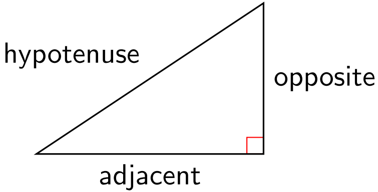
Trig Functions of Any Angle

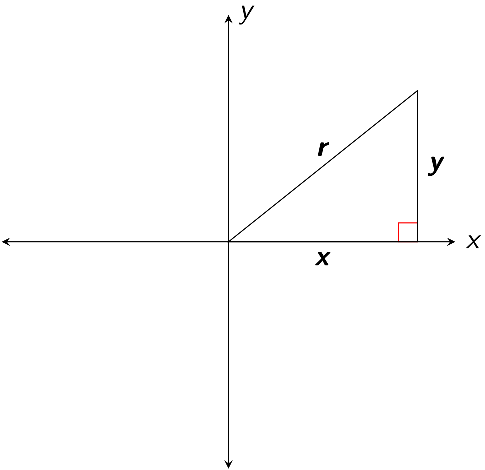
From Right Triangle Trig to the Coordinate Plane

In this section, adjacent, opposite and hypotenuse



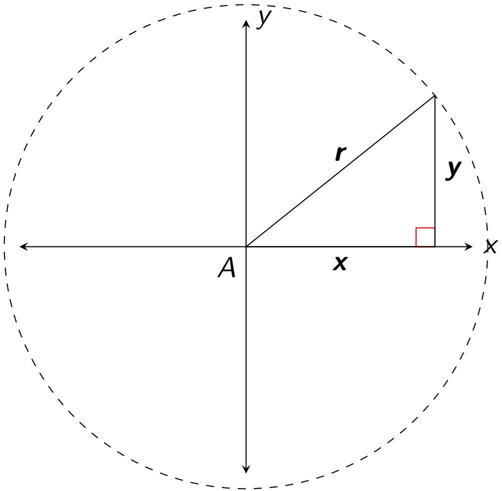
From Right Triangle Trig to the Coordinate Plane

become



From Right Triangle Trig to the Coordinate Plane

where r represents the radius of the circle shown below.



From Right Triangle Trig to the Coordinate Plane

The biggest difference between this section and the last is that in this section, trig ratios can be positive, negative, zero, or undefined, since x and y can each be positive, negative, or zero.

Objectives

1. Calculate the 6 Trig Ratios for a Point in the Coordinate Plane
2. Find the Exact Values of the Trig Ratios of Special Angles in the Coordinate Plane

The 6 Trig Ratios

In the coordinate plane, the six trig ratios become

$$\sin A = \frac{y}{r}$$

$$\cos A = \frac{x}{r}$$

$$\tan A = \frac{y}{x}$$

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$$\begin{array}{c|c} \sin A = \dfrac{y}{r} & \csc A = \dfrac{r}{y} \\[2em] \cos A = \dfrac{x}{r} & \sec A = \dfrac{r}{x} \\[2em] \tan A = \dfrac{y}{x} & \cot A = \dfrac{x}{y} \end{array}$$

Example 1a

For the given point, find the exact values of the 6 trig functions for the angle drawn in standard form.

(a) $(3, -4)$

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Example 1b

(b) $(-8, -15)$

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Example 1b

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$$r = \sqrt{289} = 17$$

$\sin A = \frac{-15}{17}$

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Example 1b

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$$8^2 + 15^2 = r^2$$

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Example 1c

(c) $(-1, 5)$

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Example 1c

(c) $(-1, 5)$

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$$r = \sqrt{26}$$

$\sin A = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$

Example 1c

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$$r = \sqrt{26}$$

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Objectives

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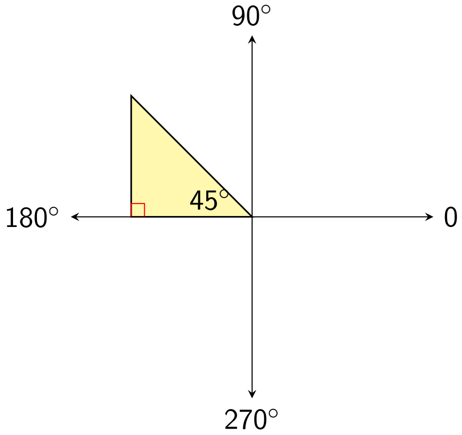
If we put a 45-45-90 triangle in the second quadrant of the coordinate plane, it would resemble the following:



