Polynomial and Rational Inequalities

Objectives

1. Solve polynomial inequalities

2. Solve rational inequalities

Polynomial Inequalities

When solving polynomial inequalities, first find the zeros of the polynomial (may have to get $=$ to 0 first).

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Then, set up a number line and use test values.

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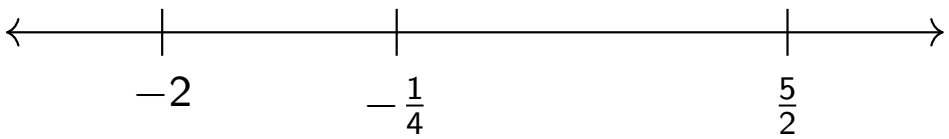

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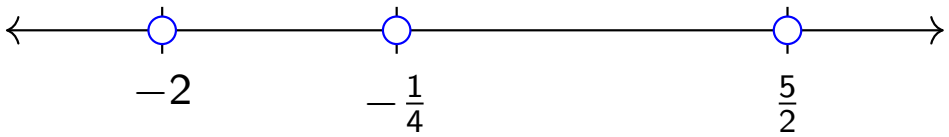

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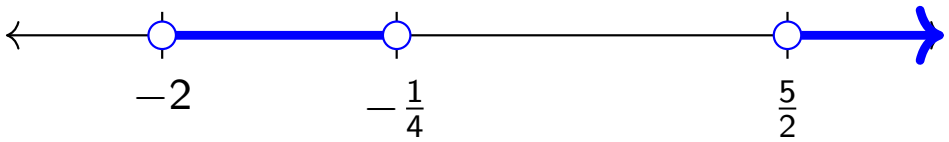

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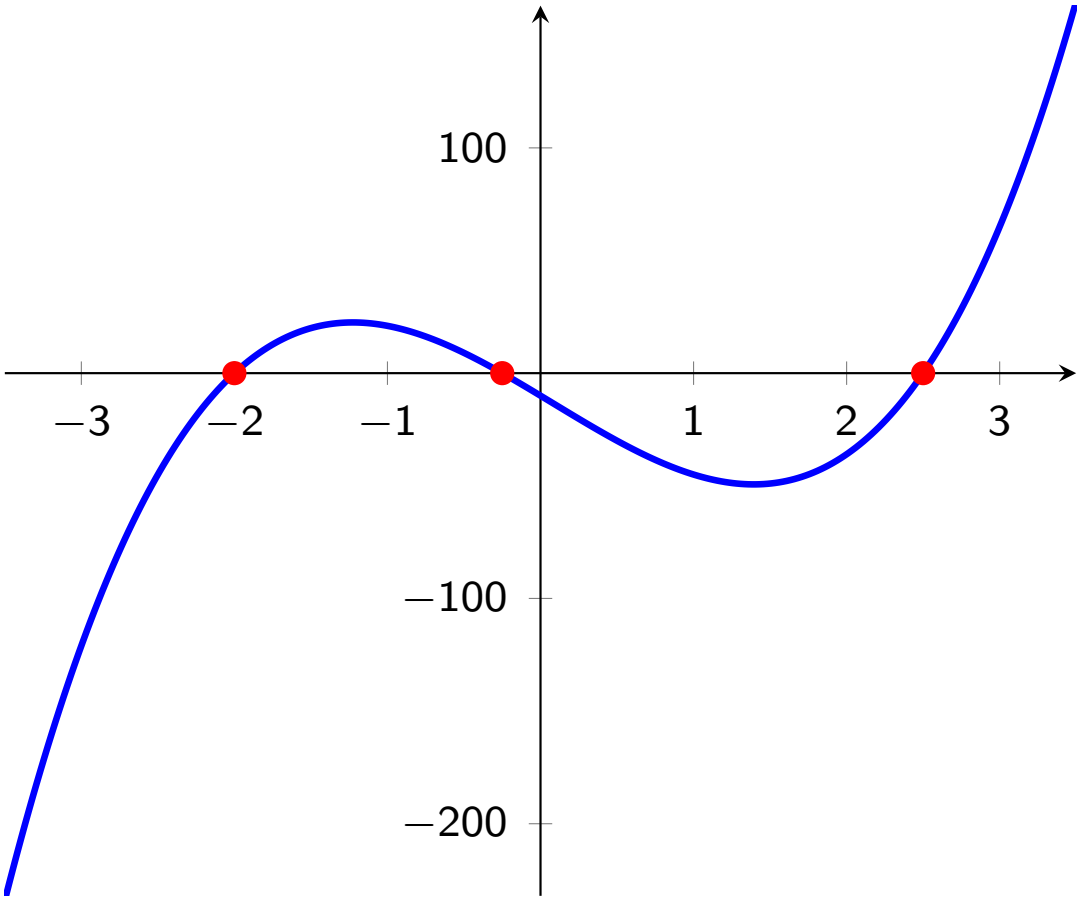

