Honors PreCalculus

$$\sum$$

Additional Notes, Examples, and Practice Problems

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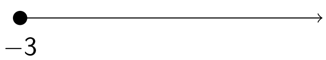
September 16, 2023

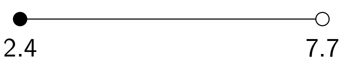
Contents

Chapter 1

Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

1. $(-5, 8]$
2. $\{x|x \leq 1\}$
3. 

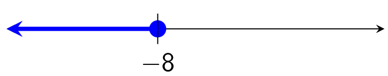
4. $\{x|x \neq 4, 11\}$
5. 

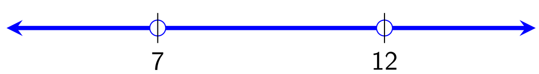
6. $(9, \infty)$

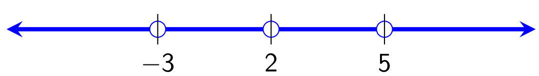
Write each using interval notation and graph on a number line.

7. $\{x|x \geq 2\}$
8. $\{x|x < -8\}$
9. $\{x|x \neq 3\}$
10. $\{x|x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

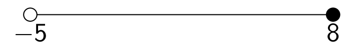
11. 

12. 

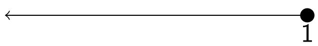
13. 

1.1 Answer Key

1. $\{x| -5 < x \leq 8\}$

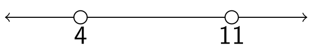


2. $(-\infty, 1]$



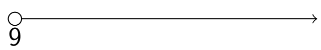
3. $[-3, \infty)$ $\{x|x \geq -3\}$

4. $(-\infty, 4) \cup (4, 11) \cup (11, \infty)$

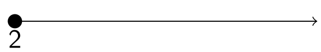


5. $[2.4, 7.7)$ $\{x|2.4 \leq x < 7.7\}$

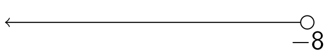
6. $\{x|x > 9\}$



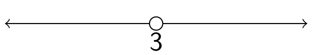
7. $[2, \infty)$



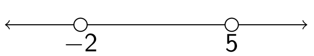
8. $(-\infty, -8)$



9. $(-\infty, 3) \cup (3, \infty)$



10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$



11. $(-\infty, -8]$ $\{x|x \leq -8\}$

12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x|x \neq 7, 12\}$

13. $(-\infty, -3) \cup (-3, 2) \cup (2, 5) \cup (5, \infty)$ $\{x|x \neq -3, 2, 5\}$

Chapter 2

Functions and Their Graphs

2.1 Evaluating Functions

The essence of functions can be summarized as follows:

1. Take an input
2. Process it according to how the function is defined
3. Return a result as output

We can give functions numerical or algebraic values.

Regardless of what type of input we give a function, you will need to remember your **order of operations** when processing that input. For each step in the order of operations, perform each step completely before moving on to the next step.

1. Simplify any expressions inside parentheses.
2. Evaluate any exponential expressions
3. Perform any multiplication or division *in the order in which they appear.*
4. Perform any addition or subtraction *in the order in which they appear.*

Example 1. Given $f(x) = -6x^2 - 5x + 2$, evaluate each of the following.

(a) $f(7)$ (b) $f(3x)$ (c) $f(5x - 6)$ (d) $f(x + h)$

Solution 1. $f(x)$'s whole purpose is to perform the following process:

1. Square the input you give it
2. Multiply that result by -6
3. Multiply the input by 5 and subtract that from the previous step
4. Add 2 to the result from the previous step

(a) $f(7)$

$$\begin{aligned} f(7) &= -6(7)^2 - 5(7) + 2 \\ &= -6(49) - 5(7) + 2 \qquad 7^2 = 49 \\ &= -294 - 5(7) + 2 \qquad -6(49) = -294 \\ &= -294 - 35 + 2 \qquad 5(7) = 35 \\ &= -329 + 2 \qquad -294 - 35 = -329 \\ &= -327 \end{aligned}$$

(b) $f(3x)$

$$\begin{aligned} f(3x) &= -6(3x)^2 - 5(3x) + 2 \\ &= -6(9x^2) - 5(3x) + 2 \qquad (3x)^2 = 9x^2 \\ &= -54x^2 - 5(3x) + 2 \qquad -6(9x^2) = -54x^2 \\ &= -54x^2 - 15x + 2 \qquad 5(3x) = 15x \\ &= -54x^2 - 15x + 2 \qquad \text{no like terms to combine} \end{aligned}$$

(c) $f(5x - 6)$

$$\begin{aligned} f(5x-6) &= -6(5x-6)^2 - 5(5x-6) + 2 \\ &= -6(25x^2 - 60x + 36) - 5(5x-6) + 2 \qquad (5x-6)^2 = 25x^2 - 60x + 36 \\ &= -150x^2 + 360x - 216 - 5(5x-6) + 2 \qquad \text{distribute the } -6 \\ &= -150x^2 + 360x - 216 - 25x + 30 + 2 \qquad \text{distribute the } -5 \\ &= -150x^2 + 335x - 184 \qquad \text{combine like terms} \end{aligned}$$

(d) $f(x + h)$

$$\begin{aligned} f(x+h) &= -6(x+h)^2 - 5(x+h) + 2 \\ &= -6(x^2 + 2hx + h^2) - 5(x+h) + 2 \qquad (x+h)^2 = (x+h)(x+h) = x^2 + 2hx + h^2 \\ &= -6x^2 - 12hx - 6h^2 - 5(x+h) + 2 \qquad \text{Distribute the } -6 \\ &= -6x^2 - 12hx - 6h^2 - 5x - 5h + 2 \qquad \text{Distribute the } -5 \end{aligned}$$

Domain

Recall that domain is the set of all possible inputs a function can accept. Currently, there are 2 domain issues that can arise:

- Dividing by 0
- Taking an even root of a negative number

Example 2. Find the domain of each. Write your answers in interval notation.

(a) $f(x) = \frac{8+\frac{5}{x}}{7-\frac{x}{x+9}}$ (b) $g(x) = \frac{9x^2-3}{x^2-6x+5}$ (c) $h(x) = \frac{2}{\sqrt{4x+3}}$

Solution 2. (a) $f(x) = \frac{8+\frac{5}{x}}{7-\frac{x}{x+9}}$

If we look at each fraction bar in the function, we find the following expressions represent denominators: x, $x + 9$, and $7 - \frac{x}{x+9}$. We want to make sure each of these does not equal 0.

For the first one, it's just a simple $x \neq 0$.

For the second, we set $x + 9 \neq 0$ from which we get $x \neq -9$.

For the third, we set $7 - \frac{x}{x+9} \neq 0$ and solve:

$$\begin{aligned}
7 - \frac{x}{x+9} &\neq 0 \\
7 &\neq \frac{x}{x+9} && \text{Add } \frac{x}{x+9} \text{ to both sides} \\
7(x+9) &\neq x && \text{Multiply both sides by } x+9 \\
7x + 63 &\neq x && \text{Distribute the 7} \\
63 &\neq -6x && \text{Subtract } 7x \text{ from both sides} \\
x &\neq -\frac{21}{2} && \text{Divide both sides by } -6 \text{ and simplify}
\end{aligned}$$

So we have the following for the domain: $x \neq -\frac{21}{2}, -9, 0$. Writing this in interval notation gives us

$$\left(-\infty, -\tfrac{21}{2}\right) \cup \left(-\tfrac{21}{2}, -9\right) \cup (-9, 0) \cup (0, \infty)$$

(b) $g(x) = \frac{9x^2-3}{x^2-6x+5}$

Here, we have the expression $x^2 - 6x + 5$ in a denominator. We want to make sure that denominator does not equal 0.

$$\begin{aligned}
x^2 - 6x + 5 &\neq 0 \\
(x-1)(x-5) &\neq 0 && \text{Factor the left side} \\
x - 1 \neq 0 \qquad x - 5 &\neq 0 && \text{Set each factor} \neq 0 \\
x &\neq 1, 5 && \text{Solve each}
\end{aligned}$$

The answer $x \neq 1, 5$ written in interval notation is $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$

(c) $h(x) = \frac{2}{\sqrt{4x+3}}$

For this one, we have a square root (which is an even root) in the denominator. Normally, we would want to set $4x + 3 \geq 0$, since we can't take the square root of a negative.

However, because the square root expression is in a denominator, we can not let it equal 0. Thus, we will set $4x + 3 > 0$ and solve to get $x > -\frac{3}{4}$

$x > -\frac{3}{4}$ written in interval notation is $\left(-\frac{3}{4}, \infty\right)$

2.2 Exercises

Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

1. $f(5)$
2. $f(-2)$
3. $f(0)$
4. $g(1)$
5. $g(-5)$
6. $g(1/4)$
7. $f(-x)$
8. $g(-x)$
9. $f(2x)$
10. $g(2x)$
11. $f(x-3)$
12. $g(x-3)$

13. $f\left(\frac{1}{3}x\right)$
14. $g\left(\frac{1}{3}x\right)$
15. $f(2x+1)$
16. $g(2x+1)$
17. $f(-x+7)$
18. $g(-x+7)$

Domain of Functions

Find the domain of each write your answers in interval notation.

1. $f(x) = -8x^2 - 7x + 1$
2. $g(x) = \sqrt{5x+12} - 2$
3. $h(x) = \frac{x+2}{9x-7}$
4. $f(x) = -5x + 4$
5. $f(x) = x^2 + 2$
6. $f(x) = \frac{2x+1}{3x-5}$
7. $f(x) = \sqrt{3x-12}$
8. $f(x) = \frac{x}{x^2-16}$
9. $f(x) = \frac{x+4}{x^3-4x}$
10. $f(x) = \frac{x}{\sqrt{x-4}}$
11. $f(x) = \frac{x^2+1}{2x^2+8}$
12. $f(x) = -\frac{x+7}{x^2-5x-6}$
13. $g(x) = \sqrt{2x+3}$
14. $h(x) = \sqrt[3]{2x+3}$
15. $f(x) = -\frac{7x-10}{x^2+3x+2}$
16. $g(x) = \sqrt{-9x+8}$
17. $h(x) = -\sqrt[3]{4x+1}$
18. $f(x) = \sqrt[3]{8x+1}$
19. $g(x) = \frac{x^2-1}{\sqrt{x+3}}$
20. $h(x) = \frac{3}{9+\frac{4}{x+7}}$
21. $f(x) = \frac{x+1}{\sqrt{10x+8}}$
22. $g(x) = \frac{5}{1+\frac{3}{x+2}}$
23. $i(x) = \frac{7}{3-\frac{4}{x+1}}$
24. $n(x) = \frac{7x+14}{\sqrt{2x-1}}$
25. $a(x) = \dfrac{\frac{x}{x-2}}{\frac{3}{x-2}+6}$
26. $d(x) = \frac{7x-5}{\sqrt[3]{5x+2}}$

2.3 Answer Key

Evaluating Functions

1. -55
2. -20
3. 0
4. -4
5. -5.2
6. -1
7. $-3x^2 - 4x$
8. $-\frac{1}{x} - 5 = \frac{-1-5x}{x}$
9. $-12x^2 + 8x$
10. $\frac{1-10x}{2x}$
11. $-3x^2 + 22x - 39$
12. $\frac{16-5x}{x-3}$
13. $-\frac{1}{3}x^2 + \frac{4}{3}x$
14. $\frac{3-5x}{x}$
15. $-12x^2 - 4x + 1$
16. $-\frac{10x+4}{2x+1}$
17. $-3x^2 + 38x - 119$
18. $\frac{5x-34}{-x+7}$

Domain of Functions

1. $(-\infty, \infty)$
2. $\left[\frac{-12}{5}, \infty\right)$
3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
4. $(-\infty, \infty)$
5. $(-\infty, \infty)$
6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
7. $[4, \infty)$
8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$
10. $(4, \infty)$
11. $(-\infty, \infty)$
12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
13. $\left[-\frac{3}{2}, \infty\right)$
14. $(-\infty, \infty)$
15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
16. $\left(-\infty, \frac{8}{9}\right]$
17. $(-\infty, \infty)$
18. $(-\infty, \infty)$
19. $(-3, \infty)$
20. $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup (-7, \infty)$
21. $\left(-\frac{4}{5}, \infty\right)$
22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$
23. $(-\infty, -1) \cup \left(-1, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$
24. $\left(\frac{1}{2}, \infty\right)$
25. $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, 2\right) \cup (2, \infty)$
26. $\left(-\infty, -\frac{2}{5}\right) \cup \left(-\frac{2}{5}, \infty\right)$

Chapter 3

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$
2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$
3. $f(x) = -x^4 + 3x^2 - 2x + 6$
4. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$
5. $f(x) = -4x^3 + 2x^2 + 10x + 4$
6. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$
7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \geq 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$
2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$
3. $f(x) = x^3 + 2x^2 - 4x - 8$
4. $g(x) = x^4 - 2x^2 + 1$
5. $h(x) = \sqrt{x+1} - 2$
6. $f(x) = -4x^3 + 2x^2 + 10x + 4$
7. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

3.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3 \\ 0.2x + 7 & \text{if } -3 \le x < 2 \\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

1. $f(3)$
2. $f(0)$
3. $f(-2)$
4. $f(-3)$
5. $f(0.5)$

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4 \\ \sqrt{2x+7} & -4 < x < 0 \\ |-x-1| & x \ge 0 \end{cases}$$

6. $f(3)$
7. $f(-2)$
8. $f(0)$
9. $f(-5)$

Find the value of each given the piecewise function below. Round to 3 decimal places when necessary.

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \le -3 \\ \sqrt{-4x+1} & \text{if } -3 < x \le 0 \\ \frac{5x^2}{x+7} & \text{if } x > 0 \end{cases}$$

10. $f(7)$
11. $f(-3)$
12. $f(1)$
13. $f(0)$
14. $f(-1)$
15. $f(-3/2)$

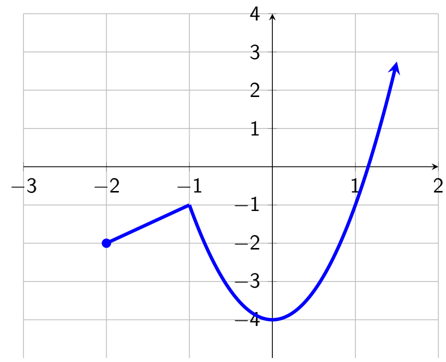
Find the value of each given the piecewise function below. Round to 3 decimal places where applicable.

$$f(x) = \begin{cases} |-3x-5| & \text{if } x \le -2 \\ 5e^{2x+1} & \text{if } -2 < x < 1 \\ \log_2\left(x^2 - 3x + 4\right) & \text{if } 1 \le x \le 4 \\ -3\sin(3\pi x) + 7 & \text{if } x > 4 \end{cases}$$

16. $f(-1)$
17. $f(8)$
18. $f(-3)$
19. $f(3)$
20. $f(-1.2)$

3.4 Miscellaneous

Use the graph of $y = f(x)$ below to answer the following questions. Write your answers using interval notation.

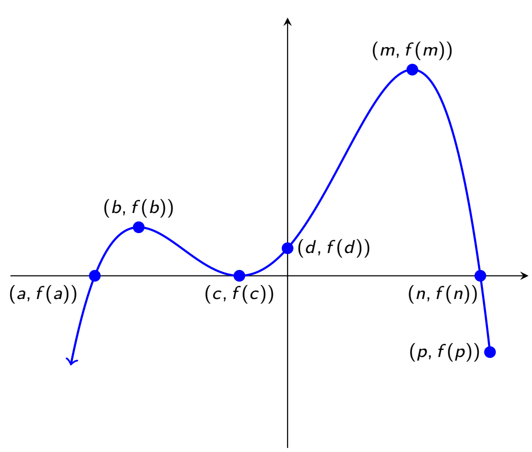


1. Domain of f
2. Range of f
3. Relative Minimum
4. Relative Maximum
5. $f(1)$
6. $f(0)$
7. Increasing Interval(s)
8. Decreasing Interval(s)
9. Absolute Maximum
10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

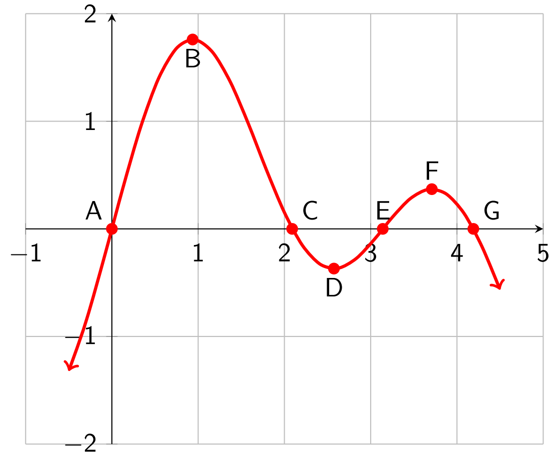
11. $f(7)$
12. $f(-2)$
13. Rel. Max
14. Rel. Min
15. Global Max
16. Global Min
17. Increasing Interval(s)
18. Decreasing Interval(s)

Use the graph of $f(x)$ to answer each.



19. Relative maxima of $f(x)$
20. Relative minima of $f(x)$
21. Absolute maxima of $f(x)$
22. Absolute minima of $f(x)$
23. Intervals where f is increasing
24. Intervals where f is decreasing
25. Zeros of f

Given the labeled points A through G on the graph of $f(x)$ below, find each of the following.



26. Increasing interval(s)

27. Decreasing interval(s)

28. Relative max

29. Relative min

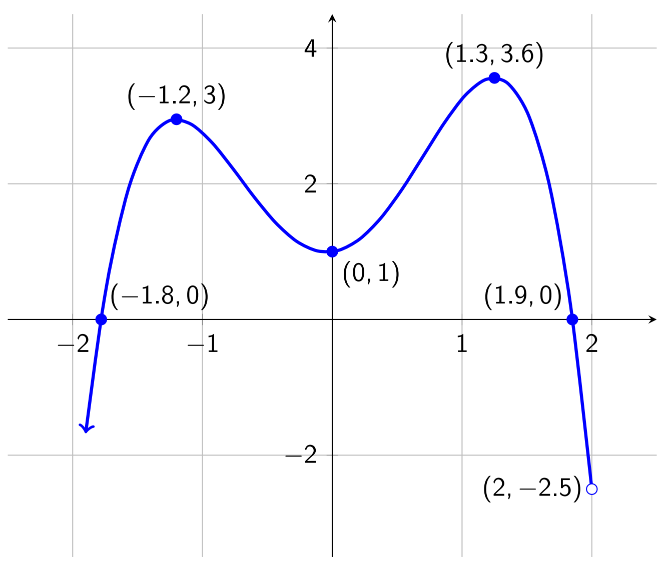
30. Global max

31. Global min

32. Zeros of f

33. Number of solutions to $f(x) = 1$

Answer each of the following about the function $f(x)$ below.



34. Domain of f

35. Range of f

36. Relative maxima

37. Relative minima

38. Absolute maximum

39. Absolute minimum

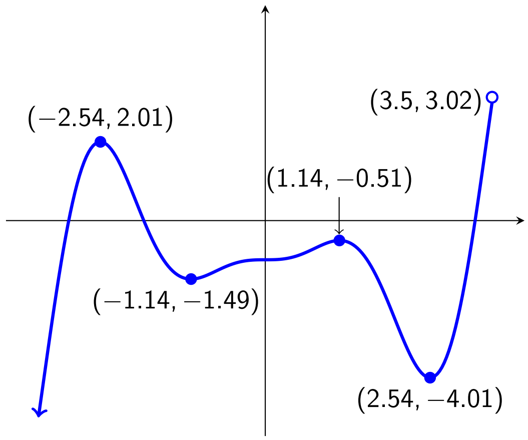
40. Increasing intervals

41. Decreasing intervals

42. Zeros of $f(x)$

43. Number of solutions to $f(x) = 2$

Find each of the following given the graph of $g(x)$ below.



