

# Honors PreCalculus

Extra Practice

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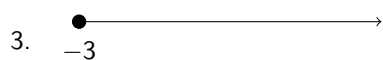
# Chapter 1

## Basic Set Theory and Interval Notation

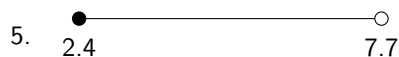
You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

1.  $(-5, 8]$

2.  $\{x|x \leq 1\}$



4.  $\{x|x \neq 4, 11\}$



6.  $(9, \infty)$

Write each using interval notation and graph on a number line.

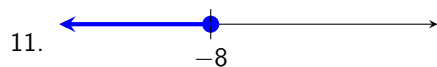
7.  $\{x|x \geq 2\}$

8.  $\{x|x < -8\}$

9.  $\{x|x \neq 3\}$

10.  $\{x|x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

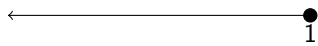


# BASIC SET THEORY AND INTERVAL NOTATION KEY

1.  $\{x | -5 < x \leq 8\}$

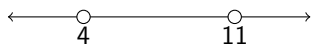


2.  $(-\infty, 1]$



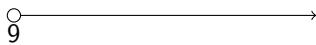
3.  $[-3, \infty)$      $\{x | x \geq -3\}$

4.  $(-\infty, 4) \cup (4, 11) \cup (11, \infty)$



5.  $[2.4, 7.7)$      $\{x | 2.4 \leq x < 7.7\}$

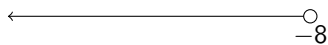
6.  $\{x | x > 9\}$



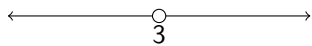
7.  $[2, \infty)$



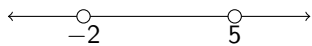
8.  $(-\infty, -8)$



9.  $(-\infty, 3) \cup (3, \infty)$



10.  $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$



11.  $(-\infty, -8]$      $\{x | x \leq -8\}$

12.  $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$      $\{x | x \neq 7, 12\}$

## Chapter 2

# Functions and Their Graphs

### 2.1 Evaluating Functions

Given  $f(x) = -3x^2 + 4x$  and  $g(x) = \frac{1}{x} - 5$ , evaluate each.

1.  $f(5)$
2.  $f(-2)$
3.  $f(0)$
4.  $g(1)$
5.  $g(-5)$
6.  $g(1/4)$

### 2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

- |                                    |  |
|------------------------------------|--|
| 1. $f(x) = -8x^2 - 7x + 1$         | 15. $f(x) = -\frac{7x-10}{x^2+3x+2}$                 |
| 2. $g(x) = \sqrt{5x+12} - 2$       | 16. $g(x) = \sqrt{-9x+8}$                            |
| 3. $h(x) = \frac{x+2}{9x-7}$       | 17. $h(x) = -\sqrt[3]{4x+1}$                         |
| 4. $f(x) = -5x + 4$                | 18. $f(x) = \sqrt[3]{8x+1}$                          |
| 5. $f(x) = x^2 + 2$                | 19. $g(x) = \frac{x^2-1}{\sqrt{x+3}}$                |
| 6. $f(x) = \frac{2x+1}{3x-5}$      | 20. $h(x) = \frac{3}{9+\frac{4}{x+7}}$               |
| 7. $f(x) = \sqrt{3x-12}$           | 21. $f(x) = \frac{x+1}{\sqrt{10x+8}}$                |
| 8. $f(x) = \frac{x}{x^2-16}$       | 22. $g(x) = \frac{5}{1+\frac{3}{x+2}}$               |
| 9. $f(x) = \frac{x+4}{x^3-4x}$     | 23. $i(x) = \frac{7}{3-\frac{4}{x+1}}$               |
| 10. $f(x) = \frac{x}{\sqrt{x-4}}$  | 24. $n(x) = \frac{7x+14}{\sqrt{2x-1}}$               |
| 11. $f(x) = \frac{x^2+1}{2x^2+8}$  | 25. $a(x) = \frac{\frac{x}{x-2}}{\frac{3}{x-2} + 6}$ |
| 12. $f(x) = -\frac{x+7}{x^2-5x-6}$ | 26. $d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$             |
| 13. $g(x) = \sqrt{2x+3}$           |  |
| 14. $h(x) = \sqrt[3]{2x+3}$        |  |

## 2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3 \\ 0.2x + 7 & \text{if } -3 \leq x < 2 \\ \sqrt{5x} & \text{if } x \geq 2 \end{cases}$$

1.  $f(3)$
2.  $f(0)$
3.  $f(-2)$
4.  $f(-3)$
5.  $f(0.5)$

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \leq -4 \\ \sqrt{2x + 7} & -4 < x < 0 \\ |-x - 1| & x \geq 0 \end{cases}$$

6.  $f(3)$
7.  $f(-2)$
8.  $f(0)$
9.  $f(-5)$

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \leq -3 \\ \sqrt{-4x + 1} & \text{if } -3 < x \leq 0 \\ \frac{5x^2}{x+7} & \text{if } x > 0 \end{cases}$$

10.  $f(7)$
11.  $f(-3)$
12.  $f(1)$
13.  $f(0)$
14.  $f(-1)$
15.  $f(-3/2)$

## Evaluating Functions

1.  $-55$
2.  $-20$
3.  $0$
4.  $-4$
5.  $-5.2$
6.  $-1$

## Domain of Functions

- |  |   |
|--|---|
| 1. $(-\infty, \infty)$                                       | 14. $(-\infty, \infty)$   |
| 2. $[-\frac{12}{5}, \infty)$                                 | 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$                       |
| 3. $(-\infty, \frac{7}{9}) \cup (\frac{7}{9}, \infty)$       | 16. $(-\infty, \frac{8}{9}]$  |
| 4. $(-\infty, \infty)$                                       | 17. $(-\infty, \infty)$   |
| 5. $(-\infty, \infty)$                                       | 18. $(-\infty, \infty)$   |
| 6. $(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$       | 19. $(-3, \infty)$  |
| 7. $[4, \infty)$   | 20. $(-\infty, -\frac{67}{9}) \cup (-\frac{67}{9}, -7) \cup (-7, \infty)$ |
| 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$             | 21. $(-\frac{4}{5}, \infty)$  |
| 9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$ | 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$                        |
| 10. $(4, \infty)$  | 23. $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$     |
| 11. $(-\infty, \infty)$                                      | 24. $(\frac{1}{2}, \infty)$   |
| 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$            | 25. $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, 2) \cup (2, \infty)$       |
| 13. $[-\frac{3}{2}, \infty)$                                 | 26. $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$                 |

## Piecewise Functions

- |                              |                             |                              |
|------------------------------|-----------------------------|------------------------------|
| 1. $\sqrt{15} \approx 3.873$ | 6. $4$                      | 11. $4$                      |
| 2. $7$                       | 7. $\sqrt{3} \approx 1.732$ | 12. $\frac{5}{8}$            |
| 3. $6.6$                     | 8. $1$                      | 13. $1$                      |
| 4. $6.4$                     | 9. $18$                     | 14. $\sqrt{5} \approx 2.236$ |
| 5. $7.1$                     | 10. $17.5$                  | 15. $\sqrt{7} \approx 2.646$ |



## Chapter 3

# Properties of Functions

### 3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.  $f(x) = x^2 - 3x^2 + 5$

2.  $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3.  $f(x) = -x^4 + 3x^2 - 2x + 6$

4.  $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

5.  $f(x) = -4x^3 + 2x^2 + 10x + 4$

6.  $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

7. The concentration  $C$  of a medication in the bloodstream  $t$  hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \geq 0$$

After how many hours will the concentration be the highest?

### 3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.  $f(x) = x^2 - 3x^2 + 5$

2.  $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3.  $f(x) = x^3 + 2x^2 - 4x - 8$

4.  $g(x) = x^4 - 2x^2 + 1$

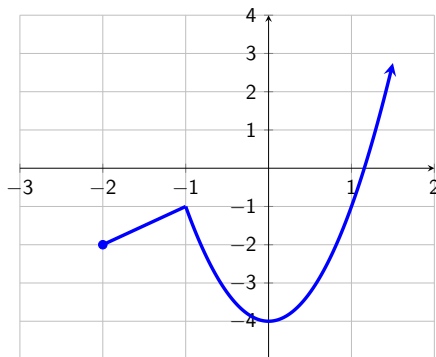
5.  $h(x) = \sqrt{x+1} - 2$

6.  $f(x) = -4x^3 + 2x^2 + 10x + 4$

7.  $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

### 3.3 Miscellaneous

Use the graph of  $y = f(x)$  below to answer the following questions. Write your answers using interval notation.

