Dividing Polynomials

Objectives

1. Divide polynomials without a remainder
2. Divide polynomials with a remainder
3. Use the Remainder Theorem and Factor Theorem

Division Basics

In the expression $a \div b = c$, a is the dividend, b is the divisor, and c is the quotient.

When dividing polynomials, it will help to write your terms in standard form (descending powers). You may also need to fill in any missing terms using 0 as a coefficient.

Before we get to division, let's review an organizational technique for multiplying polynomials.

Multiplication Example

To find the product of $(x-2)(x^2+6x+7)$, we can use the following method:

	x^2	$6x$	7
x			
-2			

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$$\begin{array}{c|c|c|c|}
 & x^2 & 6x & 7 \\ \hline
x & \mathbf{x^3} & & \\ \hline
-2 & & & \\ \hline
\end{array}$$

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$$\begin{array}{c|c|c|c|}
 & x^2 & 6x & 7 \\
\hline
x & \color{blue}{\boldsymbol{x^3}} & & \\
\hline
-2 & \color{red}{\boldsymbol{-2x^2}} & & \\
\hline
\end{array}$$

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	x^2	$6x$	7
x	$\mathbf{x^3}$	$\mathbf{6x^2}$	
-2	$\mathbf{-2x^2}$		

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-2	$\mathbf{-2x^2}$	$\mathbf{-12x}$	

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	x^2	$6x$	7
x	x^3	$6x^2$	$7x$
-2	$-2x^2$	$-12x$	-14

When we combine all of terms inside the box, we get the product, $x^3+4x^2-5x-14$.

Division

We can reverse the process using division to find the quotient

$$(x^3 + 4x^2 - 5x - 14) \div (x - 2)$$

x	x^3		
-2			

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x	x^3	$6x^2$	
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	x^2	$6x$	
x	x^3	$6x^2$	
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	x^2	$6x$	7
x	x^3	$6x^2$	$7x$
-2	$-2x^2$	$-12x$	-14

$$x^2 + 6x + 7$$

Example 1

Divide $(x^3 + 8) \div (x + 2)$

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x			
2			

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x	x^3		
2			

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	x^2		
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x	x^3	$-2x^2$	
2	$2x^2$		

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$$(x^3 + 0x^2 + 0x + 8) \div (x + 2)$$

	x^2	$-2x$	
x	x^3	$-2x^2$	
2	$2x^2$	$-4x$	

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	x^2	$-2x$	
x	x^3	$-2x^2$	$4x$
2	$2x^2$	$-4x$	

Example 1

Divide $(x^3 + 8) \div (x + 2)$

$$(x^3 + 0x^2 + 0x + 8) \div (x + 2)$$

	x^2	$-2x$	4
x	x^3	$-2x^2$	$4x$
2	$2x^2$	$-4x$	

Example 1

Divide $(x^3 + 8) \div (x + 2)$

$$(x^3 + 0x^2 + 0x + 8) \div (x + 2)$$

	x^2	$-2x$	4
x	x^3	$-2x^2$	$4x$
2	$2x^2$	$-4x$	8

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	x^2	$-2x$	4
x	x^3	$-2x^2$	$4x$
2	$2x^2$	$-4x$	8

$$x^2 - 2x + 4$$

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Dividing Polynomials With a Remainder

In the previous examples, everything “balanced out” from within the grid.

In other words, after combining like terms, all terms in the dividend were accounted for.

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In the next group of examples, we will need to figure out what to add to our quotient to “balance out the problem.”

We will write our answers in the form

$$\text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

Example 2

Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

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x			
-3			

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(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

x	$5x^3$		
-3			

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Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$		
x	$5x^3$		
-3			

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Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$		
x	$5x^3$		
-3	$-15x^2$		

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Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$		
x	$5x^3$	$13x^2$	
-3	$-15x^2$		

Example 2

Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$	$13x$	
x	$5x^3$	$13x^2$	
-3	$-15x^2$		

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Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$	$13x$	
x	$5x^3$	$13x^2$	
-3	$-15x^2$	$-39x$	

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$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$	$13x$	
x	$5x^3$	$13x^2$	$39x$
-3	$-15x^2$	$-39x$	

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Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$	$13x$	39
x	$5x^3$	$13x^2$	$39x$
-3	$-15x^2$	$-39x$	

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Divide each.

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	$5x^2$	$13x$	39
x	$5x^3$	$13x^2$	$39x$
-3	$-15x^2$	$-39x$	-107

Example 2

Divide each.

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$(5x^3 - 2x^2 + 0x + 1) \div (x - 3)$$

	$5x^2$	$13x$	39
x	$5x^3$	$13x^2$	$39x$
-3	$-15x^2$	$-39x$	-107

remainder: 108

Example 2

(a) $(5x^3 - 2x^2 + 1) \div (x - 3)$

$$5x^2 + 13x + 39 \text{ remainder: } 108$$

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$$5x^2 + 13x + 39 + \frac{108}{x - 3}$$