44. Domain of g
45. Range of g
46. Number of zeros of g
47. Relative maxima
48. Relative minima
49. Global maximum
50. Global minimum
51. Increasing intervals
52. Decreasing intervals

3.5 Answer Key

Maxima and Minima

1. Rel max @ $(0, 5)$; No rel min
2. Rel max @ $(2.158, 3.248)$; Rel min @ $(-1.158, -4.048)$
3. Rel Max $(-1.366, 10.848)$ and $(1, 6)$; Rel Min $(0.366, 5.652)$
4. Rel Max $(-1.716, 11.598)$; Rel Min $(1.132, -3.929)$
5. Rel Max: $(1.095, 12.096)$; Rel Min $(-0.761, -0.680)$
6. Rel Max: $(1.366, 0.348)$; Rel Min: $(-0.366, -4.848)$ and $(2, 0)$
7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
2. Increasing: $(-1.158, 2.158)$ Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
3. Inc: $(-\infty, -2) \cup \left(\frac{2}{3}, \infty\right)$ Dec: $\left(-2, \frac{2}{3}\right)$
4. Inc; $(-1, 0) \cup (1, \infty)$ Dec: $(-\infty, -1) \cup (0, 1)$
5. Inc: $(-1, \infty)$ No intervals where it is decreasing
6. Inc: $(-0.761, 1.095)$; Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Piecewise Functions

1. $\sqrt{15} \approx 3.873$
2. 7
3. 6.6
4. 6.4
5. 7.1
6. 4
7. $\sqrt{3} \approx 1.732$
8. 1
9. 18
10. 17.5
11. 4
12. $\frac{5}{8}$
13. 1
14. $\sqrt{5} \approx 2.236$
15. $\sqrt{7} \approx 2.646$
16. 1.839
17. 7
18. 4
19. 2
20. 1.233

Miscellaneous

1. $[-2, \infty)$
2. $[-4, \infty)$
3. $(0, -4)$
4. $(-1, -1)$
5. -1
6. $-4)$
7. $(-2, -1) \cup (0, \infty)$
8. $(-1, 0)$
9. $(0, -4)$
10. None
11. -426
12. 51
13. (1.408, 0.272)
14. (0.592, −0.272)
15. None
16. None
17. (0.592, 1.408)
18. $(-\infty,\ 0.592) \cup (1.408,\ \infty)$
19. $(b, f(b))$ and $(m, f(m))$
20. $(c, f(c))$
21. $(m, f(m))$
22. None
23. $(-\infty, b) \cup (c, m)$
24. $(b, c) \cup (m, p)$
25. $x = a$, $x = c$, $x = n$
26. $(\infty, B) \cup (D, F)$
27. $(B, D) \cup (F, \infty)$
28. B and F
29. D
30. B
31. None
32. A, C, E, G
33. 2
34. $(-\infty, 2)$
35. $(-\infty, 3.6]$
36. $(-1.2, 3)$ and $(1.3, 3.6)$
37. $(0, 1)$
38. $(1.3, 3.6)$
39. Does not exist
40. $(-\infty, -1.2) \cup (0, 1.3)$
41. $(-1.2, 0) \cup (1.3, 2)$
42. $(-1.8, 0)$ and $(1.9, 0)$
43. 4
44. $(-\infty, 3.5)$
45. $(-\infty, 3.02)$
46. 3
47. $(-2.54, 2.01)$ and $(1.14, -0.51)$
48. $(-1.14, -1.49)$ and $(2.54, -4.01)$
49. None
50. None
51. $(-\infty, -2.54) \cup (-1.14, 1.14) \cup (2.54, 3.5)$
52. $(-2.54, -1.14) \cup (1.14, 2.54)$

Chapter 4

Linear Functions and Slope

4.1 Equations of Lines

Write the equation of each line **in point-slope form** that goes through each pair of points.

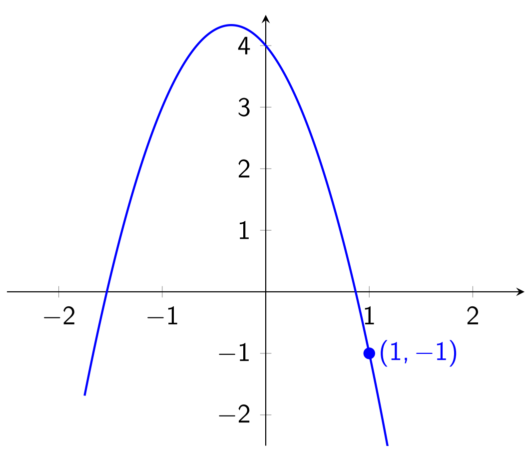
1. $(-2, 1)$, $(7, 8)$
2. $(0, 4)$, $(9, -15)$
3. $(-1, -2)$, $(-3, -13)$

4.2 Average Rate of Change

Recall that when you evaluate a function at a point, such as evaluating $f(x) = -3x^2 - 2x + 4$ when $x = 1$, you get

$$\begin{aligned} f(x) &= -3x^2 - 2x + 4 \\ f(1) &= -3(1)^2 - 2(1) + 4 \\ &= -1 \end{aligned}$$

If we look at the graph of the function, the point $(1, -1)$ is on the graph.



The average rate of change of a function $f(x)$ is the **slope** of the line connecting two points on the graph:

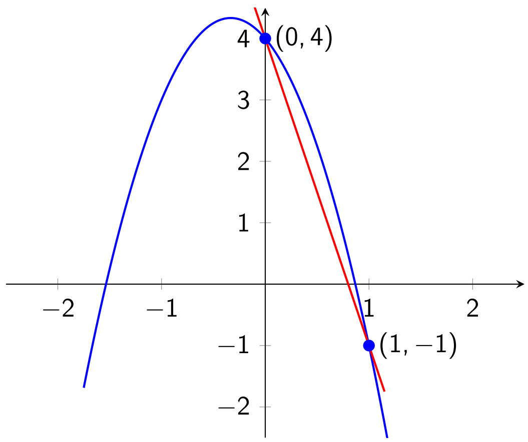
$$\text{Average rate of change} = \text{slope} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example 1. Find the average rate of change of $f(x) = -3x^2 - 2x + 4$ in each interval.

(a) $[0, 1]$

(b) $[-2, 1]$

Solution 1.
(a) Given $f(x) = -3x^2 - 2x + 4$, we want to find the slope of the red line connecting the points $(0, f(0))$ and $(1, f(1))$.

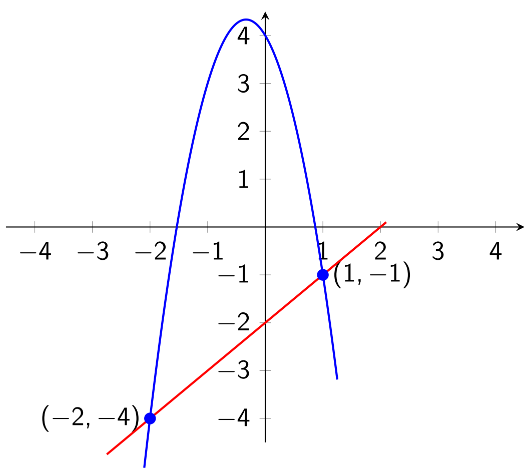


Noting that $f(0) = 4$, we have

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{4 - (-1)}{1 - 0}$$
$$= -5$$

Thus, the average rate of change of $f(x) = -3x^2 - 2x + 4$ over the interval $[0, 1]$ is $\boxed{-5}$

(b) This time, we want to find the slope of the red line connecting the points $(-2, f(-2))$ and $(1, f(1))$.



Noting that $f(-2) = -4$, we have

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{-1 - (-4)}{1 - (-2)}$$
$$= 1$$

Thus, the average rate of change of $f(x) = -3x^2 - 2x + 4$ over the interval $[-2, 1]$ is $\boxed{1}$

4.3 Exercises

For the function $f(x) = x^2$, compute the average rate of change for each interval.

1. $[1, 1.1]$
2. $[1, 1.01]$
3. $[1, 1.001]$
4. $[1, 1.0001]$
5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

6. $[-2, -1]$
7. $[5, 6]$
8. $[0, 1]$
9. $[5, 5.001]$
10. $[5, 5.0001]$
11. $[5, 5.00001]$
12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

13. $[7, 7.001]$
14. $[7, 7.0001]$
15. $[7, 7.00001]$
16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

17. $[1, 1.0001]$
18. $[1, 1.00001]$
19. $[1, 1.000001]$
20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = 6\sqrt{x}$, find the average rate of change of each over the given interval.

21. [25, 25.1]
22. [25, 25.01]
23. [25, 25.001]
24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -7x^3 + 6\sqrt{3x} + 4$ over each interval. Round your answers to 4 decimal places.

25. $[0, 1]$
26. $[10, 11]$
27. $[8, 15]$

4.4 Answer Key

Equations of Lines

1. $y - 1 = \frac{7}{9}(x + 2)$ or $y - 8 = \frac{7}{9}(x - 7)$
2. $y - 4 = -\frac{19}{9}(x - 0)$ or $y + 15 = -\frac{19}{9}(x - 9)$
3. $y + 2 = \frac{11}{2}(x + 1)$ or $y + 13 = \frac{11}{2}(x + 3)$

Average Rate of Change

1. 2.1
2. 2.01
3. 2.001
4. 2.0001
5. 2
6. 25
7. -59
8. 1
9. -53.006
10. -53.0006
11. -53.00006
12. -53
13. -42.003
14. -42.0003
15. -42.00003
16. -42
17. -0.499988
18. -0.4999988
19. -0.49999988
20. -0.5
21. 0.5994
22. 0.59999
23. 0.6
24. 0.6
25. 3.3923
26. $-2,315.3960$
27. -2861.4492

Chapter 5

Function Transformations

5.1 Ordered Transformations

Effect	What You Do	Function Notation	Actual Equation
1. Shift up 2 units	Add 2 to the function; replace $f(x)$ with $f(x)+2$	$f(x)+2$	$\sqrt{x}+2$
2. Shift down 2 units	Subtract 2 from function; replace $f(x)$ with $f(x)-2$	$f(x)-2$	$\sqrt{x}-2$
3. Shift right 4 units	Replace x with $x-4$	$f(x-4)$	$\sqrt{x-4}$
4. Shift left 5 units	Replace x with $x+5$	$f(x+5)$	$\sqrt{x+5}$
5. Reflect across the x-axis	Multiply the entire function by -1; replace $f(x)$ with $-f(x)$	$-f(x)$	$-\sqrt{x}$
6. Reflect across the y-axis	Replace x with $-x$	$f(-x)$	$\sqrt{-x}$
7. Vertical stretch by 7	Multiply entire function by 7; replace $f(x)$ with $7f(x)$	$7f(x)$	$7\sqrt{x}$
8. Vertical compression by 2	Multiply entire function by $\frac{1}{2}$; replace $f(x)$ with $\frac{1}{2}f(x)$	$\frac{1}{2}f(x)$	$\frac{1}{2}\sqrt{x}$
9. Horizontal stretch by 3	Replace x with $\frac{1}{3}x$	$f\left(\frac{1}{3}x\right)$	$\sqrt{\frac{1}{3}x}$
10. Horizontal compression by 6	Replace x with $6x$	$f(6x)$	$\sqrt{6x}$

Example 1. Given $f(x)=x^2$, write the equation for $g(x)$ if $g(x)$ is obtained by the following *ordered* sequence of transformations to $f(x)$.

(a)

1. Shift left 2 units
2. Vertical stretch by factor of 3
3. Reflect across y-axis
4. Shift down 5 units

(b)

1. Horizontal stretch by factor of 3
2. Shift right 2 units
3. Shift down 5 units
4. Reflect across y-axis

(c)

1. Shift up 2 units
2. Reflect across x-axis
3. Horizontal compression by factor of 3
4. Shift right 4 units

Solution 1.

(a)

1. Shifting left by 2 units replaces x with $x+2$, *i.e.* $g(x)=f(x+2)$:

$$\begin{aligned} f(x) &\to f(x+2) \\ &= (x+2)^2 \end{aligned}$$

2. We now take this new function, $g(x) = (x+2)^2$, and stretch the points away from the x-axis by a factor of 3. To do this, we multiply our function by 3 (or replace our current function $g(x)$ with $3g(x)$.

$$g(x) = 3(x+2)^2$$

3. Reflecting across the y-axis replaces our x with $-x$, *i.e.* $g(x) \to g(-x)$

$$g(-x) = 3(-x+2)^2$$

4. Shifting down 5 units subtracts 5 from our function: $g(x) - 5$, i.e. we replace $g(x)$ with $g(x) - 5$:

$$g(x) \to g(x) - 5 = 3(-x+2)^2 - 5$$

Thus, our equation is $\boxed{g(x) = 3(-x+2)^2 - 5}$

(b)

1. We first replace our x with $\frac{1}{3}x$

$$\begin{aligned} f(x) &\to f\left(\tfrac{1}{3}x\right) \\ &= \left(\tfrac{1}{3}x\right)^2 \end{aligned}$$

2. Next, we replace x in our current function with $x-2$, *i.e.* $f(x) \to f(x-2)$.

$$\begin{aligned} f(x) &\to f(x-2) \\ &= \left(\tfrac{1}{3}(x-2)\right)^2 \end{aligned}$$

3. Next, we subtract 5 from our current function: $f(x) \to f(x) - 5$:

$$\begin{aligned} f(x) &\to f(x) - 5 \\ &= \left(\tfrac{1}{3}(x-2)\right)^2 - 5 \end{aligned}$$

4. Lastly, reflect the graph across the y-axis by replacing x with $-x$

$$\begin{aligned} g(x) &\to f(-x) \\ &= \left(\tfrac{1}{3}(-x-2)\right)^2 - 5 \end{aligned}$$

Thus, our transformed function is $\boxed{g(x) = \left(\tfrac{1}{3}(-x-2)\right)^2 - 5}$

(c)

1. Shifting up 2 units adds 2 to our current function, *i.e.* $f(x) \to f(x) + 2$

$$\begin{aligned} f(x) &\to f(x) + 2 \\ &= x^2 + 2 \end{aligned}$$

2. Reflecting the graph across the x-axis multiplies our current function by -1, *i.e.* $f(x) \to -f(x)$

$$\begin{aligned} f(x) &\to -f(x) \\ &= -(x^2 + 2) \end{aligned}$$

3. Compressing the graph horizontally by a factor of 3 replaces x with $3x$, *i.e.* $f(x) \to f(3x)$

$$\begin{aligned} f(x) &\to f(3x) \\ &= -((3x)^2 + 2) \end{aligned}$$

4. Finally, shifting the graph right 4 units replaces the x in our current function with $x-4$

$$\begin{aligned} g(x) &\to f(x-4) \\ &= -((3(x-4))^2 + 2) \end{aligned}$$

Thus, our transformed function is $\boxed{g(x) = -((3(x-4))^2 + 2)}$

5.2 Exercises

Write the function for $g(x)$ if it is the result of $f(x)$ after the following ordered sequence of transformations.

1. (1) Vertical stretch by 3
   (2) Shift left 1 unit
   (3) Reflect across y-axis
2. (1) Horizontal compression by 2
   (2) Shift up 1 unit
3. (1) Reflect across x-axis
   (2) Vertical compression by 4
   (3) Move right 7 units

Write the function $g(x)$ that is a result of the following ordered sequence of transformations to $f(x) = |x|$.

4. (1) Reflect across x-axis
   (2) Shift right 3 units
   (3) Horizontal stretch by factor of 5
5. (1) Shift down 2 units
   (2) Reflect across y-axis
   (3) Shift up 1 unit
6. (1) Horizontal compression by factor of 7
   (2) Vertical compression by factor of 4
   (3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function $g(x)$ after the following ordered sequence of transformations.

7. (1) Shift up 2 units
   (2) Horizontal stretch by 5
   (3) Shift left 3 units
8. (1) Vertical compression by factor of 3
   (2) Reflect across y-axis
   (3) Horizontal compression by 5
9. (1) Shift right 8 units
   (2) Reflect across x-axis
   (3) Horizontal compression by factor of 4

Write the final equation of $g(x)$ if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

10. (1) Shift right 2 units
    (2) Horizontal stretch by factor 3
    (3) Shift down 2 units
    (4) Reflect across x-axis
11. (1) Horizontal stretch by factor 3
    (2) Shift left 1 unit
    (3) Shift up 2 units
    (4) Reflect across y-axis
12. (1) Vertical stretch by factor 5
    (2) Horizontal stretch by factor 2
    (3) Shift up 3 units
    (4) Reflect across x-axis

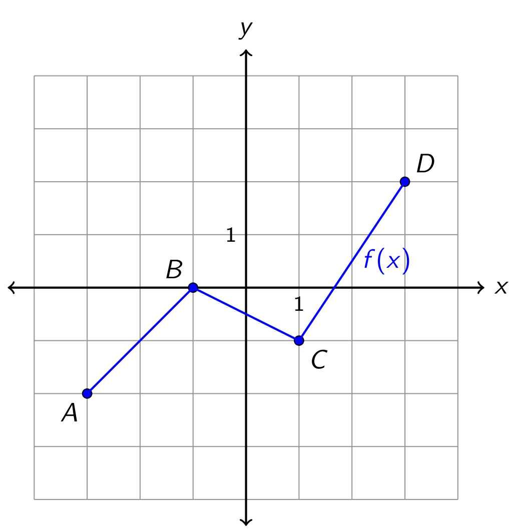
Find the equation for $g(x)$ if $g(x)$ is found by performing the following *ordered* sequence of transformations to $f(x) = \frac{1}{x}$.

13. (1) Shift left 3 spaces
    (2) Reflect across y-axis
    (3) Shift down 5 spaces
    (4) Vertical stretch by factor of 7
14. (1) Shift up 3 spaces
    (2) Reflect across x-axis
    (3) Shift right 5 spaces
    (4) Horizontal compression by factor of 7

Given $f(x) = x^3$, determine the equation for $g(x)$ after the following *ordered* sequence of transformations to $f(x)$.

15. (1) Vertical stretch by factor of 4
    (2) Shift up 3 units
    (3) Reflect across y-axis
    (4) Shift down 5 units
16. (1) Horizontal compression by factor of 3
    (2) Shift right 4 units
    (3) Shift up 1 unit
17. (1) Reflect across x-axis
    (2) Shift down 5 units
    (3) Vertical compression by factor of 5
    (4) Horizontal stretch by factor of 9

Given the graph of $f(x)$ below, find the new coordinates of each point after the following transformations.



18. $-2f(x+1)$

19. $f\left(-\frac{1}{2}x\right)-3$

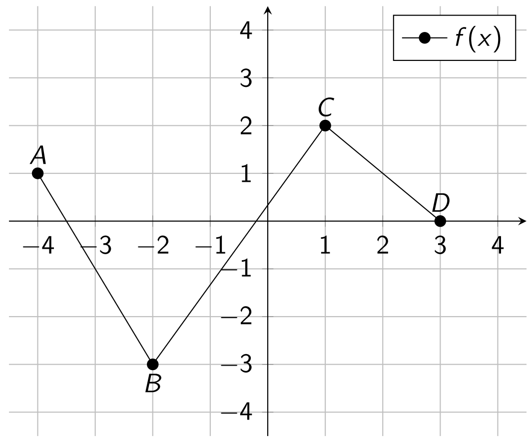
20. $\frac{1}{2}f(-x-2)+2$

21. $f(2x+2)-1$

22. $-3f(-x+1)+2$

23. $5f\left(-\frac{1}{2}x\right)$

Given $f(x)$ below, determine the coordinates of A', B', C', and D' after the final transformations done to f.



24. $f(-2x-5)+4$

25. $\frac{1}{3}f(x+4)-1$

26. $-3f\left(-\frac{1}{2}x-3\right)$

27. $f(4x+3)+8$

5.3 Answer Key

1. $g(x) = 3f(-x+1)$
2. $g(x) = f(2x) + 1$
3. $g(x) = -\frac{1}{4}f(x-7)$
4. $g(x) = -\left|\frac{1}{5}x - 3\right|$
5. $g(x) = |-x| - 1$
6. $g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$
7. $g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$
8. $g(x) = \frac{1}{3}\sqrt{-5x}$
9. $g(x) = -\sqrt{4x-8}$
10. $g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$
11. $g(x) = \sqrt{\frac{1}{3}(-x+1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$
12. $g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$
13. $g(x) = \frac{7}{-x+3} - 35$
14. $g(x) = -\frac{1}{7x-5} - 3$
15. $g(x) = 4(-x)^3 - 2$
16. $g(x) = (3(x-4))^3 + 1$
17. $g(x) = \frac{1}{5}\left(-\frac{1}{9}x\right)^3 - 5$
18. $A'(-4,4),\ B'(-2,0),\ C'(0,2),\ D'(2,-4)$
19. $A'(6,-5),\ B'(2,-3),\ C'(-2,-4),\ D'(-6,-1)$
20. $A'(1,1),\ B'(-1,2),\ C'(-3,1.5),\ D'(-5,3)$
21. $A'(-2.5,-3),\ B'(-1.5,-1),\ C'(-0.5,-2),\ D'(0.5,1)$
22. $A'(4,8),\ B'(2,2),\ C'(0,5),\ D'(-2,-4)$
23. $A'(6,-10),\ B'(2,0),\ C'(-2,-5),\ D'(-6,10)$
24. $A'\left(-\frac{1}{2},5\right),\ B'\left(-\frac{3}{2},1\right),\ C'(-3,6),\ D'(-4,4)$
25. $A'\left(-8,-\frac{2}{3}\right),\ B'(-6,-2),\ C'\left(-3,-\frac{1}{3}\right),\ D'(-1,-1)$
26. $A'(2,-3),\ B'(-2,9),\ C'(-8,-6),\ D'(-12,0)$
27. $A'\left(-\frac{7}{4},9\right),\ B'\left(-\frac{5}{4},5\right),\ C'\left(-\frac{1}{2},10\right),\ D'(0,8)$

Chapter 6

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

6.2 Operations with Functions: Domain

When adding, subtracting, multiplying functions, the **domain** of this is the overlap (on a number line) of the domains of each function.

In other words, if you graph the domain of each function on a number line, the overlap is the part of the number line those shaded areas have in common.

You can also find the domain of the sum, difference, product, or quotient *before* you simplify the expression.

Note: Division is the same with the added condition that the denominator is not allowed to equal 0.

Example 1. Given $f(x) = \frac{3}{x^2-4x+3}$, $g(x) = \sqrt{5x+10}$, and $h(x) = 2x+2$, state the domain of each.

(a) $(f+g)(x)$

(b) $(g \cdot h)(x)$

Solution 1.

