

Honors PreCalculus

Calculus



Algebra

Review

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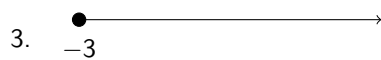
Chapter 1

Basic Set Theory and Interval Notation

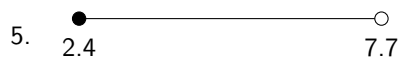
You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

1. $(-5, 8]$

2. $\{x|x \leq 1\}$



4. $\{x|x \neq 4, 11\}$



6. $(9, \infty)$

Write each using interval notation and graph on a number line.

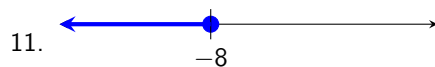
7. $\{x|x \geq 2\}$

8. $\{x|x < -8\}$

9. $\{x|x \neq 3\}$

10. $\{x|x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.

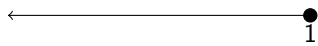


BASIC SET THEORY AND INTERVAL NOTATION KEY

1. $\{x | -5 < x \leq 8\}$

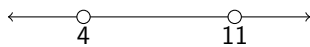


2. $(-\infty, 1]$



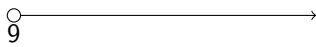
3. $[-3, \infty)$ $\{x | x \geq -3\}$

4. $(-\infty, 4) \cup (4, 11) \cup (11, \infty)$



5. $[2.4, 7.7)$ $\{x | 2.4 \leq x < 7.7\}$

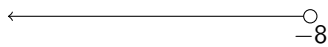
6. $\{x | x > 9\}$



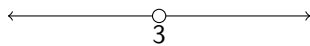
7. $[2, \infty)$



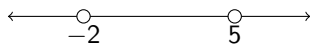
8. $(-\infty, -8)$



9. $(-\infty, 3) \cup (3, \infty)$



10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$



11. $(-\infty, -8]$ $\{x | x \leq -8\}$

12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x | x \neq 7, 12\}$

Chapter 2

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

1. $f(5)$
2. $f(-2)$
3. $f(0)$
4. $g(1)$
5. $g(-5)$
6. $g(1/4)$

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

- | | |
|------------------------------------|--|
| 1. $f(x) = -8x^2 - 7x + 1$ | 13. $g(x) = \sqrt{2x + 3}$ |
| 2. $g(x) = \sqrt{5x + 12} - 2$ | 14. $h(x) = \sqrt[3]{2x + 3}$ |
| 3. $h(x) = \frac{x+2}{9x-7}$ | 15. $f(x) = -\frac{7x-10}{x^2+3x+2}$ |
| 4. $f(x) = -5x + 4$ | 16. $g(x) = \sqrt{-9x + 8}$ |
| 5. $f(x) = x^2 + 2$ | 17. $h(x) = -\sqrt[3]{4x + 1}$ |
| 6. $f(x) = \frac{2x+1}{3x-5}$ | 18. $f(x) = \sqrt[3]{8x + 1}$ |
| 7. $f(x) = \sqrt{3x - 12}$ | 19. $g(x) = \frac{x^2-1}{\sqrt{x+3}}$ |
| 8. $f(x) = \frac{x}{x^2-16}$ | 20. $h(x) = \frac{3}{9+\frac{4}{x+7}}$ |
| 9. $f(x) = \frac{x+4}{x^3-4x}$ | 21. $f(x) = \frac{x+1}{\sqrt{10x+8}}$ |
| 10. $f(x) = \frac{x}{\sqrt{x-4}}$ | 22. $g(x) = \frac{5}{1+\frac{3}{x+2}}$ |
| 11. $f(x) = \frac{x^2+1}{2x^2+8}$ | 23. $i(x) = \frac{7}{3-\frac{4}{x+1}}$ |
| 12. $f(x) = -\frac{x+7}{x^2-5x-6}$ | 24. $n(x) = \frac{7x+14}{\sqrt{2x-1}}$ |

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3 \\ 0.2x + 7 & \text{if } -3 \leq x < 2 \\ \sqrt{5x} & \text{if } x \geq 2 \end{cases}$$

1. $f(3)$
2. $f(0)$
3. $f(-2)$
4. $f(-3)$
5. $f(0.5)$

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \leq -4 \\ \sqrt{2x + 7} & -4 < x < 0 \\ |-x - 1| & x \geq 0 \end{cases}$$

