Honors PreCalculus

Hundreds of Extra Practice Problems

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Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2. $\{x | x \le 1\}$
- 3. _3
- 4. $\{x | x \neq 4, 11\}$
- 5. 2.4
- 6. $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7. $\{x | x \ge 2\}$
- 8. $\{x|x<-8\}$
- 9. $\{x | x \neq 3\}$
- 10. $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

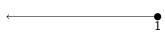
- 11.
- $\begin{array}{cccc}
 & & & & & & \downarrow \\
 & & & & & \uparrow \\
 & & & & & & \uparrow \\
 & & & & & & \uparrow
 \end{array}$

BASIC SET THEORY AND INTERVAL NOTATION KEY

1.
$$\{x | -5 < x \le 8\}$$



2. $(-\infty, 1]$



3. $[-3, \infty)$ $\{x | x \ge -3\}$

4.
$$(-\infty, 4) \cup (4, 11) \cup (11, \infty)$$



5. [2.4, 7.7) $\{x | 2.4 \le x < 7.7\}$

6. $\{x|x > 9\}$



7. $[2, \infty)$



8. $(-\infty, -8)$

9. $(-\infty,3)\cup(3,\infty)$

$$\longleftrightarrow$$
 3

10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$

$$\begin{array}{ccc} & \bigcirc & \bigcirc & \bigcirc \\ -2 & & 5 \end{array}$$

11. $(-\infty, -8]$ $\{x | x \le -8\}$

12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x | x \neq 7, 12\}$

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

- 1. f(5)
- 2. f(-2)
- 3. f(0)
- 4. g(1)
- 5. g(-5)
- 6. g(1/4)

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1.
$$f(x) = -8x^2 - 7x + 1$$

2.
$$g(x) = \sqrt{5x + 12} - 2$$

3.
$$h(x) = \frac{x+2}{9x-7}$$

4.
$$f(x) = -5x + 4$$

5.
$$f(x) = x^2 + 2$$

6.
$$f(x) = \frac{2x+1}{3x-5}$$

7.
$$f(x) = \sqrt{3x - 12}$$

8.
$$f(x) = \frac{x}{x^2 - 16}$$

9.
$$f(x) = \frac{x+4}{x^3-4x}$$

10.
$$f(x) = \frac{x}{\sqrt{x-4}}$$

11.
$$f(x) = \frac{x^2+1}{2x^2+8}$$

12.
$$f(x) = -\frac{x+7}{x^2-5x-6}$$

13.
$$g(x) = \sqrt{2x+3}$$

14.
$$h(x) = \sqrt[3]{2x+3}$$

15.
$$f(x) = -\frac{7x-10}{x^2+3x+2}$$

16.
$$g(x) = \sqrt{-9x + 8}$$

17.
$$h(x) = -\sqrt[3]{4x+1}$$

18.
$$f(x) = \sqrt[3]{8x+1}$$

19.
$$g(x) = \frac{x^2-1}{\sqrt{x+3}}$$

20.
$$h(x) = \frac{3}{9 + \frac{4}{x+7}}$$

21.
$$f(x) = \frac{x+1}{\sqrt{10x+8}}$$

22.
$$g(x) = \frac{5}{1 + \frac{3}{x+2}}$$

23.
$$i(x) = \frac{7}{3 - \frac{4}{x+1}}$$

24.
$$n(x) = \frac{7x+14}{\sqrt{2x-1}}$$

25.
$$a(x) = \frac{\frac{x}{x-2}}{\frac{3}{x-2}+6}$$

26.
$$d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$$

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. f(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4\\ \sqrt{2x + 7} & -4 < x < 0\\ |-x - 1| & x \ge 0 \end{cases}$$

- 6. f(3)
- 7. f(-2)
- 8. f(0)
- 9. f(-5)

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \le -3\\ \sqrt{-4x + 1} & \text{if } -3 < x \le 0\\ \frac{5x^2}{x + 7} & \text{if } x > 0 \end{cases}$$

- 10. f(7)
- 11. f(-3)
- 12. f(1)
- 13. f(0)
- 14. f(-1)
- 15. f(-3/2)

Evaluating Functions

- 1. -55
- 2. -20
- 3. 0
- 4. -4
- 5. -5.2
- 6. -1

Domain of Functions

- 1. $(-\infty, \infty)$
- 2. $\left[\frac{-12}{5}, \infty\right)$
- 3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
- 4. $(-\infty, \infty)$
- 5. $(-\infty, \infty)$
- 6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
- 7. $[4, \infty)$
- 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
- 9. $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$
- 10. $(4, \infty)$
- 11. $(-\infty, \infty)$
- 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
- 13. $\left[-\frac{3}{2}, \infty\right)$

- 14. $(-\infty, \infty)$
- 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
- 16. $\left(-\infty, \frac{8}{9}\right]$
- 17. $(-\infty, \infty)$
- 18. $(-\infty, \infty)$
- 19. $(-3, \infty)$
- 20. $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup \left(-7, \infty\right)$
- 21. $\left(-\frac{4}{5}, \infty\right)$
- 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$
- 23. $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$
- 24. $(\frac{1}{2}, \infty)$
- 25. $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 2\right) \cup \left(2, \infty\right)$
- 26. $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$

Piecewise Functions

- 1. $\sqrt{15} \approx 3.873$
- 2. 7
- 3. 6.6
- 4. 6.4
- 5. 7.1

- 6. 4
- 7. $\sqrt{3} \approx 1.732$
- 8. 1
- 9. 18
- 10. 17.5

- 11. 4
- 12. $\frac{5}{8}$
- 13. 1
- 14. $\sqrt{5} \approx 2.236$
- 15. $\sqrt{7} \approx 2.646$

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

5.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

6.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = x^3 + 2x^2 - 4x - 8$$

4.
$$g(x) = x^4 - 2x^2 + 1$$

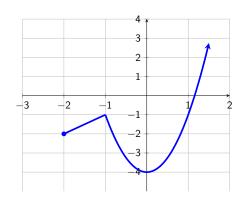
5.
$$h(x) = \sqrt{x+1} - 2$$

6.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

7.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

3.3 Miscellaneous

Use the graph of y = f(x) below to answer the following questions. Write your answers using interval notation.



- 1. Domain of f
- 2. Range of f
- 3. Relative Minimum
- 4. Relative Maximum
- 5. f(1)

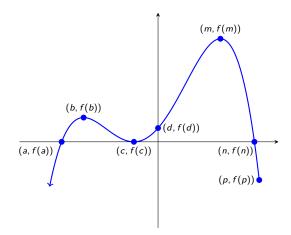
- 6. f(0)
- 7. Increasing Interval(s)
- 8. Decreasing Interval(s)
- 9. Absolute Maximum
- 10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

- 11. f(7)
- 12. f(-2)
- 13. Rel. Max
- 14. Rel. Min

- 15. Global Max
- 16. Global Min
- 17. Increasing Interval(s)
- 18. Decreasing Interval(s)

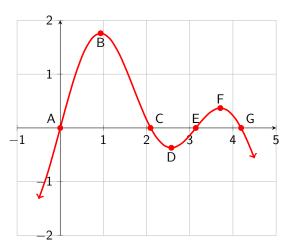
Use the graph of f(x) to answer each.



- 19. Relative maxima of f(x)
- 20. Relative minima of f(x)
- 21. Absolute maxima of f(x)
- 22. Absolute minima of f(x)

- 23. Intervals where f is increasing
- 24. Intervals where f is decreasing
- 25. Zeros of *f*

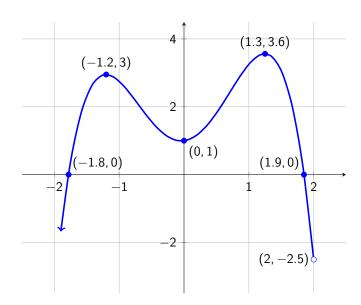
Given the labeled points A through G on the graph of f(x) below, find each of the following.



- 26. Increasing interval(s)
- 28. Relative max
- 30. Global max
- 32. Zeros of *f*

- 27. Decreasing interval(s)
- 29. Relative min
- 31. Global min
- 33. Number of solutions to f(x) = 1

Answer each of the following about the function f(x) below.



