Honors PreCalculus

Hundreds of Extra Practice Problems

Contents

1	Basic Set Theory and Interval Notation	3
2	Functions and Their Graphs 2.1 Evaluating Functions	5 5 5 6
3	Properties of Functions 3.1 Maxima and Minima	8 8 8 9
4	Linear Functions and Slope 4.1 Equations of Lines	12 12 12
5	Function Transformations	15
6	Function Operations 6.1 Adding, Subtracting, Multiplying, and Dividing Functions	18 18 19 19
7	Polynomials and Their Graphs	21
8	Dividing Polynomials 8.1 Dividing Polynomials	23 23 23
9	Rational Functions and Their Graphs	25
10	Polynomial and Rational Inequalities 10.1 Polynomial Inequalities	27 27 27 27
11	Function Compositions	29
12	Inverse Functions	31
13	Exponential Functions 13.1 End Behavior	33
14	Logarithmic Functions	35
15	Properties of Logarithms	37
16	Exponential Equations 16.1 Applications	39 39
17	Logarithmic Equations and Inequalities	41

18 Sequences	43
19 Series	45
20 Angles and Radian Measure	47
21 Trig Functions of Any Angle	49
22 Graphs of Sine and Cosine Functions	51
23 Graphs of Other Trig Functions	53
24 Inverse Trig Functions	55
25 Trig Equations and Inequalities	57
26 Law of Sines and Cosines	59
27 Area of Triangles	61
28 Polar Coordinates	63
29 Vectors	65
30 Numerical and Graphical Limits	67
31 Algebraic Limits	69
32 Continuity	71
33 Derivatives	73
A Factoring	75
B Complex Fractions	77

Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2. $\{x | x \le 1\}$
- 3. _3
- 4. $\{x | x \neq 4, 11\}$
- 5. 2.4
- 6. $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7. $\{x | x \ge 2\}$
- 8. $\{x|x<-8\}$
- 9. $\{x | x \neq 3\}$
- 10. $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

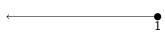
- 11.
- $\begin{array}{cccc}
 & & & & & & \downarrow \\
 & & & & & \uparrow \\
 & & & & & & \uparrow \\
 & & & & & & \uparrow
 \end{array}$

BASIC SET THEORY AND INTERVAL NOTATION KEY

1.
$$\{x | -5 < x \le 8\}$$



2. $(-\infty, 1]$



3. $[-3, \infty)$ $\{x | x \ge -3\}$

4.
$$(-\infty, 4) \cup (4, 11) \cup (11, \infty)$$



5. [2.4, 7.7) $\{x | 2.4 \le x < 7.7\}$

6. $\{x|x > 9\}$



7. $[2, \infty)$



8. $(-\infty, -8)$

9. $(-\infty,3)\cup(3,\infty)$

$$\longleftrightarrow$$
 3

10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$

$$\begin{array}{ccc} & \bigcirc & \bigcirc & \bigcirc \\ -2 & & 5 \end{array}$$

11. $(-\infty, -8]$ $\{x | x \le -8\}$

12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x | x \neq 7, 12\}$

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

- 1. f(5)
- 2. f(-2)
- 3. f(0)
- 4. g(1)
- 5. g(-5)
- 6. g(1/4)

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1.
$$f(x) = -8x^2 - 7x + 1$$

2.
$$g(x) = \sqrt{5x + 12} - 2$$

3.
$$h(x) = \frac{x+2}{9x-7}$$

4.
$$f(x) = -5x + 4$$

5.
$$f(x) = x^2 + 2$$

6.
$$f(x) = \frac{2x+1}{3x-5}$$

7.
$$f(x) = \sqrt{3x - 12}$$

8.
$$f(x) = \frac{x}{x^2 - 16}$$

9.
$$f(x) = \frac{x+4}{x^3-4x}$$

10.
$$f(x) = \frac{x}{\sqrt{x-4}}$$

11.
$$f(x) = \frac{x^2+1}{2x^2+8}$$

12.
$$f(x) = -\frac{x+7}{x^2-5x-6}$$

13.
$$g(x) = \sqrt{2x+3}$$

14.
$$h(x) = \sqrt[3]{2x+3}$$

15.
$$f(x) = -\frac{7x-10}{x^2+3x+2}$$

16.
$$g(x) = \sqrt{-9x + 8}$$

17.
$$h(x) = -\sqrt[3]{4x+1}$$

18.
$$f(x) = \sqrt[3]{8x+1}$$

19.
$$g(x) = \frac{x^2-1}{\sqrt{x+3}}$$

20.
$$h(x) = \frac{3}{9 + \frac{4}{x+7}}$$

21.
$$f(x) = \frac{x+1}{\sqrt{10x+8}}$$

22.
$$g(x) = \frac{5}{1 + \frac{3}{x+2}}$$

23.
$$i(x) = \frac{7}{3 - \frac{4}{x+1}}$$

24.
$$n(x) = \frac{7x+14}{\sqrt{2x-1}}$$

25.
$$a(x) = \frac{\frac{x}{x-2}}{\frac{3}{x-2}+6}$$

26.
$$d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$$

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. f(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4\\ \sqrt{2x + 7} & -4 < x < 0\\ |-x - 1| & x \ge 0 \end{cases}$$

- 6. f(3)
- 7. f(-2)
- 8. f(0)
- 9. f(-5)

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \le -3\\ \sqrt{-4x + 1} & \text{if } -3 < x \le 0\\ \frac{5x^2}{x + 7} & \text{if } x > 0 \end{cases}$$

- 10. f(7)
- 11. f(-3)
- 12. f(1)
- 13. f(0)
- 14. f(-1)
- 15. f(-3/2)

Evaluating Functions

- 1. -55
- 2. -20
- 3. 0
- 4. -4
- 5. -5.2
- 6. -1

Domain of Functions

- 1. $(-\infty, \infty)$
- 2. $\left[\frac{-12}{5}, \infty\right)$
- 3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
- 4. $(-\infty, \infty)$
- 5. $(-\infty, \infty)$
- 6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
- 7. $[4, \infty)$
- 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
- 9. $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$
- 10. $(4, \infty)$
- 11. $(-\infty, \infty)$
- 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
- 13. $\left[-\frac{3}{2}, \infty\right)$

- 14. $(-\infty, \infty)$
- 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
- 16. $\left(-\infty, \frac{8}{9}\right]$
- 17. $(-\infty, \infty)$
- 18. $(-\infty, \infty)$
- 19. $(-3, \infty)$
- 20. $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup \left(-7, \infty\right)$
- 21. $\left(-\frac{4}{5}, \infty\right)$
- 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$
- 23. $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$
- 24. $(\frac{1}{2}, \infty)$
- 25. $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 2\right) \cup \left(2, \infty\right)$
- 26. $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$

Piecewise Functions

- 1. $\sqrt{15} \approx 3.873$
- 2. 7
- 3. 6.6
- 4. 6.4
- 5. 7.1

- 6. 4
- 7. $\sqrt{3} \approx 1.732$
- 8. 1
- 9. 18
- 10. 17.5

- 11. 4
- 12. $\frac{5}{8}$
- 13. 1
- 14. $\sqrt{5} \approx 2.236$
- 15. $\sqrt{7} \approx 2.646$

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

5.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

6.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = x^3 + 2x^2 - 4x - 8$$

4.
$$g(x) = x^4 - 2x^2 + 1$$

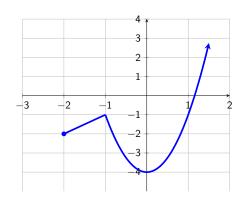
5.
$$h(x) = \sqrt{x+1} - 2$$

6.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

7.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

3.3 Miscellaneous

Use the graph of y = f(x) below to answer the following questions. Write your answers using interval notation.



- 1. Domain of f
- 2. Range of f
- 3. Relative Minimum
- 4. Relative Maximum
- 5. f(1)

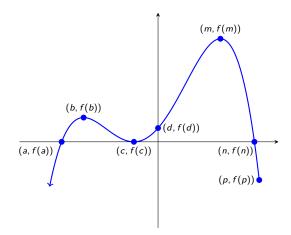
- 6. f(0)
- 7. Increasing Interval(s)
- 8. Decreasing Interval(s)
- 9. Absolute Maximum
- 10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

- 11. f(7)
- 12. f(-2)
- 13. Rel. Max
- 14. Rel. Min

- 15. Global Max
- 16. Global Min
- 17. Increasing Interval(s)
- 18. Decreasing Interval(s)

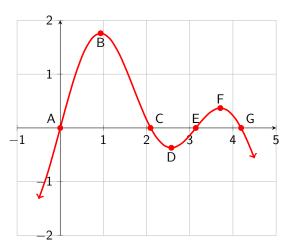
Use the graph of f(x) to answer each.



- 19. Relative maxima of f(x)
- 20. Relative minima of f(x)
- 21. Absolute maxima of f(x)
- 22. Absolute minima of f(x)

- 23. Intervals where f is increasing
- 24. Intervals where f is decreasing
- 25. Zeros of *f*

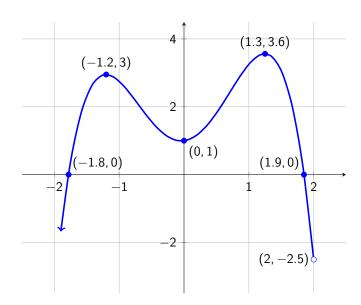
Given the labeled points A through G on the graph of f(x) below, find each of the following.