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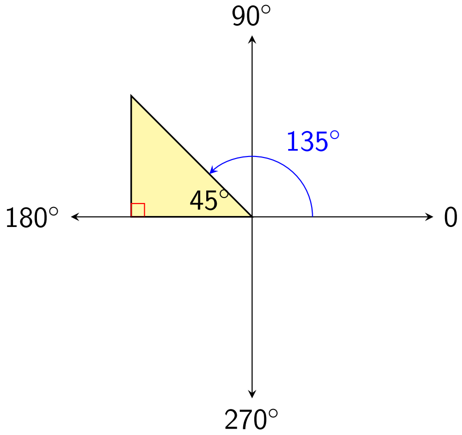
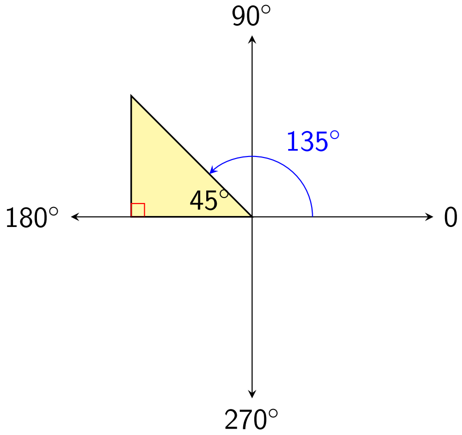


The hypotenuse, r, would have rotated $180^\circ - 45^\circ = 135^\circ$.

If we put a 45-45-90 triangle in the second quadrant of the coordinate plane, it would resemble the following:



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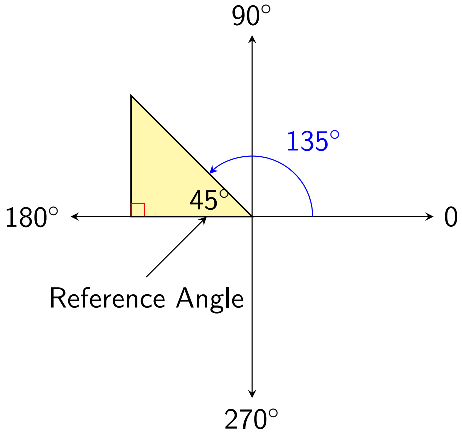
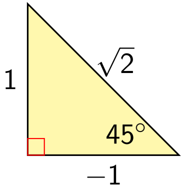


The hypotenuse, r, would have rotated $180^\circ - 45^\circ = 135^\circ$.

In the second quadrant, x-coordinates are negative and y-coordinates are positive (r is always positive). Thus, the values would be



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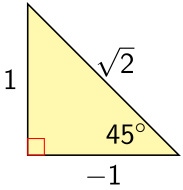
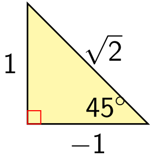


So $\sin 135^\circ = \frac{\sqrt{2}}{2}$, $\cos 135^\circ = -\frac{\sqrt{2}}{2}$, and $\tan 135^\circ = -1$.

Example 2a

Find the exact values of the 6 trig functions for each of the following.

(a) $135^\circ = \frac{3\pi}{4}$



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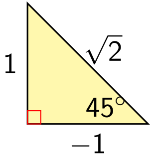

$\sin 135^\circ = \frac{\sqrt{2}}{2}$

$\csc 135^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$

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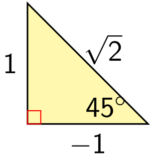

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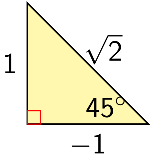

$$\sin 135^\circ = \frac{\sqrt{2}}{2} \qquad \csc 135^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

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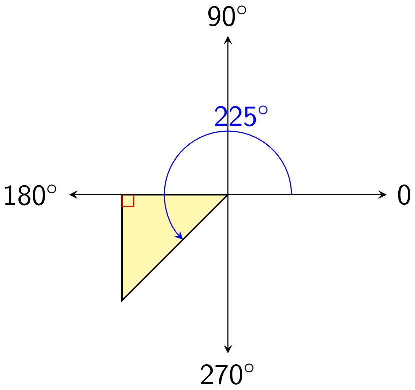
$$\tan 135^\circ = -1 \qquad \cot 135^\circ = -1$$

