Contents

1	Basic Set Theory and Interval Notation	2
2	Functions and Their Graphs 2.1 Evaluating Functions	4 4 4 5
3	Properties of Functions 3.1 Maxima and Minima	7 7 7 7
4	Linear Functions and Slope 4.1 Average Rate of Change	10 10
5	Function Transformations	12
6	Function Operations 6.1 Adding, Subtracting, Multiplying, and Dividing Functions	15 15 15 15
7	Polynomials and Their Graphs	17
8	Dividing Polynomials 8.1 Dividing Polynomials	19 19 19
9	Rational Functions and Their Graphs	21
10	Polynomial and Rational Inequalities 10.1 Polynomial Inequalities	23 23 23
11	Function Compositions	25
12	Inverse Functions	27
	Exponential Functions 13.1 End Behavior	29 29
14	Logarithmic Functions	31
15	Properties of Logarithms	33
Α	Factoring	35
В	Complex Fractions	37

Basic Set Theory and Interval Notation

You are given either interval notation, set-builder notation, or a graph. Write each of the following in its other 2 forms.

- 1. (-5, 8]
- 2. $\{x | x \le 1\}$
- 3. _3
- 4. $\{x | x \neq 4, 11\}$
- 5. 2.4 7.
- 6. $(9, \infty)$

Write each using interval notation and graph on a number line.

- 7. $\{x | x \ge 2\}$
- 8. $\{x|x<-8\}$
- 9. $\{x | x \neq 3\}$
- 10. $\{x | x \neq -2, 5\}$

You are given the graph of an interval. Write the interval and set-builder notation for it.



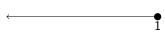


BASIC SET THEORY AND INTERVAL NOTATION KEY

1.
$$\{x | -5 < x \le 8\}$$



2. $(-\infty, 1]$



3.
$$[-3, \infty)$$
 $\{x | x \ge -3\}$

4.
$$(-\infty, 4) \cup (4, 11) \cup (11, \infty)$$

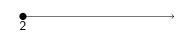


5. [2.4, 7.7)
$$\{x | 2.4 \le x < 7.7\}$$

6.
$$\{x|x > 9\}$$



7. $[2, \infty)$



8.
$$(-\infty, -8)$$



9.
$$(-\infty,3)\cup(3,\infty)$$

$$\longleftrightarrow$$
 3

10.
$$(\infty, -2) \cup (-2, 5) \cup (5, \infty)$$

$$\overset{\longleftarrow}{\underset{-2}{\smile}}\overset{\bigcirc}{\underset{5}{\smile}}$$

11.
$$(-\infty, -8]$$
 $\{x | x \le -8\}$

12.
$$(-\infty, 7) \cup (7, 12) \cup (12, \infty)$$
 $\{x | x \neq 7, 12\}$

Functions and Their Graphs

2.1 Evaluating Functions

Given $f(x) = -3x^2 + 4x$ and $g(x) = \frac{1}{x} - 5$, evaluate each.

- 1. f(5)
- 2. f(-2)
- 3. f(0)
- 4. g(1)
- 5. g(-5)
- 6. g(1/4)

2.2 Domain of Functions

Find the domain of each write your answers in interval notation.

1.
$$f(x) = -8x^2 - 7x + 1$$

2.
$$g(x) = \sqrt{5x + 12} - 2$$

3.
$$h(x) = \frac{x+2}{9x-7}$$

4.
$$f(x) = -5x + 4$$

5.
$$f(x) = x^2 + 2$$

6.
$$f(x) = \frac{2x+1}{3x-5}$$

7.
$$f(x) = \sqrt{3x - 12}$$

8.
$$f(x) = \frac{x}{x^2 - 16}$$

9.
$$f(x) = \frac{x+4}{x^3-4x}$$

10.
$$f(x) = \frac{x}{\sqrt{x-4}}$$

11.
$$f(x) = \frac{x^2+1}{2x^2+8}$$

12.
$$f(x) = -\frac{x+7}{x^2-5x-6}$$

13.
$$g(x) = \sqrt{2x+3}$$

14.
$$h(x) = \sqrt[3]{2x+3}$$

15.
$$f(x) = -\frac{7x-10}{x^2+3x+2}$$

16.
$$g(x) = \sqrt{-9x + 8}$$

17.
$$h(x) = -\sqrt[3]{4x+1}$$

18.
$$f(x) = \sqrt[3]{8x+1}$$

19.
$$g(x) = \frac{x^2 - 1}{\sqrt{x + 3}}$$

20.
$$h(x) = \frac{3}{9 + \frac{4}{x+7}}$$

21.
$$f(x) = \frac{x+1}{\sqrt{10x+8}}$$

22.
$$g(x) = \frac{5}{1 + \frac{3}{x+2}}$$

23.
$$i(x) = \frac{7}{3 - \frac{4}{x+1}}$$

24.
$$n(x) = \frac{7x+14}{\sqrt{2x-1}}$$

2.3 Piecewise Functions

Find the value of each given the piecewise function below. Use exact answers when possible.