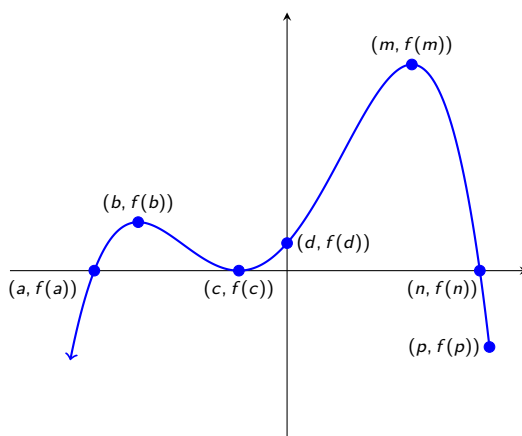


1. Domain of  $f$
2. Range of  $f$
3. Relative Minimum
4. Relative Maximum
5.  $f(1)$
6.  $f(0)$
7. Increasing Interval(s)
8. Decreasing Interval(s)
9. Absolute Maximum
10. Absolute Minimum

Find each of the following given  $f(x) = -2x^3 + 6x^2 - 5x + 1$ . Round to 3 decimal places and use interval notation when applicable.

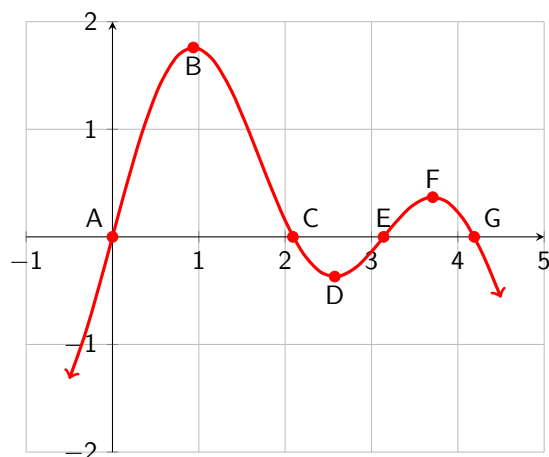
11.  $f(7)$
12.  $f(-2)$
13. Rel. Max
14. Rel. Min
15. Global Max
16. Global Min
17. Increasing Interval(s)
18. Decreasing Interval(s)

Use the graph of  $f(x)$  to answer each.



19. Relative maxima of  $f(x)$
20. Relative minima of  $f(x)$
21. Absolute maxima of  $f(x)$
22. Absolute minima of  $f(x)$
23. Intervals where  $f$  is increasing
24. Intervals where  $f$  is decreasing
25. Zeros of  $f$

Given the labeled points A through G on the graph of  $f(x)$  below, find each of the following.



26. Increasing interval(s)

28. Relative max

30. Global max

32. Zeros of  $f$

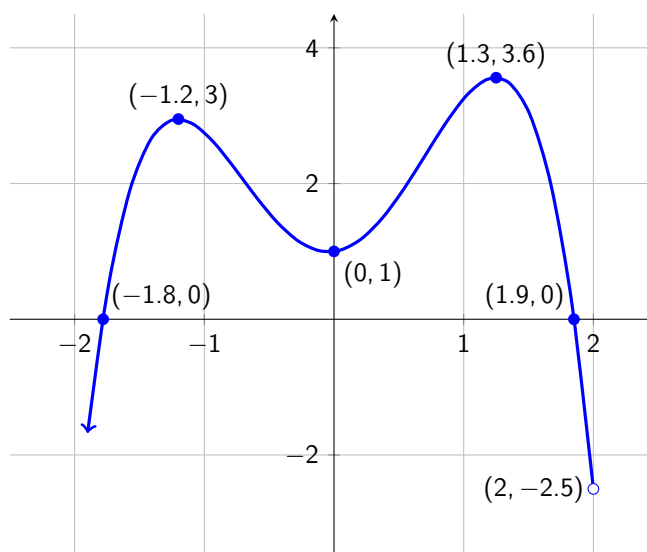
27. Decreasing interval(s)

29. Relative min

31. Global min

33. Number of solutions to  $f(x) = 1$

Answer each of the following about the function  $f(x)$  below.



34. Domain of  $f$

35. Range of  $f$

36. Relative maxima

37. Relative minima

38. Absolute maximum

39. Absolute minimum

40. Increasing intervals

41. Decreasing intervals

42. Zeros of  $f(x)$

43. Number of solutions to  $f(x) = 2$

## Maxima and Minima

1. Rel max @  $(0, 5)$ ; No rel min
2. Rel max @  $(2.158, 3.248)$ ; Rel min @  $(-1.158, -4.048)$
3. Rel Max  $(-1.366, 10.848)$  and  $(1, 6)$ ; Rel Min  $(0.366, 5.652)$
4. Rel Max  $(-1.716, 11.598)$ ; Rel Min  $(1.132, -3.929)$
5. Rel Max:  $(1.095, 12.096)$ ; Rel Min  $(-0.761, -0.680)$
6. Rel Max:  $(1.366, 0.348)$ ; Rel Min:  $(-0.366, -4.848)$  and  $(2, 0)$
7. About 2.16 hours

## Increasing, Decreasing, and Constant Intervals

1. Increasing:  $(-\infty, 0)$  Decreasing:  $(0, \infty)$
2. Increasing:  $(-1.158, 2.158)$  Decreasing:  $(-\infty, -1.158) \cup (2.158, \infty)$
3. Inc:  $(-\infty, -2) \cup (\frac{2}{3}, \infty)$  Dec:  $(-2, \frac{2}{3})$
4. Inc:  $(-1, 0) \cup (1, \infty)$  Dec:  $(-\infty, -1) \cup (0, 1)$
5. Inc:  $(-1, \infty)$  No intervals where it is decreasing
6. Inc:  $(-0.761, 1.095)$ ; Dec:  $(-\infty, -0.761) \cup (1.095, \infty)$
7. Inc:  $(-0.366, 1.366) \cup (2, \infty)$ ; Dec:  $(-\infty, -0.366) \cup (1.366, 2)$

## Miscellaneous

- |                                |   |                                |  |
|--------------------------------|---|--------------------------------|--|
| 1. $[-2, \infty)$              | 12. 51                                      | 22. None                       | 33. 2                                  |
| 2. $[-4, \infty)$              | 13. $(1.408, 0.272)$                        | 23. $(-\infty, b) \cup (c, m)$ | 34. $(-\infty, 2)$                     |
| 3. $(0, -4)$                   | 14. $(0.592, -0.272)$                       | 24. $(b, c) \cup (m, p)$       | 35. $(-\infty, -2.5) \cup (-2.5, 3.6]$ |
| 4. $(-1, -1)$                  | 15. None                                    | 25. $x = a, x = c, x = n$      | 36. $(-1.2, 3)$ and $(1.3, 3.6)$       |
| 5. $-1$                        | 16. None                                    | 26. $(\infty, B) \cup (D, F)$  | 37. $(0, 1)$                           |
| 6. $-4$                        | 17. $(0.592, 1.408)$                        | 27. $(B, D) \cup (F, \infty)$  | 38. $(1.3, 3.6)$                       |
| 7. $(-2, -1) \cup (0, \infty)$ | 18. $(-\infty, 0.592) \cup (1.408, \infty)$ | 28. $B$ and $F$                | 39. Does not exist                     |
| 8. $(-1, 0)$                   | 19. $(b, f(b))$ and $(m, f(m))$             | 29. $D$                        | 40. $(-\infty, -1.2) \cup (0, 1.3)$    |
| 9. $(0, -4)$                   | 20. $(c, f(c))$                             | 30. $B$                        | 41. $(-1.2, 0) \cup (1.3, 2)$          |
| 10. None                       | 21. $(m, f(m))$                             | 31. None                       | 42. $(-1.8, 0)$ and $(1.9, 0)$         |
| 11. $-426$                     |   | 32. $A, C, E, G$               | 43. 4                                  |

## Chapter 4

# Linear Functions and Slope

### 4.1 Equations of Lines

Write the equation of each line **in point-slope form** that goes through each pair of points.