6. $f(3)$
7. $f(-2)$
8. $f(0)$
9. $f(-5)$

Evaluating Functions

1. -55
2. -20
3. 0
4. -4
5. -5.2
6. -1

Domain of Functions

- | | |
|--|---|
| 1. $(-\infty, \infty)$ | 13. $[-\frac{3}{2}, \infty)$ |
| 2. $[\frac{-12}{5}, \infty)$ | 14. $(-\infty, \infty)$ |
| 3. $(-\infty, \frac{7}{9}) \cup (\frac{7}{9}, \infty)$ | 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$ |
| 4. $(-\infty, \infty)$ | 16. $(-\infty, \frac{8}{9}]$ |
| 5. $(-\infty, \infty)$ | 17. $(-\infty, \infty)$ |
| 6. $(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$ | 18. $(-\infty, \infty)$ |
| 7. $[4, \infty)$ | 19. $(-3, \infty)$ |
| 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$ | 20. $(-\infty, -\frac{67}{9}) \cup (-\frac{67}{9}, -7) \cup (-7, \infty)$ |
| 9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$ | 21. $(-\frac{4}{5}, \infty)$ |
| 10. $(4, \infty)$ | 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$ |
| 11. $(-\infty, \infty)$ | 23. $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ |
| 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$ | 24. $(\frac{1}{2}, \infty)$ |

Piecewise Functions

1. $\sqrt{15} \approx 3.873$
2. 7
3. 6.6
4. 6.4
5. 7.1
6. 4
7. $\sqrt{3} \approx 1.732$
8. 1
9. 18

Chapter 3

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$

2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3. $f(x) = -x^4 + 3x^2 - 2x + 6$

4. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

5. $f(x) = -4x^3 + 2x^2 + 10x + 4$

6. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \geq 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$

2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$

3. $f(x) = x^3 + 2x^2 - 4x - 8$

4. $g(x) = x^4 - 2x^2 + 1$

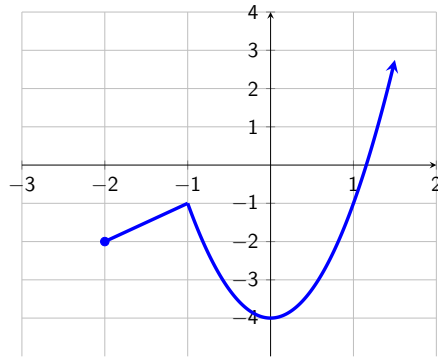
5. $h(x) = \sqrt{x+1} - 2$

6. $f(x) = -4x^3 + 2x^2 + 10x + 4$

7. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

3.3 Miscellaneous

Use the graph of $y = f(x)$ below to answer questions 1–10. Write your answers using interval notation.

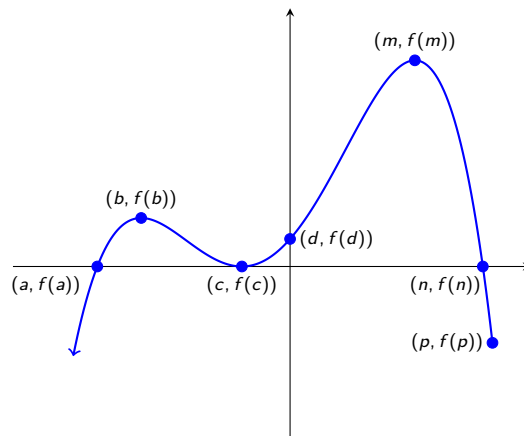


1. Domain of f
2. Range of f
3. Relative Minimum
4. Relative Maximum
5. $f(1)$
6. $f(0)$
7. Increasing Interval(s)
8. Decreasing Interval(s)
9. Absolute Maximum
10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

11. $f(7)$
12. $f(-2)$
13. Rel. Max
14. Rel. Min
15. Global Max
16. Global Min
17. Increasing Interval(s)
18. Decreasing Interval(s)

Use the graph of $f(x)$ to answer each.



19. Relative maxima of $f(x)$
20. Relative minima of $f(x)$
21. Absolute maxima of $f(x)$
22. Absolute minima of $f(x)$
23. Intervals where f is increasing
24. Intervals where f is decreasing
25. Zeros of f

Maxima and Minima

1. Rel max @ $(0, 5)$; No rel min
2. Rel max @ $(2.158, 3.248)$; Rel min @ $(-1.158, -4.048)$
3. Rel Max $(-1.366, 10.848)$ and $(1, 6)$; Rel Min $(0.366, 5.652)$
4. Rel Max $(-1.716, 11.598)$; Rel Min $(1.132, -3.929)$
5. Rel Max: $(1.095, 12.096)$; Rel Min $(-0.761, -0.680)$
6. Rel Max: $(1.366, 0.348)$; Rel Min: $(-0.366, -4.848)$ and $(2, 0)$
7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
2. Increasing: $(-1.158, 2.158)$ Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
3. Inc: $(-\infty, -2) \cup (\frac{2}{3}, \infty)$ Dec: $(-2, \frac{2}{3})$
4. Inc: $(-1, 0) \cup (1, \infty)$ Dec: $(-\infty, -1) \cup (0, 1)$
5. Inc: $(-1, \infty)$ No intervals where it is decreasing
6. Inc: $(-0.761, 1.095)$; Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Miscellaneous