- 26. Increasing interval(s)
- 28. Relative max
- 30. Global max
- 32. Zeros of *f*

- 27. Decreasing interval(s)
- 29. Relative min
- 31. Global min
- 33. Number of solutions to f(x) = 1

Answer each of the following about the function f(x) below.



- 34. Domain of f
- 35. Range of f
- 36. Relative maxima
- 37. Relative minima
- 38. Absolute maximum

- 39. Absolute minimum
- 40. Increasing intervals
- 41. Decreasing intervals
- 42. Zeros of f(x)
- 43. Number of solutions to f(x) = 2

Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. Rel Max (-1.366, 10.848) and (1,6); Rel Min (0.366, 5.652)
- 4. Rel Max (-1.716, 11.598); Rel Min (1.132, -3.929)
- 5. Rel Max: (1.095, 12.096); Rel Min (-0.761, -0.680)
- 6. Rel Max: (1.366, 0.348); Rel Min: (-0.366, -4.848) and (2, 0)
- 7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

- 1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
- 3. Inc: $(-\infty, -2) \cup (\frac{2}{3}, \infty)$ Dec: $(-2, \frac{2}{3})$
- 4. Inc; $(-1,0) \cup (1,\infty)$ Dec: $(-\infty,-1) \cup (0,1)$
- 5. Inc: $(-1, \infty)$ No intervals where it is decreasing
- 6. Inc: (-0.761, 1.095); Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
- 7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Miscellaneous

- 1. $[-2, \infty)$
- 2. $[-4, \infty)$
- 3. (0, -4)
- 4. (-1, -1)
- 5. -1
- 6. -4)
- 7. $(-2, -1) \cup (0, \infty)$
- 8. (-1,0)
- 9. (0, -4)
- 10. None
- 11. -426

- 12. 51
- 13. (1.408, 0.272)
- 14. (0.592, -0.272)
- 15. None
- 16. None
- 17. (0.592, 1.408)
- 18. $(-\infty, 0.592) \cup$ $(1.408, \infty)$
- 19. (b, f(b)) and (m, f(m))
- 20. (c, f(c))
- 21. (m, f(m))

- 22. None
- 23. $(-\infty, b) \cup (c, m)$
- 24. $(b, c) \cup (m, p)$
- 25. x = a, x = c, x = n
- 26. $(\infty, B) \cup (D, F)$
- 27. (B, D) ∪ (F, ∞)
- 28. *B* and *F*
- 29. D
- 30. B
- 31. None
- 32. A, C, E, G

- 33. 2
- 34. $(-\infty, 2)$
- 35. $(-\infty, -2.5) \cup (-2.5, 3.6]$
- 36. (-1.2, 3) and (1.3, 3.6)
- 37. (0, 1)
- 38. (1.3, 3.6)
- 39. Does not exist
- 40. $(-\infty, -1.2) \cup (0, 1.3)$
- 41. $(-1.2, 0) \cup (1.3, 2)$
- 42. (-1.8, 0) and (1.9, 0)
- 43. 4

Linear Functions and Slope

4.1 Equations of Lines

Write the equation of each line in point-slope form that goes through each pair of points.

- 1. (-2,1), (7,8)
- 2. (0,4), (9,-15)
- 3. (-1, -2), (-3, -13)

4.2 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

- 6. [-2, -1]
- 7. [5, 6]
- 8. [0, 1]
- 9. [5, 5.001]
- 10. [5, 5.0001]
- 11. [5, 5.00001]
- 12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

- 13. [7, 7.001]
- 14. [7, 7.0001]
- 15. [7, 7.00001]
- 16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

- 17. [1, 1.0001]
- 18. [1, 1.00001]
- 19. [1, 1.000001]
- 20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = 6\sqrt{x}$, find the average rate of change of each over the given interval.

- 21. [25, 25.1]
- 22. [25, 25.01]
- 23. [25, 25.001]
- 24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -7x^3 + 6\sqrt{3x} + 4$ over each interval. Round your answers to 4 decimal places.

- 25. [0, 1]
- 26. [10, 11]
- 27. [8, 15]

Equations of Lines

- 1. $y-1=\frac{7}{9}(x+2)$ or $y-8=\frac{7}{9}(x-7)$
- 2. $y-4=-\frac{19}{9}(x-0)$ or $y+15=-\frac{19}{9}(x-9)$
- 3. $y + 2 = \frac{11}{2}(x+1)$ or $y + 13 = \frac{11}{2}(x+3)$

Average Rate of Change

- 1. 2.1
- 2. 2.01
- 3. 2.001
- 4. 2.0001
- 5. 2
- 6. 25
- 7. -59
- 8. 1
- 9. -53.006

- 10. -53.0006
- 11. -53.00006
- 12. -53
- 13. -42.003
- 14. -42.0003
- 15. -42.00003
- 16. -42
- 17. -0.499988
- 18. -0.4999988

- 19. -0.49999988
- 20. -0.5
- 21. 0.5994
- 22. 0.59999
- 23. 0.6
- 24. 0.6
- 25. 3.3923
- 26. -2,315.3960
- 27. -2861.4492

Function Transformations

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
 - (2) Shift left 1 unit
 - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
 - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
 - (2) Vertical compression by 4
 - (3) Move right 7 units

Write the function g(x) that is a result of the following ordered sequence of transformations to f(x) = |x|.

- 4. (1) Reflect across x-axis
 - (2) Shift right 3 units
 - (3) Horizontal stretch by factor of 5
- 5. (1) Shift down 2 units
 - (2) Reflect across y-axis
 - (3) Shift up 1 unit
- 6. (1) Horizontal compression by factor of 7
 - (2) Vertical compression by factor of 4
 - (3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function g(x) after the following ordered sequence of transformations.

- 7. (1) Shift up 2 units
 - (2) Horizontal stretch by 5
 - (3) Shift left 3 units
- 8. (1) Vertical compression by factor of 3
 - (2) Reflect across y-axis
 - (3) Horizontal compression by 5
- 9. (1) Shift right 8 units
 - (2) Reflect across x-axis
 - (3) Horizontal compression by factor of 4

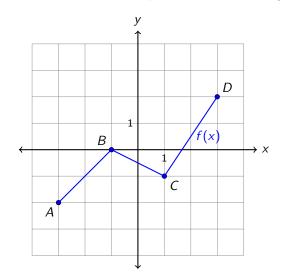
Write the final equation of g(x) if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

- 10. (1) Shift right 2 units
 - (2) Horizontal stretch by factor 3
 - (3) Shift down 2 units
 - (4) Reflect across x-axis
- 11. (1) Horizontal stretch by factor 3
 - (2) Shift left 1 unit
 - (3) Shift up 2 units
 - (4) Reflect across y-axis
- 12. (1) Vertical stretch by factor 5
 - (2) Horizontal stretch by factor 2
 - (3) Shift up 3 units
 - (4) Reflect across x-axis

Find the equation for g(x) if g(x) is found by performing the following *ordered* sequence of transformations to $f(x) = \frac{1}{x}$.

- 13. (1) Shift left 3 spaces
 - (2) Reflect across y-axis
 - (3) Shift down 5 spaces
 - (4) Vertical stretch by factor of 7
- 14. (1) Shift up 3 spaces
 - (2) Reflect across x-axis
 - (3) Shift right 5 spaces
 - (4) Horizontal compression by factor of 7

Given the graph of f(x) below, find the new coordinates of each point after the following transformations.



15.
$$-2f(x+1)$$

16.
$$f\left(-\frac{1}{2}x\right) - 3$$

17.
$$\frac{1}{2}f(-x-2)+2$$

18.
$$f(2x+2)-1$$

19.
$$-3f(-x+1)+2$$

20.
$$5f\left(-\frac{1}{2}x\right)$$

FUNCTION TRANSFORMATIONS KEY

1.
$$g(x) = 3f(-x+1)$$

2.
$$g(x) = f(2x) + 1$$

3.
$$g(x) = -\frac{1}{4}f(x-7)$$

4.
$$g(x) = -\left|\frac{1}{5}x - 3\right|$$

5.
$$g(x) = |-x| - 1$$

6.
$$g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$$

7.
$$g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$$

8.
$$g(x) = \frac{1}{3}\sqrt{-5x}$$

9.
$$g(x) = -\sqrt{4x - 8}$$

10.
$$g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$$

11.
$$g(x) = \sqrt{\frac{1}{3}(-x+1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$$

12.
$$g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$$

13.
$$g(x) = \frac{7}{-x+3} - 35$$

14.
$$g(x) = -\frac{1}{7x-5} - 3$$

15.
$$A'(-4,4)$$
, $B'(-2,0)$, $C'(0,2)$, $D'(2,-4)$

16.
$$A'(6,-5)$$
, $B'(2,-3)$, $C'(-2,-4)$, $D'(-6,-1)$

17.
$$A'(1,1)$$
, $B'(-1,2)$, $C'(-3,1.5)$, $D'(-5,3)$

18.
$$A'(-2.5, -3)$$
, $B'(-1.5, -1)$, $C'(-0.5, -2)$, $D'(0.5, 1)$

19.
$$A'(4,8)$$
, $B'(2,2)$, $C'(0,5)$, $D'(-2,-4)$

20.
$$A'(6,-10)$$
, $B'(2,0)$, $C'(-2,-5)$, $D'(-6,10)$

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given f(x) = x + 5, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

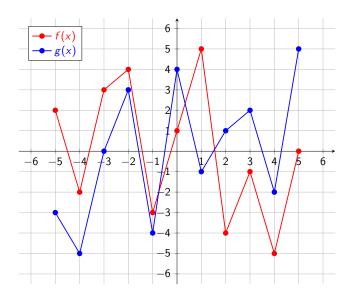
- 1. (g f)(x)
- 2. (fh)(14)
- 3. (f+g)(x)

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-3	0	-1	3	1	2	4	-4	$\overline{-2}$
g(x)	3	-1	0	1	4	-2	-4	2	-3

- 4. (f+g)(-2) 5. (f-g)(0) 6. (fg)(1)
- 7. $\left(\frac{f}{g}\right)$ (3)
- 8. (f+f)(-4)

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 9. (f+g)(2)
- 10. (f-g)(1)
- 11. (g-f)(-3)
- 12. (fg)(4)
- 13. $\left(\frac{f}{g}\right)(0)$

6.2 Operations with Functions: Domain

Given $f(x) = \sqrt{2x+7}$ and g(x) = 3x+3, find the domain of each.