1.  $(-2, 1), (7, 8)$
2.  $(0, 4), (9, -15)$
3.  $(-1, -2), (-3, -13)$

### 4.2 Average Rate of Change

For the function  $f(x) = x^2$ , compute the average rate of change for each interval.

1.  $[1, 1.1]$
2.  $[1, 1.01]$
3.  $[1, 1.001]$
4.  $[1, 1.0001]$
5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function  $f(x) = -6x^2 + 7x + 4$  over each specified interval.

6.  $[-2, -1]$
7.  $[5, 6]$
8.  $[0, 1]$
9.  $[5, 5.001]$
10.  $[5, 5.0001]$
11.  $[5, 5.00001]$
12. What value are your last 3 answers getting closer to?

For the function  $f(x) = -3x^2 + 5$ , determine the average rate of change of each over the given interval.

13.  $[7, 7.001]$
14.  $[7, 7.0001]$
15.  $[7, 7.00001]$
16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = \sqrt{x}$ , find the average rate of change of each over the given interval.

17.  $[1, 1.0001]$

18.  $[1, 1.00001]$

19.  $[1, 1.000001]$

20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = 6\sqrt{x}$ , find the average rate of change of each over the given interval.

21.  $[25, 25.1]$

22.  $[25, 25.01]$

23.  $[25, 25.001]$

24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function  $f(x) = -7x^3 + 6\sqrt{3x} + 4$  over each interval. Round your answers to 4 decimal places.

25.  $[0, 1]$

26.  $[10, 11]$

27.  $[8, 15]$

## Equations of Lines

1.  $y - 1 = \frac{7}{9}(x + 2)$  or  $y - 8 = \frac{7}{9}(x - 7)$
2.  $y - 4 = -\frac{19}{9}(x - 0)$  or  $y + 15 = -\frac{19}{9}(x - 9)$
3.  $y + 2 = \frac{11}{2}(x + 1)$  or  $y + 13 = \frac{11}{2}(x + 3)$

## Average Rate of Change

- |            |                |                 |
|------------|----------------|-----------------|
| 1. 2.1     | 10. -53.0006   | 19. -0.49999988 |
| 2. 2.01    | 11. -53.00006  | 20. -0.5        |
| 3. 2.001   | 12. -53        | 21. 0.5994      |
| 4. 2.0001  | 13. -42.003    | 22. 0.59999     |
| 5. 2       | 14. -42.0003   | 23. 0.6         |
| 6. 25      | 15. -42.00003  | 24. 0.6         |
| 7. -59     | 16. -42        | 25. 3.3923      |
| 8. 1       | 17. -0.499988  | 26. -2,315.3960 |
| 9. -53.006 | 18. -0.4999988 | 27. -2861.4492  |

## Chapter 5

# Function Transformations

Write the function for  $g(x)$  if it is the result of  $f(x)$  after the following ordered sequence of transformations.

1. (1) Vertical stretch by 3  
(2) Shift left 1 unit  
(3) Reflect across  $y$ -axis
2. (1) Horizontal compression by 2  
(2) Shift up 1 unit
3. (1) Reflect across  $x$ -axis  
(2) Vertical compression by 4  
(3) Move right 7 units

Write the function  $g(x)$  that is a result of the following ordered sequence of transformations to  $f(x) = |x|$ .

4. (1) Reflect across  $x$ -axis  
(2) Shift right 3 units  
(3) Horizontal stretch by factor of 5
5. (1) Shift down 2 units  
(2) Reflect across  $y$ -axis  
(3) Shift up 1 unit
6. (1) Horizontal compression by factor of 7  
(2) Vertical compression by factor of 4  
(3) Shift left 9 units

Given  $f(x) = \sqrt{x}$ , determine the resulting function  $g(x)$  after the following ordered sequence of transformations.

7. (1) Shift up 2 units  
(2) Horizontal stretch by 5  
(3) Shift left 3 units
8. (1) Vertical compression by factor of 3  
(2) Reflect across  $y$ -axis  
(3) Horizontal compression by 5
9. (1) Shift right 8 units  
(2) Reflect across  $x$ -axis  
(3) Horizontal compression by factor of 4



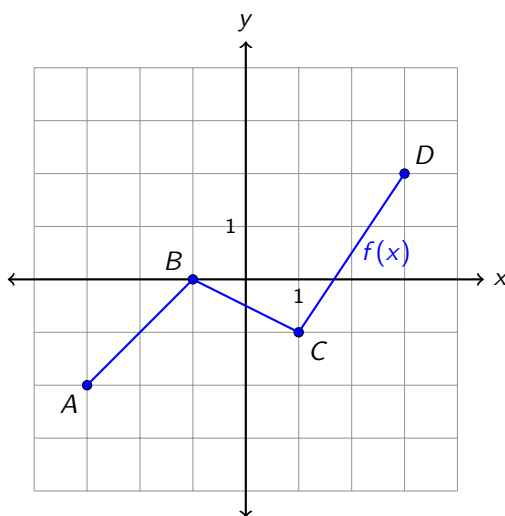
Write the final equation of  $g(x)$  if it is found by taking  $f(x) = \sqrt{x}$  after the following ordered sequence of transformations.

10. (1) Shift right 2 units  
(2) Horizontal stretch by factor 3  
(3) Shift down 2 units  
(4) Reflect across  $x$ -axis
11. (1) Horizontal stretch by factor 3  
(2) Shift left 1 unit  
(3) Shift up 2 units  
(4) Reflect across  $y$ -axis
12. (1) Vertical stretch by factor 5  
(2) Horizontal stretch by factor 2  
(3) Shift up 3 units  
(4) Reflect across  $x$ -axis

Find the equation for  $g(x)$  if  $g(x)$  is found by performing the following *ordered* sequence of transformations to  $f(x) = \frac{1}{x}$ .

13. (1) Shift left 3 spaces  
(2) Reflect across  $y$ -axis  
(3) Shift down 5 spaces  
(4) Vertical stretch by factor of 7
14. (1) Shift up 3 spaces  
(2) Reflect across  $x$ -axis  
(3) Shift right 5 spaces  
(4) Horizontal compression by factor of 7

Given the graph of  $f(x)$  below, find the new coordinates of each point after the following transformations.



15.  $-2f(x + 1)$
16.  $f\left(-\frac{1}{2}x\right) - 3$
17.  $\frac{1}{2}f(-x - 2) + 2$
18.  $f(2x + 2) - 1$

# FUNCTION TRANSFORMATIONS KEY

1.  $g(x) = 3f(-x + 1)$
2.  $g(x) = f(2x) + 1$
3.  $g(x) = -\frac{1}{4}f(x - 7)$
4.  $g(x) = -\left|\frac{1}{5}x - 3\right|$
5.  $g(x) = |-x| - 1$
6.  $g(x) = \frac{1}{4}|7(x + 9)| = \frac{1}{4}|7x + 63|$
7.  $g(x) = \sqrt{\frac{1}{5}(x + 3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$
8.  $g(x) = \frac{1}{3}\sqrt{-5x}$
9.  $g(x) = -\sqrt{4x - 8}$
10.  $g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$
11.  $g(x) = \sqrt{\frac{1}{3}(-x + 1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$
12.  $g(x) = -\left(5\sqrt{\frac{1}{2}x + 3}\right) = -5\sqrt{\frac{1}{2}x + 3}$
13.  $g(x) = \frac{7}{-x+3} - 35$
14.  $g(x) = -\frac{1}{7x-5} - 3$
15.  $A'(-4, 4), B'(-2, 0), C'(0, 2), D'(2, -4)$
16.  $A'(6, -5), B'(2, -3), C'(-2, -4), D'(-6, -1)$
17.  $A'(1, 1), B'(-1, 2), C'(-3, 1.5), D'(-5, 3)$
18.  $A'(-2.5, -3), B'(-1.5, -1), C'(-0.5, -2), D'(0.5, 1)$

## Chapter 6

# Function Operations

### 6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given  $f(x) = x + 5$ ,  $g(x) = x^2 - 1$ , and  $h(x) = \sqrt{x - 10}$ , simplify or evaluate each.

