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Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2. $\{x | x \le 1\}$
- 3. _3
- 4. $\{x | x \neq 4, 11\}$
- 5. 2.4 7.
- 6. $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7. $\{x | x \ge 2\}$
- 8. $\{x|x<-8\}$
- 9. $\{x | x \neq 3\}$
- 10. $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.



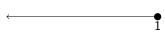


BASIC SET THEORY AND INTERVAL NOTATION KEY

1.
$$\{x | -5 < x \le 8\}$$



2. $(-\infty, 1]$



3.
$$[-3, \infty)$$
 $\{x | x \ge -3\}$

4.
$$(-\infty, 4) \cup (4, 11) \cup (11, \infty)$$

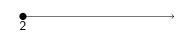


5. [2.4, 7.7)
$$\{x | 2.4 \le x < 7.7\}$$

6.
$$\{x|x > 9\}$$



7. $[2, \infty)$



8.
$$(-\infty, -8)$$



9.
$$(-\infty,3)\cup(3,\infty)$$

$$\longleftrightarrow$$
 3

10.
$$(\infty, -2) \cup (-2, 5) \cup (5, \infty)$$

$$\overset{\longleftarrow}{\underset{-2}{\smile}}\overset{\bigcirc}{\underset{5}{\smile}}$$

11.
$$(-\infty, -8]$$
 $\{x | x \le -8\}$

12.
$$(-\infty, 7) \cup (7, 12) \cup (12, \infty)$$
 $\{x | x \neq 7, 12\}$

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

- 1. f(5)
- 2. f(-2)
- 3. f(0)
- 4. g(1)
- 5. g(-5)
- 6. g(1/4)

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1.
$$f(x) = -8x^2 - 7x + 1$$

2.
$$g(x) = \sqrt{5x + 12} - 2$$

3.
$$h(x) = \frac{x+2}{9x-7}$$

4.
$$f(x) = -5x + 4$$

5.
$$f(x) = x^2 + 2$$

6.
$$f(x) = \frac{2x+1}{3x-5}$$

7.
$$f(x) = \sqrt{3x - 12}$$

8.
$$f(x) = \frac{x}{x^2-16}$$

9.
$$f(x) = \frac{x+4}{x^3-4x}$$

10.
$$f(x) = \frac{x}{\sqrt{x-4}}$$

11.
$$f(x) = \frac{x^2+1}{2x^2+8}$$

12.
$$f(x) = -\frac{x+7}{x^2-5x-6}$$

13.
$$g(x) = \sqrt{2x+3}$$

14.
$$h(x) = \sqrt[3]{2x+3}$$

15.
$$f(x) = -\frac{7x-10}{x^2+3x+2}$$

16.
$$g(x) = \sqrt{-9x + 8}$$

17.
$$h(x) = -\sqrt[3]{4x+1}$$

18.
$$f(x) = \sqrt[3]{8x+1}$$

19.
$$g(x) = \frac{x^2-1}{\sqrt{x+3}}$$

20.
$$h(x) = \frac{3}{9 + \frac{4}{x+7}}$$

21.
$$f(x) = \frac{x+1}{\sqrt{10x+8}}$$

22.
$$g(x) = \frac{5}{1 + \frac{3}{x+2}}$$

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. f(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4 \\ \sqrt{2x + 7} & -4 < x < 0 \\ |-x - 1| & x \ge 0 \end{cases}$$

- 6. f(3)
- 7. f(-2)
- 8. *f*(0)
- 9. f(-5)

Evaluating Functions

- 1. -55
- 2. -20
- 3. 0
- 4. -4
- 5. -5.2
- 6. -1

Domain of Functions

- 1. $(-\infty, \infty)$
- 2. $\left[\frac{-12}{5}, \infty\right)$
- 3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
- 4. $(-\infty, \infty)$
- 5. $(-\infty, \infty)$
- 6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
- 7. $[4,\infty)$
- 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
- 9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$
- 10. $(4, \infty)$
- 11. $(-\infty, \infty)$

