Right Triangle Trigonometry

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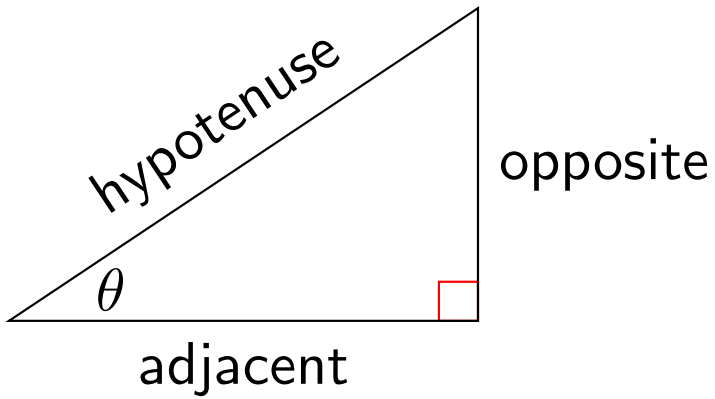
The 3 Main Trig Ratios



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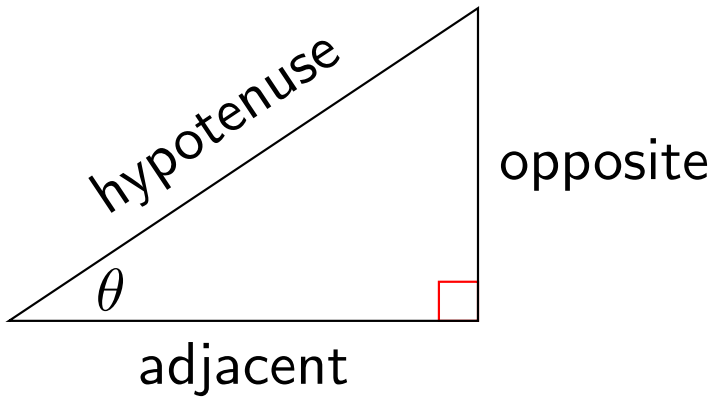

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

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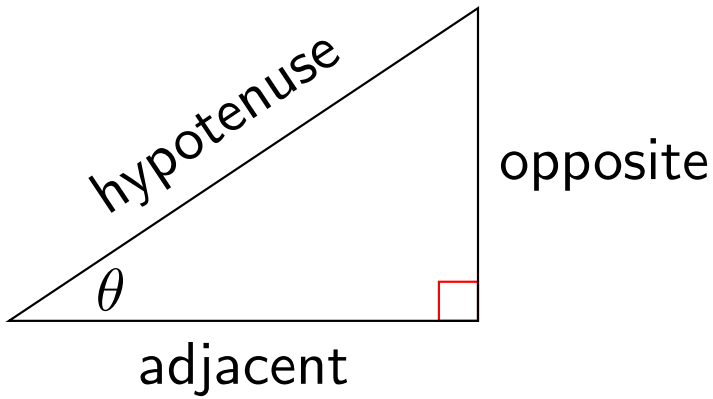

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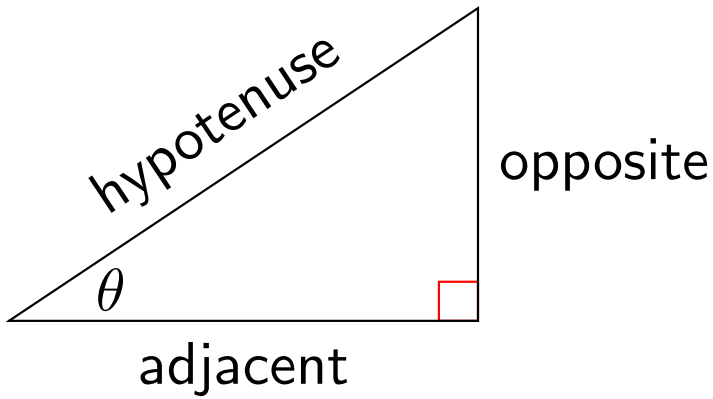

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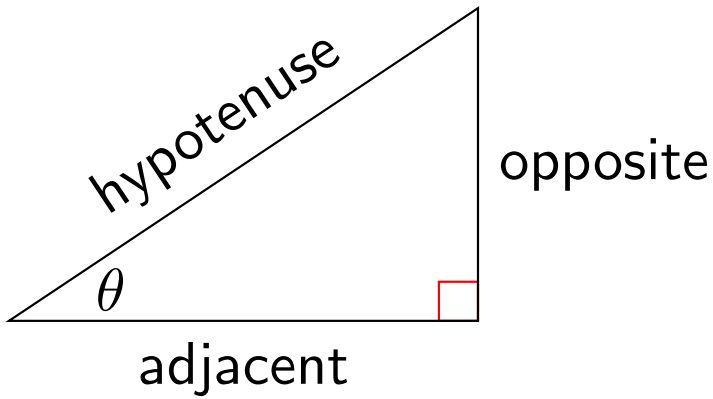

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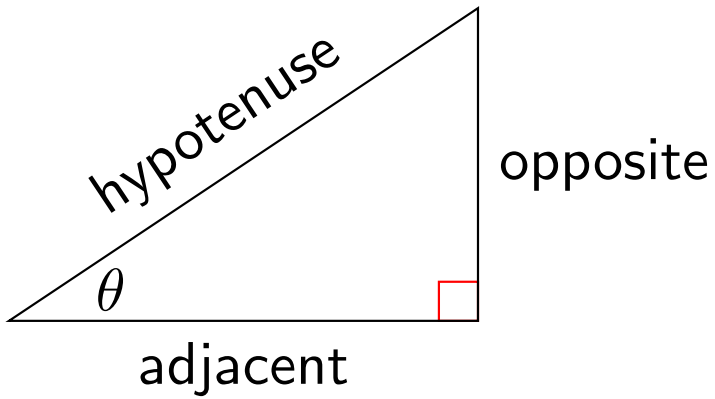
We usually remember this as SOH-CAH-TOA.

Reciprocals for SOH-CAH-TOA



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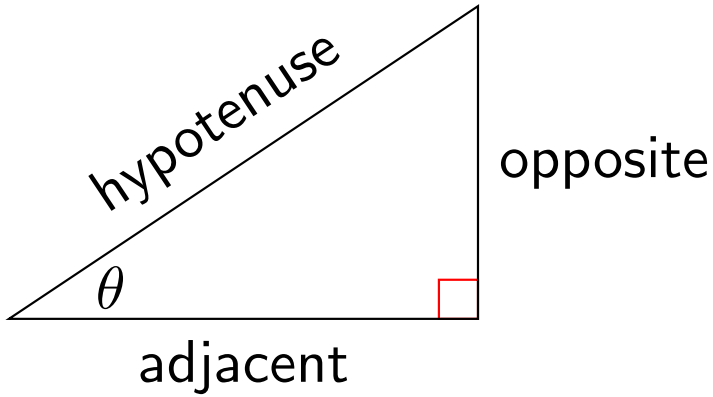

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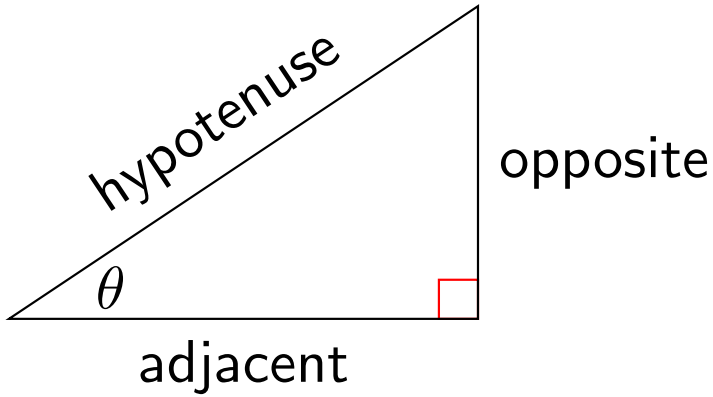