Example 2b

(b) $225^\circ = \frac{5\pi}{4}$



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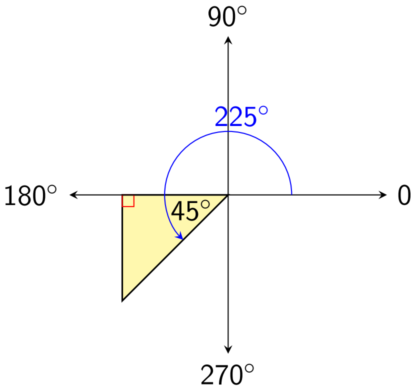
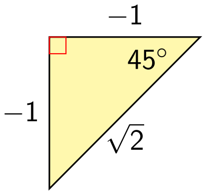


Example 2b



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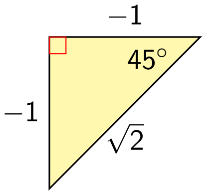
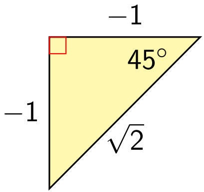


$\sin 225^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Example 2b



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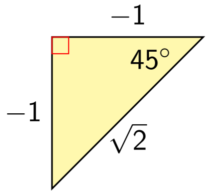

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$$\tan 225^\circ = \frac{-1}{-1} = 1$$

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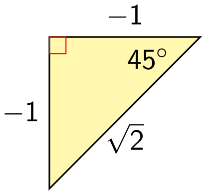

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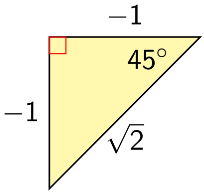
$\csc 225^\circ = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

Example 2b



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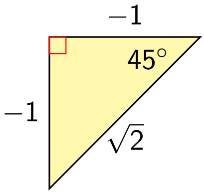

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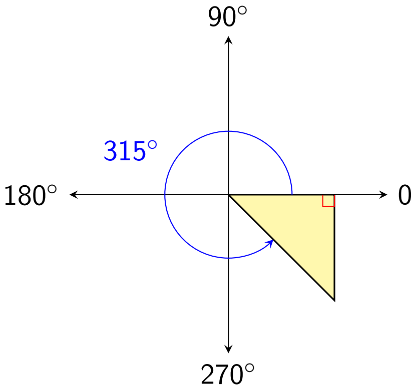
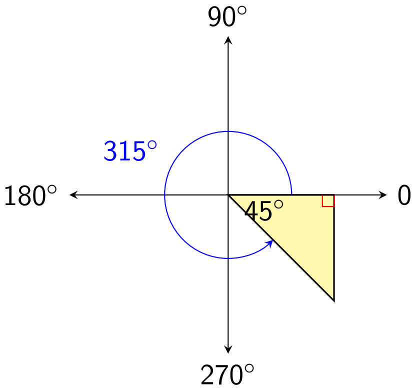
Example 2c

(c) $315^\circ = \frac{7\pi}{4}$

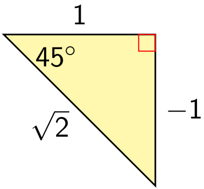


Example 2c

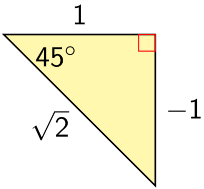
(c) $315^\circ = \frac{7\pi}{4}$



Example 2c

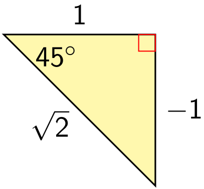


Example 2c



$\sin 315^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

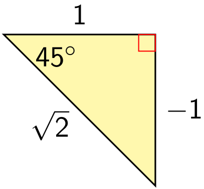
Example 2c



$$\sin 315^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos 315^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Example 2c



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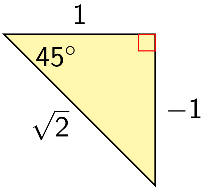
Example 2c



