

Vectors

Table of Contents

- 1 Write the Component Form of a Vector and Find Magnitude
- 2 Perform Operations with Vectors
- 3 Finding Unit Vectors
- 4 Find Component Form from Magnitude and Direction and Vice Versa

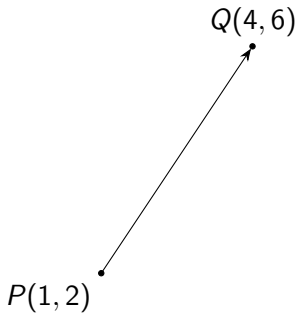
Intro

A **vector** is a mathematical object with both magnitude (length) and direction.

It is represented geometrically by a directed line segment.

Intro

The following shows an example of a vector $\vec{v} = \overrightarrow{PQ}$.



P is the **initial point** and Q is the **terminal point**.

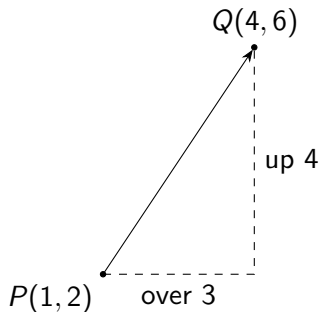
Magnitude

The **magnitude** of a vector (denoted $\|\vec{v}\|$ or $|\vec{v}|$) is the distance between P and Q .

Think Pythagorean Theorem.

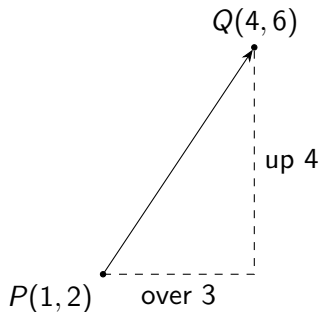
Component Form

If we connect the point P to point Q we get the following diagram:



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This can be represented by $\vec{v} = \langle 3, 4 \rangle$ or by $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.

Component Form

To find the component form for $P(x_0, y_0)$ and $Q(x_1, y_1)$, just take the difference in coordinates:

$$\vec{v} = \overrightarrow{PQ} = \langle x_1 - x_0, y_1 - y_0 \rangle$$

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or

$$(x_1 - x_0)\mathbf{i} + (y_1 - y_0)\mathbf{j}$$

Example 1a

Find the component form and exact magnitude of \overrightarrow{AB} for each.

(a) $A(2, -1) \quad B(-5, 3)$

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$$= \sqrt{10}$$

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Vector Addition

We can add vectors by adding the horizontal components, and adding the vertical components.

Think of these as *combining like terms*.

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For instance, when adding $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = \langle 1, -2 \rangle$, combine horizontal components and vertical components:

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$$\vec{v} + \vec{w} = \langle 3 + 1, 4 + (-2) \rangle$$

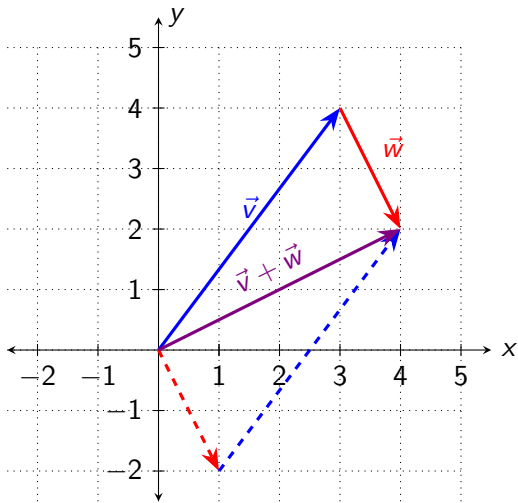
Vector Addition

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$$\begin{aligned}\vec{v} + \vec{w} &= \langle 3 + 1, 4 + (-2) \rangle \\ &= \langle 4, 2 \rangle\end{aligned}$$

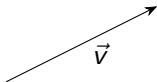
Vector Addition

Visually, the previous problem is



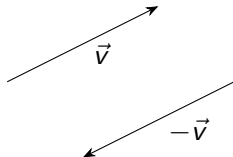
Scalar Multiplication

If we multiply our vector by a real number (a **scalar**), we get a new vector.



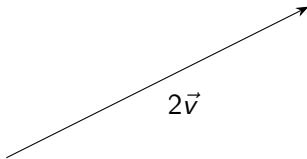
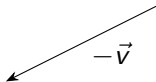
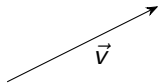
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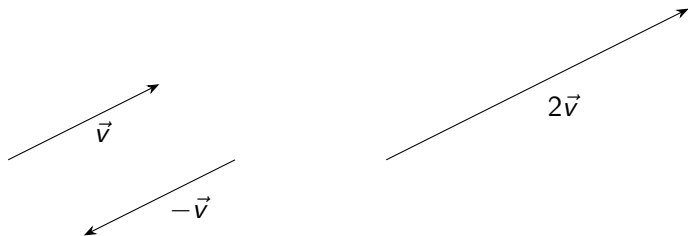
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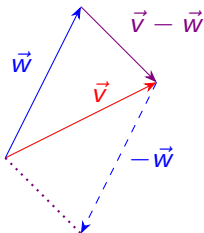
If we multiply our vector by a real number (a **scalar**), we get a new vector.



Scalar multiplication “distributes” the scalar to both horizontal and vertical component.