- Domain of $f(x)$:

$$\begin{aligned} x^2 - 4x + 3 &\neq 0 \\ (x-1)(x-3) &\neq 0 \\ x \neq 1 &\text{ or } x \neq 3 \end{aligned}$$

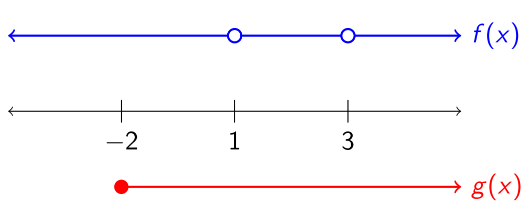
- Domain of $g(x)$:

$$\begin{aligned} 5x + 10 &\geq 0 \\ x &\geq -2 \end{aligned}$$

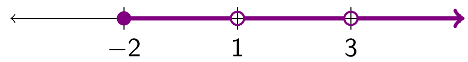
- Domain of $h(x)$: $\mathbb{R}$

(a) $(f+g)(x) = \frac{3}{x^2-4x+3} + \sqrt{5x+10}$

Combining the domains of $f(x)$ and $g(x)$ from above on a number line:



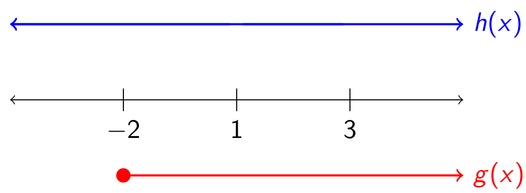
Looking at where the intervals of $f(x)$ and $g(x)$ overlap gives us the following.



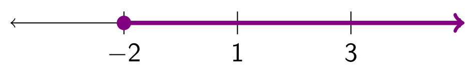
In interval notation, our solution is $[-2,1)\cup(1,3)\cup(3,\infty)$

(b) $(g\cdot h)(x)=\left(\sqrt{5x+10}\right)(2x+2)$

Combining the domains of $g(x)$ and $h(x)$ on a number line:



Looking at where the intervals of $g(x)$ and $h(x)$ overlap gives us the following.



In interval notation, our solution is $[-2,\infty)$

6.3 Difference Quotient

6.4 Exercises

Adding, Subtracting, Multiplying, and Dividing Functions

Given $f(x) = x + 5$, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

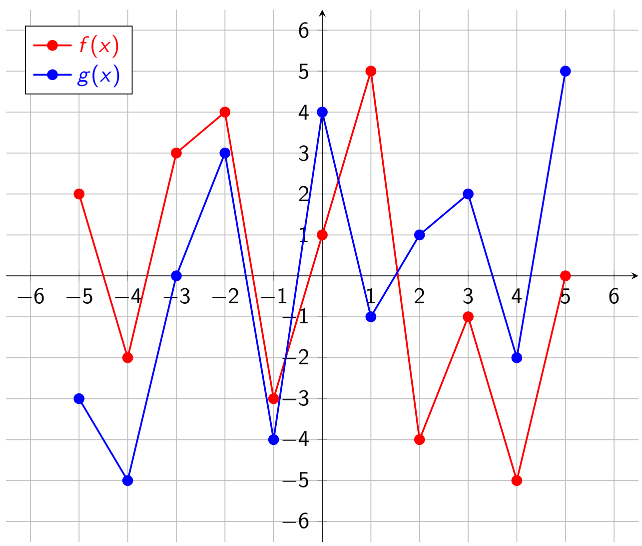
1. $(g - f)(x)$
2. $(fh)(14)$
3. $(f + g)(x)$

Find each of the following given the table below.

x	-4	-3	-2	-1	**0**	**1**	**2**	**3**	**4**
f(x)	-3	0	-1	3	1	2	4	-4	-2
g(x)	3	-1	0	1	4	-2	-4	2	-3

4. $(f + g)(-2)$ 5. $(f - g)(0)$ 6. $(fg)(1)$ 7. $\left(\frac{f}{g}\right)(3)$ 8. $(f + f)(-4)$

Find each of the following given the graphs of $f(x)$ (in red) and $g(x)$ (in blue) below:



9. $(f + g)(2)$ 10. $(f - g)(1)$ 11. $(g - f)(-3)$ 12. $(fg)(4)$ 13. $\left(\frac{f}{g}\right)(0)$

Use the table below to find each.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	-1	3	-4	5	0	4	-5	2	-2	-3	1
$g(x)$	4	-2	-4	1	-1	3	0	-3	-5	2	5

14. $(f + g)(-1)$ 15. $(f - g)(2)$ 16. $(fg)(-3)$ 17. $\left(\frac{f}{g}\right)(5)$ 18. $(ff)(-4)$

Operations with Functions: Domain

Given $f(x) = \sqrt{2x + 7}$ and $g(x) = 3x + 3$, find the domain of each.

1. $(f + g)(x)$
2. $\left(\frac{f}{g}\right)(x)$
3. $\left(\frac{g}{f}\right)(x)$

Difference Quotient

Write the difference quotient for each.

1. $f(x) = 2x - 7$
2. $g(x) = x^2 + 4x$
3. $h(x) = -1$
4. $f(x) = \frac{3}{x+2}$
5. $g(x) = \sqrt{3x}$
6. $f(x) = x^2 - 2x + 5$
7. $g(x) = \frac{5}{x}$
8. $f(x) = -2x^2 + 3x - 5$
9. $g(x) = \frac{6}{2x+3}$
10. $h(x) = \sqrt{7x+5}$
11. $f(x) = -x^2 + x$
12. $f(x) = 3x - 1$
13. $f(x) = x^3 + 5x$
14. $f(x) = \frac{6}{x+7}$
15. $g(x) = \frac{9}{x}$
16. $h(x) = \frac{5}{2x-1}$

6.5 Answer Key

Adding, Subtracting, Multiplying, and Dividing Functions

1. $x^2 - x - 6$
2. 38
3. $x^2 + x + 4$
4. -1
5. -3
6. -4
7. -2
8. -6
9. -3
10. 6
11. -3
12. 10
13. $\frac{1}{4}$
14. -1
15. 5
16. 16
17. $\frac{1}{5}$
18. 9

Operations with Functions: Domain

1. $\left[-\frac{7}{2}, \infty\right)$
2. $\left[-\frac{7}{2}, -1\right) \cup (-1, \infty)$
3. $\left(-\frac{7}{2}, \infty\right)$

Difference Quotient

1. 2
2. $2x + h + 4$
3. 0
4. $\frac{-3}{(x+2)(x+h+2)}$
5. $\frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$
6. $2x + h - 2$
7. $\frac{-5}{x(x+h)}$
8. $-4x - 2h + 3$
9. $\frac{-12}{(2x+3)(2x+2h+3)}$
10. $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$
11. $-2x - h + 1$
12. 3
13. $3x^2 + 3xh + h^2 + 5$
14. $\frac{-6}{(x+7)(x+h+7)}$
15. $\frac{-9}{x(x+h)}$
16. $\frac{-10}{(2x-1)(2x+2h-1)}$

Chapter 7

Polynomials and Their Graphs

Find the degree, leading term, leading coefficient, and constant term of the following polynomials.

1. $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$
2. $g(x) = 4x^2 - 16x^6 + 3x$
3. $h(x) = 1 + x^{11} - 4x^8$
4. $f(x) = -x^4 + 3x^2 - 2x + 6$
5. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$
6. $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$
7. $g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$
8. $h(x) = 7(x+1)^2(x-2)^3$
9. $j(x) = -\frac{1}{2}\left(3x+2\right)^2\left(x-1\right)^5$

Determine the end behavior of each.

10. $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$
11. $g(x) = 4x^2 - 16x^6 + 3x$
12. $h(x) = 1 + x^{11} - 4x^8$
13. $f(x) = -x^4 + 3x^2 - 2x + 6$
14. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$
15. $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$
16. $g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$
17. $h(x) = 5(x+1)^2(x-2)^3$
18. $j(x) = -\frac{1}{2}\left(3x+2\right)^2\left(x-1\right)^5$

Find the zeros of each. Round to 2 decimal places when necessary.

19. $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$
20. $g(x) = 4x^2 - 16x^6 + 3x$
21. $h(x) = 1 + x^{11} - 4x^8$
22. $f(x) = -x^4 + 3x^2 - 2x + 6$
23. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$
24. $f(x) = -6x^3 + 2x^2 + 7x^4 - 1$
25. $g(x) = \frac{1}{3}x^3 - \frac{\pi}{8}x^2 + x\sqrt{2} - 3^4$
26. $h(x) = 5(x+1)^2(x-2)^3$
27. $j(x) = -\frac{1}{2}\left(3x+2\right)^2\left(x-1\right)^5$

7.1 Answer Key

1. Degree $= 5$, Leading Term $= -x^5$, Leading Coefficient $= -1$, Constant $=$ none (or 0)
2. Degree $= 6$, Leading Term $= -16x^6$, Leading Coefficient $= -16$, Constant $=$ none (or 0)
3. Degree $= 11$, Leading Term $= x^{11}$, Leading Coefficient $= 1$, Constant $= 1$
4. Degree $= 4$, Leading Term $= -x^4$, Leading Coefficient $= -1$, Constant $= 6$
5. Degree $= 5$, Leading Term $= 0.25x^5$, Leading Coefficient $= 0.25$, Constant $=$ none (or 0)
6. Degree $= 3$, Leading Term $= -6x^3$, Leading Coefficient $= -6$, Constant $= -1$
7. Degree $= 3$, Leading Term $= \frac{1}{3}x^3$, Leading Coefficient $= \frac{1}{3}$, Constant $= 3^4$
8. Degree $= 5$, Leading Term $= 7x^5$, Leading Coefficient $= 7$, Constant $= -56$
9. Degree $= 7$, Leading Term $= -\frac{9}{2}x^7$, Leading Coefficient $= -\frac{9}{2}$, Constant $= 2$
10. $\lim_{x\to-\infty} f(x) = \infty \quad \lim_{x\to\infty} f(x) = -\infty$
11. $\lim_{x\to-\infty} g(x) = -\infty, \quad \lim_{x\to\infty} g(x) = \infty$
12. $\lim_{x\to-\infty} h(x) = -\infty \quad \lim_{x\to\infty} h(x) = \infty$
13. $\lim_{x\to-\infty} f(x) = -\infty \quad \lim_{x\to\infty} f(x) = -\infty$
14. $\lim_{x\to-\infty} g(x) = -\infty \quad \lim_{x\to\infty} g(x) = \infty$
15. $\lim_{x\to-\infty} f(x) = \infty \quad \lim_{x\to\infty} f(x) = -\infty$
16. $\lim_{x\to-\infty} g(x) = -\infty \quad \lim_{x\to\infty} g(x) = \infty$
17. $\lim_{x\to-\infty} h(x) = -\infty \quad \lim_{x\to\infty} h(x) = \infty$
18. $\lim_{x\to-\infty} j(x) = \infty \quad \lim_{x\to\infty} j(x) = -\infty$
19. $(-1.92, 0)$, $(0, 0)$
20. $(0, 0)$, $(0.83, 0)$
21. $(-0.83, 0)$, $(0.86, 0)$, $(1.58, 0)$
22. $(-2.25, 0)$, $(1.90, 0)$
23. $(-2.48, 0)$, $(0, 0)$, $(1.85, 0)$
24. $(-0.42, 0)$, $(0.79, 0)$
25. $(6.42, 0)$
26. $(-1, 0)$, $(2, 0)$
27. $\left(-\frac{2}{3}, 0\right)$, $(1, 0)$

Chapter 8

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

1. $(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$
2. $(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$
3. $\left(4x^3 - 3x^2 + x + 1\right) \div (x + 2)$
4. $\left(5x^4 - x^2 + x - 2\right) \div \left(x^2 + 2\right)$
5. $\left(10x^3 + 27x^2 + 8x - 11\right) \div (2x + 3)$
6. $\left(7x^3 + 23x^2 + 12x + 1\right) \div \left(x^2 + 3x + 1\right)$
7. $\left(28x^3 - 27x^2 - 4x + 17\right) \div (4x + 3)$
8. $\left(7x^3 - 27x + 4\right) \div \left(x^2 - 5\right)$
9. $\left(11x^6 - 24x^5 + 15x^4 - 19x^3 - 16x^2 + 21x - 8\right) \div (x - 2)$
10. $\left(12x^5 - 15x^4 - 11x^3 + 16x^2 - 15x + 17\right) \div \left(3x^2 - 5\right)$
11. $\left(6x^4 + 20x^3 - 13x^2 + 20x + 25\right) \div (x + 4)$
12. $\left(24x^5 + 30x^4 - 21x^3 - 4x^2 + 3x - 25\right) \div \left(6x^3 + 3x^2 + 3\right)$
13. $\left(3x^5 - 22x^4 + 12x^3 + 10x^2 - 7x + 24\right) \div \left(3x^2 - x - 4\right)$
14. $\left(3x^4 - 23x^2 - 15x^3 + 28x + 24\right) \div (x - 6)$
15. $\left(-29x^2 + 6x^6 - 29x^3 + 25x^4 - 15x^5 - 25 - 29x\right) \div \left(3x^3 - 6x^2 - 3 - x\right)$
16. $\left(12x^6 + 16x^5 - 5x^4 + 12x^3 - 17x^2 - x - 23\right) \div (x + 2)$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

1. $\left(2x^{53} - 9x^{44} + 13x^8\right) \div (x - 1)$
2. $\left(x^{71} + 15x^{58} - 3x^{14} + 2\right) \div (x + 1)$
3. $\left(x^{23} - 5x^{20} + 17x^8 - 5\right) \div (x + 2)$
4. $\left(-7x^{17} + 40x^{15} - 6x^8 + 4x^3\right) \div (x - 3)$

8.3 Answer Key

Dividing Polynomials

1. $4x^2 - 2x + 5$
2. $4y^2 + 8y + 7 - \frac{72}{3y+5}$
3. $4x^2 - 11x + 23 - \frac{45}{x+2}$
4. $5x^2 - 11 + \frac{x+20}{x^2+2}$
5. $5x^2 + 6x - 5 + \frac{4}{2x+3}$
6. $7x + 2 + \frac{-x-1}{x^2+3x+1}$
7. $7x^2 - 12x + 8 - \frac{7}{4x+3}$
8. $7x + \frac{8x+4}{x^2-5}$
9. $11x^5 - 2x^4 + 11x^3 + 3x^2 - 10x + 1 - \frac{6}{x-2}$
10. $4x^3 - 5x^2 + 3x - 3 + \frac{2}{3x^2-5}$
11. $6x^3 - 4x^2 + 3x + 8 - \frac{7}{x+4}$
12. $4x^2 + 3x - 5 + \frac{-x^2-6x-10}{6x^3+3x^2+3}$
13. $x^3 - 7x^2 + 3x - 5 + \frac{4}{3x^2-x-4}$
14. $3x^3 + 3x^2 - 5x - 2 + \frac{12}{x-6}$
15. $2x^3 - x^2 + 7x + 6 + \frac{11x^2-2x-7}{3x^3-6x^2-3-x}$
16. $12x^5 - 8x^4 + 11x^3 - 10x^2 + 3x - 7 - \frac{9}{x+2}$

Remainder and Factor Theorems

1. 6
2. 13
3. $-13,627,141$
4. $-330,064,119$

Chapter 9

Rational Functions and Their Graphs

9.1 Domain, Vertical Asymptotes, and Holes

Much of these concepts boils down to factoring the numerator and denominator **completely**.

Domain

For the domain, find the values of x that cause the denominator to equal 0.

Example 1. State the domain of

$$f(x) = \frac{2x^3 + 5x^2 - 3x}{5x^2 + 17x + 6}$$

Solution 1. We want to find the values of x that cause the denominator to equal 0 and avoid those values.

$$\begin{aligned} 5x^2 + 17x + 6 &\neq 0 \\ (5x+2)(x+3) &\neq 0 \qquad 5x^2 + 17x + 6 \text{ factors as } (5x+2)(x+3) \\ 5x + 2 \neq 0 &\text{ and } x + 3 \neq 0 \qquad \text{set each factor } \neq 0 \\ x \neq -\tfrac{2}{5} &\text{ and } x \neq -3 \end{aligned}$$

So the domain of the answer is $x \neq -\frac{2}{5}$ and $x \neq -3$.

Vertical Asymptotes and Holes

At each of these domain values, there is either a **vertical asymptote** or a **hole in the graph**.

In order to know which each value is, we need to factor the numerator as well and simplify our expression.

Example 2. Simplify

$$f(x) = \frac{2x^3 + 5x^2 - 3x}{5x^2 + 17x + 6}$$

Solution 2.

$$\begin{aligned}\frac{2x^3+5x^2-3x}{5x^2+17x+6} &= \frac{x(2x^2+5x-3)}{(5x+2)(x-3)} && \text{Factor } x \text{ out of numerator as a GCF.}\\ &= \frac{x(2x-1)(x+3)}{(5x+2)(x+3)} && 2x^2+5x-3=(2x-1)(x+3)\\ &= \frac{x(2x-1)}{5x+2} && \text{The } x+3 \text{ terms divide out to } 1\end{aligned}$$

Now, we are going to take each of those domain values of x ($-\frac{2}{5}$ and -3) and evaluate the simplified expression $\frac{x(2x-1)}{5x+2}$

- If we get an error message (like *undef*), then that value of x is a **vertical asymptote**.
- If we get an actual number back, then that value of x is a **hole** in the graph. The y-coordinate of the hole in the graph is the number we got back.

Example 3. Evaluate $\frac{x(2x-1)}{5x+2}$ for $x=-\frac{2}{5}$ and $x=-3$.

Solution 3.
When $x=-\frac{2}{5}$, we get

$$\frac{-\frac{2}{5}\left(2(-\frac{2}{5})-1\right)}{5\left(-\frac{2}{5}\right)+2} = \text{undefined}$$

When $x=-3$, we get

$$\frac{-3(2(-3)-1)}{5(-3)+2} = -\frac{21}{13}$$

Thus, $x=-\frac{2}{5}$ is the equation of a vertical asymptote, and there is a hole in the graph at $\left(-3,-\frac{21}{13}\right)$

9.2 End Behavior

End behavior has nothing to do with domain.

So anything from the previous part is not going to help here.

Instead, end behavior focuses on the behavior of the function as x approaches $-\infty$ and x approaches ∞.

You will want to focus on the *degrees* of the numerator and denominator.

For rational functions, each function will fall into one of the following three cases.

The bigger degree is	End Behavior
In the denominator	$y=0$
Degrees are equal	$y=$ ratio of leading coefficients
In the numerator	$y=$ use polynomial division (up to you get the constant)

Example 4. Find the end behavior asymptote for each.

(a) $f(x)=\frac{3x+1}{2x^2-5x-3}$ (b) $g(x)=\frac{x^3-4x^2+7}{5x^3+3x}$ (c) $h(x)=\frac{x^3+2x^2-15x}{x^2-5x-14}$

Solution 4.

(a) The bigger degree is in the denominator. Thus, the end behavior is $y = 0$.

(b) The degrees are equal (both are 3). Thus, the end behavior is the ratio of leading coefficients: $y = \frac{1}{5}$.

(c) The bigger degree is in the numerator, so you will need to use polynomial division.

The good news is, you don't need to complete the polynomial division. You only need to stop once you get the constant of the quotient; the remainder approaches 0 as $x \to -\infty$ and $x \to \infty$.

$$\left(x^3 + 2x^2 - 15x\right) \div \left(x^2 - 5x - 14\right)$$

	x	7
x^2	x^3	$(7x^2)$
$-5x$	$-5x^2$	
-14	$-14x$	

Thus, the end behavior asymptote is $y = x + 7$.

9.3 Exercises

Find the domain, coordinates of any holes, and equations of all asymptotes.

1. $f(x) = \frac{2x^2+5x-3}{2x^2-15x+7}$
2. $g(x) = \frac{3x^3+7x^2-20x}{x^2-x-12}$
3. $f(x) = \frac{3x}{x+4}$
4. $g(x) = \frac{x^2+3x+2}{x-1}$
5. $h(x) = \frac{x^2+3x-4}{x^3-2x^2+x}$
6. $f(x) = \frac{2x^3-13x^2+6x+45}{x^2-4x-5}$
7. $g(x) = \frac{5x^2-19x-4}{x^3+2x^2-24x}$
8. $h(x) = \frac{2x^2-x-3}{8x^2+51x+18}$
9. $f(x) = \frac{6x^3-21x^2-51x+30}{3x^2+7x+2}$
10. $g(x) = \frac{10x^2-29x-21}{10x^3-33x^2-7x}$
11. $f(x) = \frac{x^3+x^2-6x}{3x^2-3x-6}$
12. $f(x) = \frac{x^2-4x+3}{2x^2+2x-12}$
13. $f(x) = \frac{x-4}{-2x^2+4x+16}$
14. $f(x) = \frac{x^3-2x^2-8x}{x^3-2x^2-3x}$
15. $f(x) = \frac{x^2+x-2}{3x^2+3x-18}$
16. $f(x) = \frac{x^2-3x+2}{4x^2-12x}$
17. $f(x) = \frac{8x^2+26x+15}{2x^2-x-15}$
18. $g(x) = \frac{x^2-1}{2+2x}$
19. $f(x) = \frac{10x^2+28x-6}{12x^2+45x+27}$
20. $g(x) = \frac{x-5}{x^2-7x+10}$
21. $h(x) = \frac{2x^3-5x^2-42x}{3x-18}$

State the end behavior of each.