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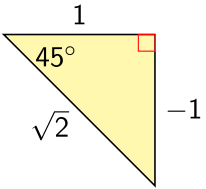
$$\tan 315^\circ = \frac{-1}{1} = -1$$

$$\csc 315^\circ = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

Example 2c



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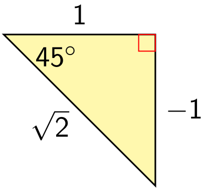
Example 2c



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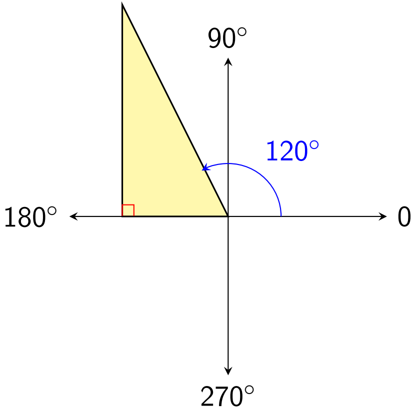
$\cot 315^\circ = \frac{1}{-1} = -1$

Example 2d

(d) $120^\circ = \frac{2\pi}{3}$



Example 2d

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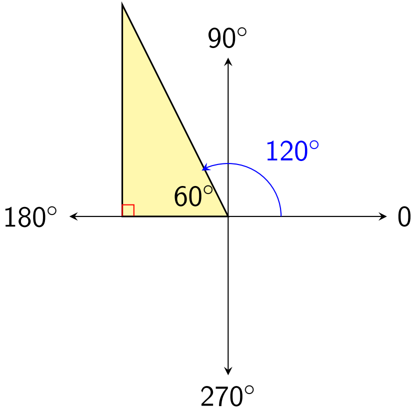
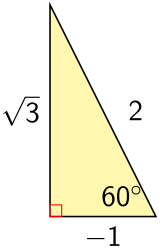
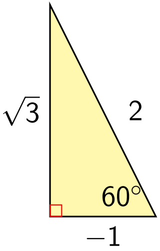


Example 2d



Example 2d



$\sin 120^\circ = \frac{\sqrt{3}}{2}$

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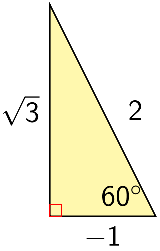

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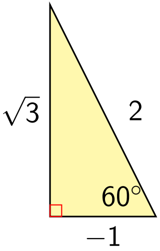

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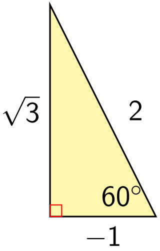

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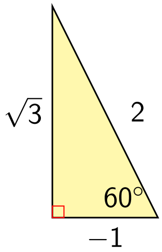

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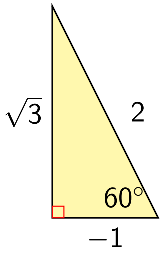

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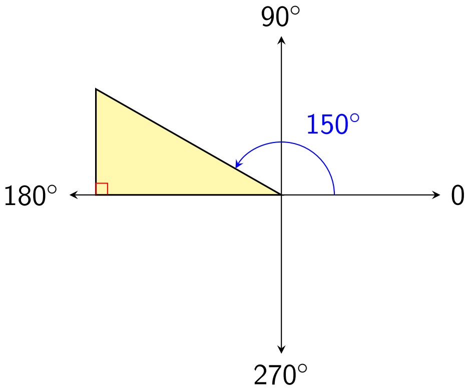
Example 2e

(e) $150^\circ = \frac{5\pi}{6}$



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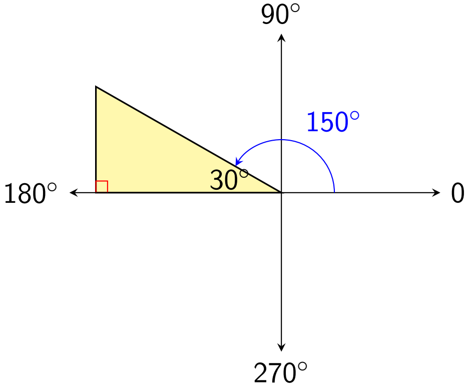

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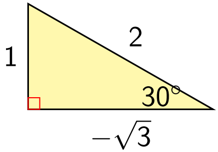

