Linear Functions and Slope

Objectives

1. Calculate the slope of a line connecting two points
2. Write the point-slope form of a line
3. Write the equation of a line in slope-intercept form
4. Determine the average rate of change of a function
5. Find the least-squares regression line

Slope

The slope of the line connect points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

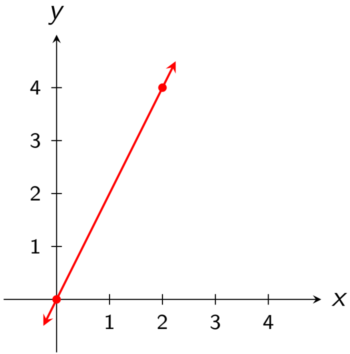
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

provided $x_2 \neq x_1$

Example 1

Find the slope of the line connecting each pair of points.

(a) $(0, 0)$ and $(2, 4)$



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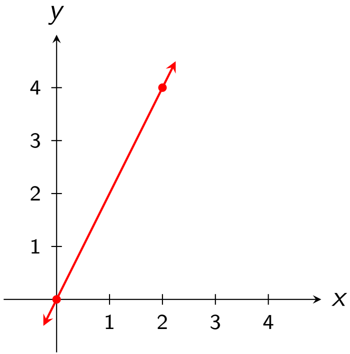

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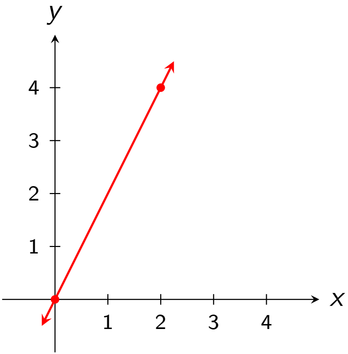

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 0}{2 - 0}$$

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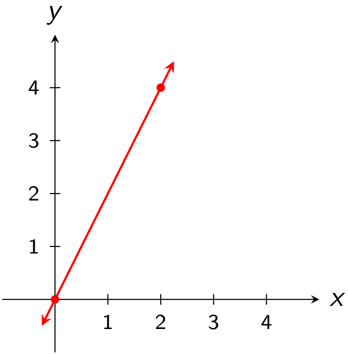


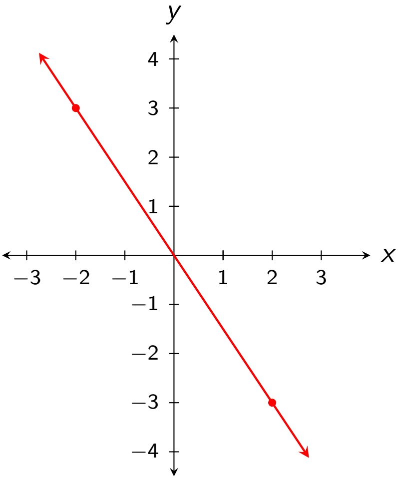
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 0}{2 - 0}$$

$$= 2$$

Example 1

(b) $(-2, 3)$ and $(2, -3)$



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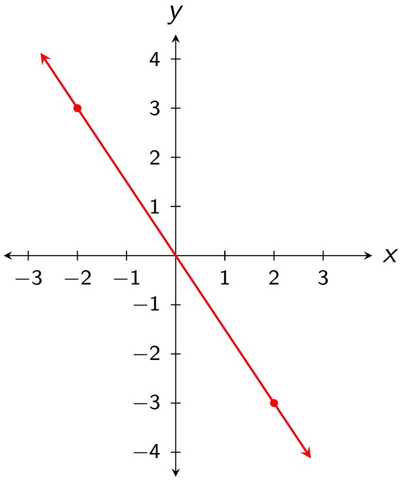

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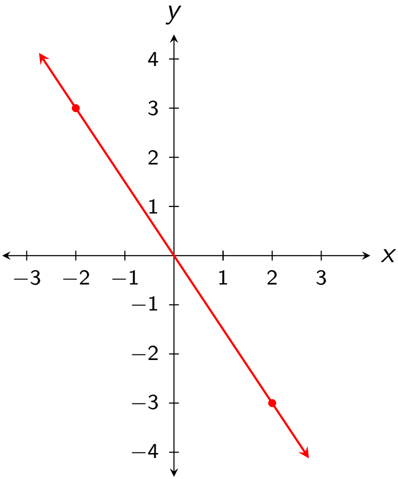