- 34. Domain of f
- 35. Range of f
- 36. Relative maxima
- 37. Relative minima
- 38. Absolute maximum

- 39. Absolute minimum
- 40. Increasing intervals
- 41. Decreasing intervals
- 42. Zeros of f(x)
- 43. Number of solutions to f(x) = 2

Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. Rel Max (-1.366, 10.848) and (1,6); Rel Min (0.366, 5.652)
- 4. Rel Max (-1.716, 11.598); Rel Min (1.132, -3.929)
- 5. Rel Max: (1.095, 12.096); Rel Min (-0.761, -0.680)
- 6. Rel Max: (1.366, 0.348); Rel Min: (-0.366, -4.848) and (2, 0)
- 7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

- 1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
- 3. Inc: $(-\infty, -2) \cup (\frac{2}{3}, \infty)$ Dec: $(-2, \frac{2}{3})$
- 4. Inc; $(-1,0) \cup (1,\infty)$ Dec: $(-\infty,-1) \cup (0,1)$
- 5. Inc: $(-1, \infty)$ No intervals where it is decreasing
- 6. Inc: (-0.761, 1.095); Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
- 7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Miscellaneous

- 1. $[-2, \infty)$
- 2. $[-4, \infty)$
- 3. (0, -4)
- 4. (-1, -1)
- 5. -1
- 6. -4)
- 7. $(-2, -1) \cup (0, \infty)$
- 8. (-1,0)
- 9. (0, -4)
- 10. None
- 11. -426

- 12. 51
- 13. (1.408, 0.272)
- 14. (0.592, -0.272)
- 15. None
- 16. None
- 17. (0.592, 1.408)
- 18. $(-\infty, 0.592) \cup$ $(1.408, \infty)$
- 19. (b, f(b)) and (m, f(m))
- 20. (c, f(c))
- 21. (m, f(m))

- 22. None
- 23. $(-\infty, b) \cup (c, m)$
- 24. $(b, c) \cup (m, p)$
- 25. x = a, x = c, x = n
- 26. $(\infty, B) \cup (D, F)$
- 27. (B, D) ∪ (F, ∞)
- 28. *B* and *F*
- 29. D
- 30. B
- 31. None
- 32. A, C, E, G

- 33. 2
- 34. $(-\infty, 2)$
- 35. $(-\infty, -2.5) \cup (-2.5, 3.6]$
- 36. (-1.2, 3) and (1.3, 3.6)
- 37. (0, 1)
- 38. (1.3, 3.6)
- 39. Does not exist
- 40. $(-\infty, -1.2) \cup (0, 1.3)$
- 41. $(-1.2, 0) \cup (1.3, 2)$
- 42. (-1.8, 0) and (1.9, 0)
- 43. 4

Linear Functions and Slope

4.1 Equations of Lines

Write the equation of each line in point-slope form that goes through each pair of points.

- 1. (-2,1), (7,8)
- 2. (0,4), (9,-15)
- 3. (-1, -2), (-3, -13)

4.2 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

- 6. [-2, -1]
- 7. [5, 6]
- 8. [0, 1]
- 9. [5, 5.001]
- 10. [5, 5.0001]
- 11. [5, 5.00001]
- 12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

- 13. [7, 7.001]
- 14. [7, 7.0001]
- 15. [7, 7.00001]
- 16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

- 17. [1, 1.0001]
- 18. [1, 1.00001]
- 19. [1, 1.000001]
- 20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = 6\sqrt{x}$, find the average rate of change of each over the given interval.

- 21. [25, 25.1]
- 22. [25, 25.01]
- 23. [25, 25.001]
- 24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -7x^3 + 6\sqrt{3x} + 4$ over each interval. Round your answers to 4 decimal places.

- 25. [0, 1]
- 26. [10, 11]
- 27. [8, 15]

Equations of Lines

- 1. $y-1=\frac{7}{9}(x+2)$ or $y-8=\frac{7}{9}(x-7)$
- 2. $y-4=-\frac{19}{9}(x-0)$ or $y+15=-\frac{19}{9}(x-9)$
- 3. $y + 2 = \frac{11}{2}(x+1)$ or $y + 13 = \frac{11}{2}(x+3)$

Average Rate of Change

- 1. 2.1
- 2. 2.01
- 3. 2.001
- 4. 2.0001
- 5. 2
- 6. 25
- 7. -59
- 8. 1
- 9. -53.006

- 10. -53.0006
- 11. -53.00006
- 12. -53
- 13. -42.003
- 14. -42.0003
- 15. -42.00003
- 16. -42
- 17. -0.499988
- 18. -0.4999988

- 19. -0.49999988
- 20. -0.5
- 21. 0.5994
- 22. 0.59999
- 23. 0.6
- 24. 0.6
- 25. 3.3923
- 26. -2,315.3960
- 27. -2861.4492

Function Transformations

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
 - (2) Shift left 1 unit
 - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
 - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
 - (2) Vertical compression by 4
 - (3) Move right 7 units

Write the function g(x) that is a result of the following ordered sequence of transformations to f(x) = |x|.

- 4. (1) Reflect across x-axis
 - (2) Shift right 3 units
 - (3) Horizontal stretch by factor of 5
- 5. (1) Shift down 2 units
 - (2) Reflect across y-axis
 - (3) Shift up 1 unit
- 6. (1) Horizontal compression by factor of 7
 - (2) Vertical compression by factor of 4
 - (3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function g(x) after the following ordered sequence of transformations.

- 7. (1) Shift up 2 units
 - (2) Horizontal stretch by 5
 - (3) Shift left 3 units
- 8. (1) Vertical compression by factor of 3
 - (2) Reflect across y-axis
 - (3) Horizontal compression by 5
- 9. (1) Shift right 8 units
 - (2) Reflect across x-axis
 - (3) Horizontal compression by factor of 4

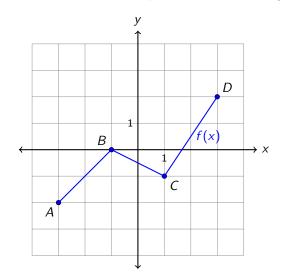
Write the final equation of g(x) if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

- 10. (1) Shift right 2 units
 - (2) Horizontal stretch by factor 3
 - (3) Shift down 2 units
 - (4) Reflect across x-axis
- 11. (1) Horizontal stretch by factor 3
 - (2) Shift left 1 unit
 - (3) Shift up 2 units
 - (4) Reflect across y-axis
- 12. (1) Vertical stretch by factor 5
 - (2) Horizontal stretch by factor 2
 - (3) Shift up 3 units
 - (4) Reflect across x-axis

Find the equation for g(x) if g(x) is found by performing the following *ordered* sequence of transformations to $f(x) = \frac{1}{x}$.

- 13. (1) Shift left 3 spaces
 - (2) Reflect across y-axis
 - (3) Shift down 5 spaces
 - (4) Vertical stretch by factor of 7
- 14. (1) Shift up 3 spaces
 - (2) Reflect across x-axis
 - (3) Shift right 5 spaces
 - (4) Horizontal compression by factor of 7

Given the graph of f(x) below, find the new coordinates of each point after the following transformations.



15.
$$-2f(x+1)$$

16.
$$f\left(-\frac{1}{2}x\right) - 3$$

17.
$$\frac{1}{2}f(-x-2)+2$$

18.
$$f(2x+2)-1$$

19.
$$-3f(-x+1)+2$$

20.
$$5f\left(-\frac{1}{2}x\right)$$

FUNCTION TRANSFORMATIONS KEY

1.
$$g(x) = 3f(-x+1)$$

2.
$$g(x) = f(2x) + 1$$

3.
$$g(x) = -\frac{1}{4}f(x-7)$$

4.
$$g(x) = -\left|\frac{1}{5}x - 3\right|$$

5.
$$g(x) = |-x| - 1$$

6.
$$g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$$

7.
$$g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$$

8.
$$g(x) = \frac{1}{3}\sqrt{-5x}$$

9.
$$g(x) = -\sqrt{4x - 8}$$

10.
$$g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$$

11.
$$g(x) = \sqrt{\frac{1}{3}(-x+1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$$

12.
$$g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$$

13.
$$g(x) = \frac{7}{-x+3} - 35$$

14.
$$g(x) = -\frac{1}{7x-5} - 3$$

15.
$$A'(-4,4)$$
, $B'(-2,0)$, $C'(0,2)$, $D'(2,-4)$

16.
$$A'(6,-5)$$
, $B'(2,-3)$, $C'(-2,-4)$, $D'(-6,-1)$

17.
$$A'(1,1)$$
, $B'(-1,2)$, $C'(-3,1.5)$, $D'(-5,3)$

18.
$$A'(-2.5, -3)$$
, $B'(-1.5, -1)$, $C'(-0.5, -2)$, $D'(0.5, 1)$

19.
$$A'(4,8)$$
, $B'(2,2)$, $C'(0,5)$, $D'(-2,-4)$

20.
$$A'(6,-10)$$
, $B'(2,0)$, $C'(-2,-5)$, $D'(-6,10)$

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given f(x) = x + 5, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

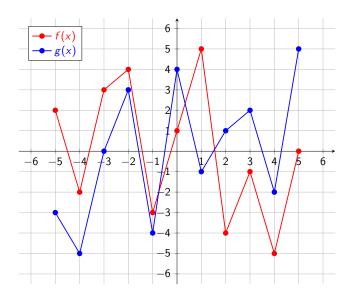
- 1. (g f)(x)
- 2. (fh)(14)
- 3. (f+g)(x)

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-3	0	-1	3	1	2	4	-4	$\overline{-2}$
g(x)	3	-1	0	1	4	-2	-4	2	-3

- 4. (f+g)(-2) 5. (f-g)(0) 6. (fg)(1)
- 7. $\left(\frac{f}{g}\right)$ (3)
- 8. (f+f)(-4)

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 9. (f+g)(2)
- 10. (f-g)(1)
- 11. (g-f)(-3)
- 12. (fg)(4)
- 13. $\left(\frac{f}{g}\right)(0)$

6.2 Operations with Functions: Domain

Given $f(x) = \sqrt{2x+7}$ and g(x) = 3x+3, find the domain of each.