- | | |
|--------------------------------|---|
| 1. $[-2, \infty)$ | 14. $(0.592, -0.272)$ |
| 2. $[-4, \infty)$ | 15. None |
| 3. $(0, -4)$ | 16. None |
| 4. $(-1, -1)$ | 17. $(0.592, 1.408)$ |
| 5. -1 | 18. $(-\infty, 0.592) \cup (1.408, \infty)$ |
| 6. -4 | 19. $(b, f(b))$ and $(m, f(m))$ |
| 7. $(-2, -1) \cup (0, \infty)$ | 20. $(c, f(c))$ |
| 8. $(-1, 0)$ | 21. $(m, f(m))$ |
| 9. $(0, -4)$ | 22. None |
| 10. None | 23. $(-\infty, b) \cup (c, m)$ |
| 11. -426 | 24. $(b, c) \cup (m, p)$ |
| 12. 51 | 25. $x = a, x = c, x = n$ |
| 13. $(1.408, 0.272)$ | |

Chapter 4

Linear Functions and Slope

4.1 Equations of Lines

Write the equation of each line **in point-slope form** that goes through each pair of points.

1. $(-2, 1), (7, 8)$
2. $(0, 4), (9, -15)$
3. $(-1, -2), (-3, -13)$

4.2 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

1. $[1, 1.1]$
2. $[1, 1.01]$
3. $[1, 1.001]$
4. $[1, 1.0001]$
5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

6. $[-2, -1]$
7. $[5, 6]$
8. $[0, 1]$
9. $[5, 5.001]$
10. $[5, 5.0001]$
11. $[5, 5.00001]$
12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

13. $[7, 7.001]$
14. $[7, 7.0001]$
15. $[7, 7.00001]$
16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

17. $[1, 1.0001]$

18. $[1, 1.00001]$

19. $[1, 1.000001]$

20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = 6\sqrt{x}$, find the average rate of change of each over the given interval.

21. $[25, 25.1]$

22. $[25, 25.01]$

23. $[25, 25.001]$

24. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Equations of Lines

1. $y - 1 = \frac{7}{9}(x + 2)$ or $y - 8 = \frac{7}{9}(x - 7)$
2. $y - 4 = -\frac{19}{9}(x - 0)$ or $y + 15 = -\frac{19}{9}(x - 9)$
3. $y + 2 = \frac{11}{2}(x + 1)$ or $y + 13 = \frac{11}{2}(x + 3)$

Average Rate of Change

1. 2.1
2. 2.01
3. 2.001
4. 2.0001
5. 2
6. 25
7. -59
8. 1
9. -53.006
10. -53.0006
11. -53.00006
12. -53
13. -42.003
14. -42.0003
15. -42.00003
16. -42
17. -0.499988
18. -0.4999988
19. -0.49999988
20. -0.5
21. 0.5994
22. 0.59999
23. 0.6
24. 0.6

Chapter 5

Function Transformations

Write the function for $g(x)$ if it is the result of $f(x)$ after the following ordered sequence of transformations.

1. (1) Vertical stretch by 3
(2) Shift left 1 unit
(3) Reflect across y -axis
2. (1) Horizontal compression by 2
(2) Shift up 1 unit
3. (1) Reflect across x -axis
(2) Vertical compression by 4
(3) Move right 7 units

Write the function $g(x)$ that is a result of the following ordered sequence of transformations to $f(x) = |x|$.

4. (1) Reflect across x -axis
(2) Shift right 3 units
(3) Horizontal stretch by factor of 5
5. (1) Shift down 2 units
(2) Reflect across y -axis
(3) Shift up 1 unit
6. (1) Horizontal compression by factor of 7
(2) Vertical compression by factor of 4
(3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function $g(x)$ after the following ordered sequence of transformations.

7. (1) Shift up 2 units
(2) Horizontal stretch by 5
(3) Shift left 3 units
8. (1) Vertical compression by factor of 3
(2) Reflect across y -axis
(3) Horizontal compression by 5
9. (1) Shift right 8 units
(2) Reflect across x -axis
(3) Horizontal compression by factor of 4

Write the final equation of $g(x)$ if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

10. (1) Shift right 2 units
(2) Horizontal stretch by factor 3
(3) Shift down 2 units
(4) Reflect across x -axis
11. (1) Horizontal stretch by factor 3
(2) Shift left 1 unit
(3) Shift up 2 units
(4) Reflect across y -axis
12. (1) Vertical stretch by factor 5
(2) Horizontal stretch by factor 2
(3) Shift up 3 units
(4) Reflect across x -axis

Find the equation for $g(x)$ if $g(x)$ is found by performing the following *ordered* sequence of transformations to $f(x) = \frac{1}{x}$.