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < -3\\ 0.2x + 7 & \text{if } -3 \le x < 2\\ \sqrt{5x} & \text{if } x \ge 2 \end{cases}$$

- 1. f(3)
- 2. f(0)
- 3. f(-2)
- 4. f(-3)
- 5. f(0.5)

Find each of the following given the piecewise function

$$f(x) = \begin{cases} x^2 - 7 & x \le -4\\ \sqrt{2x + 7} & -4 < x < 0\\ |-x - 1| & x \ge 0 \end{cases}$$

- 6. f(3)
- 7. f(-2)
- 8. f(0)
- 9. f(-5)

Evaluating Functions

- 1. -55
- 2. -20
- 3. 0
- 4. -4
- 5. -5.2
- 6. -1

Domain of Functions

- 1. $(-\infty, \infty)$
- 2. $\left[\frac{-12}{5}, \infty\right)$
- 3. $\left(-\infty, \frac{7}{9}\right) \cup \left(\frac{7}{9}, \infty\right)$
- 4. $(-\infty, \infty)$
- 5. $(-\infty, \infty)$
- 6. $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$
- 7. $[4,\infty)$
- 8. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
- 9. $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$
- 10. $(4, \infty)$
- 11. $(-\infty, \infty)$
- 12. $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$

- 13. $\left[-\frac{3}{2},\infty\right)$
- 14. $(-\infty, \infty)$
- 15. $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$
- 16. $\left(-\infty, \frac{8}{9}\right]$
- 17. $(-\infty, \infty)$
- 18. $(-\infty, \infty)$
- 19. $(-3, \infty)$
- 20. $\left(-\infty, -\frac{67}{9}\right) \cup \left(-\frac{67}{9}, -7\right) \cup \left(-7, \infty\right)$
- 21. $\left(-\frac{4}{5}, \infty\right)$
- 22. $(\infty, -5) \cup (-5, -2) \cup (-2, \infty)$
- 23. $(-\infty, -1) \cup (-1, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$
- 24. $(\frac{1}{2}, \infty)$

Piecewise Functions

- $1. \ \sqrt{15} \approx 3.873$
- 2. 7
- 3. 6.6
- 4. 6.4
- 5. 7.1
- 6. 4
- 7. $\sqrt{3} \approx 1.732$
- 8. 1
- 9. 18

Properties of Functions

3.1 Maxima and Minima

Find the coordinates of the any relative maxima or minima. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

4.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

5.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

6.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

7. The concentration C of a medication in the bloodstream t hours after being administered can be modeled by

$$C(t) = -0.002t^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085, \quad t \ge 0$$

After how many hours will the concentration be the highest?

3.2 Increasing, Decreasing, and Constant Intervals

Find the intervals in which each is increasing or decreasing. Round to 3 decimal places when necessary.

1.
$$f(x) = x^2 - 3x^2 + 5$$

2.
$$g(x) = -0.4x^3 + 0.6x^2 + 3x - 2$$

3.
$$f(x) = x^3 + 2x^2 - 4x - 8$$

4.
$$g(x) = x^4 - 2x^2 + 1$$

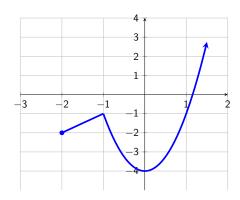
5.
$$h(x) = \sqrt{x+1} - 2$$

6.
$$f(x) = -4x^3 + 2x^2 + 10x + 4$$

7.
$$g(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$$

3.3 Miscellaneous

Use the graph of y = f(x) below to answer questions 1–10. Write your answers using interval notation.



