# Honors PreCalculus

**Extra Practice** 

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# **Basic Set Theory and Interval Notation**

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2.  $\{x | x \le 1\}$
- 3. \_3
- 4.  $\{x | x \neq 4, 11\}$
- 5. 2.4
- 6.  $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7.  $\{x | x \ge 2\}$
- 8.  $\{x|x<-8\}$
- 9.  $\{x | x \neq 3\}$
- 10.  $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

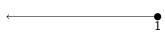
- 11.
- $\begin{array}{cccc}
   & & & & & & \downarrow \\
   & & & & & \uparrow \\
   & & & & & & \uparrow \\
   & & & & & & \uparrow
  \end{array}$

BASIC SET THEORY AND INTERVAL NOTATION KEY

1. 
$$\{x | -5 < x \le 8\}$$



2.  $(-\infty, 1]$ 



3.  $[-3, \infty)$   $\{x | x \ge -3\}$ 

4. 
$$(-\infty, 4) \cup (4, 11) \cup (11, \infty)$$



5. [2.4, 7.7)  $\{x | 2.4 \le x < 7.7\}$ 

6.  $\{x|x > 9\}$ 



7.  $[2, \infty)$ 



8.  $(-\infty, -8)$ 

9.  $(-\infty,3)\cup(3,\infty)$ 

$$\longleftrightarrow$$
 3

10.  $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$ 

$$\begin{array}{ccc} & \bigcirc & \bigcirc & \bigcirc \\ -2 & & 5 \end{array}$$

11.  $(-\infty, -8]$   $\{x | x \le -8\}$ 

12.  $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$   $\{x | x \neq 7, 12\}$ 

# **Functions and Their Graphs**

#### 2.1 Evaluating Functions

Given  $f(x) = -3x^2 + 4x$  and  $g(x) = \frac{1}{x} - 5$ , evaluate each.

- 1. f(5)
- 2. f(-2)
- 3. f(0)
- 4. g(1)
- 5. g(-5)
- 6. g(1/4)

#### 2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1. 
$$f(x) = -8x^2 - 7x + 1$$

2. 
$$g(x) = \sqrt{5x + 12} - 2$$

3. 
$$h(x) = \frac{x+2}{9x-7}$$

4. 
$$f(x) = -5x + 4$$

5. 
$$f(x) = x^2 + 2$$

6. 
$$f(x) = \frac{2x+1}{3x-5}$$

7. 
$$f(x) = \sqrt{3x - 12}$$

8. 
$$f(x) = \frac{x}{x^2 - 16}$$

9. 
$$f(x) = \frac{x+4}{x^3-4x}$$

10. 
$$f(x) = \frac{x}{\sqrt{x-4}}$$

11. 
$$f(x) = \frac{x^2+1}{2x^2+8}$$

12. 
$$f(x) = -\frac{x+7}{x^2-5x-6}$$

13. 
$$g(x) = \sqrt{2x+3}$$

14. 
$$h(x) = \sqrt[3]{2x+3}$$

15. 
$$f(x) = -\frac{7x-10}{x^2+3x+2}$$

16. 
$$g(x) = \sqrt{-9x + 8}$$

17. 
$$h(x) = -\sqrt[3]{4x+1}$$

18. 
$$f(x) = \sqrt[3]{8x+1}$$

19. 
$$g(x) = \frac{x^2-1}{\sqrt{x+3}}$$

20. 
$$h(x) = \frac{3}{9 + \frac{4}{x+7}}$$

21. 
$$f(x) = \frac{x+1}{\sqrt{10x+8}}$$

22. 
$$g(x) = \frac{5}{1 + \frac{3}{x+2}}$$

23. 
$$i(x) = \frac{7}{3 - \frac{4}{x+1}}$$

24. 
$$n(x) = \frac{7x+14}{\sqrt{2x-1}}$$

25. 
$$a(x) = \frac{\frac{x}{x-2}}{\frac{3}{x-2}+6}$$

26. 
$$d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$$

#### 2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. f(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4\\ \sqrt{2x + 7} & -4 < x < 0\\ |-x - 1| & x \ge 0 \end{cases}$$

- 6. f(3)
- 7. f(-2)
- 8. f(0)
- 9. f(-5)

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \le -3\\ \sqrt{-4x + 1} & \text{if } -3 < x \le 0\\ \frac{5x^2}{x + 7} & \text{if } x > 0 \end{cases}$$

- 10. f(7)
- 11. f(-3)
- 12. f(1)
- 13. f(0)
- 14. f(-1)
- 15. f(-3/2)

### **Evaluating Functions**

- 1. -55
- 2. -20
- 3. 0
- 4. -4
- 5. -5.2
- 6. -1

#### **Domain of Functions**

- 1.  $(-\infty, \infty)$
- 2.  $\left[\frac{-12}{5}, \infty\right)$
- 3.  $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
- 4.  $(-\infty, \infty)$
- 5.  $(-\infty, \infty)$
- 6.  $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
- 7.  $[4, \infty)$
- 8.  $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
- 9.  $(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,\infty)$
- 10.  $(4, \infty)$
- 11.  $(-\infty, \infty)$
- 12.  $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
- 13.  $\left[-\frac{3}{2}, \infty\right)$

- 14.  $(-\infty, \infty)$
- 15.  $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
- 16.  $\left(-\infty, \frac{8}{9}\right]$
- 17.  $(-\infty, \infty)$
- 18.  $(-\infty, \infty)$
- 19.  $(-3, \infty)$
- 20.  $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup \left(-7, \infty\right)$
- 21.  $\left(-\frac{4}{5}, \infty\right)$
- 22.  $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$
- 23.  $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$
- 24.  $(\frac{1}{2}, \infty)$
- 25.  $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 2\right) \cup \left(2, \infty\right)$
- 26.  $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)$

#### **Piecewise Functions**

- 1.  $\sqrt{15} \approx 3.873$
- 2. 7
- 3. 6.6
- 4. 6.4
- 5. 7.1

- 6. 4
- 7.  $\sqrt{3} \approx 1.732$
- 8. 1
- 9. 18
- 10. 17.5

- 11. 4
- 12.  $\frac{5}{8}$
- 13. 1
- 14.  $\sqrt{5} \approx 2.236$
- 15.  $\sqrt{7} \approx 2.646$

## **Properties of Functions**

#### 3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1. 
$$f(x) = x^2 - 3x^2 + 5$$

2. 
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3. 
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4. 
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

5. 
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

6. 
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

#### 3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1. 
$$f(x) = x^2 - 3x^2 + 5$$

2. 
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3. 
$$f(x) = x^3 + 2x^2 - 4x - 8$$

4. 
$$g(x) = x^4 - 2x^2 + 1$$

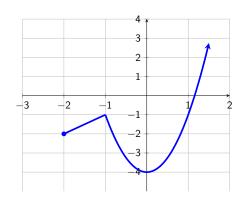
5. 
$$h(x) = \sqrt{x+1} - 2$$

6. 
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

7. 
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

#### 3.3 Miscellaneous

Use the graph of y = f(x) below to answer the following questions. Write your answers using interval notation.