13. (1) Shift left 3 spaces
(2) Reflect across y -axis
(3) Shift down 5 spaces
(4) Vertical stretch by factor of 7
14. (1) Shift up 3 spaces
(2) Reflect across x -axis
(3) Shift right 5 spaces
(4) Horizontal compression by factor of 7

FUNCTION TRANSFORMATIONS KEY

1. $g(x) = 3f(-x + 1)$
2. $g(x) = f(2x) + 1$
3. $g(x) = -\frac{1}{4}f(x - 7)$
4. $g(x) = -\left|\frac{1}{5}x - 3\right|$
5. $g(x) = |-x| - 1$
6. $g(x) = \frac{1}{4}|7(x + 9)| = \frac{1}{4}|7x + 63|$
7. $g(x) = \sqrt{\frac{1}{5}(x + 3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$
8. $g(x) = \frac{1}{3}\sqrt{-5x}$
9. $g(x) = -\sqrt{4x - 8}$
10. $g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$
11. $g(x) = \sqrt{\frac{1}{3}(-x + 1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$
12. $g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$
13. $g(x) = \frac{7}{-x+3} - 35$
14. $g(x) = -\frac{1}{7x-5} - 3$

Chapter 6

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given $f(x) = x + 5$, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

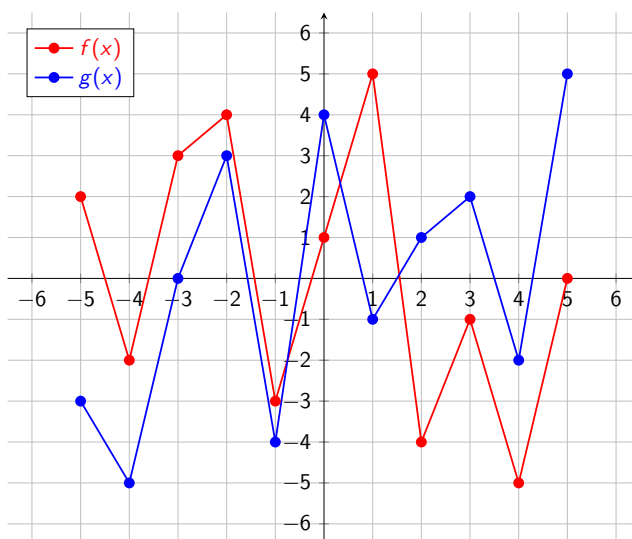
1. $(g - f)(x)$
2. $(fh)(14)$
3. $(f + g)(x)$

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-3	0	-1	3	1	2	4	-4	-2
$g(x)$	3	-1	0	1	4	-2	-4	2	-3

4. $(f + g)(-2)$
5. $(f - g)(0)$
6. $(fg)(1)$
7. $\left(\frac{f}{g}\right)(3)$
8. $(f + f)(-4)$

Find each of the following given the graphs of $f(x)$ (in red) and $g(x)$ (in blue) below:



9. $(f + g)(2)$
10. $(f - g)(1)$
11. $(g - f)(-3)$
12. $(fg)(4)$
13. $\left(\frac{f}{g}\right)(0)$

6.2 Operations with Functions: Domain

Given $f(x) = \sqrt{2x + 7}$ and $g(x) = 3x + 3$, find the domain of each.

1. $(f + g)(x)$

2. $\left(\frac{f}{g}\right)(x)$

3. $\left(\frac{g}{f}\right)(x)$

6.3 Difference Quotient

Write the difference quotient for each.

1. $f(x) = 2x - 7$

2. $g(x) = x^2 + 4x$

3. $h(x) = -1$

4. $f(x) = \frac{3}{x+2}$

5. $g(x) = \sqrt{3x}$

6. $f(x) = x^2 - 2x + 5$

7. $g(x) = \frac{5}{x}$

8. $f(x) = -2x^2 + 3x - 5$

9. $g(x) = \frac{6}{2x+3}$

10. $h(x) = \sqrt{7x+5}$

11. $f(x) = -x^2 + x$

12. $f(x) = 3x - 1$

13. $f(x) = x^3 + 5x$

Adding, Subtracting, Multiplying, and Dividing Functions

1. $x^2 - x - 6$
2. 38
3. $x^2 + x + 4$
4. -1
5. -3
6. -4
7. -2
8. -6
9. -3
10. 6
11. -3
12. 10
13. $\frac{1}{4}$

Operations with Functions: Domain

1. $[-\frac{7}{2}, \infty)$
2. $[-\frac{7}{2}, -1) \cup (-1, \infty)$
3. $(-\frac{7}{2}, \infty)$

Difference Quotient

1. 2
2. $2x + h + 4$
3. 0
4. $\frac{-3}{(x+2)(x+h+2)}$
5. $\frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$
6. $2x + h - 2$
7. $\frac{-5}{x(x+h)}$
8. $-4x - 2h + 3$
9. $\frac{-12}{(2x+3)(2x+2h+3)}$
10. $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$
11. $-2x - h + 1$
12. 3
13. $3x^2 + 3xh + h^2 + 5$

Chapter 7

Polynomials and Their Graphs

Determine the end behavior of each.

