Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2. $\{x | x \le 1\}$
- 3. ₋₃
- 4. $\{x | x \neq 4, 11\}$
- 5. 2.4
- 6. $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7. $\{x | x \ge 2\}$
- 8. $\{x|x<-8\}$
- 9. $\{x | x \neq 3\}$
- 10. $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.



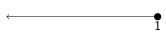


BASIC SET THEORY AND INTERVAL NOTATION KEY

1.
$$\{x | -5 < x \le 8\}$$



2. $(-\infty, 1]$



- 3. $[-3, \infty)$ $\{x | x \ge -3\}$
- $4. \ (-\infty, 4) \cup (4, 11) \cup (11, \infty)$ $\longleftrightarrow 0$ 4 11
- 5. [2.4, 7.7) $\{x | 2.4 \le x < 7.7\}$
- 6. $\{x|x > 9\}$



7. $[2, \infty)$



8. $(-\infty, -8)$



9. $(-\infty,3)\cup(3,\infty)$

$$\longleftrightarrow$$
 3

10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$

$$\begin{array}{ccc} & \bigcirc & \bigcirc & \bigcirc \\ -2 & & 5 \end{array}$$

11. $(-\infty, -8]$ $\{x | x \le -8\}$

12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x | x \neq 7, 12\}$

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

- 1. f(5)
- 2. f(-2)
- 3. f(0)
- 4. g(1)
- 5. g(-5)
- 6. g(1/4)

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

- 1. $f(x) = -8x^2 7x + 1$
- 2. $g(x) = \sqrt{5x + 12} 2$
- 3. $h(x) = \frac{x+2}{9x-7}$
- 4. f(x) = -5x + 4
- 5. $f(x) = x^2 + 2$
- 6. $f(x) = \frac{2x+1}{3x-5}$
- 7. $f(x) = \sqrt{3x 12}$
- 8. $f(x) = \frac{x}{x^2 16}$
- 9. $f(x) = \frac{x+4}{x^3-4x}$
- 10. $f(x) = \frac{x}{\sqrt{x-4}}$
- 11. $f(x) = \frac{x^2+1}{2x^2+8}$
- 12. $f(x) = -\frac{x+7}{x^2-5x-6}$
- 13. $g(x) = \sqrt{2x+3}$
- 14. $h(x) = \sqrt[3]{2x+3}$

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. *f*(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Evaluating Functions

- 1. -55
- 2. -20
- 3. 0
- 4. -4
- 5. -5.2
- 6. -1

Domain of Functions

- 1. $(-\infty, \infty)$
- 2. $\left[\frac{-12}{5}, \infty\right)$
- 3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
- 4. $(-\infty, \infty)$
- 5. $(-\infty, \infty)$
- 6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
- 7. $[4,\infty)$
- 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
- 9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$
- 10. $(4, \infty)$
- 11. $(-\infty, \infty)$
- 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
- 13. $\left[-\frac{3}{2},\infty\right)$
- 14. $(-\infty, \infty)$

Piecewise Functions

- $1. \ \sqrt{15} \approx 3.873$
- 2. 7
- 3. 6.6
- 4. 6.4
- 5. 7.1

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

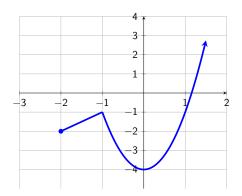
Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.3 Miscellaneous

Use the graph of y = f(x) below to answer questions 1–10. Write your answers using interval notation.



- 1. Domain of f
- 2. Range of f
- 3. Relative Minimum
- 4. Relative Maximum
- 5. f(1)

- 6. *f*(0)
- 7. Increasing Interval(s)
- 8. Decreasing Interval(s)
- 9. Absolute Maximum
- 10. Absolute Minimum

Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

- 1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$

Miscellaneous

- 1. $[-2, \infty)$
- 2. $[-4, \infty)$
- 3. (0, -4)
- 4. (-1, -1)
- 5. -1
- 6. -4)
- 7. $(-2, -1) \cup (0, \infty)$
- 8. (-1,0)
- 9. (0, -4)
- 10. None

Linear Functions and Slope

4.1 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

- 5. [-2, -1]
- 6. [5, 6]
- 7. [0, 1]

Average Rate of Change

- 1. 2.1
- 2. 2.01
- 3. 2.001
- 4. 2.0001
- 5. 2
- 6. 25
- 7. -59
- 8. 1

Function Transformations

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
 - (2) Shift left 1 unit
 - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
 - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
 - (2) Vertical compression by 4
 - (3) Move right 7 units

FUNCTION TRANSFORMATIONS KEY

1.
$$g(x) = 3f(-x+1)$$

2.
$$g(x) = f(2x) + 1$$

3.
$$g(x) = -\frac{1}{4}f(x-7)$$