- 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
- 13. $\left[-\frac{3}{2}, \infty\right)$
- 14. $(-\infty, \infty)$
- 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
- 16. $\left(-\infty, \frac{8}{9}\right]$
- 17. $(-\infty, \infty)$
- 18. $(-\infty, \infty)$
- 19. $(-3, \infty)$
- 20. $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup \left(-7, \infty\right)$
- 21. $\left(-\frac{4}{5},\infty\right)$
- 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$

Piecewise Functions

- 1. $\sqrt{15} \approx 3.873$
- 2. 7
- 3. 6.6
- 4. 6.4
- 5. 7.1
- 6. 4
- 7. $\sqrt{3} \approx 1.732$
- 8. 1
- 9. 18

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

5.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

6.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = x^3 + 2x^2 - 4x - 8$$

4.
$$g(x) = x^4 - 2x^2 + 1$$

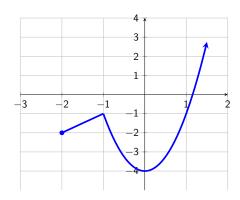
5.
$$h(x) = \sqrt{x+1} - 2$$

6.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

7.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

3.3 Miscellaneous

Use the graph of y = f(x) below to answer questions 1–10. Write your answers using interval notation.



- 1. Domain of f
- 2. Range of f
- 3. Relative Minimum
- 4. Relative Maximum
- 5. f(1)

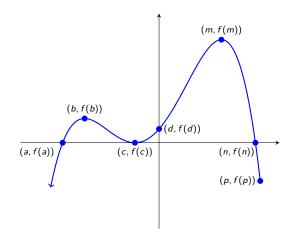
- 6. f(0)
- 7. Increasing Interval(s)
- 8. Decreasing Interval(s)
- 9. Absolute Maximum
- 10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

- 11. f(7)
- 12. f(-2)
- 13. Rel. Max
- 14. Rel. Min

- 15. Global Max
- 16. Global Min
- 17. Increasing Interval(s)
- 18. Decreasing Interval(s)

Use the graph of f(x) to answer each.



- 19. Relative maxima of f(x)
- 20. Relative minima of f(x)
- 21. Absolute maxima of f(x)
- 22. Absolute minima of f(x)

- 23. Intervals where f is increasing
- 24. Intervals where f is decreasing
- 25. Zeros of *f*

Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. Rel Max (-1.366, 10.848) and (1, 6); Rel Min (0.366, 5.652)
- 4. Rel Max (-1.716, 11.598); Rel Min (1.132, -3.929)
- 5. Rel Max: (1.095, 12.096); Rel Min (-0.761, -0.680)
- 6. Rel Max: (1.366, 0.348); Rel Min: (-0.366, -4.848) and (2, 0)
- 7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

- 1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
- 3. Inc: $(-\infty, -2) \cup (\frac{2}{3}, \infty)$ Dec: $(-2, \frac{2}{3})$
- 4. Inc; $(-1,0) \cup (1,\infty)$ Dec: $(-\infty,-1) \cup (0,1)$
- 5. Inc: $(-1, \infty)$ No intervals where it is decreasing
- 6. Inc: (-0.761, 1.095); Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
- 7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Miscellaneous

- 1. $[-2, \infty)$
- 2. $[-4, \infty)$
- 3. (0, -4)
- 4. (-1, -1)
- 5. -1
- 6. -4)
- 7. $(-2, -1) \cup (0, \infty)$
- 8. (-1,0)
- 9. (0, -4)
- 10. None
- 11. -426
- 12. 51
- 13. (1.408, 0.272)

- 14. (0.592, -0.272)
- 15. None
- 16. None
- 17. (0.592, 1.408)
- 18. $(-\infty, 0.592) \cup (1.408, \infty)$
- 19. (b, f(b)) and (m, f(m))
- 20. (c, f(c))
- 21. (m, f(m))
- 22. None
- 23. $(-\infty, b) \cup (c, m)$
- 24. $(b, c) \cup (m, p)$
- 25. x = a, x = c, x = n