1. $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$

2. $g(x) = 4x^2 - 16x^6 + 3x$

3. $h(x) = 1 + x^{11} - 4x^8$

4. $f(x) = -x^4 + 3x^2 - 2x + 6$

5. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

POLYNOMIALS AND THEIR GRAPHS

1. $\lim_{x \rightarrow -\infty} f(x) = \infty$ $\lim_{x \rightarrow \infty} f(x) = -\infty$

2. $\lim_{x \rightarrow -\infty} g(x) = -\infty$, $\lim_{x \rightarrow \infty} g(x) = \infty$

3. $\lim_{x \rightarrow -\infty} h(x) = -\infty$ $\lim_{x \rightarrow \infty} h(x) = \infty$

4. $\lim_{x \rightarrow -\infty} f(x) = -\infty$ $\lim_{x \rightarrow \infty} f(x) = -\infty$

5. $\lim_{x \rightarrow -\infty} g(x) = -\infty$ $\lim_{x \rightarrow \infty} g(x) = \infty$

Chapter 8

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

1. $(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$

2. $(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$

3. $(4x^3 - 3x^2 + x + 1) \div (x + 2)$

4. $(5x^4 - x^2 + x - 2) \div (x^2 + 2)$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

1. $(2x^{53} - 9x^{44} + 13x^8) \div (x - 1)$

2. $(x^{71} + 15x^{58} - 3x^{14} + 2) \div (x + 1)$

Dividing Polynomials

1. $4x^2 - 2x + 5$

2. $4y^2 + 8y + 7 - \frac{72}{3y + 5}$

3. $4x^2 - 11x + 23 - \frac{45}{x + 2}$

4. $5x^2 - 11 + \frac{x + 20}{x^2 + 2}$

Remainder and Factor Theorems

1. 6

2. 13

Chapter 9

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1. $f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$

2. $g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$

3. $f(x) = \frac{3x}{x + 4}$

4. $g(x) = \frac{x^2 + 3x + 2}{x - 1}$

5. $h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$

6. $f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$

7. $g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$

8. $h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$

RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain: $x \neq -\frac{1}{2}, 7$; V.A.: $x = 7$; Hole @ $\left(-\frac{1}{2}, -\frac{7}{13}\right)$; H.A.: $y = 1$
2. Domain: $x \neq -3, 4$; V.A.: $x = -3$; Obl. Asymp: $y = 3x + 10$
3. Domain: $x \neq -4$; V.A.: $x = -4$; H.A.: $y = 3$
4. Domain: $x \neq 1$; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$
5. Domain: $x \neq 0, 1$; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$
6. Domain: $x \neq -1, 5$; V.A. $x = -1$; Hole @ $\left(5, \frac{13}{3}\right)$; Obl. Asym $y = 2x - 5$
7. Domain: $x \neq -6, 0, 4$; V.A. $x = -6, x = 0$; Hole @ $\left(4, \frac{21}{40}\right)$; H.A. $y = 0$
8. Domain: $x \neq -6, -\frac{3}{8}$; V.A. $x = -6, x = -\frac{3}{8}$; H.A. $y = \frac{1}{4}$

Chapter 10

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1. $6x^3 - 4x^2 - 10x \geq 0$

2. $x^4 < 9x^2$

3. $3x^3 - 7x^2 - 22x + 8 < 0$

4. $3x^2 - 4x + 1 \leq 0$

10.2 Rational Inequalities

Solve each. Write your answers using interval notation.

1. $\frac{3x - 4}{x + 1} < 0$

2. $\frac{x^2 + 3x + 2}{x - 7} \leq 0$

3. $\frac{x^2 - 4x + 4}{x^2 - 1} \geq 0$

4. $\frac{x + 2}{x - 4} \leq 1$

5. $\frac{x^2 - 7x - 8}{x^2 - 4x - 32} \geq 0$

Polynomial Inequalities

1. $[-1, 0] \cup \left[\frac{5}{3}, \infty\right)$
2. $(-3, 0) \cup (0, 3)$
3. $(-\infty, -2) \cup \left(\frac{1}{3}, 4\right)$
4. $\left[\frac{1}{3}, 1\right]$

Rational Inequalities

1. $\left(-1, \frac{4}{3}\right)$
2. $(-\infty, -2] \cup [-1, 7)$
3. $(-\infty, -1) \cup (1, \infty)$
4. $(-\infty, 4)$
5. $(-\infty, -4) \cup [-1, 8) \cup (8, \infty)$

Chapter 11

Function Compositions

Given $f(x) = x - 5$, $g(x) = 4 + \sqrt{2x + 1}$, and $h(x) = \frac{3}{x + 7}$, simplify each and state the domain.

