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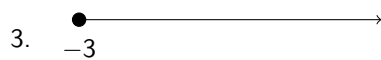
# Chapter 1

## Basic Set Theory and Interval Notation

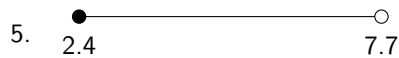
You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

1.  $(-5, 8]$

2.  $\{x|x \leq 1\}$



4.  $\{x|x \neq 4, 11\}$



6.  $(9, \infty)$

Write each using interval notation and graph on a number line.

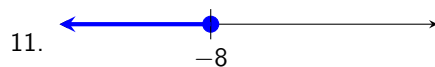
7.  $\{x|x \geq 2\}$

8.  $\{x|x < -8\}$

9.  $\{x|x \neq 3\}$

10.  $\{x|x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

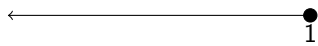


# BASIC SET THEORY AND INTERVAL NOTATION KEY

1.  $\{x | -5 < x \leq 8\}$

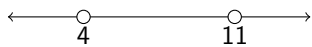


2.  $(-\infty, 1]$



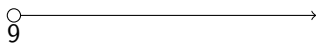
3.  $[-3, \infty)$   $\{x | x \geq -3\}$

4.  $(-\infty, 4) \cup (4, 11) \cup (11, \infty)$

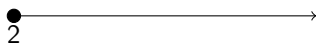


5.  $[2.4, 7.7)$   $\{x | 2.4 \leq x < 7.7\}$

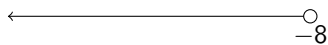
6.  $\{x | x > 9\}$



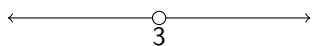
7.  $[2, \infty)$



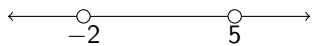
8.  $(-\infty, -8)$



9.  $(-\infty, 3) \cup (3, \infty)$



10.  $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$



11.  $(-\infty, -8]$   $\{x | x \leq -8\}$

12.  $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$   $\{x | x \neq 7, 12\}$

## Chapter 2

# Functions and Their Graphs

### 2.1 Evaluating Functions

Given  $f(x) = -3x^2 + 4x$  and  $g(x) = \frac{1}{x} - 5$ , evaluate each.

1.  $f(5)$
2.  $f(-2)$
3.  $f(0)$
4.  $g(1)$
5.  $g(-5)$
6.  $g(1/4)$

### 2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

- |                                    |  |
|------------------------------------|--|
| 1. $f(x) = -8x^2 - 7x + 1$         | 13. $g(x) = \sqrt{2x + 3}$             |
| 2. $g(x) = \sqrt{5x + 12} - 2$     | 14. $h(x) = \sqrt[3]{2x + 3}$          |
| 3. $h(x) = \frac{x+2}{9x-7}$       | 15. $f(x) = -\frac{7x-10}{x^2+3x+2}$   |
| 4. $f(x) = -5x + 4$                | 16. $g(x) = \sqrt{-9x + 8}$            |
| 5. $f(x) = x^2 + 2$                | 17. $h(x) = -\sqrt[3]{4x + 1}$         |
| 6. $f(x) = \frac{2x+1}{3x-5}$      | 18. $f(x) = \sqrt[3]{8x + 1}$          |
| 7. $f(x) = \sqrt{3x - 12}$         | 19. $g(x) = \frac{x^2-1}{\sqrt{x+3}}$  |
| 8. $f(x) = \frac{x}{x^2-16}$       | 20. $h(x) = \frac{3}{9+\frac{4}{x+7}}$ |
| 9. $f(x) = \frac{x+4}{x^3-4x}$     | 21. $f(x) = \frac{x+1}{\sqrt{10x+8}}$  |
| 10. $f(x) = \frac{x}{\sqrt{x-4}}$  | 22. $g(x) = \frac{5}{1+\frac{3}{x+2}}$ |
| 11. $f(x) = \frac{x^2+1}{2x^2+8}$  | 23. $i(x) = \frac{7}{3-\frac{4}{x+1}}$ |
| 12. $f(x) = -\frac{x+7}{x^2-5x-6}$ | 24. $n(x) = \frac{7x+14}{\sqrt{2x-1}}$ |

## 2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3 \\ 0.2x + 7 & \text{if } -3 \leq x < 2 \\ \sqrt{5x} & \text{if } x \geq 2 \end{cases}$$

