# Honors PreCalculus

# **Extra Practice Problems**

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# **Basic Set Theory and Interval Notation**

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2.  $\{x | x \le 1\}$
- 3. \_3
- 4.  $\{x | x \neq 4, 11\}$
- 5. 2.4 7.7
- 6.  $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7.  $\{x | x \ge 2\}$
- 8.  $\{x|x<-8\}$
- 9.  $\{x | x \neq 3\}$
- 10.  $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

- 11.

### 1.1 Answer Key

1.  $\{x | -5 < x \le 8\}$ 



2.  $(-\infty, 1]$ 



- 3.  $[-3, \infty)$   $\{x | x \ge -3\}$
- $4. \ (-\infty, 4) \cup (4, 11) \cup (11, \infty)$   $\longleftrightarrow \qquad \qquad \downarrow$   $4 \qquad \qquad \downarrow$  11
- 5. [2.4, 7.7)  $\{x | 2.4 \le x < 7.7\}$
- 6.  $\{x|x > 9\}$

9

7.  $[2,\infty)$ 



8.  $(-\infty, -8)$ 



9.  $(-\infty,3)\cup(3,\infty)$ 



10.  $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$ 

$$\begin{array}{ccc} & & & \bigcirc & & \bigcirc \\ & -2 & & 5 & \end{array}$$

- 11.  $(-\infty, -8]$   $\{x | x \le -8\}$
- 12.  $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$   $\{x | x \neq 7, 12\}$
- 13.  $(-\infty, -3) \cup (-3, 2) \cup (2, 5) \cup (5, \infty)$   $\{x | x \neq -3, 2, 5\}$

# **Functions and Their Graphs**

#### 2.1 Evaluating Functions

Given  $f(x) = -3x^2 + 4x$  and  $g(x) = \frac{1}{x} - 5$ , evaluate each.

1. f(5)

2. f(-2)

3. f(0)

4. g(1)

5. g(-5)

6. g(1/4)

7. f(-x)

8. g(-x)

9. f(2x)

10. g(2x)

11. f(x-3)

12. g(x-3)

13.  $f(\frac{1}{3}x)$ 

14.  $g\left(\frac{1}{2}x\right)$ 

15. f(2x+1)

16. g(2x+1)

17. f(-x+7)

18. g(-x+7)

#### 2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1.  $f(x) = -8x^2 - 7x + 1$ 

2.  $g(x) = \sqrt{5x + 12} - 2$ 

4. f(x) = -5x + 4

5.  $f(x) = x^2 + 2$ 

6.  $f(x) = \frac{2x+1}{3x-5}$ 

3.  $h(x) = \frac{x+2}{9x-7}$ 

7.  $f(x) = \sqrt{3x - 12}$ 

8.  $f(x) = \frac{x}{x^2 - 16}$ 

9.  $f(x) = \frac{x+4}{x^3-4x}$ 

10.  $f(x) = \frac{x}{\sqrt{x-4}}$ 

11.  $f(x) = \frac{x^2+1}{2x^2+8}$ 

12.  $f(x) = -\frac{x+7}{x^2-5x-6}$ 

13.  $g(x) = \sqrt{2x+3}$ 

14.  $h(x) = \sqrt[3]{2x+3}$ 

15.  $f(x) = -\frac{7x-10}{x^2+3x+2}$ 

16.  $g(x) = \sqrt{-9x + 8}$ 

17.  $h(x) = -\sqrt[3]{4x+1}$ 

18.  $f(x) = \sqrt[3]{8x+1}$ 

19.  $g(x) = \frac{x^2-1}{\sqrt{x+3}}$ 

20.  $h(x) = \frac{3}{9 + \frac{4}{x+7}}$ 

21.  $f(x) = \frac{x+1}{\sqrt{10x+8}}$ 

22.  $g(x) = \frac{5}{1 + \frac{3}{x+2}}$ 

23.  $i(x) = \frac{7}{3 - \frac{4}{x+1}}$ 

24.  $n(x) = \frac{7x+14}{\sqrt{2x-1}}$ 

25.  $a(x) = \frac{\frac{x}{x-2}}{\frac{3}{x-2}+6}$ 

26.  $d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$ 

#### 2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. f(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4\\ \sqrt{2x + 7} & -4 < x < 0\\ |-x - 1| & x \ge 0 \end{cases}$$

- 6. f(3)
- 7. f(-2)
- 8. f(0)
- 9. f(-5)

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \le -3\\ \sqrt{-4x + 1} & \text{if } -3 < x \le 0\\ \frac{5x^2}{x + 7} & \text{if } x > 0 \end{cases}$$

- 10. f(7)
- 11. f(-3)
- 12. f(1)
- 13. f(0)
- 14. f(-1)
- 15. f(-3/2)

### 2.4 Answer Key

### **Evaluating Functions**

1. -55

4. -4

7.  $-3x^2 - 4x$ 

10.  $\frac{1-10x}{2x}$ 

13.  $-\frac{1}{3}x^2 + \frac{4}{3}x$ 

16.  $-\frac{10x+4}{2x+1}$ 

2. -20

5. -5.2

8.  $-\frac{1}{x} - 5 = \frac{-1-5x}{x}$ 

11.  $-3x^2 + 22x - 39$ 

14.  $\frac{3-5x}{x}$ 

17.  $-3x^2 + 38x - 119$ 

3. 0

6. -1

9.  $-12x^2 + 8x$ 

12.  $\frac{16-5x}{x-3}$ 

15.  $-12x^2 - 4x + 1$ 

18.  $\frac{5x-34}{-x+7}$ 

### **Domain of Functions**

1.  $(-\infty, \infty)$ 

4.  $(-\infty, \infty)$ 

7.  $[4, \infty)$ 

10.  $(4, \infty)$ 

13.  $\left[-\frac{3}{2},\infty\right)$ 

16.  $\left(-\infty, \frac{8}{9}\right]$ 

19.  $(-3, \infty)$ 

25.  $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, 2) \cup (2, \infty)$ 

2.  $\left[\frac{-12}{5}, \infty\right)$ 

5.  $(-\infty, \infty)$ 

8.  $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$ 

11.  $(-\infty, \infty)$ 

14.  $(-\infty, \infty)$ 

17.  $(-\infty, \infty)$ 

20.  $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup \left(-7, \infty\right)$ 

 $22. \ (\infty, -5) \cup (-5, -2) \cup (-2, \infty)$   $23. \ (-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$   $24. \ (\frac{1}{2}, \infty)$ 

26.  $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$ 

3.  $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$ 

6.  $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$ 

9.  $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$ 

12.  $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$ 

15.  $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$ 

18.  $(-\infty, \infty)$ 

21.  $\left(-\frac{4}{5}, \infty\right)$ 

#### **Piecewise Functions**

1.  $\sqrt{15} \approx 3.873$ 

4. 6.4

7.  $\sqrt{3} \approx 1.732$ 

10. 17.5

13. 1

2. 7

5. 7.1

8. 1

11. 4

14.  $\sqrt{5} \approx 2.236$ 

3. 6.6

6. 4

9. 18

12.  $\frac{5}{8}$ 

15.  $\sqrt{7} \approx 2.646$ 

# **Properties of Functions**

#### 3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1. 
$$f(x) = x^2 - 3x^2 + 5$$

2. 
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3. 
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4. 
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

5. 
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

6. 
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

#### 3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1. 
$$f(x) = x^2 - 3x^2 + 5$$

2. 
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3. 
$$f(x) = x^3 + 2x^2 - 4x - 8$$

4. 
$$g(x) = x^4 - 2x^2 + 1$$

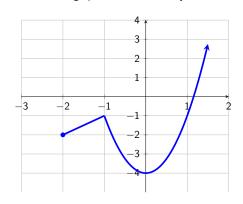
5. 
$$h(x) = \sqrt{x+1} - 2$$

6. 
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

7. 
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

#### 3.3 Miscellaneous

Use the graph of y = f(x) below to answer the following questions. Write your answers using interval notation.



- 1. Domain of f
- 3. Relative Minimum
- 5. f(1)
- 7. Increasing Interval(s)
- 9. Absolute Maximum

- 2. Range of f
- 4. Relative Maximum
- 6. f(0)
- 8. Decreasing Interval(s)
- 10. Absolute Minimum

Find each of the following given  $f(x) = -2x^3 + 6x^2 - 5x + 1$ . Round to 3 decimal places and use interval notation when applicable.

11. f(7)

12. f(-2)

- 13. Rel. Max
- 14. Rel. Min

- 15. Global Max
- 16. Global Min
- 17. Increasing Interval(s)
- 18. Decreasing Interval(s)

Use the graph of f(x) to answer each.



- 19. Relative maxima of f(x)
- 20. Relative minima of f(x)
- 21. Absolute maxima of f(x)

- 22. Absolute minima of f(x)
- 23. Intervals where f is increasing
- 24. Intervals where f is decreasing

25. Zeros of f

Given the labeled points A through G on the graph of f(x) below, find each of the following.



- 26. Increasing interval(s)
- 28. Relative max
- 30. Global max
- 32. Zeros of f

- 27. Decreasing interval(s)
- 29. Relative min
- 31. Global min
- 33. Number of solutions to f(x) = 1

Answer each of the following about the function f(x) below.



- 34. Domain of f
- 36. Relative maxima
- 38. Absolute maximum
- 40. Increasing intervals
- 42. Zeros of f(x)

- 35. Range of f
- 37. Relative minima
- 39. Absolute minimum
- 41. Decreasing intervals
- 43. Number of solutions to f(x) = 2

#### 3.4 Answer Key

#### Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. Rel Max (-1.366, 10.848) and (1, 6); Rel Min (0.366, 5.652)
- 4. Rel Max (-1.716, 11.598); Rel Min (1.132, -3.929)
- 5. Rel Max: (1.095, 12.096); Rel Min (-0.761, -0.680)
- 6. Rel Max: (1.366, 0.348); Rel Min: (-0.366, -4.848) and (2, 0)
- 7. About 2.16 hours

#### Increasing, Decreasing, and Constant Intervals

- 1. Increasing:  $(-\infty, 0)$  Decreasing:  $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing:  $(-\infty, -1.158) \cup (2.158, \infty)$
- 3. Inc:  $(-\infty, -2) \cup (\frac{2}{3}, \infty)$  Dec:  $(-2, \frac{2}{3})$
- 4. Inc;  $(-1,0) \cup (1,\infty)$  Dec:  $(-\infty,-1) \cup (0,1)$
- 5. Inc:  $(-1, \infty)$  No intervals where it is decreasing
- 6. Inc: (-0.761, 1.095); Dec:  $(-\infty, -0.761) \cup (1.095, \infty)$
- 7. Inc:  $(-0.366, 1.366) \cup (2, \infty)$ ; Dec:  $(-\infty, -0.366) \cup (1.366, 2)$ ;

#### Miscellaneous

- 1.  $[-2, \infty)$
- 4. (-1, -1)
- 7.  $(-2, -1) \cup (0, \infty)$
- 10. None
- 13. (1.408, 0.272)
- 16. None
- 19. (b, f(b)) and (m, f(m))
- 22. None
- 25. x = a, x = c, x = n
- 28.  $\it B$  and  $\it F$
- 31. None
- 34.  $(-\infty, 2)$
- 37. (0, 1)
- 40.  $(-\infty, -1.2) \cup (0, 1.3)$
- 43. 4

- 2.  $[-4, \infty)$
- 5. -1
- 8. (-1,0)
- 11. -426
- 14. (0.592, -0.272)
- 17. (0.592, 1.408)
- 20. (c, f(c))
- 23.  $(-\infty, b) \cup (c, m)$
- 26.  $(\infty, B) \cup (D, F)$
- 29. D
- 32. A, C, E, G
- 35.  $(-\infty, -2.5) \cup (-2.5, 3.6]$
- 38. (1.3, 3.6)
- 41.  $(-1.2,0) \cup (1.3,2)$

- 3. (0, -4)
- 6. -4)
- 9. (0, -4)
- 12. 51
- 15. None
- 18.  $(-\infty, 0.592) \cup (1.408, \infty)$
- 21. (m, f(m))
- 24.  $(b, c) \cup (m, p)$
- 27. (B, D) ∪ (F, ∞)
- 30. B
- 33. 2
- 36. (-1.2, 3) and (1.3, 3.6)
- 39. Does not exist
- 42. (-1.8,0) and (1.9,0)

# **Linear Functions and Slope**

#### 4.1 Equations of Lines

Write the equation of each line in point-slope form that goes through each pair of points.

- 1. (-2,1), (7,8)
- 2. (0,4), (9,-15)
- 3. (-1, -2), (-3, -13)

#### 4.2 Average Rate of Change

For the function  $f(x) = x^2$ , compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function  $f(x) = -6x^2 + 7x + 4$  over each specified interval.

