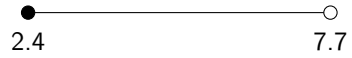
Chapter 1

Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

1. $(-5, 8]$
2. $\{x|x \leq 1\}$
3. 

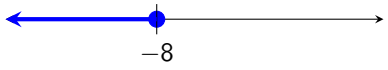
4. $\{x|x \neq 4, 11\}$
5. 

6. $(9, \infty)$

Write each using interval notation and graph on a number line.

7. $\{x|x \geq 2\}$
8. $\{x|x < -8\}$
9. $\{x|x \neq 3\}$
10. $\{x|x \neq -2, 5\}$

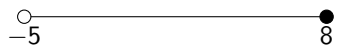
You are given the graph of an interval. Write the interval and set-builder notation for it.

11. 

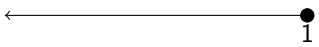
12. 

Basic Set Theory and Interval Notation Key

1. $\{x| -5 < x \leq 8\}$



2. $(-\infty, 1]$

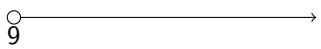


3. $[-3, \infty)$ $\{x|x \geq -3\}$

4. $(-\infty, 4) \cup (4, 11) \cup (11, \infty)$

5. $[2.4, 7.7)$ $\{x|2.4 \leq x < 7.7\}$

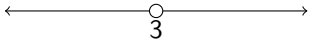
6. $\{x|x > 9\}$



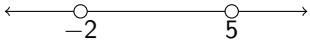
7. $[2, \infty)$

8. $(-\infty, -8)$

9. $(-\infty, 3) \cup (3, \infty)$



10. $(\infty, -2) \cup (-2, 5) \cup (5, \infty)$



11. $(-\infty, -8]$ $\{x|x \leq -8\}$

12. $(-\infty, 7) \cup (7, 12) \cup (12, \infty)$ $\{x|x \neq 7, 12\}$

Chapter 2

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

1. $f(5)$
2. $f(-2)$
3. $f(0)$
4. $g(1)$
5. $g(-5)$
6. $g(1/4)$

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1. $f(x) = -8x^2 - 7x + 1$
2. $g(x) = \sqrt{5x + 12} - 2$
3. $h(x) = \frac{x+2}{9x-7}$
4. $f(x) = -5x + 4$
5. $f(x) = x^2 + 2$
6. $f(x) = \frac{2x+1}{3x-5}$
7. $f(x) = \sqrt{3x - 12}$
8. $f(x) = \frac{x}{x^2-16}$
9. $f(x) = \frac{x+4}{x^3-4x}$
10. $f(x) = \frac{x}{\sqrt{x-4}}$
11. $f(x) = \frac{x^2+1}{2x^2+8}$
12. $f(x) = -\frac{x+7}{x^2-5x-6}$
13. $g(x) = \sqrt{2x + 3}$
14. $h(x) = \sqrt[3]{2x + 3}$
15. $f(x) = -\frac{7x-10}{x^2+3x+2}$
16. $g(x) = \sqrt{-9x + 8}$

17. $h(x) = -\sqrt[3]{4x+1}$
18. $f(x) = \sqrt[3]{8x+1}$
19. $g(x) = \frac{x^2-1}{\sqrt{x+3}}$
20. $h(x) = \frac{3}{9+\frac{4}{x+7}}$
21. $f(x) = \frac{x+1}{\sqrt{10x+8}}$
22. $g(x) = \frac{5}{1+\frac{3}{x+2}}$

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3 \\ 0.2x + 7 & \text{if } -3 \leq x < 2 \\ \sqrt{5x} & \text{if } x \geq 2 \end{cases}$$

1. $f(3)$
2. $f(0)$
3. $f(-2)$
4. $f(-3)$
5. $f(0.5)$

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \leq -4 \\ \sqrt{2x+7} & -4 < x < 0 \\ |-x-1| & x \geq 0 \end{cases}$$

