Average Rate of Change

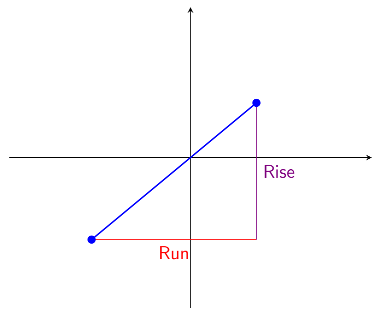
Summary

1. The slope-intercept form of the equation of a line is $y = mx + b$.
2. Slope represents variable costs; y-intercept represents fixed costs.
3. The average rate of change measures the slope of the line connecting 2 points on a graph.

Slope-Intercept Form

Recall that the slope of a line connecting 2 points is

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



We often write the equation of a line in **slope-intercept form**: $y = mx + b$, where b is the y-intercept.

- The y-intercept can be thought of as the **fixed cost** of doing business (cost no matter what)
- The slope can be thought of as the **variable cost** (cost based on how much is produced).

Example 1. A local company manufactures goods with weekly fixed costs of \$6,000 and variable costs of \$4.55 per item.

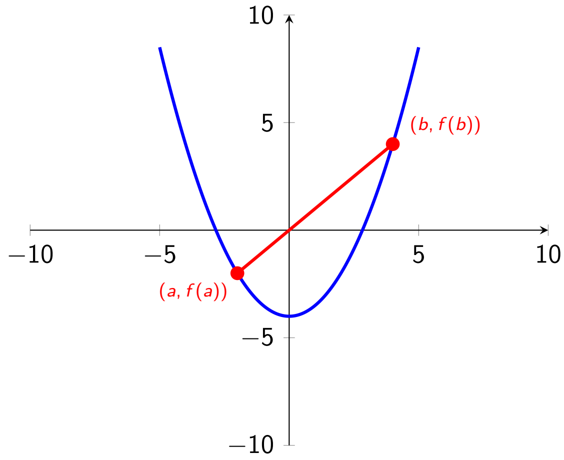
(a) Determine the linear cost function $C(x)$ and interpret $C(1500)$

(b) At 1500 per week production level, what is the cost of manufacturing the 1501st item? (*Note*: This is called the **marginal cost**)

Average Rate of Change

The **average rate of change** over an interval $[a, b]$ of a function is the slope of the line connecting the endpoints of the interval.

$$\frac{\Delta f}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$



Example 2. Find the average rate of change for each.

(a) $f(x) = 3x - 7 \qquad [2, 5]$

(b) $f(x) = 2x^2 - 5x + 4 \qquad [-2, 2]$

Example 3. Find the average rate of change of $f(x) = x^2 + 4x - 1$ over each interval.

(a) [3, 3.01]

(b) [3, 3.001]

(c) [3, 3.0001]

(d) Do the outputs get closer and closer to a value as the inputs get closer together? If so, what is that value?