- 1. Domain of f
- 2. Range of f
- 3. Relative Minimum
- 4. Relative Maximum
- 5. f(1)

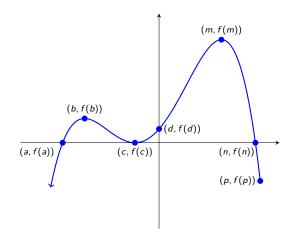
- 6. f(0)
- 7. Increasing Interval(s)
- 8. Decreasing Interval(s)
- 9. Absolute Maximum
- 10. Absolute Minimum

Find each of the following given $f(x) = -2x^3 + 6x^2 - 5x + 1$. Round to 3 decimal places and use interval notation when applicable.

- 11. f(7)
- 12. f(-2)
- 13. Rel. Max
- 14. Rel. Min

- 15. Global Max
- 16. Global Min
- 17. Increasing Interval(s)
- 18. Decreasing Interval(s)

Use the graph of f(x) to answer each.



- 19. Relative maxima of f(x)
- 20. Relative minima of f(x)
- 21. Absolute maxima of f(x)
- 22. Absolute minima of f(x)

- 23. Intervals where f is increasing
- 24. Intervals where f is decreasing
- 25. Zeros of *f*

Maxima and Minima

- 1. Rel max @ (0,5); No rel min
- 2. Rel max @ (2.158, 3.248); Rel min @ (-1.158, -4.048)
- 3. Rel Max (-1.366, 10.848) and (1,6); Rel Min (0.366, 5.652)
- 4. Rel Max (-1.716, 11.598); Rel Min (1.132, -3.929)
- 5. Rel Max: (1.095, 12.096); Rel Min (-0.761, -0.680)
- 6. Rel Max: (1.366, 0.348); Rel Min: (-0.366, -4.848) and (2, 0)
- 7. About 2.16 hours

Increasing, Decreasing, and Constant Intervals

- 1. Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
- 2. Increasing: (-1.158, 2.158) Decreasing: $(-\infty, -1.158) \cup (2.158, \infty)$
- 3. Inc: $(-\infty, -2) \cup (\frac{2}{3}, \infty)$ Dec: $(-2, \frac{2}{3})$
- 4. Inc; $(-1,0) \cup (1,\infty)$ Dec: $(-\infty,-1) \cup (0,1)$
- 5. Inc: $(-1, \infty)$ No intervals where it is decreasing
- 6. Inc: (-0.761, 1.095); Dec: $(-\infty, -0.761) \cup (1.095, \infty)$
- 7. Inc: $(-0.366, 1.366) \cup (2, \infty)$; Dec: $(-\infty, -0.366) \cup (1.366, 2)$;

Miscellaneous

- 1. $[-2, \infty)$
- $2. [-4, \infty)$
- 3. (0, -4)
- 4. (-1, -1)
- 5. -1
- 6. -4)
- 7. $(-2, -1) \cup (0, \infty)$
- 8. (-1,0)
- 9. (0, -4)
- 10. None
- 11. -426
- 12. 51
- 13. (1.408, 0.272)

- 14. (0.592, -0.272)
- 15. None
- 16. None
- 17. (0.592, 1.408)
- 18. $(-\infty, 0.592) \cup (1.408, \infty)$
- 19. (b, f(b)) and (m, f(m))
- 20. (c, f(c))
- 21. (m, f(m))
- 22. None
- 23. $(-\infty, b) \cup (c, m)$
- 24. $(b, c) \cup (m, p)$
- 25. x = a, x = c, x = n

Linear Functions and Slope

4.1 Average Rate of Change

For the function $f(x) = x^2$, compute the average rate of change for each interval.

- 1. [1, 1.1]
- 2. [1, 1.01]
- 3. [1, 1.001]
- 4. [1, 1.0001]
- 5. For your answers in the previous four problems, what value do your average rates of change get closer and closer to?

Find the average rate of change of the function $f(x) = -6x^2 + 7x + 4$ over each specified interval.

- 6. [-2, -1]
- 7. [5, 6]
- 8. [0, 1]
- 9. [5, 5.001]
- 10. [5, 5.0001]
- 11. [5, 5.00001]
- 12. What value are your last 3 answers getting closer to?

For the function $f(x) = -3x^2 + 5$, determine the average rate of change of each over the given interval.

- 13. [7, 7.001]
- 14. [7, 7.0001]
- 15. [7, 7.00001]
- 16. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Given $f(x) = \sqrt{x}$, find the average rate of change of each over the given interval.

- 17. [1, 1.0001]
- 18. [1, 1.00001]
- 19. [1, 1.000001]
- 20. For your answers in the previous three problems, what value do your average rates of change get closer and closer to?

