Honors PreCalculus

Extra Practice Problems

Contents

1	Basic Set Theory and Interval Notation						
	.1 Answer Key	. 5					
2	unctions and Their Graphs	6					
	.1 Evaluating Functions	. 6					
	.2 Domain of Functions						
	.3 Piecewise Functions						
	.4 Answer Key						
	Trillipped recy						
3	Properties of Functions	ć					
	.1 Maxima and Minima	. 9					
	.2 Increasing, Decreasing, and Constant Intervals	. 9					
	.3 Miscellaneous	. 10					
	.4 Answer Key	. 12					
4	inear Functions and Slope	14					
	.1 Equations of Lines						
	.2 Average Rate of Change						
	.3 Answer Key	. 16					
5	function Transformations	17					
3							
	.1 Answer Key	. 19					
6	function Operations	20					
•	.1 Adding, Subtracting, Multiplying, and Dividing Functions						
	.2 Operations with Functions: Domain						
	.3 Difference Quotient						
	.4 Answer Key						
	.4 Allswer Ney	. 22					
7	Polynomials and Their Graphs	23					
	.1 Answer Key	. 24					
8	Dividing Polynomials	25					
	.1 Dividing Polynomials						
	.2 Remainder and Factor Theorems	. 25					
	.3 Answer Key	. 26					
_		0-					
9	Rational Functions and Their Graphs	27					
	.1 Answer Key	. 28					
10	Polynomial and Rational Inequalities	29					
10	0.1 Polynomial Inequalities						
	0.2 Domain						
	0.3 Rational Inequalities						
	0.4 Answer Key	. 30					
11	unction Compositions	31					
	1.1 Answer Key	. 32					

12	Inverse Functions 12.1 Answer Key	33 34
13	Exponential Functions 13.1 Transforming Exponential Functions	35 35 35 36
14	Logarithmic Functions 14.1 Answer Key	37 38
15	Properties of Logarithms 15.1 Answer Key	39 40
16	Exponential Equations 16.1 Applications	41 41 42
17	Logarithmic Equations and Inequalities 17.1 Answer Key	43 44
18	Sequences 18.1 Answer Key	45 46
19	Series 19.1 Answer Key	47 48
20	Angles and Radian Measure 20.1 Answer Key	49 50
21	Trig Functions of Any Angle 21.1 Answer Key	51 52
22	Graphs of Sine and Cosine Functions 22.1 Answer Key	53 54
23	Graphs of Other Trig Functions 23.1 Answer Key	55 56
24	Inverse Trig Functions 24.1 Answer Key	57 58
25	Trig Equations and Inequalities 25.1 Answer Key	59
26	Law of Sines and Cosines 26.1 Answer Key	61
27	Area of Triangles 27.1 Answer Key	63
28	Polar Coordinates 28.1 Answer Key	65
29	Vectors 29.1 Answer Key	67
30	Numerical and Graphical Limits 30.1 Answer Key	69 70
31	Limits and Algebra 31.1 Answer Key	71

	Continuity 32.1 Answer Key	7 3
	Derivatives 33.1 Answer Key	75
	Factoring A.1 Answer Key	77
В	Complex Fractions B.1 Answer Key	79

Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2. $\{x | x \le 1\}$
- 3. _3
- 4. $\{x | x \neq 4, 11\}$
- 5. 2.4 7.7
- 6. $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7. $\{x | x \ge 2\}$
- 8. $\{x|x<-8\}$
- 9. $\{x | x \neq 3\}$
- 10. $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

- 11.

1.1 Answer Key

1. $\{x | -5 < x \le 8\}$



2. $(-\infty, 1]$



- 3. $[-3, \infty)$ $\{x | x \ge -3\}$
- $4. \ (-\infty, 4) \cup (4, 11) \cup (11, \infty)$ $\longleftrightarrow \qquad \qquad \downarrow$ $4 \qquad \qquad \downarrow$ 11
- 5. [2.4, 7.7) $\{x|2.4 \le x < 7.7\}$
- 6. $\{x|x > 9\}$

9

7. $[2,\infty)$



8. $(-\infty, -8)$



9. $(-\infty,3)\cup(3,\infty)$



10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$

$$\begin{array}{ccc} & & & \bigcirc & & \bigcirc \\ & -2 & & 5 & \end{array}$$

- 11. $(-\infty, -8]$ $\{x | x \le -8\}$
- 12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x | x \neq 7, 12\}$
- 13. $(-\infty, -3) \cup (-3, 2) \cup (2, 5) \cup (5, \infty)$ $\{x | x \neq -3, 2, 5\}$

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

1. f(5)

2. f(-2)

3. f(0)

4. g(1)

5. g(-5)

6. g(1/4)

7. f(-x)

8. g(-x)

9. f(2x)

10. g(2x)

11. f(x-3)

12. g(x-3)

13. $f(\frac{1}{3}x)$

14. $g\left(\frac{1}{2}x\right)$

15. f(2x+1)

16. g(2x+1)

17. f(-x+7)

18. g(-x+7)

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1. $f(x) = -8x^2 - 7x + 1$

2. $g(x) = \sqrt{5x + 12} - 2$

4. f(x) = -5x + 4

5. $f(x) = x^2 + 2$

6. $f(x) = \frac{2x+1}{3x-5}$

3. $h(x) = \frac{x+2}{9x-7}$

7. $f(x) = \sqrt{3x - 12}$

8. $f(x) = \frac{x}{x^2 - 16}$

9. $f(x) = \frac{x+4}{x^3-4x}$

10. $f(x) = \frac{x}{\sqrt{x-4}}$

11. $f(x) = \frac{x^2+1}{2x^2+8}$

12. $f(x) = -\frac{x+7}{x^2-5x-6}$

13. $g(x) = \sqrt{2x+3}$

14. $h(x) = \sqrt[3]{2x+3}$

15. $f(x) = -\frac{7x-10}{x^2+3x+2}$

16. $g(x) = \sqrt{-9x + 8}$

17. $h(x) = -\sqrt[3]{4x+1}$

18. $f(x) = \sqrt[3]{8x+1}$

19. $g(x) = \frac{x^2-1}{\sqrt{x+3}}$

20. $h(x) = \frac{3}{9 + \frac{4}{x+7}}$

21. $f(x) = \frac{x+1}{\sqrt{10x+8}}$

22. $g(x) = \frac{5}{1 + \frac{3}{x+2}}$

23. $i(x) = \frac{7}{3 - \frac{4}{x+1}}$

24. $n(x) = \frac{7x+14}{\sqrt{2x-1}}$

25. $a(x) = \frac{\frac{x}{x-2}}{\frac{3}{x-2}+6}$

26. $d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. f(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4\\ \sqrt{2x + 7} & -4 < x < 0\\ |-x - 1| & x \ge 0 \end{cases}$$

- 6. f(3)
- 7. f(-2)
- 8. f(0)
- 9. f(-5)

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \le -3\\ \sqrt{-4x + 1} & \text{if } -3 < x \le 0\\ \frac{5x^2}{x + 7} & \text{if } x > 0 \end{cases}$$

- 10. f(7)
- 11. f(-3)
- 12. f(1)
- 13. f(0)
- 14. f(-1)
- 15. f(-3/2)

2.4 Answer Key

Evaluating Functions

1. -55

4. -4

7. $-3x^2 - 4x$

10. $\frac{1-10x}{2x}$

13. $-\frac{1}{3}x^2 + \frac{4}{3}x$

16. $-\frac{10x+4}{2x+1}$

2. -20

5. -5.2

8. $-\frac{1}{x} - 5 = \frac{-1-5x}{x}$

11. $-3x^2 + 22x - 39$

14. $\frac{3-5x}{x}$

17. $-3x^2 + 38x - 119$

3. 0

6. -1

9. $-12x^2 + 8x$

12. $\frac{16-5x}{x-3}$

15. $-12x^2 - 4x + 1$

18. $\frac{5x-34}{-x+7}$

Domain of Functions

1. $(-\infty, \infty)$

4. $(-\infty, \infty)$

7. $[4, \infty)$

10. $(4, \infty)$

13. $\left[-\frac{3}{2},\infty\right)$

16. $\left(-\infty, \frac{8}{9}\right]$

19. $(-3, \infty)$

25. $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, 2) \cup (2, \infty)$

2. $\left[\frac{-12}{5}, \infty\right)$

5. $(-\infty, \infty)$

8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

11. $(-\infty, \infty)$

14. $(-\infty, \infty)$

17. $(-\infty, \infty)$

20. $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup \left(-7, \infty\right)$

 $22. \ (\infty, -5) \cup (-5, -2) \cup (-2, \infty)$ $23. \ (-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ $24. \ (\frac{1}{2}, \infty)$

26. $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$

3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$

6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$

9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$

12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$

15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

18. $(-\infty, \infty)$

21. $\left(-\frac{4}{5}, \infty\right)$

Piecewise Functions

1. $\sqrt{15} \approx 3.873$

4. 6.4

7. $\sqrt{3} \approx 1.732$

10. 17.5

13. 1

2. 7

5. 7.1

8. 1

11. 4

14. $\sqrt{5} \approx 2.236$

3. 6.6

6. 4

9. 18

12. $\frac{5}{8}$

15. $\sqrt{7} \approx 2.646$

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

5.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

6.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = x^3 + 2x^2 - 4x - 8$$

4.
$$g(x) = x^4 - 2x^2 + 1$$

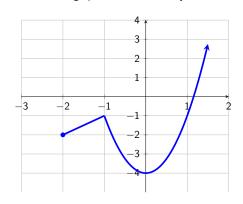
5.
$$h(x) = \sqrt{x+1} - 2$$

6.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

7.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

3.3 Miscellaneous

Use the graph of y = f(x) below to answer the following questions. Write your answers using interval notation.



- 1. Domain of f
- 3. Relative Minimum
- 5. f(1)
- 7. Increasing Interval(s)
- 9. Absolute Maximum

- 2. Range of f
- 4. Relative Maximum
- 6. f(0)
- 8. Decreasing Interval(s)
- 10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

11. f(7)

12. f(-2)

- 13. Rel. Max
- 14. Rel. Min

- 15. Global Max
- 16. Global Min
- 17. Increasing Interval(s)
- 18. Decreasing Interval(s)

Use the graph of f(x) to answer each.



- 19. Relative maxima of f(x)
- 20. Relative minima of f(x)
- 21. Absolute maxima of f(x)

- 22. Absolute minima of f(x)
- 23. Intervals where f is increasing
- 24. Intervals where f is decreasing

25. Zeros of f

Given the labeled points A through G on the graph of f(x) below, find each of the following.



- 26. Increasing interval(s)
- 28. Relative max
- 30. Global max
- 32. Zeros of f

- 27. Decreasing interval(s)
- 29. Relative min
- 31. Global min
- 33. Number of solutions to f(x) = 1

Answer each of the following about the function f(x) below.



- 34. Domain of f
- 36. Relative maxima
- 38. Absolute maximum
- 40. Increasing intervals
- 42. Zeros of f(x)

- 35. Range of f
- 37. Relative minima
- 39. Absolute minimum
- 41. Decreasing intervals
- 43. Number of solutions to f(x) = 2

3.4 Answer Key

Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. Rel Max (-1.366, 10.848) and (1, 6); Rel Min (0.366, 5.652)
- 4. Rel Max (-1.716, 11.598); Rel Min (1.132, -3.929)
- 5. Rel Max: (1.095, 12.096); Rel Min (-0.761, -0.680)
- 6. Rel Max: (1.366, 0.348); Rel Min: (-0.366, -4.848) and (2, 0)
- 7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

- 1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
- 3. Inc: $(-\infty, -2) \cup (\frac{2}{3}, \infty)$ Dec: $(-2, \frac{2}{3})$
- 4. Inc; $(-1,0) \cup (1,\infty)$ Dec: $(-\infty,-1) \cup (0,1)$
- 5. Inc: $(-1, \infty)$ No intervals where it is decreasing
- 6. Inc: (-0.761, 1.095); Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
- 7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Miscellaneous

- 1. $[-2, \infty)$
- 4. (-1, -1)
- 7. $(-2, -1) \cup (0, \infty)$
- 10. None
- 13. (1.408, 0.272)
- 16. None
- 19. (b, f(b)) and (m, f(m))
- 22. None
- 25. x = a, x = c, x = n
- 28. $\it B$ and $\it F$
- 31. None
- 34. $(-\infty, 2)$
- 37. (0, 1)
- 40. $(-\infty, -1.2) \cup (0, 1.3)$
- 43. 4

- 2. $[-4, \infty)$
- 5. -1
- 8. (-1,0)
- 11. -426
- 14. (0.592, -0.272)
- 17. (0.592, 1.408)
- 20. (c, f(c))
- 23. $(-\infty, b) \cup (c, m)$
- 26. $(\infty, B) \cup (D, F)$
- 29. D
- 32. A, C, E, G
- 35. $(-\infty, -2.5) \cup (-2.5, 3.6]$
- 38. (1.3, 3.6)
- 41. $(-1.2,0) \cup (1.3,2)$

- 3. (0, -4)
- 6. -4)
- 9. (0, -4)
- 12. 51
- 15. None
- 18. $(-\infty, 0.592) \cup (1.408, \infty)$
- 21. (m, f(m))
- 24. $(b, c) \cup (m, p)$
- 27. (B, D) ∪ (F, ∞)
- 30. B
- 33. 2
- 36. (-1.2, 3) and (1.3, 3.6)
- 39. Does not exist
- 42. (-1.8,0) and (1.9,0)

Linear Functions and Slope

4.1 Equations of Lines

Write the equation of each line in point-slope form that goes through each pair of points.