- 6. [-2, -1]
- 7. [5, 6]
- 8. [0, 1]
- 9. [5, 5.001]
- 10. [5, 5.0001]
- 11. [5, 5.00001]
- 12. What value are your last 3 answers getting closer to?

For the function  $f(x) = -3x^2 + 5$ , determine the average rate of change of each over the given interval.

- 13. [7, 7.001]
- 14. [7, 7.0001]
- 15. [7, 7.00001]
- 16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = \sqrt{x}$ , find the average rate of change of each over the given interval.

- 17. [1, 1.0001]
- 18. [1, 1.00001]
- 19. [1, 1.000001]
- 20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = 6\sqrt{x}$ , find the average rate of change of each over the given interval.

- 21. [25, 25.1]
- 22. [25, 25.01]
- 23. [25, 25.001]
- 24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function  $f(x) = -7x^3 + 6\sqrt{3x} + 4$  over each interval. Round your answers to 4 decimal places.

- 25. [0, 1]
- 26. [10, 11]
- 27. [8, 15]

### 4.3 Answer Key

### **Equations of Lines**

- 1.  $y-1=\frac{7}{9}(x+2)$  or  $y-8=\frac{7}{9}(x-7)$
- 2.  $y-4=-\frac{19}{9}(x-0)$  or  $y+15=-\frac{19}{9}(x-9)$
- 3.  $y + 2 = \frac{11}{2}(x+1)$  or  $y + 13 = \frac{11}{2}(x+3)$

### **Average Rate of Change**

- 1. 2.1
- 4. 2.0001
- 7. -59
- 10. -53.0006
- 13. -42.003
- 16. -42
- 19. -0.49999988
- 22. 0.59999
- 25. 3.3923

- 2. 2.01
- 5. 2
- 8. 1
- 11. -53.00006
- 14. -42.0003
- 17. -0.499988
- 20. -0.5
- 23. 0.6
- 26. -2,315.3960

- 3. 2.001
- 6. 25
- 9. -53.006
- 12. -53
- 15. -42.00003
- 18. -0.4999988
- 21. 0.5994
- 24. 0.6
- 27. -2861.4492

### **Function Transformations**

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
  - (2) Shift left 1 unit
  - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
  - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
  - (2) Vertical compression by 4
  - (3) Move right 7 units

Write the function g(x) that is a result of the following ordered sequence of transformations to f(x) = |x|.

- 4. (1) Reflect across x-axis
  - (2) Shift right 3 units
  - (3) Horizontal stretch by factor of 5
- 5. (1) Shift down 2 units
  - (2) Reflect across y-axis
  - (3) Shift up 1 unit
- 6. (1) Horizontal compression by factor of 7
  - (2) Vertical compression by factor of 4
  - (3) Shift left 9 units

Given  $f(x) = \sqrt{x}$ , determine the resulting function g(x) after the following ordered sequence of transformations.

- 7. (1) Shift up 2 units
  - (2) Horizontal stretch by 5
  - (3) Shift left 3 units
- 8. (1) Vertical compression by factor of 3
  - (2) Reflect across y-axis
  - (3) Horizontal compression by 5
- 9. (1) Shift right 8 units
  - (2) Reflect across x-axis
  - (3) Horizontal compression by factor of 4

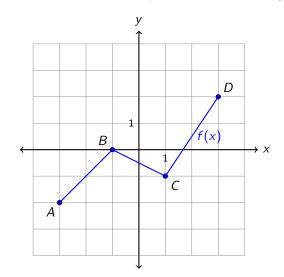
Write the final equation of g(x) if it is found by taking  $f(x) = \sqrt{x}$  after the following ordered sequence of transformations.

- 10. (1) Shift right 2 units
  - (2) Horizontal stretch by factor 3
  - (3) Shift down 2 units
  - (4) Reflect across x-axis
- 11. (1) Horizontal stretch by factor 3
  - (2) Shift left 1 unit
  - (3) Shift up 2 units
  - (4) Reflect across y-axis
- 12. (1) Vertical stretch by factor 5
  - (2) Horizontal stretch by factor 2
  - (3) Shift up 3 units
  - (4) Reflect across x-axis

Find the equation for g(x) if g(x) is found by performing the following *ordered* sequence of transformations to  $f(x) = \frac{1}{x}$ .

- 13. (1) Shift left 3 spaces
  - (2) Reflect across y-axis
  - (3) Shift down 5 spaces
  - (4) Vertical stretch by factor of 7
- 14. (1) Shift up 3 spaces
  - (2) Reflect across x-axis
  - (3) Shift right 5 spaces
  - (4) Horizontal compression by factor of 7

Given the graph of f(x) below, find the new coordinates of each point after the following transformations.



15. 
$$-2f(x+1)$$

16. 
$$f\left(-\frac{1}{2}x\right) - 3$$

17. 
$$\frac{1}{2}f(-x-2)+2$$

18. 
$$f(2x+2)-1$$

19. 
$$-3f(-x+1)+2$$

20. 
$$5f\left(-\frac{1}{2}x\right)$$

### 5.1 Answer Key

1. 
$$g(x) = 3f(-x+1)$$

2. 
$$g(x) = f(2x) + 1$$

3. 
$$g(x) = -\frac{1}{4}f(x-7)$$

4. 
$$g(x) = -\left|\frac{1}{5}x - 3\right|$$

5. 
$$g(x) = |-x| - 1$$

6. 
$$g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$$

7. 
$$g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$$

8. 
$$g(x) = \frac{1}{3}\sqrt{-5x}$$

9. 
$$g(x) = -\sqrt{4x - 8}$$

10. 
$$g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$$

11. 
$$g(x) = \sqrt{\frac{1}{3}(-x+1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$$

12. 
$$g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$$

13. 
$$g(x) = \frac{7}{-x+3} - 35$$

14. 
$$g(x) = -\frac{1}{7x-5} - 3$$

15. 
$$A'(-4,4)$$
,  $B'(-2,0)$ ,  $C'(0,2)$ ,  $D'(2,-4)$ 

16. 
$$A'(6,-5)$$
,  $B'(2,-3)$ ,  $C'(-2,-4)$ ,  $D'(-6,-1)$ 

17. 
$$A'(1,1)$$
,  $B'(-1,2)$ ,  $C'(-3,1.5)$ ,  $D'(-5,3)$ 

18. 
$$A'(-2.5, -3)$$
,  $B'(-1.5, -1)$ ,  $C'(-0.5, -2)$ ,  $D'(0.5, 1)$ 

19. 
$$A'(4,8)$$
,  $B'(2,2)$ ,  $C'(0,5)$ ,  $D'(-2,-4)$ 

20. 
$$A'(6, -10)$$
,  $B'(2, 0)$ ,  $C'(-2, -5)$ ,  $D'(-6, 10)$ 

# **Function Operations**

#### 6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given f(x) = x + 5,  $g(x) = x^2 - 1$ , and  $h(x) = \sqrt{x - 10}$ , simplify or evaluate each.

- 1. (g f)(x)
- 2. (fh)(14)
- 3. (f+g)(x)

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-3	0	-1	3	1	2	4	-4	$\overline{-2}$
g(x)	3	-1	0	1	4	-2	-4	2	-3

- 4. (f+g)(-2) 5. (f-g)(0) 6. (fg)(1)
- 7.  $\left(\frac{f}{g}\right)$  (3)
- 8. (f+f)(-4)

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 9. (f+g)(2)
- 10. (f-g)(1)
- 11. (g-f)(-3)
- 12. (fg)(4)
- 13.  $\left(\frac{f}{g}\right)(0)$

### 6.2 Operations with Functions: Domain

Given  $f(x) = \sqrt{2x+7}$  and g(x) = 3x+3, find the domain of each.

- 1. (f+g)(x)
- $2. \left(\frac{f}{g}\right)(x)$
- 3.  $\left(\frac{g}{f}\right)(x)$

### 6.3 Difference Quotient

Write the difference quotient for each.

- 1. f(x) = 2x 7
- 2.  $g(x) = x^2 + 4x$
- 3. h(x) = -1
- 4.  $f(x) = \frac{3}{x+2}$
- 5.  $g(x) = \sqrt{3x}$
- 6.  $f(x) = x^2 2x + 5$
- 7.  $g(x) = \frac{5}{x}$

- 8.  $f(x) = -2x^2 + 3x 5$
- 9.  $g(x) = \frac{6}{2x+3}$
- 10.  $h(x) = \sqrt{7x+5}$
- 11.  $f(x) = -x^2 + x$
- 12. f(x) = 3x 1
- 13.  $f(x) = x^3 + 5x$

### 6.4 Answer Key

### Adding, Subtracting, Multiplying, and Dividing Functions

1.  $x^2 - x - 6$ 

2. 38

3.  $x^2 + x + 4$ 

4. -1

5. -3

6. -4

7. -2

8. -6

9. -3

10. 6

11. -3

12. 10

13.  $\frac{1}{4}$ 

### **Operations with Functions: Domain**

1.  $\left[-\frac{7}{2},\infty\right)$ 

2.  $\left[-\frac{7}{2}, -1\right) \cup (-1, \infty)$ 

3.  $\left(-\frac{7}{2},\infty\right)$ 

### **Difference Quotient**

1. 2

2. 2x + h + 4

3. 0

4.  $\frac{-3}{(x+2)(x+h+2)}$ 

 $5. \ \frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$ 

6. 2x + h - 2

7.  $\frac{-5}{x(x+h)}$ 

8. -4x - 2h + 3

9.  $\frac{-12}{(2x+3)(2x+2h+3)}$ 

10.  $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$ 

11. -2x - h + 1

12. 3

13.  $3x^2 + 3xh + h^2 + 5$ 

# **Polynomials and Their Graphs**

Find the degree, leading term, leading coefficient, and constant term of the following polynomials.

1. 
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

2. 
$$g(x) = 4x^2 - 16x^6 + 3x$$

3. 
$$h(x) = 1 + x^{11} - 4x^8$$

4. 
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

5. 
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$
 8.  $h(x) = 7(x+1)^2(x-2)^3$ 

6. 
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

7. 
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$

8. 
$$h(x) = 7(x+1)^2(x-2)^3$$

6. 
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$
 9.  $j(x) = -\frac{1}{2}(3x+2)^2(x-1)^5$ 

Determine the end behavior of each.

10. 
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

11. 
$$g(x) = 4x^2 - 16x^6 + 3x$$

12. 
$$h(x) = 1 + x^{11} - 4x^8$$

13. 
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

14. 
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

15. 
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

16. 
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$

17. 
$$h(x) = 5(x+1)^2(x-2)^3$$

18. 
$$j(x) = -\frac{1}{2} (3x+2)^2 (x-1)^5$$

Find the zeros of each. Round to 2 decimal places when necessary.

19. 
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

20. 
$$g(x) = 4x^2 - 16x^6 + 3x$$

21. 
$$h(x) = 1 + x^{11} - 4x^8$$

22. 
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

23. 
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

24. 
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

25. 
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$

26. 
$$h(x) = 5(x+1)^2(x-2)^3$$

27. 
$$j(x) = -\frac{1}{2} (3x+2)^2 (x-1)^5$$

#### 7.1 Answer Key

- 1. Degree = 5, Leading Term =  $-x^5$ , Leading Coefficient = -1, Constant = none (or 0)
- 2. Degree = 6, Leading Term =  $-16x^6$ , Leading Coefficient = -16, Constant = none (or 0)
- 3. Degree = 11, Leading Term =  $x^{11}$ , Leading Coefficient = 1, Constant = 1
- 4. Degree = 4, Leading Term =  $-x^4$ , Leading Coefficient = -1, Constant = 6
- 5. Degree = 5, Leading Term =  $0.25x^5$ , Leading Coefficient = 0.25, Constant = none (or 0)
- 6. Degree = 3, Leading Term =  $-6x^3$ , Leading Coefficient = -6, Constant = -1
- 7. Degree = 3, Leading Term =  $\frac{1}{3}x^3$ , Leading Coefficient =  $\frac{1}{3}$ , Constant =  $3^4$
- 8. Degree = 5, Leading Term =  $7x^5$ , Leading Coefficient = 7, Constant = -56
- 9. Degree = 7, Leading Term =  $-\frac{9}{2}x^7$ , Leading Coefficient =  $-\frac{9}{2}$ , Constant = 2
- 10.  $\lim_{x\to-\infty} f(x) = \infty$   $\lim_{x\to\infty} f(x) = -\infty$
- 11.  $\lim_{x\to-\infty} g(x) = -\infty$ ,  $\lim_{x\to\infty} g(x) = \infty$
- 12.  $\lim_{x\to-\infty} h(x) = -\infty$   $\lim_{x\to\infty} h(x) = \infty$
- 13.  $\lim_{x\to-\infty} f(x) = -\infty$   $\lim_{x\to\infty} f(x) = -\infty$
- 14.  $\lim_{x\to-\infty} g(x) = -\infty$   $\lim_{x\to\infty} g(x) = \infty$
- 15.  $\lim_{x\to-\infty} f(x) = \infty$   $\lim_{x\to\infty} f(x) = -\infty$
- 16.  $\lim_{x\to-\infty} g(x) = -\infty$   $\lim_{x\to\infty} g(x) = \infty$
- 17.  $\lim_{x\to-\infty} h(x) = -\infty$   $\lim_{x\to\infty} h(x) = \infty$
- 18.  $\lim_{x\to-\infty} j(x) = \infty$   $\lim_{x\to\infty} j(x) = -\infty$
- 19. (-1.92, 0), (0, 0)
- 20. (0,0), (0.83,0)
- 21. (-0.83, 0), (0.86, 0), (1.58, 0)
- 22. (-2.25, 0), (1.90, 0)
- 23. (-2.48, 0), (0, 0), (1.85, 0)
- 24. (-0.42, 0), (0.79, 0)
- 25. (6.42, 0)
- 26. (-1,0), (2,0)
- 27.  $\left(-\frac{2}{3},0\right)$ ,  $\left(1,0\right)$

# **Dividing Polynomials**

#### 8.1 Dividing Polynomials

Divide each.