6. $f(3)$
7. $f(-2)$
8. $f(0)$
9. $f(-5)$

Evaluating Functions

1. -55
2. -20
3. 0
4. -4
5. -5.2
6. -1

Domain of Functions

1. $(-\infty, \infty)$
2. $\left[\frac{-12}{5}, \infty\right)$
3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
4. $(-\infty, \infty)$
5. $(-\infty, \infty)$
6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
7. $[4, \infty)$
8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$
10. $(4, \infty)$
11. $(-\infty, \infty)$
12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$
13. $\left[-\frac{3}{2}, \infty\right)$
14. $(-\infty, \infty)$
15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
16. $\left(-\infty, \frac{8}{9}\right]$
17. $(-\infty, \infty)$
18. $(-\infty, \infty)$
19. $(-3, \infty)$
20. $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup (-7, \infty)$
21. $\left(-\frac{4}{5}, \infty\right)$
22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$

Piecewise Functions

1. $\sqrt{15} \approx 3.873$
2. 7
3. 6.6
4. 6.4
5. 7.1
6. 4
7. $\sqrt{3} \approx 1.732$
8. 1
9. 18

Chapter 3

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$
2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$
3. $f(x) = -x^4 + 3x^2 - 2x + 6$
4. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$
5. $f(x) = -4x^3 + 2x^2 + 10x + 4$
6. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$
7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \geq 0$$

   After how many hours will the concentration be the highest?

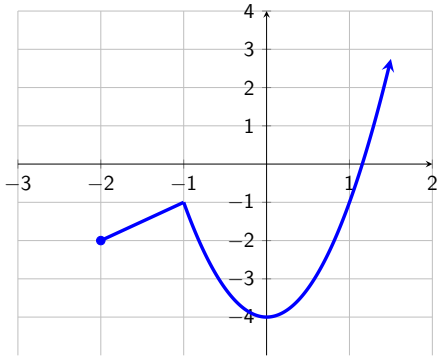
3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1. $f(x) = x^2 - 3x^2 + 5$
2. $g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$
3. $f(x) = x^3 + 2x^2 - 4x - 8$
4. $g(x) = x^4 - 2x^2 + 1$
5. $h(x) = \sqrt{x+1} - 2$
6. $f(x) = -4x^3 + 2x^2 + 10x + 4$
7. $g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$

3.3 Miscellaneous

Use the graph of $y = f(x)$ below to answer questions 1–10. Write your answers using interval notation.



1. Domain of f
2. Range of f
3. Relative Minimum
4. Relative Maximum
5. $f(1)$
6. $f(0)$
7. Increasing Interval(s)
8. Decreasing Interval(s)
9. Absolute Maximum
10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

11. $f(7)$
12. $f(-2)$
13. Rel. Max
14. Rel. Min
15. Global Max
16. Global Min
17. Increasing Interval(s)
18. Decreasing Interval(s)

Maxima and Minima

1. Rel max @ $(0, 5)$; No rel min
2. Rel max @ $(2.158, 3.248)$; Rel min @ $(-1.158, -4.048)$
3. Rel Max $(-1.366, 10.848)$ and $(1, 6)$; Rel Min $(0.366, 5.652)$
4. Rel Max $(-1.716, 11.598)$; Rel Min $(1.132, -3.929)$
5. Rel Max: $(1.095, 12.096)$; Rel Min $(-0.761, -0.680)$
6. Rel Max: $(1.366, 0.348)$; Rel Min: $(-0.366, -4.848)$ and $(2, 0)$
7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
2. Increasing: $(-1.158, 2.158)$ Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
3. Inc: $(-\infty, -2) \cup \left(\frac{2}{3}, \infty\right)$ Dec: $\left(-2, \frac{2}{3}\right)$
4. Inc; $(-1, 0) \cup (1, \infty)$ Dec: $(-\infty, -1) \cup (0, 1)$
5. Inc: $(-1, \infty)$ No intervals where it is decreasing
6. Inc: $(-0.761, 1.095)$; Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Miscellaneous

1. $[-2, \infty)$
2. $[-4, \infty)$
3. $(0, -4)$
4. $(-1, -1)$
5. -1
6. $-4)$
7. $(-2, -1) \cup (0, \infty)$
8. $(-1, 0)$
9. $(0, -4)$
10. None
11. -426
12. 51
13. (1.408, 0.272)
14. (0.592, -0.272)
15. None
16. None
17. (0.592, 1.408)
18. $(-\infty,\ 0.592) \cup (1.408,\ \infty)$