- 1. Domain of f
- 2. Range of f
- 3. Relative Minimum
- 4. Relative Maximum
- 5. f(1)

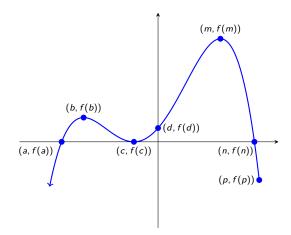
- 6. f(0)
- 7. Increasing Interval(s)
- 8. Decreasing Interval(s)
- 9. Absolute Maximum
- 10. Absolute Minimum

Find each of the following given  $f(x) = -2x^3 + 6x^2 - 5x + 1$ . Round to 3 decimal places and use interval notation when applicable.

- 11. f(7)
- 12. f(-2)
- 13. Rel. Max
- 14. Rel. Min

- 15. Global Max
- 16. Global Min
- 17. Increasing Interval(s)
- 18. Decreasing Interval(s)

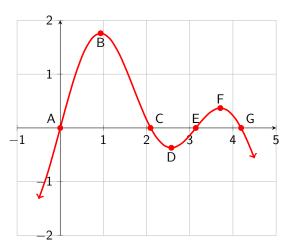
Use the graph of f(x) to answer each.



- 19. Relative maxima of f(x)
- 20. Relative minima of f(x)
- 21. Absolute maxima of f(x)
- 22. Absolute minima of f(x)

- 23. Intervals where f is increasing
- 24. Intervals where f is decreasing
- 25. Zeros of *f*

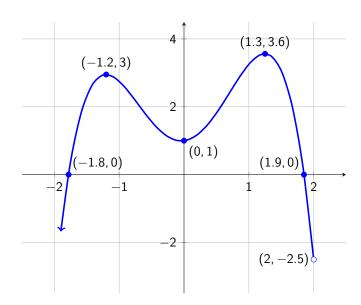
Given the labeled points A through G on the graph of f(x) below, find each of the following.



- 26. Increasing interval(s)
- 28. Relative max
- 30. Global max
- 32. Zeros of *f*

- 27. Decreasing interval(s)
- 29. Relative min
- 31. Global min
- 33. Number of solutions to f(x) = 1

Answer each of the following about the function f(x) below.



- 34. Domain of f
- 35. Range of f
- 36. Relative maxima
- 37. Relative minima
- 38. Absolute maximum

- 39. Absolute minimum
- 40. Increasing intervals
- 41. Decreasing intervals
- 42. Zeros of f(x)
- 43. Number of solutions to f(x) = 2

#### Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. Rel Max (-1.366, 10.848) and (1,6); Rel Min (0.366, 5.652)
- 4. Rel Max (-1.716, 11.598); Rel Min (1.132, -3.929)
- 5. Rel Max: (1.095, 12.096); Rel Min (-0.761, -0.680)
- 6. Rel Max: (1.366, 0.348); Rel Min: (-0.366, -4.848) and (2, 0)
- 7. About 2.16 hours

#### Increasing, Decreasing, and Constant Intervals

- 1. Increasing:  $(-\infty, 0)$  Decreasing:  $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing:  $(-\infty, -1.158) \cup (2.158, \infty)$
- 3. Inc:  $(-\infty, -2) \cup (\frac{2}{3}, \infty)$  Dec:  $(-2, \frac{2}{3})$
- 4. Inc;  $(-1,0) \cup (1,\infty)$  Dec:  $(-\infty,-1) \cup (0,1)$
- 5. Inc:  $(-1, \infty)$  No intervals where it is decreasing
- 6. Inc: (-0.761, 1.095); Dec:  $(-\infty, -0.761) \cup (1.095, \infty)$
- 7. Inc:  $(-0.366, 1.366) \cup (2, \infty)$ ; Dec:  $(-\infty, -0.366) \cup (1.366, 2)$ ;

#### Miscellaneous

- 1.  $[-2, \infty)$
- 2.  $[-4, \infty)$
- 3. (0, -4)
- 4. (-1, -1)
- 5. -1
- 6. -4)
- 7.  $(-2, -1) \cup (0, \infty)$
- 8. (-1,0)
- 9. (0, -4)
- 10. None
- 11. -426

- 12. 51
- 13. (1.408, 0.272)
- 14. (0.592, -0.272)
- 15. None
- 16. None
- 17. (0.592, 1.408)
- 18.  $(-\infty, 0.592) \cup$  $(1.408, \infty)$
- 19. (b, f(b)) and (m, f(m))
- 20. (c, f(c))
- 21. (m, f(m))

- 22. None
- 23.  $(-\infty, b) \cup (c, m)$
- 24.  $(b, c) \cup (m, p)$
- 25. x = a, x = c, x = n
- 26.  $(\infty, B) \cup (D, F)$
- 27. (B, D) ∪ (F, ∞)
- 28. *B* and *F*
- 29. D
- 30. B
- 31. None
- 32. A, C, E, G

- 33. 2
- 34.  $(-\infty, 2)$
- 35.  $(-\infty, -2.5) \cup (-2.5, 3.6]$
- 36. (-1.2, 3) and (1.3, 3.6)
- 37. (0, 1)
- 38. (1.3, 3.6)
- 39. Does not exist
- 40.  $(-\infty, -1.2) \cup (0, 1.3)$
- 41.  $(-1.2, 0) \cup (1.3, 2)$
- 42. (-1.8, 0) and (1.9, 0)
- 43. 4

### **Linear Functions and Slope**

#### 4.1 Equations of Lines

Write the equation of each line in point-slope form that goes through each pair of points.

- 1. (-2,1), (7,8)
- 2. (0,4), (9,-15)
- 3. (-1, -2), (-3, -13)

#### 4.2 Average Rate of Change

For the function  $f(x) = x^2$ , compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function  $f(x) = -6x^2 + 7x + 4$  over each specified interval.

- 6. [-2, -1]
- 7. [5, 6]
- 8. [0, 1]
- 9. [5, 5.001]
- 10. [5, 5.0001]
- 11. [5, 5.00001]
- 12. What value are your last 3 answers getting closer to?

For the function  $f(x) = -3x^2 + 5$ , determine the average rate of change of each over the given interval.

- 13. [7, 7.001]
- 14. [7, 7.0001]
- 15. [7, 7.00001]
- 16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = \sqrt{x}$ , find the average rate of change of each over the given interval.

- 17. [1, 1.0001]
- 18. [1, 1.00001]
- 19. [1, 1.000001]
- 20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given  $f(x) = 6\sqrt{x}$ , find the average rate of change of each over the given interval.

- 21. [25, 25.1]
- 22. [25, 25.01]
- 23. [25, 25.001]
- 24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function  $f(x) = -7x^3 + 6\sqrt{3x} + 4$  over each interval. Round your answers to 4 decimal places.

- 25. [0, 1]
- 26. [10, 11]
- 27. [8, 15]

#### **Equations of Lines**

- 1.  $y-1=\frac{7}{9}(x+2)$  or  $y-8=\frac{7}{9}(x-7)$
- 2.  $y-4=-\frac{19}{9}(x-0)$  or  $y+15=-\frac{19}{9}(x-9)$
- 3.  $y + 2 = \frac{11}{2}(x+1)$  or  $y + 13 = \frac{11}{2}(x+3)$

### **Average Rate of Change**

- 1. 2.1
- 2. 2.01
- 3. 2.001
- 4. 2.0001
- 5. 2
- 6. 25
- 7. -59
- 8. 1
- 9. -53.006

- 10. -53.0006
- 11. -53.00006
- 12. -53
- 13. -42.003
- 14. -42.0003
- 15. -42.00003
- 16. -42
- 17. -0.499988
- 18. -0.4999988

- 19. -0.49999988
- 20. -0.5
- 21. 0.5994
- 22. 0.59999
- 23. 0.6
- 24. 0.6
- 25. 3.3923
- 26. -2,315.3960
- 27. -2861.4492

### **Function Transformations**

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
  - (2) Shift left 1 unit
  - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
  - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
  - (2) Vertical compression by 4
  - (3) Move right 7 units

Write the function g(x) that is a result of the following ordered sequence of transformations to f(x) = |x|.