1. 
$$(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$$

2. 
$$(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$$

3. 
$$(4x^3 - 3x^2 + x + 1) \div (x + 2)$$

4. 
$$(5x^4 - x^2 + x - 2) \div (x^2 + 2)$$

5. 
$$(10x^3 + 27x^2 + 8x - 11) \div (2x + 3)$$

6. 
$$(7x^3 + 23x^2 + 12x + 1) \div (x^2 + 3x + 1)$$

7. 
$$(28x^3 - 27x^2 - 4x + 17) \div (4x + 3)$$

8. 
$$(7x^3 - 27x + 4) \div (x^2 - 5)$$

9. 
$$(11x^6 - 24x^5 + 15x^4 - 19x^3 - 16x^2 + 21x - 8) \div (x - 2)$$

10. 
$$(12x^5 - 15x^4 - 11x^3 + 16x^2 - 15x + 17) \div (3x^2 - 5)$$

11. 
$$(6x^4 + 20x^3 - 13x^2 + 20x + 25) \div (x + 4)$$

12. 
$$(24x^5 + 30x^4 - 21x^3 - 4x^2 + 3x - 25) \div (6x^3 + 3x^2 + 3)$$

13. 
$$(3x^5 - 22x^4 + 12x^3 + 10x^2 - 7x + 24) \div (3x^2 - x - 4)$$

14. 
$$(3x^4 - 23x^2 - 15x^3 + 28x + 24) \div (x - 6)$$

15. 
$$(-29x^2 + 6x^6 - 29x^3 + 25x^4 - 15x^5 - 25 - 29x) \div (3x^3 - 6x^2 - 3 - x)$$

16. 
$$(12x^6 + 16x^5 - 5x^4 + 12x^3 - 17x^2 - x - 23) \div (x+2)$$

#### 8.2 Remainder and Factor Theorems

Determine the remainder of each.

1. 
$$(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$$

2. 
$$(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x+1)$$

3. 
$$(x^{23} - 5x^{20} + 17x^8 - 5) \div (x+2)$$

4. 
$$(-7x^{17} + 40x^{15} - 6x^8 + 4x^3) \div (x-3)$$

### 8.3 Answer Key

### **Dividing Polynomials**

- 1.  $4x^2 2x + 5$
- 2.  $4y^2 + 8y + 7 \frac{72}{3y+5}$
- 3.  $4x^2 11x + 23 \frac{45}{x+2}$
- 4.  $5x^2 11 + \frac{x+20}{x^2+2}$
- 5.  $5x^2 + 6x 5 + \frac{4}{2x+3}$
- 6.  $7x + 2 + \frac{-x-1}{x^2+3x+1}$
- 7.  $7x^2 12x + 8 \frac{7}{4x+3}$
- 8.  $7x + \frac{8x+4}{x^2-5}$
- 9.  $11x^5 2x^4 + 11x^3 + 3x^2 10x + 1 \frac{6}{x-2}$
- 10.  $4x^3 5x^2 + 3x 3 + \frac{2}{3x^2 5}$
- 11.  $6x^3 4x^2 + 3x + 8 \frac{7}{x+4}$
- 12.  $4x^2 + 3x 5 + \frac{-x^2 6x 10}{6x^3 + 3x^2 + 3}$
- 13.  $x^3 7x^2 + 3x 5 + \frac{4}{3x^2 x 4}$
- 14.  $3x^3 + 3x^2 5x 2 + \frac{12}{x-6}$
- 15.  $2x^3 x^2 + 7x + 6 + \frac{11x^2 2x 7}{3x^3 6x^2 3 x}$
- 16.  $12x^5 8x^4 + 11x^3 10x^2 + 3x 7 \frac{9}{x+2}$

#### **Remainder and Factor Theorems**

- 1. 6
- 2. 13
- 3. -13,627,141
- 4. -330,064,119

# Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1. 
$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

2. 
$$g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$3. \ f(x) = \frac{3x}{x+4}$$

4. 
$$g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

5. 
$$h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

6. 
$$f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

7. 
$$g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

8. 
$$h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

9. 
$$f(x) = \frac{6x^3 - 21x^2 - 51x + 30}{3x^2 + 7x + 2}$$

10. 
$$g(x) = \frac{10x^2 - 29x - 21}{10x^3 - 33x^2 - 7x}$$

11. 
$$f(x) = \frac{x^3 + x^2 - 6x}{3x^2 - 3x - 6}$$

12. 
$$f(x) = \frac{x^2 - 4x + 3}{2x^2 + 2x - 12}$$

13. 
$$f(x) = \frac{x-4}{-2x^2+4x+16}$$

14. 
$$f(x) = \frac{x^3 - 2x^2 - 8x}{x^3 - 2x^2 - 3x}$$

15. 
$$f(x) = \frac{x^2 + x - 2}{3x^2 + 3x - 18}$$

16. 
$$f(x) = \frac{x^2 - 3x + 2}{4x^2 - 12x}$$

State the end behavior of each.

17. 
$$k(x) = \frac{5x^3 - 7x^2 + 8}{-3x^3 + 6x - 4}$$

18. 
$$m(x) = \frac{2x-1}{3x^2+7x+1}$$

#### 9.1 Answer Key

- 1. Domain:  $x \neq \frac{1}{2}$ , 7; V.A.:  $x = \frac{1}{2}$ , x = 7; H.A.: y = 1
- 2. Domain:  $x \neq -3$ , 4; V.A.: x = -3x = 4; Obl. Asymp: y = 3x + 10
- 3. Domain:  $x \neq -4$ ; V.A.: x = -4; H.A.: y = 3
- 4. Domain:  $x \neq 1$ ; V.A.: x = 1; Obl. Asymp: y = x + 4
- 5. Domain:  $x \neq 0, 1$ ; V.A.: x = 0 and x = 1; H.A.: y = 0
- 6. Domain:  $x \neq -1, 5$ ; V.A. x = -1; Hole @  $(5, \frac{13}{3})$ ; Obl. Asym y = 2x 5
- 7. Domain:  $x \neq -6, 0, 4$ ; V.A. x = -6, x = 0; Hole @  $(4, \frac{21}{40})$ ; H.A. y = 0
- 8. Domain:  $x \neq -6, -\frac{3}{8}$ ; V.A.  $x = -6, x = -\frac{3}{8}$ ; H.A.  $y = \frac{1}{4}$
- 9. Domain:  $x \neq -2$ ,  $-\frac{1}{3}$ ; Hole @ (-2, -21); V.A.:  $x = -\frac{1}{3}$ ; Obl. Asymp:  $y = 2x \frac{35}{3}$
- 10. Domain:  $x \neq -\frac{1}{5}$ , 0,  $\frac{7}{2}$ ; Hole @  $(\frac{7}{2}, \frac{82}{259})$ ; V.A.  $x = -\frac{1}{5}$  and x = 0; H.A. y = 0
- 11. Domain:  $x \neq -1$ , 2; V.A. x = -1; Hole @  $\left(2, \frac{10}{9}\right)$ ; Obl. Asymp:  $y = \frac{1}{3}x + \frac{2}{3}$
- 12. Domain:  $x \neq -3$ , 2; V.A. x = -3 and x = 2; H.A.  $y = \frac{1}{2}$
- 13. Domain:  $x \neq -2$ , 4; V.A. x = 2; Hole @  $\left(4, -\frac{1}{12}\right)$ ; H.A. y = 0
- 14. Domain:  $x \neq -1$ , 0, 3; V.A. x = -1 and x = 3; Hole @  $(0, \frac{8}{3})$ ; H.A. y = 1
- 15. Domain:  $x \neq -3$ , 2; V.A. x = -3 and x = 2; H.A.  $y = \frac{1}{3}$
- 16. Domain:  $x \neq 0$ , 3; V.A. x = 0 and x = 3; H.A.  $y = \frac{1}{4}$
- 17.  $\lim_{x \to -\infty} k(x) = \infty = \lim_{x \to \infty} k(x) = -\frac{5}{3}$
- 18.  $\lim_{x \to -\infty} m(x) = \infty = \lim_{x \to \infty} m(x) = 0$

# Polynomial and Rational Inequalities

### 10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1. 
$$6x^3 - 4x^2 - 10x > 0$$

2. 
$$x^4 < 9x^2$$

3. 
$$3x^3 - 7x^2 - 22x + 8 < 0$$

4. 
$$3x^2 - 4x + 1 < 0$$

5. 
$$12x^4 + 76x^3 + 43x^2 - 346x - 280 > 0$$

6. 
$$-2x^4 + 49x^2 + 21x^3 - 1029x + 2401 > 0$$

7. 
$$-x^2 - 7x - 6 < 0$$

8. 
$$x^2 + 4x + 4 < 0$$

9. 
$$-x^4 - 6x^3 + 61x^2 + 234x - 1008 \ge 0$$

10. 
$$-x^2 + 3x + 1 > 3$$

11. 
$$-3x^4 + 123x^3 + 142x^2 - 424x + 320 < 122x^3$$

12. 
$$-x^4 - 1120 + 77x^2 - 36x + 15x^3 > 15x^3$$

13. 
$$-3x^4 - 22x^3 + 271x^2 + 152x - 96 > 267x^2$$

#### 10.2 Domain

State the domain of each. Write your answers using interval notation.

1. 
$$b(x) = \sqrt{21x^2 - 23x - 20}$$

2. 
$$f(x) = \frac{3}{\sqrt{3x^2+2x-1}}$$

3. 
$$g(x) = \sqrt[4]{2x^3 + 9x^2 + 12x + 4}$$

#### 10.3 Rational Inequalities

Solve each. Write your answers using interval notation.

1. 
$$\frac{3x-4}{x+1} < 0$$

$$2. \ \frac{x^2 + 3x + 2}{x - 7} \le 0$$

3. 
$$\frac{x^2-4x+4}{x^2-1} \ge 0$$

4. 
$$\frac{x+2}{x-4} \leq 1$$

$$5. \ \frac{x^2 - 7x - 8}{x^2 - 4x - 32} \ge 0$$

6. 
$$\frac{4+3x}{5-x} \le 2$$

7. 
$$\frac{x-4}{x+7} < 0$$

8. 
$$\frac{x+5}{x+7} < 0$$

9. 
$$\frac{2x-26}{5x+20} > -3$$

$$10. \ \frac{2x-50}{5x+15} \le -1$$

11. 
$$\frac{x+5}{x^2-2x-15} \le 0$$

12. 
$$-\frac{2}{x} \ge -\frac{3}{x+1}$$

13. 
$$-\frac{3}{x+6} > -\frac{4}{x+7}$$

### 10.4 Answer Key

### **Polynomial Inequalities**

1.  $[-1,0] \cup \left[\frac{5}{3},\infty\right)$ 

2.  $(-3,0) \cup (0,3)$ 

3.  $(-\infty, -2) \cup (\frac{1}{3}, 4)$ 

4.  $\left[\frac{1}{3}, 1\right]$ 

5.  $(-\infty, -4] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$ 

6.  $\left[-7, \frac{7}{2}\right] \cup 7$ 

7.  $(-\infty, -6] \cup [-1, \infty)$ 

8. Ø

9.  $[-8, -7] \cup [3, 6]$ 

10. (1, 2)

11.  $(-\infty, -8] \cup \left[\frac{4}{3}, 2\right] \cup [5, \infty)$ 

12.  $[-8, -4] \cup [5, 7]$ 

13.  $[-6, -4] \cup \left[\frac{2}{3}, 2\right]$ 

#### **Domain**

1.  $\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$ 

2.  $\left(-\infty, -1\right) \cup \left(\frac{1}{3}, \infty\right)$ 

3.  $\{-2\} \cup \left[-\frac{1}{2}, \infty\right)$ 

### **Rational Inequalities**

1.  $\left(-1, \frac{4}{3}\right)$ 

2.  $(-\infty, -2] \cup [-1, 7)$ 

3.  $(-\infty, -1) \cup (1, \infty)$ 

4.  $(-\infty, 4)$ 

5.  $(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$ 

6.  $(-\infty, 1.2] \cup (5, \infty)$ 

7. (-7, 4)

8. (-7, -5)

9.  $(-\infty, -4) \cup (-2, \infty)$ 

10. (-3, 5]

11.  $(-\infty, -5] \cup (-3, 5)$ 

12.  $(-1,0) \cup [2,\infty)$ 

13.  $(-7, -6) \cup (-3, \infty)$ 

# **Function Compositions**

Given f(x) = x - 5,  $g(x) = 4 + \sqrt{2x + 1}$ , and  $h(x) = \frac{3}{x + 7}$ , simplify each and state the domain.

