

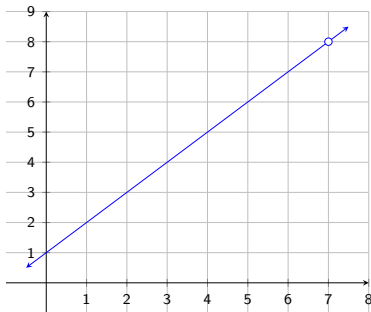
Limits and Algebra

Intro

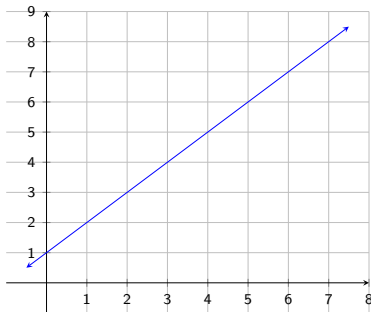
The graphs of $f(x) = \frac{x^2 - 6x - 7}{x - 7}$ and $g(x) = x + 1$ are not the same.

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$$f(x) = \frac{x^2 - 6x - 7}{x - 7}$$



$$g(x) = x + 1$$

Objectives

- 1 Find Limits via Factoring
- 2 Limits with Complex Fractions
- 3 Limits with Radicals

Algebraic Limits

Some limits that can't be evaluated directly can be evaluated after
cancelling out common factors.

Algebraic Limits

Some limits that can't be evaluated directly can be evaluated after **cancelling out common factors**.

This is called **removable discontinuity**.

Example 1

(a) Evaluate $\lim_{x \rightarrow -3} \frac{x^2 + 4x + 3}{x + 3}$

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(a) Evaluate $\lim_{x \rightarrow -3} \frac{x^2 + 4x + 3}{x + 3}$

$$\begin{aligned}\lim_{x \rightarrow -3} \frac{x^2 + 4x + 3}{x + 3} &= \lim_{x \rightarrow -3} \frac{(x + 3)(x + 1)}{x + 3} \\ &= \lim_{x \rightarrow -3} \frac{\cancel{(x + 3)}(x + 1)}{\cancel{(x + 3)}}\end{aligned}$$

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$$= \lim_{x \rightarrow -3} (x + 1)$$

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$$= -2$$

Example 1

(b) Evaluate $\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + 7x + 10}$

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(b) Evaluate $\lim_{x \rightarrow -2} \frac{x+2}{x^2+7x+10}$

$$\begin{aligned}\lim_{x \rightarrow -2} \frac{x+2}{x^2+7x+10} &= \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x+5)} \\ &= \lim_{x \rightarrow -2} \frac{\cancel{x+2}}{\cancel{(x+2)}(x+5)}\end{aligned}$$

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$$\begin{aligned}\lim_{x \rightarrow -2} \frac{x+2}{x^2+7x+10} &= \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x+5)} \\ &= \lim_{x \rightarrow -2} \frac{\cancel{x+2}}{(\cancel{x+2})(x+5)} \\ &= \lim_{x \rightarrow -2} \frac{1}{x+5}\end{aligned}$$

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$$\begin{aligned}\lim_{x \rightarrow -2} \frac{x+2}{x^2+7x+10} &= \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x+5)} \\&= \lim_{x \rightarrow -2} \frac{\cancel{x+2}}{\cancel{(x+2)}(x+5)} \\&= \lim_{x \rightarrow -2} \frac{1}{x+5} \\&= \frac{1}{-2+5}\end{aligned}$$

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$$\begin{aligned}\lim_{x \rightarrow -2} \frac{x+2}{x^2+7x+10} &= \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x+5)} \\&= \lim_{x \rightarrow -2} \frac{\cancel{x+2}}{\cancel{(x+2)}(x+5)} \\&= \lim_{x \rightarrow -2} \frac{1}{x+5} \\&= \frac{1}{-2+5} \\&= \frac{1}{3}\end{aligned}$$

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Complex Fractions

Simplify the complex fraction by multiplying every term by the **least common tiny denominator**.

Example 2

Evaluate each.

$$(a) \quad \lim_{x \rightarrow -5} \left(\frac{\frac{1}{x} + \frac{1}{5}}{x + 5} \right)$$

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$$\begin{aligned} \lim_{x \rightarrow -5} \left(\frac{\frac{1}{x} + \frac{1}{5}}{x + 5} \right) &= \lim_{x \rightarrow -5} \left(\frac{\frac{1}{x} + \frac{1}{5}}{x + 5} \right) \left(\frac{5x}{5x} \right) \\ &= \lim_{x \rightarrow -5} \frac{5 + x}{5x(x + 5)} \end{aligned}$$

Example 2

$$= \lim_{x \rightarrow -5} \frac{\cancel{5+x}}{5x(\cancel{x+5})}$$

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$$= \lim_{x \rightarrow -5} \frac{1}{5x}$$

$$= \frac{1}{5(-5)}$$

$$= -\frac{1}{25}$$

Example 2

$$(b) \quad \lim_{x \rightarrow 3} \left(\frac{\frac{1}{3} - \frac{1}{x}}{3 - x} \right)$$

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Example 2

$$= \lim_{x \rightarrow 3} \frac{\cancel{3-x}}{3x(\cancel{x-3})}$$

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$$= \frac{-1}{3(3)}$$

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$$= \lim_{x \rightarrow 3} \frac{-1}{3x}$$

$$= \frac{-1}{3(3)}$$

$$= \frac{-1}{9}$$

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