- 1. (f+g)(x)
- $2. \left(\frac{f}{g}\right)(x)$
- 3. $\left(\frac{g}{f}\right)(x)$

6.3 Difference Quotient

Write the difference quotient for each.

- 1. f(x) = 2x 7
- 2. $g(x) = x^2 + 4x$
- 3. h(x) = -1
- 4. $f(x) = \frac{3}{x+2}$
- 5. $g(x) = \sqrt{3x}$
- 6. $f(x) = x^2 2x + 5$
- 7. $g(x) = \frac{5}{x}$

- 8. $f(x) = -2x^2 + 3x 5$
- 9. $g(x) = \frac{6}{2x+3}$
- 10. $h(x) = \sqrt{7x+5}$
- 11. $f(x) = -x^2 + x$
- 12. f(x) = 3x 1
- 13. $f(x) = x^3 + 5x$

Adding, Subtracting, Multiplying, and Dividing Functions

1.
$$x^2 - x - 6$$

2. 38

3.
$$x^2 + x + 4$$

4. -1

5. -3

6. -4

7. -2

8. -6

9. -3

10. 6

11. -3

12. 10

13. $\frac{1}{4}$

Operations with Functions: Domain

1.
$$\left[-\frac{7}{2},\infty\right)$$

2.
$$\left[-\frac{7}{2}, -1\right) \cup (-1, \infty)$$

3.
$$\left(-\frac{7}{2}, \infty\right)$$

Difference Quotient

2.
$$2x + h + 4$$

4.
$$\frac{-3}{(x+2)(x+h+2)}$$

$$5. \ \frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$$

6.
$$2x + h - 2$$

$$7. \ \frac{-5}{x(x+h)}$$

8.
$$-4x - 2h + 3$$

9.
$$\frac{-12}{(2x+3)(2x+2h+3)}$$

10.
$$\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$$

11.
$$-2x - h + 1$$

13.
$$3x^2 + 3xh + h^2 + 5$$

Polynomials and Their Graphs

Determine the end behavior of each.

1.
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

2.
$$g(x) = 4x^2 - 16x^6 + 3x$$

3.
$$h(x) = 1 + x^{11} - 4x^8$$

4.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

5.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

6.
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

7.
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$

POLYNOMIALS AND THEIR GRAPHS

1.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

2.
$$\lim_{x \to -\infty} g(x) = -\infty$$
, $\lim_{x \to \infty} g(x) = \infty$

3.
$$\lim_{x \to -\infty} h(x) = -\infty$$
 $\lim_{x \to \infty} h(x) = \infty$

4.
$$\lim_{x \to -\infty} f(x) = -\infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

5.
$$\lim_{x \to -\infty} g(x) = -\infty$$
 $\lim_{x \to \infty} g(x) = \infty$

6.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

7.
$$\lim_{x \to -\infty} g(x) = -\infty$$
 $\lim_{x \to \infty} g(x) = \infty$

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

1.
$$(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$$

2.
$$(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$$

3.
$$(4x^3 - 3x^2 + x + 1) \div (x + 2)$$

4.
$$(5x^4 - x^2 + x - 2) \div (x^2 + 2)$$

5.
$$(10x^3 + 27x^2 + 8x - 11) \div (2x + 3)$$

6.
$$(7x^3 + 23x^2 + 12x + 1) \div (x^2 + 3x + 1)$$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

1.
$$(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$$

2.
$$(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x+1)$$

3.
$$(x^{23} - 5x^{20} + 17x^8 - 5) \div (x+2)$$

4.
$$\left(-7x^{17}+40x^{15}-6x^8+4x^3\right)\div(x-3)$$

Dividing Polynomials

1.
$$4x^2 - 2x + 5$$

2.
$$4y^2 + 8y + 7 - \frac{72}{3y + 5}$$

3.
$$4x^2 - 11x + 23 - \frac{45}{x+2}$$

4.
$$5x^2 - 11 + \frac{x + 20}{x^2 + 2}$$

$$5. \ 5x^2 + 6x - 5 + \frac{4}{2x + 3}$$

6.
$$7x + 2 + \frac{-x - 1}{x^2 + 3x + 1}$$

Remainder and Factor Theorems

- 1. 6
- 2. 13
- 3. -13, 627, 141
- 4. -330, 064, 119

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1.
$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

2.
$$g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$3. \ f(x) = \frac{3x}{x+4}$$

4.
$$g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

5.
$$h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

6.
$$f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

7.
$$g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

8.
$$h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

9.
$$f(x) = \frac{6x^3 - 21x^2 - 51x + 30}{3x^2 + 7x + 2}$$

10.
$$g(x) = \frac{10x^2 - 29x - 21}{10x^3 - 33x^2 - 7x}$$

State the end behavior of each.

11.
$$k(x) = \frac{5x^3 - 7x^2 + 8}{-3x^3 + 6x - 4}$$

12.
$$m(x) = \frac{2x-1}{3x^2+7x+1}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain:
$$x \neq -\frac{1}{2}$$
, 7; V.A.: $x = 7$; Hole @ $\left(-\frac{1}{2}, -\frac{7}{13}\right)$; H.A.: $y = 1$

2. Domain:
$$x \neq -3$$
, 4; V.A.: $x = -3x = 4$; Obl. Asymp: $y = 3x + 10$

3. Domain:
$$x \neq -4$$
; V.A.: $x = -4$; H.A.: $y = 3$

4. Domain:
$$x \neq 1$$
; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$

5. Domain:
$$x \neq 0, 1$$
; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$

6. Domain:
$$x \neq -1, 5$$
; V.A. $x = -1$; Hole @ $\left(5, \frac{13}{3}\right)$; Obl. Asym $y = 2x - 5$

7. Domain:
$$x \neq -6, 0, 4$$
; V.A. $x = -6, x = 0$; Hole @ $\left(4, \frac{21}{40}\right)$; H.A. $y = 0$

8. Domain:
$$x \neq -6, -\frac{3}{8}$$
; V.A. $x = -6, x = -\frac{3}{8}$; H.A. $y = \frac{1}{4}$

9. Domain:
$$x \neq -2$$
, $-\frac{1}{3}$; Hole @ $(-2, -21)$; V.A.: $x = -\frac{1}{3}$; S.A. $y = 2x - \frac{35}{3}$

10. Domain:
$$x \neq -\frac{1}{5}$$
, 0, $\frac{7}{2}$; Hole @ $\left(\frac{7}{2}, \frac{82}{259}\right)$; V.A. $x = -\frac{1}{5}$ and $x = 0$; H.A. $y = 0$

11.
$$\lim_{x \to -\infty} k(x) = \infty = \lim_{x \to \infty} k(x) = -\frac{5}{3}$$

12.
$$\lim_{x \to -\infty} m(x) = \infty = \lim_{x \to \infty} m(x) = 0$$

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1.
$$6x^3 - 4x^2 - 10x > 0$$

2.
$$x^4 < 9x^2$$

3.
$$3x^3 - 7x^2 - 22x + 8 < 0$$

4.
$$3x^2 - 4x + 1 < 0$$

5.
$$12x^4 + 76x^3 + 43x^2 - 346x - 280 > 0$$

6.
$$-2x^4 + 49x^2 + 21x^3 - 1029x + 2401 > 0$$

7.
$$-x^2 - 7x - 6 < 0$$

8.
$$x^2 + 4x + 4 < 0$$

9.
$$-x^4 - 6x^3 + 61x^2 + 234x - 1008 \ge 0$$

10.
$$-x^2 + 3x + 1 > 3$$

11.
$$-3x^4 + 123x^3 + 142x^2 - 424x + 320 \le 122x^3$$

12.
$$-x^4 - 1120 + 77x^2 - 36x + 15x^3 > 15x^3$$

$$13. -3x^4 - 22x^3 + 271x^2 + 152x - 96 > 267x^2$$

10.2 Domain

State the domain of each. Write your answers using interval notation.

1.
$$b(x) = \sqrt{21x^2 - 23x - 20}$$

10.3 Rational Inequalities

Solve each. Write your answers using interval notation.