- 1.  $(f \circ g)(x)$
- 2.  $(g \circ f)(x)$
- 3. h(h(x))

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4	
f(x)	-3	0	-1	3	1	2	4	-4	-2	
g(x)	3	-1	0	1	4	-2	-4	2	-3	

- 4.  $(f \circ g)(-1)$  5.  $(g \circ g)(0)$  6.  $(f \circ f)(2)$  7.  $(g \circ g)(-3)$  8. f(g(0))

Given  $f(x) = \sqrt{3x+2}$ ,  $g(x) = x^2 - 1$ , and h(x) = 9x - 2, find each of the following.

- 9.  $(g \circ f)(x)$
- 10. f(g(x))
- 11.  $(h \circ h)(x)$

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 12.  $(f \circ g)(-1)$  13.  $(g \circ f)(-4)$
- 14. f(g(3))
- 15. g(g(-2)) 16.  $(f \circ f)(-5)$

### 11.1 Answer Key

- 1.  $-1 + \sqrt{2x+1}$  Domain:  $\left[-\frac{1}{2}, \infty\right)$
- 2.  $4 + \sqrt{2x 9}$  Domain:  $\left[\frac{9}{2}, \infty\right)$
- 3.  $\frac{3x+21}{7x+52}$  Domain:  $\left(-\infty,-\frac{52}{7}\right)\cup\left(-\frac{52}{7},-7\right)\cup\left(-7,\infty\right)$
- 4. 2
- 5. -3
- 6. -2
- 7. 1
- 8. -2
- 9. 3x + 1
- 10.  $\sqrt{3x^2-1}$
- 11. 81x 20
- 12. -2
- 13. 3
- 14. -4
- 15. 2
- 16. -4

# **Inverse Functions**

Find the inverse of each. Then state the domain and range of the function and the inverse.

1. 
$$f(x) = \sqrt{-2x+3} + 1$$

2. 
$$g(x) = (x+4)^2 - 1$$
,  $x \le -4$ 

3. 
$$h(x) = \frac{9x}{4x-1}$$

4. 
$$f(x) = \sqrt{x} - 3$$

$$5. g(x) = \frac{1}{1-x}$$

6. 
$$h(x) = x^2 + 6x + 4$$
,  $x \le -3$ 

7. 
$$f(x) = \sqrt{5x-4}$$

8. 
$$g(x) = x^2 - 2x + 3, x \le 1$$

9. 
$$h(x) = \frac{3}{x-1}$$

### 12.1 Answer Key

1. $f^{-1}$	(x) = -	$\frac{1}{2} ((x -$	$(-1)^2 - 3$	3)
-------------	---------	---------------------	--------------	----

	Domain	Range
f(x)	$(-\infty, 1.5]$	$[1,\infty)$
$f^{-1}(x)$	$[1,\infty)$	$(-\infty, 1.5]$

2. 
$$g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
g(x)	$(-\infty, -4]$	$[-1,\infty)$
$g^{-1}(x)$	$[-1,\infty)$	$(-\infty, -4]$

3. 
$$h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
h(x)	$(-\infty,1/4)\cup(1/4,\infty)$	$(\infty,9/4)\cup(9/4,\infty)$
$h^{-1}(x)$	$(\infty,9/4)\cup(9/4,\infty)$	$(-\infty,1/4)\cup(1/4,\infty)$

4. 
$$f^{-1}(x) = (x+3)^2$$

	Dom	Ran
f(x)	$[0,\infty)$	$[-3,\infty)$
$f^{-1}(x)$	$[-3,\infty)$	[0, ∞)

5. 
$$g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
g(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$g^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

6. 
$$h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
h(x)	$(-\infty, -3]$	$[-5,\infty)$
$h^{-1}(x)$	$[-5,\infty)$	$(-\infty, -3]$

7. 
$$f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

	Dom	Ran
f(x)	$\left[rac{4}{5},\infty ight)$	[0, ∞)
$f^{-1}(x)$	[0, ∞)	$\left[\frac{4}{5},\infty\right)$

8. 
$$g^{-1}(x) = -\sqrt{x-2} + 1$$

	Dom	Ran
g(x)	$(-\infty,1]$	[2, ∞)
$g^{-1}(x)$	[2, ∞)	$(-\infty,1]$

9. 
$$h^{-1}(x) = \frac{3}{x} + 1$$

	Dom	Ran
h(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$h^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

# **Exponential Functions**

### 13.1 Transforming Exponential Functions

Given  $f(x) = e^x$ , determine the specific transformations done to f(x) to produce g(x).

- 1.  $g(x) = -3e^{x+1}$
- 2.  $g(x) = \frac{1}{4}e^{-5x} 2$
- 3.  $g(x) = e^{2x+7}$
- 4.  $g(x) = 5e^{-x-2} + 1$
- 5.  $g(x) = 0.1e^{0.25x-3} 4$

#### 13.2 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

- 1.  $f(x) = 3 + e^{2x}$
- 2.  $h(x) = 5^{-x}$
- 3.  $h(x) = -\frac{2}{3}e^{x+7} + 1$
- 4.  $f(x) = -7e^x + 4$
- 5.  $g(x) = \frac{1}{3}e^{2x+1} 5$
- 6.  $h(x) = -\frac{1}{2}e^{-4x} + 1$

#### **Transforming Exponential Functions**

- 1. Shift left 1 unit, vertical stretch by factor of 3, reflect across x-axis
- 2. Horizontal compression by factor of 5, reflect across y-axis, vertical compression by factor of 4, shift down 2 units
- 3. Shift left 7 units, horizontal compression by factor of 2
- 4. Shift right 2 units, reflect across y-axis, vertical stretch by factor of 5, shift up 1 unit
- 5. Shift right 3 units, horizontal compression by factor of 4, vertical compression by factor of 10, shift down 4 units

#### **End Behavior**

1. 
$$\lim_{x \to -\infty} f(x) = 3$$
  $\lim_{x \to \infty} f(x) = \infty$ 

2. 
$$\lim_{x \to -\infty} f(x) = \infty$$
  $\lim_{x \to \infty} f(x) = 0$ 

3. 
$$\lim_{x \to -\infty} h(x) = 1$$
  $\lim_{x \to \infty} h(x) = -\infty$ 

4. 
$$\lim_{x \to -\infty} f(x) = 4$$
  $\lim_{x \to \infty} f(x) = -\infty$ 

5. 
$$\lim_{x \to -\infty} f(x) = -5$$
  $\lim_{x \to \infty} f(x) = \infty$ 

6. 
$$\lim_{x \to -\infty} f(x) = -\infty$$
  $\lim_{x \to \infty} f(x) = 1$ 

# **Logarithmic Functions**

Write each of the following in exponential or logarithmic form.

- 1. ln(a) = 7
- 2.  $\log_4(x+1) = 9$
- 3.  $\log(5x) = 30$
- 4. ln(w) = c
- 5.  $5^x = 19$
- 6.  $8^{-3} = \frac{1}{512}$
- 7.  $e^{14} = x$
- 8.  $(1.1)^{-t} = 50$

Find the domain of each. Write your answers in interval notation.

- 9.  $b(x) = \log_7(x^2 8x + 6)$
- 10.  $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$
- 11.  $f(x) = -7 \ln (x^2 + 9x + 8)$
- 12.  $g(x) = \log (5x^2 + 13x 6)$
- 13.  $h(x) = 3 \log_2 (x^3 + 2x^2 x 2)$
- 14.  $c(x) = \ln(4x^2 15x 4)$

State the end behavior of each.

15. 
$$j(x) = 5 \log_3 (2x - 5) - 2$$

- 1.  $e^7 = a$
- 2.  $4^9 = x + 1$
- 3.  $10^{30} = 5x$
- 4.  $e^c = w$
- 5.  $\log_5(19) = x$
- $6. \log_8\left(\frac{1}{512}\right) = -3$
- 7. ln(x) = 14
- 8.  $\log_{1.1}(50) = -t$
- 9.  $(-\infty, 0.838) \cup (7.162, \infty)$
- 10.  $(-3, -2) \cup (-1, \infty)$
- 11.  $(-\infty, -8) \cup (-1, \infty)$
- 12.  $(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$
- 13.  $(-2, -1) \cup (1, \infty)$
- 14.  $\left(-\infty, -\frac{1}{4}\right) \cup \left(4, \infty\right)$
- 15.  $\lim_{x \to (5/2)^+} j(x) = -\infty \quad \lim_{x \to \infty} j(x) = \infty$

# **Properties of Logarithms**

Expand or condense each completely. Simplify numerical answers.

1. 
$$\log_b \left( \frac{x^2}{y^8} \right)$$

2. 
$$\ln (ez)^3$$

3. 
$$\log_5(x) + \log_5(9) - 2\log_5(w)$$

4. 
$$\log_2(2^ab^3)$$

5. 
$$\ln\left(\frac{w^7}{e^6}\right)$$

6. 
$$5\log_4(m) - 3\log_4(n) + 2\log_4(p)$$

Write an equivalent expression for each of the following using natural logarithms.

7. 
$$\log_7(10)$$

8. 
$$\log_9(x)$$

9. 
$$\log_b(c)$$

10. 
$$\log_3(10)$$

11. 
$$\log_{17}(\pi)$$

12. 
$$\log_w(x)$$

Suppose that  $\log_a(b) = 5$ ,  $\log_a(c) = 12$ , and  $\log_a(d) = 9$ . Evaluate each of the following.

13. 
$$\log_a(bc)$$

14. 
$$\log_a(c^3)$$

15. 
$$\log_a \left(\frac{d}{c}\right)$$

16. 
$$\log_a \left( \frac{bd}{c} \right)$$

17. 
$$\log_a (b^7 c)$$

18. 
$$\log_a \left(\frac{c^2}{d}\right)$$

19. 
$$\log_a \left( \sqrt{bc} \right)$$

20. 
$$\log_a ((bd)^2)$$

21. 
$$\log_a \left( \sqrt[3]{d^2} \right)$$

22. 
$$\log_a \left( \sqrt{b^5} \right)$$

23. 
$$\log_a \left( \frac{b^6 c}{d^3} \right)$$

24. 
$$\log_a \left( b^2 c^3 d^4 \right)$$

- $1. \ 2\log_b(x) 8\log_b(y)$
- 2.  $3 + 3 \ln(z)$
- 3.  $\log_5\left(\frac{9x}{w^2}\right)$
- 4.  $a + 3 \log_2(b)$
- 5.  $7 \ln(w) 6$
- 6.  $\log_4\left(\frac{m^5p^2}{n^3}\right)$
- 7.  $\frac{\ln(10)}{\ln(7)}$
- $8. \ \frac{\ln(x)}{\ln(9)}$
- 9.  $\frac{\ln(c)}{\ln(b)}$
- 10.  $\frac{\ln(10)}{\ln(3)}$
- $11. \ \frac{\ln(\pi)}{\ln(17)}$
- $12. \ \frac{\ln(x)}{\ln(w)}$
- 13. 17
- 14. 36
- 15. -3
- 16. 2
- 17. 47
- 18. 15
- 19. 17/2
- 20. 28
- 21. 6
- 22. 25/2
- 23. 15
- 24. 82

## **Exponential Equations**

Solve each. Round to 3 decimal places when necessary.

