Law of Sines and Law of Cosines

Objectives

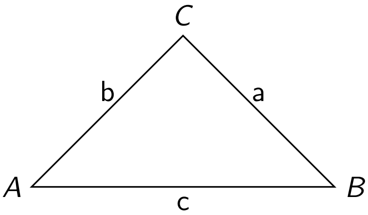
1. Solve triangles using the Law of Sines
2. Solve triangles using the Law of Cosines

Law of Sines

An oblique triangle is one that does not contain a right angle.

To solve oblique, as well as right triangles, you can use either the Law of Sines or the Law of Cosines.

Derivation of Law of Sines



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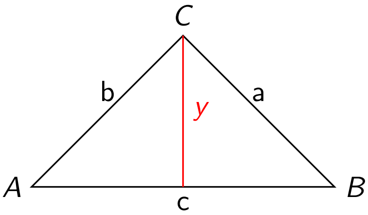

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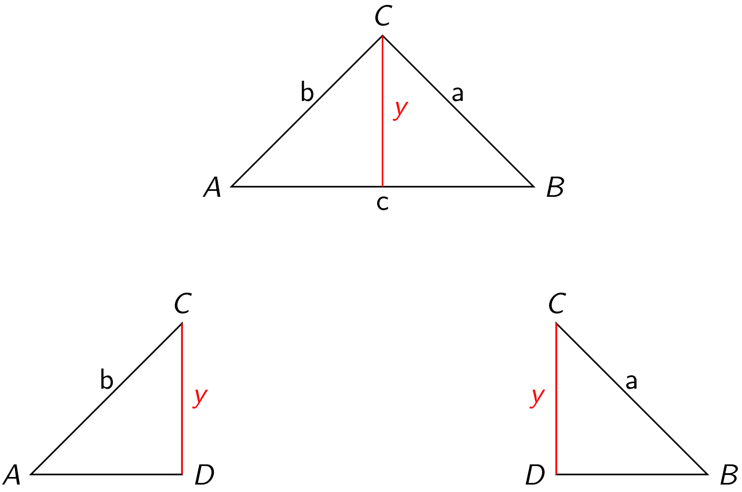

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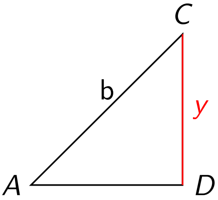

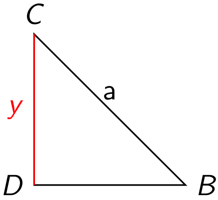

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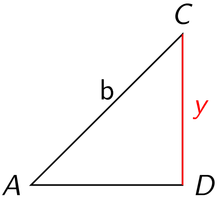
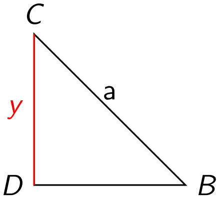




