

Honors PreCalculus

Extra Practice

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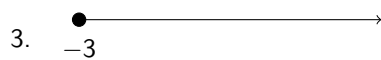
Chapter 1

Basic Set Theory and Interval Notation

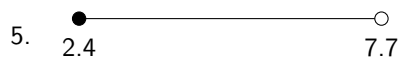
You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

1. $(-5, 8]$

2. $\{x|x \leq 1\}$



4. $\{x|x \neq 4, 11\}$



6. $(9, \infty)$

Write each using interval notation and graph on a number line.

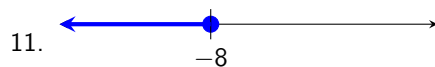
7. $\{x|x \geq 2\}$

8. $\{x|x < -8\}$

9. $\{x|x \neq 3\}$

10. $\{x|x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

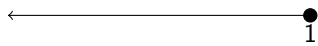


BASIC SET THEORY AND INTERVAL NOTATION KEY

1. $\{x | -5 < x \leq 8\}$

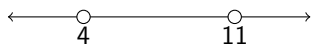


2. $(-\infty, 1]$



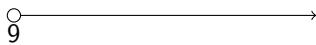
3. $[-3, \infty)$ $\{x | x \geq -3\}$

4. $(-\infty, 4) \cup (4, 11) \cup (11, \infty)$



5. $[2.4, 7.7)$ $\{x | 2.4 \leq x < 7.7\}$

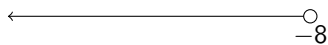
6. $\{x | x > 9\}$



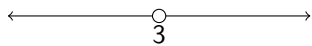
7. $[2, \infty)$



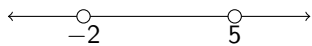
8. $(-\infty, -8)$



9. $(-\infty, 3) \cup (3, \infty)$



10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$



11. $(-\infty, -8]$ $\{x | x \leq -8\}$

12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x | x \neq 7, 12\}$

Chapter 2

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

1. $f(5)$
2. $f(-2)$
3. $f(0)$
4. $g(1)$
5. $g(-5)$
6. $g(1/4)$

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

- | | |
|------------------------------------|--|
| 1. $f(x) = -8x^2 - 7x + 1$ | 15. $f(x) = -\frac{7x-10}{x^2+3x+2}$ |
| 2. $g(x) = \sqrt{5x+12} - 2$ | 16. $g(x) = \sqrt{-9x+8}$ |
| 3. $h(x) = \frac{x+2}{9x-7}$ | 17. $h(x) = -\sqrt[3]{4x+1}$ |
| 4. $f(x) = -5x + 4$ | 18. $f(x) = \sqrt[3]{8x+1}$ |
| 5. $f(x) = x^2 + 2$ | 19. $g(x) = \frac{x^2-1}{\sqrt{x+3}}$ |
| 6. $f(x) = \frac{2x+1}{3x-5}$ | 20. $h(x) = \frac{3}{9+\frac{4}{x+7}}$ |
| 7. $f(x) = \sqrt{3x-12}$ | 21. $f(x) = \frac{x+1}{\sqrt{10x+8}}$ |
| 8. $f(x) = \frac{x}{x^2-16}$ | 22. $g(x) = \frac{5}{1+\frac{3}{x+2}}$ |
| 9. $f(x) = \frac{x+4}{x^3-4x}$ | 23. $i(x) = \frac{7}{3-\frac{4}{x+1}}$ |
| 10. $f(x) = \frac{x}{\sqrt{x-4}}$ | 24. $n(x) = \frac{7x+14}{\sqrt{2x-1}}$ |
| 11. $f(x) = \frac{x^2+1}{2x^2+8}$ | 25. $a(x) = \frac{\frac{x}{x-2}}{\frac{3}{x-2} + 6}$ |
| 12. $f(x) = -\frac{x+7}{x^2-5x-6}$ | 26. $d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$ |
| 13. $g(x) = \sqrt{2x+3}$ | |
| 14. $h(x) = \sqrt[3]{2x+3}$ | |

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3 \\ 0.2x + 7 & \text{if } -3 \leq x < 2 \\ \sqrt{5x} & \text{if } x \geq 2 \end{cases}$$

1. $f(3)$
2. $f(0)$
3. $f(-2)$
4. $f(-3)$
5. $f(0.5)$

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \leq -4 \\ \sqrt{2x + 7} & -4 < x < 0 \\ |-x - 1| & x \geq 0 \end{cases}$$

6. $f(3)$
7. $f(-2)$
8. $f(0)$
9. $f(-5)$

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \leq -3 \\ \sqrt{-4x + 1} & \text{if } -3 < x \leq 0 \\ \frac{5x^2}{x+7} & \text{if } x > 0 \end{cases}$$

10. $f(7)$
11. $f(-3)$
12. $f(1)$
13. $f(0)$
14. $f(-1)$
15. $f(-3/2)$

Evaluating Functions

1. -55
2. -20
3. 0
4. -4
5. -5.2
6. -1

Domain of Functions

- | | |
|--|---|
| 1. $(-\infty, \infty)$ | 14. $(-\infty, \infty)$ |
| 2. $[-\frac{12}{5}, \infty)$ | 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$ |
| 3. $(-\infty, \frac{7}{9}) \cup (\frac{7}{9}, \infty)$ | 16. $(-\infty, \frac{8}{9}]$ |
| 4. $(-\infty, \infty)$ | 17. $(-\infty, \infty)$ |
| 5. $(-\infty, \infty)$ | 18. $(-\infty, \infty)$ |
| 6. $(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$ | 19. $(-3, \infty)$ |
| 7. $[4, \infty)$ | 20. $(-\infty, -\frac{67}{9}) \cup (-\frac{67}{9}, -7) \cup (-7, \infty)$ |
| 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$ | 21. $(-\frac{4}{5}, \infty)$ |
| 9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$ | 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$ |
| 10. $(4, \infty)$ | 23. $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ |
| 11. $(-\infty, \infty)$ | 24. $(\frac{1}{2}, \infty)$ |
| 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$ | 25. $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, 2) \cup (2, \infty)$ |
| 13. $[-\frac{3}{2}, \infty)$ | 26. $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$ |

Piecewise Functions

- | | | |
|------------------------------|-----------------------------|------------------------------|
| 1. $\sqrt{15} \approx 3.873$ | 6. 4 | 11. 4 |
| 2. 7 | 7. $\sqrt{3} \approx 1.732$ | 12. $\frac{5}{8}$ |
| 3. 6.6 | 8. 1 | 13. 1 |
| 4. 6.4 | 9. 18 | 14. $\sqrt{5} \approx 2.236$ |
| 5. 7.1 | 10. 17.5 | 15. $\sqrt{7} \approx 2.646$ |

Chapter 3

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$

2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3. $f(x) = -x^4 + 3x^2 - 2x + 6$

4. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

5. $f(x) = -4x^3 + 2x^2 + 10x + 4$

6. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \geq 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$

2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3. $f(x) = x^3 + 2x^2 - 4x - 8$

4. $g(x) = x^4 - 2x^2 + 1$

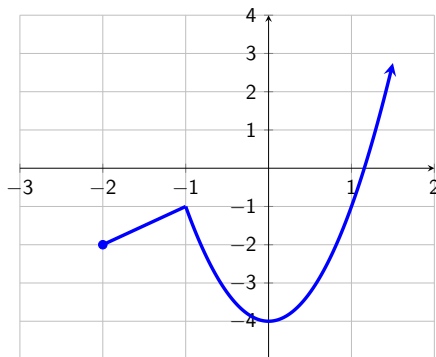
5. $h(x) = \sqrt{x+1} - 2$

6. $f(x) = -4x^3 + 2x^2 + 10x + 4$

7. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

3.3 Miscellaneous

Use the graph of $y = f(x)$ below to answer the following questions. Write your answers using interval notation.