- 4. (1) Reflect across x-axis
  - (2) Shift right 3 units
  - (3) Horizontal stretch by factor of 5
- 5. (1) Shift down 2 units
  - (2) Reflect across y-axis
  - (3) Shift up 1 unit
- 6. (1) Horizontal compression by factor of 7
  - (2) Vertical compression by factor of 4
  - (3) Shift left 9 units

Given  $f(x) = \sqrt{x}$ , determine the resulting function g(x) after the following ordered sequence of transformations.

- 7. (1) Shift up 2 units
  - (2) Horizontal stretch by 5
  - (3) Shift left 3 units
- 8. (1) Vertical compression by factor of 3
  - (2) Reflect across y-axis
  - (3) Horizontal compression by 5
- 9. (1) Shift right 8 units
  - (2) Reflect across x-axis
  - (3) Horizontal compression by factor of 4

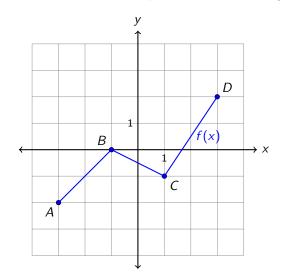
Write the final equation of g(x) if it is found by taking  $f(x) = \sqrt{x}$  after the following ordered sequence of transformations.

- 10. (1) Shift right 2 units
  - (2) Horizontal stretch by factor 3
  - (3) Shift down 2 units
  - (4) Reflect across x-axis
- 11. (1) Horizontal stretch by factor 3
  - (2) Shift left 1 unit
  - (3) Shift up 2 units
  - (4) Reflect across y-axis
- 12. (1) Vertical stretch by factor 5
  - (2) Horizontal stretch by factor 2
  - (3) Shift up 3 units
  - (4) Reflect across x-axis

Find the equation for g(x) if g(x) is found by performing the following *ordered* sequence of transformations to  $f(x) = \frac{1}{x}$ .

- 13. (1) Shift left 3 spaces
  - (2) Reflect across y-axis
  - (3) Shift down 5 spaces
  - (4) Vertical stretch by factor of 7
- 14. (1) Shift up 3 spaces
  - (2) Reflect across x-axis
  - (3) Shift right 5 spaces
  - (4) Horizontal compression by factor of 7

Given the graph of f(x) below, find the new coordinates of each point after the following transformations.



15. 
$$-2f(x+1)$$

16. 
$$f\left(-\frac{1}{2}x\right) - 3$$

17. 
$$\frac{1}{2}f(-x-2)+2$$

18. 
$$f(2x+2)-1$$

FUNCTION TRANSFORMATIONS KEY

1. 
$$g(x) = 3f(-x+1)$$

2. 
$$g(x) = f(2x) + 1$$

3. 
$$g(x) = -\frac{1}{4}f(x-7)$$

4. 
$$g(x) = -\left|\frac{1}{5}x - 3\right|$$

5. 
$$g(x) = |-x| - 1$$

6. 
$$g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$$

7. 
$$g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$$

8. 
$$g(x) = \frac{1}{3}\sqrt{-5x}$$

9. 
$$g(x) = -\sqrt{4x - 8}$$

10. 
$$g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$$

11. 
$$g(x) = \sqrt{\frac{1}{3}(-x+1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$$

12. 
$$g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$$

13. 
$$g(x) = \frac{7}{-x+3} - 35$$

14. 
$$g(x) = -\frac{1}{7x-5} - 3$$

15. 
$$A'(-4,4)$$
,  $B'(-2,0)$ ,  $C'(0,2)$ ,  $D'(2,-4)$ 

16. 
$$A'(6,-5)$$
,  $B'(2,-3)$ ,  $C'(-2,-4)$ ,  $D'(-6,-1)$ 

17. 
$$A'(1,1)$$
,  $B'(-1,2)$ ,  $C'(-3,1.5)$ ,  $D'(-5,3)$ 

18. 
$$A'(-2.5, -3)$$
,  $B'(-1.5, -1)$ ,  $C'(-0.5, -2)$ ,  $D'(0.5, 1)$ 

# **Function Operations**

#### 6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given f(x) = x + 5,  $g(x) = x^2 - 1$ , and  $h(x) = \sqrt{x - 10}$ , simplify or evaluate each.

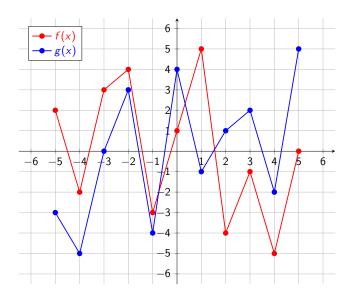
- 1. (g f)(x)
- 2. (fh)(14)
- 3. (f+g)(x)

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-3	0	-1	3	1	2	4	-4	$\overline{-2}$
g(x)	3	-1	0	1	4	-2	-4	2	-3

- 4. (f+g)(-2) 5. (f-g)(0) 6. (fg)(1)
- 7.  $\left(\frac{f}{g}\right)$  (3)
- 8. (f+f)(-4)

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 9. (f+g)(2)
- 10. (f-g)(1)
- 11. (g-f)(-3)
- 12. (fg)(4)
- 13.  $\left(\frac{f}{g}\right)(0)$

### 6.2 Operations with Functions: Domain

Given  $f(x) = \sqrt{2x+7}$  and g(x) = 3x+3, find the domain of each.

- 1. (f+g)(x)
- $2. \left(\frac{f}{g}\right)(x)$
- 3.  $\left(\frac{g}{f}\right)(x)$

#### 6.3 Difference Quotient

Write the difference quotient for each.

- 1. f(x) = 2x 7
- 2.  $g(x) = x^2 + 4x$
- 3. h(x) = -1
- 4.  $f(x) = \frac{3}{x+2}$
- 5.  $g(x) = \sqrt{3x}$
- 6.  $f(x) = x^2 2x + 5$
- 7.  $g(x) = \frac{5}{x}$

- 8.  $f(x) = -2x^2 + 3x 5$
- 9.  $g(x) = \frac{6}{2x+3}$
- 10.  $h(x) = \sqrt{7x+5}$
- 11.  $f(x) = -x^2 + x$
- 12. f(x) = 3x 1
- 13.  $f(x) = x^3 + 5x$

### Adding, Subtracting, Multiplying, and Dividing Functions

1. 
$$x^2 - x - 6$$

2. 38

3. 
$$x^2 + x + 4$$

4. -1

5. -3

6. -4

7. -2

8. -6

9. -3

10. 6

11. -3

12. 10

13.  $\frac{1}{4}$ 

### **Operations with Functions: Domain**

1. 
$$\left[-\frac{7}{2},\infty\right)$$

2. 
$$\left[-\frac{7}{2}, -1\right) \cup (-1, \infty)$$

3. 
$$\left(-\frac{7}{2}, \infty\right)$$

#### **Difference Quotient**

2. 
$$2x + h + 4$$

4. 
$$\frac{-3}{(x+2)(x+h+2)}$$

$$5. \ \frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$$

6. 
$$2x + h - 2$$

$$7. \ \frac{-5}{x(x+h)}$$

8. 
$$-4x - 2h + 3$$

9. 
$$\frac{-12}{(2x+3)(2x+2h+3)}$$

10. 
$$\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$$

11. 
$$-2x - h + 1$$

13. 
$$3x^2 + 3xh + h^2 + 5$$

# **Polynomials and Their Graphs**

Determine the end behavior of each.