Vector Subtraction

Vector subtraction $\vec{v} - \vec{w}$ can be thought of as $\vec{v} + (-\vec{w})$ and is illustrated below:



Example 2a

If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$, and $\vec{w} = \langle 6, -9 \rangle$, find each of the following.

(a) $\mathbf{v} + \mathbf{w}$

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$$\begin{aligned}\mathbf{v} + \mathbf{w} &= \langle 5, 4 \rangle + \langle 6, -9 \rangle \\ &= \langle 5 + 6, 4 + (-9) \rangle \\ &= \langle 11, -5 \rangle\end{aligned}$$

Example 2b

(b) $\mathbf{v} - \mathbf{w}$

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$$\begin{aligned}\mathbf{v} - \mathbf{w} &= \langle 5, 4 \rangle - \langle 6, -9 \rangle \\ &= \langle 5 - 6, 4 - (-9) \rangle\end{aligned}$$

Example 2b

(b) $\mathbf{v} - \mathbf{w}$

$$\begin{aligned}\mathbf{v} - \mathbf{w} &= \langle 5, 4 \rangle - \langle 6, -9 \rangle \\ &= \langle 5 - 6, 4 - (-9) \rangle \\ &= \langle -1, 13 \rangle\end{aligned}$$

Example 2c

(c) $6\vec{v}$

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$$6\vec{v} = 6\langle 5, 4 \rangle$$

Example 2c

$$(c) \quad 6\vec{v}$$

$$\begin{aligned} 6\vec{v} &= 6\langle 5, 4 \rangle \\ &= \langle 30, 24 \rangle \end{aligned}$$

Example 2d

$$(d) \quad 2\mathbf{v} + 4\mathbf{w}$$

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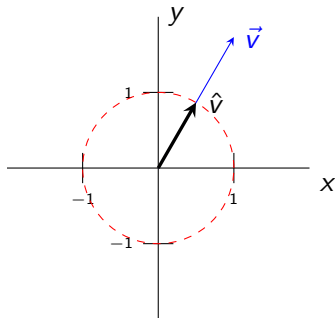
$$= 34\mathbf{i} - 28\mathbf{j}$$

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Unit Vectors

A **unit vector**, \hat{v} , is a vector that has a magnitude of 1.



Notice the unit vector \hat{v} is parallel to \vec{v} .

\mathbf{i} and \mathbf{j} are unit vectors with $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$

Unit Vectors

We get \hat{v} by dividing the magnitude of \vec{v} by itself:

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Since we are dividing the vector (the hypotenuse) by the magnitude, we also divide the x and y components as well:

$$\hat{v} = \left\langle \frac{x}{\|\vec{v}\|}, \frac{y}{\|\vec{v}\|} \right\rangle$$

Example 3a

Find a unit vector for each given vector.

(a) $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$

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$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{5^2 + 12^2} \\ &= 13\end{aligned}$$

$$\hat{\mathbf{v}} = \frac{5}{13}\mathbf{i} - \frac{12}{13}\mathbf{j}$$

Example 3b

$$(b) \quad \vec{w} = \langle 4, -3 \rangle$$

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$$|\vec{w}| = \sqrt{4^2 + 3^2}$$

$$= 5$$

$$\hat{w} = \left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$$

Example 3c

(c) $\mathbf{d} = 7\mathbf{i} + \mathbf{j}$

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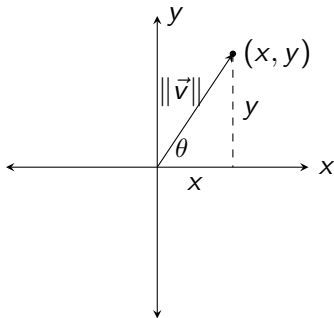
$$= \frac{7\sqrt{2}}{10}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

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Magnitude and Direction

If we place a vector $\langle x, y \rangle$ in the coordinate plane, we can put the initial point at the origin and the terminal point at (x, y) .



Magnitude and Direction

From Trig Functions of Any Angle,

$$x = \|\vec{v}\| \cos \theta \text{ and } y = \|\vec{v}\| \sin \theta$$

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. Thus,

$$\langle x, y \rangle = \langle \|\vec{v}\| \cos \theta, \|\vec{v}\| \sin \theta \rangle = \|\vec{v}\| \langle \cos \theta, \sin \theta \rangle$$

Example 4

Find the horizontal and vertical component form of the vector whose magnitude and direction angle are given by $|u| = 12$ and $\theta = 150^\circ$.

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$$\langle -6\sqrt{3}, 6 \rangle$$

Example 5

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ ($0 \leq \theta < 2\pi$) and write in $\|\vec{v}\|\langle \cos \theta, \sin \theta \rangle$ form.

Example 5

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ ($0 \leq \theta < 2\pi$) and write in $\|\vec{v}\|\langle \cos \theta, \sin \theta \rangle$ form.

$$\|\vec{v}\| = \sqrt{3^2 + (3\sqrt{3})^2}$$

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$$\begin{aligned}\|\vec{v}\| &= \sqrt{3^2 + (3\sqrt{3})^2} \\ &= \sqrt{36} = 6\end{aligned}$$

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$$\theta' = \tan^{-1} \left| \frac{-3\sqrt{3}}{3} \right|$$

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$$6 \left\langle \cos \left(\frac{5\pi}{3} \right), \sin \left(\frac{5\pi}{3} \right) \right\rangle$$