- 1. (f+g)(x)
- $2. \left(\frac{f}{g}\right)(x)$
- 3. $\left(\frac{g}{f}\right)(x)$

6.3 Difference Quotient

Write the difference quotient for each.

- 1. f(x) = 2x 7
- 2. $g(x) = x^2 + 4x$
- 3. h(x) = -1
- 4. $f(x) = \frac{3}{x+2}$
- 5. $g(x) = \sqrt{3x}$
- 6. $f(x) = x^2 2x + 5$
- 7. $g(x) = \frac{5}{x}$

- 8. $f(x) = -2x^2 + 3x 5$
- 9. $g(x) = \frac{6}{2x+3}$
- 10. $h(x) = \sqrt{7x+5}$
- 11. $f(x) = -x^2 + x$
- 12. f(x) = 3x 1
- 13. $f(x) = x^3 + 5x$

Adding, Subtracting, Multiplying, and Dividing Functions

1.
$$x^2 - x - 6$$

2. 38

3.
$$x^2 + x + 4$$

4. -1

5. -3

6. -4

7. -2

8. -6

9. -3

10. 6

11. -3

12. 10

13. $\frac{1}{4}$

Operations with Functions: Domain

1.
$$\left[-\frac{7}{2},\infty\right)$$

2.
$$\left[-\frac{7}{2}, -1\right) \cup (-1, \infty)$$

3.
$$\left(-\frac{7}{2}, \infty\right)$$

Difference Quotient

2.
$$2x + h + 4$$

4.
$$\frac{-3}{(x+2)(x+h+2)}$$

$$5. \ \frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$$

6.
$$2x + h - 2$$

$$7. \ \frac{-5}{x(x+h)}$$

8.
$$-4x - 2h + 3$$

9.
$$\frac{-12}{(2x+3)(2x+2h+3)}$$

10.
$$\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$$

11.
$$-2x - h + 1$$

13.
$$3x^2 + 3xh + h^2 + 5$$

Polynomials and Their Graphs

Determine the end behavior of each.

1.
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

2.
$$g(x) = 4x^2 - 16x^6 + 3x$$

3.
$$h(x) = 1 + x^{11} - 4x^8$$

4.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

5.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

6.
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

7.
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$

POLYNOMIALS AND THEIR GRAPHS

1.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

2.
$$\lim_{x \to -\infty} g(x) = -\infty$$
, $\lim_{x \to \infty} g(x) = \infty$

3.
$$\lim_{x \to -\infty} h(x) = -\infty$$
 $\lim_{x \to \infty} h(x) = \infty$

4.
$$\lim_{x \to -\infty} f(x) = -\infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

5.
$$\lim_{x \to -\infty} g(x) = -\infty$$
 $\lim_{x \to \infty} g(x) = \infty$

6.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

7.
$$\lim_{x \to -\infty} g(x) = -\infty$$
 $\lim_{x \to \infty} g(x) = \infty$

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

1.
$$(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$$

2.
$$(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$$

3.
$$(4x^3 - 3x^2 + x + 1) \div (x + 2)$$

4.
$$(5x^4 - x^2 + x - 2) \div (x^2 + 2)$$

5.
$$(10x^3 + 27x^2 + 8x - 11) \div (2x + 3)$$

6.
$$(7x^3 + 23x^2 + 12x + 1) \div (x^2 + 3x + 1)$$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

1.
$$(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$$

2.
$$(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x+1)$$

3.
$$(x^{23} - 5x^{20} + 17x^8 - 5) \div (x+2)$$

4.
$$\left(-7x^{17}+40x^{15}-6x^8+4x^3\right)\div(x-3)$$

Dividing Polynomials

1.
$$4x^2 - 2x + 5$$

2.
$$4y^2 + 8y + 7 - \frac{72}{3y + 5}$$

3.
$$4x^2 - 11x + 23 - \frac{45}{x+2}$$

4.
$$5x^2 - 11 + \frac{x + 20}{x^2 + 2}$$

$$5. \ 5x^2 + 6x - 5 + \frac{4}{2x + 3}$$

6.
$$7x + 2 + \frac{-x - 1}{x^2 + 3x + 1}$$

Remainder and Factor Theorems

- 1. 6
- 2. 13
- 3. -13, 627, 141
- 4. -330, 064, 119

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1.
$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

2.
$$g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$3. \ f(x) = \frac{3x}{x+4}$$

4.
$$g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

5.
$$h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

6.
$$f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

7.
$$g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

8.
$$h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

9.
$$f(x) = \frac{6x^3 - 21x^2 - 51x + 30}{3x^2 + 7x + 2}$$

10.
$$g(x) = \frac{10x^2 - 29x - 21}{10x^3 - 33x^2 - 7x}$$

State the end behavior of each.

11.
$$k(x) = \frac{5x^3 - 7x^2 + 8}{-3x^3 + 6x - 4}$$

12.
$$m(x) = \frac{2x-1}{3x^2+7x+1}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain:
$$x \neq -\frac{1}{2}$$
, 7; V.A.: $x = 7$; Hole @ $\left(-\frac{1}{2}, -\frac{7}{13}\right)$; H.A.: $y = 1$

2. Domain:
$$x \neq -3$$
, 4; V.A.: $x = -3x = 4$; Obl. Asymp: $y = 3x + 10$

3. Domain:
$$x \neq -4$$
; V.A.: $x = -4$; H.A.: $y = 3$

4. Domain:
$$x \neq 1$$
; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$

5. Domain:
$$x \neq 0, 1$$
; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$

6. Domain:
$$x \neq -1, 5$$
; V.A. $x = -1$; Hole @ $\left(5, \frac{13}{3}\right)$; Obl. Asym $y = 2x - 5$

7. Domain:
$$x \neq -6, 0, 4$$
; V.A. $x = -6, x = 0$; Hole @ $\left(4, \frac{21}{40}\right)$; H.A. $y = 0$

8. Domain:
$$x \neq -6, -\frac{3}{8}$$
; V.A. $x = -6, x = -\frac{3}{8}$; H.A. $y = \frac{1}{4}$

9. Domain:
$$x \neq -2$$
, $-\frac{1}{3}$; Hole @ $(-2, -21)$; V.A.: $x = -\frac{1}{3}$; S.A. $y = 2x - \frac{35}{3}$

10. Domain:
$$x \neq -\frac{1}{5}$$
, 0, $\frac{7}{2}$; Hole @ $\left(\frac{7}{2}, \frac{82}{259}\right)$; V.A. $x = -\frac{1}{5}$ and $x = 0$; H.A. $y = 0$

11.
$$\lim_{x \to -\infty} k(x) = \infty = \lim_{x \to \infty} k(x) = -\frac{5}{3}$$

12.
$$\lim_{x \to -\infty} m(x) = \infty = \lim_{x \to \infty} m(x) = 0$$

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1.
$$6x^3 - 4x^2 - 10x > 0$$

2.
$$x^4 < 9x^2$$

3.
$$3x^3 - 7x^2 - 22x + 8 < 0$$

4.
$$3x^2 - 4x + 1 < 0$$

5.
$$12x^4 + 76x^3 + 43x^2 - 346x - 280 > 0$$

6.
$$-2x^4 + 49x^2 + 21x^3 - 1029x + 2401 > 0$$

7.
$$-x^2 - 7x - 6 < 0$$

8.
$$x^2 + 4x + 4 < 0$$

9.
$$-x^4 - 6x^3 + 61x^2 + 234x - 1008 \ge 0$$

10.
$$-x^2 + 3x + 1 > 3$$

11.
$$-3x^4 + 123x^3 + 142x^2 - 424x + 320 \le 122x^3$$

12.
$$-x^4 - 1120 + 77x^2 - 36x + 15x^3 > 15x^3$$

$$13. -3x^4 - 22x^3 + 271x^2 + 152x - 96 > 267x^2$$

10.2 Domain

State the domain of each. Write your answers using interval notation.

1.
$$b(x) = \sqrt{21x^2 - 23x - 20}$$

10.3 Rational Inequalities

Solve each. Write your answers using interval notation.