- 1. (-2,1), (7,8)
- 2. (0,4), (9,-15)
- 3. (-1, -2), (-3, -13)

4.2 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

- 6. [-2, -1]
- 7. [5, 6]
- 8. [0, 1]
- 9. [5, 5.001]
- 10. [5, 5.0001]
- 11. [5, 5.00001]
- 12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

- 13. [7, 7.001]
- 14. [7, 7.0001]
- 15. [7, 7.00001]
- 16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

- 17. [1, 1.0001]
- 18. [1, 1.00001]
- 19. [1, 1.000001]
- 20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = 6\sqrt{x}$, find the average rate of change of each over the given interval.

- 21. [25, 25.1]
- 22. [25, 25.01]
- 23. [25, 25.001]
- 24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -7x^3 + 6\sqrt{3x} + 4$ over each interval. Round your answers to 4 decimal places.

- 25. [0, 1]
- 26. [10, 11]
- 27. [8, 15]

4.3 Answer Key

Equations of Lines

- 1. $y-1=\frac{7}{9}(x+2)$ or $y-8=\frac{7}{9}(x-7)$
- 2. $y-4=-\frac{19}{9}(x-0)$ or $y+15=-\frac{19}{9}(x-9)$
- 3. $y + 2 = \frac{11}{2}(x+1)$ or $y + 13 = \frac{11}{2}(x+3)$

Average Rate of Change

- 1. 2.1
- 4. 2.0001
- 7. -59
- 10. -53.0006
- 13. -42.003
- 16. -42
- 19. -0.49999988
- 22. 0.59999
- 25. 3.3923

- 2. 2.01
- 5. 2
- 8. 1
- 11. -53.00006
- 14. -42.0003
- 17. -0.499988
- 20. -0.5
- 23. 0.6
- 26. -2,315.3960

- 3. 2.001
- 6. 25
- 9. -53.006
- 12. -53
- 15. -42.00003
- 18. -0.4999988
- 21. 0.5994
- 24. 0.6
- 27. -2861.4492

Function Transformations

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
 - (2) Shift left 1 unit
 - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
 - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
 - (2) Vertical compression by 4
 - (3) Move right 7 units

Write the function g(x) that is a result of the following ordered sequence of transformations to f(x) = |x|.

- 4. (1) Reflect across x-axis
 - (2) Shift right 3 units
 - (3) Horizontal stretch by factor of 5
- 5. (1) Shift down 2 units
 - (2) Reflect across y-axis
 - (3) Shift up 1 unit
- 6. (1) Horizontal compression by factor of 7
 - (2) Vertical compression by factor of 4
 - (3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function g(x) after the following ordered sequence of transformations.

- 7. (1) Shift up 2 units
 - (2) Horizontal stretch by 5
 - (3) Shift left 3 units
- 8. (1) Vertical compression by factor of 3
 - (2) Reflect across y-axis
 - (3) Horizontal compression by 5
- 9. (1) Shift right 8 units
 - (2) Reflect across x-axis
 - (3) Horizontal compression by factor of 4

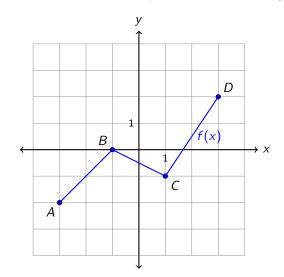
Write the final equation of g(x) if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

- 10. (1) Shift right 2 units
 - (2) Horizontal stretch by factor 3
 - (3) Shift down 2 units
 - (4) Reflect across x-axis
- 11. (1) Horizontal stretch by factor 3
 - (2) Shift left 1 unit
 - (3) Shift up 2 units
 - (4) Reflect across y-axis
- 12. (1) Vertical stretch by factor 5
 - (2) Horizontal stretch by factor 2
 - (3) Shift up 3 units
 - (4) Reflect across x-axis

Find the equation for g(x) if g(x) is found by performing the following *ordered* sequence of transformations to $f(x) = \frac{1}{x}$.

- 13. (1) Shift left 3 spaces
 - (2) Reflect across y-axis
 - (3) Shift down 5 spaces
 - (4) Vertical stretch by factor of 7
- 14. (1) Shift up 3 spaces
 - (2) Reflect across x-axis
 - (3) Shift right 5 spaces
 - (4) Horizontal compression by factor of 7

Given the graph of f(x) below, find the new coordinates of each point after the following transformations.



15.
$$-2f(x+1)$$

16.
$$f\left(-\frac{1}{2}x\right) - 3$$

17.
$$\frac{1}{2}f(-x-2)+2$$

18.
$$f(2x+2)-1$$

19.
$$-3f(-x+1)+2$$

20.
$$5f\left(-\frac{1}{2}x\right)$$

5.1 Answer Key

1.
$$g(x) = 3f(-x+1)$$

2.
$$g(x) = f(2x) + 1$$

3.
$$g(x) = -\frac{1}{4}f(x-7)$$

4.
$$g(x) = -\left|\frac{1}{5}x - 3\right|$$

5.
$$g(x) = |-x| - 1$$

6.
$$g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$$

7.
$$g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$$

8.
$$g(x) = \frac{1}{3}\sqrt{-5x}$$

9.
$$g(x) = -\sqrt{4x - 8}$$

10.
$$g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$$

11.
$$g(x) = \sqrt{\frac{1}{3}(-x+1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$$

12.
$$g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$$

13.
$$g(x) = \frac{7}{-x+3} - 35$$

14.
$$g(x) = -\frac{1}{7x-5} - 3$$

15.
$$A'(-4,4)$$
, $B'(-2,0)$, $C'(0,2)$, $D'(2,-4)$

16.
$$A'(6,-5)$$
, $B'(2,-3)$, $C'(-2,-4)$, $D'(-6,-1)$

17.
$$A'(1,1)$$
, $B'(-1,2)$, $C'(-3,1.5)$, $D'(-5,3)$

18.
$$A'(-2.5, -3)$$
, $B'(-1.5, -1)$, $C'(-0.5, -2)$, $D'(0.5, 1)$

19.
$$A'(4,8)$$
, $B'(2,2)$, $C'(0,5)$, $D'(-2,-4)$

20.
$$A'(6, -10)$$
, $B'(2, 0)$, $C'(-2, -5)$, $D'(-6, 10)$

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given f(x) = x + 5, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

- 1. (g f)(x)
- 2. (fh)(14)
- 3. (f+g)(x)

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-3	0	-1	3	1	2	4	-4	$\overline{-2}$
g(x)	3	-1	0	1	4	-2	-4	2	-3

- 4. (f+g)(-2) 5. (f-g)(0) 6. (fg)(1)
- 7. $\left(\frac{f}{g}\right)$ (3)
- 8. (f+f)(-4)

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 9. (f+g)(2)
- 10. (f-g)(1)
- 11. (g-f)(-3)
- 12. (fg)(4)
- 13. $\left(\frac{f}{g}\right)(0)$

6.2 Operations with Functions: Domain

Given $f(x) = \sqrt{2x+7}$ and g(x) = 3x+3, find the domain of each.

- 1. (f + g)(x)
- $2. \left(\frac{f}{g}\right)(x)$
- 3. $\left(\frac{g}{f}\right)(x)$

6.3 Difference Quotient

Write the difference quotient for each.

- 1. f(x) = 2x 7
- 4. $f(x) = \frac{3}{x+2}$
- 7. $g(x) = \frac{5}{x}$
- 10. $h(x) = \sqrt{7x+5}$
- 13. $f(x) = x^3 + 5x$

- 2. $g(x) = x^2 + 4x$
- 5. $g(x) = \sqrt{3x}$
- 8. $f(x) = -2x^2 + 3x 5$
- 11. $f(x) = -x^2 + x$

- 3. h(x) = -1
- 6. $f(x) = x^2 2x + 5$
- 9. $g(x) = \frac{6}{2x+3}$
- 12. f(x) = 3x 1

6.4 Answer Key

Adding, Subtracting, Multiplying, and Dividing Functions

1. $x^2 - x - 6$

2. 38

3. $x^2 + x + 4$

4. -1

5. -3

6. -4

7. -2

8. -6

9. -3

10. 6

11. -3

12. 10

13. $\frac{1}{4}$

Operations with Functions: Domain

1. $\left[-\frac{7}{2},\infty\right)$

2. $\left[-\frac{7}{2}, -1\right) \cup \left(-1, \infty\right)$

3. $\left(-\frac{7}{2},\infty\right)$

Difference Quotient

1. 2

4. $\frac{-3}{(x+2)(x+h+2)}$

7. $\frac{-5}{x(x+h)}$

10. $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$

13. $3x^2 + 3xh + h^2 + 5$

2. 2x + h + 4

 $5. \ \frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$

8. -4x - 2h + 3

11. -2x - h + 1

3. 0

6. 2x + h - 2

9. $\frac{-12}{(2x+3)(2x+2h+3)}$

12. 3

Polynomials and Their Graphs

Find the degree, leading term, leading coefficient, and constant term of the following polynomials.

1.
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

2.
$$g(x) = 4x^2 - 16x^6 + 3x$$

3.
$$h(x) = 1 + x^{11} - 4x^8$$

4.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$
 5. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$ 6. $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$

6.
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

7.
$$g(x) = \frac{1}{2}x^3 - \frac{\pi}{9}x^2 + x\sqrt{2} - 3^4$$

8.
$$h(x) = 7(x+1)^2(x-2)^3$$

7.
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$
 8. $h(x) = 7(x+1)^2(x-2)^3$ 9. $j(x) = -\frac{1}{2}(3x+2)^2(x-1)^5$

Determine the end behavior of each.

10.
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

11.
$$g(x) = 4x^2 - 16x^6 + 3x$$

12.
$$h(x) = 1 + x^{11} - 4x^8$$

13.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

13.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$
 14. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$ 15. $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$

15.
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

16.
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$
 17. $h(x) = 5(x+1)^2(x-2)^3$

17.
$$h(x) = 5(x+1)^2(x-2)^3$$

18.
$$j(x) = -\frac{1}{2} (3x+2)^2 (x-1)^5$$

Find the zeros of each. Round to 2 decimal places when necessary.

