Functions and Their Graphs

Objectives

1. Determine if a relation is a function.
2. Evaluate functions using function notation.
3. Find the domain of a function.
4. Find the intercepts of a function

Vocab

A relation is a set of ordered pairs.

The domain is the set of all input values (usually x).

The range is the set of all output values (usually y).

Relations and Functions

A relation is a function if each element in the domain has only 1 corresponding element in the range.

In other words, each x-coordinate has only 1 y-coordinate.

Example 1

Which of the following describes y as a function of x? For the ones that do, state the domain and range.

(a) $\{(-2, 1), (1, 3), (1, 4), (3, -1)\}$

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Not a function ($x = 1$ has 2 different y-coordinates)

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Domain: -2, 1, 2, 3

Range: -1, 1, 3

Vertical Line Test

If you are given the equation, you can graph it and use the Vertical Line Test:

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1. Graph the equation.
2. Draw vertical lines through the graph.
3. If each vertical line hits the graph only once (or not at all), it is a function.

Example 2

Determine if each defines y as a function of x.

(a) $x^2 + y = 4$

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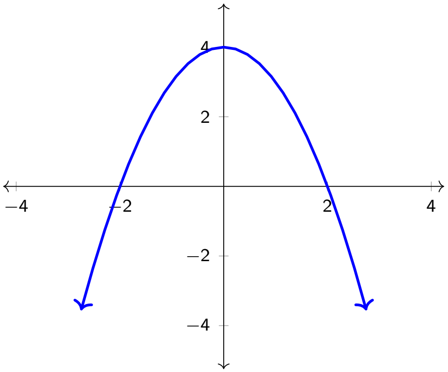

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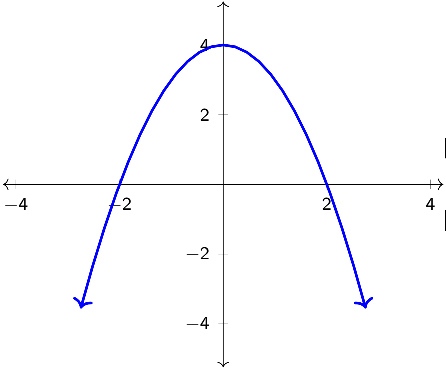

Passes Vertical Line Test

Is a function.

Example 2

(b) $x^2 + y^2 = 4$

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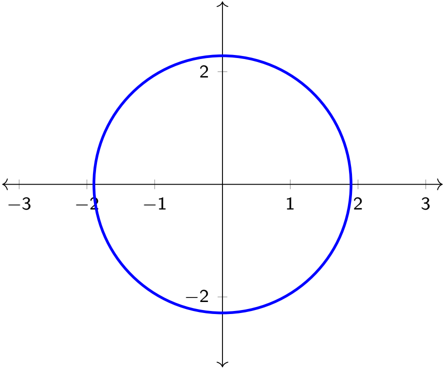

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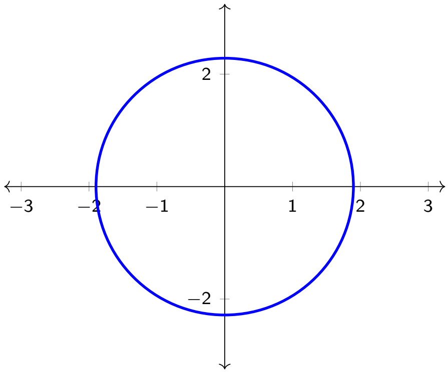
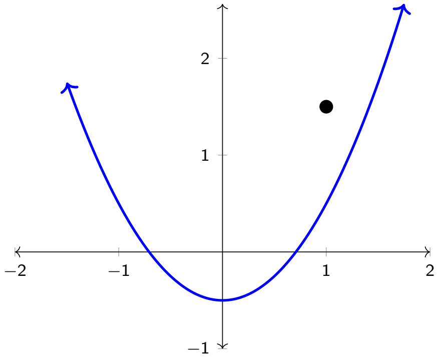


Fails Vertical Line Test

Not a function

Example 2

(c)