1.
$$\frac{3x-4}{x+1} < 0$$

$$5. \ \frac{x^2 - 7x - 8}{x^2 - 4x - 32} \ge 0$$

10.
$$\frac{2x-50}{5x+15} \le -1$$

$$2. \ \frac{x^2 + 3x + 2}{x - 7} \le 0$$

$$6. \ \frac{4+3x}{5-x} \le 2$$

7. $\frac{x-4}{x+7} < 0$

11.
$$\frac{x+5}{x^2-2x-15} \le 0$$

$$3. \ \frac{x^2 - 4x + 4}{x^2 - 1} \ge 0$$

8.
$$\frac{x+5}{x+7} < 0$$

12.
$$-\frac{2}{x} \ge -\frac{3}{x+1}$$

4.
$$\frac{x+2}{x-4} \le 1$$

9.
$$\frac{2x-26}{5x+20} > -3$$

13.
$$-\frac{3}{x+6} > -\frac{4}{x+7}$$

Polynomial Inequalities

1.
$$[-1,0] \cup \left[\frac{5}{3},\infty\right)$$

2.
$$(-3,0) \cup (0,3)$$

$$3. \ \left(-\infty, -2\right) \cup \left(\frac{1}{3}, 4\right)$$

4.
$$\left[\frac{1}{3}, 1\right]$$

5.
$$\left(-\infty, -4\right] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$$

$$6. \left[-7, \frac{7}{2} \right] \cup 7$$

7.
$$(-\infty, -6] \cup [-1, \infty)$$

9.
$$[-8, -7] \cup [3, 6]$$

11.
$$\left(-\infty, -8\right] \cup \left[\frac{4}{3}, 2\right] \cup \left[5, \infty\right)$$

12.
$$[-8, -4] \cup [5, 7]$$

13.
$$[-6, -4] \cup \left[\frac{2}{3}, 2\right]$$

Domain

1.
$$\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$$

Rational Inequalities

1.
$$\left(-1, \frac{4}{3}\right)$$

2.
$$(-\infty, -2] \cup [-1, 7)$$

3.
$$(-\infty, -1) \cup (1, \infty)$$

4.
$$(-\infty, 4)$$

5.
$$(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$$

6.
$$(-\infty, 1.2] \cup (5, \infty)$$

7.
$$(-7, 4)$$

8.
$$(-7, -5)$$

9.
$$(-\infty, -4) \cup (-2, \infty)$$

10.
$$(-3, 5]$$

11.
$$(-\infty, -5] \cup (-3, 5)$$

12.
$$(-1,0) \cup [2,\infty)$$

13.
$$(-7, -6) \cup (-3, \infty)$$

Function Compositions

Given f(x) = x - 5, $g(x) = 4 + \sqrt{2x + 1}$, and $h(x) = \frac{3}{x + 7}$, simplify each and state the domain.

- 1. $(f \circ g)(x)$
- 2. $(g \circ f)(x)$
- 3. h(h(x))

Find each of the following given the table below.

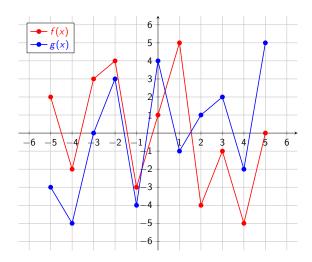
x	-4	-3	-2	-1	0	1	2	3	4	
f(x)	-3	0	-1	3	1	2	4	-4	-2	
g(x)	3	-1	0	1	4	-2	-4	2	-3	

- 4. $(f \circ g)(-1)$ 5. $(g \circ g)(0)$ 6. $(f \circ f)(2)$ 7. $(g \circ g)(-3)$ 8. f(g(0))

Given $f(x) = \sqrt{3x+2}$, $g(x) = x^2 - 1$, and h(x) = 9x - 2, find each of the following.

- 9. $(g \circ f)(x)$
- 10. f(g(x))
- 11. $(h \circ h)(x)$

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 12. $(f \circ g)(-1)$ 13. $(g \circ f)(-4)$
- 14. f(g(3))
- 15. g(g(-2)) 16. $(f \circ f)(-5)$

FUNCTION COMPOSITIONS KEY

- 1. $-1 + \sqrt{2x+1}$ Domain: $\left[-\frac{1}{2}, \infty\right)$
- 2. $4 + \sqrt{2x 9}$ Domain: $\left[\frac{9}{2}, \infty\right)$
- 3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup \left(-7, \infty\right)$
- 4. 2
- 5. -3
- 6. -2
- 7. 1
- 8. -2
- 9. 3x + 1
- 10. $\sqrt{3x^2-1}$
- 11. 81x 20
- 12. -2
- 13. 3
- 14. -4
- 15. 2
- 16. -4

Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1.
$$f(x) = \sqrt{-2x+3} + 1$$

2.
$$g(x) = (x+4)^2 - 1$$
, $x \le -4$

3.
$$h(x) = \frac{9x}{4x - 1}$$

4.
$$f(x) = \sqrt{x} - 3$$

$$5. g(x) = \frac{1}{1-x}$$

6.
$$h(x) = x^2 + 6x + 4, x \le -3$$

7.
$$f(x) = \sqrt{5x - 4}$$

8.
$$g(x) = x^2 - 2x + 3, x \le 1$$

9.
$$h(x) = \frac{3}{x-1}$$

INVERSE FUNCTIONS KEY

1. f	$^{-1}(x) = -$	$-\frac{1}{2}((x-$	$(1)^2 - 3)$
------	----------------	--------------------	--------------

	Domain	Range
f(x)	$(-\infty, 1.5]$	$[1,\infty)$
$f^{-1}(x)$	$[1,\infty)$	$(-\infty, 1.5]$

2.
$$g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
g(x)	$(-\infty, -4]$	$[-1,\infty)$
$g^{-1}(x)$	$[-1,\infty)$	$(-\infty, -4]$

3.
$$h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
h(x)	$(-\infty,1/4)\cup(1/4,\infty)$	$(\infty,9/4)\cup(9/4,\infty)$
$h^{-1}(x)$	$(\infty,9/4)\cup(9/4,\infty)$	$(-\infty,1/4)\cup(1/4,\infty)$

4.
$$f^{-1}(x) = (x+3)^2$$

	Dom	Ran
f(x)	$[0,\infty)$	$[-3,\infty)$
$f^{-1}(x)$	$[-3,\infty)$	$[0,\infty)$

5.
$$g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
g(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$g^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

6.
$$h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
h(x)	$(-\infty, -3]$	$[-5,\infty)$
$h^{-1}(x)$	$[-5,\infty)$	$(-\infty, -3]$

7.
$$f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

	Dom	Ran
f(x)	$\left[\frac{4}{5},\infty\right)$	$[0,\infty)$
$f^{-1}(x)$	[0, ∞)	$\left[\frac{4}{5},\infty\right)$

8.
$$g^{-1}(x) = -\sqrt{x-2} + 1$$

	Dom	Ran
g(x)	$(-\infty,1]$	[2, ∞)
$g^{-1}(x)$	[2, ∞)	$(-\infty,1]$

9.
$$h^{-1}(x) = \frac{3}{x} + 1$$

_		Dom	Ran
	h(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
	$h^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

Exponential Functions

13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

- 1. $f(x) = 3 + e^{2x}$
- 2. $h(x) = 5^{-x}$
- 3. $h(x) = -\frac{2}{3}e^{x+7} + 1$
- 4. $f(x) = -7e^x + 4$
- 5. $g(x) = \frac{1}{3}e^{2x+1} 5$
- 6. $h(x) = -\frac{1}{2}e^{-4x} + 1$

EXPONENTIAL FUNCTIONS

1.
$$\lim_{x \to -\infty} f(x) = 3$$
 $\lim_{x \to \infty} f(x) = \infty$

2.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = 0$

3.
$$\lim_{x \to -\infty} h(x) = 1$$
 $\lim_{x \to \infty} h(x) = -\infty$

4.
$$\lim_{x \to -\infty} f(x) = 4$$
 $\lim_{x \to \infty} f(x) = -\infty$

5.
$$\lim_{x \to -\infty} f(x) = -5$$
 $\lim_{x \to \infty} f(x) = \infty$

6.
$$\lim_{x \to -\infty} f(x) = -\infty$$
 $\lim_{x \to \infty} f(x) = 1$

Logarithmic Functions

Write each of the following in exponential or logarithmic form.

- 1. ln(a) = 7
- 2. $\log_4(x+1) = 9$
- 3. $\log(5x) = 30$
- 4. ln(w) = c
- 5. $5^x = 19$
- 6. $8^{-3} = \frac{1}{512}$
- 7. $e^{14} = x$
- 8. $(1.1)^{-t} = 50$

Find the domain of each. Write your answers in interval notation.

- 9. $b(x) = \log_7(x^2 8x + 6)$
- 10. $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$
- 11. $f(x) = -7 \ln (x^2 + 9x + 8)$
- 12. $g(x) = \log (5x^2 + 13x 6)$
- 13. $h(x) = 3 \log_2 (x^3 + 2x^2 x 2)$
- 14. $c(x) = \ln (4x^2 15x 4)$

State the end behavior of each.