1.  $f(3)$
2.  $f(0)$
3.  $f(-2)$
4.  $f(-3)$
5.  $f(0.5)$

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \leq -4 \\ \sqrt{2x + 7} & -4 < x < 0 \\ |-x - 1| & x \geq 0 \end{cases}$$

6.  $f(3)$
7.  $f(-2)$
8.  $f(0)$
9.  $f(-5)$

## Evaluating Functions

1.  $-55$
2.  $-20$
3.  $0$
4.  $-4$
5.  $-5.2$
6.  $-1$

## Domain of Functions

- |  |   |
|--|---|
| 1. $(-\infty, \infty)$                                       | 13. $[-\frac{3}{2}, \infty)$  |
| 2. $[\frac{-12}{5}, \infty)$                                 | 14. $(-\infty, \infty)$   |
| 3. $(-\infty, \frac{7}{9}) \cup (\frac{7}{9}, \infty)$       | 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$                       |
| 4. $(-\infty, \infty)$                                       | 16. $(-\infty, \frac{8}{9}]$  |
| 5. $(-\infty, \infty)$                                       | 17. $(-\infty, \infty)$   |
| 6. $(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$       | 18. $(-\infty, \infty)$   |
| 7. $[4, \infty)$   | 19. $(-3, \infty)$  |
| 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$             | 20. $(-\infty, -\frac{67}{9}) \cup (-\frac{67}{9}, -7) \cup (-7, \infty)$ |
| 9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$ | 21. $(-\frac{4}{5}, \infty)$  |
| 10. $(4, \infty)$  | 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$                        |
| 11. $(-\infty, \infty)$                                      | 23. $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$     |
| 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$            | 24. $(\frac{1}{2}, \infty)$   |

## Piecewise Functions

1.  $\sqrt{15} \approx 3.873$
2.  $7$
3.  $6.6$
4.  $6.4$
5.  $7.1$
6.  $4$
7.  $\sqrt{3} \approx 1.732$
8.  $1$
9.  $18$

## Chapter 3

# Properties of Functions

### 3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.  $f(x) = x^2 - 3x^2 + 5$

2.  $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3.  $f(x) = -x^4 + 3x^2 - 2x + 6$

4.  $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

5.  $f(x) = -4x^3 + 2x^2 + 10x + 4$

6.  $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

7. The concentration  $C$  of a medication in the bloodstream  $t$  hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \geq 0$$

After how many hours will the concentration be the highest?

### 3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.  $f(x) = x^2 - 3x^2 + 5$

2.  $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3.  $f(x) = x^3 + 2x^2 - 4x - 8$

4.  $g(x) = x^4 - 2x^2 + 1$

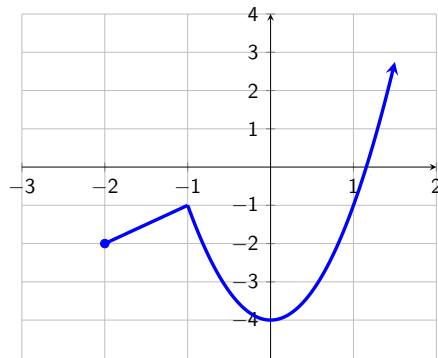
5.  $h(x) = \sqrt{x+1} - 2$

6.  $f(x) = -4x^3 + 2x^2 + 10x + 4$

7.  $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

### 3.3 Miscellaneous

Use the graph of  $y = f(x)$  below to answer questions 1–10. Write your answers using interval notation.

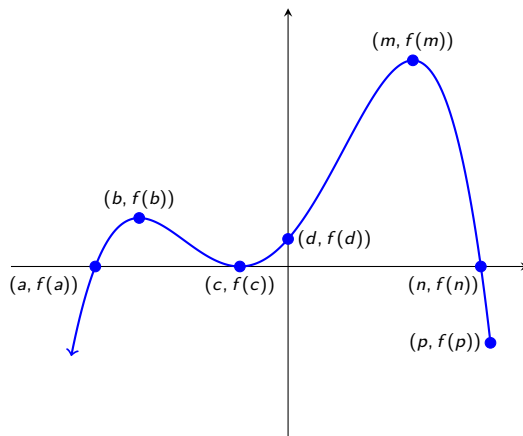


1. Domain of  $f$
2. Range of  $f$
3. Relative Minimum
4. Relative Maximum
5.  $f(1)$
6.  $f(0)$
7. Increasing Interval(s)
8. Decreasing Interval(s)
9. Absolute Maximum
10. Absolute Minimum

Find each of the following given  $f(x) = -2x^3 + 6x^2 - 5x + 1$ . Round to 3 decimal places and use interval notation when applicable.