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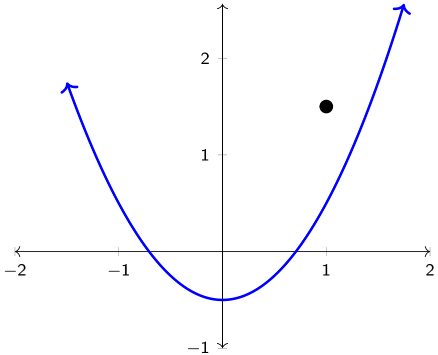
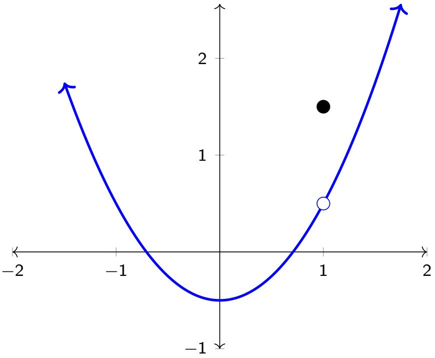


Fails Vertical Line Test

Not a Function

Example 2

(d)



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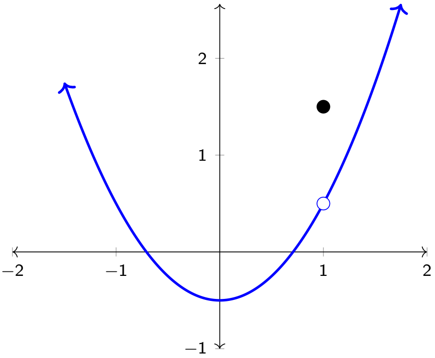

Passes Vertical Line Test

Is a Function

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Function Notation

We typically use function notation such as $f(x)$ or $g(x)$ when writing functions rather than y.

To evaluate a function, substitute the value in parentheses into the function.

Example 3

For $f(x) = -x^2 + 3x + 4$, evaluate each.

(a) $f(-1)$

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$$\begin{aligned} f(-1) &= -(-1)^2 + 3(-1) + 4 \\ &= -1 - 3 + 4 \end{aligned}$$

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(a) $f(-1)$

$$\begin{aligned} f(-1) &= -(-1)^2 + 3(-1) + 4 \\ &= -1 - 3 + 4 \\ &= 0 \end{aligned}$$

Example 3

(b) $f(0)$

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$$f(0) = -(0)^2 + 3(0) + 4$$

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$$\begin{aligned} f(0) &= -(0)^2 + 3(0) + 4 \\ &= 0 + 0 + 4 \\ &= 4 \end{aligned}$$

Example 3

(c) $f(2)$

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$$f(2) = -(2)^2 + 3(2) + 4$$

Example 3

(c) $f(2)$

$$\begin{aligned} f(2) &= -(2)^2 + 3(2) + 4 \\ &= -4 + 6 + 4 \end{aligned}$$

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(c) $f(2)$

$$\begin{aligned} f(2) &= -(2)^2 + 3(2) + 4 \\ &= -4 + 6 + 4 \\ &= 6 \end{aligned}$$

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Domains

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- Taking an even ($\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$, etc.) of a negative number.

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As we progress in the course, we will look at various functions; many having restrictions on what input values they will allow.

Restricted Domains:

- Taking an even ($\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$, etc.) of a negative number.
- Dividing by 0.

Example 4

Find the domain of each.

(a) $g(x) = \sqrt{4 - 3x}$

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$$\begin{aligned} 4 - 3x &\geq 0 \\ -3x &\geq -4 \\ x &\leq \frac{4}{3} \end{aligned}$$

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(a) $g(x) = \sqrt{4 - 3x}$

$$4 - 3x \geq 0$$

$$-3x \geq -4$$

$$x \leq \frac{4}{3}$$

$$\left(-\infty, \frac{4}{3}\right]$$

Example 4

(b) $h(x) = \sqrt[5]{4 - 3x}$

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$$(-\infty, \infty)$$

Example 4

(c) $$f(x) = \frac{2}{1 - \dfrac{4x}{x-3}}$$

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$$\begin{aligned} x - 3 &\neq 0 \\ x &\neq 3 \end{aligned}$$

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$$= \frac{2(x-3)}{x-3-4x}$$

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$$\begin{aligned} x - 3 - 4x &\neq 0 \\ -3x - 3 &\neq 0 \\ x &\neq -1 \end{aligned}$$

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(c)

$$x \neq -1, 3$$

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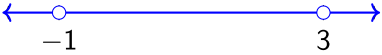