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When solving rational inequalities, your critical values will be where the **denominator = 0** and the **solution to the inequality as an equation**.

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Note: The critical values of the **denominator** will always be open circles on the number line.

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Solve each. Write your answers using interval notation.

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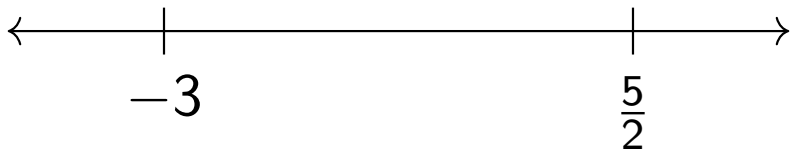

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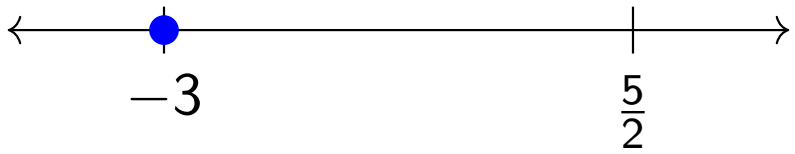

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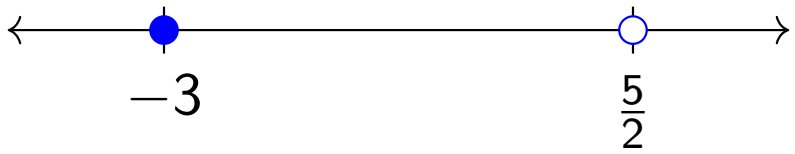

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$$\left[-3, \frac{5}{2}\right)$$