11.  $f(7)$
12.  $f(-2)$
13. Rel. Max
14. Rel. Min
15. Global Max
16. Global Min
17. Increasing Interval(s)
18. Decreasing Interval(s)

Use the graph of  $f(x)$  to answer each.



19. Relative maxima of  $f(x)$
20. Relative minima of  $f(x)$
21. Absolute maxima of  $f(x)$
22. Absolute minima of  $f(x)$
23. Intervals where  $f$  is increasing
24. Intervals where  $f$  is decreasing
25. Zeros of  $f$



## Maxima and Minima

1. Rel max @  $(0, 5)$ ; No rel min
2. Rel max @  $(2.158, 3.248)$ ; Rel min @  $(-1.158, -4.048)$
3. Rel Max  $(-1.366, 10.848)$  and  $(1, 6)$ ; Rel Min  $(0.366, 5.652)$
4. Rel Max  $(-1.716, 11.598)$ ; Rel Min  $(1.132, -3.929)$
5. Rel Max:  $(1.095, 12.096)$ ; Rel Min  $(-0.761, -0.680)$
6. Rel Max:  $(1.366, 0.348)$ ; Rel Min:  $(-0.366, -4.848)$  and  $(2, 0)$
7. About 2.16 hours

## Increasing, Decreasing, and Constant Intervals

1. Increasing:  $(-\infty, 0)$  Decreasing:  $(0, \infty)$
2. Increasing:  $(-1.158, 2.158)$  Decreasing:  $(-\infty, -1.158) \cup (2.158, \infty)$
3. Inc:  $(-\infty, -2) \cup (\frac{2}{3}, \infty)$  Dec:  $(-2, \frac{2}{3})$
4. Inc:  $(-1, 0) \cup (1, \infty)$  Dec:  $(-\infty, -1) \cup (0, 1)$
5. Inc:  $(-1, \infty)$  No intervals where it is decreasing
6. Inc:  $(-0.761, 1.095)$ ; Dec:  $(-\infty, -0.761) \cup (1.095, \infty)$
7. Inc:  $(-0.366, 1.366) \cup (2, \infty)$ ; Dec:  $(-\infty, -0.366) \cup (1.366, 2)$ ;

## Miscellaneous

- |                                |   |
|--------------------------------|---|
| 1. $[-2, \infty)$              | 14. $(0.592, -0.272)$                       |
| 2. $[-4, \infty)$              | 15. None                                    |
| 3. $(0, -4)$                   | 16. None                                    |
| 4. $(-1, -1)$                  | 17. $(0.592, 1.408)$                        |
| 5. $-1$                        | 18. $(-\infty, 0.592) \cup (1.408, \infty)$ |
| 6. $-4$                        | 19. $(b, f(b))$ and $(m, f(m))$             |
| 7. $(-2, -1) \cup (0, \infty)$ | 20. $(c, f(c))$                             |
| 8. $(-1, 0)$                   | 21. $(m, f(m))$                             |
| 9. $(0, -4)$                   | 22. None                                    |
| 10. None                       | 23. $(-\infty, b) \cup (c, m)$              |
| 11. $-426$                     | 24. $(b, c) \cup (m, p)$                    |
| 12. 51                         | 25. $x = a, x = c, x = n$                   |
| 13. $(1.408, 0.272)$           |   |

## Chapter 4

# Linear Functions and Slope

### 4.1 Equations of Lines

Write the equation of each line **in point-slope form** that goes through each pair of points.

1.  $(-2, 1), (7, 8)$
2.  $(0, 4), (9, -15)$
3.  $(-1, -2), (-3, -13)$

### 4.2 Average Rate of Change

For the function  $f(x) = x^2$ , compute the average rate of change for each interval.

1.  $[1, 1.1]$
2.  $[1, 1.01]$
3.  $[1, 1.001]$
4.  $[1, 1.0001]$
5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function  $f(x) = -6x^2 + 7x + 4$  over each specified interval.

6.  $[-2, -1]$
7.  $[5, 6]$
8.  $[0, 1]$
9.  $[5, 5.001]$
10.  $[5, 5.0001]$
11.  $[5, 5.00001]$
12. What value are your last 3 answers getting closer to?