19.
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

20.
$$g(x) = 4x^2 - 16x^6 + 3x$$

21.
$$h(x) = 1 + x^{11} - 4x^8$$

22
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

22.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$
 23. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$ 24. $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$

$$24 f(x) = -6x^3 + 2x^2 + 7x^4 - 7x^4$$

25.
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$
 26. $h(x) = 5(x+1)^2(x-2)^3$

26.
$$h(x) = 5(x+1)^2(x-2)^3$$

27.
$$j(x) = -\frac{1}{2} (3x+2)^2 (x-1)^5$$

7.1 Answer Key

- 1. Degree = 5, Leading Term = $-x^5$, Leading Coefficient = -1, Constant = none (or 0)
- 2. Degree = 6, Leading Term = $-16x^6$, Leading Coefficient = -16, Constant = none (or 0)
- 3. Degree = 11, Leading Term = x^{11} , Leading Coefficient = 1, Constant = 1
- 4. Degree = 4, Leading Term = $-x^4$, Leading Coefficient = -1, Constant = 6
- 5. Degree = 5, Leading Term = $0.25x^5$, Leading Coefficient = 0.25, Constant = none (or 0)
- 6. Degree = 3, Leading Term = $-6x^3$, Leading Coefficient = -6, Constant = -1
- 7. Degree = 3, Leading Term = $\frac{1}{3}x^3$, Leading Coefficient = $\frac{1}{3}$, Constant = 3^4
- 8. Degree = 5, Leading Term = $7x^5$, Leading Coefficient = 7, Constant = -56
- 9. Degree = 7, Leading Term = $-\frac{9}{2}x^7$, Leading Coefficient = $-\frac{9}{2}$, Constant = 2
- 10. $\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$
- 11. $\lim_{x \to -\infty} g(x) = -\infty$, $\lim_{x \to \infty} g(x) = \infty$
- 12. $\lim_{x \to -\infty} h(x) = -\infty$ $\lim_{x \to \infty} h(x) = \infty$
- 13. $\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = -\infty$
- 14. $\lim_{x \to -\infty} g(x) = -\infty$ $\lim_{x \to \infty} g(x) = \infty$
- 15. $\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$
- 16. $\lim_{x \to -\infty} g(x) = -\infty$ $\lim_{x \to \infty} g(x) = \infty$
- 17. $\lim_{x \to -\infty} h(x) = -\infty$ $\lim_{x \to \infty} h(x) = \infty$
- 18. $\lim_{x \to -\infty} j(x) = \infty$ $\lim_{x \to \infty} j(x) = -\infty$
- 19. (-1.92, 0), (0, 0)
- 20. (0,0), (0.83,0)
- 21. (-0.83, 0), (0.86, 0), (1.58, 0)
- 22. (-2.25, 0), (1.90, 0)
- 23. (-2.48, 0), (0, 0), (1.85, 0)
- 24. (-0.42, 0), (0.79, 0)
- 25. (6.42, 0)
- 26. (-1,0), (2,0)
- 27. $\left(-\frac{2}{3},0\right)$, $\left(1,0\right)$

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

1.
$$(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$$

2.
$$(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$$

3.
$$(4x^3 - 3x^2 + x + 1) \div (x + 2)$$

4.
$$(5x^4 - x^2 + x - 2) \div (x^2 + 2)$$

5.
$$(10x^3 + 27x^2 + 8x - 11) \div (2x + 3)$$

6.
$$(7x^3 + 23x^2 + 12x + 1) \div (x^2 + 3x + 1)$$

7.
$$(28x^3 - 27x^2 - 4x + 17) \div (4x + 3)$$

8.
$$(7x^3 - 27x + 4) \div (x^2 - 5)$$

9.
$$(11x^6 - 24x^5 + 15x^4 - 19x^3 - 16x^2 + 21x - 8) \div (x - 2)$$

10.
$$(12x^5 - 15x^4 - 11x^3 + 16x^2 - 15x + 17) \div (3x^2 - 5)$$

11.
$$(6x^4 + 20x^3 - 13x^2 + 20x + 25) \div (x + 4)$$

12.
$$(24x^5 + 30x^4 - 21x^3 - 4x^2 + 3x - 25) \div (6x^3 + 3x^2 + 3)$$

13.
$$(3x^5 - 22x^4 + 12x^3 + 10x^2 - 7x + 24) \div (3x^2 - x - 4)$$

14.
$$(3x^4 - 23x^2 - 15x^3 + 28x + 24) \div (x - 6)$$

15.
$$(-29x^2 + 6x^6 - 29x^3 + 25x^4 - 15x^5 - 25 - 29x) \div (3x^3 - 6x^2 - 3 - x)$$

16.
$$(12x^6 + 16x^5 - 5x^4 + 12x^3 - 17x^2 - x - 23) \div (x+2)$$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

1.
$$(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$$

2.
$$(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x+1)$$

3.
$$(x^{23} - 5x^{20} + 17x^8 - 5) \div (x+2)$$

4.
$$(-7x^{17} + 40x^{15} - 6x^8 + 4x^3) \div (x-3)$$

8.3 Answer Key

Dividing Polynomials

- 1. $4x^2 2x + 5$
- 2. $4y^2 + 8y + 7 \frac{72}{3y+5}$
- 3. $4x^2 11x + 23 \frac{45}{x+2}$
- 4. $5x^2 11 + \frac{x+20}{x^2+2}$
- 5. $5x^2 + 6x 5 + \frac{4}{2x+3}$
- 6. $7x + 2 + \frac{-x-1}{x^2+3x+1}$
- 7. $7x^2 12x + 8 \frac{7}{4x+3}$
- 8. $7x + \frac{8x+4}{x^2-5}$
- 9. $11x^5 2x^4 + 11x^3 + 3x^2 10x + 1 \frac{6}{x-2}$
- 10. $4x^3 5x^2 + 3x 3 + \frac{2}{3x^2 5}$
- 11. $6x^3 4x^2 + 3x + 8 \frac{7}{x+4}$
- 12. $4x^2 + 3x 5 + \frac{-x^2 6x 10}{6x^3 + 3x^2 + 3}$
- 13. $x^3 7x^2 + 3x 5 + \frac{4}{3x^2 x 4}$
- 14. $3x^3 + 3x^2 5x 2 + \frac{12}{x-6}$
- 15. $2x^3 x^2 + 7x + 6 + \frac{11x^2 2x 7}{3x^3 6x^2 3 x}$
- 16. $12x^5 8x^4 + 11x^3 10x^2 + 3x 7 \frac{9}{x+2}$

Remainder and Factor Theorems

- 1. 6
- 2. 13
- 3. -13,627,141
- 4. -330,064,119

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1.
$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

3.
$$f(x) = \frac{3x}{x+4}$$

5.
$$h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

7.
$$g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

9.
$$f(x) = \frac{6x^3 - 21x^2 - 51x + 30}{3x^2 + 7x + 2}$$

11.
$$f(x) = \frac{x^3 + x^2 - 6x}{3x^2 - 3x - 6}$$

13.
$$f(x) = \frac{x-4}{-2x^2+4x+16}$$

15.
$$f(x) = \frac{x^2 + x - 2}{3x^2 + 3x - 18}$$

2.
$$g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

4.
$$g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

6.
$$f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

8.
$$h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

10.
$$g(x) = \frac{10x^2 - 29x - 21}{10x^3 - 33x^2 - 7x}$$

12.
$$f(x) = \frac{x^2 - 4x + 3}{2x^2 + 2x - 12}$$

14.
$$f(x) = \frac{x^3 - 2x^2 - 8x}{x^3 - 2x^2 - 3x}$$

16.
$$f(x) = \frac{x^2 - 3x + 2}{4x^2 - 12x}$$

State the end behavior of each.

17.
$$k(x) = \frac{5x^3 - 7x^2 + 8}{-3x^3 + 6x - 4}$$

18.
$$m(x) = \frac{2x-1}{3x^2+7x+1}$$

9.1 Answer Key

- 1. Domain: $x \neq \frac{1}{2}$, 7; V.A.: $x = \frac{1}{2}$, x = 7; H.A.: y = 1
- 2. Domain: $x \neq -3$, 4; V.A.: x = -3x = 4; Obl. Asymp: y = 3x + 10
- 3. Domain: $x \neq -4$; V.A.: x = -4; H.A.: y = 3
- 4. Domain: $x \neq 1$; V.A.: x = 1; Obl. Asymp: y = x + 4
- 5. Domain: $x \neq 0, 1$; V.A.: x = 0 and x = 1; H.A.: y = 0
- 6. Domain: $x \neq -1, 5$; V.A. x = -1; Hole @ $(5, \frac{13}{3})$; Obl. Asym y = 2x 5
- 7. Domain: $x \neq -6, 0, 4$; V.A. x = -6, x = 0; Hole @ $(4, \frac{21}{40})$; H.A. y = 0
- 8. Domain: $x \neq -6, -\frac{3}{8}$; V.A. $x = -6, x = -\frac{3}{8}$; H.A. $y = \frac{1}{4}$
- 9. Domain: $x \neq -2$, $-\frac{1}{3}$; Hole @ (-2, -21); V.A.: $x = -\frac{1}{3}$; Obl. Asymp: $y = 2x \frac{35}{3}$
- 10. Domain: $x \neq -\frac{1}{5}$, 0, $\frac{7}{2}$; Hole @ $(\frac{7}{2}, \frac{82}{259})$; V.A. $x = -\frac{1}{5}$ and x = 0; H.A. y = 0
- 11. Domain: $x \neq -1$, 2; V.A. x = -1; Hole @ $\left(2, \frac{10}{9}\right)$; Obl. Asymp: $y = \frac{1}{3}x + \frac{2}{3}$
- 12. Domain: $x \neq -3$, 2; V.A. x = -3 and x = 2; H.A. $y = \frac{1}{2}$
- 13. Domain: $x \neq -2$, 4; V.A. x = 2; Hole @ $\left(4, -\frac{1}{12}\right)$; H.A. y = 0
- 14. Domain: $x \neq -1$, 0, 3; V.A. x = -1 and x = 3; Hole @ $(0, \frac{8}{3})$; H.A. y = 1
- 15. Domain: $x \neq -3$, 2; V.A. x = -3 and x = 2; H.A. $y = \frac{1}{3}$
- 16. Domain: $x \neq 0$, 3; V.A. x = 0 and x = 3; H.A. $y = \frac{1}{4}$
- 17. $\lim_{x \to -\infty} k(x) = \infty = \lim_{x \to \infty} k(x) = -\frac{5}{3}$
- 18. $\lim_{x \to -\infty} m(x) = \infty = \lim_{x \to \infty} m(x) = 0$

Polynomial and Rational Inequalities

Polynomial Inequalities 10.1

Solve each. Write your answers using interval notation.

1.
$$6x^3 - 4x^2 - 10x > 0$$

3.
$$3x^3 - 7x^2 - 22x + 8 < 0$$

5.
$$12x^4 + 76x^3 + 43x^2 - 346x - 280 \ge 0$$

7.
$$-x^2 - 7x - 6 \le 0$$

9.
$$-x^4 - 6x^3 + 61x^2 + 234x - 1008 \ge 0$$

11.
$$-3x^4 + 123x^3 + 142x^2 - 424x + 320 < 122x^3$$

13.
$$-3x^4 - 22x^3 + 271x^2 + 152x - 96 \ge 267x^2$$

2.
$$x^4 < 9x^2$$

4.
$$3x^2 - 4x + 1 \le 0$$

6.
$$-2x^4 + 49x^2 + 21x^3 - 1029x + 2401 \ge 0$$

8.
$$x^2 + 4x + 4 < 0$$

10.
$$-x^2 + 3x + 1 > 3$$

12.
$$-x^4 - 1120 + 77x^2 - 36x + 15x^3 > 15x^3$$

10.2 **Domain**

State the domain of each. Write your answers using interval notation.

1.
$$b(x) = \sqrt{21x^2 - 23x - 20}$$

2.
$$f(x) = \frac{3}{\sqrt{3x^2+2x-1}}$$

3.
$$g(x) = \sqrt[4]{2x^3 + 9x^2 + 12x + 4}$$

Rational Inequalities

Solve each. Write your answers using interval notation.