22. $k(x) = \frac{5x^3-7x^2+8}{-3x^3+6x-4}$
23. $m(x) = \frac{2x-1}{3x^2+7x+1}$

Answer each of the following given $h(x) = \frac{6x^3+40x^2-14x}{3x^2+11x-4}$

24. End behavior

25. Domain of h
26. Equation(s) for any vertical asymptotes
27. Exact coordinates of any holes
28. What is the approximate value of $h\left(5^{933}\right)$?

9.4 Answer Key

1. Domain: $x \neq \frac{1}{2}$, 7; V.A.: $x = \frac{1}{2}$, $x = 7$; H.A.: $y = 1$
2. Domain: $x \neq -3$, 4; V.A.: $x = -3\, x = 4$; Obl. Asymp: $y = 3x + 10$
3. Domain: $x \neq -4$; V.A.: $x = -4$; H.A.: $y = 3$
4. Domain: $x \neq 1$; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$
5. Domain: $x \neq 0, 1$; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$
6. Domain: $x \neq -1, 5$; V.A. $x = -1$; Hole @ $\left(5, \frac{13}{3}\right)$; Obl. Asym $y = 2x - 5$
7. Domain: $x \neq -6, 0, 4$; V.A. $x = -6$, $x = 0$; Hole @ $\left(4, \frac{21}{40}\right)$; H.A. $y = 0$
8. Domain: $x \neq -6, -\frac{3}{8}$; V.A. $x = -6$, $x = -\frac{3}{8}$; H.A. $y = \frac{1}{4}$
9. Domain: $x \neq -2$, $-\frac{1}{3}$; Hole @ $(-2, -21)$; V.A.: $x = -\frac{1}{3}$; Obl. Asymp: $y = 2x - \frac{35}{3}$
10. Domain: $x \neq -\frac{1}{5}$, 0, $\frac{7}{2}$; Hole @ $\left(\frac{7}{2}, \frac{82}{259}\right)$; V.A. $x = -\frac{1}{5}$ and $x = 0$; H.A. $y = 0$
11. Domain: $x \neq -1$, 2; V.A. $x = -1$; Hole @ $\left(2, \frac{10}{9}\right)$; Obl. Asymp: $y = \frac{1}{3}x + \frac{2}{3}$
12. Domain: $x \neq -3$, 2; V.A. $x = -3$ and $x = 2$; H.A. $y = \frac{1}{2}$
13. Domain: $x \neq -2$, 4; V.A. $x = 2$; Hole @ $\left(4, -\frac{1}{12}\right)$; H.A. $y = 0$
14. Domain: $x \neq -1$, 0, 3; V.A. $x = -1$ and $x = 3$; Hole @ $\left(0, \frac{8}{3}\right)$; H.A. $y = 1$
15. Domain: $x \neq -3$, 2; V.A. $x = -3$ and $x = 2$; H.A. $y = \frac{1}{3}$
16. Domain: $x \neq 0$, 3; V.A. $x = 0$ and $x = 3$; H.A. $y = \frac{1}{4}$
17. Domain: $x \neq -\frac{5}{2}, 3$; V.A. $x = 3$; Hole @ $\left(-\frac{5}{2}, \frac{14}{11}\right)$; H.A. $y = 4$
18. Domain: $x \neq -1$; No vertical asymptote; Hole @ $(-1, -1)$; Obl. Asymp: $y = \frac{1}{2}x - \frac{1}{2}$
19. Domain: $x \neq -3, -\frac{3}{4}$; Vert. Asymp: $x = -\frac{3}{4}$; Hole @ $\left(-3, \frac{32}{27}\right)$; Horiz. Asymp: $y = \frac{5}{6}$
20. Domain: $x \neq 2, 5$; Vert. Asymp: $x = 2$; Hole @ $\left(5, \frac{1}{3}\right)$; Horiz. Asymp: $y = 0$
21. Domain: $x \neq 6$; Hole @ $(6, 38)$; Oblique Asymp: $y = \frac{2}{3}x^2 + \frac{7}{3}x$
22. $\lim_{x \to -\infty} k(x) = \infty \quad \lim_{x \to \infty} k(x) = -\dfrac{5}{3}$
23. $\lim_{x \to -\infty} m(x) = \infty \quad \lim_{x \to \infty} m(x) = 0$
24. $y = 2x + 6$
25. $x \neq -4, \frac{1}{3}$
26. $x = -4$
27. $\left(\frac{1}{3}, \frac{44}{39}\right)$
28. $2\left(5^{933}\right) + 6$

Chapter 10

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

10.2 Rational Inequalities

When solving rational inequalities, you will need to find the critical values for your number line. To do so,

- Find the domain restrictions (like in Graphs of Rational Functions)
- Solve the inequality as an equation (might want to multiply both sides by least common denominator)

We will test each of these critical values in the original problem and see if they make the original inequality true.

Example 1. Solve each.

(a) $\frac{x+3}{x-2} \leq 0$ (b) $\frac{2x^2-3x-5}{x^2-5x-6} \geq 0$ (c) $\frac{x+7}{3x-1} < 4$

Solution 1.
(a) First, the domain:

$$\begin{aligned} x - 2 &\neq 0 \\ x &\neq 2 \end{aligned}$$

Next, the solution as an equation:

$$\begin{aligned} \frac{x+3}{x-2} &= 0 \\ x + 3 &= 0 \qquad \text{multiply both sides by the denominator} \\ x &= -3 \end{aligned}$$

We can set up a number line with these 2 critical values (2 and -3):

Next, we want to use *test values* in each interval; as well as at the critical values against whether or not the outputs at those values are less than or equal to 0.

x	$\frac{x+3}{x-2} \leq 0$?
-4	$\frac{1}{6}$ (no)
-3	0 (yes)
0	$-\frac{3}{2}$ (yes)
2	undefined (no)
3	6 (no)

These results yield the following number line:

In interval notation, our solution is $[-3, 2)$.

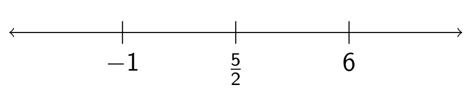
(b) First, the domain:

$$\begin{aligned} x^2 - 5x - 6 &\neq 0 \\ (x-6)(x+1) &\neq 0 \\ x - 6 \neq 0 &\text{ and } x + 1 \neq 0 \\ x \neq 6 &\text{ and } x \neq -1 \end{aligned}$$

Next, the solution as an equation:

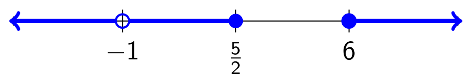
$$\begin{aligned} \frac{2x^2 - 3x - 5}{x^2 - 5x - 6} &= 0 \\ 2x^2 - 3x - 5 &= 0 \qquad \text{multiply both sides by the denominator} \\ (2x-5)(x+1) &= 0 \\ 2x - 5 = 0 &\text{ or } x + 1 = 0 \\ x = \tfrac{5}{2} &\text{ or } x = -1 \end{aligned}$$

We have 3 distinct critical values: -1, $\frac{5}{2}$, and 6.



We then check the critical values in the original inequality, along with other test values:

x	$\frac{2x^2-3x-5}{x^2-5x-6} \geq 0$?
-2	$\frac{9}{8}$ (yes)
-1	undefined (no)
0	$\frac{5}{6}$ (yes)
$\frac{5}{2}$	0 (yes)
3	$-\frac{1}{3}$ (no)
6	undefined (no)
7	9 (yes)



In interval notation, our solution is $(-\infty, -1) \cup \left(-1, \frac{5}{2}\right] \cup [6, \infty)$

(c) First, the domain:

$$\begin{aligned} 3x - 1 &\neq 0 \\ x &\neq \tfrac{1}{3} \end{aligned}$$

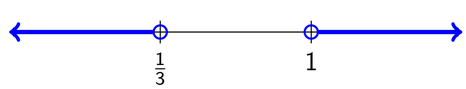
Next, the solution to the inequality as an equation:

$$\begin{aligned} \frac{x+7}{3x-1} &= 4 \\ x + 7 &= 4(3x - 1) && \text{multiply both sides by the denominator} \\ x + 7 &= 12x - 4 \\ x &= 1 \end{aligned}$$

Our number line looks like

Testing the critical values and others in the original inequality:

x	$\frac{x+7}{3x-1} < 4$?
0	-7 (yes)
$\frac{1}{3}$	undefined (no)
$\frac{1}{2}$	15 (no)
1	4 (no)
2	$\frac{9}{5}$ (yes)



In interval notation, our solution is $\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$

10.3 Exercises

Polynomial Inequalities

Solve each. Write your answers using interval notation.

1. $6x^3 - 4x^2 - 10x \geq 0$
2. $x^4 < 9x^2$
3. $3x^3 - 7x^2 - 22x + 8 < 0$
4. $3x^2 - 4x + 1 \leq 0$
5. $12x^4 + 76x^3 + 43x^2 - 346x - 280 \geq 0$
6. $-2x^4 + 49x^2 + 21x^3 - 1029x + 2401 \geq 0$
7. $-x^2 - 7x - 6 \leq 0$
8. $x^2 + 4x + 4 < 0$
9. $-x^4 - 6x^3 + 61x^2 + 234x - 1008 \geq 0$
10. $-x^2 + 3x + 1 > 3$
11. $-3x^4 + 123x^3 + 142x^2 - 424x + 320 \leq 122x^3$
12. $-x^4 - 1120 + 77x^2 - 36x + 15x^3 \geq 15x^3$
13. $-3x^4 - 22x^3 + 271x^2 + 152x - 96 \geq 267x^2$
14. $15x^3 + 27x^2 + 8x \leq 14x$
15. $x^3 + 6x^2 > -2x^2 + 64x + 512$

Domain

State the domain of each. Write your answers using interval notation.

1. $b(x) = \sqrt{21x^2 - 23x - 20}$
2. $f(x) = \frac{3}{\sqrt{3x^2+2x-1}}$
3. $g(x) = \sqrt[4]{2x^3 + 9x^2 + 12x + 4}$

Rational Inequalities

Solve each. Write your answers using interval notation.

1. $\frac{3x-4}{x+1} < 0$
2. $\frac{x^2+3x+2}{x-7} \leq 0$
3. $\frac{x^2-4x+4}{x^2-1} \geq 0$
4. $\frac{x+2}{x-4} \leq 1$
5. $\frac{x^2-7x-8}{x^2-4x-32} \geq 0$
6. $\frac{4+3x}{5-x} \leq 2$
7. $\frac{x-4}{x+7} < 0$
8. $\frac{x+5}{x+7} < 0$
9. $\frac{2x-26}{5x+20} > -3$
10. $\frac{2x-50}{5x+15} \leq -1$
11. $\frac{x+5}{x^2-2x-15} \leq 0$
12. $-\frac{2}{x} \geq -\frac{3}{x+1}$
13. $-\frac{3}{x+6} > -\frac{4}{x+7}$
14. $\frac{2x^2+3x-2}{x^2+5x+6} < 0$
15. $\frac{x-4}{2x+4} \geq 1$
16. $\frac{6x^2+5x-21}{x-4} < 0$
17. $\frac{2x+1}{4x-3} \geq x - 1$

10.4 Answer Key

Polynomial Inequalities

1. $[-1, 0] \cup \left[\frac{5}{3}, \infty\right)$
2. $(-3, 0) \cup (0, 3)$
3. $(-\infty, -2) \cup \left(\frac{1}{3}, 4\right)$
4. $\left[\frac{1}{3}, 1\right]$
5. $(-\infty, -4] \cup \left[-\frac{7}{2}, -\frac{5}{6}\right] \cup [2, \infty)$
6. $\left[-7, \frac{7}{2}\right] \cup 7$
7. $(-\infty, -6] \cup [-1, \infty)$
8. $\emptyset$
9. $[-8, -7] \cup [3, 6]$
10. $(1, 2)$
11. $(-\infty, -8] \cup \left[\frac{4}{3}, 2\right] \cup [5, \infty)$
12. $[-8, -4] \cup [5, 7]$
13. $[-6, -4] \cup \left[\frac{2}{3}, 2\right]$
14. $(-\infty, -2] \cup \left[0, \frac{1}{5}\right]$
15. $(8, \infty)$

Domain

1. $\left(-\infty, -\frac{12}{21}\right] \cup \left[\frac{5}{3}, \infty\right)$
2. $(-\infty, -1) \cup \left(\frac{1}{3}, \infty\right)$
3. $\{-2\} \cup \left[-\frac{1}{2}, \infty\right)$

Rational Inequalities

1. $\left(-1, \frac{4}{3}\right)$
2. $(-\infty, -2] \cup [-1, 7)$
3. $(-\infty, -1) \cup (1, \infty)$
4. $(-\infty, 4)$
5. $(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$
6. $(-\infty, 1.2] \cup (5, \infty)$
7. $(-7, 4)$
8. $(-7, -5)$
9. $(-\infty, -4) \cup (-2, \infty)$
10. $(-3, 5]$
11. $(-\infty, -5] \cup (-3, 5)$
12. $(-1, 0) \cup [2, \infty)$
13. $(-7, -6) \cup (-3, \infty)$
14. $(-3, -2) \cup \left(-2, \frac{1}{2}\right)$
15. $[-8, -2)$
16. $\left(-\infty, -\frac{7}{3}\right) \cup \left(\frac{3}{2}, 4\right)$
17. $\left(-\infty, \frac{1}{4}\right] \cup \left(\frac{3}{4}, 2\right]$

Chapter 11

Function Compositions

Given $f(x) = x - 5$, $g(x) = 4 + \sqrt{2x+1}$, and $h(x) = \dfrac{3}{x+7}$, simplify each and state the domain.

1. $(f \circ g)(x)$

2. $(g \circ f)(x)$

3. $h(h(x))$

Find each of the following given the table below.

x	-4	-3	-2	-1	**0**	**1**	**2**	**3**	**4**
f(x)	-3	0	-1	3	1	2	4	-4	-2
g(x)	3	-1	0	1	4	-2	-4	2	-3

4. $(f \circ g)(-1)$ 5. $(g \circ g)(0)$ 6. $(f \circ f)(2)$ 7. $(g \circ g)(-3)$ 8. $f(g(0))$

Use the table below to answer each.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	1	-1	-2	4	0	-4	-3	3	2
$g(x)$	0	-2	1	-4	-3	2	-1	4	3

9. $(f \circ g)(-1)$ 10. $f(g(3))$ 11. $(g \circ f)(0)$ 12. $f(f(4))$ 13. $g(f(g(1)))$

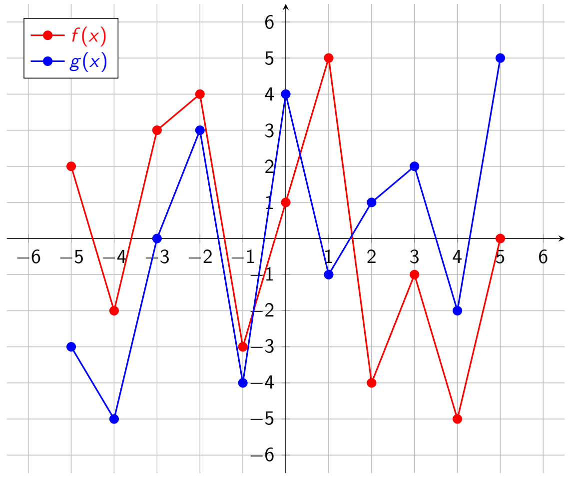
Given $f(x) = \sqrt{3x+2}$, $g(x) = x^2 - 1$, and $h(x) = 9x - 2$, find each of the following.

14. $(g \circ f)(x)$

15. $f(g(x))$

16. $(h \circ h)(x)$

Find each of the following given the graphs of $f(x)$ (in red) and $g(x)$ (in blue) below:



17. $(f \circ g)(-1)$ 18. $(g \circ f)(-4)$ 19. $f(g(3))$ 20. $g(g(-2))$ 21. $(f \circ f)(-5)$

Given $f(x) = \sqrt{2x-9}$ and $g(x) = \frac{2x}{x-3}$, simplify each and state the domain of the composition.

22. $f(g(x))$ 23. $(g \circ f)(x)$ 24. $g(g(x))$

Given $f(x) = \sqrt{3x-4}$, $g(x) = \frac{2x}{x+1}$, and $h(x) = 5x - 9$, simplify each of the following and state the domain.

25. $(g \circ f)(x)$ 26. $(h(g(x))$ 27. $(f \circ h)(x)$

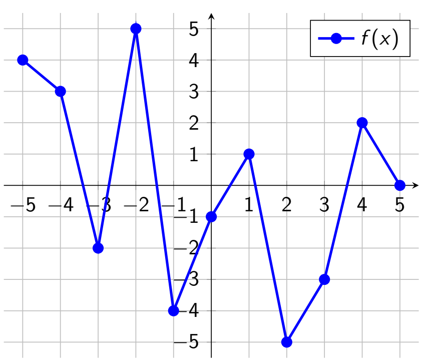
Given $f(x) = \frac{5x}{2x-7}$ and $g(x) = \frac{x}{x+8}$, simplify each and state the domain of the composition.

28. $(f \circ g)(x)$ 29. $g(f(x))$ 30. $(g \circ g)(x)$

Given $f(x) = x^2 - 5$, $g(x) = \frac{x}{2x-3}$, and $h(x) = \sqrt{6x+7}$, find the composition of each. Then state the domain of the composition using interval notation.

31. $(f \circ f)(x)$ 32. $(g \circ h)(x)$ 33. $h(g(x)$

Given the graph of $f(x)$, the table of $g(x)$, and $h(x) = \sqrt{x+12}$, find the value of each.



x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$g(x)$	5	4	-3	0	-4	-1	1	-2	2	3	-5

34. $(f \circ g)(-2)$ 35. $(h \circ h)(4)$ 36. $(g \circ f)(-5)$ 37. $f(g(h(13)))$ 38. $(g \circ g \circ f)(3)$

11.1 Answer Key

1. $-1+\sqrt{2x+1}$ Domain: $\left[-\frac{1}{2},\infty\right)$
2. $4+\sqrt{2x-9}$ Domain: $\left[\frac{9}{2},\infty\right)$
3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty,-\frac{52}{7}\right)\cup\left(-\frac{52}{7},-7\right)\cup(-7,\infty)$
4. 2
5. -3
6. -2
7. 1
8. -2
9. 1
10. 2
11. -3
12. -3
13. -2
14. $3x+1$
15. $\sqrt{3x^2-1}$
16. $81x-20$
17. -2
18. 3
19. -4
20. 2
21. -4
22. $f(g(x))=\sqrt{\frac{-5x+27}{x-3}}$; $\left(3,\frac{27}{5}\right]$
23. $(g\circ f)(x)=\frac{2\sqrt{2x-9}}{\sqrt{2x-9}-3}$; $\left[\frac{9}{2},9\right)\cup(9,\infty)$
24. $g(g(x))=\frac{4x}{9-x}$; $(-\infty,3)\cup(3,9)\cup(9,\infty)$
25. $\frac{2\sqrt{3x-4}}{\sqrt{3x-4}+1}$; $\left[\frac{4}{3},\infty\right)$
26. $\frac{x-9}{x+1}$; $(-\infty,-1)\cup(-1,\infty)$
27. $\sqrt{15x-31}$; $\left[\frac{31}{15},\infty\right)$
28. $\frac{5x}{-5x-56}$, $\left(-\infty,-\frac{56}{5}\right)\cup\left(-\frac{56}{5},-8\right)\cup(-8,\infty)$
29. $\frac{5x}{21x-56}$, $\left(-\infty,\frac{8}{3}\right)\cup\left(\frac{8}{3},\frac{7}{2}\right)\cup\left(\frac{7}{2},\infty\right)$
30. $\frac{x}{9x+64}$, $(-\infty,-8)\cup\left(-8,-\frac{64}{9}\right)\cup\left(-\frac{64}{9},\infty\right)$
31. x^4-10x^2+20; $(-\infty,\infty)$
32. $\frac{\sqrt{6x+7}}{2\sqrt{6x+7}-3}$; $\left[-\frac{7}{6},-\frac{19}{24}\right)\cup\left(-\frac{19}{24},\infty\right)$

33. $\sqrt{\frac{20x-21}{2x-3}}$; $\left(-\infty, \frac{21}{20}\right] \cup \left(\frac{3}{2}, \infty\right)$
34. -1
35. 4
36. 3
37. 4
38. -3

Chapter 12

Inverse Functions

12.1 Finding the Inverse of a Function

When finding the inverse of a function,

1. Write $y =$ instead of $f(x) =$.
2. Switch x and y variables.
3. Solve the new equation for y.

Example 1. Determine the inverse of each.

