

Discrete Probability Distributions

Summary

1. Discrete probability distributions lists discrete data values along with their associated probabilities.

Probability Distribution

A **probability distribution** is a listing of each outcome of a probability experiment with their probabilities.

A **discrete probability distribution** is one in which the outcomes of each experiment are discrete (countable) values.

Familiar Characteristics:

- $0 \leq \text{each probability} \leq 1$
- The sum of all probabilities in a distribution equals 1
- $P(A \text{ or } B) = P(A) + P(B)$

Probability Distribution of Rolling 2 Dice

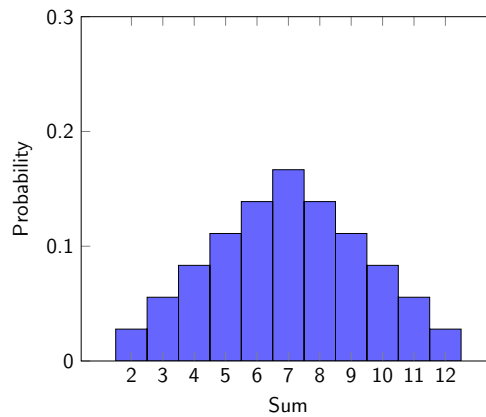
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We can create a probability distribution of the sums of rolling two dice.

We use the notation $P(X = x)$ where X is our **random variable** and x represents the outcomes, such as 2, 3, 4, ..., 12.

x	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Probability Histogram of Rolling 2 Dice



Example 1.

- (a) Create a probability distribution for flipping a coin three times, where X represents the number of times heads is flipped.
- (b) Create a probability distribution histogram for the number of times heads appears when flipping a coin 3 times.

Example 2. The distribution below represents the percentage of households that have x dogs according to a recent study.

x	$P(X = x)$
0	44%
1	27%
2	18%
3 or more	11%

How many households have at least 1 dog?

Expected Value of a Probability Distribution

Expected Value

The **expected value** of a probability distribution is the outcome we would expect to happen if the experiment was performed a very large number of times.

In other words, it is a **weighted mean** of the distribution of outcomes:

$$E(X) = \sum (x \cdot P(x))$$

Example 3. Determine the expected value of rolling two dice.

Example 4. The distribution below represents the percentage of households that have x dogs according to a recent study.

x	$P(X = x)$
0	44%
1	27%
2	18%
3	11%

What is the expected number of dogs per household?

Variance and Standard Deviation of a Probability Distribution

Recall that

- **Variance** is the average squared deviation from the mean.
- **Standard deviation** is the square root of variance.

With probability distributions, the **mean** is the **expected value**.

	Variance	Standard Deviation
Typical Formula	$\sigma^2 = \sum ((x - \mu)^2 \cdot P(x))$	$\sigma = \sqrt{\sum ((x - \mu)^2 \cdot P(x))}$
Alternative Formula	$\sigma^2 = \sum ((x - E(x))^2 \cdot P(x))$	$\sigma = \sqrt{\sum ((x - E(x))^2 \cdot P(x))}$

Fortunately, we will use technology for this.

Example 5. What is the standard deviation of rolling two dice?

Example 6. Using the range rule of thumb, what are the minimum and maximum “usual” values for rolling 2 dice?