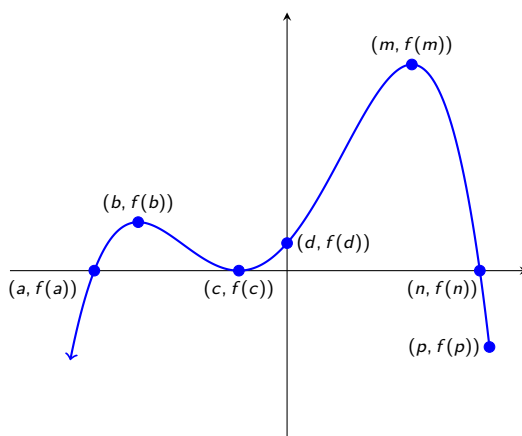


1. Domain of f
2. Range of f
3. Relative Minimum
4. Relative Maximum
5. $f(1)$
6. $f(0)$
7. Increasing Interval(s)
8. Decreasing Interval(s)
9. Absolute Maximum
10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

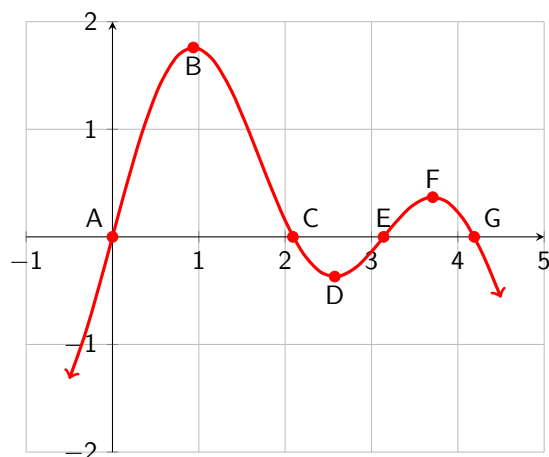
11. $f(7)$
12. $f(-2)$
13. Rel. Max
14. Rel. Min
15. Global Max
16. Global Min
17. Increasing Interval(s)
18. Decreasing Interval(s)

Use the graph of $f(x)$ to answer each.



19. Relative maxima of $f(x)$
20. Relative minima of $f(x)$
21. Absolute maxima of $f(x)$
22. Absolute minima of $f(x)$
23. Intervals where f is increasing
24. Intervals where f is decreasing
25. Zeros of f

Given the labeled points A through G on the graph of $f(x)$ below, find each of the following.



26. Increasing interval(s)

28. Relative max

30. Global max

32. Zeros of f

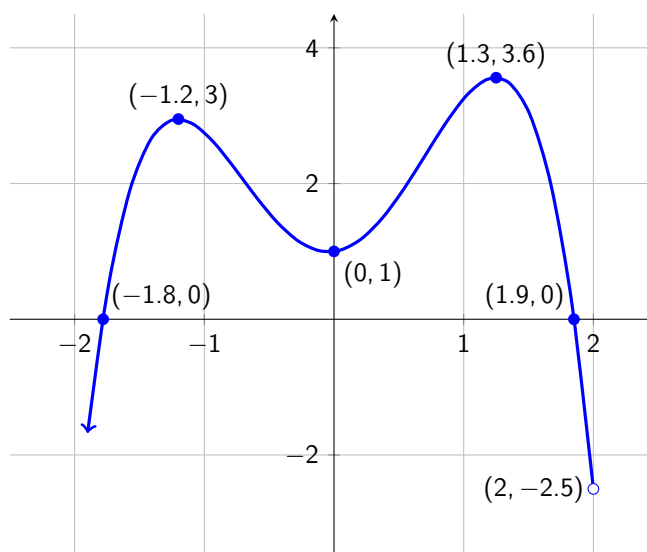
27. Decreasing interval(s)

29. Relative min

31. Global min

33. Number of solutions to $f(x) = 1$

Answer each of the following about the function $f(x)$ below.



34. Domain of f

35. Range of f

36. Relative maxima

37. Relative minima

38. Absolute maximum

39. Absolute minimum

40. Increasing intervals

41. Decreasing intervals

42. Zeros of $f(x)$

43. Number of solutions to $f(x) = 2$

Maxima and Minima

1. Rel max @ $(0, 5)$; No rel min
2. Rel max @ $(2.158, 3.248)$; Rel min @ $(-1.158, -4.048)$
3. Rel Max $(-1.366, 10.848)$ and $(1, 6)$; Rel Min $(0.366, 5.652)$
4. Rel Max $(-1.716, 11.598)$; Rel Min $(1.132, -3.929)$
5. Rel Max: $(1.095, 12.096)$; Rel Min $(-0.761, -0.680)$
6. Rel Max: $(1.366, 0.348)$; Rel Min: $(-0.366, -4.848)$ and $(2, 0)$
7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
2. Increasing: $(-1.158, 2.158)$ Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
3. Inc: $(-\infty, -2) \cup (\frac{2}{3}, \infty)$ Dec: $(-2, \frac{2}{3})$
4. Inc: $(-1, 0) \cup (1, \infty)$ Dec: $(-\infty, -1) \cup (0, 1)$
5. Inc: $(-1, \infty)$ No intervals where it is decreasing
6. Inc: $(-0.761, 1.095)$; Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$

Miscellaneous

- | | | | |
|--------------------------------|---|--------------------------------|--|
| 1. $[-2, \infty)$ | 12. 51 | 22. None | 33. 2 |
| 2. $[-4, \infty)$ | 13. $(1.408, 0.272)$ | 23. $(-\infty, b) \cup (c, m)$ | 34. $(-\infty, 2)$ |
| 3. $(0, -4)$ | 14. $(0.592, -0.272)$ | 24. $(b, c) \cup (m, p)$ | 35. $(-\infty, -2.5) \cup (-2.5, 3.6]$ |
| 4. $(-1, -1)$ | 15. None | 25. $x = a, x = c, x = n$ | 36. $(-1.2, 3)$ and $(1.3, 3.6)$ |
| 5. -1 | 16. None | 26. $(\infty, B) \cup (D, F)$ | 37. $(0, 1)$ |
| 6. -4 | 17. $(0.592, 1.408)$ | 27. $(B, D) \cup (F, \infty)$ | 38. $(1.3, 3.6)$ |
| 7. $(-2, -1) \cup (0, \infty)$ | 18. $(-\infty, 0.592) \cup (1.408, \infty)$ | 28. B and F | 39. Does not exist |
| 8. $(-1, 0)$ | 19. $(b, f(b))$ and $(m, f(m))$ | 29. D | 40. $(-\infty, -1.2) \cup (0, 1.3)$ |
| 9. $(0, -4)$ | 20. $(c, f(c))$ | 30. B | 41. $(-1.2, 0) \cup (1.3, 2)$ |
| 10. None | 21. $(m, f(m))$ | 31. None | 42. $(-1.8, 0)$ and $(1.9, 0)$ |
| 11. -426 | | 32. A, C, E, G | 43. 4 |

Chapter 4

Linear Functions and Slope

4.1 Equations of Lines

Write the equation of each line **in point-slope form** that goes through each pair of points.

1. $(-2, 1), (7, 8)$
2. $(0, 4), (9, -15)$
3. $(-1, -2), (-3, -13)$

4.2 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

1. $[1, 1.1]$
2. $[1, 1.01]$
3. $[1, 1.001]$
4. $[1, 1.0001]$
5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

6. $[-2, -1]$
7. $[5, 6]$
8. $[0, 1]$
9. $[5, 5.001]$
10. $[5, 5.0001]$
11. $[5, 5.00001]$
12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

13. $[7, 7.001]$
14. $[7, 7.0001]$
15. $[7, 7.00001]$
16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

17. $[1, 1.0001]$

18. $[1, 1.00001]$

19. $[1, 1.000001]$

20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = 6\sqrt{x}$, find the average rate of change of each over the given interval.

21. $[25, 25.1]$

22. $[25, 25.01]$

23. $[25, 25.001]$

24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -7x^3 + 6\sqrt{3x} + 4$ over each interval. Round your answers to 4 decimal places.

