Derivatives

Objectives

1. Find the derivative of a function

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Derivative Definition

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Other Notations: $\dfrac{dy}{dx}$ $\quad y' \quad \dot{x}$

Example 1

Find the derivative of each.

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$$f(x+h) - f(x) = 2x^2 + 4hx + 2h^2 - x - h + 3 - (2x^2 - x + 3)$$

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(a) $f(x) = 2x^2 - x + 3$

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\begin{aligned}
f(x+h) &= 2(x+h)^2 - (x+h) + 3 \\
&= 2(x^2 + 2hx + h^2) - x - h + 3 \\
&= 2x^2 + 4hx + 2h^2 - x - h + 3 \\
f(x+h) - f(x) &= 2x^2 + 4hx + 2h^2 - x - h + 3 - (2x^2 - x + 3) \\
&= 2x^2 + 4hx + 2h^2 - x - h + 3 - 2x^2 + x - 3 \\
&= 4hx - h + 2h^2
\end{aligned}
$$

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$$f'(x) = 4x - 1$$

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$$\begin{aligned} f(x+h) &= -3(x+h)^2 \\ &= -3(x^2 + 2hx + h^2) \end{aligned}$$

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(b) $f(x) = -3x^2$

$$\begin{aligned} f(x+h) &= -3(x+h)^2 \\ &= -3(x^2 + 2hx + h^2) \\ &= -3x^2 - 6hx - 3h^2 \end{aligned}$$

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(b) $f(x) = -3x^2$

$$
\begin{aligned}
f(x+h) &= -3(x+h)^2 \\
&= -3(x^2 + 2hx + h^2) \\
&= -3x^2 - 6hx - 3h^2 \\
f(x+h) - f(x) &= -3x^2 - 6hx - 3h^2 - (-3x^2)
\end{aligned}
$$

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$$\begin{aligned} f(x+h) &= -3(x+h)^2 \\ &= -3(x^2 + 2hx + h^2) \\ &= -3x^2 - 6hx - 3h^2 \\ f(x+h) - f(x) &= -3x^2 - 6hx - 3h^2 - (-3x^2) \\ &= -3x^2 - 6hx - 3h^2 + 3x^2 \end{aligned}$$

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$$\begin{aligned} f(x+h) &= -3(x+h)^2 \\ &= -3(x^2 + 2hx + h^2) \\ &= -3x^2 - 6hx - 3h^2 \\ f(x+h) - f(x) &= -3x^2 - 6hx - 3h^2 - (-3x^2) \\ &= -3x^2 - 6hx - 3h^2 + 3x^2 \\ &= -6hx - 3h^2 \end{aligned}$$

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\frac{f(x+h)-f(x)}{h} &= \frac{-6hx-3h^2}{h} \\
&= -6x-3h \\
\lim_{h\to 0}(-6x-3h) \\
&= -6x-3(0) \\
&= -6x
\end{aligned}$$

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$$f'(x) = -6x$$

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&= 7x + 7h + 9 - 7x - 9 \\
&= 7h
\end{aligned}
$$

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$$f'(x) = 7$$

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&= x^2 + 2hx + h^2 + 3 - x^2 - 3 \\
&= 2hx + h^2
\end{aligned}$$

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$$f'(x) = 2x$$

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- Tangent slope to a curve.

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- Instant rate of change.
- Tangent slope to a curve.
- Limit as secant line becomes tangent line.

Fear Not

In calculus, you will learn shortcuts so you won't always have to do what we did in these notes to find the derivative.