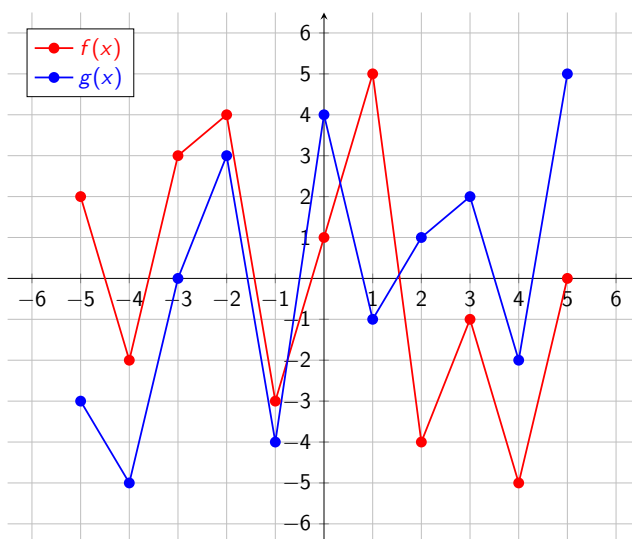
1.  $(g - f)(x)$
2.  $(fh)(14)$
3.  $(f + g)(x)$

Find each of the following given the table below.

$x$	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-3	0	-1	3	1	2	4	-4	-2
$g(x)$	3	-1	0	1	4	-2	-4	2	-3

4.  $(f + g)(-2)$
5.  $(f - g)(0)$
6.  $(fg)(1)$
7.  $\left(\frac{f}{g}\right)(3)$
8.  $(f + f)(-4)$

Find each of the following given the graphs of  $f(x)$  (in red) and  $g(x)$  (in blue) below:



9.  $(f + g)(2)$
10.  $(f - g)(1)$
11.  $(g - f)(-3)$
12.  $(fg)(4)$
13.  $\left(\frac{f}{g}\right)(0)$

## 6.2 Operations with Functions: Domain

Given  $f(x) = \sqrt{2x+7}$  and  $g(x) = 3x+3$ , find the domain of each.

1.  $(f+g)(x)$

2.  $\left(\frac{f}{g}\right)(x)$

3.  $\left(\frac{g}{f}\right)(x)$

## 6.3 Difference Quotient

Write the difference quotient for each.

1.  $f(x) = 2x - 7$

2.  $g(x) = x^2 + 4x$

3.  $h(x) = -1$

4.  $f(x) = \frac{3}{x+2}$

5.  $g(x) = \sqrt{3x}$

6.  $f(x) = x^2 - 2x + 5$

7.  $g(x) = \frac{5}{x}$

8.  $f(x) = -2x^2 + 3x - 5$

9.  $g(x) = \frac{6}{2x+3}$

10.  $h(x) = \sqrt{7x+5}$

11.  $f(x) = -x^2 + x$

12.  $f(x) = 3x - 1$

13.  $f(x) = x^3 + 5x$

## Adding, Subtracting, Multiplying, and Dividing Functions

- |                  |                   |
|------------------|-------------------|
| 1. $x^2 - x - 6$ | 8. $-6$           |
| 2. $38$          | 9. $-3$           |
| 3. $x^2 + x + 4$ | 10. $6$           |
| 4. $-1$          | 11. $-3$          |
| 5. $-3$          | 12. $10$          |
| 6. $-4$          | 13. $\frac{1}{4}$ |
| 7. $-2$          |                   |

## Operations with Functions: Domain

- $[-\frac{7}{2}, \infty)$
- $[-\frac{7}{2}, -1) \cup (-1, \infty)$
- $(-\frac{7}{2}, \infty)$

## Difference Quotient

- |                                       |  |
|---------------------------------------|--|
| 1. $2$                                | 8. $-4x - 2h + 3$                          |
| 2. $2x + h + 4$                       | 9. $\frac{-12}{(2x+3)(2x+2h+3)}$           |
| 3. $0$                                | 10. $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$ |
| 4. $\frac{-3}{(x+2)(x+h+2)}$          | 11. $-2x - h + 1$                          |
| 5. $\frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$ | 12. $3$                                    |
| 6. $2x + h - 2$                       | 13. $3x^2 + 3xh + h^2 + 5$                 |
| 7. $\frac{-5}{x(x+h)}$                |  |

## Chapter 7

# Polynomials and Their Graphs

Determine the end behavior of each.

1.  $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$

2.  $g(x) = 4x^2 - 16x^6 + 3x$

3.  $h(x) = 1 + x^{11} - 4x^8$

4.  $f(x) = -x^4 + 3x^2 - 2x + 6$

5.  $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

6.  $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$

7.  $g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$

## POLYNOMIALS AND THEIR GRAPHS

1.  $\lim_{x \rightarrow -\infty} f(x) = \infty$      $\lim_{x \rightarrow \infty} f(x) = -\infty$

2.  $\lim_{x \rightarrow -\infty} g(x) = -\infty$ ,     $\lim_{x \rightarrow \infty} g(x) = \infty$

3.  $\lim_{x \rightarrow -\infty} h(x) = -\infty$      $\lim_{x \rightarrow \infty} h(x) = \infty$

4.  $\lim_{x \rightarrow -\infty} f(x) = -\infty$      $\lim_{x \rightarrow \infty} f(x) = -\infty$

5.  $\lim_{x \rightarrow -\infty} g(x) = -\infty$      $\lim_{x \rightarrow \infty} g(x) = \infty$

6.  $\lim_{x \rightarrow -\infty} f(x) = \infty$      $\lim_{x \rightarrow \infty} f(x) = -\infty$

7.  $\lim_{x \rightarrow -\infty} g(x) = -\infty$      $\lim_{x \rightarrow \infty} g(x) = \infty$

## Chapter 8

# Dividing Polynomials

### 8.1 Dividing Polynomials

Divide each.

1.  $(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$
2.  $(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$
3.  $(4x^3 - 3x^2 + x + 1) \div (x + 2)$
4.  $(5x^4 - x^2 + x - 2) \div (x^2 + 2)$
5.  $(10x^3 + 27x^2 + 8x - 11) \div (2x + 3)$
6.  $(7x^3 + 23x^2 + 12x + 1) \div (x^2 + 3x + 1)$

### 8.2 Remainder and Factor Theorems

Determine the remainder of each.

1.  $(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$
2.  $(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x + 1)$
3.  $(x^{23} - 5x^{20} + 17x^8 - 5) \div (x + 2)$
4.  $(-7x^{17} + 40x^{15} - 6x^8 + 4x^3) \div (x - 3)$



## Dividing Polynomials

1.  $4x^2 - 2x + 5$

2.  $4y^2 + 8y + 7 - \frac{72}{3y + 5}$

3.  $4x^2 - 11x + 23 - \frac{45}{x + 2}$

4.  $5x^2 - 11 + \frac{x + 20}{x^2 + 2}$

5.  $5x^2 + 6x - 5 + \frac{4}{2x + 3}$

6.  $7x + 2 + \frac{-x - 1}{x^2 + 3x + 1}$

## Remainder and Factor Theorems

1. 6

2. 13

3.  $-13, 627, 141$

4.  $-330, 064, 119$

## Chapter 9

# Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

$$1. f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

$$6. f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

$$2. g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$7. g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

$$3. f(x) = \frac{3x}{x + 4}$$

$$8. h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

$$4. g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

$$9. f(x) = \frac{6x^3 - 21x^2 - 51x + 30}{3x^2 + 7x + 2}$$

$$5. h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

$$10. g(x) = \frac{10x^2 - 29x - 21}{10x^3 - 33x^2 - 7x}$$

State the end behavior of each.