Chapter 4

Linear Functions and Slope

4.1 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

1. $[1, 1.1]$
2. $[1, 1.01]$
3. $[1, 1.001]$
4. $[1, 1.0001]$
5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

6. $[-2, -1]$
7. $[5, 6]$
8. $[0, 1]$
9. $[5, 5.001]$
10. $[5, 5.0001]$
11. $[5, 5.00001]$
12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

13. $[7, 7.001]$
14. $[7, 7.0001]$
15. $[7, 7.00001]$
16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

17. $[1, 1.0001]$
18. $[1, 1.00001]$
19. $[1, 1.000001]$
20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Average Rate of Change

1. 2.1
2. 2.01
3. 2.001
4. 2.0001
5. 2
6. 25
7. -59
8. 1
9. -53.006
10. -53.0006
11. -53.00006
12. -53
13. -42.003
14. -42.0003
15. -42.00003
16. -42
17. -0.499988
18. -0.4999988
19. -0.49999988
20. -0.5

Chapter 5

Function Transformations

Write the function for $g(x)$ if it is the result of $f(x)$ after the following ordered sequence of transformations.

1. (1) Vertical stretch by 3
   (2) Shift left 1 unit
   (3) Reflect across y-axis
2. (1) Horizontal compression by 2
   (2) Shift up 1 unit
3. (1) Reflect across x-axis
   (2) Vertical compression by 4
   (3) Move right 7 units

Write the function $g(x)$ that is a result of the following ordered sequence of transformations to $f(x) = |x|$.

4. (1) Reflect across x-axis
   (2) Shift right 3 units
   (3) Horizontal stretch by factor of 5
5. (1) Shift down 2 units
   (2) Reflect across y-axis
   (3) Shift up 1 unit
6. (1) Horizontal compression by factor of 7
   (2) Vertical compression by factor of 4
   (3) Shift left 9 units

GIven $f(x) = \sqrt{x}$, determine the resulting function $g(x)$ after the following ordered sequence of transformations.

7. (1) Shift up 2 units
   (2) Horizontal stretch by 5
   (3) Shift left 3 units
8. (1) Vertical compression by factor of 3
   (2) Reflect across y-axis
   (3) Horizontal compression by 5
9. 1) Shift right 8 units
   2) Reflect across x-axis
   3) Horizontal compression by factor of 4

Function Transformations Key

1. $g(x) = 3f(-x+1)$
2. $g(x) = f(2x)+1$
3. $g(x) = -\frac{1}{4}f(x-7)$
4. $g(x) = -\left|\frac{1}{5}x-3\right|$
5. $g(x) = |-x|-1$
6. $g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$
7. $g(x) = \sqrt{\frac{1}{5}(x+3)}+2 = \sqrt{\frac{1}{5}x+\frac{3}{5}}+2$
8. $g(x) = \frac{1}{3}\sqrt{-5x}$
9. $g(x) = -\sqrt{4x-8}$

Chapter 6

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given $f(x) = x + 5$, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

1. $(g - f)(x)$
2. $(fh)(14)$
3. $(f + g)(x)$

6.2 Operations with Functions: Domain

Given $f(x) = \sqrt{2x + 7}$ and $g(x) = 3x + 3$, find the domain of each.

1. $(f + g)(x)$
2. $\left(\frac{f}{g}\right)(x)$
3. $\left(\frac{g}{f}\right)(x)$

6.3 Difference Quotient

Write the difference quotient for each.

1. $f(x) = 2x - 7$
2. $g(x) = x^2 + 4x$
3. $h(x) = -1$
4. $f(x) = \frac{3}{x+2}$
5. $g(x) = \sqrt{3x}$
6. $f(x) = x^2 - 2x + 5$
7. $g(x) = \frac{5}{x}$

Adding, Subtracting, Multiplying, and Dividing Functions

1. $x^2 - x - 6$
2. 38
3. $x^2 + x + 4$

Operations with Functions: Domain

1. $\left[-\frac{7}{2}, \infty\right)$
2. $\left[-\frac{7}{2}, -1\right) \cup (-1, \infty)$
3. $\left(-\frac{7}{2}, \infty\right)$

Difference Quotient

1. 2
2. $2x + h + 4$
3. 0
4. $\frac{-3}{(x+2)(x+h+2)}$
5. $\frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$
6. $2x + h - 2$
7. $\frac{-5}{x(x+h)}$

Chapter 7

Polynomials and Their Graphs

Determine the end behavior of each.