1. 
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

2. 
$$g(x) = 4x^2 - 16x^6 + 3x$$

3. 
$$h(x) = 1 + x^{11} - 4x^8$$

4. 
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

5. 
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

6. 
$$f(x) = -6x^3 + 2x^2 + 7x^4 - 1$$

7. 
$$g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$$

POLYNOMIALS AND THEIR GRAPHS

1. 
$$\lim_{x \to -\infty} f(x) = \infty$$
  $\lim_{x \to \infty} f(x) = -\infty$ 

2. 
$$\lim_{x \to -\infty} g(x) = -\infty$$
,  $\lim_{x \to \infty} g(x) = \infty$ 

3. 
$$\lim_{x \to -\infty} h(x) = -\infty$$
  $\lim_{x \to \infty} h(x) = \infty$ 

4. 
$$\lim_{x \to -\infty} f(x) = -\infty$$
  $\lim_{x \to \infty} f(x) = -\infty$ 

5. 
$$\lim_{x \to -\infty} g(x) = -\infty$$
  $\lim_{x \to \infty} g(x) = \infty$ 

6. 
$$\lim_{x \to -\infty} f(x) = \infty$$
  $\lim_{x \to \infty} f(x) = -\infty$ 

7. 
$$\lim_{x \to -\infty} g(x) = -\infty$$
  $\lim_{x \to \infty} g(x) = \infty$ 

# **Dividing Polynomials**

#### 8.1 Dividing Polynomials

Divide each.

- 1.  $(28x^3 26x^2 + 41x 15) \div (7x 3)$
- 2.  $(44y^2 + 12y^3 + 61y 37) \div (3y + 5)$
- 3.  $(4x^3 3x^2 + x + 1) \div (x + 2)$
- 4.  $(5x^4 x^2 + x 2) \div (x^2 + 2)$
- 5.  $(10x^3 + 27x^2 + 8x 11) \div (2x + 3)$
- 6.  $(7x^3 + 23x^2 + 12x + 1) \div (x^2 + 3x + 1)$

#### 8.2 Remainder and Factor Theorems

Determine the remainder of each.

- 1.  $(2x^{53} 9x^{44} + 13x^8) \div (x 1)$
- 2.  $(x^{71} + 15x^{58} 3x^{14} + 2) \div (x + 1)$

### **Dividing Polynomials**

1. 
$$4x^2 - 2x + 5$$

2. 
$$4y^2 + 8y + 7 - \frac{72}{3y + 5}$$

3. 
$$4x^2 - 11x + 23 - \frac{45}{x+2}$$

4. 
$$5x^2 - 11 + \frac{x+20}{x^2+2}$$

$$5. \ 5x^2 + 6x - 5 + \frac{4}{2x+3}$$

6. 
$$7x + 2 + \frac{-x - 1}{x^2 + 3x + 1}$$

### **Remainder and Factor Theorems**

- 1. 6
- 2. 13

# Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1. 
$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

2. 
$$g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$3. \ f(x) = \frac{3x}{x+4}$$

4. 
$$g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

5. 
$$h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

6. 
$$f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

7. 
$$g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

8. 
$$h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

9. 
$$f(x) = \frac{6x^3 - 21x^2 - 51x + 30}{3x^2 + 7x + 2}$$

10. 
$$g(x) = \frac{10x^2 - 29x - 21}{10x^3 - 33x^2 - 7x}$$

State the end behavior of each.

11. 
$$k(x) = \frac{5x^3 - 7x^2 + 8}{-3x^3 + 6x - 4}$$

12. 
$$m(x) = \frac{2x-1}{3x^2+7x+1}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain: 
$$x \neq -\frac{1}{2}$$
, 7; V.A.:  $x = 7$ ; Hole @  $\left(-\frac{1}{2}, -\frac{7}{13}\right)$ ; H.A.:  $y = 1$ 

2. Domain: 
$$x \neq -3$$
, 4; V.A.:  $x = -3x = 4$ ; Obl. Asymp:  $y = 3x + 10$ 

3. Domain: 
$$x \neq -4$$
; V.A.:  $x = -4$ ; H.A.:  $y = 3$ 

4. Domain: 
$$x \neq 1$$
; V.A.:  $x = 1$ ; Obl. Asymp:  $y = x + 4$ 

5. Domain: 
$$x \neq 0, 1$$
; V.A.:  $x = 0$  and  $x = 1$ ; H.A.:  $y = 0$ 

6. Domain: 
$$x \neq -1, 5$$
; V.A.  $x = -1$ ; Hole @  $\left(5, \frac{13}{3}\right)$ ; Obl. Asym  $y = 2x - 5$ 

7. Domain: 
$$x \neq -6, 0, 4$$
; V.A.  $x = -6, x = 0$ ; Hole @  $\left(4, \frac{21}{40}\right)$ ; H.A.  $y = 0$ 

8. Domain: 
$$x \neq -6, -\frac{3}{8}$$
; V.A.  $x = -6, x = -\frac{3}{8}$ ; H.A.  $y = \frac{1}{4}$ 

9. Domain: 
$$x \neq -2$$
,  $-\frac{1}{3}$ ; Hole @  $(-2, -21)$ ; V.A.:  $x = -\frac{1}{3}$ ; S.A.  $y = 2x - \frac{35}{3}$ 

10. Domain: 
$$x \neq -\frac{1}{5}$$
, 0,  $\frac{7}{2}$ ; Hole @  $\left(\frac{7}{2}, \frac{82}{259}\right)$ ; V.A.  $x = -\frac{1}{5}$  and  $x = 0$ ; H.A.  $y = 0$ 

11. 
$$\lim_{x \to -\infty} k(x) = \infty = \lim_{x \to \infty} k(x) = -\frac{5}{3}$$

12. 
$$\lim_{x \to -\infty} m(x) = \infty = \lim_{x \to \infty} m(x) = 0$$

# Polynomial and Rational Inequalities

#### 10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1. 
$$6x^3 - 4x^2 - 10x \ge 0$$

2. 
$$x^4 < 9x^2$$

3. 
$$3x^3 - 7x^2 - 22x + 8 < 0$$

4. 
$$3x^2 - 4x + 1 < 0$$

5. 
$$12x^4 + 76x^3 + 43x^2 - 346x - 280 > 0$$

#### 10.2 Domain

State the domain of each. Write your answers using interval notation.

1. 
$$b(x) = \sqrt{21x^2 - 23x - 20}$$

#### 10.3 Rational Inequalities

Solve each. Write your answers using interval notation.