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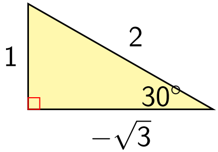

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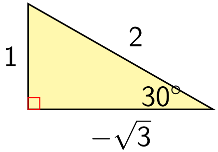

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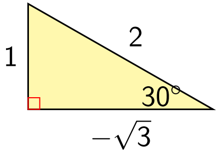

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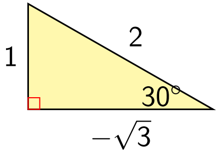

$\sin 150^\circ = \frac{1}{2}$

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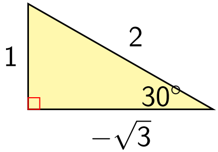

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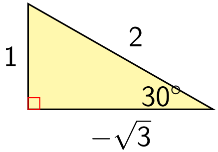

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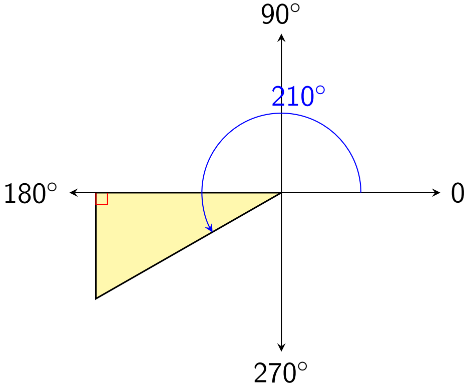
Example 2f

(a) $210^\circ = \frac{7\pi}{6}$



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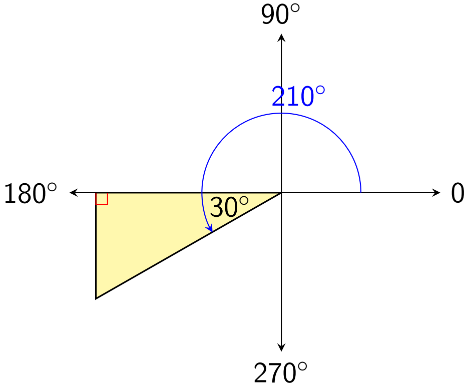

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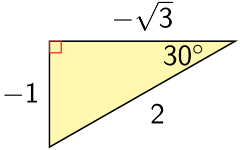

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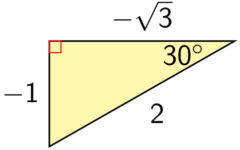

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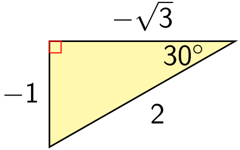

$\sin 210^\circ = \frac{-1}{2}$

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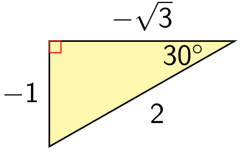

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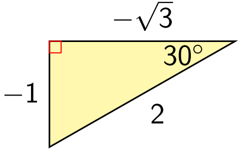

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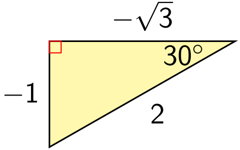

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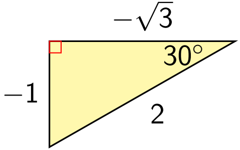

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Quadrantal Angles

A quadrantal angle is an angle whose terminal side lies on an axis.

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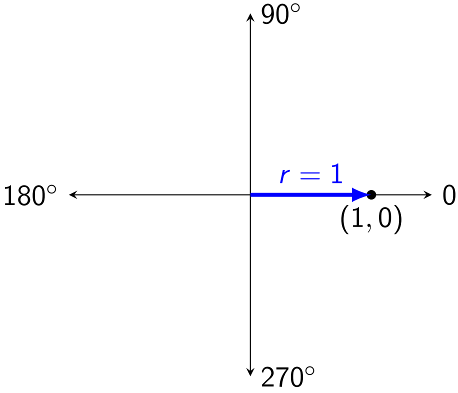
Finding the values of the trig functions for quadrantal angles can be found by using any point on the axes.

Quadrantal Angles

For simplicity, we will use combinations of 0s and 1s. Below is the point we will use for 0°.



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Example 3a

Find the exact values of each of the 6 trig functions for the following angles.