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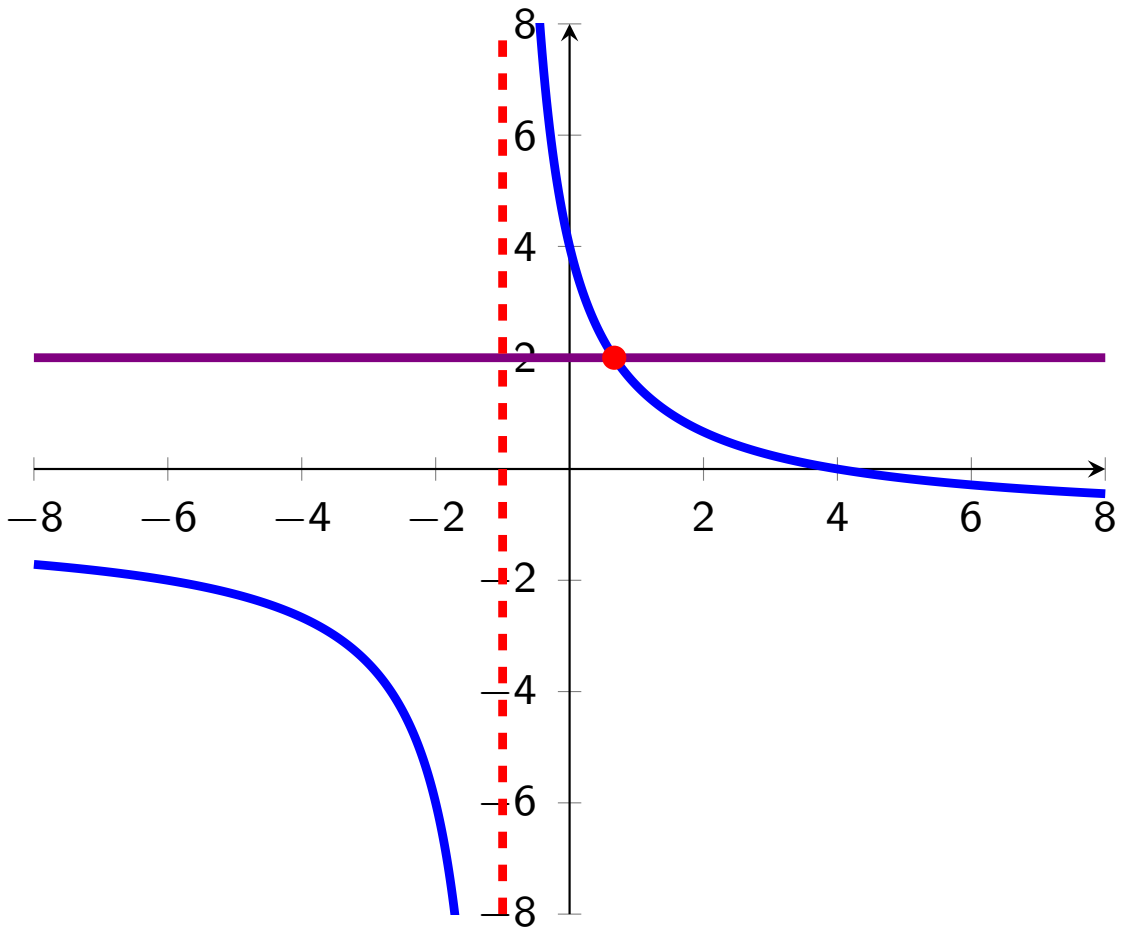

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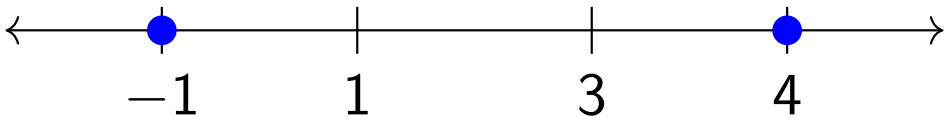

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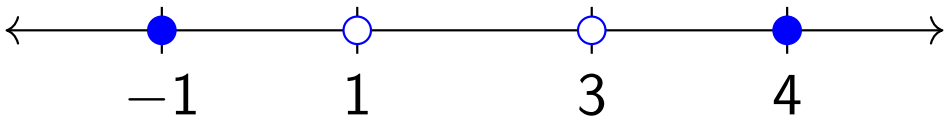

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$$(-\infty, -1] \cup (1, 3) \cup [4, \infty)$$