1. 
$$3e^{x-2} = 7$$

2. 
$$5^x + 4 > 1$$

3. 
$$2^{3x+4} = 32^{x-7}$$

4. 
$$5e^{7x} + 10 = 42$$

5. 
$$7^{4x+1} \ge 343$$

6. 
$$1000e^{0.04x} = 2000$$

7. 
$$3(4.1)^{x-2} = 8$$

8. 
$$2^{x+1} = 5^{7x-5}$$

9. 
$$8(17)^{-5x} = 22$$

10. 
$$-3(11)^{x-10} = -58$$

11. 
$$12^{-10x} + 8 = 80$$

12. 
$$-5(10)^{7x} + 9 = -46$$

13. 
$$8(8)^{10x} - 1 = 55.2$$

14. 
$$3(3)^{-5x} - 8 = 74$$

15. 
$$6(16)^{4x-9} = 19$$

16. 
$$-7(11)^{5x-7} = -3$$

17. 
$$3^{9-6x} - 7 = 26$$

#### 16.1 Applications

- 1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?
- 2. Cadmium-109 has a half-life of about 1.267 years. If 50 mg are initially present, how many years will it take for 16 mg to remain?
- 3. The half-life of bismuth-207 is about 32.9 years. If 90 mg are initially present, how many years will it take for 75 mg to remain?

1.  $x \approx 2.847$ 

2.  $(-\infty, \infty)$ 

3. x = 19.5

4.  $x \approx 0.265$ 

5.  $\left[\frac{1}{2}, \infty\right)$ 

6.  $x \approx 17.329$ 

7.  $x \approx 2.695$ 

8.  $x \approx 0.827$ 

#### **Applications**

1. Approximately 17,952 years

2. Approximately 2.0828 years

3. Approximately 8.6538 years

9.  $x \approx -0.071$ 

10.  $x \approx 11.235$ 

11.  $x \approx -0.172$ 

12.  $x \approx 0.149$ 

13.  $x \approx 0.094$ 

14.  $x \approx -0.602$ 

15.  $x \approx 2.354$ 

16.  $x \approx 1.323$ 

17.  $x \approx 0.970$ 

# Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

1. 
$$\log_5(x) + x \log_5(x) > 0$$

2. 
$$\ln(8-x^2) = \ln(2-x)$$

3. 
$$\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$$

4. 
$$\log_3(2x+1) - \log_3(x-5) = \log_3(x+1)$$

5. 
$$\log_4(x+1) + \log_4(x-5) > 2$$

6. 
$$\log(x+1) - \log(x-5) = \log(x-3)$$

7. 
$$x \log_3(x+2) - \log_3(x+2) = 0$$

8. 
$$\log_{1/2}(x+1) > -3$$

9. 
$$\log_{12}(4x+4) = \log_{12}(5x+1)$$

10. 
$$\log_{15}(-4x+2) = \log_{15}(6-2x)$$

11. 
$$\log_{11}(-5-3x^2) = \log_{11}(-2x^2+6x)$$

12. 
$$\log_{16}(x^2+4) = \log_{16}(2x+3)$$

13. 
$$\log_7(8x-1) = \log_7(x^2+14)$$

14. 
$$-7 \log_5(x+5) = -7$$

15. 
$$7 \log_8(-x) = 28$$

16. 
$$-10\log_3(x-5) = -20$$

- 1.  $(1, \infty)$
- 2. x = -2
- 3.  $x \approx 1.571$
- 4.  $x \approx 6.873$
- 5.  $(2, \infty)$
- 6. x = 7
- 7.  $x = \pm 1$
- 8. (-1,7)
- 9. x = 3
- 10. x = -2
- 11. No Solution
- 12. x = 1
- 13. x = 3, 5
- 14. x = 0
- 15. x = -4096
- 16. x = 14

## Sequences

Write the first 4 terms of each sequence.

1. 
$$a_n = 2(-3)^n$$

2. 
$$b_n = \frac{n!}{2^n}$$

3. 
$$c_{n+1} = 5c_n + 1$$
;  $c_1 = 2$ 

4. 
$$d_n = \frac{1}{2}d_{n-1} + n$$
;  $d_1 = 3$ 

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round**.

5. 
$$a_n = \{343, 667, 991, 1315, \dots\}$$
; Find the 582nd term.

6. 
$$b_n = \{300, 240, 192, 153.6, \dots\}$$
; Find the 711th term.

7. 
$$c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots \right\}$$
; Find the 8,675,309th term.

Given each sequence, find the first 4 digits after the decimal point in the scientific notation version of each term.

8. 
$$a_n = 17, 33, 49, 65, ...; a_{21,972}$$

9. 
$$b_n = 25$$
, 36, 49, 64, 81, ...  $b_{413,401}$ 

10. 
$$c_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots c_{152}$$

- 1. -6, 18, -54, 162
- $2. \ \frac{1}{2}, \ \frac{1}{2}, \ \frac{3}{4}, \ \frac{3}{2}$
- 3. 2, 11, 56, 281
- 4. 3,  $\frac{7}{2}$ ,  $\frac{19}{4}$ ,  $\frac{51}{8}$
- 5. 188,587
- 6. 6882
- 7. 7634
- 8. 5155 (3.51553  $\times$  10<sup>5</sup>)
- 9. 7090 (1.7090369403  $\times$  10<sup>11</sup>)
- 10. 1677 (1.1677487203  $\times$  10<sup>-46</sup>)

## **Series**

Find the sum of each, if possible.

1. 
$$\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^{i}$$

$$2. \sum_{i=0}^{\infty} 3\left(-\frac{2}{3}\right)^{i}$$

$$3. \sum_{k=1}^{\infty} -2\left(\frac{1}{3}\right)^k$$

4. 
$$\sum_{i=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^{j}$$

5. 
$$\sum_{i=0}^{\infty} 1.2(0.8)^{i}$$

6. 
$$\sum_{i=1}^{\infty} 1.2(0.8)^{i}$$

7. 
$$\sum_{i=0}^{\infty} 0.8(1.2)^{i}$$

8. 
$$\sum_{k=1}^{\infty} \frac{2}{3^k}$$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9. 
$$9+13+17+21+\cdots+1565$$

10. 
$$-3+6-12+24-48+\cdots+50,331,648$$

11. 
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{981}$$

12. 
$$2+4+6+8+10+\cdots+38,214$$

13. 
$$3+7+11+15+\cdots+11,491$$

14. 
$$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \dots + \frac{742}{743}$$

- 1.  $\frac{1}{4}$
- 2.  $\frac{9}{5}$
- 3. -1
- 4. Diverges
- 5. 6
- 6. 4.8
- 7. Diverges
- 8. 2
- 9. 306,930
- 10. -33, 554, 433
- 11. 7.4663
- 12. 365,096,556
- 13. 16,511,131
- 14. 733.8947

# **Angles and Radian Measure**

Sketch each of the following. Then find a coterminal between 0 and  $360^{\circ}$  (or 0 and  $2\pi$  radians) for each.

- 1.  $-\frac{3\pi}{4}$
- 2. 900°
- 3.  $\frac{27\pi}{10}$
- 4. −125°

Find the arc length and sector area formed by the central angle of each. Exact answers only.

5.



6.



7.



A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

- 8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
- 9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

1.  $\frac{5\pi}{4}$ 



2. 180°



3.  $\frac{7\pi}{10}$ 



4. 235°



- 5.  $s = \frac{10\pi}{3}$  ft.;  $A = \frac{40\pi}{3}$  sq.ft.
- 6.  $s = \frac{26\pi}{9}$  cm.;  $A = \frac{52\pi}{9}$  sq.cm.
- 7.  $s = \frac{\pi}{8}$  mm.;  $A = \frac{\pi}{64}$  sq.mm.
- 8. 26.18 rad/sec
- 9. 8.73 ft/sec

# **Trig Functions of Any Angle**

Find the exact value of each of the six trig functions of  $\theta$  if P is a point on the terminal side of  $\theta$ .

1. P(-2,3)

3.  $P(-2\sqrt{3}, 2)$ 

5. P(-2,1)

2. P(0, -4)

4. P(-3,5)

6. P(-4, -7)

Find the exact values of the 6 trig functions of the following angles.

7.  $\theta = \frac{-17\pi}{4}$ 

12.  $-\frac{\pi}{2}$ 

8.  $\theta = \frac{21\pi}{2}$ 

13.  $\frac{10\pi}{3}$ 

9.  $\theta = 24\pi$ 

14.  $-\frac{\pi}{3}$ 

10.  $-\frac{5\pi}{3}$ 

15.  $\frac{11\pi}{4}$ 

11.  $\frac{23\pi}{6}$ 

16.  $-\frac{13\pi}{2}$ 

1. 
$$\sin \theta = \frac{3\sqrt{13}}{13}$$
,  $\cos \theta = \frac{-2\sqrt{13}}{13}$ ,  $\tan \theta = -\frac{3}{2}$ ,  $\csc \theta = \frac{\sqrt{13}}{3}$ ,  $\sec \theta = -\frac{\sqrt{13}}{2}$ ,  $\cot \theta = -\frac{2}{3}$ 

2. 
$$\sin \theta = -1$$
,  $\cos \theta = 0$ ,  $\tan \theta = \text{undef.}$ ,  $\csc \theta = -1$ ,  $\sec \theta = \text{undef.}$ ,  $\cot \theta = 0$ 

$$3. \ \sin\theta=\frac{1}{2}, \cos\theta=-\frac{\sqrt{3}}{2}, \tan\theta=-\frac{\sqrt{3}}{3}, \csc\theta=2, \sec\theta=-\frac{2\sqrt{3}}{3}, \cot\theta=-\sqrt{3}$$

$$4. \ \sin\theta = \frac{5\sqrt{34}}{34}, \\ \cos\theta = -\frac{3\sqrt{34}}{34}, \\ \tan\theta = -\frac{5}{3}, \\ \csc\theta = \frac{\sqrt{34}}{5}, \\ \sec\theta = -\frac{\sqrt{34}}{3}, \\ \cot\theta = -\frac{3}{5}$$

5. 
$$\sin\theta=\frac{\sqrt{5}}{5}$$
,  $\cos\theta=-\frac{2\sqrt{5}}{5}$ ,  $\tan\theta=-\frac{1}{2}$ ,  $\csc\theta=\sqrt{5}$ ,  $\sec\theta=-\frac{\sqrt{5}}{2}$ ,  $\cot\theta=-2$ 

6. 
$$\sin \theta = -\frac{7\sqrt{65}}{65}$$
,  $\cos \theta = -\frac{4\sqrt{65}}{65}$ ,  $\tan \theta = \frac{7}{4}$ ,  $\csc \theta = -\frac{\sqrt{65}}{7}$ ,  $\sec \theta = -\frac{\sqrt{65}}{4}$ ,  $\cot \theta = \frac{4}{7}$ 

7. 
$$\sin\theta = -\frac{\sqrt{2}}{2}$$
,  $\cos\theta = \frac{\sqrt{2}}{2}$ ,  $\tan\theta = -1$ ,  $\csc\theta = -\sqrt{2}$ ,  $\sec\theta = \sqrt{2}$ ,  $\cot\theta = -1$ 

8. 
$$\sin \theta = 1$$
,  $\cos \theta = 0$ ,  $\tan \theta = \text{undefined}$ ,  $\csc \theta = 1$ ,  $\sec \theta = \text{undefined}$ ,  $\cot \theta = 0$ 