1. $(f \circ g)(x)$
2. $(g \circ f)(x)$
3. $h(h(x))$

Find each of the following given the table below.

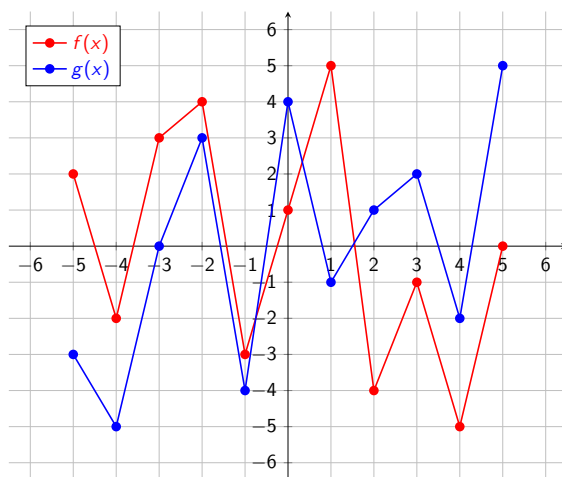
x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	-3	0	-1	3	1	2	4	-4	-2
$g(x)$	3	-1	0	1	4	-2	-4	2	-3

4. $(f \circ g)(-1)$
5. $(g \circ g)(0)$
6. $(f \circ f)(2)$
7. $(g \circ g)(-3)$
8. $f(g(0))$

Given $f(x) = \sqrt{3x + 2}$, $g(x) = x^2 - 1$, and $h(x) = 9x - 2$, find each of the following.

9. $(g \circ f)(x)$
10. $f(g(x))$
11. $(h \circ h)(x)$

Find each of the following given the graphs of $f(x)$ (in red) and $g(x)$ (in blue) below:



12. $(f \circ g)(-1)$
13. $(g \circ f)(-4)$
14. $f(g(3))$
15. $g(g(-2))$
16. $(f \circ f)(-5)$

FUNCTION COMPOSITIONS KEY

1. $-1 + \sqrt{2x+1}$ Domain: $\left[-\frac{1}{2}, \infty\right)$
2. $4 + \sqrt{2x-9}$ Domain: $\left[\frac{9}{2}, \infty\right)$
3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup (-7, \infty)$
4. 2
5. -3
6. -2
7. 1
8. -2
9. $3x+1$
10. $\sqrt{3x^2-1}$
11. $81x-20$
12. -2
13. 3
14. -4
15. 2
16. -4

Chapter 12

Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1. $f(x) = \sqrt{-2x+3} + 1$

2. $g(x) = (x+4)^2 - 1, x \leq -4$

3. $h(x) = \frac{9x}{4x-1}$

4. $f(x) = \sqrt{x} - 3$

5. $g(x) = \frac{1}{1-x}$

6. $h(x) = x^2 + 6x + 4, x \leq -3$

7. $f(x) = \sqrt{5x-4}$

8. $g(x) = x^2 - 2x + 3, x \leq 1$

9. $h(x) = \frac{3}{x-1}$

INVERSE FUNCTIONS KEY

1. $f^{-1}(x) = -\frac{1}{2}((x-1)^2 - 3)$

	Domain	Range
$f(x)$	$(-\infty, 1.5]$	$[1, \infty)$
$f^{-1}(x)$	$[1, \infty)$	$(-\infty, 1.5]$

2. $g^{-1}(x) = -\sqrt{x+1} - 4$

	Domain	Range
$g(x)$	$(-\infty, -4]$	$[-1, \infty)$
$g^{-1}(x)$	$[-1, \infty)$	$(-\infty, -4]$

3. $h^{-1}(x) = \frac{-x}{9-4x}$

	Domain	Range
$h(x)$	$(-\infty, 1/4) \cup (1/4, \infty)$	$(\infty, 9/4) \cup (9/4, \infty)$
$h^{-1}(x)$	$(\infty, 9/4) \cup (9/4, \infty)$	$(-\infty, 1/4) \cup (1/4, \infty)$

4. $f^{-1}(x) = (x+3)^2$

	Dom	Ran
$f(x)$	$[0, \infty)$	$[-3, \infty)$
$f^{-1}(x)$	$[-3, \infty)$	$[0, \infty)$

5. $g^{-1}(x) = 1 - \frac{1}{x}$

	Dom	Ran
$g(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$g^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

6. $h^{-1}(x) = -\sqrt{x+5} - 3$

	Dom	Ran
$h(x)$	$(-\infty, -3]$	$[-5, \infty)$
$h^{-1}(x)$	$[-5, \infty)$	$(-\infty, -3]$

7. $f^{-1}(x) = \frac{1}{5}x^2 + \frac{4}{5}$

	Dom	Ran
$f(x)$	$\left[\frac{4}{5}, \infty\right)$	$[0, \infty)$
$f^{-1}(x)$	$[0, \infty)$	$\left[\frac{4}{5}, \infty\right)$