1.
$$\frac{3x-4}{x+1} < 0$$

4.
$$\frac{x+2}{x-4} \le 1$$

7.
$$\frac{x-4}{x+7} < 0$$

10.
$$\frac{2x-50}{5x+15} < -1$$

10.
$$\frac{2x-50}{5x+15} \le -1$$

$$10. \ \frac{2x-50}{5x+15} \le -1$$

13. $-\frac{3}{x+6} > -\frac{4}{x+7}$

$$2. \ \frac{x^2 + 3x + 2}{x - 7} \le 0$$

$$5. \ \frac{x^2 - 7x - 8}{x^2 - 4x - 32} \ge 0$$

8.
$$\frac{x+5}{x+7} < 0$$

11.
$$\frac{x+5}{x^2-2x-15} \le 0$$

3.
$$\frac{x^2 - 4x + 4}{x^2 - 1} \ge 0$$

6.
$$\frac{4+3x}{5-x} \le 2$$

9.
$$\frac{2x-26}{5x+20} > -3$$

12.
$$-\frac{2}{x} \ge -\frac{3}{x+1}$$

10.4 Answer Key

Polynomial Inequalities

1.
$$\left[-1,0\right] \cup \left[\frac{5}{3},\infty\right)$$

3.
$$(-\infty, -2) \cup (\frac{1}{3}, 4)$$

5.
$$(-\infty, -4] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$$

7.
$$(-\infty, -6] \cup [-1, \infty)$$

9.
$$[-8, -7] \cup [3, 6]$$

11.
$$(-\infty, -8] \cup \left[\frac{4}{3}, 2\right] \cup [5, \infty)$$

13.
$$[-6, -4] \cup \left[\frac{2}{3}, 2\right]$$

Domain

1.
$$\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$$

2.
$$\left(-\infty, -1\right) \cup \left(\frac{1}{3}, \infty\right)$$

3.
$$\{-2\} \cup \left[-\frac{1}{2}, \infty\right)$$

2.
$$(-3,0) \cup (0,3)$$

4.
$$\left[\frac{1}{3}, 1\right]$$

6.
$$\left[-7, \frac{7}{2}\right] \cup 7$$

12.
$$[-8, -4] \cup [5, 7]$$

Rational Inequalities

1.
$$\left(-1, \frac{4}{3}\right)$$

3.
$$(-\infty, -1) \cup (1, \infty)$$

5.
$$(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$$

7.
$$(-7,4)$$

9.
$$(-\infty, -4) \cup (-2, \infty)$$

11.
$$(-\infty, -5] \cup (-3, 5)$$

13.
$$(-7, -6) \cup (-3, \infty)$$

2.
$$(-\infty, -2] \cup [-1, 7)$$

4.
$$(-\infty, 4)$$

6.
$$(-\infty, 1.2] \cup (5, \infty)$$

8.
$$(-7, -5)$$

10.
$$(-3, 5]$$

12.
$$(-1,0) \cup [2,\infty)$$

Function Compositions

Given f(x) = x - 5, $g(x) = 4 + \sqrt{2x + 1}$, and $h(x) = \frac{3}{x + 7}$, simplify each and state the domain.

- 1. $(f \circ g)(x)$
- 2. $(g \circ f)(x)$
- 3. h(h(x))

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4	
f(x)	-3	0	-1	3	1	2	4	-4	-2	
g(x)	3	-1	0	1	4	-2	-4	2	-3	

- 4. $(f \circ g)(-1)$ 5. $(g \circ g)(0)$ 6. $(f \circ f)(2)$ 7. $(g \circ g)(-3)$ 8. f(g(0))

Given $f(x) = \sqrt{3x+2}$, $g(x) = x^2 - 1$, and h(x) = 9x - 2, find each of the following.

- 9. $(g \circ f)(x)$
- 10. f(g(x))
- 11. $(h \circ h)(x)$

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 12. $(f \circ g)(-1)$ 13. $(g \circ f)(-4)$
- 14. f(g(3))
- 15. g(g(-2)) 16. $(f \circ f)(-5)$

11.1 Answer Key

- 1. $-1 + \sqrt{2x+1}$ Domain: $\left[-\frac{1}{2}, \infty\right)$
- 2. $4 + \sqrt{2x 9}$ Domain: $\left[\frac{9}{2}, \infty\right)$
- 3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty,-\frac{52}{7}\right)\cup\left(-\frac{52}{7},-7\right)\cup\left(-7,\infty\right)$
- 4. 2
- 5. -3
- 6. -2
- 7. 1
- 8. -2
- 9. 3x + 1
- 10. $\sqrt{3x^2-1}$
- 11. 81x 20
- 12. -2
- 13. 3
- 14. -4
- 15. 2
- 16. -4

Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1.
$$f(x) = \sqrt{-2x+3} + 1$$

2.
$$g(x) = (x+4)^2 - 1$$
, $x \le -4$

3.
$$h(x) = \frac{9x}{4x-1}$$

4.
$$f(x) = \sqrt{x} - 3$$

$$5. g(x) = \frac{1}{1-x}$$

6.
$$h(x) = x^2 + 6x + 4$$
, $x \le -3$

7.
$$f(x) = \sqrt{5x-4}$$

8.
$$g(x) = x^2 - 2x + 3, x \le 1$$

9.
$$h(x) = \frac{3}{x-1}$$

12.1 Answer Key

1. f^{-1}	(x) = -	$\frac{1}{2} ((x -$	$(-1)^2 - 3$	3)
-------------	---------	---------------------	--------------	----

	Domain	Range
f(x)	$(-\infty, 1.5]$	$[1,\infty)$
$f^{-1}(x)$	$[1,\infty)$	$(-\infty, 1.5]$

2.
$$g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
g(x)	$(-\infty, -4]$	$[-1,\infty)$
$g^{-1}(x)$	$[-1,\infty)$	$(-\infty, -4]$

3.
$$h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
h(x)	$(-\infty,1/4)\cup(1/4,\infty)$	$(\infty,9/4)\cup(9/4,\infty)$
$h^{-1}(x)$	$(\infty,9/4)\cup(9/4,\infty)$	$(-\infty,1/4)\cup(1/4,\infty)$

4.
$$f^{-1}(x) = (x+3)^2$$

	Dom	Ran
f(x)	$[0,\infty)$	$[-3,\infty)$
$f^{-1}(x)$	$[-3,\infty)$	[0, ∞)

5.
$$g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
g(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$g^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

6.
$$h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
h(x)	$(-\infty, -3]$	$[-5,\infty)$
$h^{-1}(x)$	$[-5,\infty)$	$(-\infty, -3]$

7.
$$f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

	Dom	Ran
f(x)	$\left[rac{4}{5},\infty ight)$	[0, ∞)
$f^{-1}(x)$	[0, ∞)	$\left[\frac{4}{5},\infty\right)$

8.
$$g^{-1}(x) = -\sqrt{x-2} + 1$$

	Dom	Ran
g(x)	$(-\infty,1]$	[2, ∞)
$g^{-1}(x)$	[2, ∞)	$(-\infty,1]$

9.
$$h^{-1}(x) = \frac{3}{x} + 1$$

	Dom	Ran
h(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$h^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

Exponential Functions

13.1 Transforming Exponential Functions

Given $f(x) = e^x$, determine the specific transformations done to f(x) to produce g(x).

- 1. $g(x) = -3e^{x+1}$
- 2. $g(x) = \frac{1}{4}e^{-5x} 2$
- 3. $g(x) = e^{2x+7}$
- 4. $g(x) = 5e^{-x-2} + 1$
- 5. $g(x) = 0.1e^{0.25x-3} 4$

13.2 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

- 1. $f(x) = 3 + e^{2x}$
- 2. $h(x) = 5^{-x}$
- 3. $h(x) = -\frac{2}{3}e^{x+7} + 1$
- 4. $f(x) = -7e^x + 4$
- 5. $g(x) = \frac{1}{3}e^{2x+1} 5$
- 6. $h(x) = -\frac{1}{2}e^{-4x} + 1$

Transforming Exponential Functions

- 1. Shift left 1 unit, vertical stretch by factor of 3, reflect across x-axis
- 2. Horizontal compression by factor of 5, reflect across y-axis, vertical compression by factor of 4, shift down 2 units
- 3. Shift left 7 units, horizontal compression by factor of 2
- 4. Shift right 2 units, reflect across y-axis, vertical stretch by factor of 5, shift up 1 unit
- 5. Shift right 3 units, horizontal compression by factor of 4, vertical compression by factor of 10, shift down 4 units

End Behavior

1.
$$\lim_{x \to -\infty} f(x) = 3$$
 $\lim_{x \to \infty} f(x) = \infty$

2.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = 0$

3.
$$\lim_{x \to -\infty} h(x) = 1$$
 $\lim_{x \to \infty} h(x) = -\infty$

4.
$$\lim_{x \to -\infty} f(x) = 4$$
 $\lim_{x \to \infty} f(x) = -\infty$

5.
$$\lim_{x \to -\infty} f(x) = -5$$
 $\lim_{x \to \infty} f(x) = \infty$

6.
$$\lim_{x \to -\infty} f(x) = -\infty$$
 $\lim_{x \to \infty} f(x) = 1$

Logarithmic Functions

Write each of the following in exponential or logarithmic form.

- 1. ln(a) = 7
- 2. $\log_4(x+1) = 9$
- 3. $\log(5x) = 30$
- 4. ln(w) = c
- 5. $5^x = 19$
- 6. $8^{-3} = \frac{1}{512}$
- 7. $e^{14} = x$
- 8. $(1.1)^{-t} = 50$

Find the domain of each. Write your answers in interval notation.

- 9. $b(x) = \log_7(x^2 8x + 6)$
- 10. $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$
- 11. $f(x) = -7 \ln (x^2 + 9x + 8)$
- 12. $g(x) = \log (5x^2 + 13x 6)$
- 13. $h(x) = 3 \log_2 (x^3 + 2x^2 x 2)$
- 14. $c(x) = \ln(4x^2 15x 4)$

State the end behavior of each.

15.
$$j(x) = 5 \log_3 (2x - 5) - 2$$

- 1. $e^7 = a$
- 2. $4^9 = x + 1$
- 3. $10^{30} = 5x$
- 4. $e^c = w$
- 5. $\log_5(19) = x$
- $6. \log_8\left(\frac{1}{512}\right) = -3$
- 7. ln(x) = 14
- 8. $\log_{1.1}(50) = -t$
- 9. $(-\infty, 0.838) \cup (7.162, \infty)$
- 10. $(-3, -2) \cup (-1, \infty)$
- 11. $(-\infty, -8) \cup (-1, \infty)$
- 12. $(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$
- 13. $(-2, -1) \cup (1, \infty)$
- 14. $\left(-\infty, -\frac{1}{4}\right) \cup \left(4, \infty\right)$
- 15. $\lim_{x \to (5/2)^+} j(x) = -\infty \quad \lim_{x \to \infty} j(x) = \infty$

Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

1.
$$\log_b \left(\frac{x^2}{y^8} \right)$$

2.
$$\ln (ez)^3$$

3.
$$\log_5(x) + \log_5(9) - 2\log_5(w)$$

4.
$$\log_2(2^ab^3)$$

5.
$$\ln\left(\frac{w^7}{e^6}\right)$$

6.
$$5\log_4(m) - 3\log_4(n) + 2\log_4(p)$$

Write an equivalent expression for each of the following using natural logarithms.

7.
$$\log_7(10)$$

8.
$$\log_9(x)$$

9.
$$\log_b(c)$$

10.
$$\log_3(10)$$

11.
$$\log_{17}(\pi)$$

12.
$$\log_w(x)$$

Suppose that $\log_a(b) = 5$, $\log_a(c) = 12$, and $\log_a(d) = 9$. Evaluate each of the following.

13.
$$\log_a(bc)$$

14.
$$\log_a(c^3)$$

15.
$$\log_a \left(\frac{d}{c}\right)$$

16.
$$\log_a \left(\frac{bd}{c} \right)$$

17.
$$\log_a (b^7 c)$$

18.
$$\log_a \left(\frac{c^2}{d}\right)$$

19.
$$\log_a \left(\sqrt{bc} \right)$$

20.
$$\log_a ((bd)^2)$$

21.
$$\log_a \left(\sqrt[3]{d^2} \right)$$

22.
$$\log_a \left(\sqrt{b^5} \right)$$

23.
$$\log_a \left(\frac{b^6 c}{d^3} \right)$$

24.
$$\log_a \left(b^2 c^3 d^4 \right)$$

- $1. \ 2\log_b(x) 8\log_b(y)$
- 2. $3 + 3 \ln(z)$
- 3. $\log_5\left(\frac{9x}{w^2}\right)$
- 4. $a + 3 \log_2(b)$
- 5. $7 \ln(w) 6$
- 6. $\log_4\left(\frac{m^5p^2}{n^3}\right)$
- 7. $\frac{\ln(10)}{\ln(7)}$
- $8. \ \frac{\ln(x)}{\ln(9)}$
- 9. $\frac{\ln(c)}{\ln(b)}$
- 10. $\frac{\ln(10)}{\ln(3)}$
- $11. \ \frac{\ln(\pi)}{\ln(17)}$
- $12. \ \frac{\ln(x)}{\ln(w)}$
- 13. 17
- 14. 36
- 15. -3
- 16. 2
- 17. 47
- 18. 15
- 19. 17/2
- 20. 28
- 21. 6
- 22. 25/2
- 23. 15
- 24. 82

Exponential Equations

Solve each. Round to 3 decimal places when necessary.

