Trig Identities

Objectives

1. Derive the Pythagorean identities
2. Derive the quotient identities
3. Use the sum and difference identities to verify other identities
4. Derive double-angle identities
5. Derive power-reducing identities

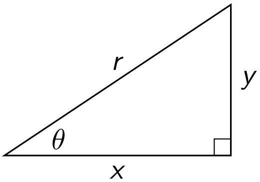
Pythagorean Identities

These are some of the most important identities in this course.

They are based on the Pythagorean Theorem.

Example 1

Verify $\cos^2\theta + \sin^2\theta = 1$



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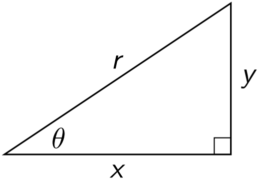

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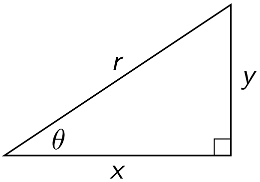

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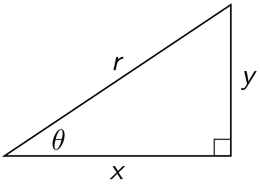

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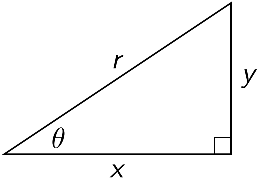

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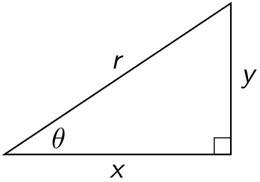

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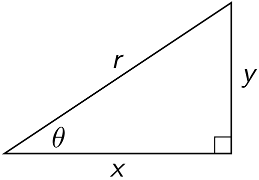

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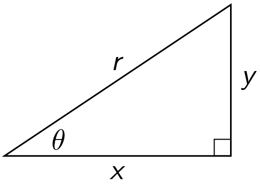

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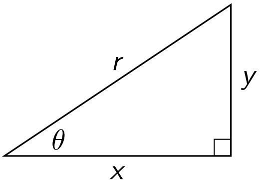

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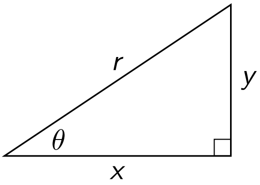

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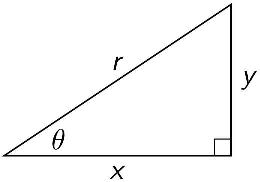

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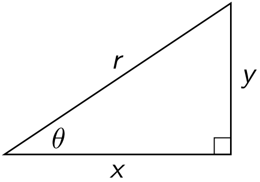

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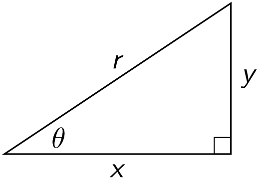

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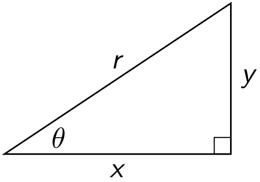

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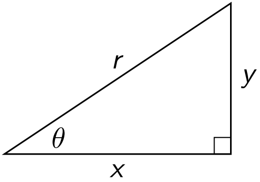

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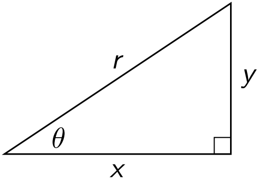

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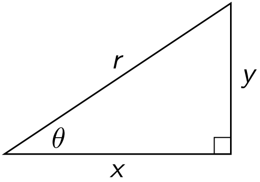

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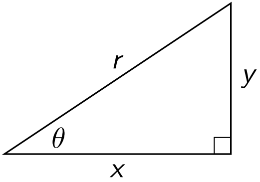

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Alternate Forms of Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

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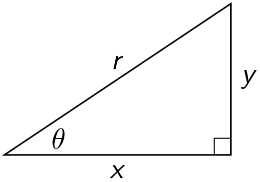
Quotient Identities

The Quotient Identities are

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Example 4

Use the triangle to verify $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$



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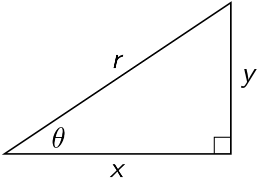

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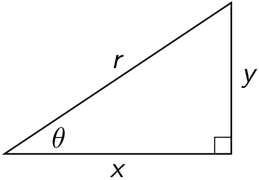

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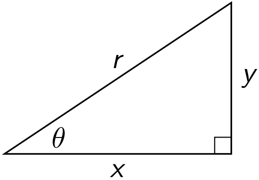

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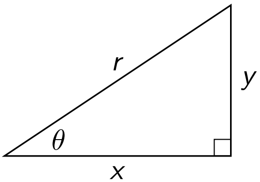

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Additional Angle Sum and Difference Identities

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$** \text{Note:} \quad \tan(A \pm B) = \frac{\sin(A \pm B)}{\cos(A \pm B)}$$

Example 5

Verify $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

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$$= 0\cos\theta + 1\sin\theta$$

Example 5

Verify $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$$\begin{aligned}
\cos\left(\frac{\pi}{2} - \theta\right) &= \cos\frac{\pi}{2} \cdot \cos\theta + \sin\frac{\pi}{2} \cdot \sin\theta \\
&= 0\cos\theta + 1\sin\theta \\
&= \sin\theta
\end{aligned}$$

Example 6

Verify $\cos(-\theta) = \cos\theta$

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$$\begin{aligned}
\cos(-\theta) &= \cos(0 - \theta) \\
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\begin{aligned}
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&= 2 \sin A \cos A
\end{aligned}
$$

Double-Angle Identities

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

Alternate Forms of $\cos(2A)$

$\cos(2A)$ has two alternate forms.

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For $\cos(2A) = 2\cos^2 A - 1$, if we solve for $\cos^2 A$, we get

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And for $\cos(2A) = 1 - 2\sin^2 A$, solving for $\sin^2 A$ gives us

$$\sin^2 A = \frac{1 - \cos(2A)}{2}$$