9. 
$$\sin\theta=0$$
,  $\cos\theta=1$ ,  $\tan\theta=0$ ,  $\csc\theta=$  undefined,  $\sec\theta=1$ ,  $\cot\theta=$  undefined

$$10. \ \sin\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}, \ \cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}, \ \tan\left(-\frac{5\pi}{3}\right) = \sqrt{3}, \ \csc\left(-\frac{5\pi}{3}\right) = \frac{2\sqrt{3}}{3}, \ \sec\left(-\frac{5\pi}{3}\right) = 2, \ \cot\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{3}$$

$$11. \ \sin\left(\frac{23\pi}{6}\right) = -\frac{1}{2}, \ \cos\left(\frac{23\pi}{6}\right) = \frac{\sqrt{3}}{2}, \ \tan\left(\frac{23\pi}{6}\right) = -\frac{\sqrt{3}}{3}, \ \csc\left(\frac{23\pi}{6}\right) = -2, \ \sec\left(\frac{23\pi}{6}\right) = \frac{2\sqrt{3}}{3}, \ \cot\left(\frac{23\pi}{6}\right) = -\sqrt{3}$$

$$12. \ \sin\left(-\frac{\pi}{2}\right) = -1, \ \cos\left(-\frac{\pi}{2}\right) = 0, \ \tan\left(-\frac{\pi}{2}\right) = \text{undefined, } \csc\left(-\frac{\pi}{2}\right) = -1, \ \sec\left(-\frac{\pi}{2}\right) = \text{undefined, } \cot\left(-\frac{\pi}{2}\right) = 0$$

$$13. \ \, \sin\left(\frac{10\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \ \, \cos\left(\frac{10\pi}{3}\right) = -\frac{1}{2}, \ \, \tan\left(\frac{10\pi}{3}\right) = \sqrt{3}, \ \, \csc\left(\frac{10\pi}{3}\right) = -\frac{2\sqrt{3}}{3}, \ \, \sec\left(\frac{10\pi}{3}\right) = -2, \ \, \cot\left(\frac{10\pi}{3}\right) = \frac{\sqrt{3}}{3}$$

$$14. \ \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \ \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}, \ \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}, \ \csc\left(-\frac{\pi}{3}\right) = -\frac{2\sqrt{3}}{3}, \ \sec\left(-\frac{\pi}{3}\right) = 2, \ \cot\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$$

$$15. \ \sin\left(\frac{11\pi}{4}\right) = \frac{\sqrt{2}}{2}, \ \cos\left(\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \ \tan\left(\frac{11\pi}{4}\right) = -1, \ \csc\left(\frac{11\pi}{4}\right) = \sqrt{2}, \ \sec\left(\frac{11\pi}{4}\right) = -\sqrt{2}, \ \cot\left(\frac{11\pi}{4}\right) = -1$$

$$16. \ \sin\left(-\frac{13\pi}{2}\right) = -1, \ \cos\left(-\frac{13\pi}{2}\right) = 0, \ \tan\left(-\frac{13\pi}{2}\right) = \text{undefined, } \csc\left(-\frac{13\pi}{2}\right) = -1, \ \sec\left(-\frac{13\pi}{2}\right) = \text{undefined, } \cot\left(-\frac{13\pi}{2}\right) = 0$$

# **Graphs of Sine and Cosine Functions**

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1. 
$$f(x) = -2\sin\left(3x - \frac{\pi}{4}\right) + 1$$

$$2. g(x) = \frac{1}{3}\cos\left(\frac{1}{2}x + 2\right)$$

3. 
$$f(x) = 2\sin\left(x - \frac{\pi}{3}\right) + 7$$

4. 
$$f(x) = -4\cos\left(\frac{2}{3}x - \frac{2\pi}{3}\right)$$

5. 
$$h(x) = \sin\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

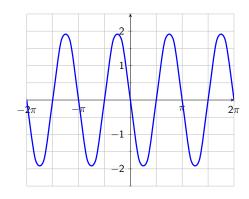
6. 
$$f(x) = 3\sin\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

7. 
$$f(x) = -4\cos\left(4x - \frac{\pi}{3}\right) + \pi$$

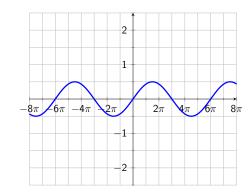
8. 
$$g(x) = \frac{4}{9}\cos\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$$

Write the equation of each of the following in the form  $y = a \sin(bx)$ .

9.



10.



1. Amp = 2, Per = 
$$\frac{2\pi}{3}$$
, P.S. =  $\frac{\pi}{12}$   $\rightarrow$ , V.S. = 1  $\uparrow$ 

2. Amp 
$$=\frac{1}{3}$$
, Per  $=4\pi$ , P.S.  $=4\leftarrow$ , V.S.  $=$  None

3. Amp = 2, Period = 
$$2\pi$$
, P.S. =  $\frac{\pi}{3}$  right, V.S. = 7 up

4. Amp = 4, Period = 
$$3\pi$$
, P.S. =  $\pi$  right, V.S. = 0 (or none)

5. Amp = 1, Per = 
$$\frac{8\pi}{3}$$
, P.S. =  $\frac{\pi}{9}$  left, V.S. = 8 down

6. Amp = 3, Period = 
$$\pi$$
, P.S. =  $\frac{\pi}{4}$  left, V.S. =  $\sqrt{3}$  down

7. Amp = 4, Period = 
$$\frac{\pi}{2}$$
, P.S. =  $\frac{\pi}{12}$  right, V.S. =  $\pi$  up

8. Amp 
$$=\frac{4}{9}$$
, Period  $=\frac{20\pi}{3}$ , P.S.  $=5\pi$  left, V.S.  $=1$  down

9. 
$$y = -2\sin(2x)$$

$$10. \ \ y = \frac{1}{2} \sin \left( \frac{1}{3} x \right)$$

# **Graphs of Other Trig Functions**

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1. 
$$h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

2. 
$$f(x) = 3 \tan \left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

3. 
$$f(x) = -4 \cot \left(4x - \frac{\pi}{3}\right) + \pi$$

4. 
$$g(x) = \frac{4}{9}\cot\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$$

5. 
$$h(x) = \sec\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

6. 
$$f(x) = 3\sec\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

7. 
$$f(x) = -4\csc\left(4x - \frac{\pi}{3}\right) + \pi$$

8. 
$$g(x) = \frac{4}{9}\csc\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$$

9. 
$$f(x) = \frac{1}{10} \cot \left(3x + \frac{3\pi}{4}\right) + 5$$

10. 
$$g(x) = 6 \csc \left(2x - \frac{\pi}{6}\right) + 5$$

11. 
$$h(x) = 4 \csc \left(8x - \frac{\pi}{6}\right) - 1$$

12. 
$$k(x) = \tan\left(\frac{1}{7}x + \frac{\pi}{4}\right) - 5$$

$$13. \ f(x) = 7 \tan \left(8x + \frac{\pi}{6}\right)$$

14. 
$$g(x) = 6 \tan \left(7x - \frac{\pi}{4}\right) - 4$$

15. 
$$h(x) = 4 \tan(5x) - 5$$

16. 
$$k(x) = \cot\left(\frac{1}{3}x - \frac{2\pi}{3}\right) - 5$$

17. 
$$f(x) = 10\left(\frac{1}{8}x - \frac{\pi}{2}\right)$$

18. 
$$g(x) = 10 \cot \left(5x - \frac{\pi}{4}\right) + 5$$

1. Amp = n/a, Per = 
$$\frac{4\pi}{3}$$
, P.S. =  $\frac{\pi}{9}$  left, V.S. = 8 down

2. Amp = n/a, Period = 
$$\frac{\pi}{2}$$
, P.S. =  $\frac{\pi}{4}$  left, V.S. =  $\sqrt{3}$  down

3. Amp = n/a, Period = 
$$\frac{\pi}{4}$$
, P.S. =  $\frac{\pi}{12}$  right, V.S. =  $\pi$  up

4. Amp = n/a, Period = 
$$\frac{10\pi}{3}$$
, P.S. =  $5\pi$  left, V.S. = 1 down

5. Amp = n/a, Per = 
$$\frac{8\pi}{3}$$
, P.S. =  $\frac{\pi}{9}$  left, V.S. = 8 down

6. Amp = n/a, Period = 
$$\pi$$
, P.S. =  $\frac{\pi}{4}$  left, V.S. =  $\sqrt{3}$  down

7. Amp = n/a, Period = 
$$\frac{\pi}{2}$$
, P.S. =  $\frac{\pi}{12}$  right, V.S. =  $\pi$  up

8. Amp = n/a, Period = 
$$\frac{20\pi}{3}$$
, P.S. =  $5\pi$  left, V.S. = 1 down

9. Amp = n/a, Period = 
$$\frac{\pi}{3}$$
, P.S. =  $\frac{\pi}{4}$  left, V.S. = 5 up

10. Amp = n/a, Period = 
$$\pi$$
, P.S. =  $\frac{\pi}{12}$  right, V.S. = 5 up

11. Amp = n/a, Period = 
$$\frac{\pi}{4}$$
, P.S. =  $\frac{\pi}{48}$  right, V.S. = 1 down

12. Amp = n/a, Period = 
$$7\pi$$
, P.S. =  $\frac{7\pi}{4}$  left, V.S. = 5 down

13. Amp = n/a, Period = 
$$\frac{\pi}{8}$$
, P.S. =  $\frac{\pi}{48}$  left, V.S. = 0 (or none)

14. Amp = n/a, Period = 
$$\frac{\pi}{7}$$
, P.S. =  $\frac{\pi}{28}$  right, V.S. = 4 down

15. Amp = n/a, Period = 
$$\frac{\pi}{5}$$
, P.S. = 0 (or none), V.S. = 5 down

16. Amp = n/a, Period = 
$$3\pi$$
, P.S. =  $2\pi$  right, V.S. = 5 down

17. Amp = n/a, Period = 
$$8\pi$$
, P.S. =  $4\pi$  right, V.S. = 0 (or none)

18. Amp = n/a, Period = 
$$\frac{\pi}{5}$$
, P.S. =  $\frac{\pi}{20}$  right, V.S. = 5 up

# **Inverse Trig Functions**

State the exact, simplified value of each or write as an expression of x.

1. 
$$\cot^{-1}(-1)$$

$$2. \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

3. 
$$tan^{-1}(0)$$

4. 
$$\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$$

5. 
$$\cos^{-1}\left(\frac{1}{2}\right)$$

6. 
$$\sec^{-1}(-2)$$

7. 
$$tan^{-1}\left(-\sqrt{3}\right)$$

8. 
$$\sec\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$$

9. 
$$\cot\left(\sec^{-1}(x)\right)$$

10. 
$$\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$$

11. 
$$\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$$

12. 
$$\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$$

13. 
$$\cos(\tan^{-1}(7x))$$

14. 
$$\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$$

15. 
$$\csc\left(\arctan\left(-\frac{3}{2}\right)\right)$$

16. 
$$\cos\left(\sin^{-1}\left(\frac{7}{8}\right)\right)$$

17. 
$$\tan\left(\cos^{-1}\left(\frac{3}{x}\right)\right)$$

18. 
$$\csc\left(\sec^{-1}\left(\sqrt{2}\right)\right)$$

19. 
$$\tan\left(\sec^{-1}\left(\frac{\sqrt{17}}{4}\right)\right)$$

20. 
$$\sin\left(\tan^{-1}\left(\frac{4x}{5}\right)\right)$$

- 1.  $\frac{3\pi}{4}$
- 2.  $-\frac{\pi}{3}$
- 3. 0
- 4.  $\frac{\pi}{6}$
- 5.  $\frac{\pi}{3}$
- 6.  $\frac{2\pi}{3}$
- 7.  $-\frac{\pi}{3}$
- 8.  $\frac{5\sqrt{21}}{21}$
- 9.  $\frac{1}{\sqrt{x^2 1}} = \frac{\sqrt{x^2 1}}{x^2 1}$
- 10.  $\frac{\sqrt{16-9x^2}}{4}$
- 11.  $-\frac{3\sqrt{5}}{2}$
- 12.  $\frac{13\sqrt{22}}{44}$
- 13.  $\frac{\sqrt{49x^2+1}}{49x^2+1}$
- 14.  $\frac{\sqrt{64-x^2}}{x}$
- 15.  $-\frac{\sqrt{13}}{3}$
- 16.  $\frac{\sqrt{15}}{8}$
- 17.  $\frac{\sqrt{x^2-9}}{3}$
- 18.  $\sqrt{2}$
- 19.  $\sqrt{17}$
- $20. \ \frac{4x}{\sqrt{16x^2 + 25}}$

# **Trig Equations and Inequalities**

Solve each in the interval  $[0, 2\pi)$ . Write your answers to inequalities using interval notation.

- 1. tan(6x) = 1
- 2.  $\cot(2x) = -\frac{\sqrt{3}}{3}$
- 3.  $\sin^2(x) = \frac{3}{4}$
- $4. \sin(2x) = \cos(x)$
- 5.  $\sin(2x) \ge \sin(x)$
- 6.  $\cos(2x) < 0$
- $7. \ 2\sin\left(x \frac{\pi}{3}\right) = -1$
- 8.  $3\tan\left(-2x+\frac{\pi}{2}\right)=\sqrt{3}$
- 9.  $\sin^2(x) < \frac{1}{2}$
- 10.  $tan^2(x) = 3 sec(x) 3$
- 11.  $2\csc(x) 3\csc^2(x) = -2\csc^2(x) + 1$
- 12.  $-2\cot(x) \csc^2(x) = 0$
- 13. tan(x) = -tan(x)cos(x)
- 14.  $3\cos(x) = 2\cos^2(x) + 1$
- 15.  $\csc(x) \cot^2(x) + 1 = 0$
- 16.  $-\sin(x) + \sin(2x) = 2\sin(2x)$
- 17.  $3\cos(x) = \sin(2x) + 2\cos(x)$

- $1.\ \ \, \frac{\pi}{24},\frac{5\pi}{24},\frac{3\pi}{8},\frac{13\pi}{24},\frac{17\pi}{24},\frac{7\pi}{8},\frac{25\pi}{24},\frac{29\pi}{24},\frac{11\pi}{8},\frac{37\pi}{24},\frac{41\pi}{24},\frac{15\pi}{8}$
- 2.  $\frac{\pi}{3}$ ,  $\frac{5\pi}{6}$ ,  $\frac{4\pi}{3}$ ,  $\frac{11\pi}{6}$
- 3.  $\frac{\pi}{3}$ ,  $\frac{2\pi}{3}$ ,  $\frac{4\pi}{3}$ ,  $\frac{5\pi}{3}$
- 4.  $\frac{\pi}{2}$ ,  $\frac{5\pi}{6}$ ,  $\frac{3\pi}{2}$
- $5. \ \left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$
- 6.  $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
- 7.  $x = \frac{\pi}{6}, \frac{3\pi}{2}$
- 8.  $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
- 9.  $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$
- 10.  $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 11.  $x = \frac{\pi}{2}$
- 12.  $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
- 13. x = 0,  $\pi$
- 14.  $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 15.  $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
- 16.  $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$
- 17.  $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

### Law of Sines and Cosines

Solve each of the following. Round your answers to 1 decimal place.