(a) $f(x) = \frac{3}{x-5}$ (b) $g(x) = \frac{6x}{x+1}$ (c) $h(x) = \sqrt{2x-1} + 7$ (d) $j(x) = x^2 - 5x$

Solution 1.
(a) First, write the function as $y = \frac{3}{x-5}$

Next, switch the x and y variables.

$$x = \frac{3}{y-5}$$

Now, solve the equation for y:

$$\begin{aligned} x &= \frac{3}{y-5} \\ x(y-5) &= 3 && \text{Multiply both side by denominator to eliminate fraction} \\ y - 5 &= \frac{3}{x} && \text{Divide both sides by } x \text{ to eliminate need for parentheses} \\ y &= \frac{3}{x} + 5 && \text{Add 5 to both sides} \end{aligned}$$

The inverse of $f(x)$ is $\boxed{f^{-1}(x) = \frac{3}{x} + 5}$

(b) Writing $g(x) = \frac{6x}{x+1}$ as $y = \frac{6x}{x+1}$.

$$\begin{aligned} y &= \frac{6x}{x+1} \\ x &= \frac{6y}{y+1} && \text{Switch } x \text{ and } y \\ x(y+1) &= 6y && \text{Multiply both sides by denominator to eliminate fraction} \\ xy + x &= 6y && \text{Distribute the } x \\ x &= 6y - xy && \text{Gather } y \text{ terms on one side} \\ x &= (6-x)y && \text{Factor out } y \text{ as greatest common factor} \\ \frac{x}{6-x} &= y && \text{Divide both sides by } 6 - x \end{aligned}$$

The inverse of $g(x)$ is $\boxed{g^{-1}(x) = \frac{x}{6-x}}$

You may have noticed two different strategies for isolating the y in the previous two examples.

In Example 1a, there wasn't an x in the numerator. However, in Example 1b, there was.

For the record, here is what you would do if you decided to divide both sides by x in Example 1b instead of distributing it:

$$\begin{aligned}
x(y+1) &= 6y \\
y+1 &= \tfrac{6y}{x} && \text{Divide both sides by } x \\
1 &= \tfrac{6y}{x} - y && \text{Get } y \text{ terms on one side} \\
1 &= \left(\tfrac{6}{x} - 1\right) y && \text{Factor out } y \text{ as greatest common factor} \\
\frac{1}{\frac{6}{x}-1} &= y && \text{Divide both sides by } \tfrac{6}{x} - 1 \\
\frac{1 \cdot x}{x \cdot \frac{6}{x} - 1 \cdot x} &= y && \text{Multiply every term by least common tiny denominator } x \\
\frac{x}{6-x} &= y && \text{Simplify}
\end{aligned}$$

You get the same answer. However, there is more work involved.

(c) For $h(x) = \sqrt{2x-1} + 7$

$$\begin{aligned}
y &= \sqrt{2x-1} + 7 \\
x &= \sqrt{2y-1} + 7 && \text{Switch } x \text{ and } y \\
x - 7 &= \sqrt{2y-1} && \text{Subtract 7 from both sides} \\
(x-7)^2 &= 2y - 1 && \text{Square both sides} \\
(x-7)^2 + 1 &= 2y && \text{Add 1 to both sides} \\
\frac{(x-7)^2+1}{2} &= y && \text{Divide both sides by 2}
\end{aligned}$$

The inverse of $h(x)$ is $\boxed{h^{-1}(x) = \frac{(x-7)^2+1}{2} \text{ or } \frac{1}{2}\left((x-7)^2+1\right)}$

(d) For $j(x) = x^2 - 5x$, start by writing it in **vertex form**, $y = a(x-h)^2 + k$, where the coordinates of the vertex are (h, k) and a is the coefficient of the x^2 term.

Note: Some math teachers call this process *completing the square*.

$$x^2 - 5x = \left(x - \tfrac{5}{2}\right)^2 - \tfrac{25}{4}$$

$$\begin{aligned}
y &= \left(x - \tfrac{5}{2}\right)^2 - \tfrac{25}{4} \\
x &= \left(y - \tfrac{5}{2}\right)^2 - \tfrac{25}{4} && \text{Switch } x \text{ and } y \\
x + \tfrac{25}{4} &= \left(y - \tfrac{5}{2}\right)^2 && \text{Add } \tfrac{25}{4} \text{ to both sides} \\
\pm\sqrt{x + \tfrac{25}{4}} &= y - \tfrac{5}{2} && \text{Take the square root of both sides} \\
\tfrac{5}{2} \pm \sqrt{x + \tfrac{25}{4}} &= y && \text{Add } \tfrac{5}{2} \text{ to both sides}
\end{aligned}$$

The inverse of $j(x)$ is $\boxed{j^{-1}(x) = \frac{5}{2} \pm \sqrt{x + \frac{25}{4}}}$

or is it ... ?

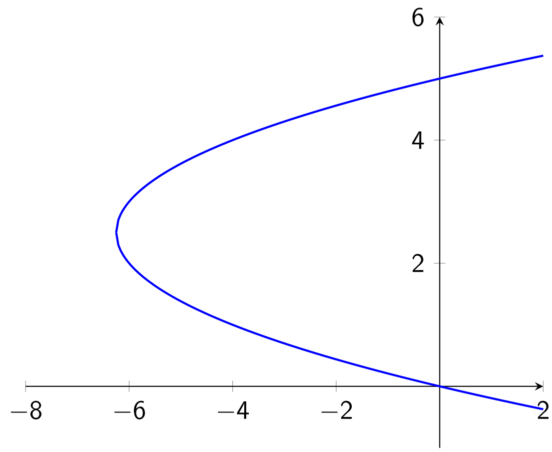
12.2 Domains and Ranges of Inverse Functions

If we look at the graph of $j^{-1}(x) = \frac{5}{2} \pm \sqrt{x + \frac{25}{4}}$, it actually consists of 2 functons:

- $\frac{5}{2} + \sqrt{x + \frac{25}{4}}$
- $\frac{5}{2} - \sqrt{x + \frac{25}{4}}$

The graph of $j^{-1}(x)$, shown below, fails the Vertical Line Test. Thus, it is not a function.

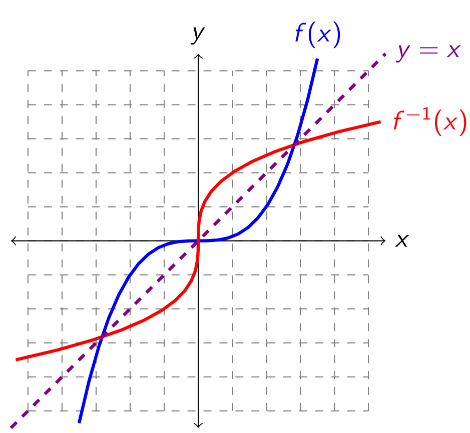
Note: This is also evident from the fact that $j(x) = x^2 - 5x$ fails the Horizontal Line Test.



When you switch the x and y, you also switch the domains and ranges of the given function and its inverse.

	Domain	**Range**
$f(x)$		
$f^{-1}(x)$		

From a visual perspective, switching the x and y also *reflects the given function across the line* $y = x$.



This is not *just* a result of switching x and y, it is a *necessity*.

> The graph of a function and the graph of its inverse **must** be reflections of each other across the line $y = x$.

This means you may need to restrict the domain and/or range of a function or its inverse in order for this to happen.

If it is easier, you can also examine the graph of the given function to get a sense of what its domain and range are. Then copy the values into the range and domain, respectively, of the inverse function.

Example 2. Determine the domain and range of each function and its inverse.

(a) $f(x) = \frac{3}{x-5}$ (b) $g(x) = \frac{6x}{x+1}$ (c) $h(x) = \sqrt{2x-1} + 7$ (d) $j(x) = x^2 - 5x; \quad x \le \frac{5}{2}$

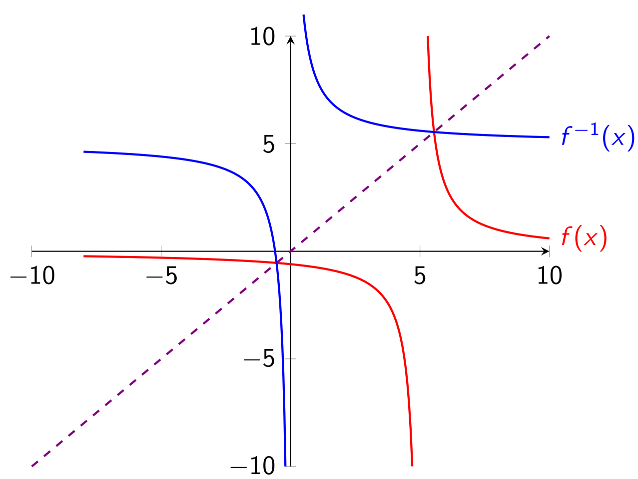
Solution 2.
(a) For $f(x) = \frac{3}{x-5}$, it might not be easy to get a sense of the domain and range from the graph. However, because of the $x - 5$ in the denominator, the domain is $(-\infty, 5) \cup (5, \infty)$. This is also the range of the inverse function, $f^{-1}(x) = \frac{3}{x} + 5$.

Speaking of $f^{-1}(x) = \frac{3}{x} + 5$, the domain of this inverse function is $(-\infty, 0) \cup (0, \infty)$; which is also the range of the given function $f(x) = \frac{3}{x-5}$.

Thus, we can fill out the table below as follows.

	Domain	Range
$f(x) = \frac{3}{x-5}$	$(-\infty, 5) \cup (5, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$f^{-1}(x) = \frac{3}{x} + 5$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 5) \cup (5, \infty)$

The graphs of $f(x)$ and $f^{-1}(x)$ are shown below, along with the line $y = x$.

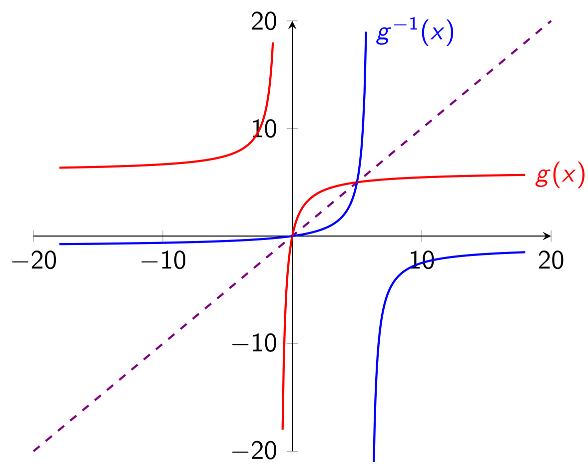


(b) Like part (a), it might not be easy to get a sense of the domain and range by looking at the graph of $g(x) = \frac{6x}{x+1}$. But we can use the $x + 1$ in the denominator to get a domain of $(-\infty, -1) \cup (-1, \infty)$. This is also the range of $g^{-1}(x) = \frac{x}{6-x}$.

The domain of $g^{-1}(x) = \frac{x}{6-x}$ is $(-\infty, 6) \cup (6, \infty)$; which is the range of the given function $g(x)$.

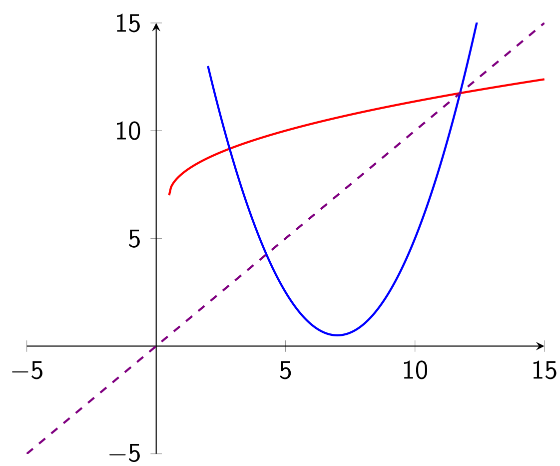
	Domain	Range
$g(x) = \frac{6x}{x+1}$	$(-\infty, -1) \cup (-1, \infty)$	$(-\infty, 6) \cup (6, \infty)$
$g^{-1}(x) = \frac{x}{6-x}$	$(-\infty, 6) \cup (6, \infty)$	$(-\infty, -1) \cup (-1, \infty)$

The graphs of $g(x)$ and $g^{-1}(x)$ are shown below, along with the line $y = x$.



(c) For $h(x) = \sqrt{2x-1}+7$, since we have $2x-1$ inside of an even root, we need to solve $2x-1 \geq 0$. This gives us $\left[\frac{1}{2}, \infty\right)$.

Our inverse function, $h^{-1}(x)$, is $h^{-1}(x) = \frac{(x-7)^2+1}{2}$. If we look at the graphs of $h(x)$ and $h^{-1}(x)$, notice they are not reflections of each other across the line $y = x$.



So we need to restrict the domain of the inverse function $h^{-1}(x)$ so that it is a reflection of $h(x) = \sqrt{2x-1}+7$ across the line $y = x$.

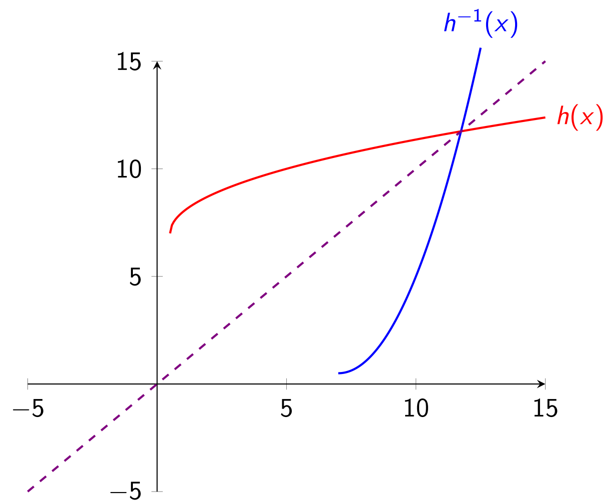
An easy way to know what domain restriction you need to use is to look at the **range** of the original function $h(x) = \sqrt{2x-1}+7$.

The lowest y-coordinate on the graph is at $\left(\frac{1}{2}, 7\right)$. Notice we can get that y-coordinate by evaluating $h(x) = \sqrt{2x-1}+7$ at $x = \frac{1}{2}$.

Since the y-coordinates of $h(x) = \sqrt{2x-1}+7$ increase beyond $y = 7$, the range of $h(x)$ is $[7, \infty)$.

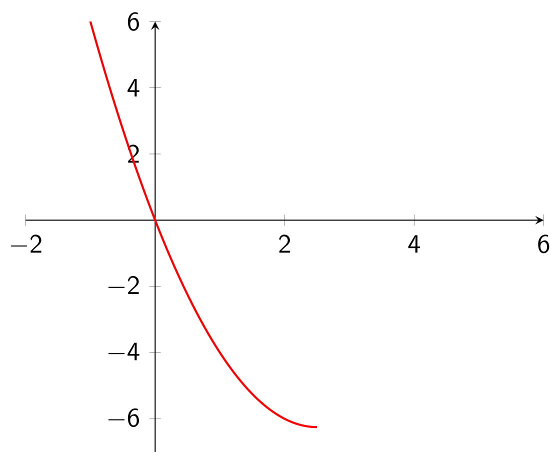
This must be the domain restriction we use on $h^{-1}(x) = \frac{(x-7)^2+1}{2}$.

	Domain	Range
$h(x) = \sqrt{2x-1}+7$	$\left[\frac{1}{2}, \infty\right)$	$[7, \infty)$
$h^{-1}(x) = \frac{(x-7)^2+1}{2}$	$[7, \infty)$	$\left[\frac{1}{2}, \infty\right)$



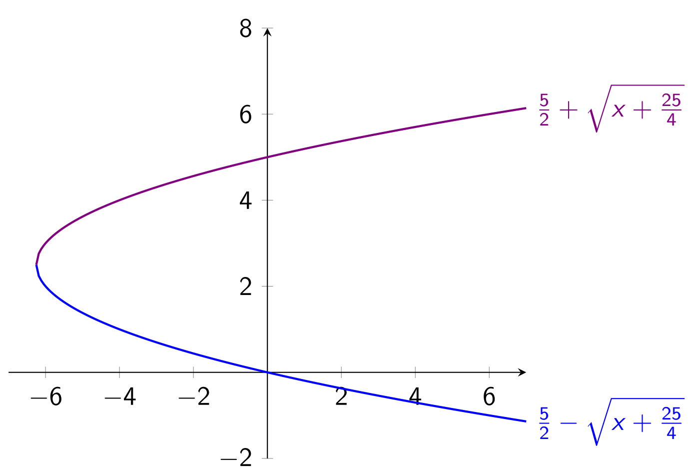
(d) Earlier, we determined the inverse of $j(x) = x^2 - 5x$ to be either $\frac{5}{2} + \sqrt{x + \frac{25}{4}}$ or $\frac{5}{2} - \sqrt{x + \frac{25}{4}}$.

With the domain restriction given in Example 2d, $x \leq \frac{5}{2}$, we can graph that piecewise function below:



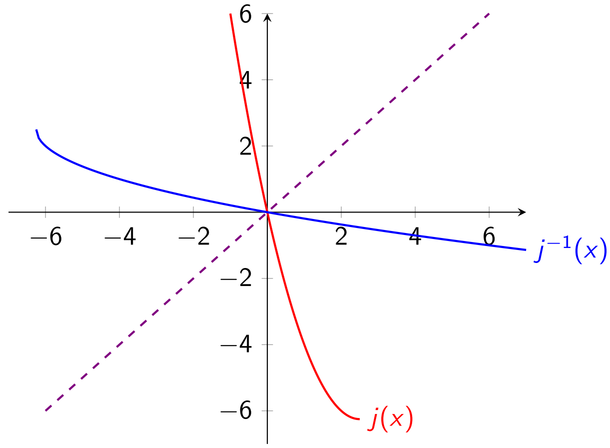
Notice the domain of the graph above is $\left(-\infty, \frac{5}{2}\right]$.

If we compare $y = \frac{5}{2} + \sqrt{x + \frac{25}{4}}$ and $y = \frac{5}{2} - \sqrt{x + \frac{25}{4}}$, the graph of $y = \frac{5}{2} - \sqrt{x + \frac{25}{4}}$ has a range of $\left(-\infty, \frac{5}{2}\right]$.



Thus, the inverse of $j(x) = x^2 - 5x$; $x \leq \frac{5}{2}$ is $\boxed{j^{-1}(x) = \frac{5}{2} - \sqrt{x + \frac{25}{4}}}$.

Also, note that the graph of $\frac{5}{2} - \sqrt{x + \frac{25}{4}}$ is the reflection of the piecewise function $x^2 - 5x$; $x \leq \frac{5}{2}$ across the line $y = x$.



To find the range of our starting function, $j(x)$, we can either look at y-coordinates on the graph of $j(x)$ or find the domain of $j^{-1}(x) = \frac{5}{2} - \sqrt{x + \frac{25}{4}}$

In any event, we can find that the range of $j(x)$ [or, equally, the domain of $j^{-1}(x)$] is $x \geq -\frac{25}{4}$.

	Domain	Range
$j(x) = x^2 - 5x$	$(-\infty, \frac{5}{2}]$	$[-\frac{25}{4}, \infty)$
$j^{-1}(x) = \frac{(x-7)^2+1}{2}$	$[-\frac{25}{4}, \infty)$	$(-\infty, \frac{5}{2}]$

12.3 Exercises

Find the inverse of each. Then state the domain and range of the function and the inverse.

1. $f(x) = \sqrt{-2x+3} + 1$
2. $g(x) = (x+4)^2 - 1$, $x \leq -4$
3. $h(x) = \frac{9x}{4x-1}$
4. $f(x) = \sqrt{x} - 3$
5. $g(x) = \frac{1}{1-x}$
6. $h(x) = x^2 + 6x + 4$, $x \leq -3$
7. $f(x) = \sqrt{5x-4}$
8. $g(x) = x^2 - 2x + 3$, $x \leq 1$
9. $h(x) = \frac{3}{x-1}$
10. $f(x) = 5 - \sqrt{2x}$
11. $g(x) = \frac{5}{x+1}$
12. $h(x) = \frac{3x}{x-2}$

12.4 Answer Key

1. $f^{-1}(x) = -\frac{1}{2}\left((x-1)^2 - 3\right)$

	Domain	Range
$f(x)$	$(-\infty, 1.5]$	$[1, \infty)$
$f^{-1}(x)$	$[1, \infty)$	$(-\infty, 1.5]$

2. $g^{-1}(x) = -\sqrt{x+1} - 4$

	Domain	Range
$g(x)$	$(-\infty, -4]$	$[-1, \infty)$
$g^{-1}(x)$	$[-1, \infty)$	$(-\infty, -4]$

3. $h^{-1}(x) = \frac{-x}{9-4x}$

	Domain	Range
$h(x)$	$(-\infty, 1/4) \cup (1/4, \infty)$	$(\infty, 9/4) \cup (9/4, \infty)$
$h^{-1}(x)$	$(\infty, 9/4) \cup (9/4, \infty)$	$(-\infty, 1/4) \cup (1/4, \infty)$

4. $f^{-1}(x) = (x+3)^2$

	Dom	Ran
$f(x)$	$[0, \infty)$	$[-3, \infty)$
$f^{-1}(x)$	$[-3, \infty)$	$[0, \infty)$

5. $g^{-1}(x) = 1 - \frac{1}{x}$

	Dom	Ran
$g(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$g^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

6. $h^{-1}(x) = -\sqrt{x+5} - 3$

	Dom	Ran
$h(x)$	$(-\infty, -3]$	$[-5, \infty)$
$h^{-1}(x)$	$[-5, \infty)$	$(-\infty, -3]$

7. $f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$

	Dom	Ran
$f(x)$	$\left[\frac{4}{5}, \infty\right)$	$[0, \infty)$
$f^{-1}(x)$	$[0, \infty)$	$\left[\frac{4}{5}, \infty\right)$

8. $g^{-1}(x) = -\sqrt{x-2} + 1$

	Dom	Ran
$g(x)$	$(-\infty, 1]$	$[2, \infty)$
$g^{-1}(x)$	$[2, \infty)$	$(-\infty, 1]$

9. $h^{-1}(x) = \frac{3}{x} + 1$

	Dom	Ran
$h(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$h^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

10. $f^{-1}(x) = \frac{1}{2}(x-5)^2$; $x \leq 5$

	Domain	Range
$f(x)$	$[0, \infty)$	$(-\infty, 5]$
$f^{-1}(x)$	$(-\infty, 5]$	$[0, \infty)$

11. $g^{-1}(x) = \frac{5}{x} - 1$

	Domain	Range
$g(x)$	$(-\infty, -1) \cup (-1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$g^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, -1) \cup (-1, \infty)$

12. $h^{-1}(x) = \frac{2x}{x-3}$

	Domain	Range
$h(x)$	$(-\infty, 2) \cup (2, \infty)$	$(-\infty, 3) \cup (3, \infty)$
$h^{-1}(x)$	$(-\infty, 3) \cup (3, \infty)$	$(-\infty, 2) \cup (2, \infty)$

Chapter 13

Exponential Functions

13.1 Transforming Exponential Functions

Given $f(x) = e^x$, determine the specific transformations done to $f(x)$ to produce $g(x)$.