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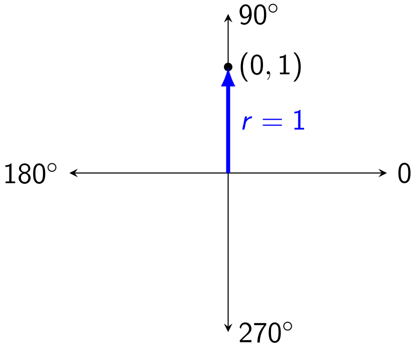

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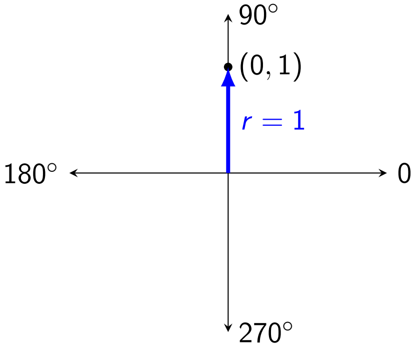

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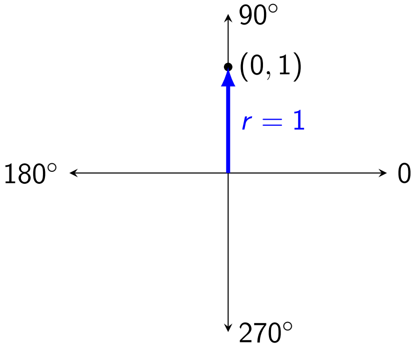

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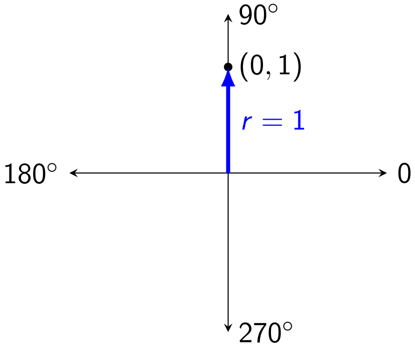

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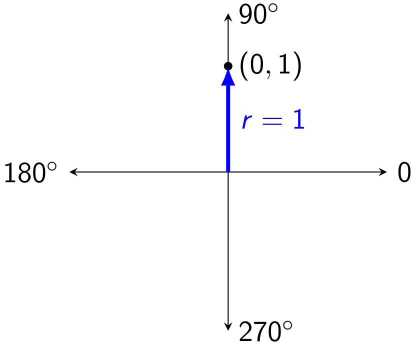

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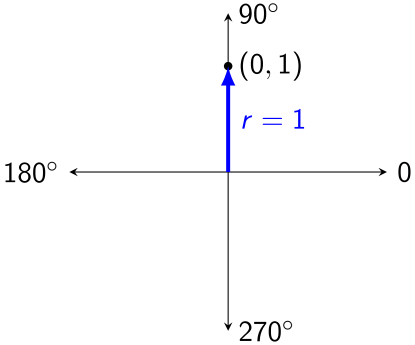

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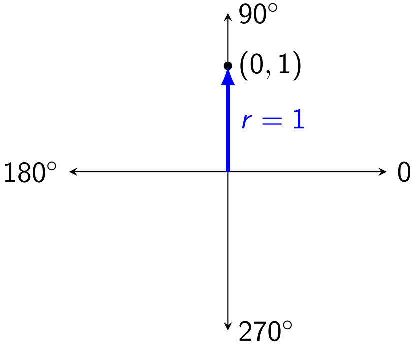

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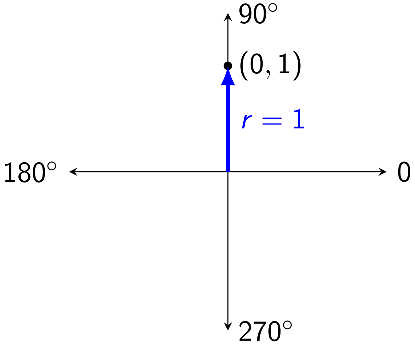

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Example 3b

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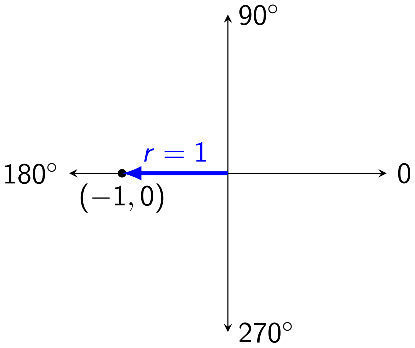