1. 
$$m \angle B = 37.8^{\circ}$$
,  $a = 15$ ,  $c = 21.1$ 

2. 
$$m \angle A = 41.9^{\circ}$$
,  $m \angle C = 59.2^{\circ}$ ,  $a = 10.2$ 

3. 
$$a = 14$$
,  $b = 19.6$ ,  $c = 13.1$ 

4. 
$$c = 29$$
,  $b = 23$ ,  $m \angle A = 55^{\circ}$ 

5. 
$$c = 8$$
,  $b = 12$ ,  $m \angle A = 90^{\circ}$ 

6. 
$$m \angle B = 67.2^{\circ}$$
,  $a = 15.6$ ,  $c = 18.9$ 

7. 
$$b = 20$$
,  $a = 30$ ,  $c = 12$ 

8. 
$$a = 14$$
,  $b = 6$ ,  $c = 12$ 

9. 
$$a = 7$$
,  $b = 14$ ,  $c = 12$ 

10. 
$$m \angle B = 119^{\circ}$$
,  $a = 11$ ,  $b = 28$ 

11. 
$$m \angle A = 125^{\circ}$$
,  $c = 21$ ,  $a = 28$ 

12. 
$$m \angle B = 108^{\circ}$$
,  $m \angle C = 61^{\circ}$ ,  $b = 25$ 

- 1.  $b \approx 13.0$ ,  $m \angle A \approx 44.8^{\circ}$ ,  $m \angle C \approx 97.4^{\circ}$
- 2.  $m \angle B = 78.9^{\circ}$ ,  $b \approx 15.0$ ,  $c \approx 13.1$
- 3.  $m \angle A \approx 45.5^{\circ}$ ,  $m \angle B \approx 92.6^{\circ}$ ,  $m \angle C \approx 41.9^{\circ}$
- 4.  $m \angle B \approx 50.1^{\circ}$ ,  $m \angle C \approx 74.9^{\circ}$ ,  $a \approx 24.6$
- 5.  $m \angle B \approx 56.3^{\circ}$ ,  $m \angle C \approx 33.7^{\circ}$ ,  $a \approx 14.4$
- 6.  $m \angle A \approx 48.2^{\circ}$ ,  $m \angle C \approx 64.6^{\circ}$ ,  $b \approx 19.3$
- 7.  $m \angle A \approx 137.8^{\circ}$ ,  $m \angle B \approx 26.6^{\circ}$ ,  $m \angle C \approx 15.6^{\circ}$
- 8.  $m \angle A \approx 96.4^{\circ}$ ,  $m \angle B \approx 25.2^{\circ}$ ,  $m \angle C \approx 58.4^{\circ}$
- 9.  $m \angle A \approx 30^{\circ}$ ,  $m \angle B \approx 91^{\circ}$ ,  $m \angle C \approx 59^{\circ}$
- 10.  $m \angle A \approx 20.1^{\circ}$ ,  $m \angle C \approx 40.9^{\circ}$ ,  $c \approx 21$
- 11.  $m \angle B \approx 17.1^{\circ}$ ,  $m \angle C \approx 37.9^{\circ}$ ,  $b \approx 10.1$
- 12.  $m \angle A \approx 11^{\circ}$ ,  $a \approx 5$ ,  $c \approx 23$

# **Area of Triangles**

Find the area of each. Round your answers to 1 decimal place.

1. 
$$m \angle B = 37.8^{\circ}$$
,  $a = 15$ ,  $c = 21.1$ 

2. 
$$m \angle A = 41.9^{\circ}$$
,  $m \angle C = 59.2^{\circ}$ ,  $a = 10.2$ 

3. 
$$a = 14$$
,  $b = 19.6$ ,  $c = 13.1$ 

4. 
$$p = 14$$
,  $k = 9$ ,  $h = 9$ 

5. 
$$m \angle T = 15^{\circ}$$
,  $m \angle S = 140^{\circ}$ ,  $r = 11.1$ 

6. 
$$m \angle Z = 67^{\circ}$$
,  $y = 6$ ,  $m \angle Y = 41^{\circ}$ 

7. 
$$m \angle R = 129^{\circ}$$
,  $r = 10$ ,  $m \angle P = 28^{\circ}$ 

8. 
$$a = 6.9$$
,  $m \angle B = 115^{\circ}$ ,  $m \angle C = 39^{\circ}$ 

9. 
$$d = 6$$
,  $3 = 12$ ,  $f = 8$ 

10. 
$$m \angle Y = 120^{\circ}$$
,  $x = 13$ ,  $m \angle Z = 21^{\circ}$ 

11. 
$$z = 10$$
,  $y = 14$ ,  $x = 6$ 

12. 
$$m \angle P = 18^{\circ}$$
,  $h = 6.9$ ,  $m \angle H = 147^{\circ}$ 

13. 
$$m \angle S = 118^{\circ}$$
,  $m \angle T = 30^{\circ}$ ,  $s = 6.3$ 

14. 
$$r = 8$$
,  $t = 7.5$ ,  $m \angle S = 50^{\circ}$ 

15. 
$$d = 15.3$$
,  $m \angle E = 105^{\circ}$ ,  $f = 5$ 

16. 
$$m \angle R = 31^{\circ}$$
,  $p = 12$ ,  $m \angle Q = 26^{\circ}$ 

17. 
$$m \angle D = 120^{\circ}$$
,  $f = 4$ ,  $m \angle E = 36^{\circ}$ 

- 1. Approximately 97.0 sq. units
- 2. Approximately 65.7 sq. units
- 3. Approximately 91.6 sq. units
- 4. Approximately 39.6 sq. units
- 5. Approximately 24.3 sq. units
- 6. Approximately 24.0 sq. units
- 7. Approximately 11.7 sq. units
- 8. Approximately 31.0 sq. units
- 9. Approximately 21.3 sq. units
- 10. Approximately 41.7 sq. units
- 11. Approximately 26.0 sq. units
- 12. Approximately 3.5 sq. units
- 13. Approximately 6.0 sq. units
- 14. Approximately 23.0 sq. units
- 15. Approximately 36.9 sq. units
- 16. Approximately 19.5 sq. units
- 17. Approximately 10.0 sq. units

#### **Polar Coordinates**

Convert each to exact rectangular coordinates.

1. 
$$\left(3, \frac{5\pi}{6}\right)$$

6.  $\left(1, \frac{3\pi}{2}\right)$ 

10.  $(-4, 150^{\circ})$ 

2. 
$$\left(-2, -\frac{\pi}{4}\right)$$

7. 
$$\left(-1, \frac{7\pi}{4}\right)$$

11. 
$$\left(2, -\frac{7\pi}{4}\right)$$

3. 
$$\left(\frac{5}{2}, 240^{\circ}\right)$$

8. 
$$\left(-2, \frac{5\pi}{3}\right)$$

9. 
$$\left(1, -\frac{4\pi}{3}\right)$$

13. 
$$\left(-1, \frac{\pi}{4}\right)$$

Convert each to exact polar coordinates. Use r>0 and  $0\leq \theta < 2\pi$ .

14. 
$$\left(-2\sqrt{3}, -2\right)$$

19. 
$$(2, -2\sqrt{3})$$

$$15. \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$20. \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

16. 
$$(7\sqrt{2}, -7\sqrt{2})$$

21. 
$$(2\sqrt{3}, -2)$$

18. 
$$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

17. (-2,0)

$$22. \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$26. \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

Convert each to either rectangular or polar coordinates.

27. 
$$2x + 5y = 9$$

29. 
$$-5x - 8y = -10$$

31. 
$$r = 4 \sec(\theta)$$

28. 
$$3y = 1$$

30. 
$$r = 8$$

32. 
$$\theta = -\frac{\pi}{6}$$

$$1. \left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$$

2. 
$$(-\sqrt{2}, \sqrt{2})$$

$$3. \left(\frac{5}{4}, \frac{5\sqrt{3}}{4}\right)$$

$$4. \left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$$

- 5. (0, 3)
- 6. (0, -1)
- $7. \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
- 8.  $\left(-1, \sqrt{3}\right)$
- 9.  $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- 10.  $(2\sqrt{3}, -2)$

- 11.  $\left(\sqrt{2}, \sqrt{2}\right)$
- 12. (2,0)

13. 
$$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

- 14.  $\left(4, \frac{7\pi}{6}\right)$
- 15.  $\left(\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$
- 16.  $\left(14, \frac{7\pi}{4}\right)$
- 17.  $(2, \pi)$
- 18.  $\left(1, \frac{11\pi}{6}\right)$
- 19.  $\left(4, \frac{5\pi}{3}\right)$
- 20.  $\left(1, \frac{\pi}{6}\right)$
- 21.  $\left(4, \frac{11\pi}{6}\right)$

22. 
$$\left(1, \frac{2\pi}{3}\right)$$

- 23.  $(4, \frac{\pi}{2})$
- 24. (3,0)
- 25. (1,0)
- 26.  $\left(1, \frac{5\pi}{4}\right)$
- $27. \ r = \frac{9}{2\cos\theta + 5\sin\theta}$
- $28. \ \ r = \frac{1}{3} \csc \theta$
- $29. \ \ r = \frac{10}{5\cos\theta + 8\sin\theta}$
- 30.  $x^2 + y^2 = 64$
- 31. x = 4
- 32.  $y = -\frac{\sqrt{3}}{3}x$

## **Vectors**

Given  $\vec{v} = 3\mathbf{i} - 5\mathbf{j}$  and  $\vec{w} = \langle -2, 1 \rangle$ , find each. Exact and simplified answers only.

1. 
$$\vec{v} + \vec{w}$$

2. 
$$-4\vec{w}$$

4. ŵ

Given  $\vec{a} = \langle 4, -3 \rangle$ ,  $\mathbf{b} = 11\mathbf{i} + 2\mathbf{j}$ , and  $\vec{c} = \langle 0, 2 \rangle$ , find each. Exact and simplified answers only.

5. 
$$\vec{a} + \vec{b}$$

6. 
$$\vec{a} - \vec{b}$$

10. 
$$3\vec{a} + 5\vec{b}$$

7. 
$$a + c$$

11. 
$$-2\vec{c} - 1.5\vec{a}$$

12. 
$$||\vec{b}||$$

Find the exact horizontal and vertical component form of each vector with the given magnitude and direction angle.

17. 
$$|u| = 8$$
,  $\theta = 120^{\circ}$ 

19. 
$$|u| = 1.5$$
,  $\theta = -300^{\circ}$ 

21. 
$$|u| = 4$$
,  $\theta = 210^{\circ}$ 

18. 
$$|u| = 5$$
,  $\theta = \frac{\pi}{4}$ 

20. 
$$|u| = \sqrt{2}, \quad \theta = \frac{3\pi}{2}$$

22. 
$$|u| = 2\sqrt{3}$$
,  $\theta = -\frac{\pi}{6}$ 

Write each of the following in  $|\vec{v}|(\cos(\theta),\sin(\theta))$  form, where  $0 \le \theta < 2\pi$ .