Average Rate of Change

- 1. 2.1
- 2. 2.01
- 3. 2.001
- 4. 2.0001
- 5. 2
- 6. 25
- 7. -59
- 8. 1
- 9. -53.006
- 10. -53.0006
- 11. -53.00006
- 12. -53
- 13. -42.003
- 14. -42.0003
- 15. -42.00003
- 16. -42
- 17. -0.499988
- 18. -0.4999988
- 19. -0.49999988
- 20. -0.5

Function Transformations

Write the function for g(x) if it is the result of f(x) after the following ordered sequence of transformations.

- 1. (1) Vertical stretch by 3
 - (2) Shift left 1 unit
 - (3) Reflect across y-axis
- 2. (1) Horizontal compression by 2
 - (2) Shift up 1 unit
- 3. (1) Reflect across x-axis
 - (2) Vertical compression by 4
 - (3) Move right 7 units

Write the function g(x) that is a result of the following ordered sequence of transformations to f(x) = |x|.

- 4. (1) Reflect across x-axis
 - (2) Shift right 3 units
 - (3) Horizontal stretch by factor of 5
- 5. (1) Shift down 2 units
 - (2) Reflect across y-axis
 - (3) Shift up 1 unit
- 6. (1) Horizontal compression by factor of 7
 - (2) Vertical compression by factor of 4
 - (3) Shift left 9 units

Given $f(x) = \sqrt{x}$, determine the resulting function g(x) after the following ordered sequence of transformations.

- 7. (1) Shift up 2 units
 - (2) Horizontal stretch by 5
 - (3) Shift left 3 units
- 8. (1) Vertical compression by factor of 3
 - (2) Reflect across y-axis
 - (3) Horizontal compression by 5
- 9. (1) Shift right 8 units
 - (2) Reflect across x-axis
 - (3) Horizontal compression by factor of 4

Write the final equation of g(x) if it is found by taking $f(x) = \sqrt{x}$ after the following ordered sequence of transformations.

- 10. (1) Shift right 2 units
 - (2) Horizontal stretch by factor 3
 - (3) Shift down 2 units
 - (4) Reflect across x-axis
- 11. (1) Horizontal stretch by factor 3
 - (2) Shift left 1 unit
 - (3) Shift up 2 units
 - (4) Reflect across y-axis
- 12. (1) Vertical stretch by factor 5
 - (2) Horizontal stretch by factor 2
 - (3) Shift up 3 units
 - (4) Reflect across x-axis

FUNCTION TRANSFORMATIONS KEY

1.
$$g(x) = 3f(-x+1)$$

2.
$$g(x) = f(2x) + 1$$

3.
$$g(x) = -\frac{1}{4}f(x-7)$$

4.
$$g(x) = -\left|\frac{1}{5}x - 3\right|$$

5.
$$g(x) = |-x| - 1$$

6.
$$g(x) = \frac{1}{4}|7(x+9)| = \frac{1}{4}|7x+63|$$

7.
$$g(x) = \sqrt{\frac{1}{5}(x+3)} + 2 = \sqrt{\frac{1}{5}x + \frac{3}{5}} + 2$$

8.
$$g(x) = \frac{1}{3}\sqrt{-5x}$$

9.
$$g(x) = -\sqrt{4x - 8}$$

10.
$$g(x) = -\left(\sqrt{\frac{1}{3}x - 2} - 2\right) = -\sqrt{\frac{1}{3}x - 2} + 2$$

11.
$$g(x) = \sqrt{\frac{1}{3}(-x+1)} + 2 = \sqrt{-\frac{1}{3}x + \frac{1}{3}} + 2$$

12.
$$g(x) = -\left(5\sqrt{\frac{1}{2}x} + 3\right) = -5\sqrt{\frac{1}{2}x} - 3$$

Function Operations

6.1 Adding, Subtracting, Multiplying, and Dividing Functions

Given f(x) = x + 5, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x - 10}$, simplify or evaluate each.

- 1. (g f)(x)
- 2. (fh)(14)
- 3. (f+g)(x)

Find each of the following given the table below.

x	ı	-3	ı				l	l	1
f(x)	-3	0	-1	3	1	2	4	-4	-2
g(x)	3	-1	0	1	4	-2	-4	2	-3

4.
$$(f+g)(-2)$$
 5. $(f-g)(0)$ 6. $(fg)(1)$

5.
$$(f-g)(0)$$

6.
$$(fg)(1)$$

7.
$$\left(\frac{f}{g}\right)$$
 (3)

8.
$$(f+f)(-4)$$

Operations with Functions: Domain

Given $f(x) = \sqrt{2x+7}$ and g(x) = 3x+3, find the domain of each.