1.
$$\frac{3x-4}{x+1} < 0$$

$$5. \ \frac{x^2 - 7x - 8}{x^2 - 4x - 32} \ge 0$$

10.
$$\frac{2x-50}{5x+15} \le -1$$

$$2. \ \frac{x^2 + 3x + 2}{x - 7} \le 0$$

$$6. \ \frac{4+3x}{5-x} \le 2$$

7. $\frac{x-4}{x+7} < 0$

11.
$$\frac{x+5}{x^2-2x-15} \le 0$$

$$3. \ \frac{x^2 - 4x + 4}{x^2 - 1} \ge 0$$

8.
$$\frac{x+5}{x+7} < 0$$

12.
$$-\frac{2}{x} \ge -\frac{3}{x+1}$$

4.
$$\frac{x+2}{x-4} \le 1$$

9.
$$\frac{2x-26}{5x+20} > -3$$

13.
$$-\frac{3}{x+6} > -\frac{4}{x+7}$$

Polynomial Inequalities

1.
$$[-1,0] \cup \left[\frac{5}{3},\infty\right)$$

2.
$$(-3,0) \cup (0,3)$$

$$3. \ \left(-\infty, -2\right) \cup \left(\frac{1}{3}, 4\right)$$

4.
$$\left[\frac{1}{3}, 1\right]$$

5.
$$\left(-\infty, -4\right] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$$

$$6. \left[-7, \frac{7}{2} \right] \cup 7$$

7.
$$(-\infty, -6] \cup [-1, \infty)$$

9.
$$[-8, -7] \cup [3, 6]$$

11.
$$\left(-\infty, -8\right] \cup \left[\frac{4}{3}, 2\right] \cup \left[5, \infty\right)$$

12.
$$[-8, -4] \cup [5, 7]$$

13.
$$[-6, -4] \cup \left[\frac{2}{3}, 2\right]$$

Domain

1.
$$\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$$

Rational Inequalities

1.
$$\left(-1, \frac{4}{3}\right)$$

2.
$$(-\infty, -2] \cup [-1, 7)$$

3.
$$(-\infty, -1) \cup (1, \infty)$$

4.
$$(-\infty, 4)$$

5.
$$(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$$

6.
$$(-\infty, 1.2] \cup (5, \infty)$$

7.
$$(-7, 4)$$

8.
$$(-7, -5)$$

9.
$$(-\infty, -4) \cup (-2, \infty)$$

10.
$$(-3, 5]$$

11.
$$(-\infty, -5] \cup (-3, 5)$$

12.
$$(-1,0) \cup [2,\infty)$$

13.
$$(-7, -6) \cup (-3, \infty)$$

Function Compositions

Given f(x) = x - 5, $g(x) = 4 + \sqrt{2x + 1}$, and $h(x) = \frac{3}{x + 7}$, simplify each and state the domain.

- 1. $(f \circ g)(x)$
- 2. $(g \circ f)(x)$
- 3. h(h(x))

Find each of the following given the table below.

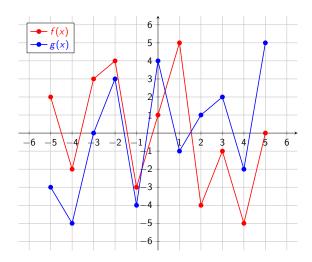
x	-4	-3	-2	-1	0	1	2	3	4	
f(x)	-3	0	-1	3	1	2	4	-4	-2	
g(x)	3	-1	0	1	4	-2	-4	2	-3	

- 4. $(f \circ g)(-1)$ 5. $(g \circ g)(0)$ 6. $(f \circ f)(2)$ 7. $(g \circ g)(-3)$ 8. f(g(0))

Given $f(x) = \sqrt{3x+2}$, $g(x) = x^2 - 1$, and h(x) = 9x - 2, find each of the following.

- 9. $(g \circ f)(x)$
- 10. f(g(x))
- 11. $(h \circ h)(x)$

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 12. $(f \circ g)(-1)$ 13. $(g \circ f)(-4)$
- 14. f(g(3))
- 15. g(g(-2)) 16. $(f \circ f)(-5)$

FUNCTION COMPOSITIONS KEY

- 1. $-1 + \sqrt{2x+1}$ Domain: $\left[-\frac{1}{2}, \infty\right)$
- 2. $4 + \sqrt{2x 9}$ Domain: $\left[\frac{9}{2}, \infty\right)$
- 3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup \left(-7, \infty\right)$
- 4. 2
- 5. -3
- 6. -2
- 7. 1
- 8. -2
- 9. 3x + 1
- 10. $\sqrt{3x^2-1}$
- 11. 81x 20
- 12. -2
- 13. 3
- 14. -4
- 15. 2
- 16. -4

Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1.
$$f(x) = \sqrt{-2x+3} + 1$$

2.
$$g(x) = (x+4)^2 - 1$$
, $x \le -4$

3.
$$h(x) = \frac{9x}{4x - 1}$$

4.
$$f(x) = \sqrt{x} - 3$$

$$5. g(x) = \frac{1}{1-x}$$

6.
$$h(x) = x^2 + 6x + 4, x \le -3$$

7.
$$f(x) = \sqrt{5x - 4}$$

8.
$$g(x) = x^2 - 2x + 3, x \le 1$$

9.
$$h(x) = \frac{3}{x-1}$$

INVERSE FUNCTIONS KEY

1. f	$^{-1}(x) = -$	$-\frac{1}{2}((x-$	$(1)^2 - 3)$
------	----------------	--------------------	--------------

	Domain	Range
f(x)	$(-\infty, 1.5]$	$[1,\infty)$
$f^{-1}(x)$	$[1,\infty)$	$(-\infty, 1.5]$

2.
$$g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
g(x)	$(-\infty, -4]$	$[-1,\infty)$
$g^{-1}(x)$	$[-1,\infty)$	$(-\infty, -4]$

3.
$$h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
h(x)	$(-\infty,1/4)\cup(1/4,\infty)$	$(\infty,9/4)\cup(9/4,\infty)$
$h^{-1}(x)$	$(\infty,9/4)\cup(9/4,\infty)$	$(-\infty,1/4)\cup(1/4,\infty)$

4.
$$f^{-1}(x) = (x+3)^2$$

	Dom	Ran
f(x)	$[0,\infty)$	$[-3,\infty)$
$f^{-1}(x)$	$[-3,\infty)$	$[0,\infty)$

5.
$$g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
g(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$g^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

6.
$$h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
h(x)	$(-\infty, -3]$	$[-5,\infty)$
$h^{-1}(x)$	$[-5,\infty)$	$(-\infty, -3]$

7.
$$f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

	Dom	Ran
f(x)	$\left[\frac{4}{5},\infty\right)$	$[0,\infty)$
$f^{-1}(x)$	[0, ∞)	$\left[\frac{4}{5},\infty\right)$

8.
$$g^{-1}(x) = -\sqrt{x-2} + 1$$

	Dom	Ran
g(x)	$(-\infty,1]$	[2, ∞)
$g^{-1}(x)$	[2, ∞)	$(-\infty,1]$

9.
$$h^{-1}(x) = \frac{3}{x} + 1$$

_		Dom	Ran
	h(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
	$h^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

Exponential Functions

13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

- 1. $f(x) = 3 + e^{2x}$
- 2. $h(x) = 5^{-x}$
- 3. $h(x) = -\frac{2}{3}e^{x+7} + 1$
- 4. $f(x) = -7e^x + 4$
- 5. $g(x) = \frac{1}{3}e^{2x+1} 5$
- 6. $h(x) = -\frac{1}{2}e^{-4x} + 1$

EXPONENTIAL FUNCTIONS

1.
$$\lim_{x \to -\infty} f(x) = 3$$
 $\lim_{x \to \infty} f(x) = \infty$

2.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = 0$

3.
$$\lim_{x \to -\infty} h(x) = 1$$
 $\lim_{x \to \infty} h(x) = -\infty$

4.
$$\lim_{x \to -\infty} f(x) = 4$$
 $\lim_{x \to \infty} f(x) = -\infty$

5.
$$\lim_{x \to -\infty} f(x) = -5$$
 $\lim_{x \to \infty} f(x) = \infty$

6.
$$\lim_{x \to -\infty} f(x) = -\infty$$
 $\lim_{x \to \infty} f(x) = 1$

Logarithmic Functions

Write each of the following in exponential or logarithmic form.

- 1. ln(a) = 7
- 2. $\log_4(x+1) = 9$
- 3. $\log(5x) = 30$
- 4. ln(w) = c
- 5. $5^x = 19$
- 6. $8^{-3} = \frac{1}{512}$
- 7. $e^{14} = x$
- 8. $(1.1)^{-t} = 50$

Find the domain of each. Write your answers in interval notation.

- 9. $b(x) = \log_7(x^2 8x + 6)$
- 10. $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$
- 11. $f(x) = -7 \ln (x^2 + 9x + 8)$
- 12. $g(x) = \log (5x^2 + 13x 6)$
- 13. $h(x) = 3 \log_2 (x^3 + 2x^2 x 2)$
- 14. $c(x) = \ln (4x^2 15x 4)$

State the end behavior of each.