23. 
$$\vec{v} = \langle \sqrt{2}, -\sqrt{2} \rangle$$

24. 
$$\vec{v} = \langle -1, \sqrt{3} \rangle$$

25. 
$$\vec{v} = \langle 0, 4 \rangle$$

1. **i** − 4**j** 

2.  $\langle 8, -4 \rangle$ 

3.  $\sqrt{34}$ 

4.  $\left\langle -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$ 

5.  $\langle 15, -1 \rangle$ 

6.  $\langle -7, -5 \rangle$ 

7.  $\langle 4, -1 \rangle$ 

8.  $\langle -11, -2 \rangle$ 

9.  $\langle 0, 4 \rangle$ 

10. (67, 1)

11.  $\langle -6, 0.5 \rangle$ 

12.  $5\sqrt{5}$ 

13. 2

14.  $\left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$ 

15.  $\left\langle \frac{11\sqrt{5}}{25}, \frac{2\sqrt{5}}{25} \right\rangle$ 

16.  $\langle 0, 1 \rangle$ 

17.  $\langle -4, 4\sqrt{3} \rangle$ 

18.  $\left\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$ 

 $19. \left\langle \frac{3}{4}, \frac{3\sqrt{3}}{4} \right\rangle$ 

20.  $\langle 0, -\sqrt{2} \rangle$ 

21.  $\langle -2\sqrt{3}, -2 \rangle$ 

22.  $\langle 3, -\sqrt{3} \rangle$ 

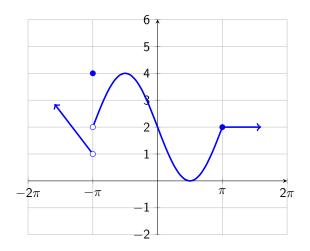
23.  $2\left\langle\cos\left(\frac{7\pi}{4}\right),\sin\left(\frac{7\pi}{4}\right)\right\rangle$ 

24.  $2\left\langle\cos\left(\frac{2\pi}{3}\right),\sin\left(\frac{2\pi}{3}\right)\right\rangle$ 

25.  $4\left\langle\cos\left(\frac{\pi}{2}\right),\sin\left(\frac{\pi}{2}\right)\right\rangle$ 

# **Numerical and Graphical Limits**

Solve using the graph of f(x) below.



$$1. \lim_{x \to -\pi^-} f(x)$$

$$3. \lim_{x \to -\pi} f(x)$$

$$5. \lim_{x \to \pi^-} f(x)$$

7. 
$$\lim_{x \to \pi} f(x)$$

$$2. \lim_{x \to -\pi^+} f(x)$$

4. 
$$f(-\pi)$$

$$6. \lim_{x \to \pi^+} f(x)$$

8. 
$$f(\pi)$$

Find each limit.

$$9. \lim_{x\to 0}\frac{e^x-1}{x}$$

$$10. \lim_{x \to 1^+} \log(x-1)$$

11. 
$$\lim_{x\to 2} \left( \frac{3e^{1/(x-2)}}{e^{1/(x-2)}+1} - 1 \right)$$

12. 
$$\lim_{x \to 0} f(x)$$
,  $f(x) = \begin{cases} -x^2 - 4x - 5, & x \le 0 \\ x - 5, & x > 0 \end{cases}$ 

13. 
$$\lim_{x \to 4} g(x)$$
,  $g(x) = \begin{cases} x+4, & x < -4 \\ \frac{x}{2} + 3, & x \ge -4 \end{cases}$ 

14. 
$$\lim_{x\to 0} h(x)$$
,  $h(x) = \begin{cases} -x-4, & x<0\\ -2x-5, & x\geq 0 \end{cases}$ 

- 1. 1
- 2. 2
- 3. Does not exist
- 4. 4
- 5. 2
- 6. 2
- 7. 2
- 8. 2
- 9.  $e \approx 2.71828$
- 10. Does not exist
- 11. Does not exist
- 12. -5
- 13. 1
- 14. Does not exist

# **Algebraic Limits**

Find each limit algebraically.

1. 
$$\lim_{x \to -4} \left( \frac{x+4}{x^2+6x+8} \right)$$

2. 
$$\lim_{x \to -3} \left( -\frac{x^2 + 2x - 3}{x + 3} \right)$$

3. 
$$\lim_{x\to 3} \left( \frac{x-3}{x^2-8x+15} \right)$$

4. 
$$\lim_{x \to -2} \left( \frac{x^2 - 2x - 8}{x + 2} \right)$$

5. 
$$\lim_{x \to -1} \left( -\frac{x^2 + 5x + 4}{x + 1} \right)$$

6. 
$$\lim_{x \to -4} \left( \frac{x^2 + 7x + 12}{x + 4} \right)$$

$$7. \lim_{x \to 2} \left( \frac{x^2 - 2x}{x - 2} \right)$$

8. 
$$\lim_{x \to -1} \left( -\frac{x^2 - 3x - 4}{x + 1} \right)$$

$$9. \lim_{x \to 0} \left( \frac{x}{\frac{1}{x-1} + 1} \right)$$

10. 
$$\lim_{x \to 0} \left( \frac{x}{\frac{1}{x+2} - \frac{1}{2}} \right)$$

11. 
$$\lim_{x \to 3} \left( \frac{x}{\frac{1}{x-3} + \frac{1}{3}} \right)$$

12. 
$$\lim_{x \to 2} \left( \frac{x}{\frac{1}{x-2} + \frac{1}{2}} \right)$$

13. 
$$\lim_{x \to 1} \left( \frac{x}{\frac{1}{x-1} + 1} \right)$$

14. 
$$\lim_{xt\neq -1} \left( \frac{x}{\frac{1}{x+1} - 1} \right)$$

15. 
$$\lim_{x \to 1} \left( \frac{\sqrt{x} - 1}{x - 1} \right)$$

$$16. \lim_{x \to 2} \left( \frac{\sqrt{x+7}-3}{x-2} \right)$$

$$17. \lim_{x \to 25} \left( \frac{x - 25}{\sqrt{x} - 5} \right)$$

$$18. \lim_{x \to 1} \left( \frac{x-1}{\sqrt{x+8}-3} \right)$$

$$19. \lim_{x \to 9} \left( \frac{x - 9}{\sqrt{x} - 3} \right)$$

20. 
$$\lim_{x \to 2} \left( \frac{\sqrt{x+14}-4}{x-1} \right)$$

- 1.  $-\frac{1}{2}$
- 2. 4
- 3.  $-\frac{1}{2}$
- 4. -6
- 5. -3
- 6. -1
- 7. 2
- 8. 5
- 9. -1
- 10. -4
- 11. 0
- 12. 0
- 13. 0
- 14. 0
- 15.  $\frac{1}{2}$
- 16.  $\frac{1}{6}$
- 17. 10
- 18. 6
- 19. 6
- 20.  $\frac{1}{8}$

# **Continuity**

For each of the following, determine whether the function graphed below is continuous at that value. If not, explain why.



1. 
$$x = -11$$

2. 
$$x = -7$$

3. 
$$x = -4$$

4. 
$$x = 2$$

$$5 y - 6$$

6. 
$$x = 10$$

Identify all discontinuities for each of the following.

7. 
$$f(x) = \frac{x^2 - 6x}{x^2 + 6x}$$

8. 
$$f(x) = \frac{x+3}{x-3}$$

$$9. \ f(x) = \frac{x+4}{3\ln(x)}$$

10. 
$$f(x) = \frac{2x+5}{x^2-9}$$

11. 
$$f(x) = \begin{cases} 2\sin(x), & x < 0 \\ 0, & x = 0 \\ 3x - 2, & x > 0 \end{cases}$$

12. 
$$f(x) = \begin{cases} e^x - 1, & x \le 0 \\ x^2, & x > 0 \end{cases}$$

13. 
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x > -1 \end{cases}$$

14. 
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x \ge -1 \end{cases}$$

- 1. Discontinuous; Not defined at x = -11
- 2. Continuous
- 3. Discontinuous; Left- and right-hand limits are not equal, nor do they equal the function value at x = -4
- 4. Discontinuous; Not defined at x = 2
- 5. Discontinuous; Left- and right-hand limits are not equal
- 6. Continuous
- 7. Discontinuous at x = 0, -6
- 8. Discontinuous at x = 3
- 9. Discontinuous at x = 1
- 10. Discontinuous at  $x = \pm 3$
- 11. Discontinuous at x = 0
- 12. Continuous for all values of x
- 13. Discontinuous at x = -1
- 14. Continuous for all values of x

## **Derivatives**

Use the definition of the derivative to find the derivative of each function with respect to x.

- 1.  $f(x) = x^2 + 2x + 4$
- 2. f(x) = -5x + 5
- 3.  $f(x) = 2x^2 5x 2$
- 4. f(x) = 5x + 2
- 5.  $f(x) = x^3 x^2$
- 6.  $f(x) = 5x^2 + 5$
- 7. f(x) = 2x + 3
- 8.  $f(x) = x^2 2$
- 9.  $f(x) = -5x^3 + 4$
- 10.  $f(x) = -4x^2 + x 5$
- 11.  $f(x) = 2x^2 3x + 5$

- 1. 2x + 2
- 2. -5
- 3. 4x 5
- 4. 5
- 5.  $3x^2 2x$
- 6. 10*x*
- 7. 2
- 8. 2*x*
- 9.  $-15x^2$
- 10. -8x + 1
- 11. 4x 3

### Appendix A

# **Factoring**

Factor each of the following completely.

1. 
$$x^2 + 2x - 15$$

2. 
$$x^2 - 8x + 12$$

3. 
$$x^2 + 15x + 56$$

4. 
$$5x^2 + 19x - 4$$

5. 
$$4x^2 - 5x - 6$$

6. 
$$9x^2 - 400$$

7. 
$$5x^2 - 7x - 6$$

8. 
$$9x^2 - 54x + 45$$

9. 
$$3x^3 + 12x^2 + 9x$$

10. 
$$9y^2 - 16$$

11. 
$$4x^2 - 28x + 49$$

12. 
$$14x^2 + 11xy - 15y^2$$
 18.  $3p^2 + 22p - 16$ 

13. 
$$6x^2 - 48x - 120$$

14. 
$$9x^4 - 54x^3 + 45x^2$$

15. 
$$16y^2 - 40y + 25$$

16. 
$$30x^2 + xy - y^2$$

17. 
$$8w^2 + 33w + 4$$

18. 
$$3p^2 + 22p - 16$$

19. 
$$18x^2 - 27x + 4$$

20. 
$$14a^2 + 15a - 9$$

21. 
$$4x^2 - 4x - 24$$

22. 
$$18t^2 - 9t - 5$$

23. 
$$6a^2 + 23a + 21$$

24. 
$$25x^2 - 1$$

1. 
$$(x+5)(x-3)$$

2. 
$$(x-6)(x-2)$$

3. 
$$(x+7)(x+8)$$

4. 
$$(5x-1)(x+4)$$

5. 
$$(4x+3)(x-2)$$

6. 
$$(3x + 20)(3x - 20)$$

7. 
$$(5x+3)(x-2)$$

8. 
$$9(x-5)(x-1)$$

9. 
$$3x(x+3)(x+1)$$

10. 
$$(3y+4)(3y-4)$$

11. 
$$(2x-7)^2$$

12. 
$$(7x - 5y)(2x + 3y)$$

13. 
$$6(x-10)(x+2)$$

14. 
$$9x^2(x-1)(x-5)$$

15. 
$$(4y-5)^2$$

16. 
$$(6x - y)(5x + y)$$

17. 
$$(8w+1)(w+4)$$

18. 
$$(3p-2)(p+8)$$

19. 
$$(6x-1)(3x-4)$$

20. 
$$(7a-3)(2a+3)$$

21. 
$$4(x-3)(x+2)$$

22. 
$$(6t-5)(3t+1)$$

23. 
$$(2a+3)(3a+7)$$

24. 
$$(5x+1)(5x-1)$$

## **Appendix B**

# **Complex Fractions**

Simplify each as much as possible.

1. 
$$\frac{5+\frac{3}{x}}{x-\frac{1}{2}}$$

$$2. \ \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

3. 
$$\frac{3}{2 - \frac{x}{x-1}}$$

4. 
$$\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$$

$$5. \ \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$6. \ \frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$7. \ \frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2 - 2x}}$$

$$8. \ \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

9. 
$$\frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$10. \ \frac{\frac{5}{x} - 5x}{x - 1}$$

11. 
$$\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

12. 
$$\frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

$$13. \ \frac{\frac{1}{x-a} + \frac{1}{a}}{X}$$

14. 
$$\frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

15. 
$$\frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

1. 
$$\frac{2(5x+3)}{x(2x-1)}$$

$$2. \ \frac{1}{x^2 - 2x + 4}$$

3. 
$$\frac{3(x-1)}{x-2}$$

$$4. \ \frac{x+3}{2+7x}$$

5. 
$$\frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \ \frac{-4x-1}{2}$$

7. 
$$\frac{8x-10}{7}$$

8. 
$$-\frac{1}{7x}$$

$$9. \ \frac{2x+1}{5x(x+1)}$$

$$10. \ \frac{-5x-5}{x}$$

11. 
$$\frac{-1}{2x+4}$$

12. 
$$\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \ \frac{1}{a(x-a)}$$

$$14. \ \frac{-2x-2}{5x^2-16x+3}$$

15. 
$$\frac{x+4}{4x-8}$$