8. $g^{-1}(x) = -\sqrt{x-2} + 1$

	Dom	Ran
$g(x)$	$(-\infty, 1]$	$[2, \infty)$
$g^{-1}(x)$	$[2, \infty)$	$(-\infty, 1]$

9. $h^{-1}(x) = \frac{3}{x} + 1$

	Dom	Ran
$h(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$h^{-1}(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$

Chapter 13

Exponential Functions

13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

1. $f(x) = 3 + e^{2x}$

2. $h(x) = 5^{-x}$

EXPONENTIAL FUNCTIONS

1. $\lim_{x \rightarrow -\infty} f(x) = 3$ $\lim_{x \rightarrow \infty} f(x) = \infty$

2. $\lim_{x \rightarrow -\infty} f(x) = \infty$ $\lim_{x \rightarrow \infty} f(x) = 0$

Chapter 14

Logarithmic Functions

Find the domain of each. Write your answers in interval notation.

1. $b(x) = \log_7(x^2 - 8x + 6)$

2. $a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$

LOGARITHMIC FUNCTIONS KEY

1. $(-\infty, 0.838) \cup (7.162, \infty)$
2. $(-3, -2) \cup (-1, \infty)$

Chapter 15

Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

1. $\log_b \left(\frac{x^2}{y^8} \right)$

2. $\ln(ez)^3$

3. $\log_5(x) + \log_5(9) - 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

4. $\log_7(10)$

5. $\log_9(x)$

6. $\log_b(c)$

Suppose that $\log_a(b) = 5$, $\log_a(c) = 12$, and $\log_a(d) = 9$. Evaluate each of the following.

7. $\log_a(bc)$

8. $\log_a(c^3)$

9. $\log_a \left(\frac{d}{c} \right)$

10. $\log_a \left(\frac{bd}{c} \right)$

PROPERTIES OF LOGARITHMS KEY

1. $2 \log_b(x) - 8 \log_b(y)$

2. $3 + 3 \ln(z)$

3. $\log_5 \left(\frac{9x}{w^2} \right)$

4. $\frac{\ln(10)}{\ln(7)}$

5. $\frac{\ln(x)}{\ln(9)}$

6. $\frac{\ln(c)}{\ln(b)}$

7. 17

8. 36

9. -3

10. 2

Chapter 16

Exponential Equations

Solve each. Round to 3 decimal places when necessary.

1. $3e^{x-2} = 7$

2. $5^x + 4 > 1$

3. $2^{3x+4} = 32^{x-7}$

4. $5e^{7x} + 10 = 42$

5. $7^{4x+1} \geq 343$

6. $1000e^{0.04x} = 2000$

16.1 Applications

1. Plutonium has a half-life of 24,360 years. If 15 grams are initially present, how long until 9.5 grams remain?

EXPONENTIAL EQUATIONS KEY

1. $x \approx 2.847$
2. $(-\infty, \infty)$
3. $x = 19.5$
4. $x \approx 0.265$
5. $\left[\frac{1}{2}, \infty\right)$
6. $x \approx 17.329$

Applications

1. Approximately 17,952 years

Chapter 17

Logarithmic Equations

Solve each. Round to 3 decimal places when necessary.

1. $\log_5(x) + x \log_5(x) > 0$

2. $\ln(8 - x^2) = \ln(2 - x)$

3. $\log_{25}\left(\frac{3x+1}{2x-2}\right) = \frac{1}{2}$

LOGARITHMIC EQUATIONS KEY

1. $(1, \infty)$
2. $x = -2$
3. $x \approx 1.571$

Chapter 18

Sequences

Write the first 4 terms of each sequence.

1. $a_n = 2(-3)^n$

2. $b_n = \frac{n!}{2^n}$

3. $c_{n+1} = 5c_n + 1$; $c_1 = 2$

4. $d_n = \frac{1}{2}d_{n-1} + n$; $d_1 = 3$

Find the indicated term of each sequence. For term values above 10 billion or below 0.00001, write the first 4 digits after the decimal point when the answer is given in scientific notation. **Do not round.**

5. $a_n = \{343, 667, 991, 1315, \dots\}$; Find the 582nd term.

6. $b_n = \{300, 240, 192, 153.6, \dots\}$; Find the 711th term.

7. $c_n = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots\right\}$; Find the 8,675,309th term.

SEQUENCES KEY

1. $-6, 18, -54, 162$

2. $\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2}$

3. $2, 11, 56, 281$

4. $3, \frac{7}{2}, \frac{19}{4}, \frac{51}{8}$

5. $188,587$

6. 6882

7. 7634

Chapter 19

Series

Find the sum of each, if possible.