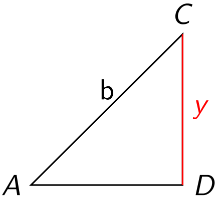
$$\sin A = \frac{y}{b}$$

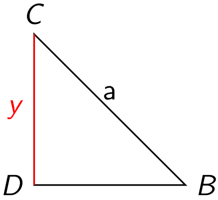
Derivation of Law of Sines

$$\sin A = \frac{y}{b}$$

$$b \sin A = y$$

Derivation of Law of Sines



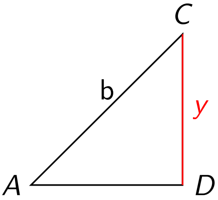


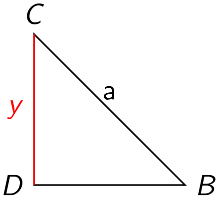
$$\sin A = \frac{y}{b}$$

$$b \sin A = y$$

$$\sin B = \frac{y}{a}$$

Derivation of Law of Sines





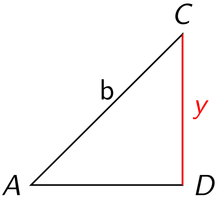
$$\sin A = \frac{y}{b}$$

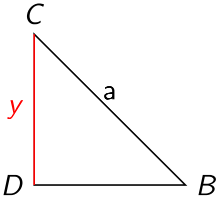
$$b \sin A = y$$

$$\sin B = \frac{y}{a}$$

$$a \sin B = y$$

Derivation of Law of Sines





$$\sin A = \frac{y}{b}$$

$$b \sin A = y$$

$$\sin B = \frac{y}{a}$$

$$a \sin B = y$$

$$b \sin A = a \sin B$$

Derivation of Law of Sines

$$b \sin A = a \sin B$$

Derivation of Law of Sines

$$b \sin A = a \sin B$$

$$\frac{b \sin A}{ab} = \frac{a \sin B}{ab}$$

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$$b \sin A = a \sin B$$

$$\frac{b \sin A}{ab} = \frac{a \sin B}{ab}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Derivation of Law of Sines

$$b \sin A = a \sin B$$

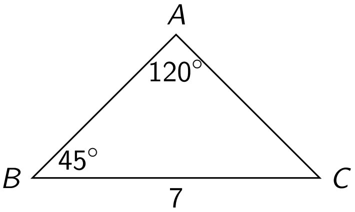
$$\frac{b \sin A}{ab} = \frac{a \sin B}{ab}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1

Solve the triangle given $m\angle A = 120^\circ$, $a = 7$, $m\angle B = 45^\circ$. Round your answers to 1 decimal place.



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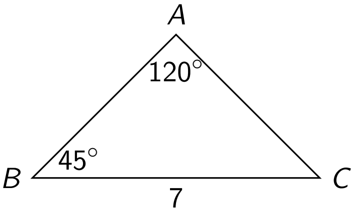

$$m\angle C = 180^\circ - 120^\circ - 45^\circ$$

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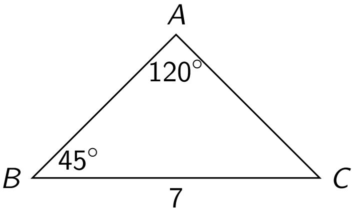
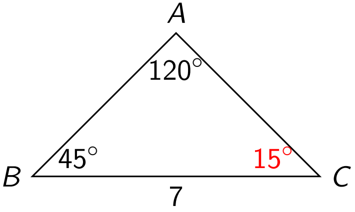
Solve the triangle given $m\angle A = 120^\circ$, $a = 7$, $m\angle B = 45^\circ$. Round your answers to 1 decimal place.



$$m\angle C = 180^\circ - 120^\circ - 45^\circ$$

$$m\angle C = 15^\circ$$

Example 1



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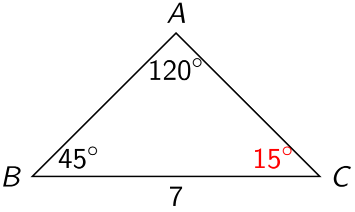
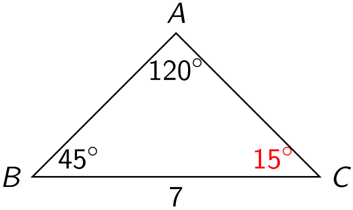


$$\frac{\sin 120^\circ}{7} = \frac{\sin 45^\circ}{b}$$

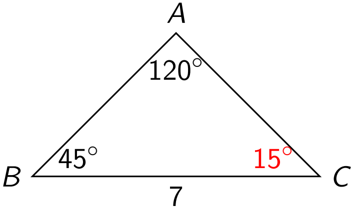
Example 1



$$\frac{\sin 120^\circ}{7} = \frac{\sin 45^\circ}{b}$$

$$b \sin 120^\circ = 7 \sin 45^\circ$$

Example 1



$$\frac{\sin 120^\circ}{7} = \frac{\sin 45^\circ}{b}$$

$$b \sin 120^\circ = 7 \sin 45^\circ$$

$$b = \frac{7 \sin 45^\circ}{\sin 120^\circ}$$

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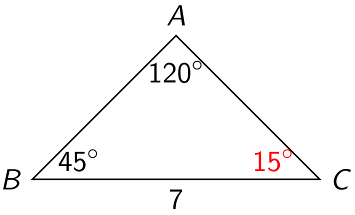