- 1. (f + g)(x)
- 2. $\left(\frac{f}{g}\right)(x)$
- 3. $\left(\frac{g}{f}\right)(x)$

Difference Quotient 6.3

Write the difference quotient for each.

1.
$$f(x) = 2x - 7$$

2.
$$g(x) = x^2 + 4x$$

3.
$$h(x) = -1$$

4.
$$f(x) = \frac{3}{x+2}$$

5.
$$g(x) = \sqrt{3x}$$

6.
$$f(x) = x^2 - 2x + 5$$

$$7. g(x) = \frac{5}{x}$$

8.
$$f(x) = -2x^2 + 3x - 5$$

9.
$$g(x) = \frac{6}{2x+3}$$

10.
$$h(x) = \sqrt{7x+5}$$

Adding, Subtracting, Multiplying, and Dividing Functions

- 1. $x^2 x 6$
- 2. 38
- 3. $x^2 + x + 4$
- 4. -1
- 5. -3
- 6. -4
- 7. -2
- 8. -6

Operations with Functions: Domain

- 1. $\left[-\frac{7}{2},\infty\right)$
- 2. $\left[-\frac{7}{2}, -1\right) \cup \left(-1, \infty\right)$
- 3. $\left(-\frac{7}{2}, \infty\right)$

Difference Quotient

- 1. 2
- 2. 2x + h + 4
- 3. 0
- 4. $\frac{-3}{(x+2)(x+h+2)}$
- $5. \ \frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$
- 6. 2x + h 2
- 7. $\frac{-5}{x(x+h)}$
- 8. -4x 2h + 3
- 9. $\frac{-12}{(2x+3)(2x+2h+3)}$
- 10. $\frac{7}{\sqrt{7x+7h+5}+\sqrt{7x+5}}$

Polynomials and Their Graphs

Determine the end behavior of each.

1.
$$f(x) = -x^5 + \sqrt{7}x^3 - 2x^2$$

2.
$$g(x) = 4x^2 - 16x^6 + 3x$$

3.
$$h(x) = 1 + x^{11} - 4x^8$$

4.
$$f(x) = -x^4 + 3x^2 - 2x + 6$$

5.
$$g(x) = 0.25x^5 - 0.1x^4 + 2x^2 - 6x$$

POLYNOMIALS AND THEIR GRAPHS

1.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

2.
$$\lim_{x \to -\infty} g(x) = -\infty$$
, $\lim_{x \to \infty} g(x) = \infty$

3.
$$\lim_{x \to -\infty} h(x) = -\infty$$
 $\lim_{x \to \infty} h(x) = \infty$

4.
$$\lim_{x \to -\infty} f(x) = -\infty$$
 $\lim_{x \to \infty} f(x) = -\infty$

5.
$$\lim_{x \to -\infty} g(x) = -\infty$$
 $\lim_{x \to \infty} g(x) = \infty$

Dividing Polynomials

8.1 Dividing Polynomials

Divide each.

- 1. $(28x^3 26x^2 + 41x 15) \div (7x 3)$
- 2. $(44y^2 + 12y^3 + 61y 37) \div (3y + 5)$
- 3. $(4x^3 3x^2 + x + 1) \div (x + 2)$
- 4. $(5x^4 x^2 + x 2) \div (x^2 + 2)$

8.2 Remainder and Factor Theorems

Determine the remainder of each.

- 1. $(2x^{53} 9x^{44} + 13x^8) \div (x 1)$
- 2. $(x^{71} + 15x^{58} 3x^{14} + 2) \div (x+1)$

Dividing Polynomials

1.
$$4x^2 - 2x + 5$$

2.
$$4y^2 + 8y + 7 - \frac{72}{3y + 5}$$

3.
$$4x^2 - 11x + 23 - \frac{45}{x+2}$$

4.
$$5x^2 - 11 + \frac{x + 20}{x^2 + 2}$$

Remainder and Factor Theorems

- 1. 6
- 2. 13

Rational Functions and Their Graphs

Find the domain, coordinates of any holes, and equations of all asymptotes.