15.
$$j(x) = 5 \log_3 (2x - 5) - 2$$

LOGARITHMIC FUNCTIONS KEY

1.
$$e^7 = a$$

2.
$$4^9 = x + 1$$

3.
$$10^{30} = 5x$$

4.
$$e^c = w$$

5.
$$\log_5(19) = x$$

$$6. \log_8\left(\frac{1}{512}\right) = -3$$

7.
$$ln(x) = 14$$

8.
$$\log_{1.1}(50) = -t$$

9.
$$(-\infty, 0.838) \cup (7.162, \infty)$$

10.
$$(-3, -2) \cup (-1, \infty)$$

11.
$$(-\infty, -8) \cup (-1, \infty)$$

12.
$$(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$$

13.
$$(-2, -1) \cup (1, \infty)$$

14.
$$\left(-\infty, -\frac{1}{4}\right) \cup (4, \infty)$$

15.
$$\lim_{x \to (5/2)^+} j(x) = -\infty \quad \lim_{x \to \infty} j(x) = \infty$$

Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

- 1. $\log_b \left(\frac{x^2}{y^8} \right)$
- 2. $\ln(ez)^3$
- 3. $\log_5(x) + \log_5(9) 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

- 4. $log_7(10)$
- 5. $\log_9(x)$
- 6. $\log_b(c)$
- 7. $\log_3(10)$
- 8. $\log_{17}(\pi)$
- 9. $\log_w(x)$

Suppose that $\log_a(b) = 5$, $\log_a(c) = 12$, and $\log_a(d) = 9$. Evaluate each of the following.

- 10. $\log_a(bc)$
- 11. $\log_a(c^3)$
- 12. $\log_a \left(\frac{d}{c}\right)$
- 13. $\log_a \left(\frac{bd}{c}\right)$

Properties of Logarithms Key

- $1. \ 2\log_b(x) 8\log_b(y)$
- 2. $3 + 3 \ln(z)$
- $3. \log_5 \left(\frac{9x}{w^2} \right)$
- 4. $\frac{\ln(10)}{\ln(7)}$
- $5. \ \frac{\ln(x)}{\ln(9)}$
- 6. $\frac{\ln(c)}{\ln(b)}$
- 7. $\frac{\ln(10)}{\ln(3)}$
- 8. $\frac{\ln(\pi)}{\ln(17)}$
- 9. $\frac{\ln(x)}{\ln(w)}$
- 10. 17
- 11. 36
- 12. -3
- 13. 2

Exponential Equations

Solve each. Round to 3 decimal places when necessary.

1.
$$3e^{x-2} = 7$$

2.
$$5^x + 4 > 1$$

3.
$$2^{3x+4} = 32^{x-7}$$

4.
$$5e^{7x} + 10 = 42$$

5.
$$7^{4x+1} \ge 343$$

6.
$$1000e^{0.04x} = 2000$$

7.
$$3(4.1)^{x-2} = 8$$

8.
$$2^{x+1} = 5^{7x-5}$$

9.
$$8(17)^{-5x} = 22$$

10.
$$-3(11)^{x-10} = -58$$

11.
$$12^{-10x} + 8 = 80$$

12.
$$-5(10)^{7x} + 9 = -46$$

13.
$$8(8)^{10x} - 1 = 55.2$$

14.
$$3(3)^{-5x} - 8 = 74$$

15.
$$6(16)^{4x-9} = 19$$

16.
$$-7(11)^{5x-7} = -3$$

17.
$$3^{9-6x} - 7 = 26$$

16.1 Applications

- 1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?
- 2. Cadmium-109 has a half-life of about 1.267 years. If 50 mg are initially present, how many years will it take for 16 mg to remain?
- 3. The half-life of bismuth-207 is about 32.9 years. If 90 mg are initially present, how many years will it take for 75 mg to remain?

EXPONENTIAL EQUATIONS KEY

1. $x \approx 2.847$

2. $(-\infty, \infty)$

3. x = 19.5

4. $x \approx 0.265$

5. $\left[\frac{1}{2}, \infty\right)$

6. $x \approx 17.329$

7. $x \approx 2.695$

8. $x \approx 0.827$

Applications

1. Approximately 17,952 years

2. Approximately 2.0828 years

3. Approximately 8.6538 years

9. $x \approx -0.071$

10. $x \approx 11.235$

11. $x \approx -0.172$

12. $x \approx 0.149$

13. $x \approx 0.094$

14. $x \approx -0.602$

15. $x \approx 2.354$

16. $x \approx 1.323$

17. $x \approx 0.970$

Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

1.
$$\log_5(x) + x \log_5(x) > 0$$

2.
$$\ln (8 - x^2) = \ln(2 - x)$$

3.
$$\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$$

4.
$$\log_3(2x+1) - \log_3(x-5) = \log_3(x+1)$$

5.
$$\log_4(x+1) + \log_4(x-5) > 2$$

6.
$$\log(x+1) - \log(x-5) = \log(x-3)$$

7.
$$x \log_3(x+2) - \log_3(x+2) = 0$$

8.
$$\log_{1/2}(x+1) > -3$$

9.
$$\log_{12}(4x+4) = \log_{12}(5x+1)$$

10.
$$\log_{15}(-4x+2) = \log_{15}(6-2x)$$

11.
$$\log_{11}(-5-3x^2) = \log_{11}(-2x^2+6x)$$

12.
$$\log_{16}(x^2+4) = \log_{16}(2x+3)$$

13.
$$\log_7(8x-1) = \log_7(x^2+14)$$

14.
$$-7 \log_5(x+5) = -7$$

15.
$$7 \log_8(-x) = 28$$

16.
$$-10\log_3(x-5) = -20$$

LOGARITHMIC EQUATIONS AND INEQUALITIES KEY

- 1. $(1, \infty)$
- 2. x = -2
- 3. $x \approx 1.571$
- 4. $x \approx 6.873$
- 5. $(2, \infty)$
- 6. x = 7
- 7. $x = \pm 1$
- 8. (-1,7)
- 9. x = 3
- 10. x = -2
- 11. No Solution
- 12. x = 1
- 13. x = 3, 5
- 14. x = 0
- 15. x = -4096
- 16. x = 14

Sequences

Write the first 4 terms of each sequence.

1.
$$a_n = 2(-3)^n$$

2.
$$b_n = \frac{n!}{2^n}$$

3.
$$c_{n+1} = 5c_n + 1$$
; $c_1 = 2$

4.
$$d_n = \frac{1}{2}d_{n-1} + n$$
; $d_1 = 3$

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round**.

5.
$$a_n = \{343, 667, 991, 1315, \dots\}$$
; Find the 582nd term.

6.
$$b_n = \{300, 240, 192, 153.6, \dots\}$$
; Find the 711th term.

7.
$$c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots \right\}$$
; Find the 8,675,309th term.

Given each sequence, find the first 4 digits after the decimal point in the scientific notation version of each term.

8.
$$a_n = 17, 33, 49, 65, ...; a_{21,972}$$

9.
$$b_n = 25$$
, 36, 49, 64, 81, ... $b_{413,401}$

10.
$$c_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots c_{152}$$

SEQUENCES KEY

- 1. -6, 18, -54, 162
- $2. \ \frac{1}{2}, \ \frac{1}{2}, \ \frac{3}{4}, \ \frac{3}{2}$
- 3. 2, 11, 56, 281
- 4. 3, $\frac{7}{2}$, $\frac{19}{4}$, $\frac{51}{8}$
- 5. 188,587
- 6. 6882
- 7. 7634
- 8. 5155 (3.51553 \times 10⁵)
- 9. 7090 (1.7090369403 \times 10¹¹)
- 10. 1677 (1.1677487203 \times 10⁻⁴⁶)

Series

Find the sum of each, if possible.

1.
$$\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^{i}$$

$$2. \sum_{i=0}^{\infty} 3\left(-\frac{2}{3}\right)^{i}$$

$$3. \sum_{k=1}^{\infty} -2\left(\frac{1}{3}\right)^k$$

4.
$$\sum_{i=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^{j}$$

5.
$$\sum_{i=0}^{\infty} 1.2(0.8)^{i}$$

6.
$$\sum_{i=1}^{\infty} 1.2(0.8)^{i}$$

7.
$$\sum_{i=0}^{\infty} 0.8(1.2)^{i}$$

8.
$$\sum_{k=1}^{\infty} \frac{2}{3^k}$$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9.
$$9+13+17+21+\cdots+1565$$

10.
$$-3+6-12+24-48+\cdots+50,331,648$$

11.
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{981}$$

12.
$$2+4+6+8+10+\cdots+38,214$$

13.
$$3+7+11+15+\cdots+11,491$$

14.
$$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \dots + \frac{742}{743}$$

SERIES KEY

- 1. $\frac{1}{4}$
- 2. $\frac{9}{5}$
- 3. -1
- 4. Diverges
- 5. 6
- 6. 4.8
- 7. Diverges
- 8. 2
- 9. 306,930
- 10. -33, 554, 433
- 11. 7.4663
- 12. 365,096,556
- 13. 16,511,131
- 14. 733.8947

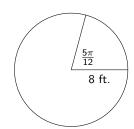
Angles and Radian Measure

Sketch each of the following. Then find a coterminal between 0 and 360° (or 0 and 2π radians) for each.

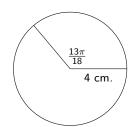
- 1. $-\frac{3\pi}{4}$
- 2. 900°
- 3. $\frac{27\pi}{10}$
- $4. -125^{\circ}$

Find the arc length and sector area formed by the central angle of each. Exact answers only.