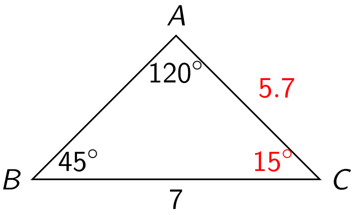
$$\frac{\sin 120^\circ}{7} = \frac{\sin 45^\circ}{b}$$

$$b \sin 120^\circ = 7 \sin 45^\circ$$

$$b = \frac{7 \sin 45^\circ}{\sin 120^\circ}$$

$$b \approx 5.7$$

Example 1



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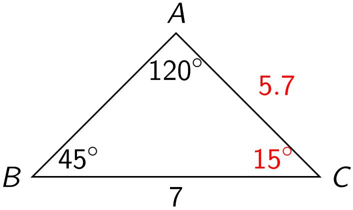

$$\frac{\sin 120^\circ}{7} = \frac{\sin 15^\circ}{c}$$

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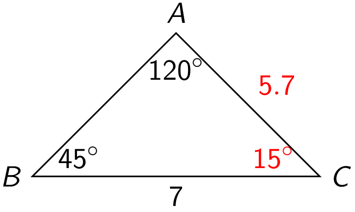

$$\frac{\sin 120^\circ}{7} = \frac{\sin 15^\circ}{c}$$

$$c \sin 120^\circ = 7 \sin 15^\circ$$

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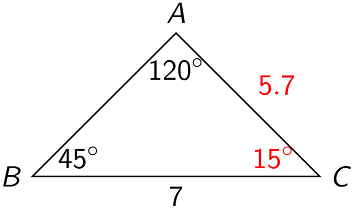

$$\frac{\sin 120^\circ}{7} = \frac{\sin 15^\circ}{c}$$

$$c \sin 120^\circ = 7 \sin 15^\circ$$

$$c = \frac{7 \sin 15^\circ}{\sin 120^\circ}$$

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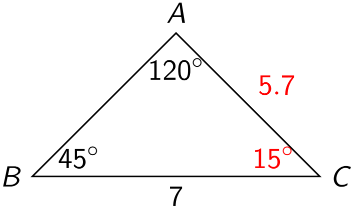


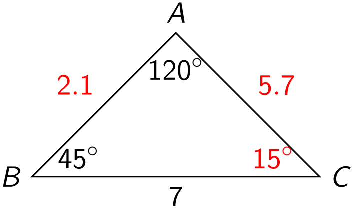
$$\frac{\sin 120^\circ}{7} = \frac{\sin 15^\circ}{c}$$

$$c \sin 120^\circ = 7 \sin 15^\circ$$

$$c = \frac{7 \sin 15^\circ}{\sin 120^\circ}$$

$$c \approx 2.1$$

Example 1



$$\frac{\sin 120^\circ}{7} = \frac{\sin 15^\circ}{c}$$

$$c \sin 120^\circ = 7 \sin 15^\circ$$

$$c = \frac{7 \sin 15^\circ}{\sin 120^\circ}$$

$$c \approx 2.1$$

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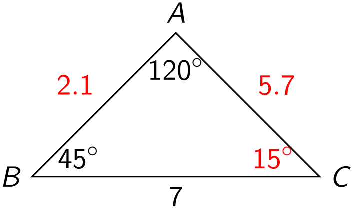

$$\frac{\sin 120^\circ}{7} = \frac{\sin 15^\circ}{c}$$

$$c \sin 120^\circ = 7 \sin 15^\circ$$

$$c = \frac{7 \sin 15^\circ}{\sin 120^\circ}$$

$$c \approx 2.1$$

$$m\angle C = 15^\circ, \quad b \approx 5.7, \quad c \approx 2.1$$

Example 2

Solve the triangle given $m\angle A = 85^\circ$, $m\angle B = 30^\circ$, $c = 5.25$

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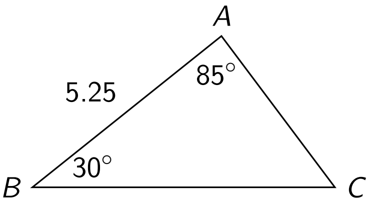
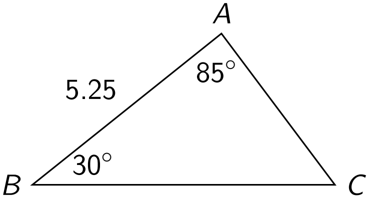


