Fundamental Theorem of Calculus

Summary

1.

Previously, we examined **antiderivatives**.

Recall that the antiderivative of $f(x) = x^2 + 3$ is

$$F(x) = \tfrac{1}{3}x^3 + 3x + C$$

Example 1. Evaluate $F(6) - F(3)$.

`The Fundamental Theorem of Calculus (Part 1)`
If F is an antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

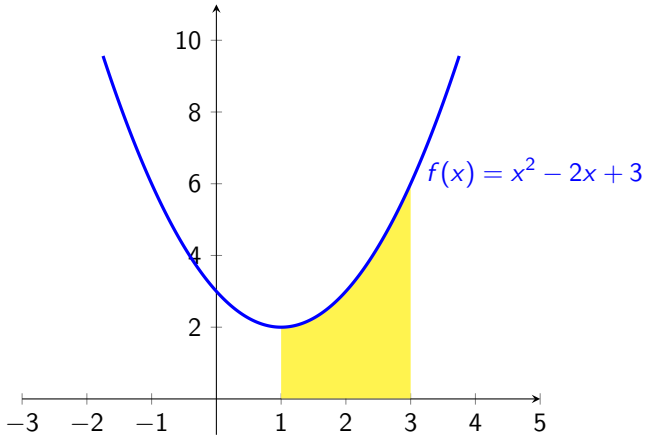
Note: The difference $F(b) - F(a)$ is also written $F(x)\Big|_a^b$

Example 2. Find the exact value of each of the following.

(a) $\displaystyle\int_{-2}^{0} (x^2 - 3x)\ dx$

(b) $\displaystyle\int_1^5 \frac{1}{\sqrt{x}}\,dx$

Example 3. Find the exact value of the shaded area below.



An area that is *below* the x-axis has an area that is **negative**.

The **net area** is the area between the graph of $f(x)$ and the x-axis.

The **gross area** is the absolute value of the region below the x-axis.

Example 4. For $f(x) = x^3 - x^2 - 4x + 4$,

(a) Sketch and evaluate the net area by computing $\displaystyle\int_{-2}^{2} f(x)\,dx$

(b) Calculate the gross area between the graph of f and the x-axis.