1. 
$$\frac{3x-4}{x+1} < 0$$

$$2. \ \frac{x^2 + 3x + 2}{x - 7} \le 0$$

$$3. \ \frac{x^2 - 4x + 4}{x^2 - 1} \ge 0$$

4. 
$$\frac{x+2}{x-4} \le 1$$

5. 
$$\frac{x^2 - 7x - 8}{x^2 - 4x - 32} \ge 0$$

6. 
$$\frac{4+3x}{5-x} \le 2$$

### **Polynomial Inequalities**

1. 
$$[-1,0] \cup \left[\frac{5}{3},\infty\right)$$

2. 
$$(-3,0) \cup (0,3)$$

$$3. \ (-\infty, -2) \cup \left(\frac{1}{3}, 4\right)$$

$$4. \left[\frac{1}{3}, 1\right]$$

5. 
$$(-\infty, -4] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$$

#### **Domain**

1. 
$$\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$$

### **Rational Inequalities**

1. 
$$\left(-1, \frac{4}{3}\right)$$

2. 
$$(-\infty, -2] \cup [-1, 7)$$

3. 
$$(-\infty, -1) \cup (1, \infty)$$

4. 
$$(-\infty, 4)$$

5. 
$$(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$$

6. 
$$(-\infty, 1.2] \cup (5, \infty)$$

### **Function Compositions**

Given f(x) = x - 5,  $g(x) = 4 + \sqrt{2x + 1}$ , and  $h(x) = \frac{3}{x + 7}$ , simplify each and state the domain.

- 1.  $(f \circ g)(x)$
- 2.  $(g \circ f)(x)$
- 3. h(h(x))

Find each of the following given the table below.

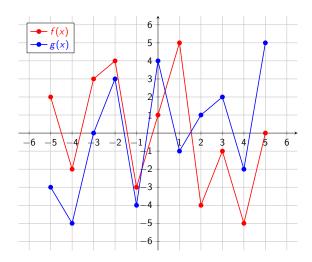
x	-4	-3	-2	-1	0	1	2	3	4	
f(x)	-3	0	-1	3	1	2	4	-4	-2	
g(x)	3	-1	0	1	4	-2	-4	2	-3	

- 4.  $(f \circ g)(-1)$  5.  $(g \circ g)(0)$  6.  $(f \circ f)(2)$  7.  $(g \circ g)(-3)$  8. f(g(0))

Given  $f(x) = \sqrt{3x+2}$ ,  $g(x) = x^2 - 1$ , and h(x) = 9x - 2, find each of the following.

- 9.  $(g \circ f)(x)$
- 10. f(g(x))
- 11.  $(h \circ h)(x)$

Find each of the following given the graphs of f(x) (in red) and g(x) (in blue) below:



- 12.  $(f \circ g)(-1)$  13.  $(g \circ f)(-4)$
- 14. f(g(3))
- 15. g(g(-2)) 16.  $(f \circ f)(-5)$

#### FUNCTION COMPOSITIONS KEY

- 1.  $-1 + \sqrt{2x+1}$  Domain:  $\left[-\frac{1}{2}, \infty\right)$
- 2.  $4 + \sqrt{2x 9}$  Domain:  $\left[\frac{9}{2}, \infty\right)$
- 3.  $\frac{3x+21}{7x+52}$  Domain:  $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup \left(-7, \infty\right)$
- 4. 2
- 5. -3
- 6. -2
- 7. 1
- 8. -2
- 9. 3x + 1
- 10.  $\sqrt{3x^2-1}$
- 11. 81x 20
- 12. -2
- 13. 3
- 14. -4
- 15. 2
- 16. -4

### **Inverse Functions**

Find the inverse of each. Then state the domain and range of the function and the inverse.

1. 
$$f(x) = \sqrt{-2x+3} + 1$$

2. 
$$g(x) = (x+4)^2 - 1$$
,  $x \le -4$ 

3. 
$$h(x) = \frac{9x}{4x - 1}$$

4. 
$$f(x) = \sqrt{x} - 3$$

$$5. g(x) = \frac{1}{1-x}$$

6. 
$$h(x) = x^2 + 6x + 4, x \le -3$$

7. 
$$f(x) = \sqrt{5x - 4}$$

8. 
$$g(x) = x^2 - 2x + 3, x \le 1$$

9. 
$$h(x) = \frac{3}{x-1}$$

#### INVERSE FUNCTIONS KEY

1. f	$^{-1}(x) = -$	$-\frac{1}{2}((x-$	$(1)^2 - 3)$
------	----------------	--------------------	--------------

	Domain	Range
f(x)	$(-\infty, 1.5]$	$[1,\infty)$
$f^{-1}(x)$	$[1,\infty)$	$(-\infty, 1.5]$

2. 
$$g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
g(x)	$(-\infty, -4]$	$[-1,\infty)$
$g^{-1}(x)$	$[-1,\infty)$	$(-\infty, -4]$

3. 
$$h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
h(x)	$(-\infty,1/4)\cup(1/4,\infty)$	$(\infty,9/4)\cup(9/4,\infty)$
$h^{-1}(x)$	$(\infty,9/4)\cup(9/4,\infty)$	$(-\infty,1/4)\cup(1/4,\infty)$

4. 
$$f^{-1}(x) = (x+3)^2$$

	Dom	Ran
f(x)	$[0,\infty)$	$[-3,\infty)$
$f^{-1}(x)$	$[-3,\infty)$	$[0,\infty)$

5. 
$$g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
g(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$g^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

6. 
$$h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
h(x)	$(-\infty, -3]$	$[-5,\infty)$
$h^{-1}(x)$	$[-5,\infty)$	$(-\infty, -3]$

7. 
$$f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$$

	Dom	Ran
f(x)	$\left[\frac{4}{5},\infty\right)$	[0, ∞)
$f^{-1}(x)$	[0, ∞)	$\left[\frac{4}{5},\infty\right)$

8. 
$$g^{-1}(x) = -\sqrt{x-2} + 1$$

	Dom	Ran
g(x)	$(-\infty,1]$	[2, ∞)
$g^{-1}(x)$	[2, ∞)	$(-\infty,1]$

9. 
$$h^{-1}(x) = \frac{3}{x} + 1$$

	Dom	Ran
h(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$h^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

# **Exponential Functions**

#### 13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

- 1.  $f(x) = 3 + e^{2x}$
- 2.  $h(x) = 5^{-x}$
- 3.  $h(x) = -\frac{2}{3}e^{x+7} + 1$

#### EXPONENTIAL FUNCTIONS

1. 
$$\lim_{x \to -\infty} f(x) = 3$$
  $\lim_{x \to \infty} f(x) = \infty$ 

2. 
$$\lim_{x \to -\infty} f(x) = \infty$$
  $\lim_{x \to \infty} f(x) = 0$ 

3. 
$$\lim_{x \to -\infty} h(x) = 1$$
  $\lim_{x \to \infty} h(x) = -\infty$ 

# **Logarithmic Functions**

Find the domain of each. Write your answers in interval notation.

1. 
$$b(x) = \log_7(x^2 - 8x + 6)$$

2. 
$$a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$$

3. 
$$f(x) = -7 \ln (x^2 + 9x + 8)$$

4. 
$$g(x) = \log (5x^2 + 13x - 6)$$

5. 
$$h(x) = 3 \log_2 (x^3 + 2x^2 - x - 2)$$

6. 
$$c(x) = \ln(4x^2 - 15x - 4)$$

State the end behavior of each.