Example 2

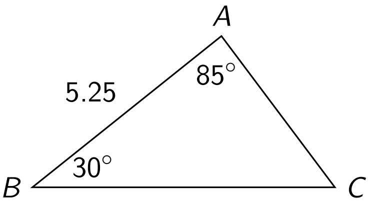
Solve the triangle given $m\angle A = 85^\circ$, $m\angle B = 30^\circ$, $c = 5.25$



$$m\angle C = 180^\circ - 30^\circ - 85^\circ$$

Example 2

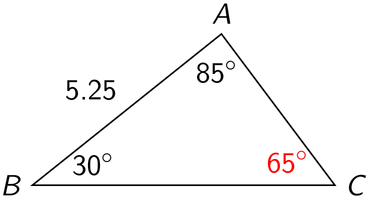
Solve the triangle given $m\angle A = 85^\circ$, $m\angle B = 30^\circ$, $c = 5.25$



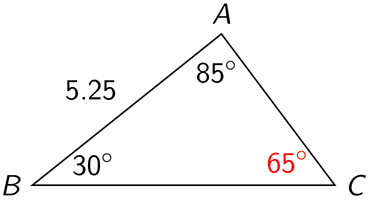
$$m\angle C = 180^\circ - 30^\circ - 85^\circ$$

$$m\angle C = 65^\circ$$

Example 2

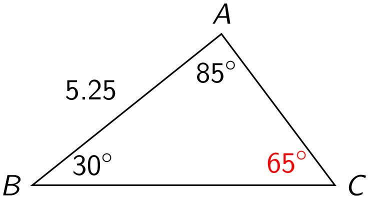


Example 2



$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 85^\circ}{a}$$

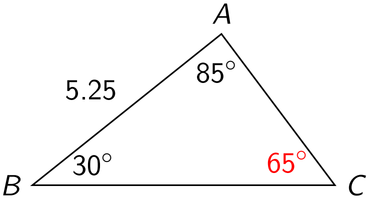
Example 2



$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 85^\circ}{a}$$

$$a \cdot \sin 65^\circ = 5.25 \sin 85^\circ$$

Example 2

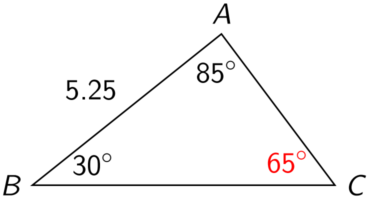


$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 85^\circ}{a}$$

$$a \cdot \sin 65^\circ = 5.25 \sin 85^\circ$$

$$a = \frac{5.25 \sin 85^\circ}{\sin 65^\circ}$$

Example 2



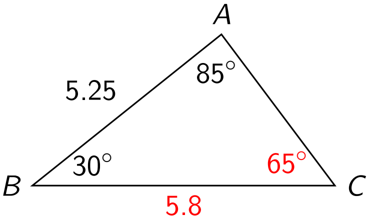
$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 85^\circ}{a}$$

$$a \cdot \sin 65^\circ = 5.25 \sin 85^\circ$$

$$a = \frac{5.25 \sin 85^\circ}{\sin 65^\circ}$$

$$a \approx 5.8$$

Example 2



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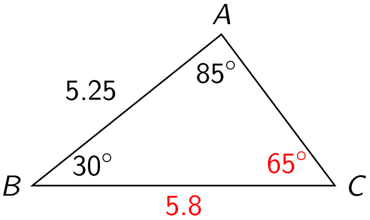
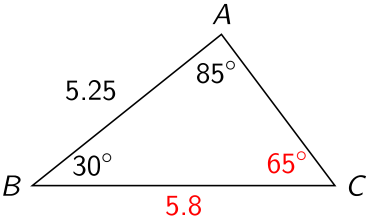