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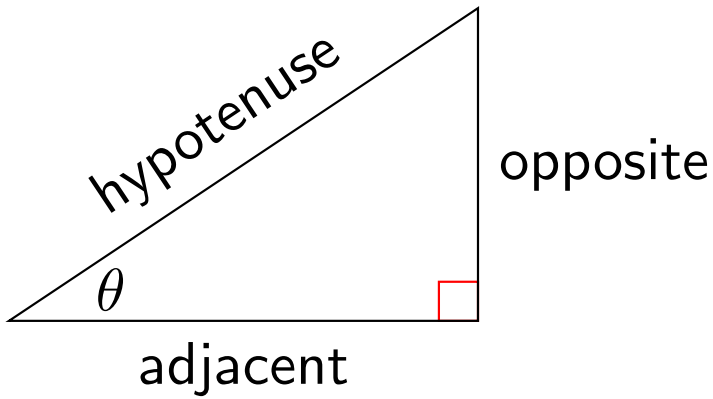

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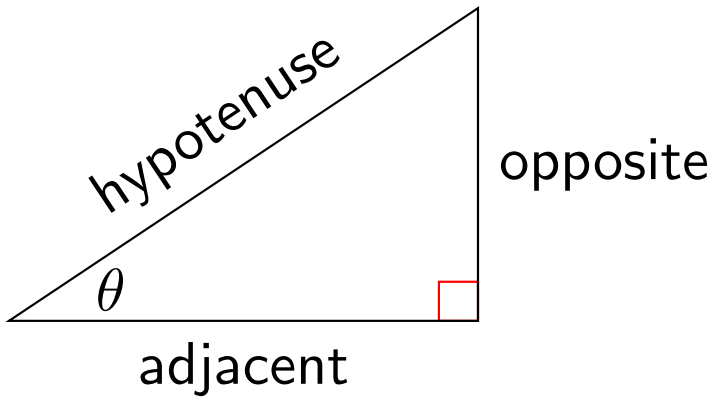

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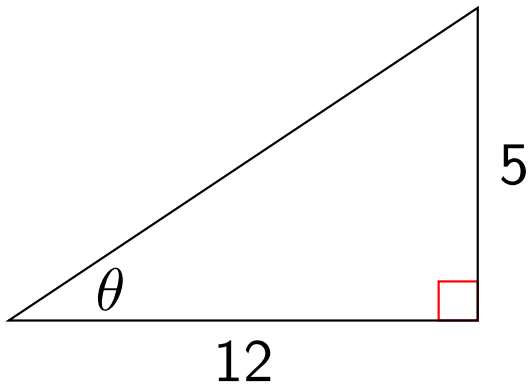
Sometimes you may need to use the Pythagorean Theorem, $a^2 + b^2 = c^2$, in order to find any missing sides.

Example 1

Find the value of each of the six trig functions of θ.

(a)



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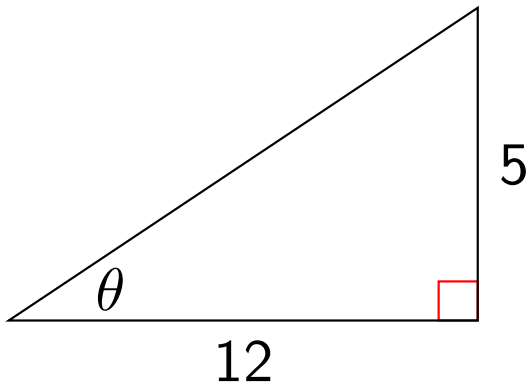

$5^2 + 12^2 = c^2$

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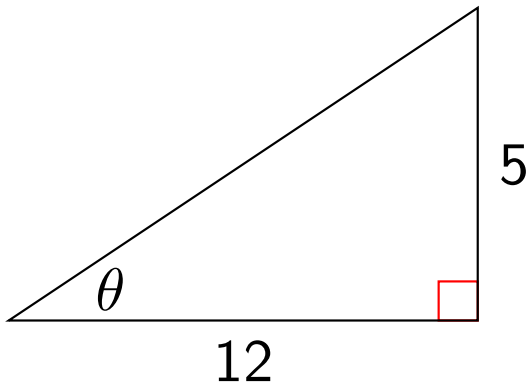

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$$c = \sqrt{169} = 13$$

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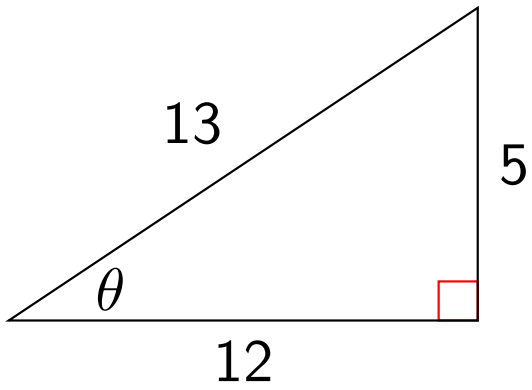

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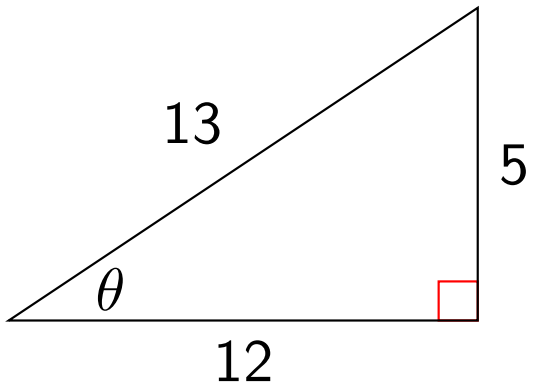

$$\sin\theta = \frac{5}{13}$$

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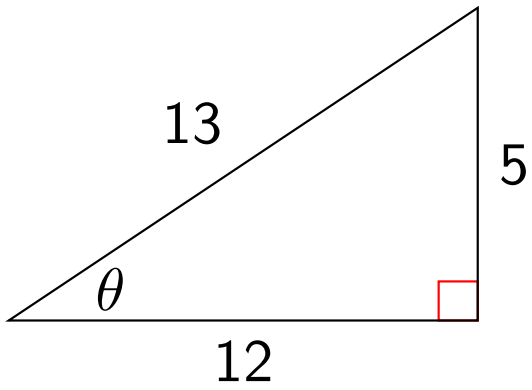

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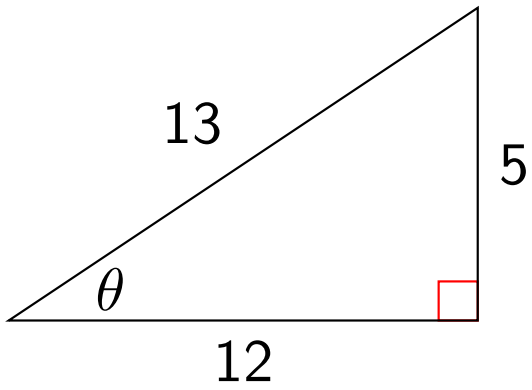

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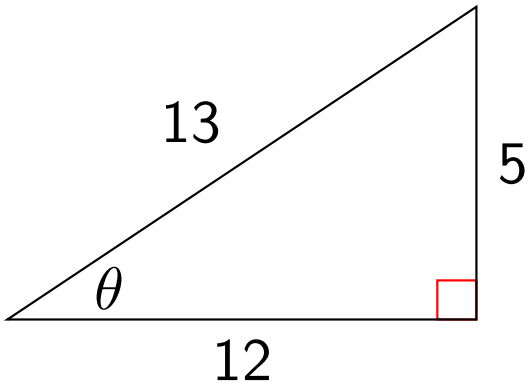