7. 
$$j(x) = 5 \log_3 (2x - 5) - 2$$

LOGARITHMIC FUNCTIONS KEY

1. 
$$(-\infty, 0.838) \cup (7.162, \infty)$$

2. 
$$(-3, -2) \cup (-1, \infty)$$

3. 
$$(-\infty, -8) \cup (-1, \infty)$$

4. 
$$(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$$

5. 
$$(-2, -1) \cup (1, \infty)$$

6. 
$$\left(-\infty, -\frac{1}{4}\right) \cup (4, \infty)$$

7. 
$$\lim_{x \to (5/2)^+} j(x) = -\infty \quad \lim_{x \to \infty} j(x) = \infty$$

## **Properties of Logarithms**

Expand or condense each completely. Simplify numerical answers.

- 1.  $\log_b \left( \frac{x^2}{y^8} \right)$
- 2.  $\ln(ez)^3$
- 3.  $\log_5(x) + \log_5(9) 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

- 4.  $log_7(10)$
- 5.  $\log_9(x)$
- 6.  $\log_b(c)$
- 7.  $\log_3(10)$
- 8.  $\log_{17}(\pi)$
- 9.  $\log_w(x)$

Suppose that  $\log_a(b) = 5$ ,  $\log_a(c) = 12$ , and  $\log_a(d) = 9$ . Evaluate each of the following.

- 10.  $\log_a(bc)$
- 11.  $\log_a(c^3)$
- 12.  $\log_a \left(\frac{d}{c}\right)$
- 13.  $\log_a \left(\frac{bd}{c}\right)$

#### Properties of Logarithms Key

- $1. \ 2\log_b(x) 8\log_b(y)$
- 2.  $3 + 3 \ln(z)$
- $3. \log_5 \left( \frac{9x}{w^2} \right)$
- 4.  $\frac{\ln(10)}{\ln(7)}$
- $5. \ \frac{\ln(x)}{\ln(9)}$
- 6.  $\frac{\ln(c)}{\ln(b)}$
- 7.  $\frac{\ln(10)}{\ln(3)}$
- 8.  $\frac{\ln(\pi)}{\ln(17)}$
- $9. \ \frac{\ln(x)}{\ln(w)}$
- 10. 17
- 11. 36
- 12. -3
- 13. 2

# **Exponential Equations**

Solve each. Round to 3 decimal places when necessary.

- 1.  $3e^{x-2} = 7$
- 2.  $5^x + 4 > 1$
- 3.  $2^{3x+4} = 32^{x-7}$
- 4.  $5e^{7x} + 10 = 42$
- 5.  $7^{4x+1} \ge 343$
- 6.  $1000e^{0.04x} = 2000$
- 7.  $3(4.1)^{x-2} = 8$
- 8.  $2^{x+1} = 5^{7x-5}$

#### 16.1 Applications

1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?

EXPONENTIAL EQUATIONS KEY

- 1.  $x \approx 2.847$
- 2.  $(-\infty, \infty)$
- 3. x = 19.5
- 4.  $x \approx 0.265$
- 5.  $\left[\frac{1}{2}, \infty\right)$
- 6.  $x \approx 17.329$
- 7.  $x \approx 2.695$
- 8.  $x \approx 0.827$

#### **Applications**

1. Approximately 17,952 years

## Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

- 1.  $\log_5(x) + x \log_5(x) > 0$
- 2.  $\ln (8-x^2) = \ln(2-x)$
- 3.  $\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$
- 4.  $\log_3(2x+1) \log_3(x-5) = \log_3(x+1)$
- 5.  $\log_4(x+1) + \log_4(x-5) > 2$

LOGARITHMIC EQUATIONS AND INEQUALITIES KEY

- 1.  $(1, \infty)$
- 2. x = -2
- 3.  $x \approx 1.571$
- 4.  $x \approx 6.873$
- 5.  $(1.472, \infty)$

## Sequences

Write the first 4 terms of each sequence.

1. 
$$a_n = 2(-3)^n$$

2. 
$$b_n = \frac{n!}{2^n}$$

3. 
$$c_{n+1} = 5c_n + 1$$
;  $c_1 = 2$ 

4. 
$$d_n = \frac{1}{2}d_{n-1} + n$$
;  $d_1 = 3$ 

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round**.

5. 
$$a_n = \{343, 667, 991, 1315, \dots\}$$
; Find the 582nd term.

6. 
$$b_n = \{300, 240, 192, 153.6, \dots\}$$
; Find the 711th term.

7. 
$$c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots \right\}$$
; Find the 8,675,309th term.

Given each sequence, find the first 4 digits after the decimal point in the scientific notation version of each term.

8. 
$$a_n = 17, 33, 49, 65, ...; a_{21,972}$$

9. 
$$b_n = 25$$
, 36, 49, 64, 81, ...  $b_{413,401}$ 

10. 
$$c_n = \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots c_{152}$$

#### SEQUENCES KEY

- 1. -6, 18, -54, 162
- $2. \ \frac{1}{2}, \ \frac{1}{2}, \ \frac{3}{4}, \ \frac{3}{2}$
- 3. 2, 11, 56, 281
- 4. 3,  $\frac{7}{2}$ ,  $\frac{19}{4}$ ,  $\frac{51}{8}$
- 5. 188,587
- 6. 6882
- 7. 7634
- 8. 5155 (3.51553  $\times$  10<sup>5</sup>)
- 9. 7090 (1.7090369403  $\times$  10<sup>11</sup>)
- 10. 1677 (1.1677487203  $\times$  10<sup>-46</sup>)

#### **Series**

Find the sum of each, if possible.

1. 
$$\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^{i}$$

$$2. \sum_{i=0}^{\infty} 3\left(-\frac{2}{3}\right)^{i}$$

$$3. \sum_{k=1}^{\infty} -2\left(\frac{1}{3}\right)^k$$

4. 
$$\sum_{i=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^{j}$$

5. 
$$\sum_{i=0}^{\infty} 1.2(0.8)^{i}$$

6. 
$$\sum_{i=1}^{\infty} 1.2(0.8)^{i}$$

7. 
$$\sum_{i=0}^{\infty} 0.8(1.2)^{i}$$

8. 
$$\sum_{k=1}^{\infty} \frac{2}{3^k}$$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9. 
$$9+13+17+21+\cdots+1565$$

10. 
$$-3+6-12+24-48+\cdots+50,331,648$$

11. 
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{981}$$

12. 
$$2+4+6+8+10+\cdots+38,214$$

13. 
$$3+7+11+15+\cdots+11,491$$

14. 
$$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \dots + \frac{742}{743}$$

#### SERIES KEY

- 1.  $\frac{1}{4}$
- 2.  $\frac{9}{5}$
- 3. -1
- 4. Diverges
- 5. 6
- 6. 4.8
- 7. Diverges
- 8. 2
- 9. 306,930
- 10. -33, 554, 433
- 11. 7.4663
- 12. 365,096,556
- 13. 16,511,131
- 14. 733.8947

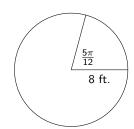
## **Angles and Radian Measure**

Sketch each of the following. Then find a coterminal between 0 and  $360^{\circ}$  (or 0 and  $2\pi$  radians) for each.