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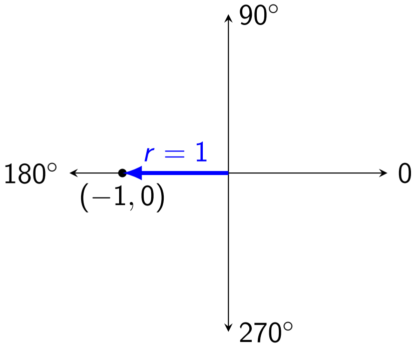

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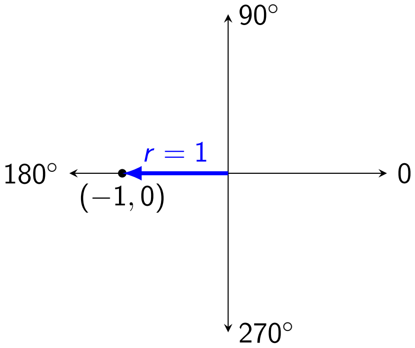

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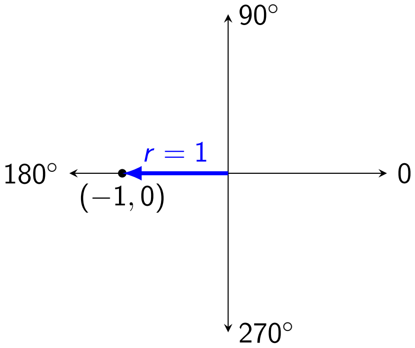

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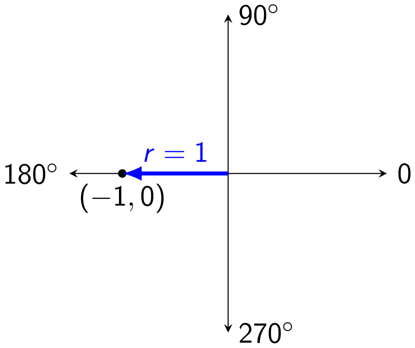

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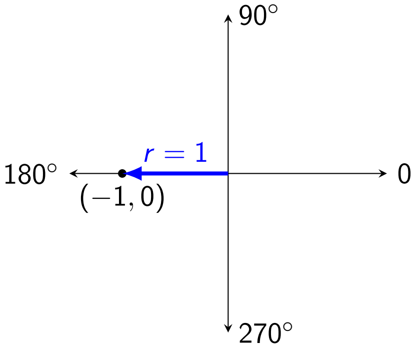

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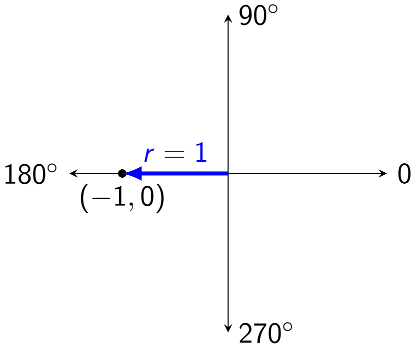

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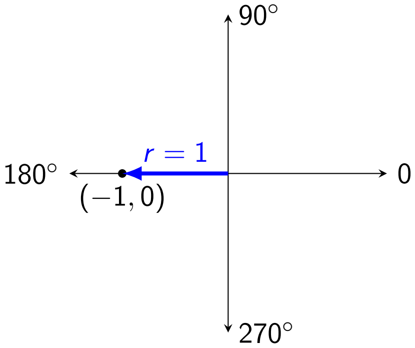

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Example 3c

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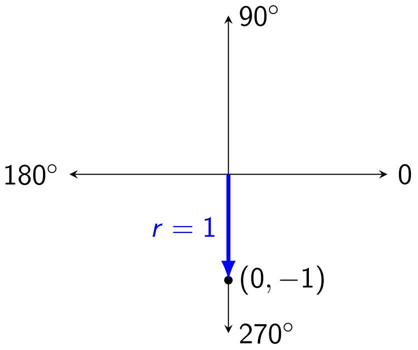