25. $[0, 1]$

26. $[10, 11]$

27. $[8, 15]$

Equations of Lines

1. $y - 1 = \frac{7}{9}(x + 2)$ or $y - 8 = \frac{7}{9}(x - 7)$
2. $y - 4 = -\frac{19}{9}(x - 0)$ or $y + 15 = -\frac{19}{9}(x - 9)$
3. $y + 2 = \frac{11}{2}(x + 1)$ or $y + 13 = \frac{11}{2}(x + 3)$

Average Rate of Change

- | | | |
|------------|----------------|-----------------|
| 1. 2.1 | 10. -53.0006 | 19. -0.49999988 |
| 2. 2.01 | 11. -53.00006 | 20. -0.5 |
| 3. 2.001 | 12. -53 | 21. 0.5994 |
| 4. 2.0001 | 13. -42.003 | 22. 0.59999 |
| 5. 2 | 14. -42.0003 | 23. 0.6 |
| 6. 25 | 15. -42.00003 | 24. 0.6 |
| 7. -59 | 16. -42 | 25. 3.3923 |
| 8. 1 | 17. -0.499988 | 26. -2,315.3960 |
| 9. -53.006 | 18. -0.4999988 | 27. -2861.4492 |

Chapter 5

Function Transformations

Write the function for $g(x)$ if it is the result of $f(x)$ after the following ordered sequence of transformations.

1. (1) Vertical stretch by 3
(2) Shift left 1 unit
(3) Reflect across y -axis
2. (1) Horizontal compression by 2
(2) Shift up 1 unit
3. (1) Reflect across x -axis
(2) Vertical compression by 4
(3) Move right 7 units

Write the function $g(x)$ that is a result of the following ordered sequence of transformations to $f(x) = |x|$.

4. (1) Reflect across x -axis
(2) Shift right 3 units
(3) Horizontal stretch by factor of 5
5. (1) Shift down 2 units
(2) Reflect across y -axis
(3) Shift up 1 unit
6. (1) Horizontal compression by factor of 7
(2) Vertical compression by factor of 4
(3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function $g(x)$ after the following ordered sequence of transformations.

7. (1) Shift up 2 units
(2) Horizontal stretch by 5
(3) Shift left 3 units
8. (1) Vertical compression by factor of 3
(2) Reflect across y -axis
(3) Horizontal compression by 5
9. (1) Shift right 8 units
(2) Reflect across x -axis
(3) Horizontal compression by factor of 4

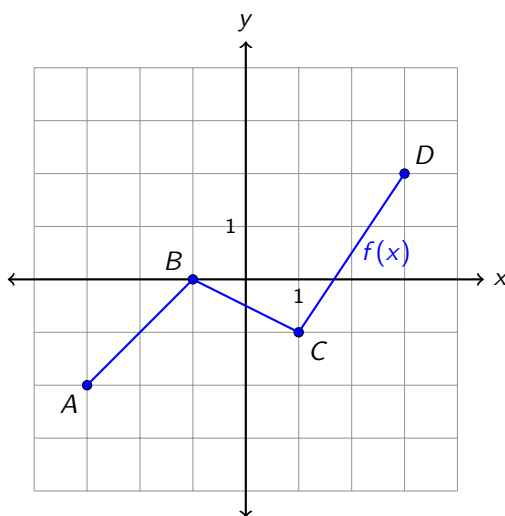
Write the final equation of $g(x)$ if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

10. (1) Shift right 2 units
(2) Horizontal stretch by factor 3
(3) Shift down 2 units
(4) Reflect across x -axis
11. (1) Horizontal stretch by factor 3
(2) Shift left 1 unit
(3) Shift up 2 units
(4) Reflect across y -axis
12. (1) Vertical stretch by factor 5
(2) Horizontal stretch by factor 2
(3) Shift up 3 units
(4) Reflect across x -axis

Find the equation for $g(x)$ if $g(x)$ is found by performing the following *ordered* sequence of transformations to $f(x) = \frac{1}{x}$.

13. (1) Shift left 3 spaces
(2) Reflect across y -axis
(3) Shift down 5 spaces
(4) Vertical stretch by factor of 7
14. (1) Shift up 3 spaces
(2) Reflect across x -axis
(3) Shift right 5 spaces
(4) Horizontal compression by factor of 7

Given the graph of $f(x)$ below, find the new coordinates of each point after the following transformations.



- | | | |
|--------------------------|----------------------------|-------------------------|
| 15. $-2f(x+1)$ | 17. $\frac{1}{2}f(-x-2)+2$ | 19. $-3f(-x+1)+2$ |
| 16. $f(-\frac{1}{2}x)-3$ | 18. $f(2x+2)-1$ | 20. $5f(-\frac{1}{2}x)$ |

FUNCTION TRANSFORMATIONS KEY

1. $g(x) = 3f(-x + 1)$
2. $g(x) = f(2x) + 1$
3. $g(x) = -\frac{1}{4}f(x - 7)$
4. $g(x) = -\left|\frac{1}{5}x - 3\right|$
5. $g(x) = |-x| - 1$
6. $g(x) = \frac{1}{4}|7(x + 9)| = \frac{1}{4}|7x + 63|$
7. $g(x) = \sqrt{\frac{1}{5}(x + 3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$
8. $g(x) = \frac{1}{3}\sqrt{-5x}$
9. $g(x) = -\sqrt{4x - 8}$
10. $g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$
11. $g(x) = \sqrt{\frac{1}{3}(-x + 1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$
12. $g(x) = -\left(5\sqrt{\frac{1}{2}x + 3}\right) = -5\sqrt{\frac{1}{2}x + 3}$
13. $g(x) = \frac{7}{-x+3} - 35$
14. $g(x) = -\frac{1}{7x-5} - 3$
15. $A'(-4, 4), B'(-2, 0), C'(0, 2), D'(2, -4)$
16. $A'(6, -5), B'(2, -3), C'(-2, -4), D'(-6, -1)$
17. $A'(1, 1), B'(-1, 2), C'(-3, 1.5), D'(-5, 3)$
18. $A'(-2.5, -3), B'(-1.5, -1), C'(-0.5, -2), D'(0.5, 1)$
19. $A'(4, 8), B'(2, 2), C'(0, 5), D'(-2, -4)$
20. $A'(6, -10), B'(2, 0), C'(-2, -5), D'(-6, 10)$

Chapter 6

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given $f(x) = x + 5$, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

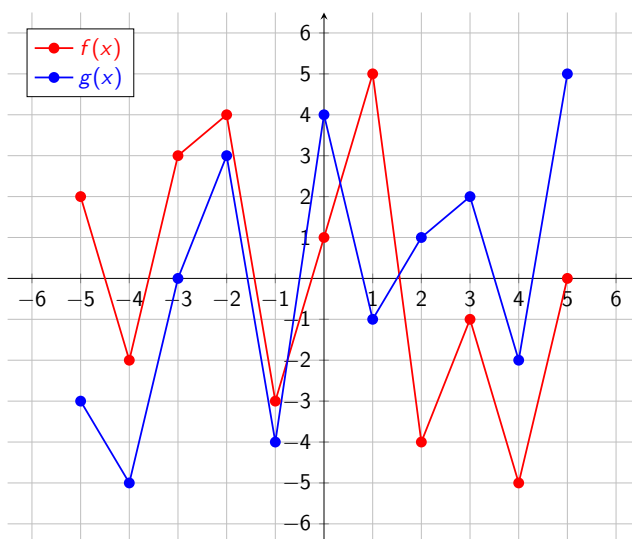
1. $(g - f)(x)$
2. $(fh)(14)$
3. $(f + g)(x)$

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-3	0	-1	3	1	2	4	-4	-2
$g(x)$	3	-1	0	1	4	-2	-4	2	-3

4. $(f + g)(-2)$
5. $(f - g)(0)$
6. $(fg)(1)$
7. $\left(\frac{f}{g}\right)(3)$
8. $(f + f)(-4)$

Find each of the following given the graphs of $f(x)$ (in red) and $g(x)$ (in blue) below:



9. $(f + g)(2)$
10. $(f - g)(1)$
11. $(g - f)(-3)$
12. $(fg)(4)$
13. $\left(\frac{f}{g}\right)(0)$

6.2 Operations with Functions: Domain

Given $f(x) = \sqrt{2x+7}$ and $g(x) = 3x+3$, find the domain of each.

1. $(f+g)(x)$

2. $\left(\frac{f}{g}\right)(x)$

3. $\left(\frac{g}{f}\right)(x)$

6.3 Difference Quotient

Write the difference quotient for each.