1. $g(x) = -3e^{x+1}$
2. $g(x) = \frac{1}{4}e^{-5x} - 2$
3. $g(x) = e^{2x+7}$
4. $g(x) = 5e^{-x-2} + 1$
5. $g(x) = 0.1e^{0.25x-3} - 4$

13.2 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

1. $f(x) = 3 + e^{2x}$
2. $h(x) = 5^{-x}$
3. $h(x) = -\frac{2}{3}e^{x+7} + 1$
4. $f(x) = -7e^x + 4$
5. $g(x) = \frac{1}{3}e^{2x+1} - 5$
6. $h(x) = -\frac{1}{2}e^{-4x} + 1$
7. $f(x) = 3^{1-2x}$
8. $g(x) = \frac{2}{3}\left(\frac{1}{2}\right)^{-x+4}$
9. $h(x) = -7(10)^{5x+4} + 3$

13.3 Answer Key

Transforming Exponential Functions

1. Shift left 1 unit, vertical stretch by factor of 3, reflect across x-axis
2. Horizontal compression by factor of 5, reflect across y-axis, vertical compression by factor of 4, shift down 2 units
3. Shift left 7 units, horizontal compression by factor of 2
4. Shift right 2 units, reflect across y-axis, vertical stretch by factor of 5, shift up 1 unit
5. Shift right 3 units, horizontal compression by factor of 4, vertical compression by factor of 10, shift down 4 units

End Behavior

1. $\lim\limits_{x\to-\infty} f(x) = 3 \quad \lim\limits_{x\to\infty} f(x) = \infty$
2. $\lim\limits_{x\to-\infty} f(x) = \infty \quad \lim\limits_{x\to\infty} f(x) = 0$
3. $\lim\limits_{x\to-\infty} h(x) = 1 \quad \lim\limits_{x\to\infty} h(x) = -\infty$
4. $\lim\limits_{x\to-\infty} f(x) = 4 \quad \lim\limits_{x\to\infty} f(x) = -\infty$
5. $\lim\limits_{x\to-\infty} f(x) = -5 \quad \lim\limits_{x\to\infty} f(x) = \infty$
6. $\lim\limits_{x\to-\infty} f(x) = -\infty \quad \lim\limits_{x\to\infty} f(x) = 1$
7. $\lim\limits_{x\to-\infty} f(x) = \infty, \quad \lim\limits_{x\to\infty} f(x) = 0$
8. $\lim\limits_{x\to-\infty} g(x) = 0, \quad \lim\limits_{x\to\infty} g(x) = \infty$
9. $\lim\limits_{x\to-\infty} h(x) = 3, \quad \lim\limits_{x\to\infty} h(x) = -\infty$

Chapter 14

Logarithmic Functions

Write each of the following in exponential or logarithmic form.

1. $\ln(a) = 7$
2. $\log_4(x+1) = 9$
3. $\log(5x) = 30$
4. $\ln(w) = c$
5. $5^x = 19$
6. $8^{-3} = \frac{1}{512}$
7. $e^{14} = x$
8. $(1.1)^{-t} = 50$

Find the domain of each. Write your answers in interval notation.

9. $b(x) = \log_7\left(x^2 - 8x + 6\right)$
10. $a(x) = \ln\left(\frac{x^2+3x+2}{5x+15}\right)$
11. $f(x) = -7\ln\left(x^2 + 9x + 8\right)$
12. $g(x) = \log\left(5x^2 + 13x - 6\right)$
13. $h(x) = 3\log_2\left(x^3 + 2x^2 - x - 2\right)$
14. $c(x) = \ln\left(4x^2 - 15x - 4\right)$

State the end behavior of each.

15. $j(x) = 5\log_3(2x - 5) - 2$

Write an equivalent expression for each using natural logarithms.

16. $\log_7(11)$
17. $\log_{12}(x)$

14.1 Answer Key

1. $e^7 = a$
2. $4^9 = x + 1$
3. $10^{30} = 5x$
4. $e^c = w$
5. $\log_5(19) = x$
6. $\log_8\left(\frac{1}{512}\right) = -3$
7. $\ln(x) = 14$
8. $\log_{1.1}(50) = -t$
9. $(-\infty, 0.838) \cup (7.162, \infty)$
10. $(-3, -2) \cup (-1, \infty)$
11. $(-\infty, -8) \cup (-1, \infty)$
12. $(-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$
13. $(-2, -1) \cup (1, \infty)$
14. $\left(-\infty, -\frac{1}{4}\right) \cup (4, \infty)$
15. $\lim\limits_{x \to (5/2)^+} j(x) = -\infty \quad \lim\limits_{x \to \infty} j(x) = \infty$
16. $\frac{\ln(11)}{\ln(7)}$
17. $\frac{\ln(x)}{\ln(12)}$

Chapter 15

Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

1. $\log_b\left(\frac{x^2}{y^8}\right)$
2. $\ln(ez)^3$
3. $\log_5(x)+\log_5(9)-2\log_5(w)$
4. $\log_2\left(2^a b^3\right)$
5. $\ln\left(\frac{w^7}{e^6}\right)$
6. $5\log_4(m)-3\log_4(n)+2\log_4(p)$

Write an equivalent expression for each of the following using natural logarithms.

7. $\log_7(10)$
8. $\log_9(x)$
9. $\log_b(c)$
10. $\log_3(10)$
11. $\log_{17}(\pi)$
12. $\log_w(x)$

Suppose that $\log_a(b)=5$, $\log_a(c)=12$, and $\log_a(d)=9$. Evaluate each of the following.

13. $\log_a(bc)$
14. $\log_a(c^3)$
15. $\log_a\left(\frac{d}{c}\right)$
16. $\log_a\left(\frac{bd}{c}\right)$
17. $\log_a\left(b^7 c\right)$
18. $\log_a\left(\frac{c^2}{d}\right)$
19. $\log_a\left(\sqrt{bc}\right)$
20. $\log_a\left((bd)^2\right)$
21. $\log_a\left(\sqrt[3]{d^2}\right)$
22. $\log_a\left(\sqrt{b^5}\right)$
23. $\log_a\left(\frac{b^6 c}{d^3}\right)$
24. $\log_a\left(b^2 c^3 d^4\right)$

15.1 Answer Key

1. $2\log_b(x) - 8\log_b(y)$
2. $3 + 3\ln(z)$
3. $\log_5\left(\frac{9x}{w^2}\right)$
4. $a + 3\log_2(b)$
5. $7\ln(w) - 6$
6. $\log_4\left(\frac{m^5p^2}{n^3}\right)$
7. $\frac{\ln(10)}{\ln(7)}$
8. $\frac{\ln(x)}{\ln(9)}$
9. $\frac{\ln(c)}{\ln(b)}$
10. $\frac{\ln(10)}{\ln(3)}$
11. $\frac{\ln(\pi)}{\ln(17)}$
12. $\frac{\ln(x)}{\ln(w)}$
13. 17
14. 36
15. -3
16. 2
17. 47
18. 15
19. $17/2$
20. 28
21. 6
22. $25/2$
23. 15
24. 82

Chapter 16

Exponential Equations

When solving exponential equations, if possible, try to isolate your exponential expression. Then take the logarithm of both sides.

Example 1. Solve each of the following. Round your answers to 4 decimal places.

(a) $5^{x+2} = 50$ (b) $2^{x-4} = 7^{x+1}$ (c) $4e^{-3x} = 11$

Solution 1. (a) The exponential function, 5^{x+2}, is already isolated on the left side. We can take $\log_5$ of both sides and then use the Power Rule for logarithms.

$$\begin{aligned}
5^{x+2} &= 50 \\
\log_5\left(5^{x+2}\right) &= \log_5(50) \\
(x+2)\log_5(5) &= \log_5(50) \\
x+2 &= \log_5(50) \qquad \text{since } \log_5(5) = 1 \\
x &= \log_5(50) - 2 \\
x &\approx 0.4307
\end{aligned}$$

To 4 decimal places, the solution to $5^{x+2} = 50$ is $\boxed{x \approx 0.4307}$

(b) We can apply the Product Rule for exponents to write a single exponential function. Then we can apply logarithms.

$$\begin{aligned}
2^{x-4} &= 7^{x+1} \\
2^x \cdot 2^{-4} &= 7^x \cdot 7^1 \qquad \text{Product Rule for exponents} \\
2^x \cdot \tfrac{1}{16} &= 7^x \cdot 7 \\
2^x &= 7^x \cdot 112 \qquad \text{Multiply both sides by 16 to clear fractions} \\
\tfrac{2^x}{7^x} &= 112 \qquad \text{Divide both sides by } 7^x \text{ since it will never equal 0} \\
\left(\tfrac{2}{7}\right)^x &= 112 \qquad \text{Raising a fraction raises both numerator and denominator to that power} \\
\log_{2/7}\left(\tfrac{2}{7}\right)^x &= \log_{2/7}(112) \\
x \cdot \log_{2/7}\left(\tfrac{2}{7}\right) &= \log_{2/7}(112) \\
x &= \log_{2/7}(112) \\
x &\approx -3.7665
\end{aligned}$$

To 4 decimal places, the solution to $2^{x-4} = 7^{x+1}$ is $\boxed{x \approx -3.7665}$

(c) This time, we want to isolate the function e^{-3x}.

$$\begin{aligned} 4e^{-3x} &= 11 \\ e^{-3x} &= \tfrac{11}{4} \\ \ln\left(e^{-3x}\right) &= \ln\left(\tfrac{11}{4}\right) \\ -3x &= \ln\left(\tfrac{11}{4}\right) \\ x &= -\tfrac{1}{3}\ln\left(\tfrac{11}{4}\right) \\ x &\approx -0.3372 \end{aligned}$$

To 4 decimal places, the solution to $4e^{-3x} = 11$ is $\boxed{x \approx -0.3372}$

16.1 Exercises

Solve each. Round to 3 decimal places when necessary.

1. $3e^{x-2} = 7$
2. $5^x + 4 > 1$
3. $2^{3x+4} = 32^{x-7}$
4. $5e^{7x} + 10 = 42$
5. $7^{4x+1} \geq 343$
6. $1000e^{0.04x} = 2000$
7. $3(4.1)^{x-2} = 8$
8. $2^{x+1} = 5^{7x-5}$
9. $8(17)^{-5x} = 22$
10. $-3(11)^{x-10} = -58$
11. $12^{-10x} + 8 = 80$
12. $-5(10)^{7x} + 9 = -46$
13. $8(8)^{10x} - 1 = 55.2$
14. $3(3)^{-5x} - 8 = 74$
15. $6(16)^{4x-9} = 19$
16. $-7(11)^{5x-7} = -3$
17. $3^{9-6x} - 7 = 26$
18. $3^{1-2x} = 7$
19. $\frac{2}{3}\left(\frac{1}{2}\right)^{-x+4} = 8$
20. $-7(10)^{5x+4} = -15$

16.2 Applications

1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?
2. Cadmium-109 has a half-life of about 1.267 years. If 50 mg are initially present, how many years will it take for 16 mg to remain?
3. The half-life of bismuth-207 is about 32.9 years. If 90 mg are initially present, how many years will it take for 75 mg to remain?

16.3 Answer Key

1. $x \approx 2.847$
2. $(-\infty, \infty)$
3. $x = 19.5$
4. $x \approx 0.265$
5. $\left[\frac{1}{2}, \infty\right)$
6. $x \approx 17.329$
7. $x \approx 2.695$
8. $x \approx 0.827$
9. $x \approx -0.071$
10. $x \approx 11.235$
11. $x \approx -0.172$
12. $x \approx 0.149$
13. $x \approx 0.094$
14. $x \approx -0.602$
15. $x \approx 2.354$
16. $x \approx 1.323$
17. $x \approx 0.970$
18. $x \approx -0.3856$
19. $x \approx 7.5850$
20. $x \approx -0.7338$

Applications

1. Approximately 17,952 years
2. Approximately 2.0828 years
3. Approximately 8.6538 years

Chapter 17

Logarithmic Equations and Inequalities

Solve each. Round to 3 decimal places when necessary.

1. $\log_5(x) + x\log_5(x) > 0$
2. $\ln\left(8 - x^2\right) = \ln(2 - x)$
3. $\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$
4. $\log_3(2x + 1) - \log_3(x - 5) = \log_3(x + 1)$
5. $\log_4(x + 1) + \log_4(x - 5) > 2$
6. $\log(x + 1) - \log(x - 5) = \log(x - 3)$
7. $x\log_3(x + 2) - \log_3(x + 2) = 0$
8. $\log_{1/2}(x + 1) > -3$
9. $\log_{12}(4x + 4) = \log_{12}(5x + 1)$
10. $\log_{15}(-4x + 2) = \log_{15}(6 - 2x)$
11. $\log_{11}(-5 - 3x^2) = \log_{11}(-2x^2 + 6x)$
12. $\log_{16}(x^2 + 4) = \log_{16}(2x + 3)$
13. $\log_7(8x - 1) = \log_7(x^2 + 14)$
14. $-7\log_5(x + 5) = -7$
15. $7\log_8(-x) = 28$
16. $-10\log_3(x - 5) = -20$

17.1 Answer Key

1. $(1, \infty)$
2. $x = -2$
3. $x \approx 1.571$
4. $x \approx 6.873$
5. $(2, \infty)$
6. $x = 7$
7. $x = \pm 1$
8. $(-1, 7)$
9. $x = 3$
10. $x = -2$
11. No Solution
12. $x = 1$
13. $x = 3,\ 5$
14. $x = 0$
15. $x = -4096$
16. $x = 14$

Chapter 18

Sequences

Write the first 4 terms of each sequence.

1. $a_n = 2(-3)^n$
2. $b_n = \frac{n!}{2^n}$
3. $c_{n+1} = 5c_n + 1;\ c_1 = 2$
4. $d_n = \frac{1}{2}d_{n-1} + n;\ d_1 = 3$

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round**.

5. $a_n = \{343,\ 667,\ 991,\ 1315, \ldots\}$; Find the 582nd term.
6. $b_n = \{300,\ 240,\ 192, 153.6,\ \ldots\}$; Find the 711th term.
7. $c_n = \left\{1,\ \frac{1}{3},\ \frac{1}{5},\ \frac{1}{7},\ \ldots\right\}$; Find the 8,675,309th term.

Given each sequence, find the first 4 digits **after the decimal point** in the **scientific notation** version of each term.

8. $a_n = 17,\ 33,\ 49,\ 65,\ \ldots;\quad a_{21,972}$
9. $b_n = 25,\ 36,\ 49,\ 64,\ 81,\ \ldots\quad b_{413,401}$
10. $c_n = \frac{1}{3},\ \frac{1}{6},\ \frac{1}{12},\ \frac{1}{24},\ \ldots\quad c_{152}$

Find the exact value of the indicated term for each sequence.

11. $a_n = \{-0.7,\ -2.8,\ -4.9,\ -7,\ \ldots\}$; find a_{941}
12. $b_n = \{\frac{1}{3},\ \frac{4}{5},\ \frac{9}{7},\ \frac{16}{9},\ \ldots\}$; find b_{137}

18.1 Answer Key

1. -6, 18, -54, 162
2. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{2}$
3. 2, 11, 56, 281
4. 3, $\frac{7}{2}$, $\frac{19}{4}$, $\frac{51}{8}$
5. 188,587
6. 6882
7. 7634
8. 5155 (3.51553×10^5)
9. 7090 $(1.7090369403 \times 10^{11})$
10. 1677 $(1.1677487203 \times 10^{-46})$
11. -1974.7
12. $\frac{18769}{275}$

Chapter 19

Series

Find the sum of each, if possible.

1. $\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^{i}$

2. $\sum_{i=0}^{\infty} 3\left(-\frac{2}{3}\right)^{i}$

3. $\sum_{k=1}^{\infty} -2\left(\frac{1}{3}\right)^{k}$

4. $\sum_{j=0}^{\infty} -\frac{1}{2}\left(\frac{3}{2}\right)^{j}$

5. $\sum_{i=0}^{\infty} 1.2(0.8)^{i}$

6. $\sum_{i=1}^{\infty} 1.2(0.8)^{i}$

7. $\sum_{i=0}^{\infty} 0.8(1.2)^{i}$

8. $\sum_{k=1}^{\infty} \frac{2}{3^{k}}$

Find the sum of each of the following. Round to 4 decimal places when necessary.

9. $9+13+17+21+\cdots+1565$

10. $-3+6-12+24-48+\cdots+50,331,648$

11. $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots+\frac{1}{981}$

12. $2+4+6+8+10+\cdots+38,214$

13. $3+7+11+15+\cdots+11,491$

14. $\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\cdots+\frac{742}{743}$

19.1 Answer Key

1. $\frac{1}{4}$
2. $\frac{9}{5}$
3. -1
4. Diverges
5. 6
6. 4.8
7. Diverges
8. 2
9. 306,930
10. $-33,554,433$
11. 7.4663
12. 365,096,556
13. 16,511,131
14. 733.8947

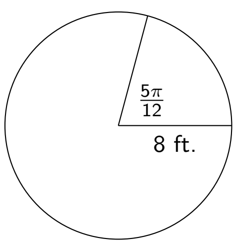
Chapter 20

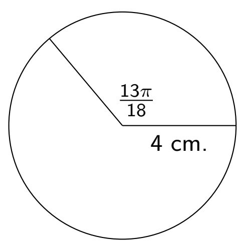
Angles and Radian Measure

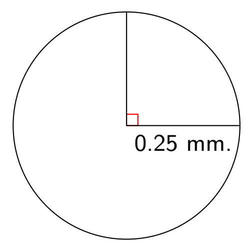
Sketch each of the following. Then find a coterminal between 0 and 360° (or 0 and 2π radians) for each.

1. $-\frac{3\pi}{4}$
2. 900°
3. $\frac{27\pi}{10}$
4. $-125°$

Find the arc length <u>and</u> sector area formed by the central angle of each. Exact answers only.

5. 

6. 

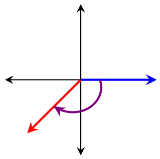
7. 

A belt runs on a pulley with radius 4 inches at 250 revolutions per minute.

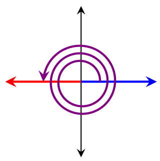
8. Find the angular velocity in rad/sec. Round your answer to 2 decimal places.
9. Find the linear velocity in ft/sec. Round your answer to 2 decimal places.

20.1 Answer Key

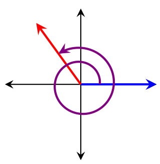
1. $\frac{5\pi}{4}$



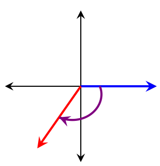
2. 180°



3. $\frac{7\pi}{10}$



4. 235°



5. $s = \frac{10\pi}{3}$ ft.; $\quad A = \frac{40\pi}{3}$ sq.ft.

6. $s = \frac{26\pi}{9}$ cm.; $\quad A = \frac{52\pi}{9}$ sq.cm.

7. $s = \frac{\pi}{8}$ mm.; $\quad A = \frac{\pi}{64}$ sq.mm.

8. 26.18 rad/sec

9. 8.73 ft/sec

Chapter 21

Trig Functions of Any Angle

Find the exact value of each of the six trig functions of θ if P is a point on the terminal side of θ.

1. $P(-2, 3)$
2. $P(0, -4)$
3. $P(-2\sqrt{3}, 2)$
4. $P(-3, 5)$
5. $P(-2, 1)$
6. $P(-4, -7)$
7. $P(-7, 4)$
8. $P(-9, -9)$
9. $P(4, -8)$

Find the exact values of the 6 trig functions of the following angles.