$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 30^\circ}{b}$$

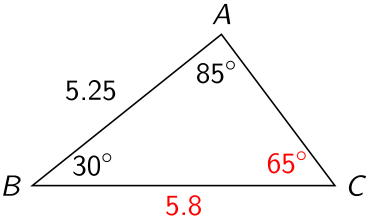
Example 2



$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 30^\circ}{b}$$

$$b \cdot \sin 65^\circ = 5.25 \sin 30^\circ$$

Example 2

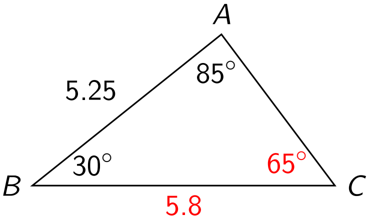


$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 30^\circ}{b}$$

$$b \cdot \sin 65^\circ = 5.25 \sin 30^\circ$$

$$b = \frac{5.25 \sin 30^\circ}{\sin 65^\circ}$$

Example 2



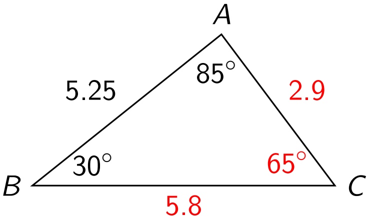
$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 30^\circ}{b}$$

$$b \cdot \sin 65^\circ = 5.25 \sin 30^\circ$$

$$b = \frac{5.25 \sin 30^\circ}{\sin 65^\circ}$$

$$b \approx 2.9$$

Example 2



$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 30^\circ}{b}$$

$$b \cdot \sin 65^\circ = 5.25 \sin 30^\circ$$

$$b = \frac{5.25 \sin 30^\circ}{\sin 65^\circ}$$

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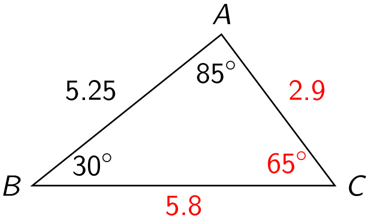
Example 2



$$\frac{\sin 65^\circ}{5.25} = \frac{\sin 30^\circ}{b}$$

$$b \cdot \sin 65^\circ = 5.25 \sin 30^\circ$$

$$b = \frac{5.25 \sin 30^\circ}{\sin 65^\circ}$$

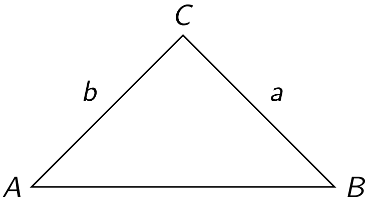
$$b \approx 2.9$$

$$m\angle C = 65^\circ, \quad a \approx 5.8, \quad b \approx 2.9$$

Objectives

1. Solve triangles using the Law of Sines
2. Solve triangles using the Law of Cosines

Derivation of the Law of Cosines



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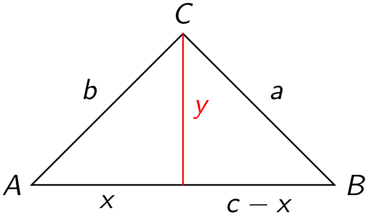

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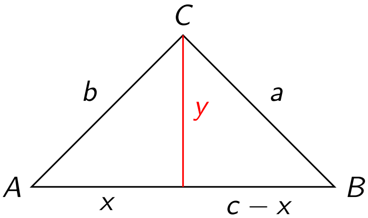
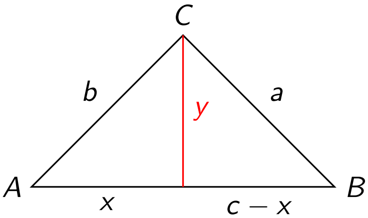