$$11. k(x) = \frac{5x^3 - 7x^2 + 8}{-3x^3 + 6x - 4}$$

$$12. m(x) = \frac{2x - 1}{3x^2 + 7x + 1}$$

# RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain:  $x \neq -\frac{1}{2}, 7$ ; V.A.:  $x = 7$ ; Hole @  $\left(-\frac{1}{2}, -\frac{7}{13}\right)$ ; H.A.:  $y = 1$
2. Domain:  $x \neq -3, 4$ ; V.A.:  $x = -3$ ; Obl. Asymp:  $y = 3x + 10$
3. Domain:  $x \neq -4$ ; V.A.:  $x = -4$ ; H.A.:  $y = 3$
4. Domain:  $x \neq 1$ ; V.A.:  $x = 1$ ; Obl. Asymp:  $y = x + 4$
5. Domain:  $x \neq 0, 1$ ; V.A.:  $x = 0$  and  $x = 1$ ; H.A.:  $y = 0$
6. Domain:  $x \neq -1, 5$ ; V.A.  $x = -1$ ; Hole @  $\left(5, \frac{13}{3}\right)$ ; Obl. Asym  $y = 2x - 5$
7. Domain:  $x \neq -6, 0, 4$ ; V.A.  $x = -6, x = 0$ ; Hole @  $\left(4, \frac{21}{40}\right)$ ; H.A.  $y = 0$
8. Domain:  $x \neq -6, -\frac{3}{8}$ ; V.A.  $x = -6, x = -\frac{3}{8}$ ; H.A.  $y = \frac{1}{4}$
9. Domain:  $x \neq -2, -\frac{1}{3}$ ; Hole @  $(-2, -21)$ ; V.A.:  $x = -\frac{1}{3}$ ; S.A.  $y = 2x - \frac{35}{3}$
10. Domain:  $x \neq -\frac{1}{5}, 0, \frac{7}{2}$ ; Hole @  $\left(\frac{7}{2}, \frac{82}{259}\right)$ ; V.A.  $x = -\frac{1}{5}$  and  $x = 0$ ; H.A.  $y = 0$
11.  $\lim_{x \rightarrow -\infty} k(x) = \infty = \lim_{x \rightarrow \infty} k(x) = -\frac{5}{3}$
12.  $\lim_{x \rightarrow -\infty} m(x) = \infty = \lim_{x \rightarrow \infty} m(x) = 0$

## Chapter 10

# Polynomial and Rational Inequalities

### 10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1.  $6x^3 - 4x^2 - 10x \geq 0$

2.  $x^4 < 9x^2$

3.  $3x^3 - 7x^2 - 22x + 8 < 0$

4.  $3x^2 - 4x + 1 \leq 0$

5.  $12x^4 + 76x^3 + 43x^2 - 346x - 280 \geq 0$

### 10.2 Domain

State the domain of each. Write your answers using interval notation.

1.  $b(x) = \sqrt{21x^2 - 23x - 20}$

### 10.3 Rational Inequalities

Solve each. Write your answers using interval notation.

1.  $\frac{3x - 4}{x + 1} < 0$

2.  $\frac{x^2 + 3x + 2}{x - 7} \leq 0$

3.  $\frac{x^2 - 4x + 4}{x^2 - 1} \geq 0$

4.  $\frac{x + 2}{x - 4} \leq 1$

5.  $\frac{x^2 - 7x - 8}{x^2 - 4x - 32} \geq 0$

6.  $\frac{4 + 3x}{5 - x} \leq 2$

**Polynomial Inequalities**

1.  $[-1, 0] \cup \left[\frac{5}{3}, \infty\right)$
2.  $(-3, 0) \cup (0, 3)$
3.  $(-\infty, -2) \cup \left(\frac{1}{3}, 4\right)$
4.  $\left[\frac{1}{3}, 1\right]$
5.  $(-\infty, -4] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$

**Domain**

1.  $\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$

**Rational Inequalities**

1.  $\left(-1, \frac{4}{3}\right)$
2.  $(-\infty, -2] \cup [-1, 7)$
3.  $(-\infty, -1) \cup (1, \infty)$
4.  $(-\infty, 4)$
5.  $(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$
6.  $(-\infty, 1.2] \cup (5, \infty)$

# Chapter 11

## Function Compositions

Given  $f(x) = x - 5$ ,  $g(x) = 4 + \sqrt{2x + 1}$ , and  $h(x) = \frac{3}{x + 7}$ , simplify each and state the domain.

1.  $(f \circ g)(x)$
2.  $(g \circ f)(x)$
3.  $h(h(x))$

Find each of the following given the table below.

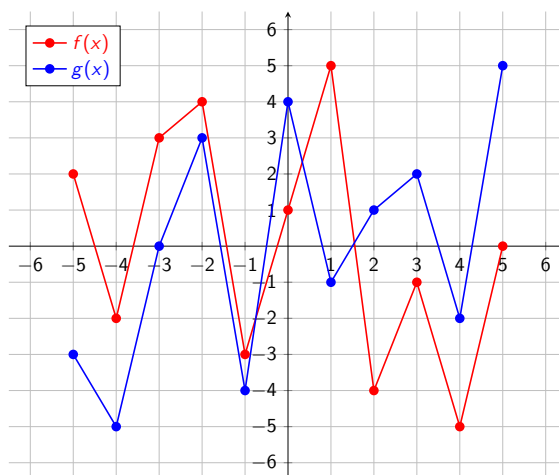
$x$	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-3	0	-1	3	1	2	4	-4	-2
$g(x)$	3	-1	0	1	4	-2	-4	2	-3

4.  $(f \circ g)(-1)$
5.  $(g \circ g)(0)$
6.  $(f \circ f)(2)$
7.  $(g \circ g)(-3)$
8.  $f(g(0))$

Given  $f(x) = \sqrt{3x + 2}$ ,  $g(x) = x^2 - 1$ , and  $h(x) = 9x - 2$ , find each of the following.

9.  $(g \circ f)(x)$
10.  $f(g(x))$
11.  $(h \circ h)(x)$

Find each of the following given the graphs of  $f(x)$  (in red) and  $g(x)$  (in blue) below:



12.  $(f \circ g)(-1)$
13.  $(g \circ f)(-4)$
14.  $f(g(3))$
15.  $g(g(-2))$
16.  $(f \circ f)(-5)$

FUNCTION COMPOSITIONS KEY

1.  $-1 + \sqrt{2x+1}$  Domain:  $\left[-\frac{1}{2}, \infty\right)$
2.  $4 + \sqrt{2x-9}$  Domain:  $\left[\frac{9}{2}, \infty\right)$
3.  $\frac{3x+21}{7x+52}$  Domain:  $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup (-7, \infty)$
4. 2
5. -3
6. -2
7. 1
8. -2
9.  $3x+1$
10.  $\sqrt{3x^2-1}$
11.  $81x-20$
12. -2
13. 3
14. -4
15. 2
16. -4

## Chapter 12

# Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1.  $f(x) = \sqrt{-2x+3} + 1$

2.  $g(x) = (x+4)^2 - 1, x \leq -4$

3.  $h(x) = \frac{9x}{4x-1}$

4.  $f(x) = \sqrt{x} - 3$

5.  $g(x) = \frac{1}{1-x}$

6.  $h(x) = x^2 + 6x + 4, x \leq -3$

7.  $f(x) = \sqrt{5x-4}$

8.  $g(x) = x^2 - 2x + 3, x \leq 1$

9.  $h(x) = \frac{3}{x-1}$



INVERSE FUNCTIONS KEY

$$1. f^{-1}(x) = -\frac{1}{2}((x-1)^2 - 3)$$

	Domain	Range
$f(x)$	$(-\infty, 1.5]$	$[1, \infty)$
$f^{-1}(x)$	$[1, \infty)$	$(-\infty, 1.5]$

$$2. g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
$g(x)$	$(-\infty, -4]$	$[-1, \infty)$
$g^{-1}(x)$	$[-1, \infty)$	$(-\infty, -4]$

$$3. h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
$h(x)$	$(-\infty, 1/4) \cup (1/4, \infty)$	$(\infty, 9/4) \cup (9/4, \infty)$
$h^{-1}(x)$	$(\infty, 9/4) \cup (9/4, \infty)$	$(-\infty, 1/4) \cup (1/4, \infty)$

$$4. f^{-1}(x) = (x+3)^2$$

	Dom	Ran
$f(x)$	$[0, \infty)$	$[-3, \infty)$
$f^{-1}(x)$	$[-3, \infty)$	$[0, \infty)$

$$5. g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
$g(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$g^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

$$6. h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
$h(x)$	$(-\infty, -3]$	$[-5, \infty)$
$h^{-1}(x)$	$[-5, \infty)$	$(-\infty, -3]$

$$7. f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

	Dom	Ran
$f(x)$	$\left[\frac{4}{5}, \infty\right)$	$[0, \infty)$
$f^{-1}(x)$	$[0, \infty)$	$\left[\frac{4}{5}, \infty\right)$

$$8. g^{-1}(x) = -\sqrt{x-2} + 1$$

	Dom	Ran
$g(x)$	$(-\infty, 1]$	$[2, \infty)$
$g^{-1}(x)$	$[2, \infty)$	$(-\infty, 1]$

$$9. h^{-1}(x) = \frac{3}{x} + 1$$

	Dom	Ran
$h(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$h^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

## Chapter 13

# Exponential Functions

### 13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

1.  $f(x) = 3 + e^{2x}$

2.  $h(x) = 5^{-x}$

3.  $h(x) = -\frac{2}{3}e^{x+7} + 1$

4.  $f(x) = -7e^x + 4$

5.  $g(x) = \frac{1}{3}e^{2x+1} - 5$

6.  $h(x) = -\frac{1}{2}e^{-4x} + 1$

## EXPONENTIAL FUNCTIONS

1.  $\lim_{x \rightarrow -\infty} f(x) = 3$      $\lim_{x \rightarrow \infty} f(x) = \infty$
2.  $\lim_{x \rightarrow -\infty} f(x) = \infty$      $\lim_{x \rightarrow \infty} f(x) = 0$
3.  $\lim_{x \rightarrow -\infty} h(x) = 1$      $\lim_{x \rightarrow \infty} h(x) = -\infty$
4.  $\lim_{x \rightarrow -\infty} f(x) = 4$      $\lim_{x \rightarrow \infty} f(x) = -\infty$
5.  $\lim_{x \rightarrow -\infty} f(x) = -5$      $\lim_{x \rightarrow \infty} f(x) = \infty$
6.  $\lim_{x \rightarrow -\infty} f(x) = -\infty$      $\lim_{x \rightarrow \infty} f(x) = 1$

## Chapter 14

# Logarithmic Functions

Write each of the following in exponential or logarithmic form.