10. $\theta = \frac{-17\pi}{4}$
11. $\theta = \frac{21\pi}{2}$
12. $\theta = 24\pi$
13. $-\frac{5\pi}{3}$
14. $\frac{23\pi}{6}$
15. $-\frac{\pi}{2}$
16. $\frac{10\pi}{3}$
17. $-\frac{\pi}{3}$
18. $\frac{11\pi}{4}$
19. $-\frac{13\pi}{2}$

21.1 Answer Key

1. $\sin\theta = \frac{3\sqrt{13}}{13}$, $\cos\theta = \frac{-2\sqrt{13}}{13}$, $\tan\theta = -\frac{3}{2}$, $\csc\theta = \frac{\sqrt{13}}{3}$, $\sec\theta = -\frac{\sqrt{13}}{2}$, $\cot\theta = -\frac{2}{3}$
2. $\sin\theta = -1$, $\cos\theta = 0$, $\tan\theta = \text{undef.}$, $\csc\theta = -1$, $\sec\theta = \text{undef.}$, $\cot\theta = 0$
3. $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$, $\tan\theta = -\frac{\sqrt{3}}{3}$, $\csc\theta = 2$, $\sec\theta = -\frac{2\sqrt{3}}{3}$, $\cot\theta = -\sqrt{3}$
4. $\sin\theta = \frac{5\sqrt{34}}{34}$, $\cos\theta = -\frac{3\sqrt{34}}{34}$, $\tan\theta = -\frac{5}{3}$, $\csc\theta = \frac{\sqrt{34}}{5}$, $\sec\theta = -\frac{\sqrt{34}}{3}$, $\cot\theta = -\frac{3}{5}$
5. $\sin\theta = \frac{\sqrt{5}}{5}$, $\cos\theta = -\frac{2\sqrt{5}}{5}$, $\tan\theta = -\frac{1}{2}$, $\csc\theta = \sqrt{5}$, $\sec\theta = -\frac{\sqrt{5}}{2}$, $\cot\theta = -2$
6. $\sin\theta = -\frac{7\sqrt{65}}{65}$, $\cos\theta = -\frac{4\sqrt{65}}{65}$, $\tan\theta = \frac{7}{4}$, $\csc\theta = -\frac{\sqrt{65}}{7}$, $\sec\theta = -\frac{\sqrt{65}}{4}$, $\cot\theta = \frac{4}{7}$
7. $\sin(\theta) = \frac{4\sqrt{65}}{65}$ $\quad\cos(\theta) = -\frac{7\sqrt{65}}{65}$ $\quad\tan(\theta) = -\frac{4}{7}$ $\quad\csc(\theta) = \frac{\sqrt{65}}{4}$ $\quad\sec(\theta) = -\frac{\sqrt{65}}{7}$ $\quad\cot(\theta) = -\frac{7}{4}$
8. $\sin(\theta) = -\frac{\sqrt{2}}{2}$ $\quad\cos(\theta) = -\frac{\sqrt{2}}{2}$ $\quad\tan(\theta) = 1$ $\quad\csc(\theta) = -\sqrt{2}$ $\quad\sec(\theta) = -\sqrt{2}$ $\quad\cot(\theta) = 1$
9. $\sin(\theta) = -\frac{2\sqrt{5}}{5}$ $\quad\cos(\theta) = \frac{\sqrt{5}}{5}$ $\quad\tan(\theta) = -2$ $\quad\csc(\theta) = -\frac{\sqrt{5}}{2}$ $\quad\sec(\theta) = \sqrt{5}$ $\quad\cot(\theta) = -\frac{1}{2}$
10. $\sin\theta = -\frac{\sqrt{2}}{2}$, $\cos\theta = \frac{\sqrt{2}}{2}$, $\tan\theta = -1$, $\csc\theta = -\sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = -1$
11. $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta = \text{undefined}$, $\csc\theta = 1$, $\sec\theta = \text{undefined}$, $\cot\theta = 0$
12. $\sin\theta = 0$, $\cos\theta = 1$, $\tan\theta = 0$, $\csc\theta = \text{undefined}$, $\sec\theta = 1$, $\cot\theta = \text{undefined}$
13. $\sin\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}$, $\cos\left(-\frac{5\pi}{3}\right) = \frac{1}{2}$, $\tan\left(-\frac{5\pi}{3}\right) = \sqrt{3}$, $\csc\left(-\frac{5\pi}{3}\right) = \frac{2\sqrt{3}}{3}$, $\sec\left(-\frac{5\pi}{3}\right) = 2$, $\cot\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{3}$
14. $\sin\left(\frac{23\pi}{6}\right) = -\frac{1}{2}$, $\cos\left(\frac{23\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\tan\left(\frac{23\pi}{6}\right) = -\frac{\sqrt{3}}{3}$, $\csc\left(\frac{23\pi}{6}\right) = -2$, $\sec\left(\frac{23\pi}{6}\right) = \frac{2\sqrt{3}}{3}$, $\cot\left(\frac{23\pi}{6}\right) = -\sqrt{3}$
15. $\sin\left(-\frac{\pi}{2}\right) = -1$, $\cos\left(-\frac{\pi}{2}\right) = 0$, $\tan\left(-\frac{\pi}{2}\right) = \text{undefined}$, $\csc\left(-\frac{\pi}{2}\right) = -1$, $\sec\left(-\frac{\pi}{2}\right) = \text{undefined}$, $\cot\left(-\frac{\pi}{2}\right) = 0$
16. $\sin\left(\frac{10\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\cos\left(\frac{10\pi}{3}\right) = -\frac{1}{2}$, $\tan\left(\frac{10\pi}{3}\right) = \sqrt{3}$, $\csc\left(\frac{10\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$, $\sec\left(\frac{10\pi}{3}\right) = -2$, $\cot\left(\frac{10\pi}{3}\right) = \frac{\sqrt{3}}{3}$
17. $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$, $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$, $\csc\left(-\frac{\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$, $\sec\left(-\frac{\pi}{3}\right) = 2$, $\cot\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$
18. $\sin\left(\frac{11\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $\cos\left(\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, $\tan\left(\frac{11\pi}{4}\right) = -1$, $\csc\left(\frac{11\pi}{4}\right) = \sqrt{2}$, $\sec\left(\frac{11\pi}{4}\right) = -\sqrt{2}$, $\cot\left(\frac{11\pi}{4}\right) = -1$
19. $\sin\left(-\frac{13\pi}{2}\right) = -1$, $\cos\left(-\frac{13\pi}{2}\right) = 0$, $\tan\left(-\frac{13\pi}{2}\right) = \text{undefined}$, $\csc\left(-\frac{13\pi}{2}\right) = -1$, $\sec\left(-\frac{13\pi}{2}\right) = \text{undefined}$, $\cot\left(-\frac{13\pi}{2}\right) = 0$

Chapter 22

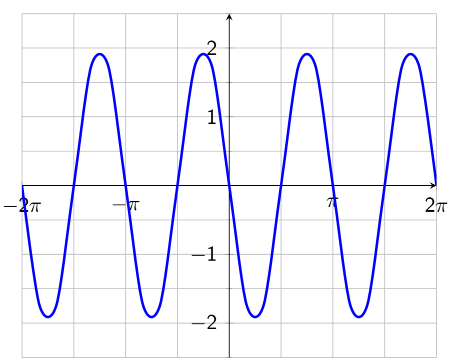
Graphs of Sine and Cosine Functions

Determine the exact values of the amplitude, period, phase shift, vertical shift, domain, and range of each. *Be specific.*

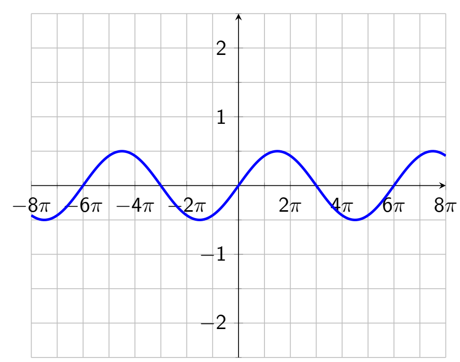
1. $f(x) = -2\sin\left(3x - \frac{\pi}{4}\right) + 1$
2. $g(x) = \frac{1}{3}\cos\left(\frac{1}{2}x + 2\right)$
3. $f(x) = 2\sin\left(x - \frac{\pi}{3}\right) + 7$
4. $f(x) = -4\cos\left(\frac{2}{3}x - \frac{2\pi}{3}\right)$
5. $h(x) = \sin\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$
6. $f(x) = 3\sin\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$
7. $f(x) = -4\cos\left(4x - \frac{\pi}{3}\right) + \pi$
8. $g(x) = \frac{4}{9}\cos\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$

Write the equation of each of the following in the form $y = a\sin(bx)$.

9.



10.



22.1 Answer Key

1. Amp $= 2$, Per $= \frac{2\pi}{3}$, P.S. $= \frac{\pi}{12} \rightarrow$, V.S. $= 1 \uparrow$, Dom: $(-\infty, \infty)$, Range: $[-1, 3]$
2. Amp $= \frac{1}{3}$, Per $= 4\pi$, P.S. $= 4 \leftarrow$, V.S. $=$ None, Dom: $(-\infty, \infty)$, Range: $\left[-\frac{1}{3}, \frac{1}{3}\right]$
3. Amp $= 2$, Period $= 2\pi$, P.S. $= \frac{\pi}{3}$ right, V.S. $= 7$ up, Dom: $(-\infty, \infty)$, Range: $[5, 9]$
4. Amp $= 4$, Period $= 3\pi$, P.S. $= \pi$ right, V.S. $= 0$ (or none), Dom: $(-\infty, \infty)$, Range: $[-4, 4]$
5. Amp $= 1$, Per $= \frac{8\pi}{3}$, P.S. $= \frac{\pi}{9}$ left, V.S. $= 8$ down, Dom: $(-\infty, \infty)$, Range: $[-8, 8]$
6. Amp $= 3$, Period $= \pi$, P.S. $= \frac{\pi}{4}$ left, V.S. $= \sqrt{3}$ down, Dom: $(-\infty, \infty)$, Range: $[-3 - \sqrt{3}, 3 - \sqrt{3}]$
7. Amp $= 4$, Period $= \frac{\pi}{2}$, P.S. $= \frac{\pi}{12}$ right, V.S. $= \pi$ up, Dom: $(-\infty, \infty)$, Range: $[-4 + \pi, 4 + \pi]$
8. Amp $= \frac{4}{9}$, Period $= \frac{20\pi}{3}$, P.S. $= 5\pi$ left, V.S. $= 1$ down, Dom: $(-\infty, \infty)$, Range: $\left[-\frac{13}{9}, -\frac{5}{9}\right]$
9. $y = -2\sin(2x)$
10. $y = \frac{1}{2}\sin\left(\frac{1}{3}x\right)$

Chapter 23

Graphs of Other Trig Functions

Determine the exact values of the amplitude, period, phase shift, vertical shift, domain, and range of each. *Be specific.*

1. $h(x) = \tan\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$
2. $f(x) = 3\tan\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$
3. $f(x) = -4\cot\left(4x - \frac{\pi}{3}\right) + \pi$
4. $g(x) = \frac{4}{9}\cot\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$
5. $h(x) = \sec\left(\frac{3}{4}x + \frac{\pi}{12}\right) - 8$
6. $f(x) = 3\sec\left(2x + \frac{\pi}{2}\right) - \sqrt{3}$
7. $f(x) = -4\csc\left(4x - \frac{\pi}{3}\right) + \pi$
8. $g(x) = \frac{4}{9}\csc\left(\frac{3}{10}x + \frac{3\pi}{2}\right) - 1$
9. $f(x) = \frac{1}{10}\cot\left(3x + \frac{3\pi}{4}\right) + 5$
10. $g(x) = 6\csc\left(2x - \frac{\pi}{6}\right) + 5$
11. $h(x) = 4\csc\left(8x - \frac{\pi}{6}\right) - 1$
12. $k(x) = \tan\left(\frac{1}{7}x + \frac{\pi}{4}\right) - 5$
13. $f(x) = 7\tan\left(8x + \frac{\pi}{6}\right)$
14. $g(x) = 6\tan\left(7x - \frac{\pi}{4}\right) - 4$
15. $h(x) = 4\tan(5x) - 5$
16. $k(x) = \cot\left(\frac{1}{3}x - \frac{2\pi}{3}\right) - 5$
17. $f(x) = 10\cot\left(\frac{1}{8}x - \frac{\pi}{2}\right)$
18. $g(x) = 10\cot\left(5x - \frac{\pi}{4}\right) + 5$
19. $f(x) = -3\tan\left(\frac{x}{5} - \frac{2\pi}{3}\right) + 1$
20. $g(x) = \frac{1}{4}\csc(2x + \pi) - 9$
21. $h(x) = -\frac{1}{10}\sec\left(\pi x - \frac{\pi}{2}\right)$

23.1 Answer Key

1. Amp = n/a, Per = $\frac{4\pi}{3}$, P.S. = $\frac{\pi}{9}$ left, V.S. = 8 down, Dom: $x \neq \frac{5\pi}{9} + \frac{4\pi}{3}k$, Range: $(-\infty, \infty)$
2. Amp = n/a, Period = $\frac{\pi}{2}$, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down, Dom: $x \neq \frac{\pi}{2}k$, Range: $(-\infty, \infty)$
3. Amp = n/a, Period = $\frac{\pi}{4}$, P.S. = $\frac{\pi}{12}$ right, V.S. = π up, Dom: $x \neq \frac{\pi}{12} + \frac{\pi}{4}k$, Range: $(-\infty, \infty)$
4. Amp = n/a, Period = $\frac{10\pi}{3}$, P.S. = 5π left, V.S. = 1 down, Dom: $x \neq -5\pi + \frac{10\pi}{3}k$, Range: $(-\infty, \infty)$
5. Amp = n/a, Per = $\frac{8\pi}{3}$, P.S. = $\frac{\pi}{9}$ left, V.S. = 8 down, Dom: $x \neq \frac{5\pi}{9} + \frac{4\pi}{3}k$, Range: $(-\infty, -9] \cup [-7, \infty)$
6. Amp = n/a, Period = π, P.S. = $\frac{\pi}{4}$ left, V.S. = $\sqrt{3}$ down, Dom: $x \neq \frac{\pi}{2}k$, Range: $(-\infty, -3-\sqrt{3}] \cup [3-\sqrt{3}, \infty)$
7. Amp = n/a, Period = $\frac{\pi}{2}$, P.S. = $\frac{\pi}{12}$ right, V.S. = π up, Dom: $x \neq \frac{\pi}{8} + \frac{\pi}{4}k$, Range: $(-\infty, -4+\pi] \cup [4+\pi, \infty)$
8. Amp = n/a, Period = $\frac{20\pi}{3}$, P.S. = 5π left, V.S. = 1 down, Dom: $x \neq -5\pi + \frac{10\pi}{3}k$, Range: $\left(-\infty, -\frac{13}{9}\right] \cup \left[-\frac{5}{9}, \infty\right)$
9. Amp = n/a, Period = $\frac{\pi}{3}$, P.S. = $\frac{\pi}{4}$ left, V.S. = 5 up, Dom: $x \neq -\frac{\pi}{4} + \frac{\pi}{3}k$, Range: $(-\infty, \infty)$
10. Amp = n/a, Period = π, P.S. = $\frac{\pi}{12}$ right, V.S. = 5 up, Dom: $x \neq \frac{\pi}{12} + \frac{\pi}{2}k$, Range: $(-\infty, -1] \cup [11, \infty)$
11. Amp = n/a, Period = $\frac{\pi}{4}$, P.S. = $\frac{\pi}{48}$ right, V.S. = 1 down, Dom: $x \neq \frac{\pi}{48} + \frac{\pi}{8}k$, Range: $(-\infty, -5] \cup [3, \infty)$
12. Amp = n/a, Period = 7π, P.S. = $\frac{7\pi}{4}$ left, V.S. = 5 down, Dom: $x \neq \frac{7\pi}{4} + 7\pi k$, Range: $(-\infty, \infty)$
13. Amp = n/a, Period = $\frac{\pi}{8}$, P.S. = $\frac{\pi}{48}$ left, V.S. = 0 (or none), Dom: $x \neq \frac{\pi}{24} + \frac{\pi}{8}k$, Range: $(-\infty, \infty)$
14. Amp = n/a, Period = $\frac{\pi}{7}$, P.S. = $\frac{\pi}{28}$ right, V.S. = 4 down, Dom: $x \neq \frac{\pi}{28} + \frac{\pi}{7}k$, Range: $(-\infty, \infty)$
15. Amp = n/a, Period = $\frac{\pi}{5}$, P.S. = 0 (or none), V.S. = 5 down, Dom: $x \neq \frac{\pi}{10} + \frac{\pi}{5}k$, Range: $(-\infty, \infty)$
16. Amp = n/a, Period = 3π, P.S. = 2π right, V.S. = 5 down, Dom: $x \neq 2\pi + 3\pi k$, Range: $(-\infty, \infty)$
17. Amp = n/a, Period = 8π, P.S. = 4π right, V.S. = 0 (or none), Dom: $x \neq 4\pi + 8\pi k$, Range: $(-\infty, \infty)$
18. Amp = n/a, Period = $\frac{\pi}{5}$, P.S. = $\frac{\pi}{20}$ right, V.S. = 5 up, Dom: $x \neq \frac{\pi}{20} + \frac{\pi}{5}k$, Range: $(-\infty, \infty)$
19. Amp = n/a, Per = 5π, P.S. = $\frac{10\pi}{3}$ right, V.S. = Up 1, Dom: $x \neq \frac{35\pi}{6} + 5\pi k$, Range: $(-\infty, \infty)$
20. Amp = n/a, Per = π, P.S. = $\frac{\pi}{2}$ left, V.S. = Down 9, Dom: $x \neq -\frac{\pi}{2} + \frac{\pi}{2}k$, Range: $\left(-\infty, -\frac{37}{4}\right] \cup \left[-\frac{35}{4}, \infty\right)$
21. Amp = n/a, Per = 2, P.S. = $\frac{1}{2}$ right, V.S. = 0, Dom: $x \neq 1 + k$, Range: $\left(-\infty, -\frac{1}{10}\right] \cup \left[\frac{1}{10}, \infty\right)$

Chapter 24

Inverse Trig Functions

State the exact, simplified value of each or write as an expression of x.

1. $\cot^{-1}(-1)$
2. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
3. $\tan^{-1}(0)$
4. $\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right)$
5. $\cos^{-1}\left(\frac{1}{2}\right)$
6. $\sec^{-1}(-2)$
7. $\tan^{-1}\left(-\sqrt{3}\right)$
8. $\sec\left(\sin^{-1}\left(\frac{2}{5}\right)\right)$
9. $\cot\left(\sec^{-1}(x)\right)$
10. $\sin\left(\cos^{-1}\left(\frac{3x}{4}\right)\right)$
11. $\cot\left(\csc^{-1}\left(-\frac{7}{2}\right)\right)$
12. $\sec\left(\arcsin\left(\frac{9}{13}\right)\right)$
13. $\cos\left(\tan^{-1}(7x)\right)$
14. $\sin\left(\sec^{-1}\left(\frac{8}{x}\right)\right)$
15. $\csc\left(\arctan\left(-\frac{3}{2}\right)\right)$
16. $\cos\left(\sin^{-1}\left(\frac{7}{8}\right)\right)$
17. $\tan\left(\cos^{-1}\left(\frac{3}{x}\right)\right)$
18. $\csc\left(\sec^{-1}\left(\sqrt{2}\right)\right)$
19. $\tan\left(\sec^{-1}\left(\frac{\sqrt{17}}{4}\right)\right)$
20. $\sin\left(\tan^{-1}\left(\frac{4x}{5}\right)\right)$
21. $\cos\left(\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)\right)$
22. $\sin\left(\cos^{-1}\left(\frac{9}{10}\right)\right)$
23. $\tan\left(\cot^{-1}(8)\right)$
24. $\csc\left(\cot^{-1}\left(\frac{\sqrt{2}}{x}\right)\right)$
25. $\sin\left(\tan^{-1}(12x)\right)$
26. $\cos\left(\sec^{-1}\left(\frac{x}{9}\right)\right)$
27. $\sin\left(\arctan\left(\frac{x}{5}\right)\right)$
28. $\tan\left(\cos^{-1}(3x)\right)$
29. $\sec\left(\arcsin\left(\sqrt{x}\right)\right)$

24.1 Answer Key

1. $\frac{3\pi}{4}$
2. $-\frac{\pi}{3}$
3. 0
4. $\frac{\pi}{6}$
5. $\frac{\pi}{3}$
6. $\frac{2\pi}{3}$
7. $-\frac{\pi}{3}$
8. $\frac{5\sqrt{21}}{21}$
9. $\frac{1}{\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}}{x^2-1}$
10. $\frac{\sqrt{16-9x^2}}{4}$
11. $-\frac{3\sqrt{5}}{2}$
12. $\frac{13\sqrt{22}}{44}$
13. $\frac{\sqrt{49x^2+1}}{49x^2+1}$
14. $\frac{\sqrt{64-x^2}}{x}$
15. $-\frac{\sqrt{13}}{3}$
16. $\frac{\sqrt{15}}{8}$
17. $\frac{\sqrt{x^2-9}}{3}$
18. $\sqrt{2}$
19. $\sqrt{17}$
20. $\frac{4x}{\sqrt{16x^2+25}}$
21. $\frac{5\sqrt{7}}{14}$
22. $\frac{\sqrt{19}}{10}$
23. $\frac{1}{8}$
24. $\frac{\sqrt{x^2+2}}{x}$
25. $\frac{12x}{\sqrt{144x^2+1}} = \frac{12x\sqrt{144x^2+1}}{144x^2+1}$
26. $\frac{9}{x}$
27. $\frac{x}{\sqrt{x^2+25}}$
28. $\frac{\sqrt{1-9x^2}}{3x}$
29. $\frac{1}{\sqrt{1-x}}$

Chapter 25

Trig Equations and Inequalities

Solve each in the interval $[0, 2\pi)$. Write your answers to inequalities using interval notation.