15.
$$j(x) = 5 \log_3 (2x - 5) - 2$$

LOGARITHMIC FUNCTIONS KEY

1.
$$e^7 = a$$

2.
$$4^9 = x + 1$$

3.
$$10^{30} = 5x$$

4.
$$e^c = w$$

5.
$$\log_5(19) = x$$

$$6. \log_8\left(\frac{1}{512}\right) = -3$$

7.
$$ln(x) = 14$$

8.
$$\log_{1.1}(50) = -t$$

9.
$$(-\infty, 0.838) \cup (7.162, \infty)$$

10.
$$(-3, -2) \cup (-1, \infty)$$

11.
$$(-\infty, -8) \cup (-1, \infty)$$

12.
$$(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$$

13.
$$(-2, -1) \cup (1, \infty)$$

14.
$$\left(-\infty, -\frac{1}{4}\right) \cup (4, \infty)$$

15.
$$\lim_{x \to (5/2)^+} j(x) = -\infty \quad \lim_{x \to \infty} j(x) = \infty$$

Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

- 1. $\log_b \left(\frac{x^2}{y^8} \right)$
- 2. $\ln(ez)^3$
- 3. $\log_5(x) + \log_5(9) 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

- 4. $log_7(10)$
- 5. $\log_9(x)$
- 6. $\log_b(c)$
- 7. $\log_3(10)$
- 8. $\log_{17}(\pi)$
- 9. $\log_w(x)$

Suppose that $\log_a(b) = 5$, $\log_a(c) = 12$, and $\log_a(d) = 9$. Evaluate each of the following.

- 10. $\log_a(bc)$
- 11. $\log_a(c^3)$
- 12. $\log_a \left(\frac{d}{c}\right)$
- 13. $\log_a \left(\frac{bd}{c}\right)$

Properties of Logarithms Key

- $1. \ 2\log_b(x) 8\log_b(y)$
- 2. $3 + 3 \ln(z)$
- $3. \log_5 \left(\frac{9x}{w^2} \right)$
- 4. $\frac{\ln(10)}{\ln(7)}$
- $5. \ \frac{\ln(x)}{\ln(9)}$
- 6. $\frac{\ln(c)}{\ln(b)}$
- 7. $\frac{\ln(10)}{\ln(3)}$
- 8. $\frac{\ln(\pi)}{\ln(17)}$
- $9. \ \frac{\ln(x)}{\ln(w)}$
- 10. 17
- 11. 36
- 12. -3
- 13. 2

Exponential Equations

Solve each. Round to 3 decimal places when necessary.

1.
$$3e^{x-2} = 7$$

2.
$$5^x + 4 > 1$$

3.
$$2^{3x+4} = 32^{x-7}$$

4.
$$5e^{7x} + 10 = 42$$

5.
$$7^{4x+1} \ge 343$$

6.
$$1000e^{0.04x} = 2000$$

7.
$$3(4.1)^{x-2} = 8$$

8.
$$2^{x+1} = 5^{7x-5}$$

9.
$$8(17)^{-5x} = 22$$

10.
$$-3(11)^{x-10} = -58$$

11.
$$12^{-10x} + 8 = 80$$

12.
$$-5(10)^{7x} + 9 = -46$$

13.
$$8(8)^{10x} - 1 = 55.2$$

14.
$$3(3)^{-5x} - 8 = 74$$

15.
$$6(16)^{4x-9} = 19$$

16.
$$-7(11)^{5x-7} = -3$$

17.
$$3^{9-6x} - 7 = 26$$

16.1 Applications

- 1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?
- 2. Cadmium-109 has a half-life of about 1.267 years. If 50 mg are initially present, how many years will it take for 16 mg to remain?
- 3. The half-life of bismuth-207 is about 32.9 years. If 90 mg are initially present, how many years will it take for 75 mg to remain?

EXPONENTIAL EQUATIONS KEY

1. $x \approx 2.847$

2. $(-\infty, \infty)$

3. x = 19.5

4. $x \approx 0.265$

5. $\left[\frac{1}{2}, \infty\right)$

6. $x \approx 17.329$

7. $x \approx 2.695$

8. $x \approx 0.827$

Applications

1. Approximately 17,952 years

2. Approximately 2.0828 years

3. Approximately 8.6538 years

9. $x \approx -0.071$

10. $x \approx 11.235$

11. $x \approx -0.172$

12. $x \approx 0.149$

13. $x \approx 0.094$

14. $x \approx -0.602$

15. $x \approx 2.354$

16. $x \approx 1.323$

17. $x \approx 0.970$

Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

1.
$$\log_5(x) + x \log_5(x) > 0$$

2.
$$\ln (8 - x^2) = \ln(2 - x)$$

3.
$$\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$$

4.
$$\log_3(2x+1) - \log_3(x-5) = \log_3(x+1)$$

5.
$$\log_4(x+1) + \log_4(x-5) > 2$$

6.
$$\log(x+1) - \log(x-5) = \log(x-3)$$

7.
$$x \log_3(x+2) - \log_3(x+2) = 0$$

8.
$$\log_{1/2}(x+1) > -3$$

9.
$$\log_{12}(4x+4) = \log_{12}(5x+1)$$

10.
$$\log_{15}(-4x+2) = \log_{15}(6-2x)$$

11.
$$\log_{11}(-5-3x^2) = \log_{11}(-2x^2+6x)$$

12.
$$\log_{16}(x^2+4) = \log_{16}(2x+3)$$

13.
$$\log_7(8x-1) = \log_7(x^2+14)$$

14.
$$-7 \log_5(x+5) = -7$$

15.
$$7 \log_8(-x) = 28$$

16.
$$-10\log_3(x-5) = -20$$

LOGARITHMIC EQUATIONS AND INEQUALITIES KEY

- 1. $(1, \infty)$
- 2. x = -2
- 3. $x \approx 1.571$
- 4. $x \approx 6.873$
- 5. $(2, \infty)$
- 6. x = 7
- 7. $x = \pm 1$
- 8. (-1,7)
- 9. x = 3
- 10. x = -2
- 11. No Solution
- 12. x = 1
- 13. x = 3, 5
- 14. x = 0
- 15. x = -4096
- 16. x = 14

Sequences

Write the first 4 terms of each sequence.

1.
$$a_n = 2(-3)^n$$

2.
$$b_n = \frac{n!}{2^n}$$

3.
$$c_{n+1} = 5c_n + 1$$
; $c_1 = 2$

4.
$$d_n = \frac{1}{2}d_{n-1} + n$$
; $d_1 = 3$

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round**.

5.
$$a_n = \{343, 667, 991, 1315, \dots\}$$
; Find the 582nd term.

6.
$$b_n = \{300, 240, 192, 153.6, \dots\}$$
; Find the 711th term.

7.
$$c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots \right\}$$
; Find the 8,675,309th term.

Given each sequence, find the first 4 digits after the decimal point in the scientific notation version of each term.

8.
$$a_n = 17, 33, 49, 65, ...; a_{21,972}$$

9.
$$b_n = 25$$
, 36, 49, 64, 81, ... $b_{413,401}$

10.
$$c_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots c_{152}$$

SEQUENCES KEY

- 1. -6, 18, -54, 162
- $2. \ \frac{1}{2}, \ \frac{1}{2}, \ \frac{3}{4}, \ \frac{3}{2}$
- 3. 2, 11, 56, 281
- 4. 3, $\frac{7}{2}$, $\frac{19}{4}$, $\frac{51}{8}$
- 5. 188,587
- 6. 6882
- 7. 7634
- 8. 5155 (3.51553 \times 10⁵)
- 9. 7090 (1.7090369403 \times 10¹¹)
- 10. 1677 (1.1677487203 \times 10⁻⁴⁶)

Series

Find the sum of each, if possible.

1.
$$\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^{i}$$

$$2. \sum_{i=0}^{\infty} 3\left(-\frac{2}{3}\right)^{i}$$

$$3. \sum_{k=1}^{\infty} -2\left(\frac{1}{3}\right)^k$$

4.
$$\sum_{i=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^{j}$$

5.
$$\sum_{i=0}^{\infty} 1.2(0.8)^{i}$$

6.
$$\sum_{i=1}^{\infty} 1.2(0.8)^{i}$$

7.
$$\sum_{i=0}^{\infty} 0.8(1.2)^{i}$$

8.
$$\sum_{k=1}^{\infty} \frac{2}{3^k}$$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9.
$$9+13+17+21+\cdots+1565$$

10.
$$-3+6-12+24-48+\cdots+50,331,648$$

11.
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{981}$$

12.
$$2+4+6+8+10+\cdots+38,214$$

13.
$$3+7+11+15+\cdots+11,491$$

14.
$$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \dots + \frac{742}{743}$$

SERIES KEY

- 1. $\frac{1}{4}$
- 2. $\frac{9}{5}$
- 3. -1
- 4. Diverges
- 5. 6
- 6. 4.8
- 7. Diverges
- 8. 2
- 9. 306,930
- 10. -33, 554, 433
- 11. 7.4663
- 12. 365,096,556
- 13. 16,511,131
- 14. 733.8947

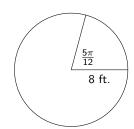
Angles and Radian Measure

Sketch each of the following. Then find a coterminal between 0 and 360° (or 0 and 2π radians) for each.

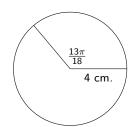
- 1. $-\frac{3\pi}{4}$
- 2. 900°
- 3. $\frac{27\pi}{10}$
- $4. -125^{\circ}$

Find the arc length and sector area formed by the central angle of each. Exact answers only.

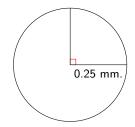
5.



6.



7.



A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

- 8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
- 9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

Angles and Radian Measure Key

1.
$$\frac{5\pi}{4}$$



2. 180°



3. $\frac{7\pi}{10}$



4. 235°



5. $s = \frac{10\pi}{3}$ ft.; $A = \frac{40\pi}{3}$ sq.ft.