Linear Functions and Slope

4.1 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

- 6. [-2, -1]
- 7. [5, 6]
- 8. [0, 1]
- 9. [5, 5.001]
- 10. [5, 5.0001]
- 11. [5, 5.00001]
- 12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

- 13. [7, 7.001]
- 14. [7, 7.0001]
- 15. [7, 7.00001]
- 16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

- 17. [1, 1.0001]
- 18. [1, 1.00001]
- 19. [1, 1.000001]
- 20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Average Rate of Change

- 1. 2.1
- 2. 2.01
- 3. 2.001
- 4. 2.0001
- 5. 2
- 6. 25
- 7. -59
- 8. 1
- 9. -53.006
- 10. -53.0006
- 11. -53.00006
- 12. -53
- 13. -42.003
- 14. -42.0003
- 15. -42.00003
- 16. -42
- 17. -0.499988
- 18. -0.4999988
- 19. -0.49999988
- 20. -0.5

Function Transformations

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
 - (2) Shift left 1 unit
 - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
 - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
 - (2) Vertical compression by 4
 - (3) Move right 7 units

Write the function g(x) that is a result of the following ordered sequence of transformations to f(x) = |x|.

- 4. (1) Reflect across x-axis
 - (2) Shift right 3 units
 - (3) Horizontal stretch by factor of 5
- 5. (1) Shift down 2 units
 - (2) Reflect across y-axis
 - (3) Shift up 1 unit
- 6. (1) Horizontal compression by factor of 7
 - (2) Vertical compression by factor of 4
 - (3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function g(x) after the following ordered sequence of transformations.

- 7. (1) Shift up 2 units
 - (2) Horizontal stretch by 5
 - (3) Shift left 3 units
- 8. (1) Vertical compression by factor of 3
 - (2) Reflect across y-axis
 - (3) Horizontal compression by 5
- 9. (1) Shift right 8 units
 - (2) Reflect across x-axis
 - (3) Horizontal compression by factor of 4

Write the final equation of g(x) if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

- 10. (1) Shift right 2 units
 - (2) Horizontal stretch by factor 3
 - (3) Shift down 2 units
 - (4) Reflect across x-axis
- 11. (1) Horizontal stretch by factor 3
 - (2) Shift left 1 unit
 - (3) Shift up 2 units
 - (4) Reflect across y-axis
- 12. (1) Vertical stretch by factor 5
 - (2) Horizontal stretch by factor 2
 - (3) Shift up 3 units
 - (4) Reflect across x-axis

FUNCTION TRANSFORMATIONS KEY

1.
$$g(x) = 3f(-x+1)$$

2.
$$g(x) = f(2x) + 1$$

3.
$$g(x) = -\frac{1}{4}f(x-7)$$

4.
$$g(x) = -\left|\frac{1}{5}x - 3\right|$$

5.
$$g(x) = |-x| - 1$$

6.
$$g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$$

7.
$$g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$$

8.
$$g(x) = \frac{1}{3}\sqrt{-5x}$$

9.
$$g(x) = -\sqrt{4x - 8}$$

10.
$$g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$$

11.
$$g(x) = \sqrt{3(-x+1)} + 2 = \sqrt{-3x+3} + 2$$

12.
$$g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$$

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given f(x) = x + 5, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

- 1. (g f)(x)
- 2. (fh)(14)
- 3. (f+g)(x)

Find each of the following given the table below.

x	ı	-3	ı				l	l	1
f(x)	-3	0	-1	3	1	2	4	-4	-2
g(x)	3	-1	0	1	4	-2	-4	2	-3

4.
$$(f+g)(-2)$$
 5. $(f-g)(0)$ 6. $(fg)(1)$

5.
$$(f-g)(0)$$

6.
$$(fg)(1)$$

7.
$$\left(\frac{f}{g}\right)$$
 (3)

8.
$$(f+f)(-4)$$

Operations with Functions: Domain

Given $f(x) = \sqrt{2x+7}$ and g(x) = 3x+3, find the domain of each.