For the function  $f(x) = -3x^2 + 5$ , determine the average rate of change of each over the given interval.

13.  $[7, 7.001]$
14.  $[7, 7.0001]$
15.  $[7, 7.00001]$

16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = \sqrt{x}$ , find the average rate of change of each over the given interval.

17.  $[1, 1.0001]$

18.  $[1, 1.00001]$

19.  $[1, 1.000001]$

20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = 6\sqrt{x}$ , find the average rate of change of each over the given interval.

21.  $[25, 25.1]$

22.  $[25, 25.01]$

23.  $[25, 25.001]$

24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

## Equations of Lines

1.  $y - 1 = \frac{7}{9}(x + 2)$  or  $y - 8 = \frac{7}{9}(x - 7)$
2.  $y - 4 = -\frac{19}{9}(x - 0)$  or  $y + 15 = -\frac{19}{9}(x - 9)$
3.  $y + 2 = \frac{11}{2}(x + 1)$  or  $y + 13 = \frac{11}{2}(x + 3)$

## Average Rate of Change

1. 2.1
2. 2.01
3. 2.001
4. 2.0001
5. 2
6. 25
7. -59
8. 1
9. -53.006
10. -53.0006
11. -53.00006
12. -53
13. -42.003
14. -42.0003
15. -42.00003
16. -42
17. -0.499988
18. -0.4999988
19. -0.49999988
20. -0.5
21. 0.5994
22. 0.59999
23. 0.6
24. 0.6

## Chapter 5

# Function Transformations

Write the function for  $g(x)$  if it is the result of  $f(x)$  after the following ordered sequence of transformations.

1. (1) Vertical stretch by 3  
(2) Shift left 1 unit  
(3) Reflect across  $y$ -axis
2. (1) Horizontal compression by 2  
(2) Shift up 1 unit
3. (1) Reflect across  $x$ -axis  
(2) Vertical compression by 4  
(3) Move right 7 units

Write the function  $g(x)$  that is a result of the following ordered sequence of transformations to  $f(x) = |x|$ .

4. (1) Reflect across  $x$ -axis  
(2) Shift right 3 units  
(3) Horizontal stretch by factor of 5
5. (1) Shift down 2 units  
(2) Reflect across  $y$ -axis  
(3) Shift up 1 unit
6. (1) Horizontal compression by factor of 7  
(2) Vertical compression by factor of 4  
(3) Shift left 9 units

Given  $f(x) = \sqrt{x}$ , determine the resulting function  $g(x)$  after the following ordered sequence of transformations.

7. (1) Shift up 2 units  
(2) Horizontal stretch by 5  
(3) Shift left 3 units
8. (1) Vertical compression by factor of 3  
(2) Reflect across  $y$ -axis  
(3) Horizontal compression by 5
9. (1) Shift right 8 units  
(2) Reflect across  $x$ -axis  
(3) Horizontal compression by factor of 4

Write the final equation of  $g(x)$  if it is found by taking  $f(x) = \sqrt{x}$  after the following ordered sequence of transformations.

10. (1) Shift right 2 units  
(2) Horizontal stretch by factor 3  
(3) Shift down 2 units  
(4) Reflect across  $x$ -axis
11. (1) Horizontal stretch by factor 3  
(2) Shift left 1 unit  
(3) Shift up 2 units  
(4) Reflect across  $y$ -axis
12. (1) Vertical stretch by factor 5  
(2) Horizontal stretch by factor 2  
(3) Shift up 3 units  
(4) Reflect across  $x$ -axis

Find the equation for  $g(x)$  if  $g(x)$  is found by performing the following *ordered* sequence of transformations to  $f(x) = \frac{1}{x}$ .

13. (1) Shift left 3 spaces  
(2) Reflect across  $y$ -axis  
(3) Shift down 5 spaces  
(4) Vertical stretch by factor of 7
14. (1) Shift up 3 spaces  
(2) Reflect across  $x$ -axis  
(3) Shift right 5 spaces  
(4) Horizontal compression by factor of 7