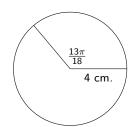
- 1.  $-\frac{3\pi}{4}$
- 2. 900°
- 3.  $\frac{27\pi}{10}$
- 4.  $-125^{\circ}$

Find the arc length and sector area formed by the central angle of each. Exact answers only.

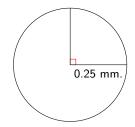
5.



6.



7.



A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

- 8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
- 9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

Angles and Radian Measure Key

1. 
$$\frac{5\pi}{4}$$



2. 180°



3.  $\frac{7\pi}{10}$ 



4. 235°



5.  $s = \frac{10\pi}{3}$  ft.;  $A = \frac{40\pi}{3}$  sq.ft.

6.  $s = \frac{26\pi}{9} \text{ cm.}; \quad A = \frac{52\pi}{9} \text{ sq.cm}.$ 

7.  $s = \frac{\pi}{8}$  mm.;  $A = \frac{\pi}{64}$  sq.mm.

8. 26.18 rad/sec

9. 8.73 ft/sec

# **Trig Functions of Any Angle**

Find the exact value of each of the six trig functions of  $\theta$  if P is a point on the terminal side of  $\theta$ .

- 1. P(-2,3)
- 2. P(0, -4)
- 3.  $P(-2\sqrt{3}, 2)$
- 4. P(-3,5)
- 5. P(-2,1)
- 6. P(-4, -7)

Find the exact values of the 6 trig functions of the following angles.

- 7.  $\theta = \frac{-17\pi}{4}$
- 8.  $\theta = \frac{21\pi}{2}$
- 9.  $\theta = 24\pi$

TRIG FUNCTIONS OF ANY ANGLE KEY

1. 
$$\sin \theta = \frac{3\sqrt{13}}{13}$$
,  $\cos \theta = \frac{-2\sqrt{13}}{13}$ ,  $\tan \theta = -\frac{3}{2}$ ,  $\csc \theta = \frac{\sqrt{13}}{3}$ ,  $\sec \theta = -\frac{\sqrt{13}}{2}$ ,  $\cot \theta = -\frac{2}{3}$ 

2. 
$$\sin \theta = -1$$
,  $\cos \theta = 0$ ,  $\tan \theta = \text{undef.}$ ,  $\csc \theta = -1$ ,  $\sec \theta = \text{undef.}$ ,  $\cot \theta = 0$ 

$$3. \ \sin\theta=\frac{1}{2}, \cos\theta=-\frac{\sqrt{3}}{2}, \tan\theta=-\frac{\sqrt{3}}{3}, \csc\theta=2, \sec\theta=-\frac{2\sqrt{3}}{3}, \cot\theta=-\sqrt{3}$$

4. 
$$\sin\theta = \frac{5\sqrt{34}}{34}$$
,  $\cos\theta = -\frac{3\sqrt{34}}{34}$ ,  $\tan\theta = -\frac{5}{3}$ ,  $\csc\theta = \frac{\sqrt{34}}{5}$ ,  $\sec\theta = -\frac{\sqrt{34}}{3}$ ,  $\cot\theta = -\frac{3}{5}$ 

5. 
$$\sin\theta = \frac{\sqrt{5}}{5}$$
,  $\cos\theta = -\frac{2\sqrt{5}}{5}$ ,  $\tan\theta = -\frac{1}{2}$ ,  $\csc\theta = \sqrt{5}$ ,  $\sec\theta = -\frac{\sqrt{5}}{2}$ ,  $\cot\theta = -2$ 

6. 
$$\sin\theta = -\frac{7\sqrt{65}}{65}$$
,  $\cos\theta = -\frac{4\sqrt{65}}{65}$ ,  $\tan\theta = \frac{7}{4}$ ,  $\csc\theta = -\frac{\sqrt{65}}{7}$ ,  $\sec\theta = -\frac{\sqrt{65}}{4}$ ,  $\cot\theta = \frac{4}{7}$ 

7. 
$$\sin\theta = -\frac{\sqrt{2}}{2}$$
,  $\cos\theta = \frac{\sqrt{2}}{2}$ ,  $\tan\theta = -1$ ,  $\csc\theta = -\sqrt{2}$ ,  $\sec\theta = \sqrt{2}$ ,  $\cot\theta = -1$ 

8. 
$$\sin \theta = 1$$
,  $\cos \theta = 0$ ,  $\tan \theta = \text{undefined}$ ,  $\csc \theta = 1$ ,  $\sec \theta = \text{undefined}$ ,  $\cot \theta = 0$ 

9. 
$$\sin \theta = 0$$
,  $\cos \theta = 1$ ,  $\tan \theta = 0$ ,  $\csc \theta =$ undefined,  $\sec \theta = 1$ ,  $\cot \theta =$ undefined

## **Graphs of Sine and Cosine Functions**

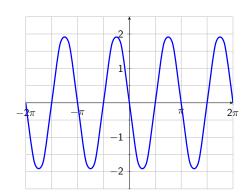
Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

1. 
$$f(x) = -2\sin\left(3x - \frac{\pi}{4}\right) + 1$$

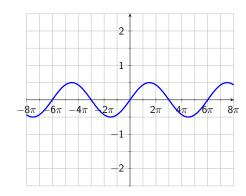
2. 
$$g(x) = \frac{1}{3}\cos\left(\frac{1}{2}x + 2\right)$$

Write the equation of each of the following in the form  $y = a \sin(bx)$ .

3.



4.



Answers

- 1. Amp = 2, Per =  $\frac{2\pi}{3}$ , P.S. =  $\frac{\pi}{12}$   $\rightarrow$ , V.S. = 1  $\uparrow$
- 2. Amp  $=\frac{1}{3}$ , Per  $=4\pi$ , P.S.  $=4\leftarrow$ , V.S. = None
- $3. \ y = -2\sin(2x)$
- $4. \ \ y = \frac{1}{2} \sin \left( \frac{1}{3} x \right)$

# **Graphs of Other Trig Functions**

Determine the amplitude, period, phase shift, and vertical shift for each. Exact answers only.

$$1. \ h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$$

GRAPHS OF OTHER TRIG FUNCTIONS KEY

1. Amp = n/a, Per = 
$$\frac{4\pi}{3}$$
, P.S. =  $\frac{\pi}{9}$   $\leftarrow$ , V.S. = 8  $\downarrow$ 

# **Inverse Trig Functions**

State the exact, simplified value of each or write as an expression of x.

- 1.  $tan^{-1}\left(-\sqrt{3}\right)$
- $2. \sec \left( \sin^{-1} \left( \frac{2}{5} \right) \right)$
- 3.  $\cot (\sec^{-1}(x))$
- 4.  $\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$
- 5.  $\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$
- 6.  $\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$
- 7.  $\cos\left(\tan^{-1}(7x)\right)$
- 8.  $\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$

INVERSE TRIG FUNCTIONS KEY

- 1.  $-\frac{\pi}{3}$
- $2. \ \frac{5\sqrt{21}}{21}$
- 3.  $\frac{1}{\sqrt{x^2 1}} = \frac{\sqrt{x^2 1}}{x^2 1}$
- 4.  $\frac{\sqrt{16-9x^2}}{4}$
- 5.  $-\frac{3\sqrt{5}}{2}$
- 6.  $\frac{13\sqrt{22}}{44}$
- $7. \ \frac{\sqrt{49x^2+1}}{49x^2+1}$
- $8. \ \frac{\sqrt{64-x^2}}{x}$

# **Trig Equations and Inequalities**

Solve each in the interval  $[0, 2\pi)$ . Write your answers to inequalities using interval notation.