1.
$$3e^{x-2} = 7$$

3.
$$2^{3x+4} = 32^{x-7}$$

5.
$$7^{4x+1} \ge 343$$

7.
$$3(4.1)^{x-2} = 8$$

9.
$$8(17)^{-5x} = 22$$

11.
$$12^{-10x} + 8 = 80$$

13.
$$8(8)^{10x} - 1 = 55.2$$

15.
$$6(16)^{4x-9} = 19$$

17.
$$3^{9-6x} - 7 = 26$$

2.
$$5^x + 4 > 1$$

4.
$$5e^{7x} + 10 = 42$$

6.
$$1000e^{0.04x} = 2000$$

8.
$$2^{x+1} = 5^{7x-5}$$

10.
$$-3(11)^{x-10} = -58$$

12.
$$-5(10)^{7x} + 9 = -46$$

14.
$$3(3)^{-5x} - 8 = 74$$

16.
$$-7(11)^{5x-7} = -3$$

16.1 Applications

- 1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?
- 2. Cadmium-109 has a half-life of about 1.267 years. If 50 mg are initially present, how many years will it take for 16 mg to remain?
- 3. The half-life of bismuth-207 is about 32.9 years. If 90 mg are initially present, how many years will it take for 75 mg to

1. $x \approx 2.847$

3. x = 19.5

5. $\left[\frac{1}{2}, \infty\right)$

7. $x \approx 2.695$

9. $x \approx -0.071$

11. $x \approx -0.172$

13. $x \approx 0.094$

15. $x \approx 2.354$

17. $x \approx 0.970$

Applications

1. Approximately 17,952 years

2. Approximately 2.0828 years

3. Approximately 8.6538 years

2. $(-\infty, \infty)$

4. $x \approx 0.265$

6. $x \approx 17.329$

8. $x \approx 0.827$

10. $x \approx 11.235$

12. $x \approx 0.149$

14. $x \approx -0.602$

16. $x \approx 1.323$

Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

1.
$$\log_5(x) + x \log_5(x) > 0$$

2.
$$\ln(8-x^2) = \ln(2-x)$$

3.
$$\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$$

4.
$$\log_3(2x+1) - \log_3(x-5) = \log_3(x+1)$$

5.
$$\log_4(x+1) + \log_4(x-5) > 2$$

6.
$$\log(x+1) - \log(x-5) = \log(x-3)$$

7.
$$x \log_3(x+2) - \log_3(x+2) = 0$$

8.
$$\log_{1/2}(x+1) > -3$$

9.
$$\log_{12}(4x+4) = \log_{12}(5x+1)$$

10.
$$\log_{15}(-4x+2) = \log_{15}(6-2x)$$

11.
$$\log_{11}(-5-3x^2) = \log_{11}(-2x^2+6x)$$

12.
$$\log_{16}(x^2+4) = \log_{16}(2x+3)$$

13.
$$\log_7(8x-1) = \log_7(x^2+14)$$

14.
$$-7 \log_5(x+5) = -7$$

15.
$$7 \log_8(-x) = 28$$

16.
$$-10\log_3(x-5) = -20$$

- 1. $(1, \infty)$
- 2. x = -2
- 3. $x \approx 1.571$
- 4. $x \approx 6.873$
- 5. $(2, \infty)$
- 6. x = 7
- 7. $x = \pm 1$
- 8. (-1,7)
- 9. x = 3
- 10. x = -2
- 11. No Solution
- 12. x = 1
- 13. x = 3, 5
- 14. x = 0
- 15. x = -4096
- 16. x = 14

Sequences

Write the first 4 terms of each sequence.

1.
$$a_n = 2(-3)^n$$

2.
$$b_n = \frac{n!}{2^n}$$

3.
$$c_{n+1} = 5c_n + 1$$
; $c_1 = 2$

4.
$$d_n = \frac{1}{2}d_{n-1} + n$$
; $d_1 = 3$

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round**.

5.
$$a_n = \{343, 667, 991, 1315, \dots\}$$
; Find the 582nd term.

6.
$$b_n = \{300, 240, 192, 153.6, \dots\}$$
; Find the 711th term.

7.
$$c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots \right\}$$
; Find the 8,675,309th term.

Given each sequence, find the first 4 digits after the decimal point in the scientific notation version of each term.

8.
$$a_n = 17, 33, 49, 65, ...; a_{21,972}$$

9.
$$b_n = 25$$
, 36, 49, 64, 81, ... $b_{413,401}$

10.
$$c_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots c_{152}$$

- 1. -6, 18, -54, 162
- $2. \ \frac{1}{2}, \ \frac{1}{2}, \ \frac{3}{4}, \ \frac{3}{2}$
- 3. 2, 11, 56, 281
- 4. 3, $\frac{7}{2}$, $\frac{19}{4}$, $\frac{51}{8}$
- 5. 188,587
- 6. 6882
- 7. 7634
- 8. 5155 (3.51553 \times 10⁵)
- 9. 7090 (1.7090369403 \times 10¹¹)
- 10. 1677 (1.1677487203 \times 10⁻⁴⁶)

Series

Find the sum of each, if possible.

1.
$$\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^{i}$$

$$2. \sum_{i=0}^{\infty} 3\left(-\frac{2}{3}\right)^{i}$$

$$3. \sum_{k=1}^{\infty} -2\left(\frac{1}{3}\right)^k$$

4.
$$\sum_{i=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^{j}$$

5.
$$\sum_{i=0}^{\infty} 1.2(0.8)^{i}$$

6.
$$\sum_{i=1}^{\infty} 1.2(0.8)^{i}$$

7.
$$\sum_{i=0}^{\infty} 0.8(1.2)^{i}$$

8.
$$\sum_{k=1}^{\infty} \frac{2}{3^k}$$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9.
$$9+13+17+21+\cdots+1565$$

10.
$$-3+6-12+24-48+\cdots+50,331,648$$

11.
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{981}$$

12.
$$2+4+6+8+10+\cdots+38,214$$

13.
$$3+7+11+15+\cdots+11,491$$

14.
$$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \dots + \frac{742}{743}$$

- 1. $\frac{1}{4}$
- 2. $\frac{9}{5}$
- 3. -1
- 4. Diverges
- 5. 6
- 6. 4.8
- 7. Diverges
- 8. 2
- 9. 306,930
- 10. -33, 554, 433
- 11. 7.4663
- 12. 365,096,556
- 13. 16,511,131
- 14. 733.8947

Angles and Radian Measure

Sketch each of the following. Then find a coterminal between 0 and 360° (or 0 and 2π radians) for each.

- 1. $-\frac{3\pi}{4}$
- 2. 900°
- 3. $\frac{27\pi}{10}$
- 4. −125°

Find the arc length and sector area formed by the central angle of each. Exact answers only.

5.



6.



7.



A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

- 8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
- 9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

1. $\frac{5\pi}{4}$



2. 180°



3. $\frac{7\pi}{10}$



4. 235°



- 5. $s = \frac{10\pi}{3}$ ft.; $A = \frac{40\pi}{3}$ sq.ft.
- 6. $s = \frac{26\pi}{9}$ cm.; $A = \frac{52\pi}{9}$ sq.cm.
- 7. $s = \frac{\pi}{8}$ mm.; $A = \frac{\pi}{64}$ sq.mm.
- 8. 26.18 rad/sec
- 9. 8.73 ft/sec

Trig Functions of Any Angle

Find the exact value of each of the six trig functions of θ if P is a point on the terminal side of θ .

1.
$$P(-2,3)$$

2.
$$P(0, -4)$$

3.
$$P(-2\sqrt{3}, 2)$$

4.
$$P(-3,5)$$

5.
$$P(-2,1)$$

6.
$$P(-4, -7)$$

Find the exact values of the 6 trig functions of the following angles.

7.
$$\theta = \frac{-17\pi}{4}$$
 8. $\theta = \frac{21\pi}{2}$ 9. $\theta = 24\pi$ 10. $-\frac{5\pi}{3}$

8.
$$\theta = \frac{21\pi}{2}$$

9.
$$\theta = 24\pi$$

10.
$$-\frac{5\pi}{3}$$

11.
$$\frac{23\pi}{6}$$

12.
$$-\frac{\pi}{2}$$

13.
$$\frac{10\pi}{3}$$

14.
$$-\frac{\pi}{3}$$

15.
$$\frac{11\pi}{4}$$

12.
$$-\frac{\pi}{2}$$
 13. $\frac{10\pi}{3}$ 14. $-\frac{\pi}{3}$ 15. $\frac{11\pi}{4}$ 16. $-\frac{13\pi}{2}$

1.
$$\sin \theta = \frac{3\sqrt{13}}{13}$$
, $\cos \theta = \frac{-2\sqrt{13}}{13}$, $\tan \theta = -\frac{3}{2}$, $\csc \theta = \frac{\sqrt{13}}{3}$, $\sec \theta = -\frac{\sqrt{13}}{2}$, $\cot \theta = -\frac{2}{3}$

2.
$$\sin \theta = -1$$
, $\cos \theta = 0$, $\tan \theta = \text{undef.}$, $\csc \theta = -1$, $\sec \theta = \text{undef.}$, $\cot \theta = 0$

$$3. \ \sin\theta=\frac{1}{2}, \cos\theta=-\frac{\sqrt{3}}{2}, \tan\theta=-\frac{\sqrt{3}}{3}, \csc\theta=2, \sec\theta=-\frac{2\sqrt{3}}{3}, \cot\theta=-\sqrt{3}$$

$$4. \ \sin\theta = \frac{5\sqrt{34}}{34}, \\ \cos\theta = -\frac{3\sqrt{34}}{34}, \\ \tan\theta = -\frac{5}{3}, \\ \csc\theta = \frac{\sqrt{34}}{5}, \\ \sec\theta = -\frac{\sqrt{34}}{3}, \\ \cot\theta = -\frac{3}{5}$$

5.
$$\sin\theta=\frac{\sqrt{5}}{5}$$
, $\cos\theta=-\frac{2\sqrt{5}}{5}$, $\tan\theta=-\frac{1}{2}$, $\csc\theta=\sqrt{5}$, $\sec\theta=-\frac{\sqrt{5}}{2}$, $\cot\theta=-2$

6.
$$\sin \theta = -\frac{7\sqrt{65}}{65}$$
, $\cos \theta = -\frac{4\sqrt{65}}{65}$, $\tan \theta = \frac{7}{4}$, $\csc \theta = -\frac{\sqrt{65}}{7}$, $\sec \theta = -\frac{\sqrt{65}}{4}$, $\cot \theta = \frac{4}{7}$

7.
$$\sin\theta = -\frac{\sqrt{2}}{2}$$
, $\cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = -1$, $\csc\theta = -\sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = -1$

8.
$$\sin \theta = 1$$
, $\cos \theta = 0$, $\tan \theta = \text{undefined}$, $\csc \theta = 1$, $\sec \theta = \text{undefined}$, $\cot \theta = 0$