1.
$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 15x + 7}$$

2.
$$g(x) = \frac{3x^3 + 7x^2 - 20x}{x^2 - x - 12}$$

$$3. \ f(x) = \frac{3x}{x+4}$$

4.
$$g(x) = \frac{x^2 + 3x + 2}{x - 1}$$

5.
$$h(x) = \frac{x^2 + 3x - 4}{x^3 - 2x^2 + x}$$

6.
$$f(x) = \frac{2x^3 - 13x^2 + 6x + 45}{x^2 - 4x - 5}$$

7.
$$g(x) = \frac{5x^2 - 19x - 4}{x^3 + 2x^2 - 24x}$$

8.
$$h(x) = \frac{2x^2 - x - 3}{8x^2 + 51x + 18}$$

RATIONAL FUNCTIONS AND THEIR GRAPHS KEY

1. Domain:
$$x \neq -\frac{1}{2}$$
, 7; V.A.: $x = 7$; Hole @ $\left(-\frac{1}{2}, -\frac{7}{13}\right)$; H.A.: $y = 1$

2. Domain:
$$x \neq -3$$
, 4; V.A.: $x = -3x = 4$; Obl. Asymp: $y = 3x + 10$

3. Domain:
$$x \neq -4$$
; V.A.: $x = -4$; H.A.: $y = 3$

4. Domain:
$$x \neq 1$$
; V.A.: $x = 1$; Obl. Asymp: $y = x + 4$

5. Domain:
$$x \neq 0, 1$$
; V.A.: $x = 0$ and $x = 1$; H.A.: $y = 0$

6. Domain:
$$x \neq -1, 5$$
; V.A. $x = -1$; Hole @ $\left(5, \frac{13}{3}\right)$; Obl. Asym $y = 2x - 5$

7. Domain:
$$x \neq -6$$
, 0, 4; V.A. $x = -6$, $x = 0$; Hole @ $\left(4, \frac{19}{40}\right)$; H.A. $y = 0$

8. Domain:
$$x \neq -6, -\frac{3}{8}$$
; V.A. $x = -6, x = -\frac{3}{8}$; H.A. $y = \frac{1}{4}$

Polynomial and Rational Inequalities

10.1 Polynomial Inequalities

Solve each. Write your answers using interval notation.

1.
$$6x^3 - 4x^2 - 10x \ge 0$$

2.
$$x^4 < 9x^2$$

10.2 Rational Inequalities

Solve each. Write your answers using interval notation.

1.
$$\frac{3x-4}{x+1} < 0$$

$$2. \ \frac{x^2 + 3x + 2}{x - 7} \le 0$$

$$3. \ \frac{x^2 - 4x + 4}{x^2 - 1} \ge 0$$

4.
$$\frac{x+2}{x-4} \le 1$$

Polynomial Inequalities

- 1. $[-1,0] \cup \left[\frac{5}{3},\infty\right)$
- 2. $(-3,0) \cup (0,3)$

Rational Inequalities

- 1. $\left(-1, \frac{4}{3}\right)$
- 2. $(-\infty, -2] \cup [-1, 7)$
- 3. $(-\infty, -1) \cup (1, \infty)$
- 4. $(-\infty, 4)$

Function Compositions

Given f(x) = x - 5, $g(x) = 4 + \sqrt{2x + 1}$, and $h(x) = \frac{3}{x + 7}$, simplify each and state the domain.

1.
$$(f \circ g)(x)$$

2.
$$(g \circ f)(x)$$

3.
$$h(h(x))$$

Find each of the following given the table below.

x	-4	-3	-2	-1	0	1	2	3	4	
f(x)	-3	0	-1	3	1	2	4	-4	-2	
g(x)	3	-1	0	1	4	-2	-4	2	-3	

4.
$$(f \circ g)(-1)$$

5.
$$(g \circ g)(0)$$

6.
$$(f \circ f)(2)$$

4.
$$(f \circ g)(-1)$$
 5. $(g \circ g)(0)$ 6. $(f \circ f)(2)$ 7. $(g \circ g)(-3)$ 8. $f(g(0))$

8.
$$f(g(0))$$

Given $f(x) = \sqrt{3x+2}$, $g(x) = x^2 - 1$, and h(x) = 9x - 2, find each of the following.

9.
$$(g \circ f)(x)$$

10.
$$f(g(x))$$

11.
$$(h \circ h)(x)$$

FUNCTION COMPOSITIONS KEY

- 1. $-1 + \sqrt{2x+1}$ Domain: $\left[-\frac{1}{2}, \infty\right)$
- 2. $4 + \sqrt{2x 9}$ Domain: $\left[\frac{9}{2}, \infty\right)$
- 3. $\frac{3x+21}{7x+52}$ Domain: $\left(-\infty, -\frac{52}{7}\right) \cup \left(-\frac{52}{7}, -7\right) \cup \left(-7, \infty\right)$
- 4. 2
- 5. -3
- 6. -2
- 7. 1
- 8. -2
- 9. 3x + 1
- 10. $\sqrt{3x^2-1}$
- 11. 81x 20

Inverse Functions

Find the inverse of each. Then state the domain and range of the function and the inverse.