1. $\sum_{i=1}^{\infty} \left(\frac{1}{5}\right)^i$

2. $\sum_{i=0}^{\infty} 3 \left(-\frac{2}{3}\right)^i$

3. $\sum_{k=1}^{\infty} -2 \left(\frac{1}{3}\right)^k$

4. $\sum_{j=0}^{\infty} -\frac{1}{2} \left(\frac{3}{2}\right)^j$

Find the sum of each of the following. Round to 4 decimal places when necessary.

5. $9 + 13 + 17 + 21 + \cdots + 1565$

6. $-3 + 6 - 12 + 24 - 48 + \cdots + 50,331,648$

7. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{981}$

8. $2 + 4 + 6 + 8 + 10 + \cdots + 38,214$

SERIES KEY

1. $\frac{1}{4}$

2. $\frac{9}{5}$

3. -1

4. Diverges

5. 306,930

6. $-33,554,433$

7. 7.4663

8. 365,096,556

Appendix A

Factoring

Factor each of the following completely.

1. $x^2 + 2x - 15$

2. $x^2 - 8x + 12$

3. $x^2 + 15x + 56$

4. $5x^2 + 19x - 4$

5. $4x^2 - 5x - 6$

6. $9x^2 - 400$

7. $5x^2 - 7x - 6$

8. $9x^2 - 54x + 45$

9. $3x^3 + 12x^2 + 9x$

10. $9y^2 - 16$

11. $4x^2 - 28x + 49$

12. $14x^2 + 11xy - 15y^2$

13. $6x^2 - 48x - 120$

14. $9x^4 - 54x^3 + 45x^2$

15. $16y^2 - 40y + 25$

16. $30x^2 + xy - y^2$

17. $8w^2 + 33w + 4$

18. $3p^2 + 22p - 16$

19. $18x^2 - 27x + 4$

20. $14a^2 + 15a - 9$

21. $4x^2 - 4x - 24$

22. $18t^2 - 9t - 5$

23. $6a^2 + 23a + 21$

24. $25x^2 - 1$

FACTORING KEY

1. $(x + 5)(x - 3)$

2. $(x - 6)(x - 2)$

3. $(x + 7)(x + 8)$

4. $(5x - 1)(x + 4)$

5. $(4x + 3)(x - 2)$

6. $(3x + 20)(3x - 20)$

7. $(5x + 3)(x - 2)$

8. $9(x - 5)(x - 1)$

9. $3x(x + 3)(x + 1)$

10. $(3y + 4)(3y - 4)$

11. $(2x - 7)^2$

12. $(7x - 5y)(2x + 3y)$

13. $6(x - 10)(x + 2)$

14. $9x^2(x - 1)(x - 5)$

15. $(4y - 5)^2$

16. $(6x - y)(5x + y)$

17. $(8w + 1)(w + 4)$

18. $(3p - 2)(p + 8)$

19. $(6x - 1)(3x - 4)$

20. $(7a - 3)(2a + 3)$

21. $4(x - 3)(x + 2)$

22. $(6t - 5)(3t + 1)$

23. $(2a + 3)(3a + 7)$

24. $(5x + 1)(5x - 1)$

Appendix B

Complex Fractions

Simplify each as much as possible.

$$1. \frac{5 + \frac{3}{x}}{x - \frac{1}{2}}$$

$$2. \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

$$3. \frac{3}{2 - \frac{x}{x-1}}$$

$$4. \frac{1 + \frac{3}{x}}{\frac{2}{x} + 7}$$

$$5. \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

$$6. \frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$7. \frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2-2x}}$$

$$8. \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

$$9. \frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$10. \frac{\frac{5}{x} - 5x}{x - 1}$$

$$11. \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

$$12. \frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

$$13. \frac{\frac{1}{x-a} + \frac{1}{a}}{x}$$

$$14. \frac{\frac{1}{x-1} - \frac{1}{x-3}}{\frac{2}{x-1} + \frac{3}{x+1}}$$

$$15. \frac{\frac{2}{x^2-4} + \frac{1}{x-2}}{\frac{4}{x+2}}$$

COMPLEX FRACTIONS KEY

$$1. \frac{2(5x+3)}{x(2x-1)}$$

$$2. \frac{1}{x^2-2x+4}$$

$$3. \frac{3(x-1)}{x-2}$$

$$4. \frac{x+3}{2+7x}$$

$$5. \frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$6. \frac{-4x-1}{2}$$

$$7. \frac{8x-10}{7}$$

$$8. -\frac{1}{7x}$$

$$9. \frac{2x+1}{5x(x+1)}$$

$$10. \frac{-5x-5}{x}$$

$$11. \frac{-1}{2x+4}$$

$$12. \frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$13. \frac{1}{a(x-a)}$$

$$14. \frac{-2x-2}{5x^2-16x+3}$$

$$15. \frac{x+4}{4x-8}$$