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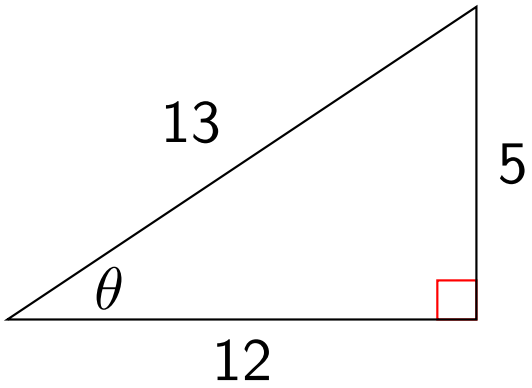

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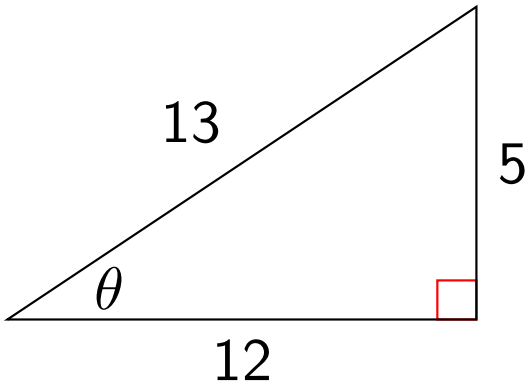

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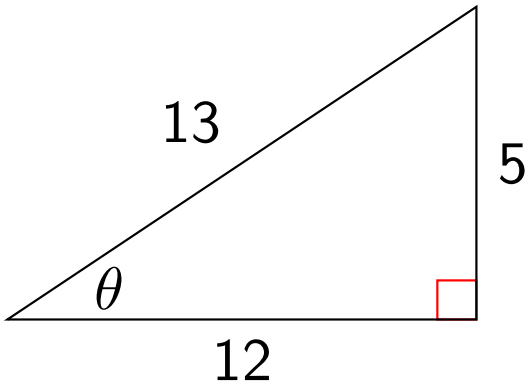

$$\csc\theta = \frac{13}{5}$$

$$\sec\theta = \frac{13}{12}$$

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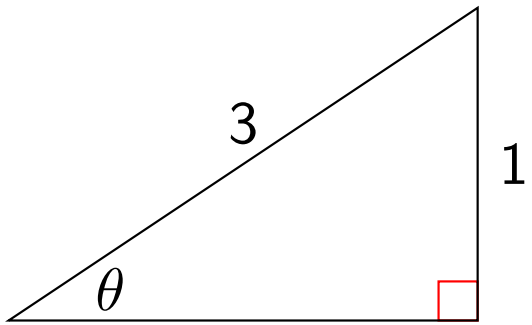

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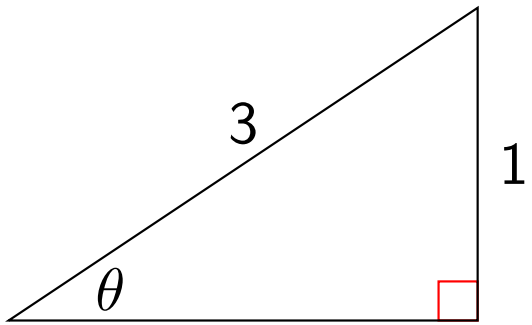

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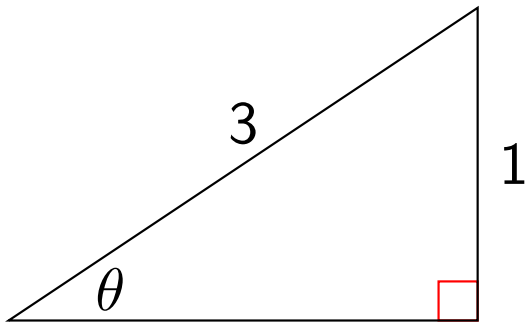

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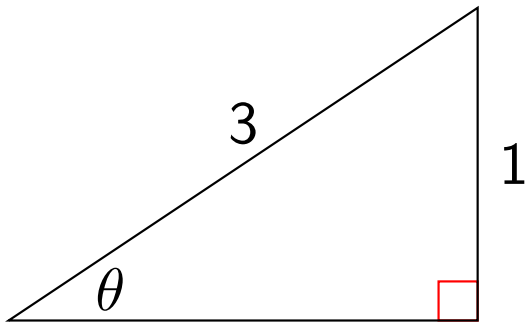

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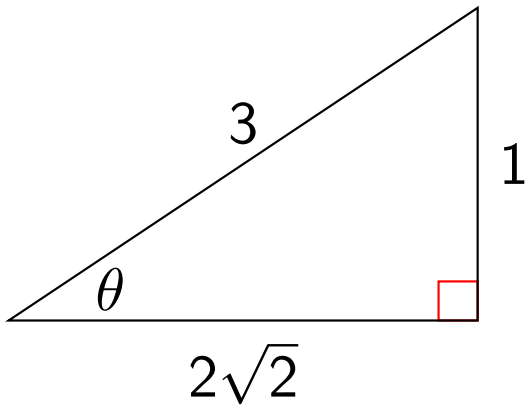

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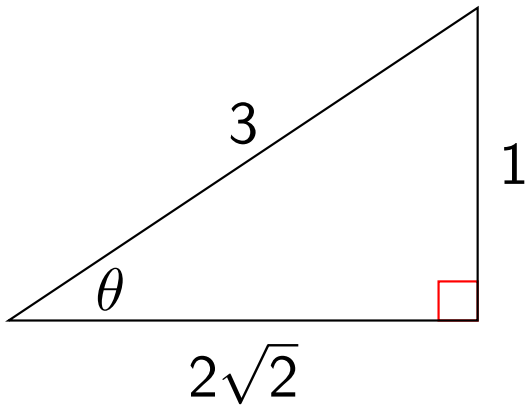

$$\sin\theta = \frac{1}{3}$$

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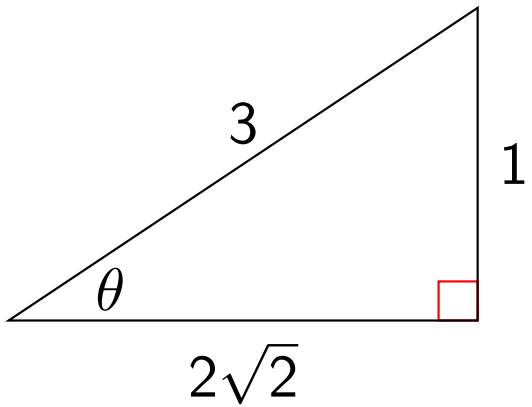

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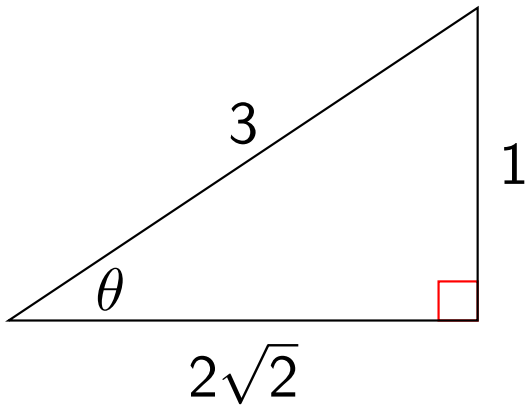