9.
$$\sin\theta=0$$
, $\cos\theta=1$, $\tan\theta=0$, $\csc\theta=$ undefined, $\sec\theta=1$, $\cot\theta=$ undefined

$$10. \ \sin\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}, \ \cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}, \ \tan\left(-\frac{5\pi}{3}\right) = \sqrt{3}, \ \csc\left(-\frac{5\pi}{3}\right) = \frac{2\sqrt{3}}{3}, \ \sec\left(-\frac{5\pi}{3}\right) = 2, \ \cot\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{3}$$

$$11. \ \sin\left(\frac{23\pi}{6}\right) = -\frac{1}{2}, \ \cos\left(\frac{23\pi}{6}\right) = \frac{\sqrt{3}}{2}, \ \tan\left(\frac{23\pi}{6}\right) = -\frac{\sqrt{3}}{3}, \ \csc\left(\frac{23\pi}{6}\right) = -2, \ \sec\left(\frac{23\pi}{6}\right) = \frac{2\sqrt{3}}{3}, \ \cot\left(\frac{23\pi}{6}\right) = -\sqrt{3}$$

$$12. \ \sin\left(-\frac{\pi}{2}\right) = -1, \ \cos\left(-\frac{\pi}{2}\right) = 0, \ \tan\left(-\frac{\pi}{2}\right) = \text{undefined, } \csc\left(-\frac{\pi}{2}\right) = -1, \ \sec\left(-\frac{\pi}{2}\right) = \text{undefined, } \cot\left(-\frac{\pi}{2}\right) = 0$$

$$13. \ \, \sin\left(\frac{10\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \ \, \cos\left(\frac{10\pi}{3}\right) = -\frac{1}{2}, \ \, \tan\left(\frac{10\pi}{3}\right) = \sqrt{3}, \ \, \csc\left(\frac{10\pi}{3}\right) = -\frac{2\sqrt{3}}{3}, \ \, \sec\left(\frac{10\pi}{3}\right) = -2, \ \, \cot\left(\frac{10\pi}{3}\right) = \frac{\sqrt{3}}{3}$$

$$14. \ \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}, \ \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}, \ \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}, \ \csc\left(-\frac{\pi}{3}\right) = -\frac{2\sqrt{3}}{3}, \ \sec\left(-\frac{\pi}{3}\right) = 2, \ \cot\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$$

$$15. \ \sin\left(\frac{11\pi}{4}\right) = \frac{\sqrt{2}}{2}, \ \cos\left(\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \ \tan\left(\frac{11\pi}{4}\right) = -1, \ \csc\left(\frac{11\pi}{4}\right) = \sqrt{2}, \ \sec\left(\frac{11\pi}{4}\right) = -\sqrt{2}, \ \cot\left(\frac{11\pi}{4}\right) = -1$$

$$16. \ \sin\left(-\frac{13\pi}{2}\right) = -1, \ \cos\left(-\frac{13\pi}{2}\right) = 0, \ \tan\left(-\frac{13\pi}{2}\right) = \text{undefined, } \csc\left(-\frac{13\pi}{2}\right) = -1, \ \sec\left(-\frac{13\pi}{2}\right) = \text{undefined, } \cot\left(-\frac{13\pi}{2}\right) = 0$$

Graphs of Sine and Cosine Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1.
$$f(x) = -2\sin\left(3x - \frac{\pi}{4}\right) + 1$$

3.
$$f(x) = 2\sin\left(x - \frac{\pi}{3}\right) + 7$$

5.
$$h(x) = \sin\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

7.
$$f(x) = -4\cos\left(4x - \frac{\pi}{3}\right) + \pi$$

$$2. g(x) = \frac{1}{3} \cos \left(\frac{1}{2}x + 2\right)$$

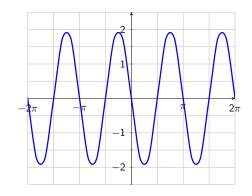
$$4. \ f(x) = -4\cos\left(\frac{2}{3}x - \frac{2\pi}{3}\right)$$

6.
$$f(x) = 3\sin\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

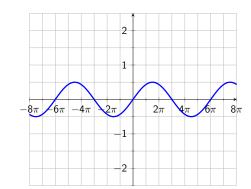
8.
$$g(x) = \frac{4}{9}\cos\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$$

Write the equation of each of the following in the form $y = a \sin(bx)$.

9.



10.



1. Amp = 2, Per =
$$\frac{2\pi}{3}$$
, P.S. = $\frac{\pi}{12}$ \rightarrow , V.S. = 1 \uparrow

2. Amp
$$=\frac{1}{3}$$
, Per $=4\pi$, P.S. $=4\leftarrow$, V.S. $=$ None

3. Amp = 2, Period =
$$2\pi$$
, P.S. = $\frac{\pi}{3}$ right, V.S. = 7 up

4. Amp = 4, Period =
$$3\pi$$
, P.S. = π right, V.S. = 0 (or none)

5. Amp = 1, Per =
$$\frac{8\pi}{3}$$
, P.S. = $\frac{\pi}{9}$ left, V.S. = 8 down

6. Amp = 3, Period =
$$\pi$$
, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down

7. Amp = 4, Period =
$$\frac{\pi}{2}$$
, P.S. = $\frac{\pi}{12}$ right, V.S. = π up

8. Amp
$$=\frac{4}{9}$$
, Period $=\frac{20\pi}{3}$, P.S. $=5\pi$ left, V.S. $=1$ down

9.
$$y = -2\sin(2x)$$

$$10. \ \ y = \frac{1}{2} \sin \left(\frac{1}{3} x \right)$$

Graphs of Other Trig Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1.
$$h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

2.
$$f(x) = 3\tan\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

3.
$$f(x) = -4 \cot \left(4x - \frac{\pi}{3}\right) + \pi$$

4.
$$g(x) = \frac{4}{9} \cot \left(\frac{3}{10} x + \frac{3\pi}{2} \right) - 1$$

5.
$$h(x) = \sec\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

6.
$$f(x) = 3\sec\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

7.
$$f(x) = -4\csc\left(4x - \frac{\pi}{3}\right) + \pi$$

8.
$$g(x) = \frac{4}{9}\csc\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$$

9.
$$f(x) = \frac{1}{10} \cot \left(3x + \frac{3\pi}{4}\right) + 5$$

10.
$$g(x) = 6 \csc \left(2x - \frac{\pi}{6}\right) + 5$$

11.
$$h(x) = 4 \csc \left(8x - \frac{\pi}{6}\right) - 1$$

12.
$$k(x) = \tan\left(\frac{1}{7}x + \frac{\pi}{4}\right) - 5$$

13.
$$f(x) = 7 \tan \left(8x + \frac{\pi}{6}\right)$$

14.
$$g(x) = 6 \tan \left(7x - \frac{\pi}{4}\right) - 4$$

15.
$$h(x) = 4\tan(5x) - 5$$

$$16. \ k(x) = \cot\left(\frac{1}{3}x - \frac{2\pi}{3}\right) - 5$$

17.
$$f(x) = 10 \cot \left(\frac{1}{8}x - \frac{\pi}{2}\right)$$

18.
$$g(x) = 10 \cot \left(5x - \frac{\pi}{4}\right) + 5$$

1. Amp = n/a, Per =
$$\frac{4\pi}{3}$$
, P.S. = $\frac{\pi}{9}$ left, V.S. = 8 down

2. Amp = n/a, Period =
$$\frac{\pi}{2}$$
, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down

3. Amp = n/a, Period =
$$\frac{\pi}{4}$$
, P.S. = $\frac{\pi}{12}$ right, V.S. = π up

4. Amp = n/a, Period =
$$\frac{10\pi}{3}$$
, P.S. = 5π left, V.S. = 1 down

5. Amp = n/a, Per =
$$\frac{8\pi}{3}$$
, P.S. = $\frac{\pi}{9}$ left, V.S. = 8 down

6. Amp = n/a, Period =
$$\pi$$
, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down

7. Amp = n/a, Period =
$$\frac{\pi}{2}$$
, P.S. = $\frac{\pi}{12}$ right, V.S. = π up

8. Amp = n/a, Period =
$$\frac{20\pi}{3}$$
, P.S. = 5π left, V.S. = 1 down

9. Amp = n/a, Period =
$$\frac{\pi}{3}$$
, P.S. = $\frac{\pi}{4}$ left, V.S. = 5 up

10. Amp = n/a, Period =
$$\pi$$
, P.S. = $\frac{\pi}{12}$ right, V.S. = 5 up

11. Amp = n/a, Period =
$$\frac{\pi}{4}$$
, P.S. = $\frac{\pi}{48}$ right, V.S. = 1 down

12. Amp = n/a, Period =
$$7\pi$$
, P.S. = $\frac{7\pi}{4}$ left, V.S. = 5 down

13. Amp = n/a, Period =
$$\frac{\pi}{8}$$
, P.S. = $\frac{\pi}{48}$ left, V.S. = 0 (or none)

14. Amp = n/a, Period =
$$\frac{\pi}{7}$$
, P.S. = $\frac{\pi}{28}$ right, V.S. = 4 down

15. Amp = n/a, Period =
$$\frac{\pi}{5}$$
, P.S. = 0 (or none), V.S. = 5 down

16. Amp = n/a, Period =
$$3\pi$$
, P.S. = 2π right, V.S. = 5 down

17. Amp = n/a, Period =
$$8\pi$$
, P.S. = 4π right, V.S. = 0 (or none)

18. Amp = n/a, Period =
$$\frac{\pi}{5}$$
, P.S. = $\frac{\pi}{20}$ right, V.S. = 5 up

Inverse Trig Functions

State the exact, simplified value of each or write as an expression of x.

1. $\cot^{-1}(-1)$

 $2. \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

3. $tan^{-1}(0)$

4. $\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

5. $\cos^{-1}\left(\frac{1}{2}\right)$

6. $\sec^{-1}(-2)$

7. $tan^{-1}\left(-\sqrt{3}\right)$

8. $\operatorname{sec}\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$

9. $\cot(\sec^{-1}(x))$

10. $\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$

11. $\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$

12. $\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$

13. $\cos(\tan^{-1}(7x))$

14. $\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$

15. $\csc\left(\arctan\left(-\frac{3}{2}\right)\right)$

16. $\cos\left(\sin^{-1}\left(\frac{7}{8}\right)\right)$

17. $\tan\left(\cos^{-1}\left(\frac{3}{x}\right)\right)$

18. $\csc\left(\sec^{-1}\left(\sqrt{2}\right)\right)$

19. $\tan\left(\sec^{-1}\left(\frac{\sqrt{17}}{4}\right)\right)$

20. $\sin\left(\tan^{-1}\left(\frac{4x}{5}\right)\right)$

- 1. $\frac{3\pi}{4}$
- 2. $-\frac{\pi}{3}$
- 3. 0
- 4. $\frac{\pi}{6}$
- 5. $\frac{\pi}{3}$
- 6. $\frac{2\pi}{3}$
- 7. $-\frac{\pi}{3}$
- 8. $\frac{5\sqrt{21}}{21}$
- 9. $\frac{1}{\sqrt{x^2 1}} = \frac{\sqrt{x^2 1}}{x^2 1}$
- 10. $\frac{\sqrt{16-9x^2}}{4}$
- 11. $-\frac{3\sqrt{5}}{2}$
- 12. $\frac{13\sqrt{22}}{44}$
- 13. $\frac{\sqrt{49x^2+1}}{49x^2+1}$
- 14. $\frac{\sqrt{64-x^2}}{x}$
- 15. $-\frac{\sqrt{13}}{3}$
- 16. $\frac{\sqrt{15}}{8}$
- 17. $\frac{\sqrt{x^2-9}}{3}$
- 18. $\sqrt{2}$
- 19. $\sqrt{17}$
- $20. \ \frac{4x}{\sqrt{16x^2 + 25}}$

Trig Equations and Inequalities

Solve each in the interval $[0, 2\pi)$. Write your answers to inequalities using interval notation.

- 1. tan(6x) = 1
- 2. $\cot(2x) = -\frac{\sqrt{3}}{3}$
- 3. $\sin^2(x) = \frac{3}{4}$
- $4. \sin(2x) = \cos(x)$
- 5. $\sin(2x) \ge \sin(x)$
- 6. $\cos(2x) < 0$
- $7. \ 2\sin\left(x \frac{\pi}{3}\right) = -1$
- 8. $3\tan\left(-2x+\frac{\pi}{2}\right)=\sqrt{3}$
- 9. $\sin^2(x) < \frac{1}{2}$
- 10. $tan^2(x) = 3 sec(x) 3$
- 11. $2\csc(x) 3\csc^2(x) = -2\csc^2(x) + 1$
- 12. $-2\cot(x) \csc^2(x) = 0$
- 13. $\tan(x) = -\tan(x)\cos(x)$
- 14. $3\cos(x) = 2\cos^2(x) + 1$
- 15. $\csc(x) \cot^2(x) + 1 = 0$
- 16. $-\sin(x) + \sin(2x) = 2\sin(2x)$
- 17. $3\cos(x) = \sin(2x) + 2\cos(x)$

- $1.\ \ \, \frac{\pi}{24},\frac{5\pi}{24},\frac{3\pi}{8},\frac{13\pi}{24},\frac{17\pi}{24},\frac{7\pi}{8},\frac{25\pi}{24},\frac{29\pi}{24},\frac{11\pi}{8},\frac{37\pi}{24},\frac{41\pi}{24},\frac{15\pi}{8}$
- 2. $\frac{\pi}{3}$, $\frac{5\pi}{6}$, $\frac{4\pi}{3}$, $\frac{11\pi}{6}$
- 3. $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$
- 4. $\frac{\pi}{2}$, $\frac{5\pi}{6}$, $\frac{3\pi}{2}$
- $5. \ \left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$
- 6. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
- 7. $x = \frac{\pi}{6}, \frac{3\pi}{2}$
- 8. $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
- 9. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$
- 10. $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 11. $x = \frac{\pi}{2}$
- 12. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
- 13. x = 0, π
- 14. $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 15. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
- 16. $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$
- 17. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Law of Sines and Cosines

Solve each of the following. Round your answers to 1 decimal place.