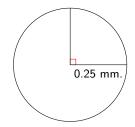
5.



6.



7.



A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

- 8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
- 9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

Angles and Radian Measure Key

1.
$$\frac{5\pi}{4}$$



2. 180°



3. $\frac{7\pi}{10}$



4. 235°



5. $s = \frac{10\pi}{3}$ ft.; $A = \frac{40\pi}{3}$ sq.ft.

6. $s = \frac{26\pi}{9} \text{ cm.}; \quad A = \frac{52\pi}{9} \text{ sq.cm}.$

7. $s = \frac{\pi}{8}$ mm.; $A = \frac{\pi}{64}$ sq.mm.

8. 26.18 rad/sec

9. 8.73 ft/sec

Trig Functions of Any Angle

Find the exact value of each of the six trig functions of θ if P is a point on the terminal side of θ .

- 1. P(-2,3)
- 2. P(0, -4)
- 3. $P(-2\sqrt{3}, 2)$
- 4. P(-3,5)
- 5. P(-2,1)
- 6. P(-4, -7)

Find the exact values of the 6 trig functions of the following angles.

- 7. $\theta = \frac{-17\pi}{4}$
- 8. $\theta = \frac{21\pi}{2}$
- 9. $\theta = 24\pi$

TRIG FUNCTIONS OF ANY ANGLE KEY

1.
$$\sin \theta = \frac{3\sqrt{13}}{13}$$
, $\cos \theta = \frac{-2\sqrt{13}}{13}$, $\tan \theta = -\frac{3}{2}$, $\csc \theta = \frac{\sqrt{13}}{3}$, $\sec \theta = -\frac{\sqrt{13}}{2}$, $\cot \theta = -\frac{2}{3}$

2.
$$\sin\theta=-1$$
, $\cos\theta=0$, $\tan\theta=$ undef., $\csc\theta=-1$, $\sec\theta=$ undef., $\cot\theta=0$

$$3. \ \sin\theta=\frac{1}{2}, \cos\theta=-\frac{\sqrt{3}}{2}, \tan\theta=-\frac{\sqrt{3}}{3}, \csc\theta=2, \sec\theta=-\frac{2\sqrt{3}}{3}, \cot\theta=-\sqrt{3}$$

4.
$$\sin\theta = \frac{5\sqrt{34}}{34}$$
, $\cos\theta = -\frac{3\sqrt{34}}{34}$, $\tan\theta = -\frac{5}{3}$, $\csc\theta = \frac{\sqrt{34}}{5}$, $\sec\theta = -\frac{\sqrt{34}}{3}$, $\cot\theta = -\frac{3}{5}$

5.
$$\sin\theta = \frac{\sqrt{5}}{5}$$
, $\cos\theta = -\frac{2\sqrt{5}}{5}$, $\tan\theta = -\frac{1}{2}$, $\csc\theta = \sqrt{5}$, $\sec\theta = -\frac{\sqrt{5}}{2}$, $\cot\theta = -2$

6.
$$\sin\theta = -\frac{7\sqrt{65}}{65}$$
, $\cos\theta = -\frac{4\sqrt{65}}{65}$, $\tan\theta = \frac{7}{4}$, $\csc\theta = -\frac{\sqrt{65}}{7}$, $\sec\theta = -\frac{\sqrt{65}}{4}$, $\cot\theta = \frac{4}{7}$

7.
$$\sin\theta = -\frac{\sqrt{2}}{2}$$
, $\cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = -1$, $\csc\theta = -\sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = -1$

8.
$$\sin \theta = 1$$
, $\cos \theta = 0$, $\tan \theta = \text{undefined}$, $\csc \theta = 1$, $\sec \theta = \text{undefined}$, $\cot \theta = 0$

9.
$$\sin \theta = 0$$
, $\cos \theta = 1$, $\tan \theta = 0$, $\csc \theta =$ undefined, $\sec \theta = 1$, $\cot \theta =$ undefined

Graphs of Sine and Cosine Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1.
$$f(x) = -2\sin\left(3x - \frac{\pi}{4}\right) + 1$$

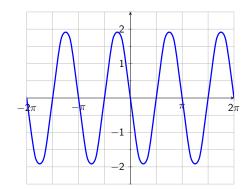
2.
$$g(x) = \frac{1}{3}\cos\left(\frac{1}{2}x + 2\right)$$

3.
$$f(x) = 2\sin\left(x - \frac{\pi}{3}\right) + 7$$

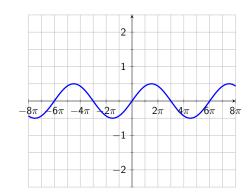
4.
$$f(x) = -4\cos\left(\frac{2}{3}x - \frac{2\pi}{3}\right)$$

Write the equation of each of the following in the form $y = a \sin(bx)$.

5.



6.



Answers

1. Amp = 2, Per =
$$\frac{2\pi}{3}$$
, P.S. = $\frac{\pi}{12}$ \rightarrow , V.S. = 1 \uparrow

2. Amp
$$=\frac{1}{3}$$
, Per $=4\pi$, P.S. $=4\leftarrow$, V.S. $=$ None

3. Amp = 2, Period =
$$2\pi$$
, P.S. = $\frac{\pi}{3}$ right, V.S. = 7 up

4. Amp = 4, Period =
$$3\pi$$
, P.S. = π right, V.S. = 0 (or none)

$$5. \ y = -2\sin(2x)$$

$$6. \ \ y = \frac{1}{2} \sin \left(\frac{1}{3} x \right)$$

Graphs of Other Trig Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

$$1. \ h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

2.
$$f(x) = 3\tan\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

GRAPHS OF OTHER TRIG FUNCTIONS KEY

1. Amp = n/a, Per =
$$\frac{4\pi}{3}$$
, P.S. = $\frac{\pi}{9}$ \leftarrow , V.S. = 8 \downarrow

2. Amp = n/a, Period =
$$\frac{\pi}{2}$$
, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down

Inverse Trig Functions

State the exact, simplified value of each or write as an expression of x.

- 1. $tan^{-1}\left(-\sqrt{3}\right)$
- $2. \sec \left(\sin^{-1} \left(\frac{2}{5} \right) \right)$
- 3. $\cot (\sec^{-1}(x))$
- 4. $\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$
- 5. $\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$
- 6. $\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$
- 7. $\cos\left(\tan^{-1}(7x)\right)$
- 8. $\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$

INVERSE TRIG FUNCTIONS KEY

- 1. $-\frac{\pi}{3}$
- $2. \ \frac{5\sqrt{21}}{21}$
- 3. $\frac{1}{\sqrt{x^2 1}} = \frac{\sqrt{x^2 1}}{x^2 1}$
- 4. $\frac{\sqrt{16-9x^2}}{4}$
- 5. $-\frac{3\sqrt{5}}{2}$
- 6. $\frac{13\sqrt{22}}{44}$
- $7. \ \frac{\sqrt{49x^2+1}}{49x^2+1}$
- $8. \ \frac{\sqrt{64-x^2}}{x}$

Trig Equations and Inequalities

Solve each in the interval $[0, 2\pi)$. Write your answers to inequalities using interval notation.

- 1. tan(6x) = 1
- 2. $\cot(2x) = -\frac{\sqrt{3}}{3}$
- 3. $\sin^2(x) = \frac{3}{4}$
- $4. \sin(2x) = \cos(x)$
- 5. $\sin(2x) \ge \sin(x)$
- 6. $\cos(2x) < 0$
- $7. \ 2\sin\left(x \frac{\pi}{3}\right) = -1$
- 8. $3\tan\left(-2x+\frac{\pi}{2}\right)=\sqrt{3}$
- 9. $\sin^2(x) < \frac{1}{2}$
- 10. $tan^2(x) = 3 sec(x) 3$
- 11. $2\csc(x) 3\csc^2(x) = -2\csc^2(x) + 1$
- 12. $-2\cot(x) \csc^2(x) = 0$
- 13. tan(x) = -tan(x)cos(x)
- 14. $3\cos(x) = 2\cos^2(x) + 1$
- 15. $\csc(x) \cot^2(x) + 1 = 0$
- 16. $-\sin(x) + \sin(2x) = 2\sin(2x)$
- 17. $3\cos(x) = \sin(2x) + 2\cos(x)$

TRIG EQUATIONS AND INEQUALITIES KEY

- $1.\ \ \, \frac{\pi}{24},\frac{5\pi}{24},\frac{3\pi}{8},\frac{13\pi}{24},\frac{17\pi}{24},\frac{7\pi}{8},\frac{25\pi}{24},\frac{29\pi}{24},\frac{11\pi}{8},\frac{37\pi}{24},\frac{41\pi}{24},\frac{15\pi}{8}$
- 2. $\frac{\pi}{3}$, $\frac{5\pi}{6}$, $\frac{4\pi}{3}$, $\frac{11\pi}{6}$
- 3. $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$
- 4. $\frac{\pi}{2}$, $\frac{5\pi}{6}$, $\frac{3\pi}{2}$
- $5. \ \left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$
- 6. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
- 7. $x = \frac{\pi}{6}, \frac{3\pi}{2}$
- 8. $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
- 9. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$
- 10. $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 11. $x = \frac{\pi}{2}$
- 12. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
- 13. x = 0, π
- 14. $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 15. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
- 16. $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$
- 17. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Law of Sines and Cosines

Solve each of the following. Round your answers to 1 decimal place.