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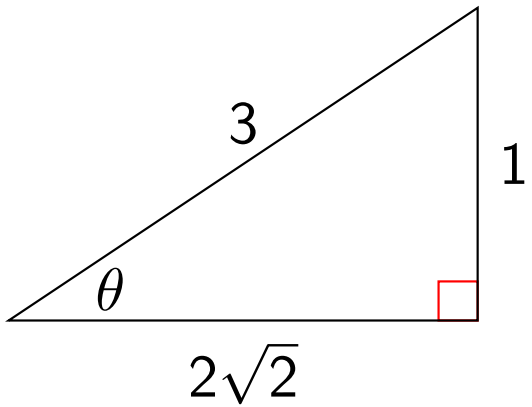

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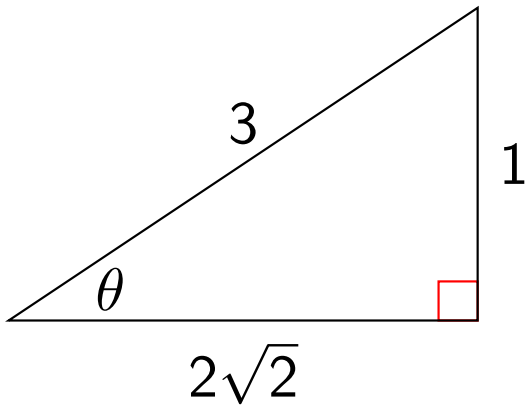

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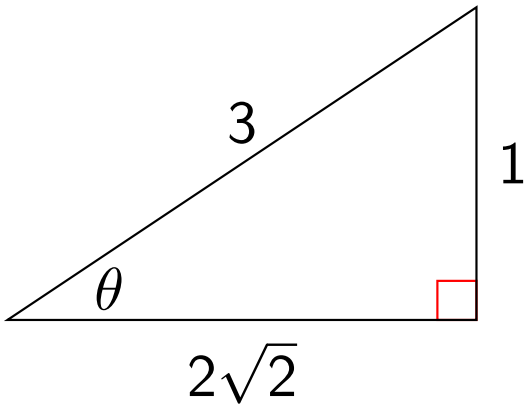

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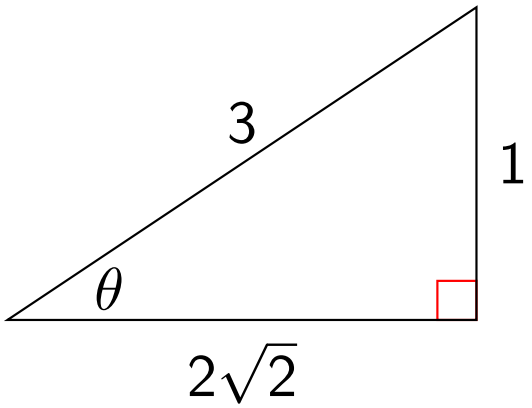

$$\csc\theta = \frac{3}{1} = 3$$

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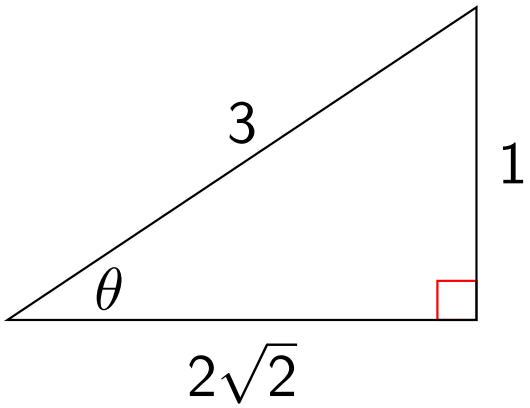

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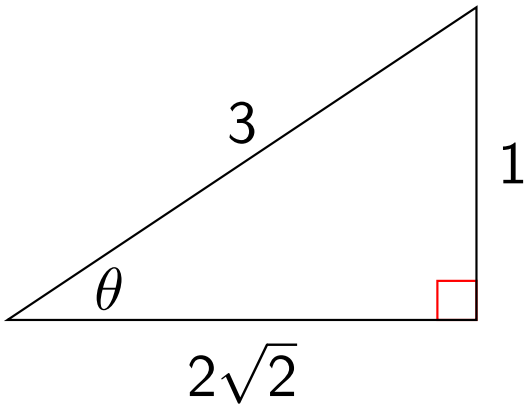

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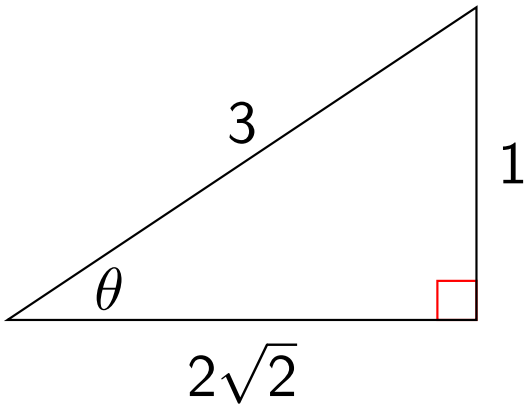

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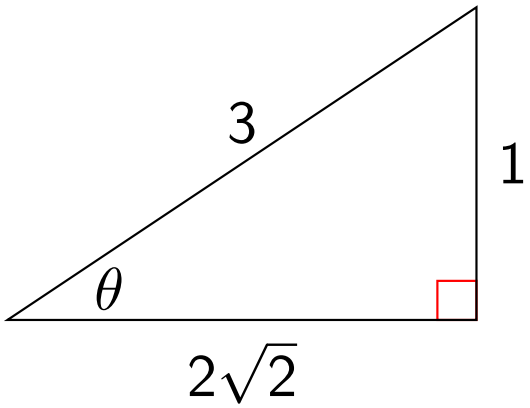

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$$\sec\theta = \frac{3}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

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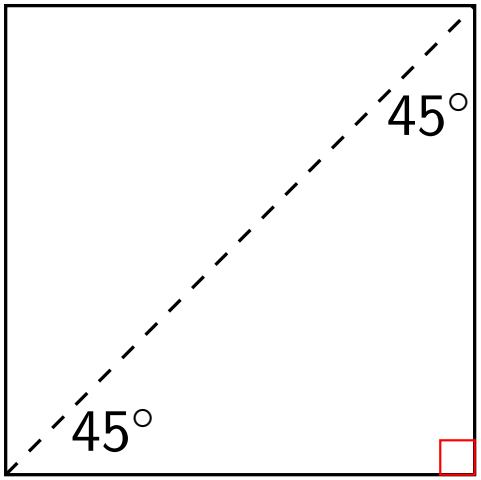
45-45-90 Triangles

45-45-90 triangles (also known as *isosceles right triangles*) can be created by drawing a diagonal across a square:



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Since each side of a square is the same length, we can use whatever length we want. For simplicity, we will use a length of 1.

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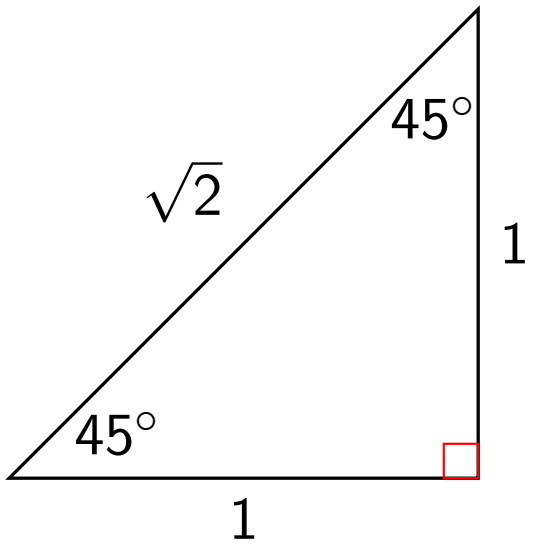
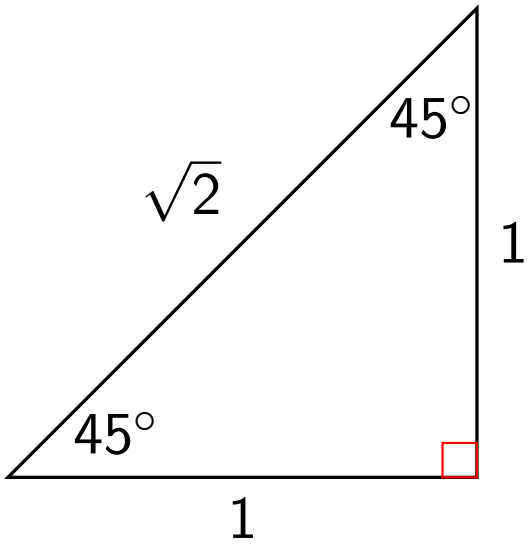


Example 2

Find the exact values of the six trig ratios for $45°$.