1.  $\ln(a) = 7$

2.  $\log_4(x + 1) = 9$

3.  $\log(5x) = 30$

4.  $\ln(w) = c$

5.  $5^x = 19$

6.  $8^{-3} = \frac{1}{512}$

7.  $e^{14} = x$

8.  $(1.1)^{-t} = 50$

Find the domain of each. Write your answers in interval notation.

9.  $b(x) = \log_7(x^2 - 8x + 6)$

10.  $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$

11.  $f(x) = -7 \ln(x^2 + 9x + 8)$

12.  $g(x) = \log(5x^2 + 13x - 6)$

13.  $h(x) = 3 \log_2(x^3 + 2x^2 - x - 2)$

14.  $c(x) = \ln(4x^2 - 15x - 4)$

State the end behavior of each.

15.  $j(x) = 5 \log_3(2x - 5) - 2$

# LOGARITHMIC FUNCTIONS KEY

1.  $e^7 = a$
2.  $4^9 = x + 1$
3.  $10^{30} = 5x$
4.  $e^c = w$
5.  $\log_5(19) = x$
6.  $\log_8\left(\frac{1}{512}\right) = -3$
7.  $\ln(x) = 14$
8.  $\log_{1.1}(50) = -t$
9.  $(-\infty, 0.838) \cup (7.162, \infty)$
10.  $(-3, -2) \cup (-1, \infty)$
11.  $(-\infty, -8) \cup (-1, \infty)$
12.  $(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$
13.  $(-2, -1) \cup (1, \infty)$
14.  $\left(-\infty, -\frac{1}{4}\right) \cup (4, \infty)$
15.  $\lim_{x \rightarrow (5/2)^+} j(x) = -\infty \quad \lim_{x \rightarrow \infty} j(x) = \infty$

## Chapter 15

# Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

1.  $\log_b \left( \frac{x^2}{y^8} \right)$

2.  $\ln(ez)^3$

3.  $\log_5(x) + \log_5(9) - 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

4.  $\log_7(10)$

5.  $\log_9(x)$

6.  $\log_b(c)$

7.  $\log_3(10)$

8.  $\log_{17}(\pi)$

9.  $\log_w(x)$

Suppose that  $\log_a(b) = 5$ ,  $\log_a(c) = 12$ , and  $\log_a(d) = 9$ . Evaluate each of the following.

10.  $\log_a(bc)$

11.  $\log_a(c^3)$

12.  $\log_a \left( \frac{d}{c} \right)$

13.  $\log_a \left( \frac{bd}{c} \right)$

PROPERTIES OF LOGARITHMS KEY

1.  $2 \log_b(x) - 8 \log_b(y)$

2.  $3 + 3 \ln(z)$

3.  $\log_5 \left( \frac{9x}{w^2} \right)$

4.  $\frac{\ln(10)}{\ln(7)}$

5.  $\frac{\ln(x)}{\ln(9)}$

6.  $\frac{\ln(c)}{\ln(b)}$

7.  $\frac{\ln(10)}{\ln(3)}$

8.  $\frac{\ln(\pi)}{\ln(17)}$

9.  $\frac{\ln(x)}{\ln(w)}$

10. 17

11. 36

12.  $-3$

13. 2

## Chapter 16

# Exponential Equations

Solve each. Round to 3 decimal places when necessary.

1.  $3e^{x-2} = 7$

2.  $5^x + 4 > 1$

3.  $2^{3x+4} = 32^{x-7}$

4.  $5e^{7x} + 10 = 42$

5.  $7^{4x+1} \geq 343$

6.  $1000e^{0.04x} = 2000$

7.  $3(4.1)^{x-2} = 8$

8.  $2^{x+1} = 5^{7x-5}$

### 16.1 Applications

1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?



## EXPONENTIAL EQUATIONS KEY

1.  $x \approx 2.847$
2.  $(-\infty, \infty)$
3.  $x = 19.5$
4.  $x \approx 0.265$
5.  $\left[\frac{1}{2}, \infty\right)$
6.  $x \approx 17.329$
7.  $x \approx 2.695$
8.  $x \approx 0.827$

## Applications

1. Approximately 17,952 years

## Chapter 17

# Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

1.  $\log_5(x) + x \log_5(x) > 0$

2.  $\ln(8 - x^2) = \ln(2 - x)$

3.  $\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$

4.  $\log_3(2x+1) - \log_3(x-5) = \log_3(x+1)$

5.  $\log_4(x+1) + \log_4(x-5) > 2$

## LOGARITHMIC EQUATIONS AND INEQUALITIES KEY

1.  $(1, \infty)$
2.  $x = -2$
3.  $x \approx 1.571$
4.  $x \approx 6.873$
5.  $(2, \infty)$

# Chapter 18

## Sequences

Write the first 4 terms of each sequence.

1.  $a_n = 2(-3)^n$

2.  $b_n = \frac{n!}{2^n}$

3.  $c_{n+1} = 5c_n + 1; c_1 = 2$

4.  $d_n = \frac{1}{2}d_{n-1} + n; d_1 = 3$

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round.**

5.  $a_n = \{343, 667, 991, 1315, \dots\}$ ; Find the 582nd term.

6.  $b_n = \{300, 240, 192, 153.6, \dots\}$ ; Find the 711th term.

7.  $c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots\right\}$ ; Find the 8,675,309th term.

Given each sequence, find the first 4 digits **after the decimal point** in the **scientific notation** version of each term.

8.  $a_n = 17, 33, 49, 65, \dots$ ;  $a_{21,972}$

9.  $b_n = 25, 36, 49, 64, 81, \dots$   $b_{413,401}$

10.  $c_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots$   $c_{152}$

## SEQUENCES KEY

1.  $-6, 18, -54, 162$
2.  $\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2}$
3.  $2, 11, 56, 281$
4.  $3, \frac{7}{2}, \frac{19}{4}, \frac{51}{8}$
5.  $188,587$
6.  $6882$
7.  $7634$
8.  $5155 (3.51553 \times 10^5)$
9.  $7090 (1.7090369403 \times 10^{11})$
10.  $1677 (1.1677487203 \times 10^{-46})$

# Chapter 19

## Series

Find the sum of each, if possible.

1.  $\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^i$

2.  $\sum_{i=0}^{\infty} 3 \left(-\frac{2}{3}\right)^i$

3.  $\sum_{k=1}^{\infty} -2 \left(\frac{1}{3}\right)^k$

4.  $\sum_{j=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^j$

5.  $\sum_{i=0}^{\infty} 1.2(0.8)^i$

6.  $\sum_{i=1}^{\infty} 1.2(0.8)^i$

7.  $\sum_{i=0}^{\infty} 0.8(1.2)^i$

8.  $\sum_{k=1}^{\infty} \frac{2}{3^k}$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9.  $9 + 13 + 17 + 21 + \cdots + 1565$

10.  $-3 + 6 - 12 + 24 - 48 + \cdots + 50,331,648$

11.  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{981}$

12.  $2 + 4 + 6 + 8 + 10 + \cdots + 38,214$

13.  $3 + 7 + 11 + 15 + \cdots + 11,491$

14.  $\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \cdots + \frac{742}{743}$

## SERIES KEY

1.  $\frac{1}{4}$
2.  $\frac{9}{5}$
3.  $-1$
4. Diverges
5. 6
6. 4.8
7. Diverges
8. 2
9. 306,930
10.  $-33,554,433$
11. 7.4663
12. 365,096,556
13. 16,511,131
14. 733.8947

## Chapter 20

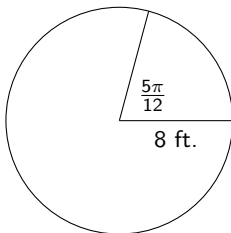
# Angles and Radian Measure

Sketch each of the following. Then find a coterminal between  $0$  and  $360^\circ$  (or  $0$  and  $2\pi$  radians) for each.