6. $s = \frac{26\pi}{9} \text{ cm.}; \quad A = \frac{52\pi}{9} \text{ sq.cm}.$

7. $s = \frac{\pi}{8}$ mm.; $A = \frac{\pi}{64}$ sq.mm.

8. 26.18 rad/sec

9. 8.73 ft/sec

Trig Functions of Any Angle

Find the exact value of each of the six trig functions of θ if P is a point on the terminal side of θ .

- 1. P(-2,3)
- 2. P(0, -4)
- 3. $P(-2\sqrt{3}, 2)$
- 4. P(-3,5)
- 5. P(-2,1)
- 6. P(-4, -7)

Find the exact values of the 6 trig functions of the following angles.

- 7. $\theta = \frac{-17\pi}{4}$
- 8. $\theta = \frac{21\pi}{2}$
- 9. $\theta = 24\pi$

TRIG FUNCTIONS OF ANY ANGLE KEY

1.
$$\sin \theta = \frac{3\sqrt{13}}{13}$$
, $\cos \theta = \frac{-2\sqrt{13}}{13}$, $\tan \theta = -\frac{3}{2}$, $\csc \theta = \frac{\sqrt{13}}{3}$, $\sec \theta = -\frac{\sqrt{13}}{2}$, $\cot \theta = -\frac{2}{3}$

2.
$$\sin\theta=-1$$
, $\cos\theta=0$, $\tan\theta=$ undef., $\csc\theta=-1$, $\sec\theta=$ undef., $\cot\theta=0$

$$3. \ \sin\theta=\frac{1}{2}, \cos\theta=-\frac{\sqrt{3}}{2}, \tan\theta=-\frac{\sqrt{3}}{3}, \csc\theta=2, \sec\theta=-\frac{2\sqrt{3}}{3}, \cot\theta=-\sqrt{3}$$

4.
$$\sin\theta = \frac{5\sqrt{34}}{34}$$
, $\cos\theta = -\frac{3\sqrt{34}}{34}$, $\tan\theta = -\frac{5}{3}$, $\csc\theta = \frac{\sqrt{34}}{5}$, $\sec\theta = -\frac{\sqrt{34}}{3}$, $\cot\theta = -\frac{3}{5}$

5.
$$\sin\theta = \frac{\sqrt{5}}{5}$$
, $\cos\theta = -\frac{2\sqrt{5}}{5}$, $\tan\theta = -\frac{1}{2}$, $\csc\theta = \sqrt{5}$, $\sec\theta = -\frac{\sqrt{5}}{2}$, $\cot\theta = -2$

6.
$$\sin\theta = -\frac{7\sqrt{65}}{65}$$
, $\cos\theta = -\frac{4\sqrt{65}}{65}$, $\tan\theta = \frac{7}{4}$, $\csc\theta = -\frac{\sqrt{65}}{7}$, $\sec\theta = -\frac{\sqrt{65}}{4}$, $\cot\theta = \frac{4}{7}$

7.
$$\sin\theta = -\frac{\sqrt{2}}{2}$$
, $\cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = -1$, $\csc\theta = -\sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = -1$

8.
$$\sin \theta = 1$$
, $\cos \theta = 0$, $\tan \theta = \text{undefined}$, $\csc \theta = 1$, $\sec \theta = \text{undefined}$, $\cot \theta = 0$

9.
$$\sin \theta = 0$$
, $\cos \theta = 1$, $\tan \theta = 0$, $\csc \theta =$ undefined, $\sec \theta = 1$, $\cot \theta =$ undefined

Graphs of Sine and Cosine Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1.
$$f(x) = -2\sin\left(3x - \frac{\pi}{4}\right) + 1$$

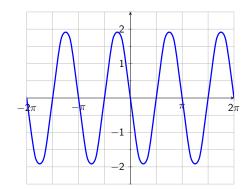
2.
$$g(x) = \frac{1}{3}\cos\left(\frac{1}{2}x + 2\right)$$

3.
$$f(x) = 2\sin\left(x - \frac{\pi}{3}\right) + 7$$

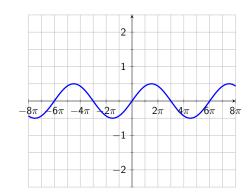
4.
$$f(x) = -4\cos\left(\frac{2}{3}x - \frac{2\pi}{3}\right)$$

Write the equation of each of the following in the form $y = a \sin(bx)$.

5.



6.



Answers

1. Amp = 2, Per =
$$\frac{2\pi}{3}$$
, P.S. = $\frac{\pi}{12}$ \rightarrow , V.S. = 1 \uparrow

2. Amp
$$=\frac{1}{3}$$
, Per $=4\pi$, P.S. $=4\leftarrow$, V.S. $=$ None

3. Amp = 2, Period =
$$2\pi$$
, P.S. = $\frac{\pi}{3}$ right, V.S. = 7 up

4. Amp = 4, Period =
$$3\pi$$
, P.S. = π right, V.S. = 0 (or none)

$$5. \ y = -2\sin(2x)$$

$$6. \ \ y = \frac{1}{2} \sin \left(\frac{1}{3} x \right)$$

Graphs of Other Trig Functions

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

$$1. \ h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

2.
$$f(x) = 3\tan\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$$

GRAPHS OF OTHER TRIG FUNCTIONS KEY

1. Amp = n/a, Per =
$$\frac{4\pi}{3}$$
, P.S. = $\frac{\pi}{9}$ \leftarrow , V.S. = 8 \downarrow

2. Amp = n/a, Period =
$$\frac{\pi}{2}$$
, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down

Inverse Trig Functions

State the exact, simplified value of each or write as an expression of x.

- 1. $tan^{-1}\left(-\sqrt{3}\right)$
- $2. \sec \left(\sin^{-1} \left(\frac{2}{5} \right) \right)$
- 3. $\cot (\sec^{-1}(x))$
- 4. $\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$
- 5. $\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$
- 6. $\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$
- 7. $\cos\left(\tan^{-1}(7x)\right)$
- 8. $\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$

INVERSE TRIG FUNCTIONS KEY

- 1. $-\frac{\pi}{3}$
- $2. \ \frac{5\sqrt{21}}{21}$
- 3. $\frac{1}{\sqrt{x^2 1}} = \frac{\sqrt{x^2 1}}{x^2 1}$
- 4. $\frac{\sqrt{16-9x^2}}{4}$
- 5. $-\frac{3\sqrt{5}}{2}$
- 6. $\frac{13\sqrt{22}}{44}$
- $7. \ \frac{\sqrt{49x^2+1}}{49x^2+1}$
- $8. \ \frac{\sqrt{64-x^2}}{x}$

Trig Equations and Inequalities

Solve each in the interval $[0, 2\pi)$. Write your answers to inequalities using interval notation.

- 1. tan(6x) = 1
- 2. $\cot(2x) = -\frac{\sqrt{3}}{3}$
- 3. $\sin^2(x) = \frac{3}{4}$
- $4. \sin(2x) = \cos(x)$
- 5. $\sin(2x) \ge \sin(x)$
- 6. $\cos(2x) < 0$
- $7. \ 2\sin\left(x \frac{\pi}{3}\right) = -1$
- 8. $3\tan\left(-2x+\frac{\pi}{2}\right)=\sqrt{3}$
- 9. $\sin^2(x) < \frac{1}{2}$
- 10. $tan^2(x) = 3 sec(x) 3$
- 11. $2\csc(x) 3\csc^2(x) = -2\csc^2(x) + 1$
- 12. $-2\cot(x) \csc^2(x) = 0$
- 13. tan(x) = -tan(x)cos(x)
- 14. $3\cos(x) = 2\cos^2(x) + 1$
- 15. $\csc(x) \cot^2(x) + 1 = 0$
- 16. $-\sin(x) + \sin(2x) = 2\sin(2x)$
- 17. $3\cos(x) = \sin(2x) + 2\cos(x)$

TRIG EQUATIONS AND INEQUALITIES KEY

- $1.\ \ \, \frac{\pi}{24},\frac{5\pi}{24},\frac{3\pi}{8},\frac{13\pi}{24},\frac{17\pi}{24},\frac{7\pi}{8},\frac{25\pi}{24},\frac{29\pi}{24},\frac{11\pi}{8},\frac{37\pi}{24},\frac{41\pi}{24},\frac{15\pi}{8}$
- 2. $\frac{\pi}{3}$, $\frac{5\pi}{6}$, $\frac{4\pi}{3}$, $\frac{11\pi}{6}$
- 3. $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$
- 4. $\frac{\pi}{2}$, $\frac{5\pi}{6}$, $\frac{3\pi}{2}$
- $5. \ \left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$
- 6. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
- 7. $x = \frac{\pi}{6}, \frac{3\pi}{2}$
- 8. $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$
- 9. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$
- 10. $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 11. $x = \frac{\pi}{2}$
- 12. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
- 13. x = 0, π
- 14. $x = 0 \frac{\pi}{3}, \frac{5\pi}{3}$
- 15. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
- 16. $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$
- 17. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Law of Sines and Cosines

Solve each of the following. Round your answers to 1 decimal place.