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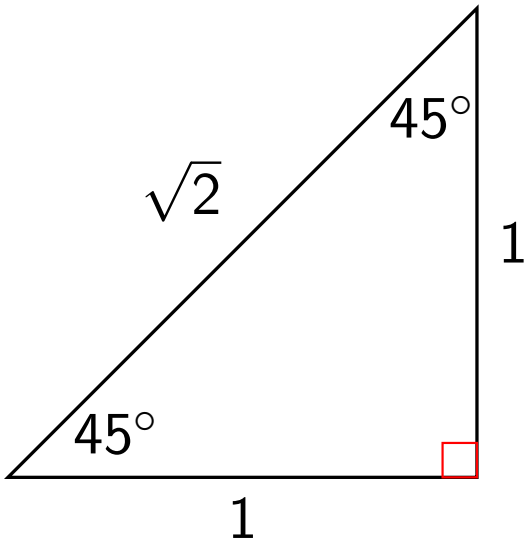

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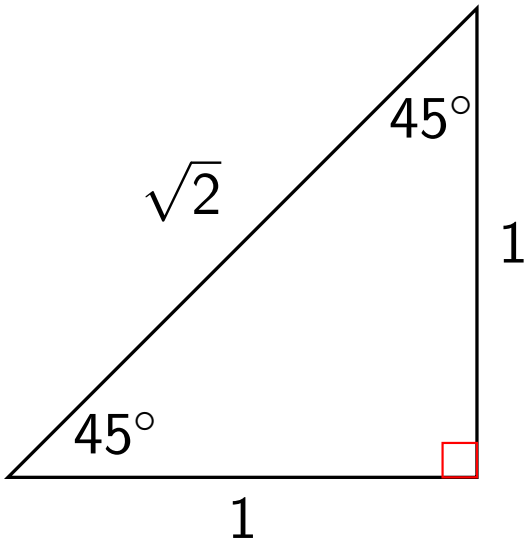

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

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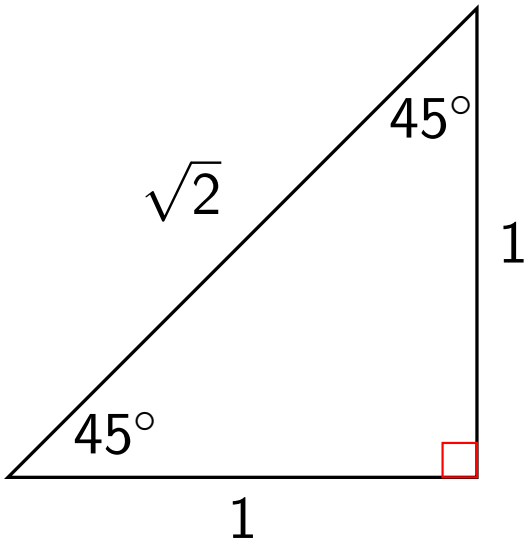


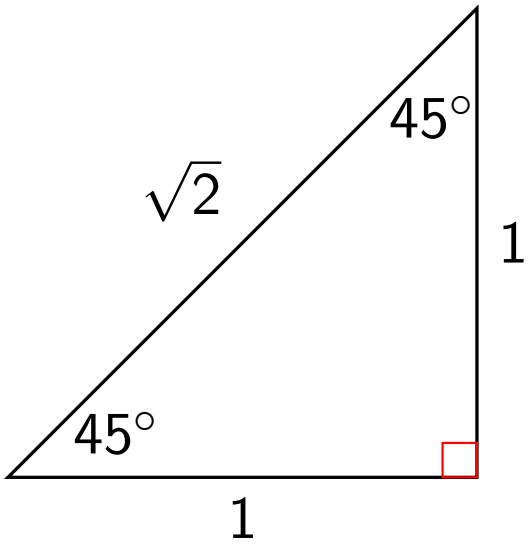
$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Example 2

Find the exact values of the six trig ratios for 45°.



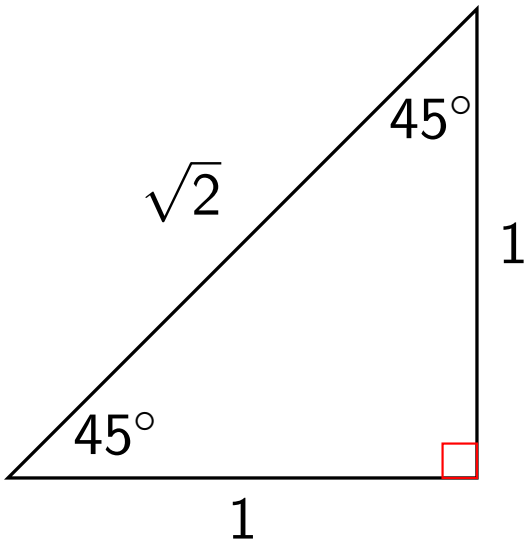
$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$

Example 2



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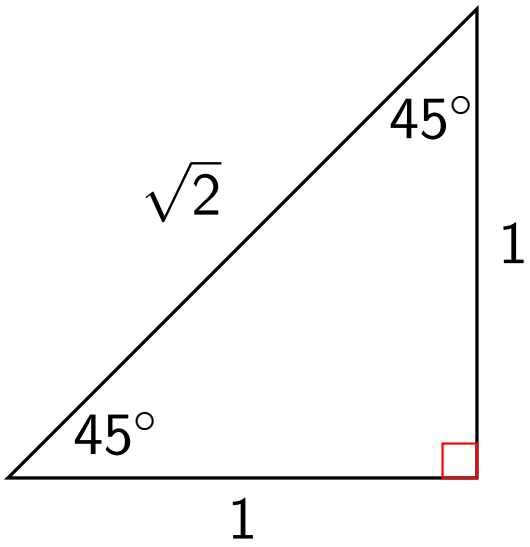

$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

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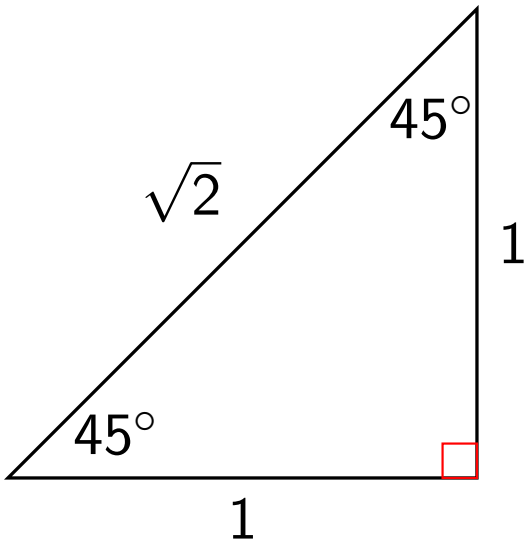