- 1. (f + g)(x)
- 2. $\left(\frac{f}{g}\right)(x)$
- 3. $\left(\frac{g}{f}\right)(x)$

Difference Quotient 6.3

Write the difference quotient for each.

1.
$$f(x) = 2x - 7$$

2.
$$g(x) = x^2 + 4x$$

3.
$$h(x) = -1$$

4.
$$f(x) = \frac{3}{x+2}$$

5.
$$g(x) = \sqrt{3x}$$

6.
$$f(x) = x^2 - 2x + 5$$

$$7. g(x) = \frac{5}{x}$$

8.
$$f(x) = -2x^2 + 3x - 5$$

9.
$$g(x) = \frac{6}{2x+3}$$

10.
$$h(x) = \sqrt{7x+5}$$

Adding, Subtracting, Multiplying, and Dividing Functions

- 1. $x^2 x 6$
- 2. 38
- 3. $x^2 + x + 4$
- 4. -1
- 5. -3
- 6. -4
- 7. -2
- 8. -6

Operations with Functions: Domain

- 1. $\left[-\frac{7}{2},\infty\right)$
- 2. $\left[-\frac{7}{2}, -1\right) \cup \left(-1, \infty\right)$
- 3. $\left(-\frac{7}{2}, \infty\right)$

Difference Quotient

- 1. 2
- 2. 2x + h + 4
- 3. 0
- 4. $\frac{-3}{(x+2)(x+h+2)}$
- $5. \ \frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$
- 6. 2x + h 2
- 7. $\frac{-5}{x(x+h)}$
- 8. -4x 2h + 3
- 9. $\frac{-12}{(2x+3)(2x+2h+3)}$
- 10. $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$

Polynomials and Their Graphs

Determine the end behavior of each.

1.
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

2.
$$g(x) = 4x^2 - 16x^6 + 3x$$

3.
$$h(x) = 1 + x^{11} - 4x^8$$

4.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

5.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

POLYNOMIALS AND THEIR GRAPHS

1.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

2.
$$\lim_{x \to -\infty} g(x) = -\infty$$
, $\lim_{x \to \infty} g(x) = \infty$

3.
$$\lim_{x \to -\infty} h(x) = -\infty$$
 $\lim_{x \to \infty} h(x) = \infty$

4.
$$\lim_{x \to -\infty} f(x) = -\infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

5.
$$\lim_{x \to -\infty} g(x) = -\infty$$
 $\lim_{x \to \infty} g(x) = \infty$

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

- 1. $(28x^3 26x^2 + 41x 15) \div (7x 3)$
- 2. $(44y^2 + 12y^3 + 61y 37) \div (3y + 5)$
- 3. $(4x^3 3x^2 + x + 1) \div (x + 2)$
- 4. $(5x^4 x^2 + x 2) \div (x^2 + 2)$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

- 1. $(2x^{53} 9x^{44} + 13x^8) \div (x 1)$
- 2. $(x^{71} + 15x^{58} 3x^{14} + 2) \div (x+1)$

Dividing Polynomials

1.
$$4x^2 - 2x + 5$$

2.
$$4y^2 + 8y + 7 - \frac{72}{3y + 5}$$

3.
$$4x^2 - 11x + 23 - \frac{45}{x+2}$$

4.
$$5x^2 - 11 + \frac{x + 20}{x^2 + 2}$$

Remainder and Factor Theorems

- 1. 6
- 2. 13

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1.
$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

2.
$$g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$3. \ f(x) = \frac{3x}{x+4}$$

4.
$$g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

5.
$$h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain:
$$x \neq -\frac{1}{2}$$
, 7; V.A.: $x = 7$; Hole @ $\left(-\frac{1}{2}, -\frac{7}{13}\right)$; H.A.: $y = 1$

2. Domain:
$$x \neq -3$$
, 4; V.A.: $x = -3x = 4$; Obl. Asymp: $y = 3x + 10$

3. Domain:
$$x \neq -4$$
; V.A.: $x = -4$; H.A.: $y = 3$

4. Domain:
$$x \neq 1$$
; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$

5. Domain:
$$x \neq 0, 1$$
; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1.
$$6x^3 - 4x^2 - 10x \ge 0$$

2.
$$x^4 < 9x^2$$

10.2 Rational Inequalities

Solve each. Write your answers using interval notation.