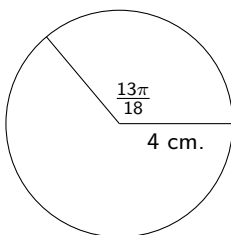
1.  $-\frac{3\pi}{4}$
2.  $900^\circ$
3.  $\frac{27\pi}{10}$
4.  $-125^\circ$

Find the arc length and sector area formed by the central angle of each. Exact answers only.

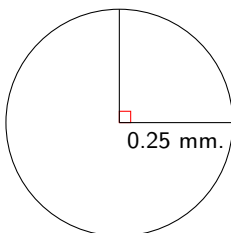
5.



6.



7.



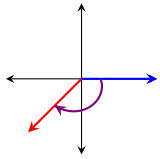
A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

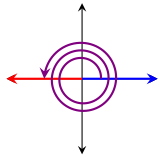


# ANGLES AND RADIAN MEASURE KEY

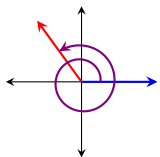
1.  $\frac{5\pi}{4}$



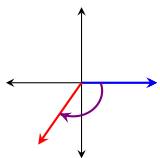
2.  $180^\circ$



3.  $\frac{7\pi}{10}$



4.  $235^\circ$



5.  $s = \frac{10\pi}{3}$  ft.;  $A = \frac{40\pi}{3}$  sq.ft.

6.  $s = \frac{26\pi}{9}$  cm.;  $A = \frac{52\pi}{9}$  sq.cm.

7.  $s = \frac{\pi}{8}$  mm.;  $A = \frac{\pi}{64}$  sq.mm.

8. 26.18 rad/sec

9. 8.73 ft/sec

## Chapter 21

# Trig Functions of Any Angle

Find the exact value of each of the six trig functions of  $\theta$  if  $P$  is a point on the terminal side of  $\theta$ .

1.  $P(-2, 3)$
2.  $P(0, -4)$
3.  $P(-2\sqrt{3}, 2)$
4.  $P(-3, 5)$
5.  $P(-2, 1)$
6.  $P(-4, -7)$

Find the exact values of the 6 trig functions of the following angles.

7.  $\theta = \frac{-17\pi}{4}$
8.  $\theta = \frac{21\pi}{2}$
9.  $\theta = 24\pi$

# TRIG FUNCTIONS OF ANY ANGLE KEY

$$1. \sin \theta = \frac{3\sqrt{13}}{13}, \cos \theta = \frac{-2\sqrt{13}}{13}, \tan \theta = -\frac{3}{2}, \csc \theta = \frac{\sqrt{13}}{3}, \sec \theta = -\frac{\sqrt{13}}{2}, \cot \theta = -\frac{2}{3}$$

$$2. \sin \theta = -1, \cos \theta = 0, \tan \theta = \text{undef.}, \csc \theta = -1, \sec \theta = \text{undef.}, \cot \theta = 0$$

$$3. \sin \theta = \frac{1}{2}, \cos \theta = -\frac{\sqrt{3}}{2}, \tan \theta = -\frac{\sqrt{3}}{3}, \csc \theta = 2, \sec \theta = -\frac{2\sqrt{3}}{3}, \cot \theta = -\sqrt{3}$$

$$4. \sin \theta = \frac{5\sqrt{34}}{34}, \cos \theta = -\frac{3\sqrt{34}}{34}, \tan \theta = -\frac{5}{3}, \csc \theta = \frac{\sqrt{34}}{5}, \sec \theta = -\frac{\sqrt{34}}{3}, \cot \theta = -\frac{3}{5}$$

$$5. \sin \theta = \frac{\sqrt{5}}{5}, \cos \theta = -\frac{2\sqrt{5}}{5}, \tan \theta = -\frac{1}{2}, \csc \theta = \sqrt{5}, \sec \theta = -\frac{\sqrt{5}}{2}, \cot \theta = -2$$

$$6. \sin \theta = -\frac{7\sqrt{65}}{65}, \cos \theta = -\frac{4\sqrt{65}}{65}, \tan \theta = \frac{7}{4}, \csc \theta = -\frac{\sqrt{65}}{7}, \sec \theta = -\frac{\sqrt{65}}{4}, \cot \theta = \frac{4}{7}$$

$$7. \sin \theta = -\frac{\sqrt{2}}{2}, \cos \theta = \frac{\sqrt{2}}{2}, \tan \theta = -1, \csc \theta = -\sqrt{2}, \sec \theta = \sqrt{2}, \cot \theta = -1$$

$$8. \sin \theta = 1, \cos \theta = 0, \tan \theta = \text{undefined}, \csc \theta = 1, \sec \theta = \text{undefined}, \cot \theta = 0$$

$$9. \sin \theta = 0, \cos \theta = 1, \tan \theta = 0, \csc \theta = \text{undefined}, \sec \theta = 1, \cot \theta = \text{undefined}$$

## Chapter 22

# Graphs of Sine and Cosine Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1.  $f(x) = -2 \sin\left(3x - \frac{\pi}{4}\right) + 1$

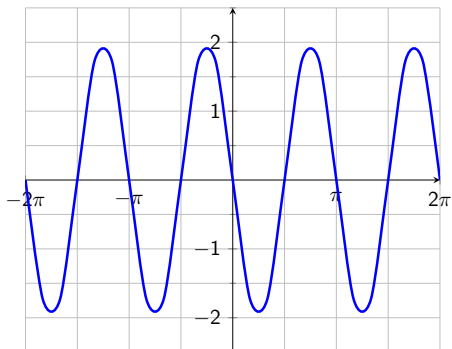
2.  $g(x) = \frac{1}{3} \cos\left(\frac{1}{2}x + 2\right)$

3.  $f(x) = 2 \sin\left(x - \frac{\pi}{3}\right) + 7$

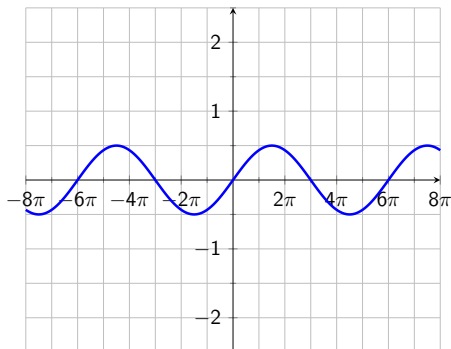
4.  $f(x) = -4 \cos\left(\frac{2}{3}x - \frac{2\pi}{3}\right)$

Write the equation of each of the following in the form  $y = a \sin(bx)$ .

5.



6.



## ANSWERS

1. Amp = 2, Per =  $\frac{2\pi}{3}$ , P.S. =  $\frac{\pi}{12} \rightarrow$ , V.S. = 1  $\uparrow$
2. Amp =  $\frac{1}{3}$ , Per =  $4\pi$ , P.S. =  $4 \leftarrow$ , V.S. = None
3. Amp = 2, Period =  $2\pi$ , P.S. =  $\frac{\pi}{3}$  right, V.S. = 7 up
4. Amp = 4, Period =  $3\pi$ , P.S. =  $\pi$  right, V.S. = 0 (or none)
5.  $y = -2\sin(2x)$
6.  $y = \frac{1}{2}\sin\left(\frac{1}{3}x\right)$

## Chapter 23

# Graphs of Other Trig Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1.  $h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$

2.  $f(x) = 3 \tan\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$

# GRAPHS OF OTHER TRIG FUNCTIONS KEY

1. Amp =  $n/a$ , Per =  $\frac{4\pi}{3}$ , P.S. =  $\frac{\pi}{9}$   $\leftarrow$ , V.S. =  $8 \downarrow$

2. Amp =  $n/a$ , Period =  $\frac{\pi}{2}$ , P.S. =  $\frac{\pi}{4}$  left, V.S. =  $\sqrt{3}$  down

## Chapter 24

# Inverse Trig Functions

State the exact, simplified value of each or write as an expression of  $x$ .