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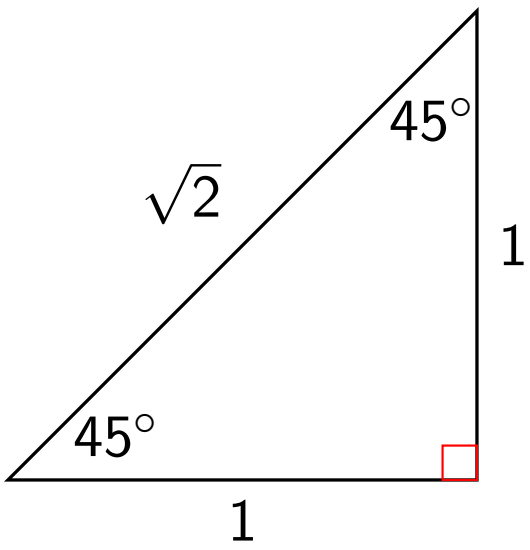

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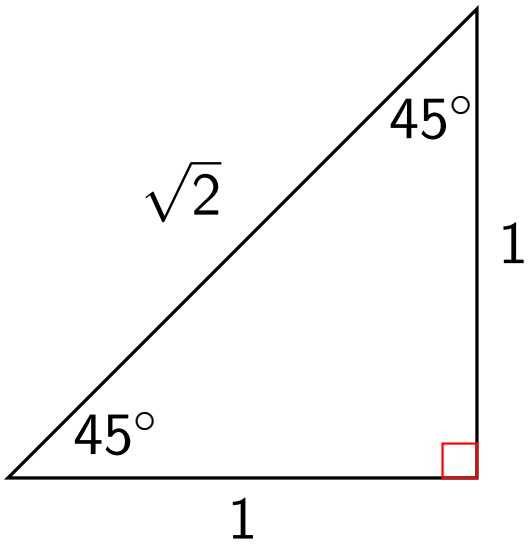

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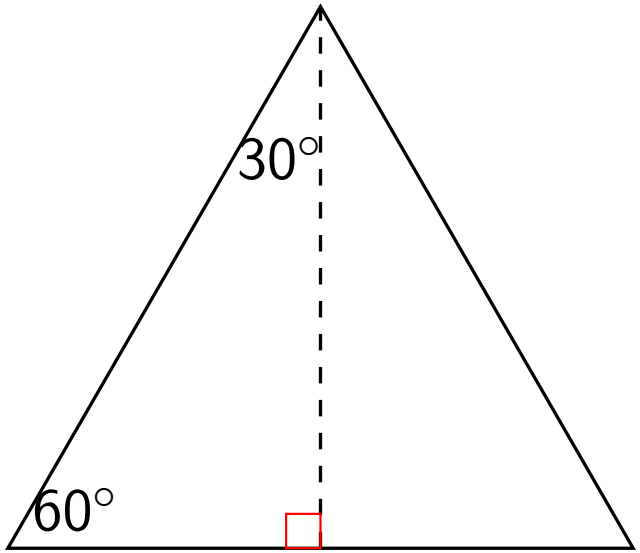
Note: Your answers from the above example will be the same if you replace 45° with $\frac{\pi}{4}$.

30-60-90 Triangles

We can create a 30-60-90 triangle by drawing an altitude in an equilateral triangle.



30-60-90 Triangles

Recall that the altitude of an equilateral triangle bisects one of the sides.

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Rather than use a length of 1 for the sides of the equilateral triangle, we will use a length of 2 (if only to avoid using fractions).

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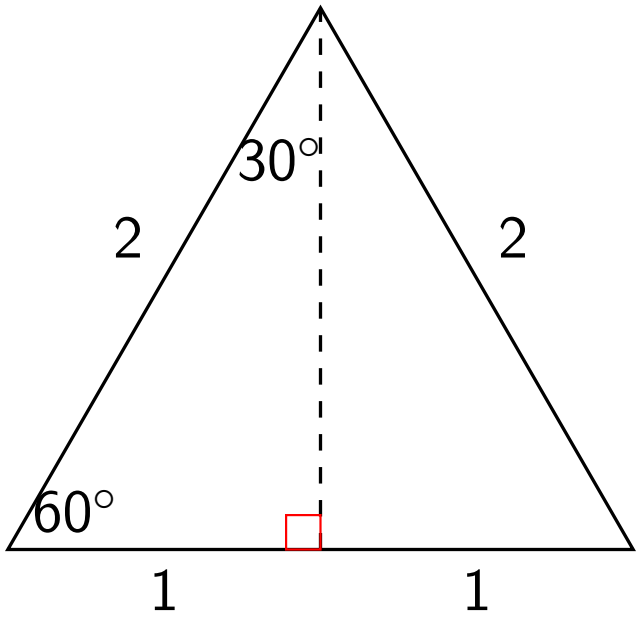
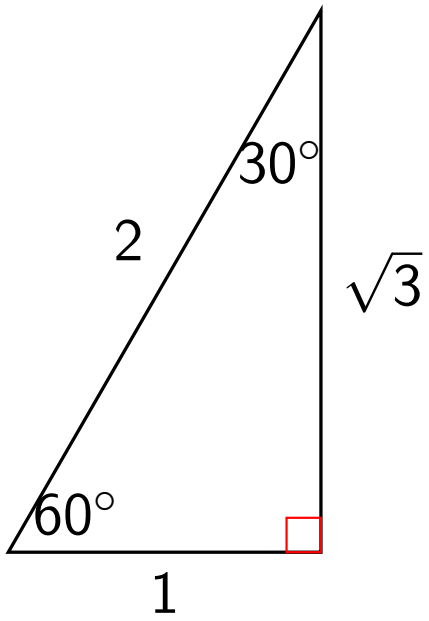


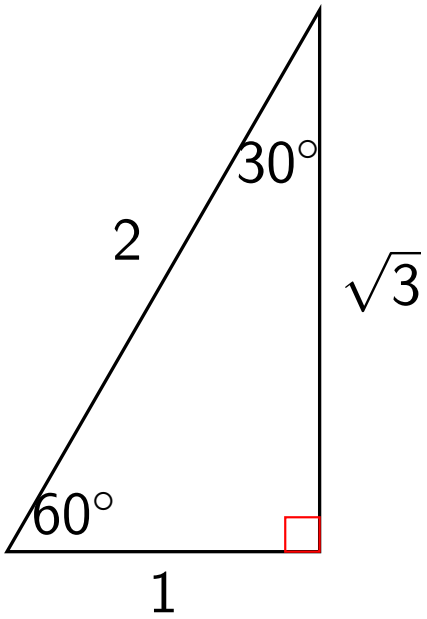
30-60-90 Triangles

We can use the Pythagorean Theorem to find the length of the altitude, $\sqrt{3}$:



Example 3

Find the exact values of the six trig ratios for $60°$.