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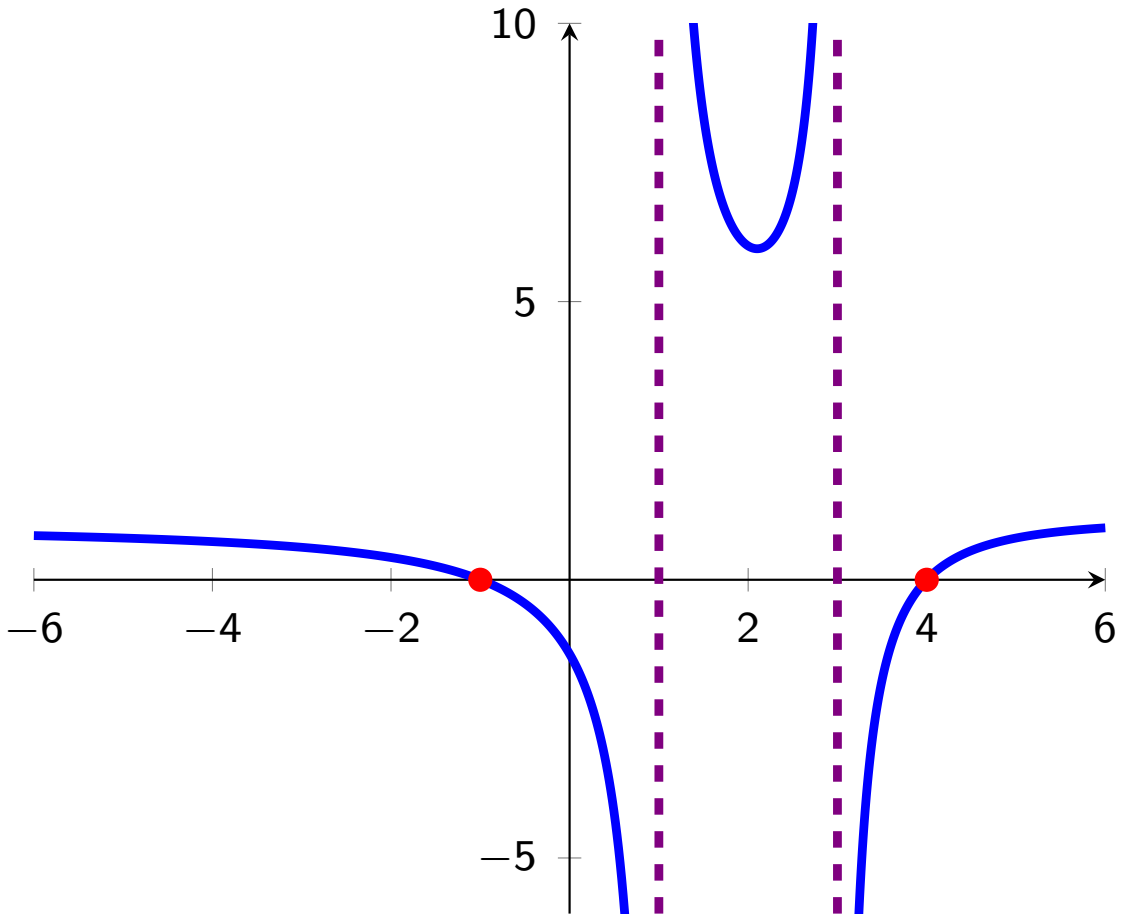

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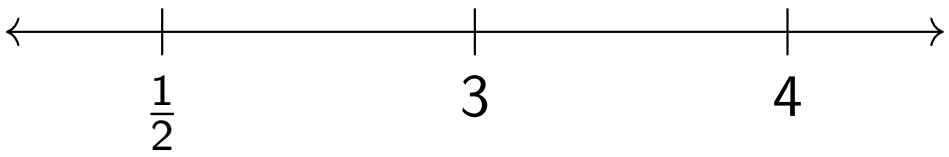

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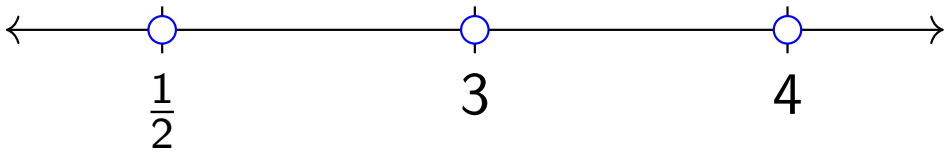

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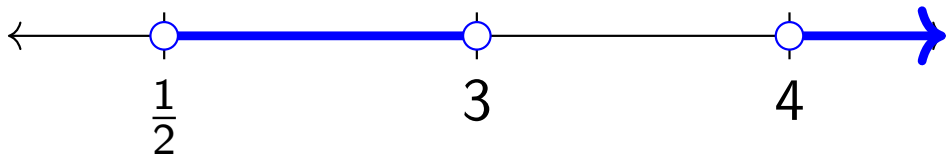

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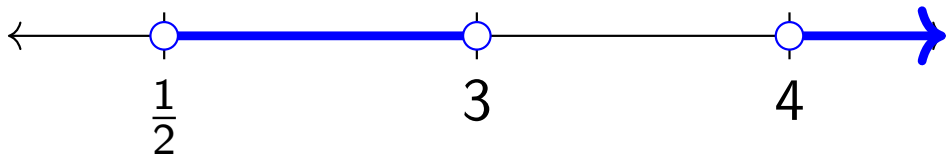

$$\left(\frac{1}{2}, 3\right) \cup (4, \infty)$$