1. $f(x) = 2x - 7$

2. $g(x) = x^2 + 4x$

3. $h(x) = -1$

4. $f(x) = \frac{3}{x+2}$

5. $g(x) = \sqrt{3x}$

6. $f(x) = x^2 - 2x + 5$

7. $g(x) = \frac{5}{x}$

8. $f(x) = -2x^2 + 3x - 5$

9. $g(x) = \frac{6}{2x+3}$

10. $h(x) = \sqrt{7x+5}$

11. $f(x) = -x^2 + x$

12. $f(x) = 3x - 1$

13. $f(x) = x^3 + 5x$

Adding, Subtracting, Multiplying, and Dividing Functions

- | | |
|------------------|-------------------|
| 1. $x^2 - x - 6$ | 8. -6 |
| 2. 38 | 9. -3 |
| 3. $x^2 + x + 4$ | 10. 6 |
| 4. -1 | 11. -3 |
| 5. -3 | 12. 10 |
| 6. -4 | 13. $\frac{1}{4}$ |
| 7. -2 | |

Operations with Functions: Domain

- $[-\frac{7}{2}, \infty)$
- $[-\frac{7}{2}, -1) \cup (-1, \infty)$
- $(-\frac{7}{2}, \infty)$

Difference Quotient

- | | |
|---|--|
| 1. 2 | 8. $-4x - 2h + 3$ |
| 2. $2x + h + 4$ | 9. $\frac{-12}{(2x+3)(2x+2h+3)}$ |
| 3. 0 | 10. $\frac{7}{\sqrt{7x+7h+5} + \sqrt{7x+5}}$ |
| 4. $\frac{-3}{(x+2)(x+h+2)}$ | 11. $-2x - h + 1$ |
| 5. $\frac{3}{\sqrt{3x+3h} + \sqrt{3x}}$ | 12. 3 |
| 6. $2x + h - 2$ | 13. $3x^2 + 3xh + h^2 + 5$ |
| 7. $\frac{-5}{x(x+h)}$ | |

Chapter 7

Polynomials and Their Graphs

Determine the end behavior of each.

1. $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$

2. $g(x) = 4x^2 - 16x^6 + 3x$

3. $h(x) = 1 + x^{11} - 4x^8$

4. $f(x) = -x^4 + 3x^2 - 2x + 6$

5. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

6. $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$

7. $g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$

POLYNOMIALS AND THEIR GRAPHS

1. $\lim_{x \rightarrow -\infty} f(x) = \infty$ $\lim_{x \rightarrow \infty} f(x) = -\infty$
2. $\lim_{x \rightarrow -\infty} g(x) = -\infty$, $\lim_{x \rightarrow \infty} g(x) = \infty$
3. $\lim_{x \rightarrow -\infty} h(x) = -\infty$ $\lim_{x \rightarrow \infty} h(x) = \infty$
4. $\lim_{x \rightarrow -\infty} f(x) = -\infty$ $\lim_{x \rightarrow \infty} f(x) = -\infty$
5. $\lim_{x \rightarrow -\infty} g(x) = -\infty$ $\lim_{x \rightarrow \infty} g(x) = \infty$
6. $\lim_{x \rightarrow -\infty} f(x) = \infty$ $\lim_{x \rightarrow \infty} f(x) = -\infty$
7. $\lim_{x \rightarrow -\infty} g(x) = -\infty$ $\lim_{x \rightarrow \infty} g(x) = \infty$

Chapter 8

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

1. $(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$
2. $(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$
3. $(4x^3 - 3x^2 + x + 1) \div (x + 2)$
4. $(5x^4 - x^2 + x - 2) \div (x^2 + 2)$
5. $(10x^3 + 27x^2 + 8x - 11) \div (2x + 3)$
6. $(7x^3 + 23x^2 + 12x + 1) \div (x^2 + 3x + 1)$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

1. $(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$
2. $(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x + 1)$
3. $(x^{23} - 5x^{20} + 17x^8 - 5) \div (x + 2)$
4. $(-7x^{17} + 40x^{15} - 6x^8 + 4x^3) \div (x - 3)$

Dividing Polynomials

1. $4x^2 - 2x + 5$

2. $4y^2 + 8y + 7 - \frac{72}{3y + 5}$

3. $4x^2 - 11x + 23 - \frac{45}{x + 2}$

4. $5x^2 - 11 + \frac{x + 20}{x^2 + 2}$

5. $5x^2 + 6x - 5 + \frac{4}{2x + 3}$

6. $7x + 2 + \frac{-x - 1}{x^2 + 3x + 1}$

Remainder and Factor Theorems

1. 6

2. 13

3. $-13, 627, 141$

4. $-330, 064, 119$

Chapter 9

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

$$1. f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

$$6. f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

$$2. g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$7. g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

$$3. f(x) = \frac{3x}{x + 4}$$

$$8. h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

$$4. g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

$$9. f(x) = \frac{6x^3 - 21x^2 - 51x + 30}{3x^2 + 7x + 2}$$

$$5. h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

$$10. g(x) = \frac{10x^2 - 29x - 21}{10x^3 - 33x^2 - 7x}$$

State the end behavior of each.

$$11. k(x) = \frac{5x^3 - 7x^2 + 8}{-3x^3 + 6x - 4}$$

$$12. m(x) = \frac{2x - 1}{3x^2 + 7x + 1}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain: $x \neq -\frac{1}{2}, 7$; V.A.: $x = 7$; Hole @ $\left(-\frac{1}{2}, -\frac{7}{13}\right)$; H.A.: $y = 1$
2. Domain: $x \neq -3, 4$; V.A.: $x = -3$; Obl. Asymp: $y = 3x + 10$
3. Domain: $x \neq -4$; V.A.: $x = -4$; H.A.: $y = 3$
4. Domain: $x \neq 1$; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$
5. Domain: $x \neq 0, 1$; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$
6. Domain: $x \neq -1, 5$; V.A. $x = -1$; Hole @ $\left(5, \frac{13}{3}\right)$; Obl. Asym $y = 2x - 5$
7. Domain: $x \neq -6, 0, 4$; V.A. $x = -6, x = 0$; Hole @ $\left(4, \frac{21}{40}\right)$; H.A. $y = 0$
8. Domain: $x \neq -6, -\frac{3}{8}$; V.A. $x = -6, x = -\frac{3}{8}$; H.A. $y = \frac{1}{4}$
9. Domain: $x \neq -2, -\frac{1}{3}$; Hole @ $(-2, -21)$; V.A.: $x = -\frac{1}{3}$; S.A. $y = 2x - \frac{35}{3}$
10. Domain: $x \neq -\frac{1}{5}, 0, \frac{7}{2}$; Hole @ $\left(\frac{7}{2}, \frac{82}{259}\right)$; V.A. $x = -\frac{1}{5}$ and $x = 0$; H.A. $y = 0$
11. $\lim_{x \rightarrow -\infty} k(x) = \infty = \lim_{x \rightarrow \infty} k(x) = -\frac{5}{3}$
12. $\lim_{x \rightarrow -\infty} m(x) = \infty = \lim_{x \rightarrow \infty} m(x) = 0$

Chapter 10

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1. $6x^3 - 4x^2 - 10x \geq 0$

2. $x^4 < 9x^2$

3. $3x^3 - 7x^2 - 22x + 8 < 0$

4. $3x^2 - 4x + 1 \leq 0$

5. $12x^4 + 76x^3 + 43x^2 - 346x - 280 \geq 0$

10.2 Domain

State the domain of each. Write your answers using interval notation.

1. $b(x) = \sqrt{21x^2 - 23x - 20}$

10.3 Rational Inequalities

Solve each. Write your answers using interval notation.

1. $\frac{3x - 4}{x + 1} < 0$

2. $\frac{x^2 + 3x + 2}{x - 7} \leq 0$

3. $\frac{x^2 - 4x + 4}{x^2 - 1} \geq 0$

4. $\frac{x + 2}{x - 4} \leq 1$

5. $\frac{x^2 - 7x - 8}{x^2 - 4x - 32} \geq 0$

6. $\frac{4 + 3x}{5 - x} \leq 2$

Polynomial Inequalities

1. $[-1, 0] \cup \left[\frac{5}{3}, \infty\right)$
2. $(-3, 0) \cup (0, 3)$
3. $(-\infty, -2) \cup \left(\frac{1}{3}, 4\right)$
4. $\left[\frac{1}{3}, 1\right]$
5. $(-\infty, -4] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$

Domain

1. $\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$

Rational Inequalities

1. $\left(-1, \frac{4}{3}\right)$
2. $(-\infty, -2] \cup [-1, 7)$
3. $(-\infty, -1) \cup (1, \infty)$
4. $(-\infty, 4)$
5. $(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$
6. $(-\infty, 1.2] \cup (5, \infty)$

Chapter 11

Function Compositions

Given $f(x) = x - 5$, $g(x) = 4 + \sqrt{2x + 1}$, and $h(x) = \frac{3}{x + 7}$, simplify each and state the domain.

