x = 3$

Example 1

$$5^x = -2$$

$$x \cdot \log_5(5) = \log_5(-2)$$

$$5^x = 3$$

$$x \cdot \log_5(5) = \log_5(3)$$

Example 1

$$\begin{aligned} 5^x &= -2 \\ x \cdot \log_5(5) &= \log_5(-2) \\ x &= \log_5(-2) \end{aligned}$$

$$\begin{aligned} 5^x &= 3 \\ x \cdot \log_5(5) &= \log_5(3) \\ x &= \log_5(3) \end{aligned}$$

Example 1

$$\begin{aligned} 5^x &= -2 \\ x \cdot \log_5(5) &= \log_5(-2) \\ x &= \log_5(-2) \\ x &= \varnothing \end{aligned}$$

$$\begin{aligned} 5^x &= 3 \\ x \cdot \log_5(5) &= \log_5(3) \\ x &= \log_5(3) \\ x &\approx 0.6826 \end{aligned}$$

Example 1

$$
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5^x &= -2 \\
x \cdot \log_5(5) &= \log_5(-2) \\
x &= \log_5(-2) \\
x &= \varnothing
\end{aligned}
\qquad\qquad
\begin{aligned}
5^x &= 3 \\
x \cdot \log_5(5) &= \log_5(3) \\
x &= \log_5(3) \\
x &\approx 0.6826
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$$

$$x \approx 0.6826$$

Example 1

(f) $$\frac{e^{x} - e^{-x}}{2} = 5$$

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$$e^x - e^{-x} = 10$$ Eliminate fraction

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(f) $\dfrac{e^x - e^{-x}}{2} = 5$

$$e^x - e^{-x} = 10 \qquad \text{Eliminate fraction}$$

$$e^x - \frac{1}{e^x} = 10 \qquad e^{-x} = \tfrac{1}{e^x}$$

Example 1

(f) $$\frac{e^x - e^{-x}}{2} = 5$$

$$e^x - e^{-x} = 10$$ Eliminate fraction

$$e^x - \frac{1}{e^x} = 10$$ $e^{-x} = \frac{1}{e^x}$

$$e^{2x} - 1 = 10e^x$$ Multiply by e^x

Example 1

(f) $\dfrac{e^x - e^{-x}}{2} = 5$

$$e^x - e^{-x} = 10 \qquad \text{Eliminate fraction}$$

$$e^x - \frac{1}{e^x} = 10 \qquad e^{-x} = \tfrac{1}{e^x}$$

$$e^{2x} - 1 = 10e^x \qquad \text{Multiply by } e^x$$

$$e^{2x} - 10e^x - 1 = 0$$

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$$e^x - e^{-x} = 10 \qquad \text{Eliminate fraction}$$

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$$e^{2x} - 1 = 10e^x \qquad \text{Multiply by } e^x$$

$$e^{2x} - 10e^x - 1 = 0$$

Let u $= e^x$

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(f) $\dfrac{e^x - e^{-x}}{2} = 5$

$$e^x - e^{-x} = 10$$ Eliminate fraction

$$e^x - \frac{1}{e^x} = 10$$ $e^{-x} = \frac{1}{e^x}$

$$e^{2x} - 1 = 10e^x$$ Multiply by e^x

$$e^{2x} - 10e^x - 1 = 0$$

Let u $= e^x$

$$u^2 - 10u - 1 = 0$$

Example 1 letting $u = e^x$

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$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

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$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$

Example 1 letting $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$

$$u = \frac{10 \pm 2\sqrt{26}}{2}$$

Example 1 letting $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$

$$u = \frac{10 \pm 2\sqrt{26}}{2}$$

$$u = 5 \pm \sqrt{26}$$

Example 1 letting $u = e^x$

$$u^2 - 10u - 1 = 0$$

$$u = \frac{10 \pm \sqrt{10^2 - 4(1)(-1)}}{2}$$

$$u = \frac{10 \pm \sqrt{104}}{2}$$

$$u = \frac{10 \pm 2\sqrt{26}}{2}$$

$$u = 5 \pm \sqrt{26}$$

$$e^x = 5 \pm \sqrt{26}$$

Example 1

$$e^x = 5 + \sqrt{26} \qquad\qquad e^x = 5 - \sqrt{26}$$

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$$e^x = 5 + \sqrt{26}$$

$$\ln(e^x) = \ln(5 + \sqrt{26})$$

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$$\ln(e^x) = \ln(5 + \sqrt{26}) \qquad\qquad \ln(e^x) = \ln(5 - \sqrt{26})$$

$$x \ln e = \ln(5 + \sqrt{26}) \qquad\qquad x \ln e = \ln(5 - \sqrt{26})$$

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$$x = \ln(5 + \sqrt{26}) \qquad x = \ln(5 - \sqrt{26})$$

$$x \approx 2.312 \qquad x = \varnothing$$

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