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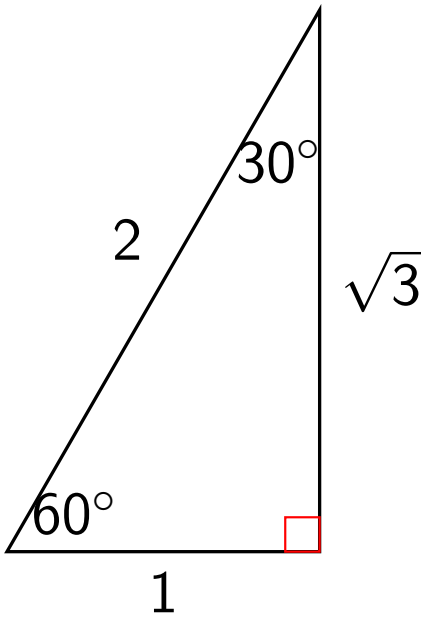

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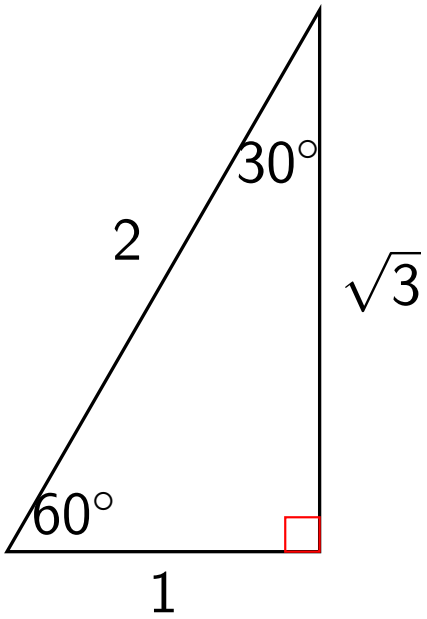

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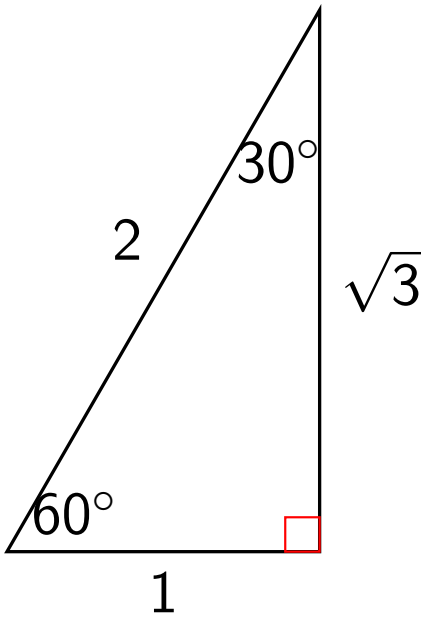

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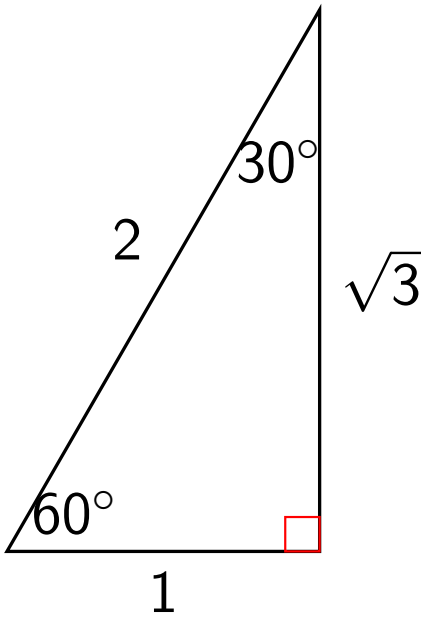

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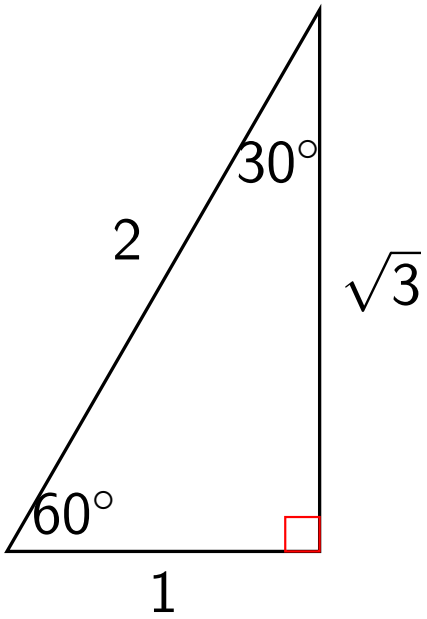

$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

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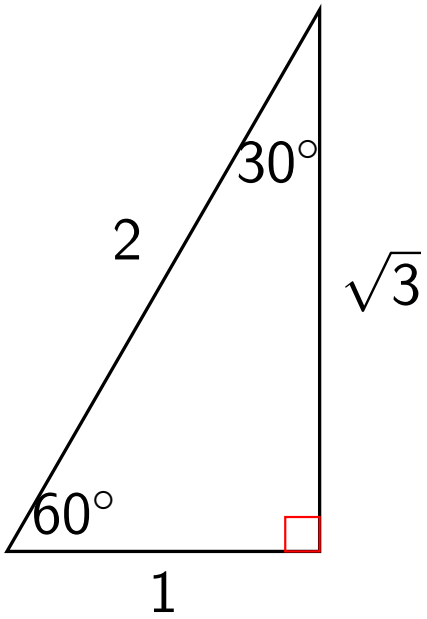

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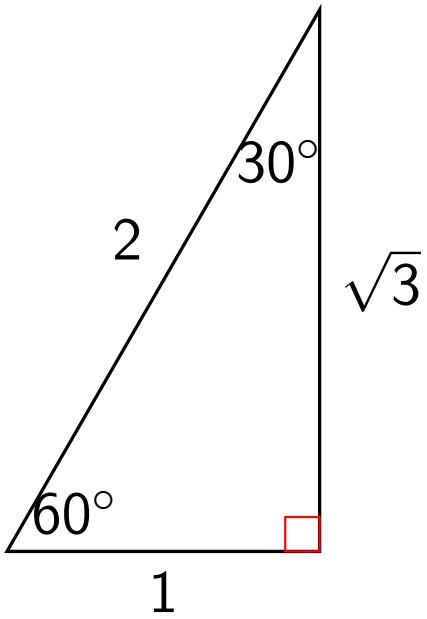

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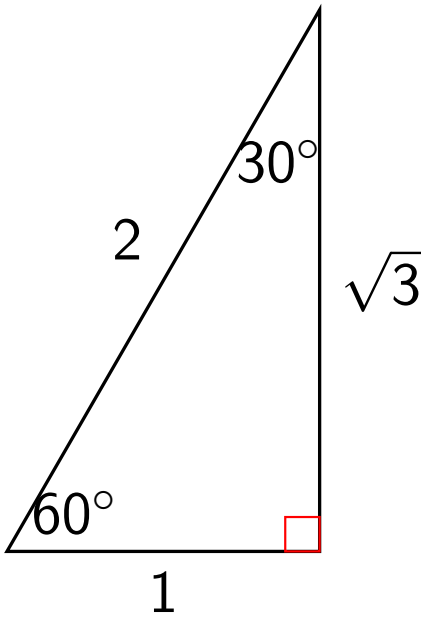

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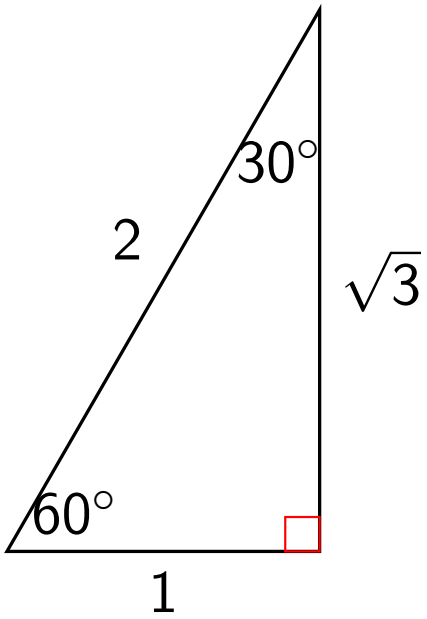

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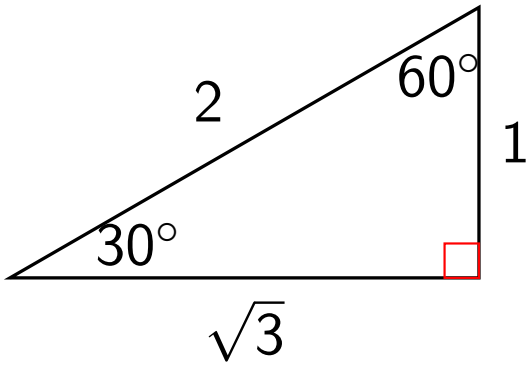
Find the exact values of the six trig ratios for $30°$.