1.
$$m \angle B = 37.8^{\circ}$$
, $a = 15$, $c = 21.1$

2.
$$m \angle A = 41.9^{\circ}$$
, $m \angle C = 59.2^{\circ}$, $a = 10.2$

3.
$$a = 14$$
, $b = 19.6$, $c = 13.1$

4.
$$c = 29$$
, $b = 23$, $m \angle A = 55^{\circ}$

5.
$$c = 8$$
, $b = 12$, $m \angle A = 90^{\circ}$

6.
$$m \angle B = 67.2^{\circ}$$
, $a = 15.6$, $c = 18.9$

7.
$$b = 20$$
, $a = 30$, $c = 12$

8.
$$a = 14$$
, $b = 6$, $c = 12$

9.
$$a = 7$$
, $b = 14$, $c = 12$

10.
$$m \angle B = 119^{\circ}$$
, $a = 11$, $b = 28$

11.
$$m \angle A = 125^{\circ}$$
, $c = 21$, $a = 28$

12.
$$m \angle B = 108^{\circ}$$
, $m \angle C = 61^{\circ}$, $b = 25$

Law of Sines and Cosines Key

- 1. $b \approx 13.0$, $m \angle A \approx 44.8^{\circ}$, $m \angle C \approx 97.4^{\circ}$
- 2. $m \angle B = 78.9^{\circ}$, $b \approx 15.0$, $c \approx 13.1$
- 3. $m \angle A \approx 45.5^{\circ}$, $m \angle B \approx 92.6^{\circ}$, $m \angle C \approx 41.9^{\circ}$
- 4. $m \angle B \approx 50.1^{\circ}$, $m \angle C \approx 74.9^{\circ}$, $a \approx 24.6$
- 5. $m \angle B \approx 56.3^{\circ}$, $m \angle C \approx 33.7^{\circ}$, $a \approx 14.4$
- 6. $m \angle A \approx 48.2^{\circ}$, $m \angle C \approx 64.6^{\circ}$, $b \approx 19.3$
- 7. $m \angle A \approx 137.8^{\circ}$, $m \angle B \approx 26.6^{\circ}$, $m \angle C \approx 15.6^{\circ}$
- 8. $m \angle A \approx 96.4^{\circ}$, $m \angle B \approx 25.2^{\circ}$, $m \angle C \approx 58.4^{\circ}$
- 9. $m \angle A \approx 30^{\circ}$, $m \angle B \approx 91^{\circ}$, $m \angle C \approx 59^{\circ}$
- 10. $m \angle A \approx 20.1^{\circ}$, $m \angle C \approx 40.9^{\circ}$, $c \approx 21$
- 11. $m \angle B \approx 17.1^{\circ}$, $m \angle C \approx 37.9^{\circ}$, $b \approx 10.1$
- 12. $m \angle A \approx 11^{\circ}$, $a \approx 5$, $c \approx 23$

Area of Triangles

Find the area of each. Round your answers to 1 decimal place.

1.
$$m \angle B = 37.8^{\circ}$$
, $a = 15$, $c = 21.1$

2.
$$m \angle A = 41.9^{\circ}$$
, $m \angle C = 59.2^{\circ}$, $a = 10.2$

3.
$$a = 14$$
, $b = 19.6$, $c = 13.1$

4.
$$p = 14$$
, $k = 9$, $h = 9$

5.
$$m \angle T = 15^{\circ}$$
, $m \angle S = 140^{\circ}$, $r = 11.1$

6.
$$m \angle Z = 67^{\circ}$$
, $y = 6$, $m \angle Y = 41^{\circ}$

7.
$$m \angle R = 129^{\circ}$$
, $r = 10$, $m \angle P = 28^{\circ}$

8.
$$a = 6.9$$
, $m \angle B = 115^{\circ}$, $m \angle C = 39^{\circ}$

9.
$$d = 6$$
, $3 = 12$, $f = 8$

10.
$$m \angle Y = 120^{\circ}$$
, $x = 13$, $m \angle Z = 21^{\circ}$

11.
$$z = 10$$
, $y = 14$, $x = 6$

12.
$$m \angle P = 18^{\circ}$$
, $h = 6.9$, $m \angle H = 147^{\circ}$

13.
$$m \angle S = 118^{\circ}$$
, $m \angle T = 30^{\circ}$, $s = 6.3$

14.
$$r = 8$$
, $t = 7.5$, $m \angle S = 50^{\circ}$

15.
$$d = 15.3$$
, $m \angle E = 105^{\circ}$, $f = 5$

16.
$$m \angle R = 31^{\circ}$$
, $p = 12$, $m \angle Q = 26^{\circ}$

17.
$$m \angle D = 120^{\circ}$$
, $f = 4$, $m \angle E = 36^{\circ}$

Area of Triangles Key

- 1. Approximately 97.0 sq. units
- 2. Approximately 65.7 sq. units
- 3. Approximately 91.6 sq. units
- 4. Approximately 39.6 sq. units
- 5. Approximately 24.3 sq. units
- 6. Approximately 24.0 sq. units
- 7. Approximately 11.7 sq. units
- 8. Approximately 31.0 sq. units
- 9. Approximately 21.3 sq. units
- 10. Approximately 41.7 sq. units
- 11. Approximately 26.0 sq. units
- 12. Approximately 3.5 sq. units
- 13. Approximately 6.0 sq. units
- 14. Approximately 23.0 sq. units
- 15. Approximately 36.9 sq. units
- 16. Approximately 19.5 sq. units
- 17. Approximately 10.0 sq. units

Polar Coordinates

Convert each to exact rectangular coordinates.

- 1. $\left(3, \frac{5\pi}{6}\right)$
- 2. $\left(-2, -\frac{\pi}{4}\right)$
- 3. $\left(\frac{5}{2}, 240^{\circ}\right)$

Convert each to exact polar coordinates. Use r>0 and $0\leq \theta < 2\pi$.

- 4. $\left(-2\sqrt{3}, -2\right)$
- $5. \left(-\frac{1}{2}, \frac{1}{2}\right)$
- 6. $(7\sqrt{2}, -7\sqrt{2})$

Convert each to either rectangular or polar coordinates.

- 7. 2x + 5y = 9
- 8. 3y = 1
- 9. -5x 8y = -10
- 10. r = 8
- 11. $r = 4 \sec(\theta)$
- 12. $\theta = -\frac{\pi}{6}$

POLAR COORDINATES KEY

- $1. \left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$
- 2. $(-\sqrt{2}, \sqrt{2})$
- $3. \left(\frac{5}{4}, \frac{5\sqrt{3}}{4}\right)$
- 4. $\left(4, \frac{7\pi}{6}\right)$
- $5. \left(\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$
- 6. $\left(14, \frac{7\pi}{4}\right)$
- $7. \ \ r = \frac{9}{2\cos\theta + 5\sin\theta}$
- 8. $r = \frac{1}{3} \csc \theta$
- 9. $r = \frac{10}{5\cos\theta + 8\sin\theta}$
- 10. $x^2 + y^2 = 64$
- 11. x = 4
- 12. $y = -\frac{\sqrt{3}}{3}x$

Vectors

Given $\vec{v} = 3\mathbf{i} - 5\mathbf{j}$ and $\vec{w} = \langle -2, 1 \rangle$, find each. Exact and simplified answers only.

1.
$$\vec{v} + \vec{w}$$

2.
$$-4\vec{w}$$

Given $\vec{a} = \langle 4, -3 \rangle$, $\mathbf{b} = 11\mathbf{i} + 2\mathbf{j}$, and $\vec{c} = \langle 0, 2 \rangle$, find each. Exact and simplified answers only.

5.
$$\vec{a} + \vec{b}$$

6.
$$\vec{a} - \vec{b}$$

10.
$$3\vec{a} + 5\vec{b}$$

7.
$$a + c$$

11.
$$-2\vec{c} - 1.5\vec{a}$$

12.
$$||\vec{b}||$$

Find the exact horizontal and vertical component form of each vector with the given magnitude and direction angle.

17.
$$|u| = 8$$
, $\theta = 120^{\circ}$

19.
$$|u| = 1.5$$
, $\theta = -300^{\circ}$

21.
$$|u| = 4$$
, $\theta = 210^{\circ}$

18.
$$|u| = 5$$
, $\theta = \frac{\pi}{4}$

20.
$$|u| = \sqrt{2}, \quad \theta = \frac{3\pi}{2}$$

22.
$$|u| = 2\sqrt{3}$$
, $\theta = -\frac{\pi}{6}$

Write each of the following in $|\vec{v}|\langle\cos(\theta),\sin(\theta)\rangle$ form, where $0 \le \theta < 2\pi$.