1.
$$f(x) = \sqrt{-2x+3} + 1$$

2.
$$g(x) = (x+4)^2 - 1$$
, $x \le -4$

3.
$$h(x) = \frac{9x}{4x - 1}$$

4.
$$f(x) = \sqrt{x} - 3$$

$$5. g(x) = \frac{1}{1-x}$$

6.
$$h(x) = x^2 + 6x + 4, x \le -3$$

INVERSE FUNCTIONS KEY

1.
$$f^{-1}(x) = -\frac{1}{2}((x-1)^2 - 3)$$

	Domain	Range
f(x)	$(-\infty, 1.5]$	$[1,\infty)$
$f^{-1}(x)$	$[1,\infty)$	$(-\infty, 1.5]$

2.
$$g^{-1}(x) = -\sqrt{x+1} - 4$$

	Domain	Range
g(x)	$(-\infty, -4]$	$[-1,\infty)$
$g^{-1}(x)$	$[-1,\infty)$	$(-\infty, -4]$

3.
$$h^{-1}(x) = \frac{-x}{9-4x}$$

	Domain	Range
h(x)	$(-\infty,1/4)\cup(1/4,\infty)$	$(\infty,9/4)\cup(9/4,\infty)$
$h^{-1}(x)$	$(\infty,9/4)\cup(9/4,\infty)$	$(-\infty,1/4)\cup(1/4,\infty)$

4.
$$f^{-1}(x) = (x+3)^2$$

	Dom	Ran
f(x)	$[0,\infty)$	$[-3,\infty)$
$f^{-1}(x)$	$[-3,\infty)$	$[0,\infty)$

5.
$$g^{-1}(x) = 1 - \frac{1}{x}$$

	Dom	Ran
g(x)	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$g^{-1}(x)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,1)\cup(1,\infty)$

6.
$$h^{-1}(x) = -\sqrt{x+5} - 3$$

	Dom	Ran
h(x)	$(-\infty, -3]$	$[-5,\infty)$
$h^{-1}(x)$	$[-5,\infty)$	$(-\infty, -3]$

Exponential Functions

13.1 End Behavior

Determine the end behavior of each. Write your answers using limit notation.

- 1. $f(x) = 3 + e^{2x}$
- 2. $h(x) = 5^{-x}$

EXPONENTIAL FUNCTIONS

1.
$$\lim_{x \to -\infty} f(x) = 3$$
 $\lim_{x \to \infty} f(x) = \infty$

2.
$$\lim_{x \to -\infty} f(x) = \infty$$
 $\lim_{x \to \infty} f(x) = 0$

Logarithmic Functions

Find the domain of each. Write your answers in interval notation.

1.
$$b(x) = \log_7(x^2 - 8x + 6)$$

2.
$$a(x) = \ln\left(\frac{x^2 + 3x + 2}{5x + 15}\right)$$

LOGARITHMIC FUNCTIONS KEY

- 1. $(-\infty, 0.838) \cup (7.162, \infty)$
- 2. $(-3, -2) \cup (-1, \infty)$

Properties of Logarithms

Expand or condense each completely. Simplify numerical answers.

- 1. $\log_b \left(\frac{x^2}{y^8} \right)$
- 2. $\ln (ez)^3$
- 3. $\log_5(x) + \log_5(9) 2\log_5(w)$

Write an equivalent expression for each of the following using natural logarithms.

- 4. $\log_7(10)$
- 5. $\log_9(x)$
- 6. $\log_b(c)$

Properties of Logarithms Key

- $1. \ 2\log_b(x) 8\log_b(y)$
- 2. $3 + 3 \ln(z)$
- $3. \log_5 \left(\frac{9x}{w^2} \right)$
- 4. $\frac{\ln(10)}{\ln(7)}$
- $5. \ \frac{\ln(x)}{\ln(9)}$
- 6. $\frac{\ln(c)}{\ln(b)}$

Appendix A

Factoring

Factor each of the following completely.