1.
$$m \angle B = 37.8^{\circ}$$
, $a = 15$, $c = 21.1$

2.
$$m \angle A = 41.9^{\circ}$$
, $m \angle C = 59.2^{\circ}$, $a = 10.2$

3.
$$a = 14$$
, $b = 19.6$, $c = 13.1$

4.
$$c = 29$$
, $b = 23$, $m \angle A = 55^{\circ}$

5.
$$c = 8$$
, $b = 12$, $m \angle A = 90^{\circ}$

6.
$$m \angle B = 67.2^{\circ}$$
, $a = 15.6$, $c = 18.9$

7.
$$b = 20$$
, $a = 30$, $c = 12$

8.
$$a = 14$$
, $b = 6$, $c = 12$

9.
$$a = 7$$
, $b = 14$, $c = 12$

10.
$$m \angle B = 119^{\circ}$$
, $a = 11$, $b = 28$

11.
$$m \angle A = 125^{\circ}$$
, $c = 21$, $a = 28$

12.
$$m \angle B = 108^{\circ}$$
, $m \angle C = 61^{\circ}$, $b = 25$

- 1. $b \approx 13.0$, $m \angle A \approx 44.8^{\circ}$, $m \angle C \approx 97.4^{\circ}$
- 2. $m \angle B = 78.9^{\circ}$, $b \approx 15.0$, $c \approx 13.1$
- 3. $m \angle A \approx 45.5^{\circ}$, $m \angle B \approx 92.6^{\circ}$, $m \angle C \approx 41.9^{\circ}$
- 4. $m \angle B \approx 50.1^{\circ}$, $m \angle C \approx 74.9^{\circ}$, $a \approx 24.6$
- 5. $m \angle B \approx 56.3^{\circ}$, $m \angle C \approx 33.7^{\circ}$, $a \approx 14.4$
- 6. $m \angle A \approx 48.2^{\circ}$, $m \angle C \approx 64.6^{\circ}$, $b \approx 19.3$
- 7. $m \angle A \approx 137.8^{\circ}$, $m \angle B \approx 26.6^{\circ}$, $m \angle C \approx 15.6^{\circ}$
- 8. $m \angle A \approx 96.4^{\circ}$, $m \angle B \approx 25.2^{\circ}$, $m \angle C \approx 58.4^{\circ}$
- 9. $m \angle A \approx 30^{\circ}$, $m \angle B \approx 91^{\circ}$, $m \angle C \approx 59^{\circ}$
- 10. $m \angle A \approx 20.1^{\circ}$, $m \angle C \approx 40.9^{\circ}$, $c \approx 21$
- 11. $m \angle B \approx 17.1^{\circ}$, $m \angle C \approx 37.9^{\circ}$, $b \approx 10.1$
- 12. $m \angle A \approx 11^{\circ}$, $a \approx 5$, $c \approx 23$

Area of Triangles

Find the area of each. Round your answers to 1 decimal place.

1.
$$m \angle B = 37.8^{\circ}$$
, $a = 15$, $c = 21.1$

2.
$$m \angle A = 41.9^{\circ}$$
, $m \angle C = 59.2^{\circ}$, $a = 10.2$

3.
$$a = 14$$
, $b = 19.6$, $c = 13.1$

4.
$$p = 14$$
, $k = 9$, $h = 9$

5.
$$m \angle T = 15^{\circ}$$
, $m \angle S = 140^{\circ}$, $r = 11.1$

6.
$$m \angle Z = 67^{\circ}$$
, $y = 6$, $m \angle Y = 41^{\circ}$

7.
$$m \angle R = 129^{\circ}$$
, $r = 10$, $m \angle P = 28^{\circ}$

8.
$$a = 6.9$$
, $m \angle B = 115^{\circ}$, $m \angle C = 39^{\circ}$

9.
$$d = 6$$
, $3 = 12$, $f = 8$

10.
$$m \angle Y = 120^{\circ}$$
, $x = 13$, $m \angle Z = 21^{\circ}$

11.
$$z = 10$$
, $y = 14$, $x = 6$

12.
$$m \angle P = 18^{\circ}$$
, $h = 6.9$, $m \angle H = 147^{\circ}$

13.
$$m \angle S = 118^{\circ}$$
, $m \angle T = 30^{\circ}$, $s = 6.3$

14.
$$r = 8$$
, $t = 7.5$, $m \angle S = 50^{\circ}$

15.
$$d = 15.3$$
, $m \angle E = 105^{\circ}$, $f = 5$

16.
$$m \angle R = 31^{\circ}$$
, $p = 12$, $m \angle Q = 26^{\circ}$

17.
$$m \angle D = 120^{\circ}$$
, $f = 4$, $m \angle E = 36^{\circ}$

- 1. Approximately 97.0 sq. units
- 2. Approximately 65.7 sq. units
- 3. Approximately 91.6 sq. units
- 4. Approximately 39.6 sq. units
- 5. Approximately 24.3 sq. units
- 6. Approximately 24.0 sq. units
- 7. Approximately 11.7 sq. units
- 8. Approximately 31.0 sq. units
- 9. Approximately 21.3 sq. units
- 10. Approximately 41.7 sq. units
- 11. Approximately 26.0 sq. units
- 12. Approximately 3.5 sq. units
- 13. Approximately 6.0 sq. units
- 14. Approximately 23.0 sq. units
- 15. Approximately 36.9 sq. units
- 16. Approximately 19.5 sq. units
- 17. Approximately 10.0 sq. units

Polar Coordinates

Convert each to exact rectangular coordinates.

1.
$$\left(3, \frac{5\pi}{6}\right)$$

2.
$$\left(-2, -\frac{\pi}{4}\right)$$

3.
$$\left(\frac{5}{2}, 240^{\circ}\right)$$

6.
$$\left(1, \frac{3\pi}{2}\right)$$

7.
$$\left(-1, \frac{7\pi}{4}\right)$$

8.
$$\left(-2, \frac{5\pi}{3}\right)$$

9.
$$\left(1, -\frac{4\pi}{3}\right)$$

11.
$$\left(2, -\frac{7\pi}{4}\right)$$

12.
$$(-2, 180^{\circ})$$

13.
$$\left(-1, \frac{\pi}{4}\right)$$

Convert each to exact polar coordinates. Use r>0 and $0\leq \theta < 2\pi.$

14.
$$\left(-2\sqrt{3}, -2\right)$$

15.
$$\left(-\frac{1}{2}, \frac{1}{2}\right)$$

16.
$$(7\sqrt{2}, -7\sqrt{2})$$

$$18. \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

19.
$$(2, -2\sqrt{3})$$

$$20. \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

21.
$$(2\sqrt{3}, -2)$$

$$22. \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$26. \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

Convert each to either rectangular or polar coordinates.

27.
$$2x + 5y = 9$$

28.
$$3y = 1$$

29.
$$-5x - 8y = -10$$

30.
$$r = 8$$

31.
$$r = 4 \sec(\theta)$$

32.
$$\theta = -\frac{\pi}{6}$$

1.
$$\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$$

$$4. \left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$$

$$7. \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

10.
$$(2\sqrt{3}, -2)$$

13.
$$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

16.
$$\left(14, \frac{7\pi}{4}\right)$$

19.
$$\left(4, \frac{5\pi}{3}\right)$$

22.
$$\left(1, \frac{2\pi}{3}\right)$$

$$28. \ \ r = \frac{1}{3} \csc \theta$$

31.
$$x = 4$$

2.
$$(-\sqrt{2}, \sqrt{2})$$

8.
$$\left(-1,\sqrt{3}\right)$$

11.
$$\left(\sqrt{2}, \sqrt{2}\right)$$

14.
$$\left(4, \frac{7\pi}{6}\right)$$

20.
$$\left(1, \frac{\pi}{6}\right)$$

23.
$$(4, \frac{\pi}{2})$$

26.
$$\left(1, \frac{5\pi}{4}\right)$$

$$29. \ \ r = \frac{10}{5\cos\theta + 8\sin\theta}$$

32.
$$y = -\frac{\sqrt{3}}{3}x$$

$$3. \left(\frac{5}{4}, \frac{5\sqrt{3}}{4}\right)$$

9.
$$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

15.
$$\left(\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$$

18.
$$\left(1, \frac{11\pi}{6}\right)$$

21.
$$\left(4, \frac{11\pi}{6}\right)$$

$$27. \ r = \frac{9}{2\cos\theta + 5\sin\theta}$$

30.
$$x^2 + y^2 = 64$$

Vectors

Given $\vec{v}=3\mathbf{i}-5\mathbf{j}$ and $\vec{w}=\langle -2,1\rangle$, find each. Exact and simplified answers only.

1.
$$\vec{v} + \vec{w}$$

2.
$$-4\vec{w}$$

Given $\vec{a} = \langle 4, -3 \rangle$, $\mathbf{b} = 11\mathbf{i} + 2\mathbf{j}$, and $\vec{c} = \langle 0, 2 \rangle$, find each. Exact and simplified answers only.

5.
$$\vec{a} + \vec{b}$$

6.
$$\vec{a} - \vec{b}$$

7.
$$a + c$$

10.
$$3\vec{a} + 5\vec{b}$$

11.
$$-2\vec{c} - 1.5\vec{a}$$

12.
$$||\vec{b}||$$

15.
$$\hat{b}$$

Find the exact horizontal and vertical component form of each vector with the given magnitude and direction angle.

17.
$$|u| = 8$$
, $\theta = 120^{\circ}$

18.
$$|u| = 5$$
, $\theta = \frac{\pi}{4}$

19.
$$|u| = 1.5$$
, $\theta = -300^{\circ}$

20.
$$|u| = \sqrt{2}, \quad \theta = \frac{3\pi}{2}$$

21.
$$|u| = 4$$
, $\theta = 210^{\circ}$

22.
$$|u| = 2\sqrt{3}$$
, $\theta = -\frac{\pi}{6}$

Write each of the following in $|\vec{v}|(\cos(\theta),\sin(\theta))$ form, where $0 \le \theta < 2\pi$.