# FUNCTION TRANSFORMATIONS KEY

1.  $g(x) = 3f(-x + 1)$
2.  $g(x) = f(2x) + 1$
3.  $g(x) = -\frac{1}{4}f(x - 7)$
4.  $g(x) = -\left|\frac{1}{5}x - 3\right|$
5.  $g(x) = |-x| - 1$
6.  $g(x) = \frac{1}{4}|7(x + 9)| = \frac{1}{4}|7x + 63|$
7.  $g(x) = \sqrt{\frac{1}{5}(x + 3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$
8.  $g(x) = \frac{1}{3}\sqrt{-5x}$
9.  $g(x) = -\sqrt{4x - 8}$
10.  $g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$
11.  $g(x) = \sqrt{\frac{1}{3}(-x + 1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$
12.  $g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$
13.  $g(x) = \frac{7}{-x+3} - 35$
14.  $g(x) = -\frac{1}{7x-5} - 3$

## Chapter 6

# Function Operations

### 6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given  $f(x) = x + 5$ ,  $g(x) = x^2 - 1$ , and  $h(x) = \sqrt{x - 10}$ , simplify or evaluate each.

1.  $(g - f)(x)$
2.  $(fh)(14)$
3.  $(f + g)(x)$

Find each of the following given the table below.

$x$	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-3	0	-1	3	1	2	4	-4	-2
$g(x)$	3	-1	0	1	4	-2	-4	2	-3

4.  $(f + g)(-2)$
5.  $(f - g)(0)$
6.  $(fg)(1)$
7.  $\left(\frac{f}{g}\right)(3)$
8.  $(f + f)(-4)$

### 6.2 Operations with Functions: Domain

Given  $f(x) = \sqrt{2x + 7}$  and  $g(x) = 3x + 3$ , find the domain of each.

1.  $(f + g)(x)$
2.  $\left(\frac{f}{g}\right)(x)$
3.  $\left(\frac{g}{f}\right)(x)$

### 6.3 Difference Quotient

Write the difference quotient for each.

1.  $f(x) = 2x - 7$
2.  $g(x) = x^2 + 4x$
3.  $h(x) = -1$
4.  $f(x) = \frac{3}{x+2}$
5.  $g(x) = \sqrt{3x}$
6.  $f(x) = x^2 - 2x + 5$
7.  $g(x) = \frac{5}{x}$
8.  $f(x) = -2x^2 + 3x - 5$
9.  $g(x) = \frac{6}{2x+3}$
10.  $h(x) = \sqrt{7x + 5}$



## Adding, Subtracting, Multiplying, and Dividing Functions

1.  $x^2 - x - 6$
2. 38
3.  $x^2 + x + 4$
4.  $-1$
5.  $-3$
6.  $-4$
7.  $-2$
8.  $-6$

## Operations with Functions: Domain

1.  $[-\frac{7}{2}, \infty)$
2.  $[-\frac{7}{2}, -1) \cup (-1, \infty)$
3.  $(-\frac{7}{2}, \infty)$

## Difference Quotient

1. 2
2.  $2x + h + 4$
3. 0
4.  $\frac{-3}{(x+2)(x+h+2)}$
5.  $\frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$
6.  $2x + h - 2$
7.  $\frac{-5}{x(x+h)}$
8.  $-4x - 2h + 3$
9.  $\frac{-12}{(2x+3)(2x+2h+3)}$
10.  $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$

## Chapter 7

# Polynomials and Their Graphs

Determine the end behavior of each.

1.  $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$

2.  $g(x) = 4x^2 - 16x^6 + 3x$

3.  $h(x) = 1 + x^{11} - 4x^8$

4.  $f(x) = -x^4 + 3x^2 - 2x + 6$

5.  $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

## POLYNOMIALS AND THEIR GRAPHS

1.  $\lim_{x \rightarrow -\infty} f(x) = \infty$      $\lim_{x \rightarrow \infty} f(x) = -\infty$

2.  $\lim_{x \rightarrow -\infty} g(x) = -\infty$ ,     $\lim_{x \rightarrow \infty} g(x) = \infty$

3.  $\lim_{x \rightarrow -\infty} h(x) = -\infty$      $\lim_{x \rightarrow \infty} h(x) = \infty$

4.  $\lim_{x \rightarrow -\infty} f(x) = -\infty$      $\lim_{x \rightarrow \infty} f(x) = -\infty$

5.  $\lim_{x \rightarrow -\infty} g(x) = -\infty$      $\lim_{x \rightarrow \infty} g(x) = \infty$

## Chapter 8

# Dividing Polynomials

### 8.1 Dividing Polynomials

Divide each.

1.  $(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$

2.  $(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$

3.  $(4x^3 - 3x^2 + x + 1) \div (x + 2)$

4.  $(5x^4 - x^2 + x - 2) \div (x^2 + 2)$

### 8.2 Remainder and Factor Theorems

Determine the remainder of each.