1.
$$x^2 + 2x - 15$$

2.
$$x^2 - 8x + 12$$

3.
$$x^2 + 15x + 56$$

4.
$$5x^2 + 19x - 4$$

5.
$$4x^2 - 5x - 6$$

6.
$$9x^2 - 400$$

7.
$$5x^2 - 7x - 6$$

8.
$$9x^2 - 54x + 45$$

9.
$$3x^3 + 12x^2 + 9x$$

10.
$$9y^2 - 16$$

11.
$$4x^2 - 28x + 49$$

12.
$$14x^2 + 11xy - 15y^2$$
 18. $3p^2 + 22p - 16$

13.
$$6x^2 - 48x - 120$$

14.
$$9x^4 - 54x^3 + 45x^2$$

15.
$$16y^2 - 40y + 25$$

16.
$$30x^2 + xy - y^2$$

17.
$$8w^2 + 33w + 4$$

18.
$$3p^2 + 22p - 16$$

19.
$$18x^2 - 27x + 4$$

20.
$$14a^2 + 15a - 9$$

21.
$$4x^2 - 4x - 24$$

22.
$$18t^2 - 9t - 5$$

23.
$$6a^2 + 23a + 21$$

24.
$$25x^2 - 1$$

FACTORING KEY

1.
$$(x+5)(x-3)$$

2.
$$(x-6)(x-2)$$

3.
$$(x+7)(x+8)$$

4.
$$(5x-1)(x+4)$$

5.
$$(4x+3)(x-2)$$

6.
$$(3x + 20)(3x - 20)$$

7.
$$(5x+3)(x-2)$$

8.
$$9(x-5)(x-1)$$

9.
$$3x(x+3)(x+1)$$

10.
$$(3y+4)(3y-4)$$

11.
$$(2x-7)^2$$

12.
$$(7x - 5y)(2x + 3y)$$

13.
$$6(x-10)(x+2)$$

14.
$$9x^2(x-1)(x-5)$$

15.
$$(4y-5)^2$$

16.
$$(6x - y)(5x + y)$$

17.
$$(8w+1)(w+4)$$

18.
$$(3p-2)(p+8)$$

19.
$$(6x-1)(3x-4)$$

20.
$$(7a-3)(2a+3)$$

21.
$$4(x-3)(x+2)$$

22.
$$(6t-5)(3t+1)$$

23.
$$(2a+3)(3a+7)$$

24.
$$(5x+1)(5x-1)$$

Appendix B

Complex Fractions

Simplify each as much as possible.

1.
$$\frac{5+\frac{3}{x}}{x-\frac{1}{2}}$$

$$5. \ \frac{\frac{4}{x} - \frac{x}{x-2}}{\frac{1}{x} + \frac{3}{x-2}}$$

9.
$$\frac{\frac{1}{x} + \frac{1}{x+1}}{5}$$

$$2. \ \frac{\frac{1}{x} + \frac{2}{x^2}}{x + \frac{8}{x^2}}$$

6.
$$\frac{\frac{3}{x+1} - 4}{\frac{2}{x+1}}$$

$$10. \ \frac{\frac{5}{x} - 5x}{x - 1}$$

11. $\frac{\frac{1}{2+x} - \frac{1}{2}}{x}$

3.
$$\frac{3}{2 - \frac{x}{x-1}}$$

7.
$$\frac{\frac{5}{x} + \frac{3}{x-2}}{\frac{7}{x^2 - 2x}}$$

12.
$$\frac{\frac{3}{x-4} + \frac{2x}{x+1}}{4x}$$

4.
$$\frac{1+\frac{3}{x}}{\frac{2}{x}+7}$$

$$8. \ \frac{\frac{1}{x} - \frac{1}{7}}{x - 7}$$

13.
$$\frac{\frac{1}{x-a} + \frac{1}{a}}{X}$$

COMPLEX FRACTIONS KEY

1.
$$\frac{2(5x+3)}{x(2x-1)}$$

5.
$$\frac{-1(x^2-4x+8)}{2(2x-1)}$$

$$10. \ \frac{-5x-5}{x}$$

$$2. \ \frac{1}{x^2 - 2x + 4}$$

6.
$$\frac{-4x-1}{2}$$

11.
$$\frac{-1}{2x+4}$$

$$3. \ \frac{3(x-1)}{x-2}$$

7.
$$\frac{8x - 10}{7}$$
8. $-\frac{1}{7x}$

12.
$$\frac{(x-1)(2x-3)}{4x(x-4)(x+1)}$$

$$4. \ \frac{x+3}{2+7x}$$

$$9. \ \frac{2x+1}{5x(x+1)}$$

$$13. \ \frac{1}{a(x-a)}$$