$$x^2 + y^2 = b^2 \qquad\qquad (c-x)^2 + y^2 = a^2$$

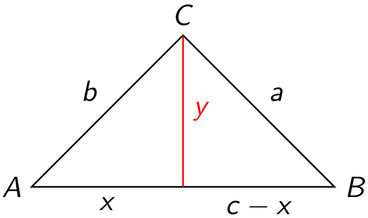
Derivation of the Law of Cosines



$$x^2 + y^2 = b^2 \qquad (c - x)^2 + y^2 = a^2$$

$$y^2 = b^2 - x^2 \qquad c^2 - 2cx + x^2 + y^2 = a^2$$

Derivation of the Law of Cosines

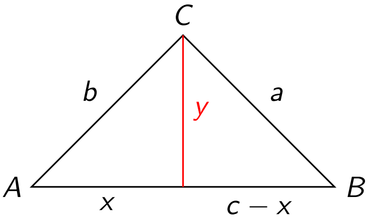


$$x^2 + y^2 = b^2 \qquad (c-x)^2 + y^2 = a^2$$

$$y^2 = b^2 - x^2 \qquad c^2 - 2cx + x^2 + y^2 = a^2$$

$$c^2 - 2cx + x^2 + b^2 - x^2 = a^2$$

Derivation of the Law of Cosines



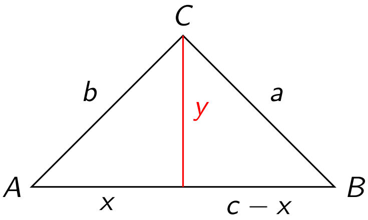
$$x^2 + y^2 = b^2 \qquad (c - x)^2 + y^2 = a^2$$

$$y^2 = b^2 - x^2 \qquad c^2 - 2cx + x^2 + y^2 = a^2$$

$$c^2 - 2cx + x^2 + b^2 - x^2 = a^2$$

$$b^2 + c^2 - 2cx = a^2$$

Derivation of Law of Cosines



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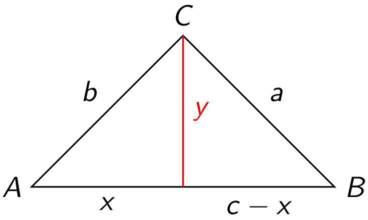
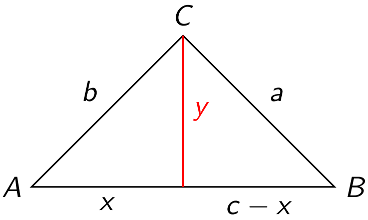


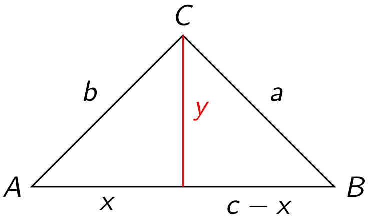
$$a^2 = b^2 + c^2 - 2cx$$

Derivation of Law of Cosines



$$a^2 = b^2 + c^2 - 2cx$$
$$\cos A = \frac{x}{b}$$

Derivation of Law of Cosines

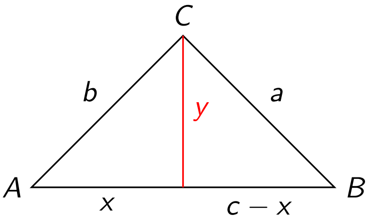


$$a^2 = b^2 + c^2 - 2cx$$

$$\cos A = \frac{x}{b}$$

$$x = b\cos A$$

Derivation of Law of Cosines



$$a^2 = b^2 + c^2 - 2cx$$

$$\cos A = \frac{x}{b}$$

$$x = b\cos A$$

$$a^2 = b^2 + c^2 - 2c(b\cos A)$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Law of Cosines

By solving each of the previous equations for the cosine of the angle, we get the following:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

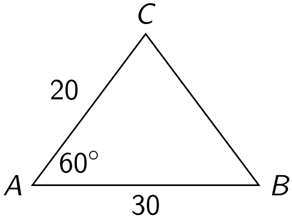
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Then take the inverse cosine to get the angle measure.

Example 3

Solve each. Round your answers to one decimal place.

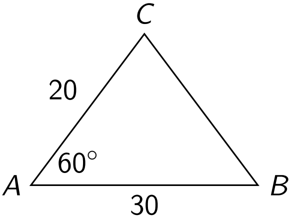
$m\angle A = 60^\circ,\ b = 20,\ c = 30$



Example 3

Solve each. Round your answers to one decimal place.

$m\angle A = 60^\circ,\ b = 20,\ c = 30$

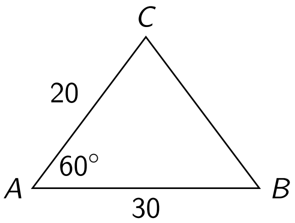


$$a^2 = 20^2 + 30^2 - 2(20)(30)\cos 60^\circ$$

Example 3

Solve each. Round your answers to one decimal place.

$m\angle A = 60^\circ,\ b = 20,\ c = 30$



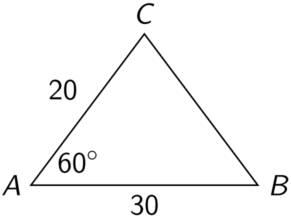
$$a^2 = 20^2 + 30^2 - 2(20)(30)\cos 60^\circ$$

$$a^2 = 700$$

Example 3

Solve each. Round your answers to one decimal place.