Example 2

(b) $(4 - 8x - 12x^2) \div (2x - 3)$

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$2x$		
-3		

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$2x$	$-12x^2$	
-3		

Example 2

(b) $(4 - 8x - 12x^2) \div (2x - 3)$

$$(-12x^2 - 8x + 4) \div (2x - 3)$$

	$-6x$	
$2x$	$-12x^2$	
-3		

Example 2

(b) $(4 - 8x - 12x^2) \div (2x - 3)$

$$(-12x^2 - 8x + 4) \div (2x - 3)$$

	$-6x$	
$2x$	$-12x^2$	
-3	$18x$	

Example 2

(b) $(4 - 8x - 12x^2) \div (2x - 3)$

$$(-12x^2 - 8x + 4) \div (2x - 3)$$

	$-6x$	
$2x$	$-12x^2$	$-26x$
-3	$18x$	

Example 2

(b) $(4 - 8x - 12x^2) \div (2x - 3)$

$$(-12x^2 - 8x + 4) \div (2x - 3)$$

	$-6x$	-13
$2x$	$-12x^2$	$-26x$
-3	$18x$	

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Example 2

(b) $(4 - 8x - 12x^2) \div (2x - 3)$

$$(-12x^2 - 8x + 4) \div (2x - 3)$$

	$-6x$	-13
$2x$	$-12x^2$	$-26x$
-3	$18x$	39

remainder: -35

Example 2

(b) $(4 - 8x - 12x^2) \div (2x - 3)$

$$-6x - 13 \text{remainder} \; -35$$

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$$-6x - 13 - \frac{35}{2x - 3}$$

Example 2

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

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$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

x^2	$3x^3$	
$0x$		
1		

Example 2

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	
x^2	$3x^3$	
$0x$		
1		

Example 2

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	
x^2	$3x^3$	
$0x$	$0x^2$	
1		

Example 2

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	
x^2	$3x^3$	
$0x$	$0x^2$	
1	$3x$	

Example 2

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	
x^2	$3x^3$	$4x^2$
$0x$	$0x^2$	
1	$3x$	

Example 2

(c) $\quad (3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	4
x^2	$3x^3$	$4x^2$
$0x$	$0x^2$	
1	$3x$	

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	4
x^2	$3x^3$	$4x^2$
$0x$	$0x^2$	$0x$
1	$3x$	

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$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	4
x^2	$3x^3$	$4x^2$
$0x$	$0x^2$	$0x$
1	$3x$	4

Example 2

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$$(3x^3 + 4x^2 + x + 7) \div (x^2 + 0x + 1)$$

	$3x$	4
x^2	$3x^3$	$4x^2$
$0x$	$0x^2$	$0x$
1	$3x$	4

remainder: $-2x + 3$

Example 2

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$3x + 4\text{remainder } -2x + 3$$

Example 2

(c) $(3x^3 + 4x^2 + x + 7) \div (x^2 + 1)$

$$3x + 4 \text{remainder } -2x + 3$$

$$3x + 4 + \frac{-2x + 3}{x^2 + 1}$$

Objectives

1. Divide polynomials without a remainder
2. Divide polynomials with a remainder
3. Use the Remainder Theorem and Factor Theorem

The Remainder Theorem

There is a quicker way to determine if some polynomial division problems result in a remainder.

The Remainder Theorem

If p is a polynomial of degree at least 1, and c is a real number, then when $p(x)$ is divided by $x - c$, the remainder is $p(c)$.

Example 3

What is the remainder when $2x^3 - 5x + 3$ is divided by $x + 2$?

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$$p(-2) = 2(-2)^3 - 5(-2) + 3$$

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$$
\begin{aligned}
p(-2) &= 2(-2)^3 - 5(-2) + 3 \\
p(-2) &= -3
\end{aligned}
$$

Example 3

What is the remainder when $2x^3 - 5x + 3$ is divided by $x + 2$?

$$p(-2) = 2(-2)^3 - 5(-2) + 3$$
$$p(-2) = -3$$

The remainder is -3

The Factor Theorem

The Factor Theorem states that if p is a nonzero polynomial, then the real number c is a zero of p if $(x - c)$ is a factor of $p(x)$.

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The Factor Theorem states that if p is a nonzero polynomial, then the real number c is a zero of p if $(x - c)$ is a factor of $p(x)$.

In other words, $p(c) = 0$.

Example 4

Use the fact that $x = 1$ is a zero of $p(x) = 2x^3 - 5x + 3$ to factor $p(x)$ and find all of the real zeros of p.

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$(2x^3 + 0x^2 - 5x + 3) \div (x - 1)$ will have a remainder of 0.

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x	$2x^3$		
-1			

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$(2x^3 + 0x^2 - 5x + 3) \div (x - 1)$ will have a remainder of 0.

	$2x^2$		
x	$2x^3$		
-1			

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	$2x^2$		
x	$2x^3$		
-1	$-2x^2$		

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$(2x^3 + 0x^2 - 5x + 3) \div (x - 1)$ will have a remainder of 0.

	$2x^2$		
x	$2x^3$	$2x^2$	
-1	$-2x^2$		

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	$2x^2$	$2x$	
x	$2x^3$	$2x^2$	
-1	$-2x^2$		

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$(2x^3 + 0x^2 - 5x + 3) \div (x - 1)$ will have a remainder of 0.

	$2x^2$	$2x$	
x	$2x^3$	$2x^2$	
-1	$-2x^2$	$-2x$	

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	$2x^2$	$2x$	
x	$2x^3$	$2x^2$	$-3x$
-1	$-2x^2$	$-2x$	

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$(2x^3 + 0x^2 - 5x + 3) \div (x - 1)$ will have a remainder of 0.

	$2x^2$	$2x$	-3
x	$2x^3$	$2x^2$	$-3x$
-1	$-2x^2$	$-2x$	

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Use the fact that $x = 1$ is a zero of $p(x) = 2x^3 - 5x + 3$ to factor $p(x)$ and find all of the real zeros of p.

$(2x^3 + 0x^2 - 5x + 3) \div (x - 1)$ will have a remainder of 0.

	$2x^2$	$2x$	-3
x	$2x^3$	$2x^2$	$-3x$
-1	$-2x^2$	$-2x$	3

Example 4

$$2x^2 + 2x - 3 = 0$$

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using the Quadratic Formula, we get

$$x = \frac{-1 \pm \sqrt{7}}{2}$$