1.  $(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$

2.  $(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x + 1)$

## Dividing Polynomials

1.  $4x^2 - 2x + 5$

2.  $4y^2 + 8y + 7 - \frac{72}{3y + 5}$

3.  $4x^2 - 11x + 23 - \frac{45}{x + 2}$

4.  $5x^2 - 11 + \frac{x + 20}{x^2 + 2}$

## Remainder and Factor Theorems

1. 6

2. 13

## Chapter 9

# Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1.  $f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$

2.  $g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$

3.  $f(x) = \frac{3x}{x + 4}$

4.  $g(x) = \frac{x^2 + 3x + 2}{x - 1}$

5.  $h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$

6.  $f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$

7.  $g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$

8.  $h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$

## RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain:  $x \neq -\frac{1}{2}, 7$ ; V.A.:  $x = 7$ ; Hole @  $\left(-\frac{1}{2}, -\frac{7}{13}\right)$ ; H.A.:  $y = 1$
2. Domain:  $x \neq -3, 4$ ; V.A.:  $x = -3$ ; Obl. Asymp:  $y = 3x + 10$
3. Domain:  $x \neq -4$ ; V.A.:  $x = -4$ ; H.A.:  $y = 3$
4. Domain:  $x \neq 1$ ; V.A.:  $x = 1$ ; Obl. Asymp:  $y = x + 4$
5. Domain:  $x \neq 0, 1$ ; V.A.:  $x = 0$  and  $x = 1$ ; H.A.:  $y = 0$
6. Domain:  $x \neq -1, 5$ ; V.A.  $x = -1$ ; Hole @  $\left(5, \frac{13}{3}\right)$ ; Obl. Asym  $y = 2x - 5$
7. Domain:  $x \neq -6, 0, 4$ ; V.A.  $x = -6, x = 0$ ; Hole @  $\left(4, \frac{21}{40}\right)$ ; H.A.  $y = 0$
8. Domain:  $x \neq -6, -\frac{3}{8}$ ; V.A.  $x = -6, x = -\frac{3}{8}$ ; H.A.  $y = \frac{1}{4}$

## Chapter 10

# Polynomial and Rational Inequalities

### 10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1.  $6x^3 - 4x^2 - 10x \geq 0$

2.  $x^4 < 9x^2$

3.  $3x^3 - 7x^2 - 22x + 8 < 0$

4.  $3x^2 - 4x + 1 \leq 0$

### 10.2 Rational Inequalities

Solve each. Write your answers using interval notation.

1.  $\frac{3x - 4}{x + 1} < 0$

2.  $\frac{x^2 + 3x + 2}{x - 7} \leq 0$

3.  $\frac{x^2 - 4x + 4}{x^2 - 1} \geq 0$

4.  $\frac{x + 2}{x - 4} \leq 1$

5.  $\frac{x^2 - 7x - 8}{x^2 - 4x - 32} \geq 0$



**Polynomial Inequalities**

1.  $[-1, 0] \cup \left[\frac{5}{3}, \infty\right)$
2.  $(-3, 0) \cup (0, 3)$
3.  $(-\infty, -2) \cup \left(\frac{1}{3}, 4\right)$
4.  $\left[\frac{1}{3}, 1\right]$

**Rational Inequalities**

1.  $\left(-1, \frac{4}{3}\right)$
2.  $(-\infty, -2] \cup [-1, 7)$
3.  $(-\infty, -1) \cup (1, \infty)$
4.  $(-\infty, 4)$
5.  $(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$

## Chapter 11

# Function Compositions

Given  $f(x) = x - 5$ ,  $g(x) = 4 + \sqrt{2x + 1}$ , and  $h(x) = \frac{3}{x + 7}$ , simplify each and state the domain.

1.  $(f \circ g)(x)$

2.  $(g \circ f)(x)$

3.  $h(h(x))$

Find each of the following given the table below.

<b>x</b>	<b>-4</b>	<b>-3</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>f(x)</b>	-3	0	-1	3	1	2	4	-4	-2
<b>g(x)</b>	3	-1	0	1	4	-2	-4	2	-3

4.  $(f \circ g)(-1)$

5.  $(g \circ g)(0)$

6.  $(f \circ f)(2)$

7.  $(g \circ g)(-3)$

8.  $f(g(0))$

Given  $f(x) = \sqrt{3x + 2}$ ,  $g(x) = x^2 - 1$ , and  $h(x) = 9x - 2$ , find each of the following.