23.
$$\vec{v} = \langle \sqrt{2}, -\sqrt{2} \rangle$$

24.
$$\vec{v} = \langle -1, \sqrt{3} \rangle$$

25.
$$\vec{v} = \langle 0, 4 \rangle$$

4.
$$\left\langle -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$$

7.
$$\langle 4, -1 \rangle$$

10.
$$\langle 67, 1 \rangle$$

16.
$$\langle 0, 1 \rangle$$

$$19. \left\langle \frac{3}{4}, \frac{3\sqrt{3}}{4} \right\rangle$$

22.
$$\langle 3, -\sqrt{3} \rangle$$

25.
$$4\left\langle\cos\left(\frac{\pi}{2}\right),\sin\left(\frac{\pi}{2}\right)\right\rangle$$

2.
$$(8, -4)$$

5.
$$\langle 15, -1 \rangle$$

8.
$$\langle -11, -2 \rangle$$

11.
$$\langle -6, 0.5 \rangle$$

14.
$$\left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$$

17.
$$\langle -4, 4\sqrt{3} \rangle$$

20.
$$\left\langle 0, -\sqrt{2} \right\rangle$$

23.
$$2\left\langle\cos\left(\frac{7\pi}{4}\right),\sin\left(\frac{7\pi}{4}\right)\right\rangle$$

6.
$$\langle -7, -5 \rangle$$

9.
$$\langle 0, 4 \rangle$$

12.
$$5\sqrt{5}$$

15.
$$\left\langle \frac{11\sqrt{5}}{25}, \frac{2\sqrt{5}}{25} \right\rangle$$

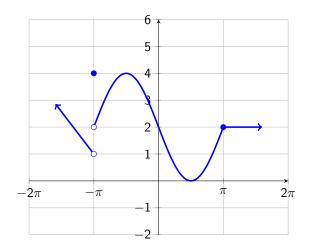
18.
$$\left\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$$

21.
$$\langle -2\sqrt{3}, -2 \rangle$$

24.
$$2\left\langle\cos\left(\frac{2\pi}{3}\right),\sin\left(\frac{2\pi}{3}\right)\right\rangle$$

Numerical and Graphical Limits

Solve using the graph of f(x) below.



$$1. \lim_{x \to -\pi^-} f(x)$$

$$2. \lim_{x \to -\pi^+} f(x)$$

$$3. \lim_{x \to -\pi} f(x)$$

4.
$$f(-\pi)$$

$$5. \lim_{x \to \pi^-} f(x)$$

$$6. \lim_{x \to \pi^+} f(x)$$

7.
$$\lim_{x \to \pi} f(x)$$

8.
$$f(\pi)$$

Find each limit.

$$9. \lim_{x\to 0}\frac{e^x-1}{x}$$

10.
$$\lim_{x \to 1^+} \log(x-1)$$

11.
$$\lim_{x \to 2} \left(\frac{3e^{1/(x-2)}}{e^{1/(x-2)} + 1} - 1 \right)$$

12.
$$\lim_{x \to 0} f(x)$$
, $f(x) = \begin{cases} -x^2 - 4x - 5, & x \le 0 \\ x - 5, & x > 0 \end{cases}$

13.
$$\lim_{x \to 4} g(x)$$
, $g(x) = \begin{cases} x+4, & x < -4 \\ \frac{x}{2} + 3, & x \ge -4 \end{cases}$

14.
$$\lim_{x \to 0} h(x), h(x) = \begin{cases} -x - 4, & x < 0 \\ -2x - 5, & x \ge 0 \end{cases}$$

- 1. 1
- 2. 2
- 3. Does not exist
- 4. 4
- 5. 2
- 6. 2
- 7. 2
- 8. 2
- 9. $e \approx 2.71828$
- 10. Does not exist
- 11. Does not exist
- 12. -5
- 13. 1
- 14. Does not exist

Limits and Algebra

Find each limit algebraically.

1.
$$\lim_{x \to -4} \left(\frac{x+4}{x^2+6x+8} \right)$$

3.
$$\lim_{x \to 3} \left(\frac{x-3}{x^2 - 8x + 15} \right)$$

5.
$$\lim_{x \to -1} \left(-\frac{x^2 + 5x + 4}{x + 1} \right)$$

7.
$$\lim_{x \to 2} \left(\frac{x^2 - 2x}{x - 2} \right)$$

9.
$$\lim_{x \to 0} \left(\frac{x}{\frac{1}{x-1} + 1} \right)$$

11.
$$\lim_{x \to 3} \left(\frac{x}{\frac{1}{x-3} + \frac{1}{3}} \right)$$

13.
$$\lim_{x \to 1} \left(\frac{x}{\frac{1}{x-1} + 1} \right)$$

15.
$$\lim_{x \to 1} \left(\frac{\sqrt{x} - 1}{x - 1} \right)$$

17.
$$\lim_{x \to 25} \left(\frac{x - 25}{\sqrt{x} - 5} \right)$$

$$19. \lim_{x \to 9} \left(\frac{x - 9}{\sqrt{x} - 3} \right)$$

2.
$$\lim_{x \to -3} \left(-\frac{x^2 + 2x - 3}{x + 3} \right)$$

4.
$$\lim_{x \to -2} \left(\frac{x^2 - 2x - 8}{x + 2} \right)$$

6.
$$\lim_{x \to -4} \left(\frac{x^2 + 7x + 12}{x + 4} \right)$$

8.
$$\lim_{x \to -1} \left(-\frac{x^2 - 3x - 4}{x + 1} \right)$$

10.
$$\lim_{x \to 0} \left(\frac{x}{\frac{1}{x+2} - \frac{1}{2}} \right)$$

12.
$$\lim_{x \to 2} \left(\frac{x}{\frac{1}{x-2} + \frac{1}{2}} \right)$$

14.
$$\lim_{xt \neq -1} \left(\frac{x}{\frac{1}{x+1} - 1} \right)$$

16.
$$\lim_{x \to 2} \left(\frac{\sqrt{x+7} - 3}{x-2} \right)$$

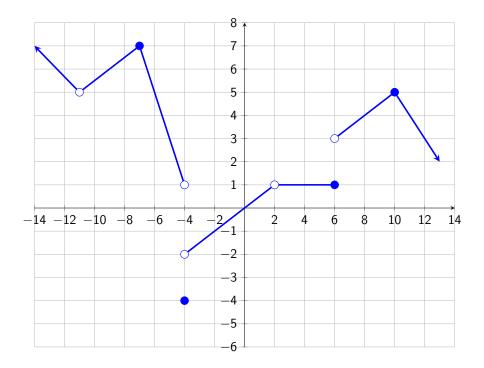
18.
$$\lim_{x \to 1} \left(\frac{x-1}{\sqrt{x+8}-3} \right)$$

20.
$$\lim_{x \to 2} \left(\frac{\sqrt{x+14}-4}{x-1} \right)$$

- 1. $-\frac{1}{2}$
- 2. 4
- 3. $-\frac{1}{2}$
- 4. -6
- 5. -3
- 6. -1
- 7. 2
- 8. 5
- 9. -1
- 10. -4
- 11. 0
- 12. 0
- 13. 0
- 14. 0
- 15. $\frac{1}{2}$
- 16. $\frac{1}{6}$
- 17. 10
- 18. 6
- 19. 6
- 20. $\frac{1}{8}$

Continuity

For each of the following, determine whether the function graphed below is continuous at that value. If not, explain why.



1.
$$x = -11$$

2.
$$x = -7$$

3.
$$x = -4$$

4.
$$x = 2$$

5.
$$x = 6$$

6.
$$x = 10$$

Identify all discontinuities for each of the following.

7.
$$f(x) = \frac{x^2 - 6x}{x^2 + 6x}$$

$$9. \ f(x) = \frac{x+4}{3\ln(x)}$$

11.
$$f(x) = \begin{cases} 2\sin(x), & x < 0 \\ 0, & x = 0 \\ 3x - 2, & x > 0 \end{cases}$$

13.
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x > -1 \end{cases}$$

8.
$$f(x) = \frac{x+3}{x-3}$$

10.
$$f(x) = \frac{2x+5}{x^2-9}$$

12.
$$f(x) = \begin{cases} e^x - 1, & x \le 0 \\ x^2, & x > 0 \end{cases}$$

14.
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x \ge -1 \end{cases}$$

- 1. Discontinuous; Not defined at x = -11
- 2. Continuous
- 3. Discontinuous; Left- and right-hand limits are not equal, nor do they equal the function value at x = -4
- 4. Discontinuous; Not defined at x = 2
- 5. Discontinuous; Left- and right-hand limits are not equal
- 6. Continuous
- 7. Discontinuous at x = 0, -6
- 8. Discontinuous at x = 3
- 9. Discontinuous at x = 1
- 10. Discontinuous at $x = \pm 3$
- 11. Discontinuous at x = 0
- 12. Continuous for all values of x
- 13. Discontinuous at x = -1
- 14. Continuous for all values of x

Derivatives

Use the definition of the derivative to find the derivative of each function with respect to x.

- 1. $f(x) = x^2 + 2x + 4$
- 2. f(x) = -5x + 5
- 3. $f(x) = 2x^2 5x 2$
- 4. f(x) = 5x + 2
- 5. $f(x) = x^3 x^2$
- 6. $f(x) = 5x^2 + 5$
- 7. f(x) = 2x + 3
- 8. $f(x) = x^2 2$
- 9. $f(x) = -5x^3 + 4$
- 10. $f(x) = -4x^2 + x 5$
- 11. $f(x) = 2x^2 3x + 5$

- 1. 2x + 2
- 2. -5
- 3. 4x 5
- 4. 5
- 5. $3x^2 2x$
- 6. 10*x*
- 7. 2
- 8. 2*x*
- 9. $-15x^2$
- 10. -8x + 1
- 11. 4x 3

Appendix A

Factoring

Factor each of the following completely.

1.
$$x^2 + 2x - 15$$

2.
$$x^2 - 8x + 12$$

3.
$$x^2 + 15x + 56$$

4.
$$5x^2 + 19x - 4$$

5.
$$4x^2 - 5x - 6$$

6.
$$9x^2 - 400$$

7.
$$5x^2 - 7x - 6$$

8.
$$9x^2 - 54x + 45$$

9.
$$3x^3 + 12x^2 + 9x$$

10.
$$9y^2 - 16$$

11.
$$4x^2 - 28x + 49$$

12.
$$14x^2 + 11xy - 15y^2$$

13.
$$6x^2 - 48x - 120$$

14.
$$9x^4 - 54x^3 + 45x^2$$

15.
$$16y^2 - 40y + 25$$

16.
$$30x^2 + xy - y^2$$

17.
$$8w^2 + 33w + 4$$

18.
$$3p^2 + 22p - 16$$

19.
$$18x^2 - 27x + 4$$

20.
$$14a^2 + 15a - 9$$

21.
$$4x^2 - 4x - 24$$

22.
$$18t^2 - 9t - 5$$

23.
$$6a^2 + 23a + 21$$

24.
$$25x^2 - 1$$

1.
$$(x+5)(x-3)$$

5.
$$(4x+3)(x-2)$$

9.
$$3x(x+3)(x+1)$$

13.
$$6(x-10)(x+2)$$

17.
$$(8w+1)(w+4)$$

21.
$$4(x-3)(x+2)$$

2.
$$(x-6)(x-2)$$
 3. $(x+7)(x+8)$

6.
$$(3x+20)(3x-20)$$
 7. $(5x+3)(x-2)$

10.
$$(3y+4)(3y-4)$$

14.
$$9x^2(x-1)(x-5)$$

18.
$$(3p-2)(p+8)$$

22.
$$(6t-5)(3t+1)$$

3.
$$(x+7)(x+8)$$

7.
$$(5x+3)(x-2)$$

11.
$$(2x-7)^2$$

15.
$$(4y-5)^2$$

19.
$$(6x-1)(3x-4)$$

22.
$$(6t-5)(3t+1)$$
 23. $(2a+3)(3a+7)$

4.
$$(5x-1)(x+4)$$

8.
$$9(x-5)(x-1)$$

12.
$$(7x - 5y)(2x + 3y)$$

16.
$$(6x - y)(5x + y)$$

20.
$$(7a-3)(2a+3)$$

24.
$$(5x+1)(5x-1)$$

Appendix B

Complex Fractions

Simplify each as much as possible.

1.
$$\frac{5+\frac{3}{x}}{x-\frac{1}{2}}$$

$$2. \ \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

3.
$$\frac{3}{2 - \frac{x}{x-1}}$$

4.
$$\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$$

$$5. \ \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$6. \ \frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$7. \ \frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2 - 2x}}$$

$$8. \ \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

9.
$$\frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$10. \ \frac{\frac{5}{x} - 5x}{x - 1}$$

11.
$$\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

12.
$$\frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

$$13. \ \frac{\frac{1}{x-a} + \frac{1}{a}}{X}$$

14.
$$\frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

15.
$$\frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

1.
$$\frac{2(5x+3)}{x(2x-1)}$$

$$2. \ \frac{1}{x^2 - 2x + 4}$$

3.
$$\frac{3(x-1)}{x-2}$$

$$4. \ \frac{x+3}{2+7x}$$

5.
$$\frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \ \frac{-4x-1}{2}$$

7.
$$\frac{8x-10}{7}$$

8.
$$-\frac{1}{7x}$$

$$9. \ \frac{2x+1}{5x(x+1)}$$

$$10. \ \frac{-5x-5}{x}$$

11.
$$\frac{-1}{2x+4}$$

12.
$$\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \ \frac{1}{a(x-a)}$$

$$14. \ \frac{-2x-2}{5x^2-16x+3}$$

15.
$$\frac{x+4}{4x-8}$$