1. $(f \circ g)(x)$
2. $(g \circ f)(x)$
3. $h(h(x))$

Find each of the following given the table below.

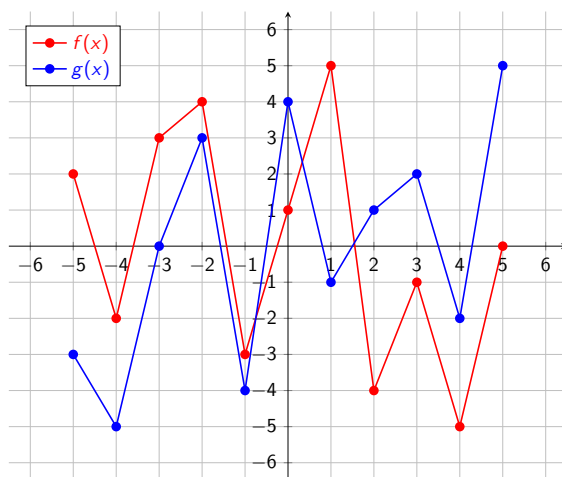
x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-3	0	-1	3	1	2	4	-4	-2
$g(x)$	3	-1	0	1	4	-2	-4	2	-3

4. $(f \circ g)(-1)$
5. $(g \circ g)(0)$
6. $(f \circ f)(2)$
7. $(g \circ g)(-3)$
8. $f(g(0))$

Given $f(x) = \sqrt{3x + 2}$, $g(x) = x^2 - 1$, and $h(x) = 9x - 2$, find each of the following.

9. $(g \circ f)(x)$
10. $f(g(x))$
11. $(h \circ h)(x)$

Find each of the following given the graphs of $f(x)$ (in red) and $g(x)$ (in blue) below:



12. $(f \circ g)(-1)$
13. $(g \circ f)(-4)$
14. $f(g(3))$
15. $g(g(-2))$
16. $(f \circ f)(-5)$

FUNCTION COMPOSITIONS KEY

1. $-1 + \sqrt{2x+1}$ Domain: $\left[-\frac{1}{2}, \infty\right)$
2. $4 + \sqrt{2x-9}$ Domain: $\left[\frac{9}{2}, \infty\right)$
3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup (-7, \infty)$
4. 2
5. -3
6. -2
7. 1
8. -2
9. $3x+1$
10. $\sqrt{3x^2-1}$
11. $81x-20$
12. -2
13. 3
14. -4
15. 2
16. -4

Chapter 12

Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1. $f(x) = \sqrt{-2x+3} + 1$

2. $g(x) = (x+4)^2 - 1, x \leq -4$

3. $h(x) = \frac{9x}{4x-1}$

4. $f(x) = \sqrt{x} - 3$

5. $g(x) = \frac{1}{1-x}$

6. $h(x) = x^2 + 6x + 4, x \leq -3$

7. $f(x) = \sqrt{5x-4}$

8. $g(x) = x^2 - 2x + 3, x \leq 1$

9. $h(x) = \frac{3}{x-1}$

INVERSE FUNCTIONS KEY

1. $f^{-1}(x) = -\frac{1}{2}((x-1)^2 - 3)$

	Domain	Range
$f(x)$	$(-\infty, 1.5]$	$[1, \infty)$
$f^{-1}(x)$	$[1, \infty)$	$(-\infty, 1.5]$

2. $g^{-1}(x) = -\sqrt{x+1} - 4$

	Domain	Range
$g(x)$	$(-\infty, -4]$	$[-1, \infty)$
$g^{-1}(x)$	$[-1, \infty)$	$(-\infty, -4]$

3. $h^{-1}(x) = \frac{-x}{9-4x}$

	Domain	Range
$h(x)$	$(-\infty, 1/4) \cup (1/4, \infty)$	$(\infty, 9/4) \cup (9/4, \infty)$
$h^{-1}(x)$	$(\infty, 9/4) \cup (9/4, \infty)$	$(-\infty, 1/4) \cup (1/4, \infty)$

4. $f^{-1}(x) = (x+3)^2$

	Dom	Ran
$f(x)$	$[0, \infty)$	$[-3, \infty)$
$f^{-1}(x)$	$[-3, \infty)$	$[0, \infty)$

5. $g^{-1}(x) = 1 - \frac{1}{x}$

	Dom	Ran
$g(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$g^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

6. $h^{-1}(x) = -\sqrt{x+5} - 3$

	Dom	Ran
$h(x)$	$(-\infty, -3]$	$[-5, \infty)$
$h^{-1}(x)$	$[-5, \infty)$	$(-\infty, -3]$

7. $f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$

	Dom	Ran
$f(x)$	$\left[\frac{4}{5}, \infty\right)$	$[0, \infty)$
$f^{-1}(x)$	$[0, \infty)$	$\left[\frac{4}{5}, \infty\right)$

8. $g^{-1}(x) = -\sqrt{x-2} + 1$

	Dom	Ran
$g(x)$	$(-\infty, 1]$	$[2, \infty)$
$g^{-1}(x)$	$[2, \infty)$	$(-\infty, 1]$

9. $h^{-1}(x) = \frac{3}{x} + 1$

	Dom	Ran
$h(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$h^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

Chapter 13

Exponential Functions

13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

1. $f(x) = 3 + e^{2x}$

2. $h(x) = 5^{-x}$

3. $h(x) = -\frac{2}{3}e^{x+7} + 1$

4. $f(x) = -7e^x + 4$

5. $g(x) = \frac{1}{3}e^{2x+1} - 5$

6. $h(x) = -\frac{1}{2}e^{-4x} + 1$

EXPONENTIAL FUNCTIONS

1. $\lim_{x \rightarrow -\infty} f(x) = 3$ $\lim_{x \rightarrow \infty} f(x) = \infty$
2. $\lim_{x \rightarrow -\infty} f(x) = \infty$ $\lim_{x \rightarrow \infty} f(x) = 0$
3. $\lim_{x \rightarrow -\infty} h(x) = 1$ $\lim_{x \rightarrow \infty} h(x) = -\infty$
4. $\lim_{x \rightarrow -\infty} f(x) = 4$ $\lim_{x \rightarrow \infty} f(x) = -\infty$
5. $\lim_{x \rightarrow -\infty} f(x) = -5$ $\lim_{x \rightarrow \infty} f(x) = \infty$
6. $\lim_{x \rightarrow -\infty} f(x) = -\infty$ $\lim_{x \rightarrow \infty} f(x) = 1$

Chapter 14

Logarithmic Functions

Write each of the following in exponential or logarithmic form.

1. $\ln(a) = 7$

2. $\log_4(x + 1) = 9$

3. $\log(5x) = 30$

4. $\ln(w) = c$

5. $5^x = 19$

6. $8^{-3} = \frac{1}{512}$

7. $e^{14} = x$

8. $(1.1)^{-t} = 50$

Find the domain of each. Write your answers in interval notation.

9. $b(x) = \log_7(x^2 - 8x + 6)$

10. $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$

11. $f(x) = -7 \ln(x^2 + 9x + 8)$

12. $g(x) = \log(5x^2 + 13x - 6)$

13. $h(x) = 3 \log_2(x^3 + 2x^2 - x - 2)$

14. $c(x) = \ln(4x^2 - 15x - 4)$

State the end behavior of each.