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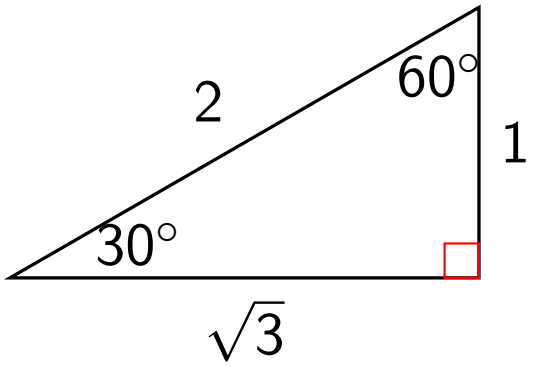

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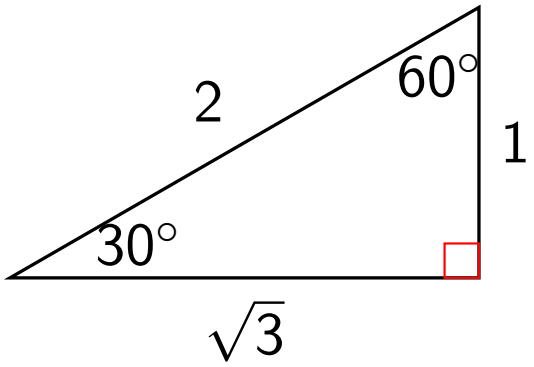

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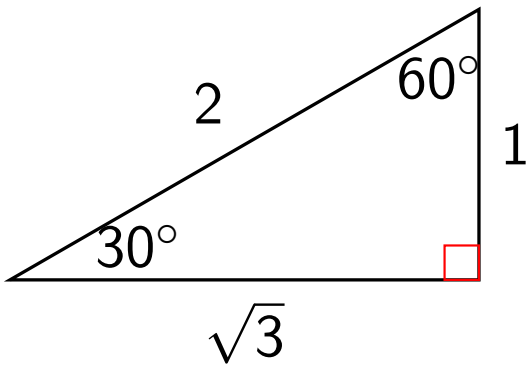
Find the exact values of the six trig ratios for 30°.



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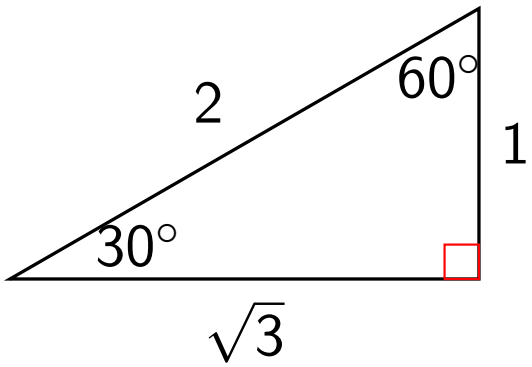

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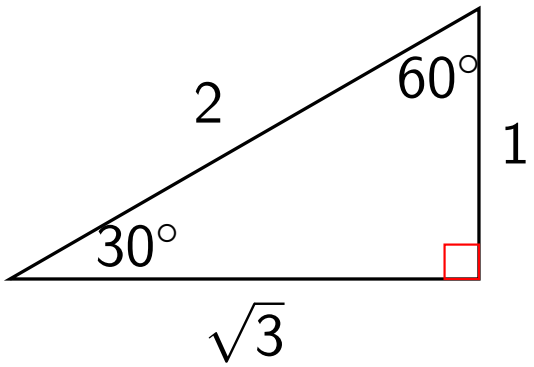

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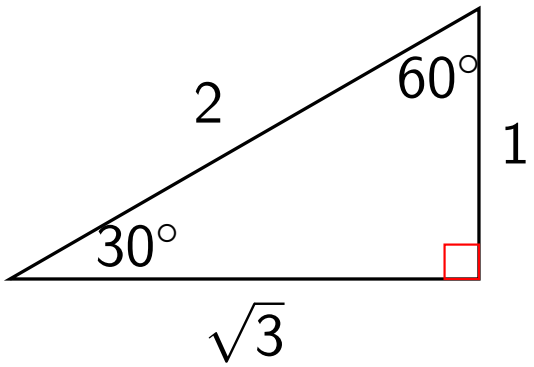

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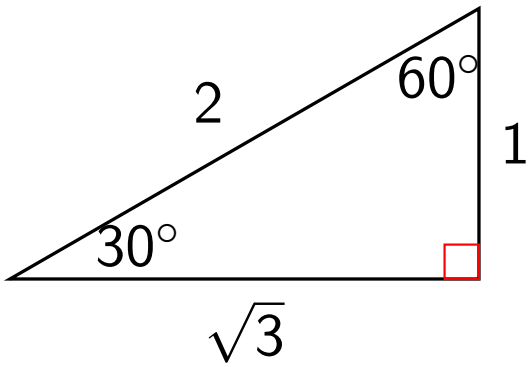

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Cofunctions

Notice how $\sin 30^\circ = \cos 60^\circ$, $\tan 30^\circ = \cot 60^\circ$, etc. This is because these ratios are cofunctions.

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Any pair of trig functions f and g for which

$$f(\theta) = g\left(90^\circ - \theta\right)$$

and vice versa are **cofunctions**.

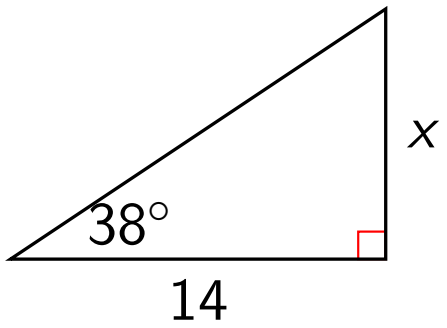
Table of Contents

Finding Missing Sides

You can use SOH-CAH-TOA and your calculator to find missing sides in right triangles.

Example 5

Find the value of x. Round your answer to 2 decimal places.



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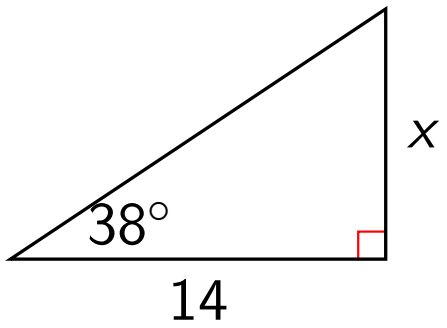

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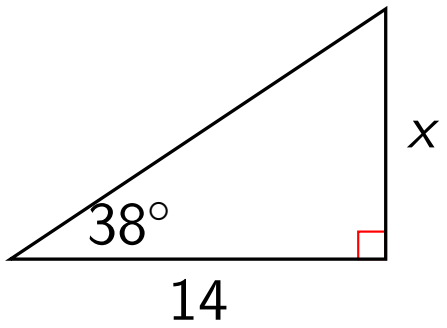

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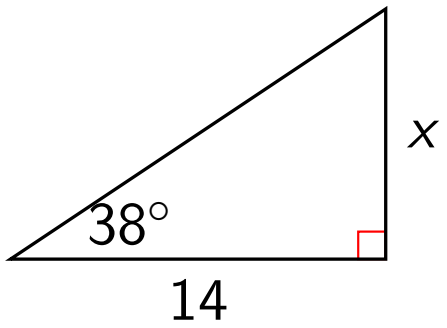

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$$x = 14(0.7813) \approx 10.94$$