1.  $\tan^{-1}(-\sqrt{3})$

2.  $\sec\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$

3.  $\cot(\sec^{-1}(x))$

4.  $\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$

5.  $\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$

6.  $\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$

7.  $\cos(\tan^{-1}(7x))$

8.  $\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$



# INVERSE TRIG FUNCTIONS KEY

1.  $-\frac{\pi}{3}$

2.  $\frac{5\sqrt{21}}{21}$

3.  $\frac{1}{\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}}{x^2-1}$

4.  $\frac{\sqrt{16-9x^2}}{4}$

5.  $-\frac{3\sqrt{5}}{2}$

6.  $\frac{13\sqrt{22}}{44}$

7.  $\frac{\sqrt{49x^2+1}}{49x^2+1}$

8.  $\frac{\sqrt{64-x^2}}{x}$

## Chapter 25

# Trig Equations and Inequalities

Solve each in the interval  $[0, 2\pi)$ . Write your answers to inequalities using interval notation.

1.  $\tan(6x) = 1$

2.  $\cot(2x) = -\frac{\sqrt{3}}{3}$

3.  $\sin^2(x) = \frac{3}{4}$

4.  $\sin(2x) = \cos(x)$

5.  $\sin(2x) \geq \sin(x)$

6.  $\cos(2x) < 0$

7.  $2 \sin\left(x - \frac{\pi}{3}\right) = -1$

8.  $3 \tan\left(-2x + \frac{\pi}{2}\right) = \sqrt{3}$

9.  $\sin^2(x) < \frac{1}{2}$

# TRIG EQUATIONS AND INEQUALITIES KEY

1.  $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{3\pi}{8}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{7\pi}{8}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{11\pi}{8}, \frac{37\pi}{24}, \frac{41\pi}{24}, \frac{15\pi}{8}$
2.  $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$
3.  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
4.  $\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
5.  $\left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$
6.  $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
7.  $x = \frac{\pi}{6}, \frac{3\pi}{2}$
8.  $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
9.  $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$

## Chapter 26

# Law of Sines and Cosines

Solve each of the following. Round your answers to 1 decimal place.

1.  $m\angle B = 37.8^\circ$ ,  $a = 15$ ,  $c = 21.1$
2.  $m\angle A = 41.9^\circ$ ,  $m\angle C = 59.2^\circ$ ,  $a = 10.2$
3.  $a = 14$ ,  $b = 19.6$ ,  $c = 13.1$

# LAW OF SINES AND COSINES KEY

1.  $b \approx 13.0$ ,  $m\angle A \approx 44.8^\circ$ ,  $m\angle C \approx 97.4^\circ$
2.  $m\angle B = 78.9^\circ$ ,  $b \approx 15.0$ ,  $c \approx 13.1$
3.  $m\angle A \approx 45.5^\circ$ ,  $m\angle B \approx 92.6^\circ$ ,  $m\angle C \approx 41.9^\circ$

## Chapter 27

# Area of Triangles

Find the area of each. Round your answers to 1 decimal place.

1.  $m\angle B = 37.8^\circ$ ,  $a = 15$ ,  $c = 21.1$
2.  $m\angle A = 41.9^\circ$ ,  $m\angle C = 59.2^\circ$ ,  $a = 10.2$
3.  $a = 14$ ,  $b = 19.6$ ,  $c = 13.1$

#### AREA OF TRIANGLES KEY

1. Approximately 97.0 sq. units
2. Approximately 65.7 sq. units
3. Approximately 91.6 sq. units

## Chapter 28

# Polar Coordinates

Convert each to exact rectangular coordinates.

1.  $\left(3, \frac{5\pi}{6}\right)$

2.  $\left(-2, -\frac{\pi}{4}\right)$

3.  $\left(\frac{5}{2}, 240^\circ\right)$

Convert each to exact polar coordinates. Use  $r > 0$  and  $0 \leq \theta < 2\pi$ .

4.  $(-2\sqrt{3}, -2)$

5.  $\left(-\frac{1}{2}, \frac{1}{2}\right)$

6.  $(7\sqrt{2}, -7\sqrt{2})$



POLAR COORDINATES KEY

1.  $\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

2.  $(-\sqrt{2}, \sqrt{2})$

3.  $\left(\frac{5}{4}, \frac{5\sqrt{3}}{4}\right)$

4.  $\left(4, \frac{7\pi}{6}\right)$

5.  $\left(\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$

6.  $\left(14, \frac{7\pi}{4}\right)$

# Appendix A

## Factoring

Factor each of the following completely.

1.  $x^2 + 2x - 15$

2.  $x^2 - 8x + 12$

3.  $x^2 + 15x + 56$

4.  $5x^2 + 19x - 4$

5.  $4x^2 - 5x - 6$

6.  $9x^2 - 400$

7.  $5x^2 - 7x - 6$

8.  $9x^2 - 54x + 45$

9.  $3x^3 + 12x^2 + 9x$

10.  $9y^2 - 16$

11.  $4x^2 - 28x + 49$

12.  $14x^2 + 11xy - 15y^2$

13.  $6x^2 - 48x - 120$

14.  $9x^4 - 54x^3 + 45x^2$

15.  $16y^2 - 40y + 25$

16.  $30x^2 + xy - y^2$

17.  $8w^2 + 33w + 4$

18.  $3p^2 + 22p - 16$

19.  $18x^2 - 27x + 4$

20.  $14a^2 + 15a - 9$

21.  $4x^2 - 4x - 24$

22.  $18t^2 - 9t - 5$

23.  $6a^2 + 23a + 21$

24.  $25x^2 - 1$

# FACTORING KEY

1.  $(x + 5)(x - 3)$

2.  $(x - 6)(x - 2)$

3.  $(x + 7)(x + 8)$

4.  $(5x - 1)(x + 4)$

5.  $(4x + 3)(x - 2)$

6.  $(3x + 20)(3x - 20)$

7.  $(5x + 3)(x - 2)$

8.  $9(x - 5)(x - 1)$

9.  $3x(x + 3)(x + 1)$

10.  $(3y + 4)(3y - 4)$

11.  $(2x - 7)^2$

12.  $(7x - 5y)(2x + 3y)$

13.  $6(x - 10)(x + 2)$

14.  $9x^2(x - 1)(x - 5)$

15.  $(4y - 5)^2$

16.  $(6x - y)(5x + y)$

17.  $(8w + 1)(w + 4)$

18.  $(3p - 2)(p + 8)$

19.  $(6x - 1)(3x - 4)$

20.  $(7a - 3)(2a + 3)$

21.  $4(x - 3)(x + 2)$

22.  $(6t - 5)(3t + 1)$

23.  $(2a + 3)(3a + 7)$

24.  $(5x + 1)(5x - 1)$

## Appendix B

# Complex Fractions

Simplify each as much as possible.

$$1. \frac{5 + \frac{3}{x}}{x - \frac{1}{2}}$$

$$2. \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

$$3. \frac{3}{2 - \frac{x}{x-1}}$$

$$4. \frac{1 + \frac{3}{x}}{\frac{2}{x} + 7}$$

$$5. \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$6. \frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$7. \frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2-2x}}$$

$$8. \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

$$9. \frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$10. \frac{\frac{5}{x} - 5x}{x - 1}$$

$$11. \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$12. \frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

$$13. \frac{\frac{1}{x-a} + \frac{1}{a}}{x}$$

$$14. \frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

$$15. \frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

COMPLEX FRACTIONS KEY

$$1. \frac{2(5x+3)}{x(2x-1)}$$

$$2. \frac{1}{x^2-2x+4}$$

$$3. \frac{3(x-1)}{x-2}$$

$$4. \frac{x+3}{2+7x}$$

$$5. \frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \frac{-4x-1}{2}$$

$$7. \frac{8x-10}{7}$$

$$8. -\frac{1}{7x}$$

$$9. \frac{2x+1}{5x(x+1)}$$

$$10. \frac{-5x-5}{x}$$

$$11. \frac{-1}{2x+4}$$

$$12. \frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \frac{1}{a(x-a)}$$

$$14. \frac{-2x-2}{5x^2-16x+3}$$

$$15. \frac{x+4}{4x-8}$$