1. $\tan(6x) = 1$
2. $\cot(2x) = -\frac{\sqrt{3}}{3}$
3. $\sin^2(x) = \frac{3}{4}$
4. $\sin(2x) = \cos(x)$
5. $\sin(2x) \geq \sin(x)$
6. $\cos(2x) < 0$
7. $2\sin\left(x - \frac{\pi}{3}\right) = -1$
8. $3\tan\left(-2x + \frac{\pi}{2}\right) = \sqrt{3}$
9. $\sin^2(x) < \frac{1}{2}$
10. $\tan^2(x) = 3\sec(x) - 3$
11. $2\csc(x) - 3\csc^2(x) = -2\csc^2(x) + 1$
12. $-2\cot(x) - \csc^2(x) = 0$
13. $\tan(x) = -\tan(x)\cos(x)$
14. $3\cos(x) = 2\cos^2(x) + 1$
15. $\csc(x) - \cot^2(x) + 1 = 0$
16. $-\sin(x) + \sin(2x) = 2\sin(2x)$
17. $3\cos(x) = \sin(2x) + 2\cos(x)$
18. $-2\sin\left(3x - \frac{2\pi}{3}\right) = 1$
19. $\tan\left(\frac{1}{2}x + \pi\right) = \frac{\sqrt{3}}{3}$
20. $2\cos\left(-\frac{2}{3}x + \frac{\pi}{4}\right) = \sqrt{2}$
21. $2\sec^2(x) - 2 = 3\sec(x)$
22. $3\cos^2(x) - 2\cos(x) = 5$
23. $\cos(2x) = \sin(x)$
24. $\sin(2x) = \sqrt{3}\sin(x)$

25.1 Answer Key

1. $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{3\pi}{8}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{7\pi}{8}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{11\pi}{8}, \frac{37\pi}{24}, \frac{41\pi}{24}, \frac{15\pi}{8}$
2. $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$
3. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
4. $\frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
5. $\left[0, \frac{\pi}{3}\right] \cup \left[\pi, \frac{5\pi}{3}\right]$
6. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$
7. $x = \frac{\pi}{6},\ \frac{3\pi}{2}$
8. $x = \frac{\pi}{6},\ \frac{2\pi}{3},\ \frac{7\pi}{6},\ \frac{5\pi}{3}$
9. $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$
10. $x = 0\ \frac{\pi}{3},\ \frac{5\pi}{3}$
11. $x = \frac{\pi}{2}$
12. $x = \frac{3\pi}{4},\ \frac{7\pi}{4}$
13. $x = 0,\ \pi$
14. $x = 0\ \frac{\pi}{3},\ \frac{5\pi}{3}$
15. $x = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{3\pi}{2}$
16. $x = 0,\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3}$
17. $x = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6},\ \frac{3\pi}{2}$
18. $x = \frac{\pi}{6}, \frac{11\pi}{18}, \frac{5\pi}{6}, \frac{23\pi}{18}, \frac{3\pi}{2}, \frac{35\pi}{18}$
19. $x = \frac{\pi}{3}$
20. $x = 0, \frac{3\pi}{4}$
21. $x = \frac{\pi}{3},\ \frac{5\pi}{3}$
22. $x = \pi$
23. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
24. $x = 0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$

Chapter 26

Law of Sines and Cosines; Area of Triangles

Solve each triangle and find its area.

1. $m\angle B = 37.8^\circ$, $a = 15$, $c = 21.1$
2. $m\angle A = 41.9^\circ$, $m\angle C = 59.2^\circ$, $a = 10.2$
3. $a = 14$, $b = 19.6$, $c = 13.1$
4. $c = 29$, $b = 23$, $m\angle A = 55^\circ$
5. $c = 8$, $b = 12$, $m\angle A = 90^\circ$
6. $m\angle B = 67.2^\circ$, $a = 15.6$, $c = 18.9$
7. $b = 20$, $a = 30$, $c = 12$
8. $a = 14$, $b = 6$, $c = 12$
9. $a = 7$, $b = 14$, $c = 12$

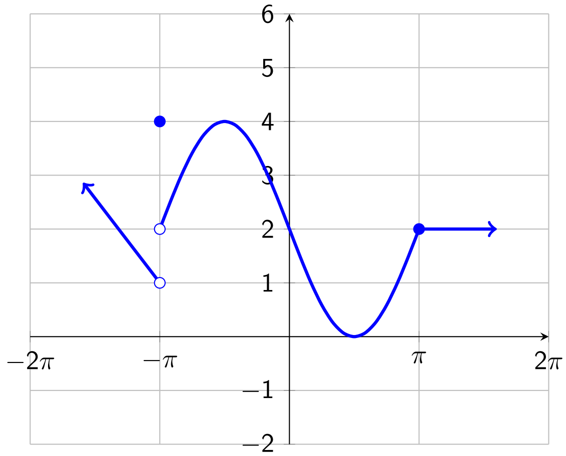
26.1 Answer Key

1. $b \approx 13.0$, $m\angle A \approx 44.8^\circ$, $m\angle C \approx 97.4^\circ$, Area ≈ 97.0 units^2
2. $m\angle B = 78.9^\circ$, $b \approx 15.0$, $c \approx 13.1$, Area ≈ 65.7 units^2
3. $m\angle A \approx 45.5^\circ$, $m\angle B \approx 92.6^\circ$, $m\angle C \approx 41.9^\circ$, Area ≈ 91.6 units^2
4. $m\angle B \approx 50.1^\circ$, $m\angle C \approx 74.9^\circ$, $a \approx 24.6$, Area ≈ 273.19 units^2
5. $m\angle B \approx 56.3^\circ$, $m\angle C \approx 33.7^\circ$, $a \approx 14.4$, Area $= 48$ units^2
6. $m\angle A \approx 48.2^\circ$, $m\angle C \approx 64.6^\circ$, $b \approx 19.3$, Area ≈ 135.9 units^2
7. $m\angle A \approx 137.8^\circ$, $m\angle B \approx 26.6^\circ$, $m\angle C \approx 15.6^\circ$, Area ≈ 80.5 units^2
8. $m\angle A \approx 96.4^\circ$, $m\angle B \approx 25.2^\circ$, $m\angle C \approx 58.4^\circ$, Area ≈ 35.8 units^2
9. $m\angle A \approx 30^\circ$, $m\angle B \approx 91^\circ$, $m\angle C \approx 59^\circ$, Area ≈ 42.0 units^2

Chapter 27

Numerical and Graphical Limits

Solve using the graph of $f(x)$ below.



1. $\lim_{x\to-\pi^-} f(x)$
2. $\lim_{x\to-\pi^+} f(x)$
3. $\lim_{x\to-\pi} f(x)$
4. $f(-\pi)$
5. $\lim_{x\to\pi^-} f(x)$
6. $\lim_{x\to\pi^+} f(x)$
7. $\lim_{x\to\pi} f(x)$
8. $f(\pi)$

Find each limit.

9. $\lim_{x\to 0} \dfrac{e^x-1}{x}$

10. $\lim_{x\to 1^+} \log(x-1)$

11. $\lim_{x\to 2} \left(\dfrac{3e^{1/(x-2)}}{e^{1/(x-2)}+1} - 1 \right)$

12. $\lim_{x\to 0} f(x)$, $f(x) = \begin{cases} -x^2-4x-5, & x \le 0 \\ x-5, & x > 0 \end{cases}$

13. $\lim_{x\to 4} g(x)$, $g(x) = \begin{cases} x+4, & x < -4 \\ \frac{x}{2}+3, & x \ge -4 \end{cases}$

14. $\lim_{x\to 0} h(x)$, $h(x) = \begin{cases} -x-4, & x < 0 \\ -2x-5, & x \ge 0 \end{cases}$

27.1 Answer Key

1. 1
2. 2
3. Does not exist
4. 4
5. 2
6. 2
7. 2
8. 2
9. $e \approx 2.71828$
10. Does not exist
11. Does not exist
12. -5
13. 1
14. Does not exist

Chapter 28

Limits and Algebra

Find each limit algebraically.

1. $\displaystyle\lim_{x\to -4}\left(\frac{x+4}{x^2+6x+8}\right)$

2. $\displaystyle\lim_{x\to -3}\left(-\frac{x^2+2x-3}{x+3}\right)$

3. $\displaystyle\lim_{x\to 3}\left(\frac{x-3}{x^2-8x+15}\right)$

4. $\displaystyle\lim_{x\to -2}\left(\frac{x^2-2x-8}{x+2}\right)$

5. $\displaystyle\lim_{x\to -1}\left(-\frac{x^2+5x+4}{x+1}\right)$

6. $\displaystyle\lim_{x\to -4}\left(\frac{x^2+7x+12}{x+4}\right)$

7. $\displaystyle\lim_{x\to 2}\left(\frac{x^2-2x}{x-2}\right)$

8. $\displaystyle\lim_{x\to -1}\left(-\frac{x^2-3x-4}{x+1}\right)$

9. $\displaystyle\lim_{x\to 0}\left(\frac{x}{\frac{1}{x-1}+1}\right)$

10. $\displaystyle\lim_{x\to 0}\left(\frac{x}{\frac{1}{x+2}-\frac{1}{2}}\right)$

11. $\displaystyle\lim_{x\to 3}\left(\frac{x}{\frac{1}{x-3}+\frac{1}{3}}\right)$

12. $\displaystyle\lim_{x\to 2}\left(\frac{x}{\frac{1}{x-2}+\frac{1}{2}}\right)$

13. $\displaystyle\lim_{x\to 1}\left(\frac{x}{\frac{1}{x-1}+1}\right)$

14. $\displaystyle\lim_{xt\varnothing -1}\left(\frac{x}{\frac{1}{x+1}-1}\right)$

15. $\displaystyle\lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x-1}\right)$

16. $\displaystyle\lim_{x\to 2}\left(\frac{\sqrt{x+7}-3}{x-2}\right)$

17. $\displaystyle\lim_{x\to 25}\left(\frac{x-25}{\sqrt{x}-5}\right)$

18. $\displaystyle\lim_{x\to 1}\left(\frac{x-1}{\sqrt{x+8}-3}\right)$

19. $\displaystyle\lim_{x\to 9}\left(\frac{x-9}{\sqrt{x}-3}\right)$

20. $\displaystyle\lim_{x\to 2}\left(\frac{\sqrt{x+14}-4}{x-1}\right)$

28.1 Answer Key

1. $-\frac{1}{2}$
2. 4
3. $-\frac{1}{2}$
4. -6
5. -3
6. -1
7. 2
8. 5
9. -1
10. -4
11. 0
12. 0
13. 0
14. 0
15. $\frac{1}{2}$
16. $\frac{1}{6}$
17. 10
18. 6
19. 6
20. $\frac{1}{8}$

Chapter 29

Derivatives

Use the definition of the derivative to find the derivative of each function with respect to x.

1. $f(x) = x^2 + 2x + 4$
2. $f(x) = -5x + 5$
3. $f(x) = 2x^2 - 5x - 2$
4. $f(x) = 5x + 2$
5. $f(x) = x^3 - x^2$
6. $f(x) = 5x^2 + 5$
7. $f(x) = 2x + 3$
8. $f(x) = x^2 - 2$
9. $f(x) = -5x^3 + 4$
10. $f(x) = -4x^2 + x - 5$
11. $f(x) = 2x^2 - 3x + 5$
12. $f(x) = 5x^2 - 7x + 4$
13. $g(x) = \frac{9}{x}$
14. $h(x) = 8\sqrt{x}$

29.1 Answer Key

1. $2x + 2$
2. -5
3. $4x - 5$
4. 5
5. $3x^2 - 2x$
6. $10x$
7. 2
8. $2x$
9. $-15x^2$
10. $-8x + 1$
11. $4x - 3$
12. $10x - 7$
13. $-\frac{9}{x^2}$
14. $\frac{4}{\sqrt{x}}$

Chapter 30

Derivative Shortcuts

Using the shortcut rules, find the derivative of each.

1. $f(x) = 3x^2 + 6x - 2$
2. $g(x) = -2\sqrt{x} + 5x^{-7}$
3. $h(x) = \frac{x^2-6x+1}{x^3+4x^2}$

30.1 Answer Key

1. $6x + 6$
2. $-x^{-1/2} - 35x^{-8}$
3. $\frac{(2x-6)(x^3+4x^2)-(3x^2+8x)(x^2-6x+1)}{(x^3+4x^2)^2}$

Chapter 31

Area Under a Curve

Given $f(x) = 3x^2 + 2x$, find the area under the curve between $x = 3$ and $x = 8$ using left endpoints of the number of rectangles listed.

1. 5 rectangles of equal width
2. 10 rectangles of equal width
3. What is the area as the number of rectangles approaches ∞?

31.1 Answer Key

1. 455
2. 496.875
3. $\int_3^8 \left(3x^2 + 2x\right) \mathrm{d}x = 540$

Appendix A

Factoring

Factor each of the following completely.

1. $x^2 + 2x - 15$
2. $x^2 - 8x + 12$
3. $x^2 + 15x + 56$
4. $5x^2 + 19x - 4$
5. $4x^2 - 5x - 6$
6. $9x^2 - 400$
7. $5x^2 - 7x - 6$
8. $9x^2 - 54x + 45$
9. $3x^3 + 12x^2 + 9x$
10. $9y^2 - 16$
11. $4x^2 - 28x + 49$
12. $14x^2 + 11xy - 15y^2$
13. $6x^2 - 48x - 120$
14. $9x^4 - 54x^3 + 45x^2$
15. $16y^2 - 40y + 25$
16. $30x^2 + xy - y^2$
17. $8w^2 + 33w + 4$
18. $3p^2 + 22p - 16$
19. $18x^2 - 27x + 4$
20. $14a^2 + 15a - 9$
21. $4x^2 - 4x - 24$
22. $18t^2 - 9t - 5$
23. $6a^2 + 23a + 21$
24. $25x^2 - 1$

A.1 Answer Key

1. $(x+5)(x-3)$
2. $(x-6)(x-2)$
3. $(x+7)(x+8)$
4. $(5x-1)(x+4)$
5. $(4x+3)(x-2)$
6. $(3x+20)(3x-20)$
7. $(5x+3)(x-2)$
8. $9(x-5)(x-1)$
9. $3x(x+3)(x+1)$
10. $(3y+4)(3y-4)$
11. $(2x-7)^2$
12. $(7x-5y)(2x+3y)$
13. $6(x-10)(x+2)$
14. $9x^2(x-1)(x-5)$
15. $(4y-5)^2$
16. $(6x-y)(5x+y)$
17. $(8w+1)(w+4)$
18. $(3p-2)(p+8)$
19. $(6x-1)(3x-4)$
20. $(7a-3)(2a+3)$
21. $4(x-3)(x+2)$
22. $(6t-5)(3t+1)$
23. $(2a+3)(3a+7)$
24. $(5x+1)(5x-1)$

Appendix B

Complex Fractions

Simplify each as much as possible.

1. $\frac{5+\frac{3}{x}}{x-\frac{1}{2}}$
2. $\frac{\frac{1}{x}+\frac{2}{x^2}}{x+\frac{8}{x^2}}$
3. $\frac{3}{2-\frac{x}{x-1}}$
4. $\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$
5. $\frac{\frac{4}{x}-\frac{x}{x-2}}{\frac{1}{x}+\frac{3}{x-2}}$
6. $\frac{\frac{3}{x+1}-4}{\frac{2}{x+1}}$
7. $\frac{\frac{5}{x}+\frac{3}{x-2}}{\frac{7}{x^2-2x}}$
8. $\frac{\frac{1}{x}-\frac{1}{7}}{x-7}$
9. $\frac{\frac{1}{x}+\frac{1}{x+1}}{5}$
10. $\frac{\frac{5}{x}-5x}{x-1}$
11. $\frac{\frac{1}{2+x}-\frac{1}{2}}{x}$
12. $\frac{\frac{3}{x-4}+\frac{2x}{x+1}}{4x}$
13. $\frac{\frac{1}{x-a}+\frac{1}{a}}{x}$
14. $\dfrac{\dfrac{1}{x-1}-\dfrac{1}{x-3}}{\dfrac{2}{x-1}+\dfrac{3}{x+1}}$
15. $\dfrac{\dfrac{2}{x^2-4}+\dfrac{1}{x-2}}{\dfrac{4}{x+2}}$

B.1 Answer Key

1. $\frac{2(5x+3)}{x(2x-1)}$
2. $\frac{1}{x^2-2x+4}$
3. $\frac{3(x-1)}{x-2}$
4. $\frac{x+3}{2+7x}$
5. $\frac{-1(x^2-4x+8)}{2(2x-1)}$
6. $\frac{-4x-1}{2}$
7. $\frac{8x-10}{7}$
8. $-\frac{1}{7x}$
9. $\frac{2x+1}{5x(x+1)}$
10. $\frac{-5x-5}{x}$
11. $\frac{-1}{2x+4}$
12. $\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$
13. $\frac{1}{a(x-a)}$
14. $\frac{-2x-2}{5x^2-16x+3}$
15. $\frac{x+4}{4x-8}$

Appendix C

Vertex Form of Quadratic Functions

Quadratic functions written are often written in the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers.

However, they can also be written in an equivalent expression, *vertex form*.

$$f(x) = a(x-h)^2 + k$$

where the coordinates of the vertex are (h, k) and a is the same as it is in $f(x) = ax^2 + bx + c$.

One way to obtain the coordinates of the vertex is through graphing.

However, in the event a graphing utility is not available, the values of h and k can be obtained using the values from $f(x) = ax^2 + bx + c$.

- $h = -\frac{b}{2a}$.
- k can be found by plugging the value of h into the function: $f(x) = ax^2 + bx + c$ (*i.e.* evaluate $f(h)$).

Example 1. Write each of the following in vertex form.

(a) $f(x) = x^2 - 6x + 5$ (b) $g(x) = -2x^2 + 8x - 1$ (c) $h(x) = \frac{3}{7}x^2 + \frac{2}{3}x$

Solution 1.
(a) In most cases, you can graph the function on a graphing utility and use built-in methods to find the coordinates of the vertex.

Solving algebraically, however, we have $a = 1$, $b = -6$, and

$$\begin{aligned} h &= -\tfrac{b}{2a} \\ &= -\tfrac{-6}{2(1)} \\ &= 3 \end{aligned}$$

To obtain the value of k, evaluate $f(x) = x^2 - 6x + 5$ at $x = 3$.

$$\begin{aligned} f(3) &= 3^2 - 6(3) + 5 \\ &= -4 \end{aligned}$$

With $a = 1$ and the coordinates of the vertex at $(3, -4)$,

$$f(x) = x^2 - 6x + 5 = \boxed{(x-3)^2 - 4}$$

Note: You can check that form is correct by multiplying $(x-3)^2$ out and then subtracting 4. It will give you $x^2 - 6x + 5$.

(b) We have $a = -2$, $b = 8$, and

$$\begin{aligned} h &= -\tfrac{b}{2a} \\ &= -\tfrac{8}{2(-2)} \\ &= 2 \end{aligned}$$

We obtain the value of k by evaluating $g(x) = -2x^2 + 8x - 1$ at $x = 2$.

$$\begin{aligned} g(2) &= -2(2)^2 + 8(2) - 1 \\ &= 7 \end{aligned}$$

With $a = -2$ and the coordinates of the vertex at $(2, 7)$,

$$g(x) = -2x^2 + 8x - 1 = \boxed{-2(x-2)^2 + 7}$$

(c) We have $a = \frac{3}{7}$, $b = \frac{2}{3}$, and

$$\begin{aligned} h &= -\tfrac{b}{2a} \\ &= -\tfrac{2/3}{2(3/7)} \\ &= -\tfrac{7}{9} \end{aligned}$$

We obtain the value of k by evaluating $h(x) = \frac{3}{7}x^2 + \frac{2}{3}x$ at $x = -\frac{7}{9}$

$$\begin{aligned} h\left(-\tfrac{7}{9}\right) &= \tfrac{3}{7}\left(-\tfrac{7}{9}\right)^2 + \tfrac{2}{3}\left(-\tfrac{7}{9}\right) \\ &= -\tfrac{7}{27} \end{aligned}$$

With $a = \frac{3}{7}$ and the coordinates of the vertex at $\left(-\frac{7}{9}, -\frac{7}{27}\right)$,

$$\begin{aligned} h(x) &= \tfrac{3}{7}x^2 + \tfrac{2}{3}x \\ &= \tfrac{3}{7}\left(x - \left(-\tfrac{7}{9}\right)\right)^2 + \left(-\tfrac{7}{27}\right) \\ &= \boxed{\tfrac{3}{7}\left(x + \tfrac{7}{9}\right)^2 - \tfrac{7}{27}} \end{aligned}$$