$m\angle A = 60^\circ,\ b = 20,\ c = 30$

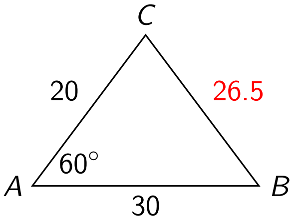


$$a^2 = 20^2 + 30^2 - 2(20)(30)\cos 60^\circ$$

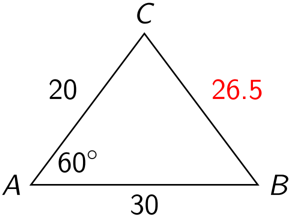
$$a^2 = 700$$

$$a \approx 26.5$$

Example 3

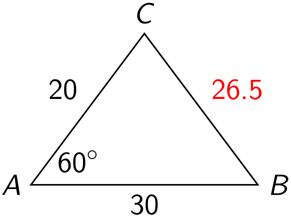


Example 3



$$\cos B = \frac{26.5^2 + 30^2 - 20^2}{2(26.5)(30)}$$

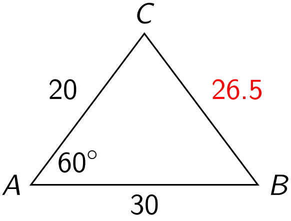
Example 3



$$\cos B = \frac{26.5^2 + 30^2 - 20^2}{2(26.5)(30)}$$

$$\cos B \approx 0.7561$$

Example 3

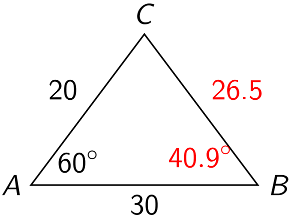


$$\cos B = \frac{26.5^2 + 30^2 - 20^2}{2(26.5)(30)}$$

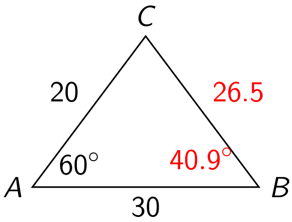
$$\cos B \approx 0.7561$$

$$B \approx 40.9^\circ$$

Example 3

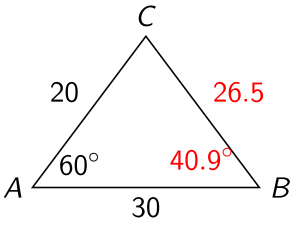


Example 3



$$m\angle C \approx 180^\circ - 60^\circ - 40.9^\circ$$

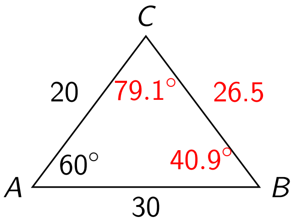
Example 3



$$m\angle C \approx 180^\circ - 60^\circ - 40.9^\circ$$

$$m\angle C \approx 79.1^\circ$$

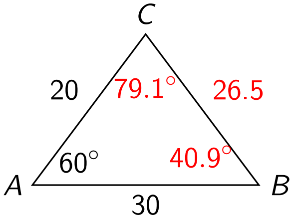
Example 3



$$m\angle C \approx 180^\circ - 60^\circ - 40.9^\circ$$

$$m\angle C \approx 79.1^\circ$$

Example 3



$$m\angle C \approx 180^\circ - 60^\circ - 40.9^\circ$$

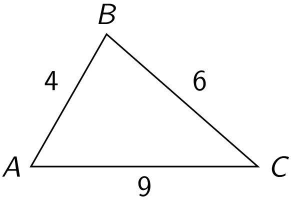
$$m\angle C \approx 79.1^\circ$$

$$a \approx 26.5, \quad m\angle B \approx 40.9^\circ, \quad m\angle C \approx 79.1^\circ$$

Example 4

Solve the triangle. Round your answers to 1 decimal place.

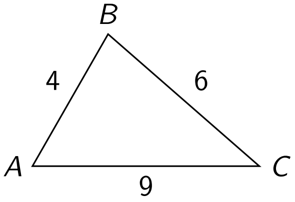
$a = 6,\ b = 9,\ c = 4$



Example 4

Solve the triangle. Round your answers to 1 decimal place.