23.
$$\vec{v} = \langle \sqrt{2}, -\sqrt{2} \rangle$$

24.
$$\vec{v} = \langle -1, \sqrt{3} \rangle$$

25.
$$\vec{v} = \langle 0, 4 \rangle$$

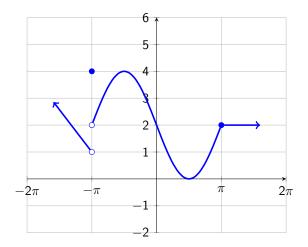
VECTORS KEY

- 1. **i** 4**j**
- 2. $\langle 8, -4 \rangle$
- 3. √34

4.
$$\left\langle -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$$

Numerical and Graphical Limits

Solve using the graph of f(x) below.



$$1. \lim_{x \to -\pi^-} f(x)$$

$$3. \lim_{x \to -\pi} f(x)$$

$$5. \lim_{x \to \pi^-} f(x)$$

7.
$$\lim_{x \to \pi} f(x)$$

$$2. \lim_{x \to -\pi^+} f(x)$$

4.
$$f(-\pi)$$

$$6. \lim_{x \to \pi^+} f(x)$$

8.
$$f(\pi)$$

Numerical and Graphical Limits Key

- 1. 1
- 2. 2
- 3. Does not exist
- 4. 4
- 5. 2
- 6. 2
- 7. 2
- 8. 2

Algebraic Limits

Evaluate each limit.

1.
$$\lim_{x \to -4} \left(\frac{x+4}{x^2+6x+8} \right)$$

2.
$$\lim_{x \to -3} \left(-\frac{x^2 + 2x - 3}{x + 3} \right)$$

3.
$$\lim_{x \to 3} \left(\frac{x-3}{x^2 - 8x + 15} \right)$$

4.
$$\lim_{x \to -2} \left(\frac{x^2 - 2x - 8}{x + 2} \right)$$

5.
$$\lim_{x \to -1} \left(-\frac{x^2 + 5x + 4}{x + 1} \right)$$

6.
$$\lim_{x \to -4} \left(\frac{x^2 + 7x + 12}{x + 4} \right)$$

7.
$$\lim_{x \to 2} \left(\frac{x^2 - 2x}{x - 2} \right)$$

8.
$$\lim_{x \to -1} \left(-\frac{x^2 - 3x - 4}{x + 1} \right)$$

ALGEBRAIC LIMITS KEY

- 1. $-\frac{1}{2}$
- 2. 4
- 3. $-\frac{1}{2}$
- 4. -6
- 5. -3
- 6. -1
- 7. 2
- 8. 5

Continuity

For each of the following, determine whether the function graphed below is continuous at that value. If not, explain why.



1.
$$x = -11$$

2.
$$x = -7$$

3.
$$x = -4$$

4.
$$x = 2$$

$$5 \times -6$$

6.
$$x = 10$$

Identify all discontinuities for each of the following.

7.
$$f(x) = \frac{x^2 - 6x}{x^2 + 6x}$$

8.
$$f(x) = \frac{x+3}{x-3}$$

$$9. \ f(x) = \frac{x+4}{3\ln(x)}$$

10.
$$f(x) = \frac{2x+5}{x^2-9}$$

11.
$$f(x) = \begin{cases} 2\sin(x), & x < 0 \\ 0, & x = 0 \\ 3x - 2, & x > 0 \end{cases}$$

12.
$$f(x) = \begin{cases} e^x - 1, & x \le 0 \\ x^2, & x > 0 \end{cases}$$

13.
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x > -1 \end{cases}$$

14.
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x \ge -1 \end{cases}$$

Continuity Key

- 1. Discontinuous; Not defined at x = -11
- 2. Continuous
- 3. Discontinuous; Left- and right-hand limits are not equal, nor do they equal the function value at x = -4
- 4. Discontinuous; Not defined at x = 2
- 5. Discontinuous; Left- and right-hand limits are not equal
- 6. Continuous
- 7. Discontinuous at x = 0, -6
- 8. Discontinuous at x = 3
- 9. Discontinuous at x = 1
- 10. Discontinuous at $x = \pm 3$
- 11. Discontinuous at x = 0
- 12. Continuous for all values of x
- 13. Discontinuous at x = -1
- 14. Continuous for all values of x

Derivatives

Use the definition of the derivative to find the derivative of each function with respect to x.

- 1. $f(x) = x^2 + 2x + 4$
- 2. f(x) = -5x + 5
- 3. $f(x) = 2x^2 5x 2$
- 4. f(x) = 5x + 2
- 5. $f(x) = x^3 x^2$
- 6. $f(x) = 5x^2 + 5$
- 7. f(x) = 2x + 3
- 8. $f(x) = x^2 2$
- 9. $f(x) = -5x^3 + 4$
- 10. $f(x) = -4x^2 + x 5$
- 11. $f(x) = 2x^2 3x + 5$

DERIVATIVES KEY

- 1. 2x + 2
- 2. -5
- 3. 4x 5
- 4. 5
- 5. $3x^2 2x$
- 6. 10*x*
- 7. 2
- 8. 2*x*
- 9. $-15x^2$
- 10. -8x + 1
- 11. 4x 3

Appendix A

Factoring

Factor each of the following completely.

1.
$$x^2 + 2x - 15$$

2.
$$x^2 - 8x + 12$$

3.
$$x^2 + 15x + 56$$

4.
$$5x^2 + 19x - 4$$

5.
$$4x^2 - 5x - 6$$

6.
$$9x^2 - 400$$

7.
$$5x^2 - 7x - 6$$

8.
$$9x^2 - 54x + 45$$

9.
$$3x^3 + 12x^2 + 9x$$

10.
$$9y^2 - 16$$

11.
$$4x^2 - 28x + 49$$

12.
$$14x^2 + 11xy - 15y^2$$
 18. $3p^2 + 22p - 16$

13.
$$6x^2 - 48x - 120$$

14.
$$9x^4 - 54x^3 + 45x^2$$

15.
$$16y^2 - 40y + 25$$

16.
$$30x^2 + xy - y^2$$

17.
$$8w^2 + 33w + 4$$

18.
$$3p^2 + 22p - 16$$

19.
$$18x^2 - 27x + 4$$

20.
$$14a^2 + 15a - 9$$

21.
$$4x^2 - 4x - 24$$

22.
$$18t^2 - 9t - 5$$

23.
$$6a^2 + 23a + 21$$

24.
$$25x^2 - 1$$

FACTORING KEY

1.
$$(x+5)(x-3)$$

2.
$$(x-6)(x-2)$$

3.
$$(x+7)(x+8)$$

4.
$$(5x-1)(x+4)$$

5.
$$(4x+3)(x-2)$$

6.
$$(3x + 20)(3x - 20)$$

7.
$$(5x+3)(x-2)$$

8.
$$9(x-5)(x-1)$$

9.
$$3x(x+3)(x+1)$$

10.
$$(3y+4)(3y-4)$$

11.
$$(2x-7)^2$$

12.
$$(7x - 5y)(2x + 3y)$$

13.
$$6(x-10)(x+2)$$

14.
$$9x^2(x-1)(x-5)$$

15.
$$(4y-5)^2$$

16.
$$(6x - y)(5x + y)$$

17.
$$(8w+1)(w+4)$$

18.
$$(3p-2)(p+8)$$

19.
$$(6x-1)(3x-4)$$

20.
$$(7a-3)(2a+3)$$

21.
$$4(x-3)(x+2)$$

22.
$$(6t-5)(3t+1)$$

23.
$$(2a+3)(3a+7)$$

24.
$$(5x+1)(5x-1)$$

Appendix B

Complex Fractions

Simplify each as much as possible.

1.
$$\frac{5+\frac{3}{x}}{x-\frac{1}{2}}$$

6.
$$\frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

11.
$$\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$2. \ \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

7.
$$\frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2 - 2x}}$$

12.
$$\frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

3.
$$\frac{3}{2 - \frac{x}{x-1}}$$

$$8. \ \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

$$13. \ \frac{\frac{1}{x-a} + \frac{1}{a}}{X}$$

4.
$$\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$$

9.
$$\frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

14.
$$\frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

$$5. \ \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$10. \ \frac{\frac{5}{x} - 5x}{x - 1}$$

15.
$$\frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

COMPLEX FRACTIONS KEY

1.
$$\frac{2(5x+3)}{x(2x-1)}$$

$$2. \ \frac{1}{x^2 - 2x + 4}$$

3.
$$\frac{3(x-1)}{x-2}$$

$$4. \ \frac{x+3}{2+7x}$$

5.
$$\frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \ \frac{-4x-1}{2}$$

7.
$$\frac{8x-10}{7}$$

8.
$$-\frac{1}{7x}$$

$$9. \ \frac{2x+1}{5x(x+1)}$$

$$10. \ \frac{-5x-5}{x}$$

11.
$$\frac{-1}{2x+4}$$

12.
$$\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \ \frac{1}{a(x-a)}$$

$$14. \ \frac{-2x-2}{5x^2-16x+3}$$

15.
$$\frac{x+4}{4x-8}$$