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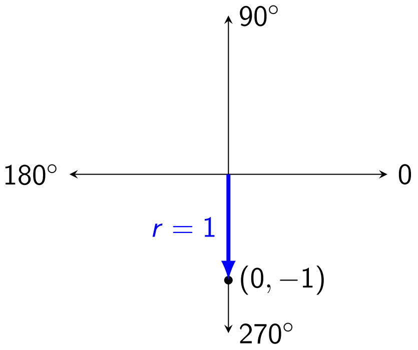

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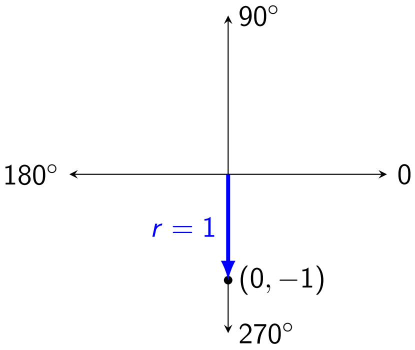

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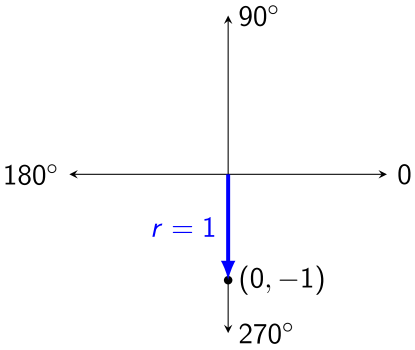

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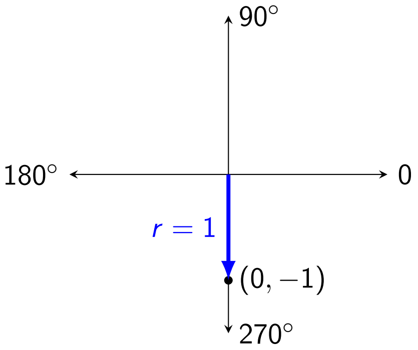
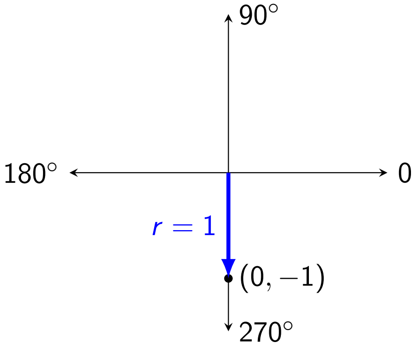
$$\tan 270^\circ = \frac{-1}{0} = \text{undefined}$$

Example 3c

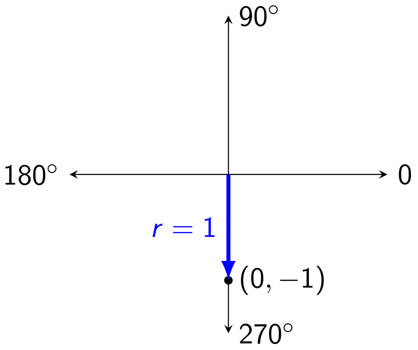


Example 3c



$$\csc 270^\circ = \frac{1}{-1} = -1$$

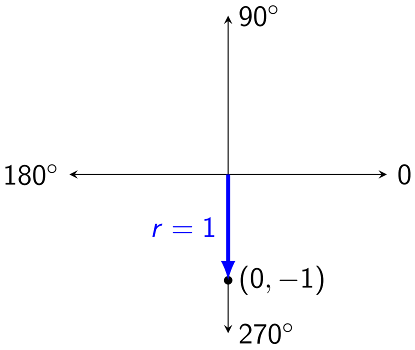
Example 3c



$$\csc 270^\circ = \frac{1}{-1} = -1$$

$$\sec 270^\circ = \frac{1}{0} = \text{undefined}$$

Example 3c



$$\csc 270^\circ = \frac{1}{-1} = -1$$

$$\sec 270^\circ = \frac{1}{0} = \text{undefined}$$

$$\cot 270^\circ = \frac{0}{-1} = 0$$

Angles Not Between 0 and 360° or 0 and 2π

For angles not within 1 standard rotation, use coterminal angles to bring the angle within 1 rotation and then apply the rules of the previous notes.

For instance, $\cos\left(\frac{9\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$