1.
$$m \angle B = 37.8^{\circ}$$
, $a = 15$, $c = 21.1$

2.
$$m \angle A = 41.9^{\circ}$$
, $m \angle C = 59.2^{\circ}$, $a = 10.2$

3.
$$a = 14$$
, $b = 19.6$, $c = 13.1$

4.
$$c = 29$$
, $b = 23$, $m \angle A = 55^{\circ}$

5.
$$c = 8$$
, $b = 12$, $m \angle A = 90^{\circ}$

6.
$$m \angle B = 67.2^{\circ}$$
, $a = 15.6$, $c = 18.9$

7.
$$b = 20$$
, $a = 30$, $c = 12$

8.
$$a = 14$$
, $b = 6$, $c = 12$

9.
$$a = 7$$
, $b = 14$, $c = 12$

10.
$$m \angle B = 119^{\circ}$$
, $a = 11$, $b = 28$

11.
$$m \angle A = 125^{\circ}$$
, $c = 21$, $a = 28$

12.
$$m \angle B = 108^{\circ}$$
, $m \angle C = 61^{\circ}$, $b = 25$

Law of Sines and Cosines Key

- 1. $b \approx 13.0$, $m \angle A \approx 44.8^{\circ}$, $m \angle C \approx 97.4^{\circ}$
- 2. $m \angle B = 78.9^{\circ}$, $b \approx 15.0$, $c \approx 13.1$
- 3. $m \angle A \approx 45.5^{\circ}$, $m \angle B \approx 92.6^{\circ}$, $m \angle C \approx 41.9^{\circ}$
- 4. $m \angle B \approx 50.1^{\circ}$, $m \angle C \approx 74.9^{\circ}$, $a \approx 24.6$
- 5. $m \angle B \approx 56.3^{\circ}$, $m \angle C \approx 33.7^{\circ}$, $a \approx 14.4$
- 6. $m \angle A \approx 48.2^{\circ}$, $m \angle C \approx 64.6^{\circ}$, $b \approx 19.3$
- 7. $m \angle A \approx 137.8^{\circ}$, $m \angle B \approx 26.6^{\circ}$, $m \angle C \approx 15.6^{\circ}$
- 8. $m \angle A \approx 96.4^{\circ}$, $m \angle B \approx 25.2^{\circ}$, $m \angle C \approx 58.4^{\circ}$
- 9. $m \angle A \approx 30^{\circ}$, $m \angle B \approx 91^{\circ}$, $m \angle C \approx 59^{\circ}$
- 10. $m \angle A \approx 20.1^{\circ}$, $m \angle C \approx 40.9^{\circ}$, $c \approx 21$
- 11. $m \angle B \approx 17.1^{\circ}$, $m \angle C \approx 37.9^{\circ}$, $b \approx 10.1$
- 12. $m \angle A \approx 11^{\circ}$, $a \approx 5$, $c \approx 23$

Area of Triangles

Find the area of each. Round your answers to 1 decimal place.

1.
$$m \angle B = 37.8^{\circ}$$
, $a = 15$, $c = 21.1$

2.
$$m \angle A = 41.9^{\circ}$$
, $m \angle C = 59.2^{\circ}$, $a = 10.2$

3.
$$a = 14$$
, $b = 19.6$, $c = 13.1$

4.
$$p = 14$$
, $k = 9$, $h = 9$

5.
$$m \angle T = 15^{\circ}$$
, $m \angle S = 140^{\circ}$, $r = 11.1$

6.
$$m \angle Z = 67^{\circ}$$
, $y = 6$, $m \angle Y = 41^{\circ}$

7.
$$m \angle R = 129^{\circ}$$
, $r = 10$, $m \angle P = 28^{\circ}$

8.
$$a = 6.9$$
, $m \angle B = 115^{\circ}$, $m \angle C = 39^{\circ}$

9.
$$d = 6$$
, $3 = 12$, $f = 8$

10.
$$m \angle Y = 120^{\circ}$$
, $x = 13$, $m \angle Z = 21^{\circ}$

11.
$$z = 10$$
, $y = 14$, $x = 6$

12.
$$m \angle P = 18^{\circ}$$
, $h = 6.9$, $m \angle H = 147^{\circ}$

13.
$$m \angle S = 118^{\circ}$$
, $m \angle T = 30^{\circ}$, $s = 6.3$

14.
$$r = 8$$
, $t = 7.5$, $m \angle S = 50^{\circ}$

15.
$$d = 15.3$$
, $m \angle E = 105^{\circ}$, $f = 5$

16.
$$m \angle R = 31^{\circ}$$
, $p = 12$, $m \angle Q = 26^{\circ}$

17.
$$m \angle D = 120^{\circ}$$
, $f = 4$, $m \angle E = 36^{\circ}$

Area of Triangles Key

- 1. Approximately 97.0 sq. units
- 2. Approximately 65.7 sq. units
- 3. Approximately 91.6 sq. units
- 4. Approximately 39.6 sq. units
- 5. Approximately 24.3 sq. units
- 6. Approximately 24.0 sq. units
- 7. Approximately 11.7 sq. units
- 8. Approximately 31.0 sq. units
- 9. Approximately 21.3 sq. units
- 10. Approximately 41.7 sq. units
- 11. Approximately 26.0 sq. units
- 12. Approximately 3.5 sq. units
- 13. Approximately 6.0 sq. units
- 14. Approximately 23.0 sq. units
- 15. Approximately 36.9 sq. units
- 16. Approximately 19.5 sq. units
- 17. Approximately 10.0 sq. units

Polar Coordinates

Convert each to exact rectangular coordinates.

- 1. $\left(3, \frac{5\pi}{6}\right)$
- 2. $\left(-2, -\frac{\pi}{4}\right)$
- 3. $\left(\frac{5}{2}, 240^{\circ}\right)$

Convert each to exact polar coordinates. Use r>0 and $0\leq \theta < 2\pi$.

- 4. $\left(-2\sqrt{3}, -2\right)$
- $5. \left(-\frac{1}{2}, \frac{1}{2}\right)$
- 6. $(7\sqrt{2}, -7\sqrt{2})$

Convert each to either rectangular or polar coordinates.

- 7. 2x + 5y = 9
- 8. 3y = 1
- 9. -5x 8y = -10
- 10. r = 8
- 11. $r = 4 \sec(\theta)$
- 12. $\theta = -\frac{\pi}{6}$

POLAR COORDINATES KEY

- $1. \left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$
- 2. $(-\sqrt{2}, \sqrt{2})$
- $3. \left(\frac{5}{4}, \frac{5\sqrt{3}}{4}\right)$
- 4. $\left(4, \frac{7\pi}{6}\right)$
- $5. \left(\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$
- 6. $\left(14, \frac{7\pi}{4}\right)$
- $7. \ \ r = \frac{9}{2\cos\theta + 5\sin\theta}$
- 8. $r = \frac{1}{3} \csc \theta$
- 9. $r = \frac{10}{5\cos\theta + 8\sin\theta}$
- 10. $x^2 + y^2 = 64$
- 11. x = 4
- 12. $y = -\frac{\sqrt{3}}{3}x$

Vectors

Given $\vec{v} = 3\mathbf{i} - 5\mathbf{j}$ and $\vec{w} = \langle -2, 1 \rangle$, find each. Exact and simplified answers only.

1.
$$\vec{v} + \vec{w}$$

2.
$$-4\vec{w}$$

Given $\vec{a} = \langle 4, -3 \rangle$, $\mathbf{b} = 11\mathbf{i} + 2\mathbf{j}$, and $\vec{c} = \langle 0, 2 \rangle$, find each. Exact and simplified answers only.

5.
$$\vec{a} + \vec{b}$$

6.
$$\vec{a} - \vec{b}$$

10.
$$3\vec{a} + 5\vec{b}$$

7.
$$a + c$$

11.
$$-2\vec{c} - 1.5\vec{a}$$

12.
$$||\vec{b}||$$

Find the exact horizontal and vertical component form of each vector with the given magnitude and direction angle.

17.
$$|u| = 8$$
, $\theta = 120^{\circ}$

19.
$$|u| = 1.5$$
, $\theta = -300^{\circ}$

21.
$$|u| = 4$$
, $\theta = 210^{\circ}$

18.
$$|u| = 5$$
, $\theta = \frac{\pi}{4}$

20.
$$|u| = \sqrt{2}, \quad \theta = \frac{3\pi}{2}$$

22.
$$|u| = 2\sqrt{3}$$
, $\theta = -\frac{\pi}{6}$

Write each of the following in $|\vec{v}|\langle\cos(\theta),\sin(\theta)\rangle$ form, where $0 \le \theta < 2\pi$.