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3 - 3}{2 - (-2)}$$

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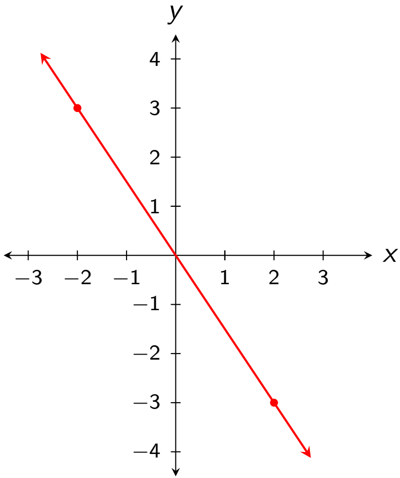

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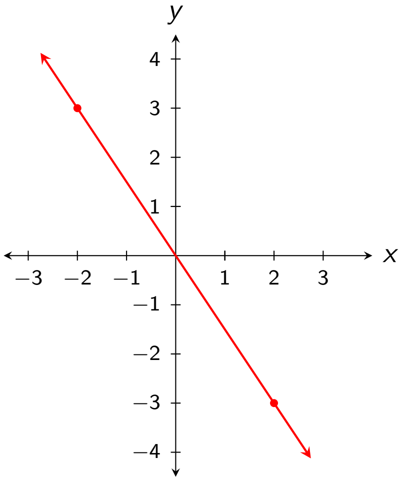

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

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$$= -\frac{3}{2}$$

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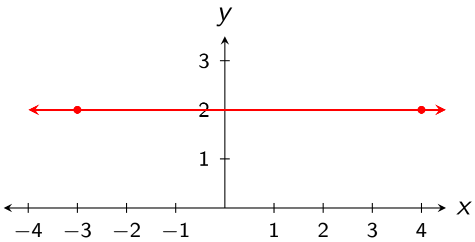
(c) $(-3, 2)$ and $(4, 2)$



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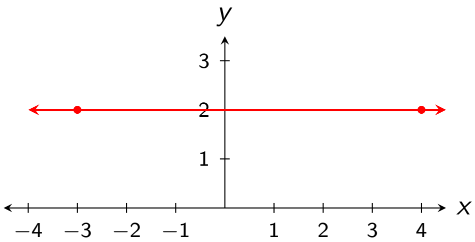

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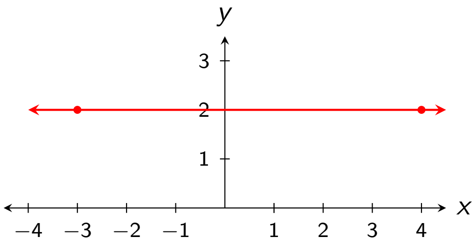

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

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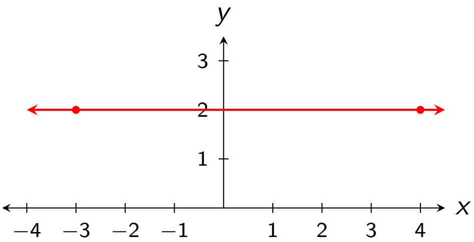

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$$= \frac{0}{7}$$

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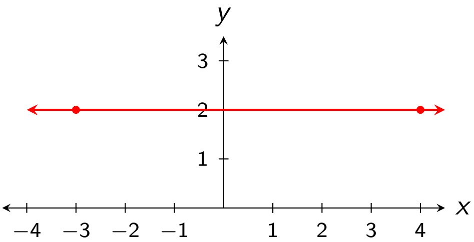

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$$= 0$$

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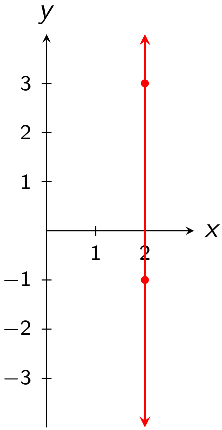
(d) $(2, 3)$ and $(2, -1)$



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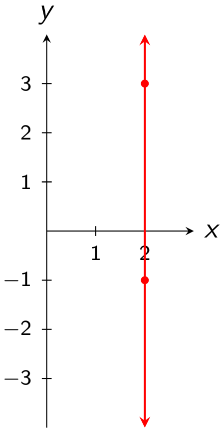

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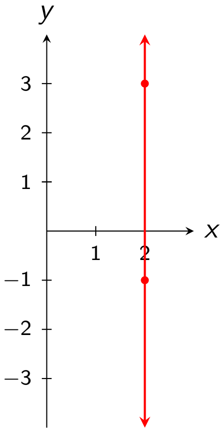

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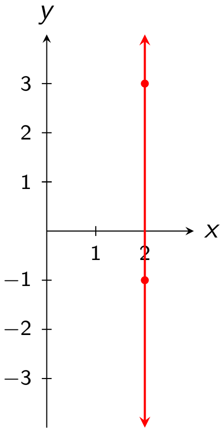

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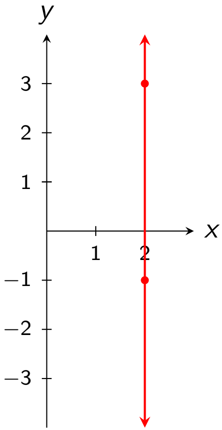

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1 - 3}{2 - 2}$$

$$= \frac{-4}{0}$$

$$= \text{undefined}$$

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Point-Slope Form of a Line

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Slope-Intercept Form

The slope-intercept form of the equation of a line is

$$y = mx + b$$

where b is the y-intercept.