1.
$$\frac{3x-4}{x+1} < 0$$

$$2. \ \frac{x^2 + 3x + 2}{x - 7} \le 0$$

$$3. \ \frac{x^2 - 4x + 4}{x^2 - 1} \ge 0$$

4.
$$\frac{x+2}{x-4} \le 1$$

Polynomial Inequalities

- 1. $[-1,0] \cup \left[\frac{5}{3},\infty\right)$
- 2. $(-3,0) \cup (0,3)$

Rational Inequalities

- 1. $\left(-1, \frac{4}{3}\right)$
- 2. $(-\infty, -2] \cup [-1, 7)$
- 3. $(-\infty, -1) \cup (1, \infty)$
- 4. $(-\infty, 4)$

Function Compositions

Given f(x) = x - 5, $g(x) = 4 + \sqrt{2x + 1}$, and $h(x) = \frac{3}{x + 7}$, simplify each and state the domain.

- 1. $(f \circ g)(x)$
- 2. $(g \circ f)(x)$
- 3. h(h(x))

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-3	0	-1	3	1	2	4	-4	-2
$\mathbf{g}(\mathbf{x})$	3	-1	0	1	4	-2	-4	2	-3

4.
$$(f \circ g)(-1)$$
 5. $(g \circ g)(0)$

5.
$$(g \circ g)(0)$$

6.
$$(f \circ f)(2)$$

7.
$$(g \circ g)(-3)$$
 8. $f(g(0))$

8.
$$f(g(0))$$

FUNCTION COMPOSITIONS KEY

- 1. $-1 + \sqrt{2x+1}$ Domain: $\left[-\frac{1}{2}, \infty\right)$
- 2. $4 + \sqrt{2x 9}$ Domain: $\left[\frac{9}{2}, \infty\right)$
- 3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup \left(-7, \infty\right)$
- 4. 2
- 5. -3
- 6. -2
- 7. 1
- 8. -2

Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1.
$$f(x) = \sqrt{-2x+3} + 1$$

2.
$$g(x) = (x+4)^2 - 1$$
, $x \le -4$

3.
$$h(x) = \frac{9x}{4x - 1}$$

4.
$$f(x) = \sqrt{x} - 3$$

$$5. g(x) = \frac{1}{1-x}$$

6.
$$h(x) = x^2 + 6x + 4, x \le -3$$

INVERSE FUNCTIONS KEY

1.
$$f^{-1}(x) = -\frac{1}{2}((x-1)^2 - 3)$$

	Domain	Range
f(x)	$(-\infty, 1.5]$	$[1,\infty)$
$f^{-1}(x)$	$[1,\infty)$	$(-\infty, 1.5]$

2.
$$g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
g(x)	$(-\infty, -4]$	$[-1,\infty)$
$g^{-1}(x)$	$[-1,\infty)$	$(-\infty, -4]$

3.
$$h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
h(x)	$(-\infty,1/4)\cup(1/4,\infty)$	$(\infty,9/4)\cup(9/4,\infty)$
$h^{-1}(x)$	$(\infty,9/4)\cup(9/4,\infty)$	$(-\infty,1/4)\cup(1/4,\infty)$

4.
$$f^{-1}(x) = (x+3)^2$$

	Dom	Ran
f(x)	$[0,\infty)$	$[-3,\infty)$
$f^{-1}(x)$	$[-3,\infty)$	$[0,\infty)$

5.
$$g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
g(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$g^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

6.
$$h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
h(x)	$(-\infty, -3]$	$[-5,\infty)$
$h^{-1}(x)$	$[-5,\infty)$	$(-\infty, -3]$

Exponential Functions

13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

- 1. $f(x) = 3 + e^{2x}$
- 2. $h(x) = 5^{-x}$

EXPONENTIAL FUNCTIONS

1.
$$\lim_{x \to -\infty} f(x) = 3$$
 $\lim_{x \to \infty} f(x) = \infty$

2.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = 0$

Appendix A

Factoring

Factor each of the following completely.