23.
$$\vec{v} = \langle \sqrt{2}, -\sqrt{2} \rangle$$

24.
$$\vec{v} = \langle -1, \sqrt{3} \rangle$$

25.
$$\vec{v} = \langle 0, 4 \rangle$$

Vectors Key

2.
$$\langle 8, -4 \rangle$$

4.
$$\left\langle -\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$$

5.
$$\langle 15, -1 \rangle$$

6.
$$\langle -7, -5 \rangle$$

7.
$$\langle 4, -1 \rangle$$

8.
$$\langle -11, -2 \rangle$$

9.
$$\langle 0, 4 \rangle$$

10.
$$\langle 67, 1 \rangle$$

11.
$$\langle -6, 0.5 \rangle$$

12.
$$5\sqrt{5}$$

14.
$$\left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$$

15.
$$\left\langle \frac{11\sqrt{5}}{25}, \frac{2\sqrt{5}}{25} \right\rangle$$

16.
$$\langle 0, 1 \rangle$$

17.
$$\langle -4, 4\sqrt{3} \rangle$$

$$18. \left\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2} \right\rangle$$

$$19. \left\langle \frac{3}{4}, \frac{3\sqrt{3}}{4} \right\rangle$$

20.
$$\langle 0, -\sqrt{2} \rangle$$

21.
$$\langle -2\sqrt{3}, -2 \rangle$$

22.
$$\langle 3, -\sqrt{3} \rangle$$

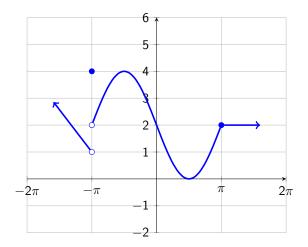
23.
$$2\left\langle\cos\left(\frac{7\pi}{4}\right),\sin\left(\frac{7\pi}{4}\right)\right\rangle$$

24.
$$2\left\langle\cos\left(\frac{2\pi}{3}\right),\sin\left(\frac{2\pi}{3}\right)\right\rangle$$

25.
$$4\left\langle\cos\left(\frac{\pi}{2}\right),\sin\left(\frac{\pi}{2}\right)\right\rangle$$

Numerical and Graphical Limits

Solve using the graph of f(x) below.



$$1. \lim_{x \to -\pi^-} f(x)$$

$$3. \lim_{x \to -\pi} f(x)$$

$$5. \lim_{x \to \pi^-} f(x)$$

7.
$$\lim_{x \to \pi} f(x)$$

$$2. \lim_{x \to -\pi^+} f(x)$$

4.
$$f(-\pi)$$

$$6. \lim_{x \to \pi^+} f(x)$$

8.
$$f(\pi)$$

Numerical and Graphical Limits Key

- 1. 1
- 2. 2
- 3. Does not exist
- 4. 4
- 5. 2
- 6. 2
- 7. 2
- 8. 2

Algebraic Limits

Evaluate each limit.

1.
$$\lim_{x \to -4} \left(\frac{x+4}{x^2+6x+8} \right)$$

2.
$$\lim_{x \to -3} \left(-\frac{x^2 + 2x - 3}{x + 3} \right)$$

3.
$$\lim_{x \to 3} \left(\frac{x-3}{x^2 - 8x + 15} \right)$$

4.
$$\lim_{x \to -2} \left(\frac{x^2 - 2x - 8}{x + 2} \right)$$

5.
$$\lim_{x \to -1} \left(-\frac{x^2 + 5x + 4}{x + 1} \right)$$

6.
$$\lim_{x \to -4} \left(\frac{x^2 + 7x + 12}{x + 4} \right)$$

7.
$$\lim_{x \to 2} \left(\frac{x^2 - 2x}{x - 2} \right)$$

8.
$$\lim_{x \to -1} \left(-\frac{x^2 - 3x - 4}{x + 1} \right)$$

ALGEBRAIC LIMITS KEY

- 1. $-\frac{1}{2}$
- 2. 4
- 3. $-\frac{1}{2}$
- 4. -6
- 5. -3
- 6. -1
- 7. 2
- 8. 5

Continuity

For each of the following, determine whether the function graphed below is continuous at that value. If not, explain why.



1.
$$x = -11$$

2.
$$x = -7$$

3.
$$x = -4$$

4.
$$x = 2$$

$$5 \times -6$$

6.
$$x = 10$$

Identify all discontinuities for each of the following.

7.
$$f(x) = \frac{x^2 - 6x}{x^2 + 6x}$$

8.
$$f(x) = \frac{x+3}{x-3}$$

$$9. \ f(x) = \frac{x+4}{3\ln(x)}$$

10.
$$f(x) = \frac{2x+5}{x^2-9}$$

11.
$$f(x) = \begin{cases} 2\sin(x), & x < 0 \\ 0, & x = 0 \\ 3x - 2, & x > 0 \end{cases}$$

12.
$$f(x) = \begin{cases} e^x - 1, & x \le 0 \\ x^2, & x > 0 \end{cases}$$

13.
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x > -1 \end{cases}$$

14.
$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & x < -1\\ 2x, & x \ge -1 \end{cases}$$

Continuity Key

- 1. Discontinuous; Not defined at x = -11
- 2. Continuous
- 3. Discontinuous; Left- and right-hand limits are not equal, nor do they equal the function value at x = -4
- 4. Discontinuous; Not defined at x = 2
- 5. Discontinuous; Left- and right-hand limits are not equal
- 6. Continuous
- 7. Discontinuous at x = 0, -6
- 8. Discontinuous at x = 3
- 9. Discontinuous at x = 1
- 10. Discontinuous at $x = \pm 3$
- 11. Discontinuous at x = 0
- 12. Continuous for all values of x
- 13. Discontinuous at x = -1
- 14. Continuous for all values of x

Derivatives

Use the definition of the derivative to find the derivative of each function with respect to x.

- 1. $f(x) = x^2 + 2x + 4$
- 2. f(x) = -5x + 5
- 3. $f(x) = 2x^2 5x 2$
- 4. f(x) = 5x + 2
- 5. $f(x) = x^3 x^2$
- 6. $f(x) = 5x^2 + 5$
- 7. f(x) = 2x + 3
- 8. $f(x) = x^2 2$
- 9. $f(x) = -5x^3 + 4$
- 10. $f(x) = -4x^2 + x 5$
- 11. $f(x) = 2x^2 3x + 5$

DERIVATIVES KEY

- 1. 2x + 2
- 2. -5
- 3. 4x 5
- 4. 5
- 5. $3x^2 2x$
- 6. 10*x*
- 7. 2
- 8. 2*x*
- 9. $-15x^2$
- 10. -8x + 1
- 11. 4x 3

Appendix A

Factoring

Factor each of the following completely.

1.
$$x^2 + 2x - 15$$

2.
$$x^2 - 8x + 12$$

3.
$$x^2 + 15x + 56$$

4.
$$5x^2 + 19x - 4$$

5.
$$4x^2 - 5x - 6$$

6.
$$9x^2 - 400$$

7.
$$5x^2 - 7x - 6$$

8.
$$9x^2 - 54x + 45$$

9.
$$3x^3 + 12x^2 + 9x$$

10.
$$9y^2 - 16$$

11.
$$4x^2 - 28x + 49$$

12.
$$14x^2 + 11xy - 15y^2$$
 18. $3p^2 + 22p - 16$

13.
$$6x^2 - 48x - 120$$

14.
$$9x^4 - 54x^3 + 45x^2$$

15.
$$16y^2 - 40y + 25$$

16.
$$30x^2 + xy - y^2$$

17.
$$8w^2 + 33w + 4$$

18.
$$3p^2 + 22p - 16$$

19.
$$18x^2 - 27x + 4$$

20.
$$14a^2 + 15a - 9$$

21.
$$4x^2 - 4x - 24$$

22.
$$18t^2 - 9t - 5$$

23.
$$6a^2 + 23a + 21$$

24.
$$25x^2 - 1$$

FACTORING KEY

1.
$$(x+5)(x-3)$$

2.
$$(x-6)(x-2)$$

3.
$$(x+7)(x+8)$$

4.
$$(5x-1)(x+4)$$

5.
$$(4x+3)(x-2)$$

6.
$$(3x + 20)(3x - 20)$$

7.
$$(5x+3)(x-2)$$

8.
$$9(x-5)(x-1)$$

9.
$$3x(x+3)(x+1)$$

10.
$$(3y+4)(3y-4)$$

11.
$$(2x-7)^2$$

12.
$$(7x - 5y)(2x + 3y)$$

13.
$$6(x-10)(x+2)$$

14.
$$9x^2(x-1)(x-5)$$

15.
$$(4y-5)^2$$

16.
$$(6x - y)(5x + y)$$

17.
$$(8w+1)(w+4)$$

18.
$$(3p-2)(p+8)$$

19.
$$(6x-1)(3x-4)$$

20.
$$(7a-3)(2a+3)$$

21.
$$4(x-3)(x+2)$$

22.
$$(6t-5)(3t+1)$$

23.
$$(2a+3)(3a+7)$$

24.
$$(5x+1)(5x-1)$$

Appendix B

Complex Fractions

Simplify each as much as possible.

1.
$$\frac{5+\frac{3}{x}}{x-\frac{1}{2}}$$

6.
$$\frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

11.
$$\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$2. \ \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

7.
$$\frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2 - 2x}}$$

12.
$$\frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

3.
$$\frac{3}{2 - \frac{x}{x-1}}$$

$$8. \ \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

$$13. \ \frac{\frac{1}{x-a} + \frac{1}{a}}{X}$$

4.
$$\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$$

9.
$$\frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

14.
$$\frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

$$5. \ \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$10. \ \frac{\frac{5}{x} - 5x}{x - 1}$$

15.
$$\frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

COMPLEX FRACTIONS KEY

1.
$$\frac{2(5x+3)}{x(2x-1)}$$

$$2. \ \frac{1}{x^2 - 2x + 4}$$

3.
$$\frac{3(x-1)}{x-2}$$

$$4. \ \frac{x+3}{2+7x}$$

5.
$$\frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \ \frac{-4x-1}{2}$$

7.
$$\frac{8x-10}{7}$$

8.
$$-\frac{1}{7x}$$

$$9. \ \frac{2x+1}{5x(x+1)}$$

$$10. \ \frac{-5x-5}{x}$$

11.
$$\frac{-1}{2x+4}$$

12.
$$\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \ \frac{1}{a(x-a)}$$

$$14. \ \frac{-2x-2}{5x^2-16x+3}$$

15.
$$\frac{x+4}{4x-8}$$