1. $f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$
2. $g(x) = 4x^2 - 16x^6 + 3x$
3. $h(x) = 1 + x^{11} - 4x^8$
4. $f(x) = -x^4 + 3x^2 - 2x + 6$
5. $g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$

1. $\lim_{x\to-\infty} f(x) = \infty \quad \lim_{x\to\infty} f(x) = -\infty$
2. $\lim_{x\to-\infty} g(x) = -\infty, \quad \lim_{x\to\infty} g(x) = \infty$
3. $\lim_{x\to-\infty} h(x) = -\infty \quad \lim_{x\to\infty} h(x) = \infty$
4. $\lim_{x\to-\infty} f(x) = -\infty \quad \lim_{x\to\infty} f(x) = -\infty$
5. $\lim_{x\to-\infty} g(x) = -\infty \quad \lim_{x\to\infty} g(x) = \infty$

Chapter 8

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

1. $(28x^3 - 26x^2 + 41x - 15) \div (7x - 3)$
2. $(44y^2 + 12y^3 + 61y - 37) \div (3y + 5)$

8.2 Remainder and Factor Theorems

1. $\left(2x^{53} - 9x^{44} + 13x^8\right) \div (x - 1)$
2. $\left(x^{71} + 15x^{58} - 3x^{14} + 2\right) \div (x + 1)$

Dividing Polynomials

1. $4x^2 - 2x + 5$
2. $4y^2 + 8y + 7 - \frac{72}{3y + 5}$

Remainder and Factor Theorems

1. 6
2. 13

Chapter 9

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1. $f(x) = \dfrac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$

2. $g(x) = \dfrac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$

3. $f(x) = \dfrac{3x}{x + 4}$

4. $g(x) = \dfrac{x^2 + 3x + 2}{x - 1}$

5. $h(x) = h(x) = \dfrac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$

Rational Functions and Their Graphs Key

1. Domain: $x \neq -\frac{1}{2}, 7$; V.A.: $x = 7$; Hole @ $\left(-\frac{1}{2}, -\frac{7}{13}\right)$; H.A.: $y = 1$
2. Domain: $x \neq -3, 4$; V.A.: $x = -3\, x = 4$; Obl. Asymp: $y = 3x + 10$
3. Domain: $x \neq -4$; V.A.: $x = -4$; H.A.: $y = 3$
4. Domain: $x \neq 1$; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$
5. Domain: $x \neq 0, 1$; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$

Chapter 10

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1. $6x^3 - 4x^2 - 10x \geq 0$

10.2 Rational Inequalities

Solve each. Write your answers using interval notation.

1. $\dfrac{3x-4}{x+1} < 0$

2. $\dfrac{x^2+3x+2}{x-7} \leq 0$

Polynomial Inequalities

1. $[-1, 0] \cup \left[\frac{5}{3}, \infty\right)$

Rational Inequalities

1. $\left(-1, \frac{4}{3}\right)$
2. $(-\infty, -2] \cup [-1, 7)$

Chapter 11

Function Compositions

Given $f(x) = x - 5$, $g(x) = 4 + \sqrt{2x+1}$, and $h(x) = \dfrac{3}{x+7}$, simplify each and state the domain.

1. $(f \circ g)(x)$
2. $(g \circ f)(x)$
3. $h(h(x))$

Function Compositions Key

1. $-1+\sqrt{2x+1}$ Domain: $\left[-\frac{1}{2},\infty\right)$
2. $4+\sqrt{2x-9}$ Domain: $\left[\frac{9}{2},\infty\right)$
3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty,-\frac{52}{7}\right)\cup\left(-\frac{52}{7},-7\right)\cup(-7,\infty)$