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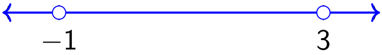

$$(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$$

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(d) $$F(x) = \frac{\sqrt[4]{2x+1}}{x^2-1}$$

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$$x \geq -\frac{1}{2}$$

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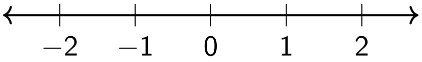

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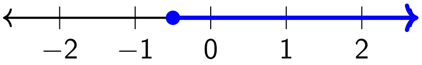

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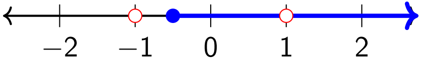

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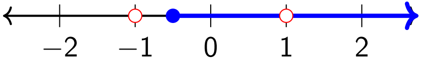

$$\left[-\frac{1}{2}, 1\right) \cup (1, \infty)$$

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(e) $r(t) = \dfrac{4}{6 - \sqrt{t+3}}$

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$$
\begin{aligned}
6 - \sqrt{t+3} &\neq 0 \\
6 &\neq \sqrt{t+3} \\
6^2 &\neq (\sqrt{t+3})^2
\end{aligned}
$$

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$$\begin{aligned} 6 - \sqrt{t+3} &\neq 0 \\ 6 &\neq \sqrt{t+3} \\ 6^2 &\neq (\sqrt{t+3})^2 \\ 36 &\neq t+3 \end{aligned}$$

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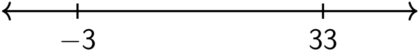

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$$[-3, 33) \cup (33, \infty)$$

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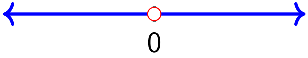

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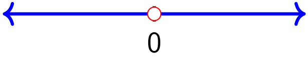

$$(-\infty, 0) \cup (0, \infty)$$

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Intercepts

The *x-intercept* of a function is where it crosses the x-axis (i.e. the y-coordinate is 0).

The *y-intercept* of a function is where it crosses the y-axis (i.e. the x-coordinate is 0).

Example 5

Find the intercepts of $f(x) = x^2 + 4x + 3$.

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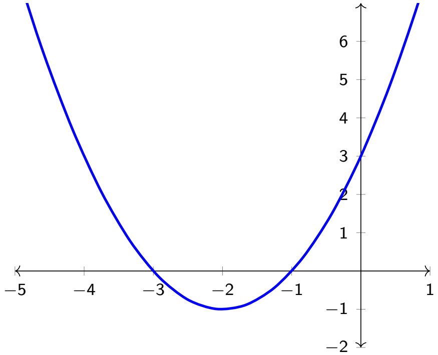

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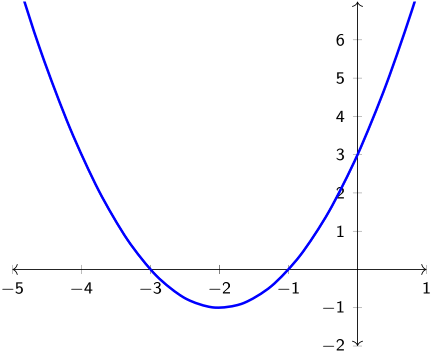

x-intercepts:

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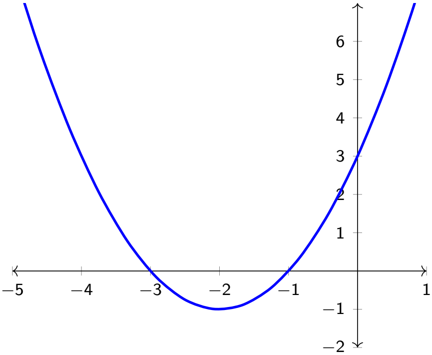

x-intercepts:

$(-3, 0)$ and $(-1, 0)$

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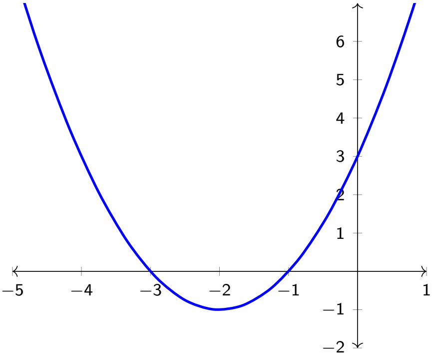

x-intercepts:

$(-3, 0)$ and $(-1, 0)$

y-intercept: $(0, 3)$