15. $j(x) = 5 \log_3(2x - 5) - 2$

LOGARITHMIC FUNCTIONS KEY

1. $e^7 = a$
2. $4^9 = x + 1$
3. $10^{30} = 5x$
4. $e^c = w$
5. $\log_5(19) = x$
6. $\log_8\left(\frac{1}{512}\right) = -3$
7. $\ln(x) = 14$
8. $\log_{1.1}(50) = -t$
9. $(-\infty, 0.838) \cup (7.162, \infty)$
10. $(-3, -2) \cup (-1, \infty)$
11. $(-\infty, -8) \cup (-1, \infty)$
12. $(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$
13. $(-2, -1) \cup (1, \infty)$
14. $\left(-\infty, -\frac{1}{4}\right) \cup (4, \infty)$
15. $\lim_{x \rightarrow (5/2)^+} j(x) = -\infty \quad \lim_{x \rightarrow \infty} j(x) = \infty$

Chapter 15

Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

1. $\log_b \left(\frac{x^2}{y^8} \right)$

2. $\ln(ez)^3$

3. $\log_5(x) + \log_5(9) - 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

4. $\log_7(10)$

5. $\log_9(x)$

6. $\log_b(c)$

7. $\log_3(10)$

8. $\log_{17}(\pi)$

9. $\log_w(x)$

Suppose that $\log_a(b) = 5$, $\log_a(c) = 12$, and $\log_a(d) = 9$. Evaluate each of the following.

10. $\log_a(bc)$

11. $\log_a(c^3)$

12. $\log_a \left(\frac{d}{c} \right)$

13. $\log_a \left(\frac{bd}{c} \right)$

PROPERTIES OF LOGARITHMS KEY

1. $2 \log_b(x) - 8 \log_b(y)$

2. $3 + 3 \ln(z)$

3. $\log_5 \left(\frac{9x}{w^2} \right)$

4. $\frac{\ln(10)}{\ln(7)}$

5. $\frac{\ln(x)}{\ln(9)}$

6. $\frac{\ln(c)}{\ln(b)}$

7. $\frac{\ln(10)}{\ln(3)}$

8. $\frac{\ln(\pi)}{\ln(17)}$

9. $\frac{\ln(x)}{\ln(w)}$

10. 17

11. 36

12. -3

13. 2

Chapter 16

Exponential Equations

Solve each. Round to 3 decimal places when necessary.

1. $3e^{x-2} = 7$
2. $5^x + 4 > 1$
3. $2^{3x+4} = 32^{x-7}$
4. $5e^{7x} + 10 = 42$
5. $7^{4x+1} \geq 343$
6. $1000e^{0.04x} = 2000$
7. $3(4.1)^{x-2} = 8$
8. $2^{x+1} = 5^{7x-5}$

16.1 Applications

1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?
2. Cadmium-109 has a half-life of about 1.267 years. If 50 mg are initially present, how many years will it take for 16 mg to remain?
3. The half-life of bismuth-207 is about 32.9 years. If 90 mg are initially present, how many years will it take for 75 mg to remain?

EXPONENTIAL EQUATIONS KEY

1. $x \approx 2.847$
2. $(-\infty, \infty)$
3. $x = 19.5$
4. $x \approx 0.265$
5. $\left[\frac{1}{2}, \infty\right)$
6. $x \approx 17.329$
7. $x \approx 2.695$
8. $x \approx 0.827$

Applications

1. Approximately 17,952 years
2. Approximately 2.0828 years
3. Approximately 8.6538 years

Chapter 17

Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

1. $\log_5(x) + x \log_5(x) > 0$

2. $\ln(8 - x^2) = \ln(2 - x)$

3. $\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$

4. $\log_3(2x+1) - \log_3(x-5) = \log_3(x+1)$

5. $\log_4(x+1) + \log_4(x-5) > 2$

6. $\log(x+1) - \log(x-5) = \log(x-3)$

7. $x \log_3(x+2) - \log_3(x+2) = 0$

8. $\log_{1/2}(x+1) > -3$

LOGARITHMIC EQUATIONS AND INEQUALITIES KEY

1. $(1, \infty)$
2. $x = -2$
3. $x \approx 1.571$
4. $x \approx 6.873$
5. $(2, \infty)$
6. $x = 7$
7. $x = \pm 1$
8. $(-1, 7)$

Chapter 18

Sequences

Write the first 4 terms of each sequence.

1. $a_n = 2(-3)^n$

2. $b_n = \frac{n!}{2^n}$

3. $c_{n+1} = 5c_n + 1; c_1 = 2$

4. $d_n = \frac{1}{2}d_{n-1} + n; d_1 = 3$

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round.**

5. $a_n = \{343, 667, 991, 1315, \dots\}$; Find the 582nd term.

6. $b_n = \{300, 240, 192, 153.6, \dots\}$; Find the 711th term.

7. $c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots\right\}$; Find the 8,675,309th term.

Given each sequence, find the first 4 digits **after the decimal point** in the **scientific notation** version of each term.

8. $a_n = 17, 33, 49, 65, \dots$; $a_{21,972}$

9. $b_n = 25, 36, 49, 64, 81, \dots$ $b_{413,401}$

10. $c_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots$ c_{152}

SEQUENCES KEY

1. $-6, 18, -54, 162$
2. $\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2}$
3. $2, 11, 56, 281$
4. $3, \frac{7}{2}, \frac{19}{4}, \frac{51}{8}$
5. $188,587$
6. 6882
7. 7634
8. $5155 (3.51553 \times 10^5)$
9. $7090 (1.7090369403 \times 10^{11})$
10. $1677 (1.1677487203 \times 10^{-46})$

Chapter 19

Series

Find the sum of each, if possible.

1. $\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^i$

2. $\sum_{i=0}^{\infty} 3 \left(-\frac{2}{3}\right)^i$

3. $\sum_{k=1}^{\infty} -2 \left(\frac{1}{3}\right)^k$

4. $\sum_{j=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^j$

5. $\sum_{i=0}^{\infty} 1.2(0.8)^i$

6. $\sum_{i=1}^{\infty} 1.2(0.8)^i$

7. $\sum_{i=0}^{\infty} 0.8(1.2)^i$

8. $\sum_{k=1}^{\infty} \frac{2}{3^k}$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9. $9 + 13 + 17 + 21 + \cdots + 1565$

10. $-3 + 6 - 12 + 24 - 48 + \cdots + 50,331,648$

11. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{981}$

12. $2 + 4 + 6 + 8 + 10 + \cdots + 38,214$

13. $3 + 7 + 11 + 15 + \cdots + 11,491$

14. $\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \cdots + \frac{742}{743}$

SERIES KEY

1. $\frac{1}{4}$
2. $\frac{9}{5}$
3. -1
4. Diverges
5. 6
6. 4.8
7. Diverges
8. 2
9. 306,930
10. $-33,554,433$
11. 7.4663
12. 365,096,556
13. 16,511,131
14. 733.8947

Chapter 20

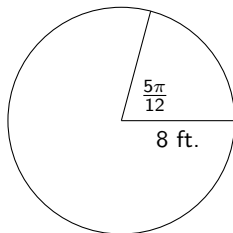
Angles and Radian Measure

Sketch each of the following. Then find a coterminal between 0 and 360° (or 0 and 2π radians) for each.

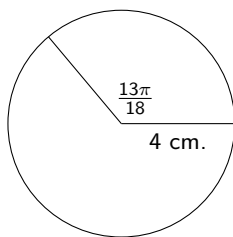
1. $-\frac{3\pi}{4}$
2. 900°
3. $\frac{27\pi}{10}$
4. -125°

Find the arc length and sector area formed by the central angle of each. Exact answers only.

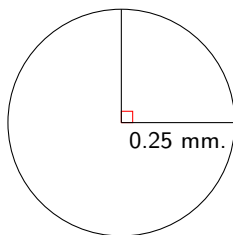
5.



6.



7.

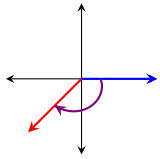


A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

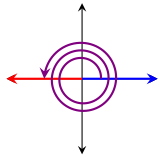
8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

ANGLES AND RADIAN MEASURE KEY

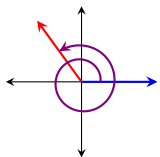
1. $\frac{5\pi}{4}$



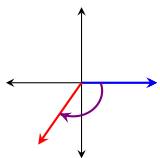
2. 180°



3. $\frac{7\pi}{10}$



4. 235°



5. $s = \frac{10\pi}{3}$ ft.; $A = \frac{40\pi}{3}$ sq.ft.

6. $s = \frac{26\pi}{9}$ cm.; $A = \frac{52\pi}{9}$ sq.cm.

7. $s = \frac{\pi}{8}$ mm.; $A = \frac{\pi}{64}$ sq.mm.

8. 26.18 rad/sec

9. 8.73 ft/sec

Chapter 21

Trig Functions of Any Angle

Find the exact value of each of the six trig functions of θ if P is a point on the terminal side of θ .