9.  $(g \circ f)(x)$

10.  $f(g(x))$

11.  $(h \circ h)(x)$

FUNCTION COMPOSITIONS KEY

1.  $-1 + \sqrt{2x+1}$  Domain:  $\left[-\frac{1}{2}, \infty\right)$

2.  $4 + \sqrt{2x-9}$  Domain:  $\left[\frac{9}{2}, \infty\right)$

3.  $\frac{3x+21}{7x+52}$  Domain:  $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup (-7, \infty)$

4. 2

5. -3

6. -2

7. 1

8. -2

9.  $3x+1$

10.  $\sqrt{3x^2-1}$

11.  $81x-20$

## Chapter 12

# Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1.  $f(x) = \sqrt{-2x+3} + 1$

2.  $g(x) = (x+4)^2 - 1, x \leq -4$

3.  $h(x) = \frac{9x}{4x-1}$

4.  $f(x) = \sqrt{x} - 3$

5.  $g(x) = \frac{1}{1-x}$

6.  $h(x) = x^2 + 6x + 4, x \leq -3$

7.  $f(x) = \sqrt{5x-4}$

8.  $g(x) = x^2 - 2x + 3, x \leq 1$

9.  $h(x) = \frac{3}{x-1}$

INVERSE FUNCTIONS KEY

$$1. f^{-1}(x) = -\frac{1}{2}((x-1)^2 - 3)$$

	Domain	Range
$f(x)$	$(-\infty, 1.5]$	$[1, \infty)$
$f^{-1}(x)$	$[1, \infty)$	$(-\infty, 1.5]$

$$2. g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
$g(x)$	$(-\infty, -4]$	$[-1, \infty)$
$g^{-1}(x)$	$[-1, \infty)$	$(-\infty, -4]$

$$3. h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
$h(x)$	$(-\infty, 1/4) \cup (1/4, \infty)$	$(\infty, 9/4) \cup (9/4, \infty)$
$h^{-1}(x)$	$(\infty, 9/4) \cup (9/4, \infty)$	$(-\infty, 1/4) \cup (1/4, \infty)$

$$4. f^{-1}(x) = (x+3)^2$$

	Dom	Ran
$f(x)$	$[0, \infty)$	$[-3, \infty)$
$f^{-1}(x)$	$[-3, \infty)$	$[0, \infty)$

$$5. g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
$g(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$g^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

$$6. h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
$h(x)$	$(-\infty, -3]$	$[-5, \infty)$
$h^{-1}(x)$	$[-5, \infty)$	$(-\infty, -3]$

$$7. f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

	Dom	Ran
$f(x)$	$\left[\frac{4}{5}, \infty\right)$	$[0, \infty)$
$f^{-1}(x)$	$[0, \infty)$	$\left[\frac{4}{5}, \infty\right)$

$$8. g^{-1}(x) = -\sqrt{x-2} + 1$$

	Dom	Ran
$g(x)$	$(-\infty, 1]$	$[2, \infty)$
$g^{-1}(x)$	$[2, \infty)$	$(-\infty, 1]$

$$9. h^{-1}(x) = \frac{3}{x} + 1$$

	Dom	Ran
$h(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$h^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

## Chapter 13

# Exponential Functions

### 13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

1.  $f(x) = 3 + e^{2x}$

2.  $h(x) = 5^{-x}$

## EXPONENTIAL FUNCTIONS

1.  $\lim_{x \rightarrow -\infty} f(x) = 3$      $\lim_{x \rightarrow \infty} f(x) = \infty$

2.  $\lim_{x \rightarrow -\infty} f(x) = \infty$      $\lim_{x \rightarrow \infty} f(x) = 0$

## Chapter 14

# Logarithmic Functions

Find the domain of each. Write your answers in interval notation.

1.  $b(x) = \log_7(x^2 - 8x + 6)$

2.  $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$



## LOGARITHMIC FUNCTIONS KEY

1.  $(-\infty, 0.838) \cup (7.162, \infty)$
2.  $(-3, -2) \cup (-1, \infty)$

## Chapter 15

# Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

1.  $\log_b \left( \frac{x^2}{y^8} \right)$

2.  $\ln(ez)^3$

3.  $\log_5(x) + \log_5(9) - 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

4.  $\log_7(10)$

5.  $\log_9(x)$

6.  $\log_b(c)$

PROPERTIES OF LOGARITHMS KEY

1.  $2 \log_b(x) - 8 \log_b(y)$

2.  $3 + 3 \ln(z)$

3.  $\log_5 \left( \frac{9x}{w^2} \right)$

4.  $\frac{\ln(10)}{\ln(7)}$

5.  $\frac{\ln(x)}{\ln(9)}$

6.  $\frac{\ln(c)}{\ln(b)}$

## Chapter 16

# Exponential Equations

Solve each. Round to 3 decimal places when necessary.