- 1. tan(6x) = 1
- $2. \cot(2x) = -\frac{\sqrt{3}}{3}$
- 3.  $\sin^2(x) = \frac{3}{4}$
- $4. \sin(2x) = \cos(x)$
- 5.  $\sin(2x) \ge \sin(x)$
- 6.  $\cos(2x) < 0$

Trig Equations and Inequalities Key

$$1.\ \ \, \frac{\pi}{24},\frac{5\pi}{24},\frac{3\pi}{8},\frac{13\pi}{24},\frac{17\pi}{24},\frac{7\pi}{8},\frac{25\pi}{24},\frac{29\pi}{24},\frac{11\pi}{8},\frac{37\pi}{24},\frac{41\pi}{24},\frac{15\pi}{8}$$

2. 
$$\frac{\pi}{3}$$
,  $\frac{5\pi}{6}$ ,  $\frac{4\pi}{3}$ ,  $\frac{11\pi}{6}$ 

3. 
$$\frac{\pi}{3}$$
,  $\frac{2\pi}{3}$ ,  $\frac{4\pi}{3}$ ,  $\frac{5\pi}{3}$ 

4. 
$$\frac{\pi}{2}$$
,  $\frac{5\pi}{6}$ ,  $\frac{3\pi}{2}$ 

$$5. \ \left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$$

$$6. \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$$

## Law of Sines and Cosines

Solve each of the following. Round your answers to 1 decimal place.

- 1.  $m \angle B = 37.8^{\circ}$ , a = 15, c = 21.1
- 2.  $m \angle A = 41.9^{\circ}$ ,  $m \angle C = 59.2^{\circ}$ , a = 10.2
- 3. a = 14, b = 19.6, c = 13.1

Law of Sines and Cosines Key

- 1.  $b \approx 13.0$ ,  $m \angle A \approx 44.8^{\circ}$ ,  $m \angle C \approx 97.4^{\circ}$
- 2.  $m \angle B = 78.9^{\circ}$ ,  $b \approx 15.0$ ,  $c \approx 13.1$
- 3.  $m \angle A \approx 45.5^{\circ}$ ,  $m \angle B \approx 92.6^{\circ}$ ,  $m \angle C \approx 41.9^{\circ}$

# **Area of Triangles**

Find the area of each. Round your answers to 1 decimal place.

- 1.  $m \angle B = 37.8^{\circ}$ , a = 15, c = 21.1
- 2.  $m \angle A = 41.9^{\circ}$ ,  $m \angle C = 59.2^{\circ}$ , a = 10.2
- 3. a = 14, b = 19.6, c = 13.1

#### Area of Triangles Key

- 1. Approximately 97.0 sq. units
- 2. Approximately 65.7 sq. units
- 3. Approximately 91.6 sq. units

#### Appendix A

## **Factoring**

Factor each of the following completely.

1. 
$$x^2 + 2x - 15$$

2. 
$$x^2 - 8x + 12$$

3. 
$$x^2 + 15x + 56$$

4. 
$$5x^2 + 19x - 4$$

5. 
$$4x^2 - 5x - 6$$

6. 
$$9x^2 - 400$$

7. 
$$5x^2 - 7x - 6$$

8. 
$$9x^2 - 54x + 45$$

9. 
$$3x^3 + 12x^2 + 9x$$

10. 
$$9y^2 - 16$$

11. 
$$4x^2 - 28x + 49$$

12. 
$$14x^2 + 11xy - 15y^2$$
 18.  $3p^2 + 22p - 16$ 

13. 
$$6x^2 - 48x - 120$$

14. 
$$9x^4 - 54x^3 + 45x^2$$

15. 
$$16y^2 - 40y + 25$$

16. 
$$30x^2 + xy - y^2$$

17. 
$$8w^2 + 33w + 4$$

18. 
$$3p^2 + 22p - 16$$

19. 
$$18x^2 - 27x + 4$$

20. 
$$14a^2 + 15a - 9$$

21. 
$$4x^2 - 4x - 24$$

22. 
$$18t^2 - 9t - 5$$

23. 
$$6a^2 + 23a + 21$$

24. 
$$25x^2 - 1$$

#### FACTORING KEY

1. 
$$(x+5)(x-3)$$

2. 
$$(x-6)(x-2)$$

3. 
$$(x+7)(x+8)$$

4. 
$$(5x-1)(x+4)$$

5. 
$$(4x+3)(x-2)$$

6. 
$$(3x + 20)(3x - 20)$$

7. 
$$(5x+3)(x-2)$$

8. 
$$9(x-5)(x-1)$$

9. 
$$3x(x+3)(x+1)$$

10. 
$$(3y+4)(3y-4)$$

11. 
$$(2x-7)^2$$

12. 
$$(7x - 5y)(2x + 3y)$$

13. 
$$6(x-10)(x+2)$$

14. 
$$9x^2(x-1)(x-5)$$

15. 
$$(4y-5)^2$$

16. 
$$(6x - y)(5x + y)$$

17. 
$$(8w+1)(w+4)$$

18. 
$$(3p-2)(p+8)$$

19. 
$$(6x-1)(3x-4)$$

20. 
$$(7a-3)(2a+3)$$

21. 
$$4(x-3)(x+2)$$

22. 
$$(6t-5)(3t+1)$$

23. 
$$(2a+3)(3a+7)$$

24. 
$$(5x+1)(5x-1)$$

## **Appendix B**

## **Complex Fractions**

Simplify each as much as possible.

1. 
$$\frac{5 + \frac{3}{x}}{x - \frac{1}{2}}$$

6. 
$$\frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

11. 
$$\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$2. \ \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

7. 
$$\frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2 - 2x}}$$

12. 
$$\frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

3. 
$$\frac{3}{2 - \frac{x}{x-1}}$$

$$8. \ \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

$$13. \ \frac{\frac{1}{x-a} + \frac{1}{a}}{X}$$

4. 
$$\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$$

9. 
$$\frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

14. 
$$\frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

$$5. \ \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$10. \ \frac{\frac{5}{x} - 5x}{x - 1}$$

15. 
$$\frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

COMPLEX FRACTIONS KEY

1. 
$$\frac{2(5x+3)}{x(2x-1)}$$

$$2. \ \frac{1}{x^2 - 2x + 4}$$

3. 
$$\frac{3(x-1)}{x-2}$$

$$4. \ \frac{x+3}{2+7x}$$

5. 
$$\frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \ \frac{-4x-1}{2}$$

7. 
$$\frac{8x-10}{7}$$

8. 
$$-\frac{1}{7x}$$

$$9. \ \frac{2x+1}{5x(x+1)}$$

$$10. \ \frac{-5x-5}{x}$$

11. 
$$\frac{-1}{2x+4}$$

12. 
$$\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \ \frac{1}{a(x-a)}$$

$$14. \ \frac{-2x-2}{5x^2-16x+3}$$

15. 
$$\frac{x+4}{4x-8}$$