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To convert from point-slope form to slope-intercept form, solve the point-slope form for y.

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Convert $y - 3 = -\frac{2}{3}(x+1)$ to slope-intercept form.

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$$y - 3 = -\frac{2}{3}x - \frac{2}{3}$$

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Convert $y-3=-\frac{2}{3}(x+1)$ to slope-intercept form.

$$y-3=-\frac{2}{3}(x+1)$$

$$y-3=-\frac{2}{3}x-\frac{2}{3}$$

$$y=-\frac{2}{3}x+\frac{7}{3}$$

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Average Rate of Change

The average rate of change over an interval $[a, b]$ of a function is the slope of the line connecting the endpoints of the interval.

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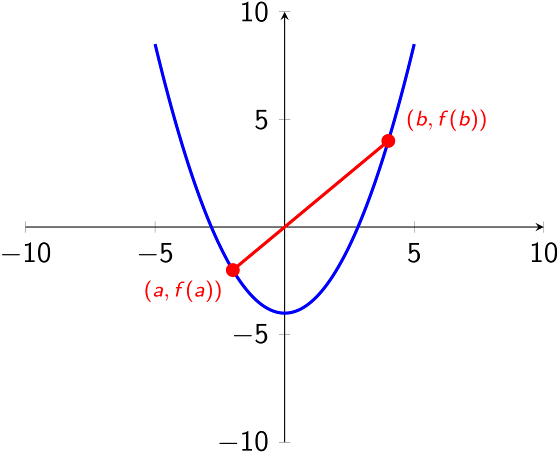
$$\frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

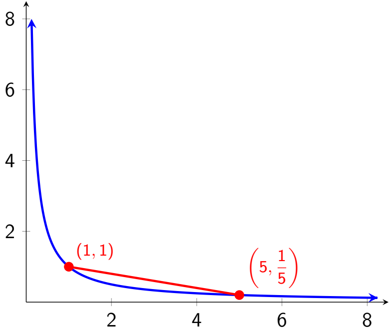
Average Rate of Change



Example 4

Find the average rate of change for each.

(a) $\quad f(x) = \dfrac{1}{x} \quad [1, 5]$



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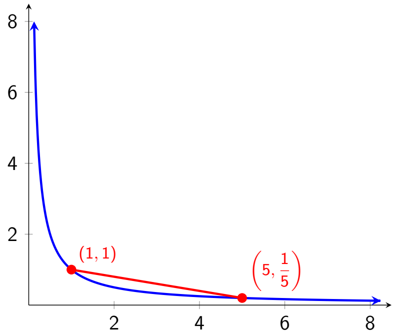

$$f(5) = \frac{1}{5} \qquad\qquad f(1) = 1$$

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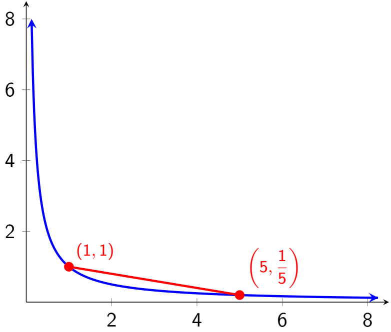

$$f(5) = \frac{1}{5} \qquad f(1) = 1$$

$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{5} - 1}{5 - 1}\left(\frac{5}{5}\right)$$

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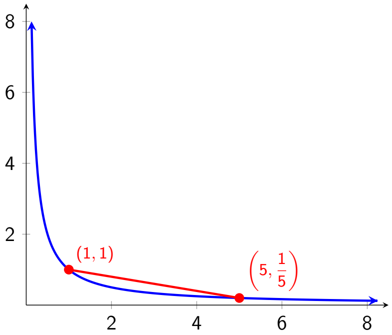

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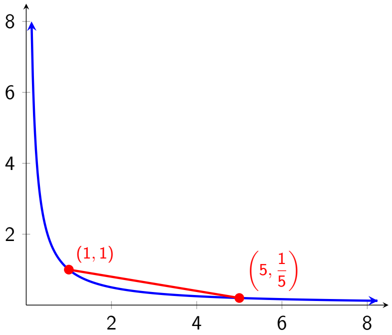