$a = 6,\ b = 9,\ c = 4$

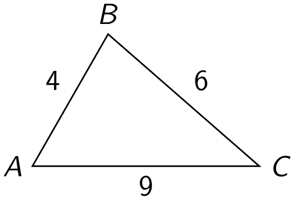


$$\cos A = \frac{9^2 + 4^2 - 6^2}{2(9)(4)}$$

Example 4

Solve the triangle. Round your answers to 1 decimal place.

$a = 6,\ b = 9,\ c = 4$



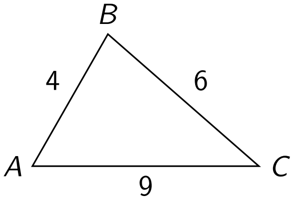
$$\cos A = \frac{9^2 + 4^2 - 6^2}{2(9)(4)}$$

$$\cos A = \frac{61}{72}$$

Example 4

Solve the triangle. Round your answers to 1 decimal place.

$a = 6,\ b = 9,\ c = 4$

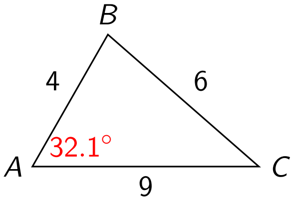


$$\cos A = \frac{9^2 + 4^2 - 6^2}{2(9)(4)}$$

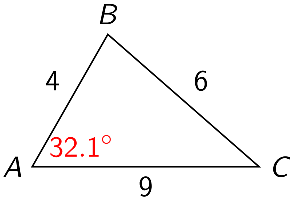
$$\cos A = \frac{61}{72}$$

$$A \approx 32.1^\circ$$

Example 4

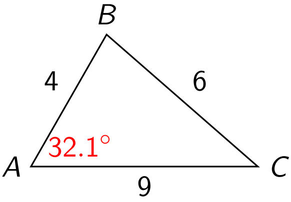


Example 4



$$\cos B = \frac{6^2 + 4^2 - 9^2}{2(6)(4)}$$

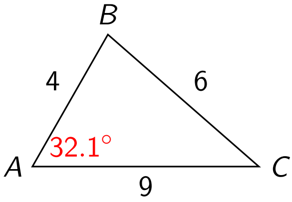
Example 4



$$\cos B = \frac{6^2 + 4^2 - 9^2}{2(6)(4)}$$

$$\cos B = -\frac{29}{48}$$

Example 4

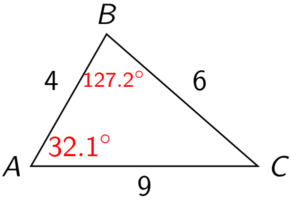


$$\cos B = \frac{6^2 + 4^2 - 9^2}{2(6)(4)}$$

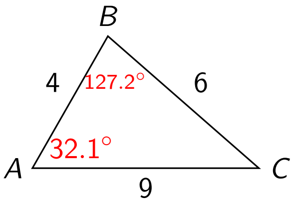
$$\cos B = -\frac{29}{48}$$

$$B \approx 127.2^\circ$$

Example 4

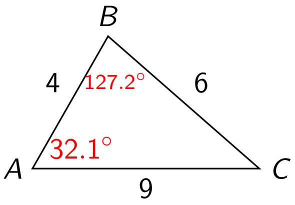


Example 4



$$m\angle C \approx 180^\circ - 127.2^\circ - 32.1^\circ$$

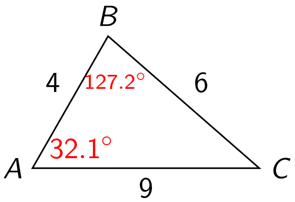
Example 4



$$m\angle C \approx 180^\circ - 127.2^\circ - 32.1^\circ$$

$$m\angle C \approx 20.7^\circ$$

Example 4



$$m\angle C \approx 180^\circ - 127.2^\circ - 32.1^\circ$$

$$m\angle C \approx 20.7^\circ$$

$$m\angle A \approx 32.1^\circ, \quad m\angle B \approx 127.2^\circ, \quad m\angle C \approx 20.7^\circ$$