1. $P(-2, 3)$
2. $P(0, -4)$
3. $P(-2\sqrt{3}, 2)$
4. $P(-3, 5)$
5. $P(-2, 1)$
6. $P(-4, -7)$

Find the exact values of the 6 trig functions of the following angles.

7. $\theta = \frac{-17\pi}{4}$
8. $\theta = \frac{21\pi}{2}$
9. $\theta = 24\pi$

TRIG FUNCTIONS OF ANY ANGLE KEY

$$1. \sin \theta = \frac{3\sqrt{13}}{13}, \cos \theta = \frac{-2\sqrt{13}}{13}, \tan \theta = -\frac{3}{2}, \csc \theta = \frac{\sqrt{13}}{3}, \sec \theta = -\frac{\sqrt{13}}{2}, \cot \theta = -\frac{2}{3}$$

$$2. \sin \theta = -1, \cos \theta = 0, \tan \theta = \text{undef.}, \csc \theta = -1, \sec \theta = \text{undef.}, \cot \theta = 0$$

$$3. \sin \theta = \frac{1}{2}, \cos \theta = -\frac{\sqrt{3}}{2}, \tan \theta = -\frac{\sqrt{3}}{3}, \csc \theta = 2, \sec \theta = -\frac{2\sqrt{3}}{3}, \cot \theta = -\sqrt{3}$$

$$4. \sin \theta = \frac{5\sqrt{34}}{34}, \cos \theta = -\frac{3\sqrt{34}}{34}, \tan \theta = -\frac{5}{3}, \csc \theta = \frac{\sqrt{34}}{5}, \sec \theta = -\frac{\sqrt{34}}{3}, \cot \theta = -\frac{3}{5}$$

$$5. \sin \theta = \frac{\sqrt{5}}{5}, \cos \theta = -\frac{2\sqrt{5}}{5}, \tan \theta = -\frac{1}{2}, \csc \theta = \sqrt{5}, \sec \theta = -\frac{\sqrt{5}}{2}, \cot \theta = -2$$

$$6. \sin \theta = -\frac{7\sqrt{65}}{65}, \cos \theta = -\frac{4\sqrt{65}}{65}, \tan \theta = \frac{7}{4}, \csc \theta = -\frac{\sqrt{65}}{7}, \sec \theta = -\frac{\sqrt{65}}{4}, \cot \theta = \frac{4}{7}$$

$$7. \sin \theta = -\frac{\sqrt{2}}{2}, \cos \theta = \frac{\sqrt{2}}{2}, \tan \theta = -1, \csc \theta = -\sqrt{2}, \sec \theta = \sqrt{2}, \cot \theta = -1$$

$$8. \sin \theta = 1, \cos \theta = 0, \tan \theta = \text{undefined}, \csc \theta = 1, \sec \theta = \text{undefined}, \cot \theta = 0$$

$$9. \sin \theta = 0, \cos \theta = 1, \tan \theta = 0, \csc \theta = \text{undefined}, \sec \theta = 1, \cot \theta = \text{undefined}$$

Chapter 22

Graphs of Sine and Cosine Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1. $f(x) = -2 \sin\left(3x - \frac{\pi}{4}\right) + 1$

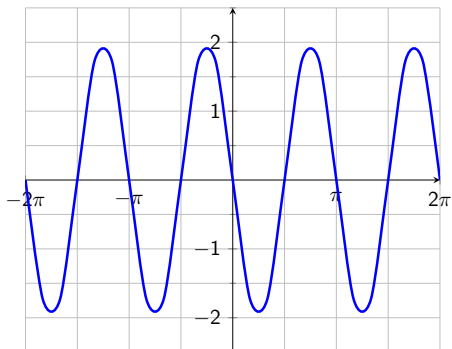
2. $g(x) = \frac{1}{3} \cos\left(\frac{1}{2}x + 2\right)$

3. $f(x) = 2 \sin\left(x - \frac{\pi}{3}\right) + 7$

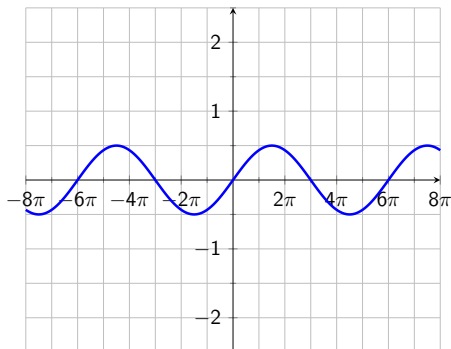
4. $f(x) = -4 \cos\left(\frac{2}{3}x - \frac{2\pi}{3}\right)$

Write the equation of each of the following in the form $y = a \sin(bx)$.

5.



6.



ANSWERS

1. Amp = 2, Per = $\frac{2\pi}{3}$, P.S. = $\frac{\pi}{12} \rightarrow$, V.S. = 1 \uparrow
2. Amp = $\frac{1}{3}$, Per = 4π , P.S. = $4 \leftarrow$, V.S. = None
3. Amp = 2, Period = 2π , P.S. = $\frac{\pi}{3}$ right, V.S. = 7 up
4. Amp = 4, Period = 3π , P.S. = π right, V.S. = 0 (or none)
5. $y = -2\sin(2x)$
6. $y = \frac{1}{2}\sin\left(\frac{1}{3}x\right)$

Chapter 23

Graphs of Other Trig Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1. $h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$

2. $f(x) = 3 \tan\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$

GRAPHS OF OTHER TRIG FUNCTIONS KEY

1. Amp = n/a , Per = $\frac{4\pi}{3}$, P.S. = $\frac{\pi}{9} \leftarrow$, V.S. = $8 \downarrow$

2. Amp = n/a , Period = $\frac{\pi}{2}$, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down

Chapter 24

Inverse Trig Functions

State the exact, simplified value of each or write as an expression of x .

1. $\tan^{-1}(-\sqrt{3})$

2. $\sec\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$

3. $\cot(\sec^{-1}(x))$

4. $\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$

5. $\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$

6. $\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$

7. $\cos(\tan^{-1}(7x))$

8. $\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$

INVERSE TRIG FUNCTIONS KEY

$$1. -\frac{\pi}{3}$$

$$2. \frac{5\sqrt{21}}{21}$$

$$3. \frac{1}{\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}}{x^2-1}$$

$$4. \frac{\sqrt{16-9x^2}}{4}$$

$$5. -\frac{3\sqrt{5}}{2}$$

$$6. \frac{13\sqrt{22}}{44}$$

$$7. \frac{\sqrt{49x^2+1}}{49x^2+1}$$

$$8. \frac{\sqrt{64-x^2}}{x}$$

Chapter 25

Trig Equations and Inequalities

Solve each in the interval $[0, 2\pi)$. Write your answers to inequalities using interval notation.

1. $\tan(6x) = 1$

2. $\cot(2x) = -\frac{\sqrt{3}}{3}$

3. $\sin^2(x) = \frac{3}{4}$

4. $\sin(2x) = \cos(x)$

5. $\sin(2x) \geq \sin(x)$

6. $\cos(2x) < 0$

7. $2 \sin\left(x - \frac{\pi}{3}\right) = -1$

8. $3 \tan\left(-2x + \frac{\pi}{2}\right) = \sqrt{3}$

9. $\sin^2(x) < \frac{1}{2}$

TRIG EQUATIONS AND INEQUALITIES KEY

1. $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{3\pi}{8}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{7\pi}{8}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{11\pi}{8}, \frac{37\pi}{24}, \frac{41\pi}{24}, \frac{15\pi}{8}$
2. $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$
3. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
4. $\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
5. $\left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$
6. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
7. $x = \frac{\pi}{6}, \frac{3\pi}{2}$
8. $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
9. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$

Chapter 26

Law of Sines and Cosines

Solve each of the following. Round your answers to 1 decimal place.

1. $m\angle B = 37.8^\circ$, $a = 15$, $c = 21.1$
2. $m\angle A = 41.9^\circ$, $m\angle C = 59.2^\circ$, $a = 10.2$
3. $a = 14$, $b = 19.6$, $c = 13.1$

LAW OF SINES AND COSINES KEY

1. $b \approx 13.0$, $m\angle A \approx 44.8^\circ$, $m\angle C \approx 97.4^\circ$
2. $m\angle B = 78.9^\circ$, $b \approx 15.0$, $c \approx 13.1$
3. $m\angle A \approx 45.5^\circ$, $m\angle B \approx 92.6^\circ$, $m\angle C \approx 41.9^\circ$

Chapter 27

Area of Triangles

Find the area of each. Round your answers to 1 decimal place.