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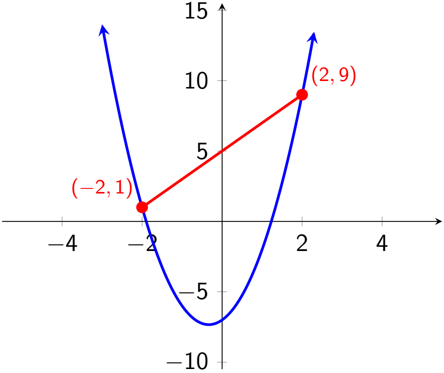
$$\frac{\Delta f}{\Delta x} = \frac{1 - 5}{25 - 5}$$

$$= -\frac{1}{5}$$

Example 4

(b) $f(x) = 3x^2 + 2x - 7 \qquad [-2, 2]$



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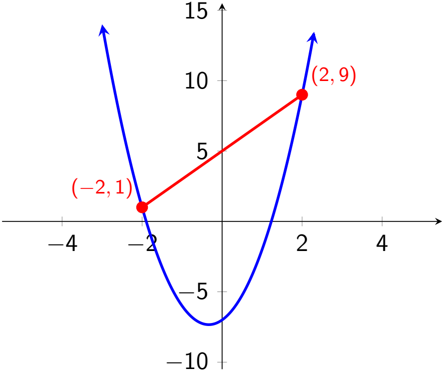

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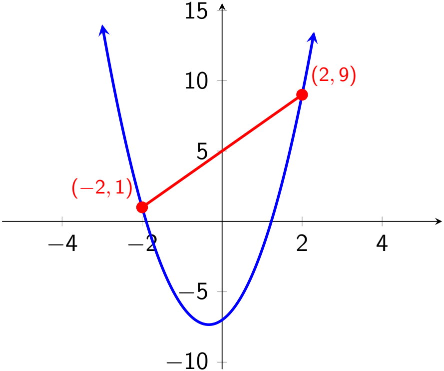

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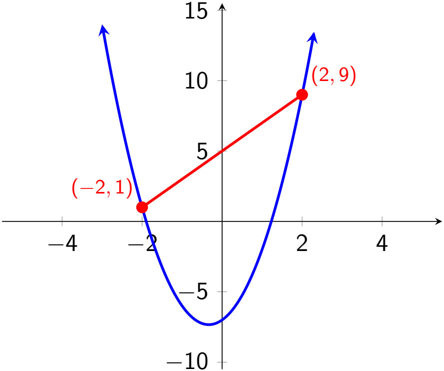

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$$\frac{\Delta f}{\Delta x} = \frac{8}{4}$$

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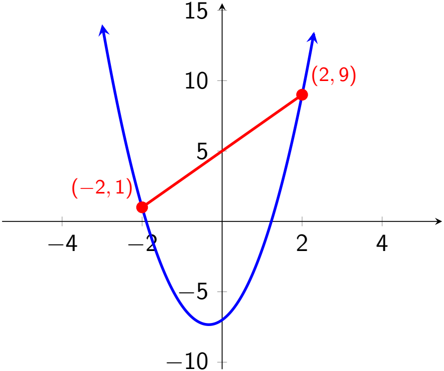

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$$= 2$$

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Least-Squares Regression Line

When we plot data points, we can sometimes create a least squares regression line to describe and predict the data.

The value of r (the correlation coefficient) determines how well the data "falls into line."

The closer r is to 1 (or -1) the better the linear fit.

Example 5

The census data for Lake County, Ohio is shown:

Year	1970	1980	1990	2000	2010
Pop	197200	212801	215499	227511	230041

(a) Find the least-squares regression line.

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The census data for Lake County, Ohio is shown:

Year	1970	1980	1990	2000	2010
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(a) Find the least-squares regression line.

$$y = 803.92x + 200532$$

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(b) Interpret the slope.

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Pop	197200	212801	215499	227511	230041

(a) Find the least-squares regression line.

$$y = 803.92x + 200532$$

(b) Interpret the slope.

Population increases by about 804 people per year.

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(c) Predict the population of Lake County in 2070.

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$$803.92x + 200532 > 250000$$

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$$\begin{aligned} 803.92x + 200532 &> 250000 \\ x &> 61.5 \end{aligned}$$

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$$\begin{aligned} 803.92x + 200532 &> 250000 \\ x &> 61.5 \end{aligned}$$

Predicted to exceed 250,000 in the year 2031.