Logarithmic Equations

Objectives

1. Solve logarithmic equations

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General Techniques for Solving Logarithmic Equations

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    - If convenient, express both sides as logs with the same base and equate the arguments of the log functions.
    - Else, rewrite the log equation as an exponential equation.

For Instance

$$\log_2(x) = 3$$

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$$2^3 = x$$

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$$2^3 = x$$

$$x = 8$$

Domain Issues

*****Important*****

The domain of $\log_b(x)$ is $x > 0$

Check your answers!!!

Example 1

Solve each. Round your answers to 4 decimal places.

(a) $\log_{117}(1 - 3x) = \log_{117}(x^2 - 3)$

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$$1-3x = x^2-3 \qquad \text{Equality Prop.}$$

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(a) $\log_{117}(1-3x) = \log_{117}(x^2-3)$

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$$x^2+3x-4=0$$

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(a) $\log_{117}(1-3x) = \log_{117}(x^2-3)$

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$$x = -4,\ 1$$

Example 1

Checking $x = -4$:

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$x = e + 3$

$x \approx 5.7183$

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$-x^2 - x + 12 = 6$ Write in expon. form

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$$2x^2 - 7x + 3 = 7 \qquad \text{Write in expon. form}$$

$$2x^2 - 7x - 4 = 0$$

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$$x = -\frac{1}{2},\ 4$$

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$\log_2(x+3) - \log_2(6-x) = 3$

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$$\log_2(x+3) - \log_2(6-x) = 3$$

$$\log_2\left(\frac{x+3}{6-x}\right) = 3 \qquad \text{Quotient Prop.}$$

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$$\frac{x+3}{6-x} = 2^3 \qquad \text{Write in expon. form}$$

$$\frac{x+3}{6-x} = 8 \qquad 2^3 = 8$$

$$x+3 = 8(6-x) \qquad \text{Eliminate the fraction}$$

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$$x = 1 \pm \sqrt{3}$$

Example 1 $1 + 2\log_4(x+1) = 2\log_2(x)$

$x = 1 + \sqrt{3}$ $x = 1 - \sqrt{3}$

Example 1 $1 + 2\log_4(x+1) = 2\log_2(x)$

$x = 1 + \sqrt{3}$ $\qquad$ $x = 1 - \sqrt{3}$

$x \approx 2.7321$ $\qquad$ $x \approx -0.7321$

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