1.
$$x^2 + 2x - 15$$

2.
$$x^2 - 8x + 12$$

3.
$$x^2 + 15x + 56$$

4.
$$5x^2 + 19x - 4$$

5.
$$4x^2 - 5x - 6$$

6.
$$9x^2 - 400$$

7.
$$5x^2 - 7x - 6$$

8.
$$9x^2 - 54x + 45$$

9.
$$3x^3 + 12x^2 + 9x$$

10.
$$9y^2 - 16$$

11.
$$4x^2 - 28x + 49$$

12.
$$14x^2 + 11xy - 15y^2$$
 18. $3p^2 + 22p - 16$

13.
$$6x^2 - 48x - 120$$

14.
$$9x^4 - 54x^3 + 45x^2$$

15.
$$16y^2 - 40y + 25$$

16.
$$30x^2 + xy - y^2$$

17.
$$8w^2 + 33w + 4$$

18.
$$3p^2 + 22p - 16$$

19.
$$18x^2 - 27x + 4$$

20.
$$14a^2 + 15a - 9$$

21.
$$4x^2 - 4x - 24$$

22.
$$18t^2 - 9t - 5$$

23.
$$6a^2 + 23a + 21$$

24.
$$25x^2 - 1$$

FACTORING KEY

1.
$$(x+5)(x-3)$$

2.
$$(x-6)(x-2)$$

3.
$$(x+7)(x+8)$$

4.
$$(5x-1)(x+4)$$

5.
$$(4x+3)(x-2)$$

6.
$$(3x+20)(3x-20)$$

7.
$$(5x+3)(x-2)$$

8.
$$9(x-5)(x-1)$$

9.
$$3x(x+3)(x+1)$$

10.
$$(3y+4)(3y-4)$$

11.
$$(2x-7)^2$$

12.
$$(7x - 5y)(2x + 3y)$$

13.
$$6(x-10)(x+2)$$

14.
$$9x^2(x-1)(x-5)$$

15.
$$(4y-5)^2$$

16.
$$(6x - y)(5x + y)$$

17.
$$(8w+1)(w+4)$$

18.
$$(3p-2)(p+8)$$

19.
$$(6x-1)(3x-4)$$

20.
$$(7a-3)(2a+3)$$

21.
$$4(x-3)(x+2)$$

22.
$$(6t-5)(3t+1)$$

23.
$$(2a+3)(3a+7)$$

24.
$$(5x+1)(5x-1)$$

Appendix B

Complex Fractions

Simplify each as much as possible.

1.
$$\frac{5 + \frac{3}{x}}{x - \frac{1}{2}}$$

$$5. \ \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

9.
$$\frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$2. \ \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

6.
$$\frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$10. \ \frac{\frac{5}{x} - 5x}{x - 1}$$

11. $\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

3.
$$\frac{3}{2 - \frac{x}{x-1}}$$

7.
$$\frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2 - 2x}}$$

12.
$$\frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

4.
$$\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$$

$$8. \ \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

13.
$$\frac{\frac{1}{x-a} + \frac{1}{a}}{X}$$

COMPLEX FRACTIONS KEY

1.
$$\frac{2(5x+3)}{x(2x-1)}$$

5.
$$\frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$10. \ \frac{-5x-5}{x}$$

$$2. \ \frac{1}{x^2 - 2x + 4}$$

6.
$$\frac{-4x-1}{2}$$

7. $\frac{8x-10}{7}$

11.
$$\frac{-1}{2x+4}$$

$$3. \ \frac{3(x-1)}{x-2}$$

8.
$$-\frac{1}{7x}$$

12.
$$\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$4. \ \frac{x+3}{2+7x}$$

$$9. \ \frac{2x+1}{5x(x+1)}$$

$$13. \ \frac{1}{a(x-a)}$$