1. $m\angle B = 37.8^\circ$, $a = 15$, $c = 21.1$

2. $m\angle A = 41.9^\circ$, $m\angle C = 59.2^\circ$, $a = 10.2$

3. $a = 14$, $b = 19.6$, $c = 13.1$

AREA OF TRIANGLES KEY

1. Approximately 97.0 sq. units
2. Approximately 65.7 sq. units
3. Approximately 91.6 sq. units

Chapter 28

Polar Coordinates

Convert each to exact rectangular coordinates.

1. $\left(3, \frac{5\pi}{6}\right)$

2. $\left(-2, -\frac{\pi}{4}\right)$

3. $\left(\frac{5}{2}, 240^\circ\right)$

Convert each to exact polar coordinates. Use $r > 0$ and $0 \leq \theta < 2\pi$.

4. $(-2\sqrt{3}, -2)$

5. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

6. $(7\sqrt{2}, -7\sqrt{2})$

Convert each to either rectangular or polar coordinates.

7. $2x + 5y = 9$

8. $3y = 1$

9. $-5x - 8y = -10$

10. $r = 8$

11. $r = 4 \sec(\theta)$

12. $\theta = -\frac{\pi}{6}$

POLAR COORDINATES KEY

$$1. \left(-\frac{3\sqrt{3}}{2}, \frac{3}{2} \right)$$

$$2. (-\sqrt{2}, \sqrt{2})$$

$$3. \left(\frac{5}{4}, \frac{5\sqrt{3}}{4} \right)$$

$$4. \left(4, \frac{7\pi}{6} \right)$$

$$5. \left(\frac{\sqrt{2}}{2}, \frac{3\pi}{4} \right)$$

$$6. \left(14, \frac{7\pi}{4} \right)$$

$$7. r = \frac{9}{2 \cos \theta + 5 \sin \theta}$$

$$8. r = \frac{1}{3} \csc \theta$$

$$9. r = \frac{10}{5 \cos \theta + 8 \sin \theta}$$

$$10. x^2 + y^2 = 64$$

$$11. x = 4$$

$$12. y = -\frac{\sqrt{3}}{3}x$$

Chapter 29

Vectors

[Vectors] Given $\vec{v} = 3\mathbf{i} - 5\mathbf{j}$ and $\vec{w} = \langle -2, 1 \rangle$, find each. Exact and simplified answers only.

1. $\vec{v} + \vec{w}$

2. $-4\vec{w}$

3. $|\mathbf{v}|$

4. \hat{w}

VECTORS KEY

1. $\mathbf{i} - 4\mathbf{j}$

2. $\langle 8, -4 \rangle$

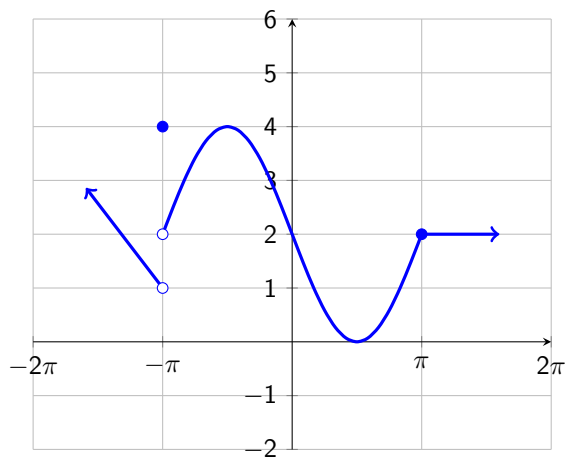
3. $\sqrt{34}$

4. $\left\langle -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$

Chapter 30

Numerical and Graphical Limits

Solve using the graph of $f(x)$ below.



1. $\lim_{x \rightarrow -\pi^-} f(x)$

3. $\lim_{x \rightarrow -\pi} f(x)$

5. $\lim_{x \rightarrow \pi^-} f(x)$

7. $\lim_{x \rightarrow \pi} f(x)$

2. $\lim_{x \rightarrow -\pi^+} f(x)$

4. $f(-\pi)$

6. $\lim_{x \rightarrow \pi^+} f(x)$

8. $f(\pi)$

NUMERICAL AND GRAPHICAL LIMITS KEY

1. 1
2. 2
3. Does not exist
4. 4
5. 2
6. 2
7. 2
8. 2

Appendix A

Factoring

Factor each of the following completely.

1. $x^2 + 2x - 15$

2. $x^2 - 8x + 12$

3. $x^2 + 15x + 56$

4. $5x^2 + 19x - 4$

5. $4x^2 - 5x - 6$

6. $9x^2 - 400$

7. $5x^2 - 7x - 6$

8. $9x^2 - 54x + 45$

9. $3x^3 + 12x^2 + 9x$

10. $9y^2 - 16$

11. $4x^2 - 28x + 49$

12. $14x^2 + 11xy - 15y^2$

13. $6x^2 - 48x - 120$

14. $9x^4 - 54x^3 + 45x^2$

15. $16y^2 - 40y + 25$

16. $30x^2 + xy - y^2$

17. $8w^2 + 33w + 4$

18. $3p^2 + 22p - 16$

19. $18x^2 - 27x + 4$

20. $14a^2 + 15a - 9$

21. $4x^2 - 4x - 24$

22. $18t^2 - 9t - 5$

23. $6a^2 + 23a + 21$

24. $25x^2 - 1$

FACTORIZING KEY

1. $(x + 5)(x - 3)$

2. $(x - 6)(x - 2)$

3. $(x + 7)(x + 8)$

4. $(5x - 1)(x + 4)$

5. $(4x + 3)(x - 2)$

6. $(3x + 20)(3x - 20)$

7. $(5x + 3)(x - 2)$

8. $9(x - 5)(x - 1)$

9. $3x(x + 3)(x + 1)$

10. $(3y + 4)(3y - 4)$

11. $(2x - 7)^2$

12. $(7x - 5y)(2x + 3y)$

13. $6(x - 10)(x + 2)$

14. $9x^2(x - 1)(x - 5)$

15. $(4y - 5)^2$

16. $(6x - y)(5x + y)$

17. $(8w + 1)(w + 4)$

18. $(3p - 2)(p + 8)$

19. $(6x - 1)(3x - 4)$

20. $(7a - 3)(2a + 3)$

21. $4(x - 3)(x + 2)$

22. $(6t - 5)(3t + 1)$

23. $(2a + 3)(3a + 7)$

24. $(5x + 1)(5x - 1)$

Appendix B

Complex Fractions

Simplify each as much as possible.

$$1. \frac{5 + \frac{3}{x}}{x - \frac{1}{2}}$$

$$2. \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

$$3. \frac{3}{2 - \frac{x}{x-1}}$$

$$4. \frac{1 + \frac{3}{x}}{\frac{2}{x} + 7}$$

$$5. \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$6. \frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$7. \frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2-2x}}$$

$$8. \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

$$9. \frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$10. \frac{\frac{5}{x} - 5x}{x - 1}$$

$$11. \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$12. \frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

$$13. \frac{\frac{1}{x-a} + \frac{1}{a}}{x}$$

$$14. \frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

$$15. \frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

COMPLEX FRACTIONS KEY

$$1. \frac{2(5x+3)}{x(2x-1)}$$

$$2. \frac{1}{x^2-2x+4}$$

$$3. \frac{3(x-1)}{x-2}$$

$$4. \frac{x+3}{2+7x}$$

$$5. \frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \frac{-4x-1}{2}$$

$$7. \frac{8x-10}{7}$$

$$8. -\frac{1}{7x}$$

$$9. \frac{2x+1}{5x(x+1)}$$

$$10. \frac{-5x-5}{x}$$

$$11. \frac{-1}{2x+4}$$

$$12. \frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \frac{1}{a(x-a)}$$

$$14. \frac{-2x-2}{5x^2-16x+3}$$

$$15. \frac{x+4}{4x-8}$$