1.  $3e^{x-2} = 7$

2.  $5^x + 4 > 1$

3.  $2^{3x+4} = 32^{x-7}$

## EXPONENTIAL EQUATIONS KEY

1.  $x \approx 2.847$
2.  $(-\infty, \infty)$
3.  $x = 19.5$

# Appendix A

## Factoring

Factor each of the following completely.

1.  $x^2 + 2x - 15$

2.  $x^2 - 8x + 12$

3.  $x^2 + 15x + 56$

4.  $5x^2 + 19x - 4$

5.  $4x^2 - 5x - 6$

6.  $9x^2 - 400$

7.  $5x^2 - 7x - 6$

8.  $9x^2 - 54x + 45$

9.  $3x^3 + 12x^2 + 9x$

10.  $9y^2 - 16$

11.  $4x^2 - 28x + 49$

12.  $14x^2 + 11xy - 15y^2$

13.  $6x^2 - 48x - 120$

14.  $9x^4 - 54x^3 + 45x^2$

15.  $16y^2 - 40y + 25$

16.  $30x^2 + xy - y^2$

17.  $8w^2 + 33w + 4$

18.  $3p^2 + 22p - 16$

19.  $18x^2 - 27x + 4$

20.  $14a^2 + 15a - 9$

21.  $4x^2 - 4x - 24$

22.  $18t^2 - 9t - 5$

23.  $6a^2 + 23a + 21$

24.  $25x^2 - 1$

# FACTORING KEY

1.  $(x + 5)(x - 3)$

2.  $(x - 6)(x - 2)$

3.  $(x + 7)(x + 8)$

4.  $(5x - 1)(x + 4)$

5.  $(4x + 3)(x - 2)$

6.  $(3x + 20)(3x - 20)$

7.  $(5x + 3)(x - 2)$

8.  $9(x - 5)(x - 1)$

9.  $3x(x + 3)(x + 1)$

10.  $(3y + 4)(3y - 4)$

11.  $(2x - 7)^2$

12.  $(7x - 5y)(2x + 3y)$

13.  $6(x - 10)(x + 2)$

14.  $9x^2(x - 1)(x - 5)$

15.  $(4y - 5)^2$

16.  $(6x - y)(5x + y)$

17.  $(8w + 1)(w + 4)$

18.  $(3p - 2)(p + 8)$

19.  $(6x - 1)(3x - 4)$

20.  $(7a - 3)(2a + 3)$

21.  $4(x - 3)(x + 2)$

22.  $(6t - 5)(3t + 1)$

23.  $(2a + 3)(3a + 7)$

24.  $(5x + 1)(5x - 1)$

## Appendix B

# Complex Fractions

Simplify each as much as possible.

$$1. \frac{5 + \frac{3}{x}}{x - \frac{1}{2}}$$

$$2. \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

$$3. \frac{3}{2 - \frac{x}{x-1}}$$

$$4. \frac{1 + \frac{3}{x}}{\frac{2}{x} + 7}$$

$$5. \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$6. \frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$7. \frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2-2x}}$$

$$8. \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

$$9. \frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$10. \frac{\frac{5}{x} - 5x}{x - 1}$$

$$11. \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$12. \frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

$$13. \frac{\frac{1}{x-a} + \frac{1}{a}}{x}$$



COMPLEX FRACTIONS KEY

$$1. \frac{2(5x+3)}{x(2x-1)}$$

$$2. \frac{1}{x^2-2x+4}$$

$$3. \frac{3(x-1)}{x-2}$$

$$4. \frac{x+3}{2+7x}$$

$$5. \frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \frac{-4x-1}{2}$$

$$7. \frac{8x-10}{7}$$

$$8. -\frac{1}{7x}$$

$$9. \frac{2x+1}{5x(x+1)}$$

$$10. \frac{-5x-5}{x}$$

$$11. \frac{-1}{2x+4}$$

$$12. \frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \frac{1}{a(x-a)}$$