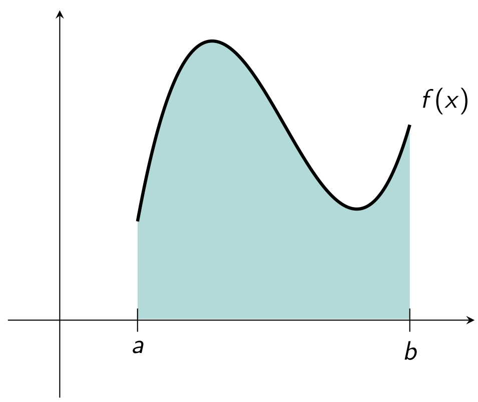
Area and the Definite Integral

Summary

1.

Suppose we have a function that lies entirely above the x-axis.

What would the area of the function that is above the x-axis be between the x-coordinates of a and b below?



Or, perhaps even more importantly, *how would we go about doing this?*

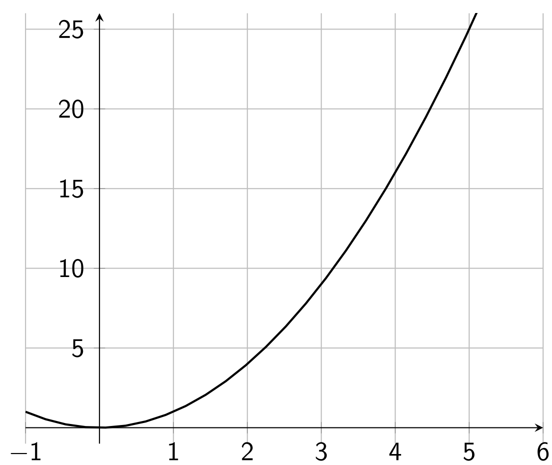
One of the most common ways to find the area under a curve is to

$$\boxed{\text{use rectangles of equal width and calculate the area of each rectangle.}}$$

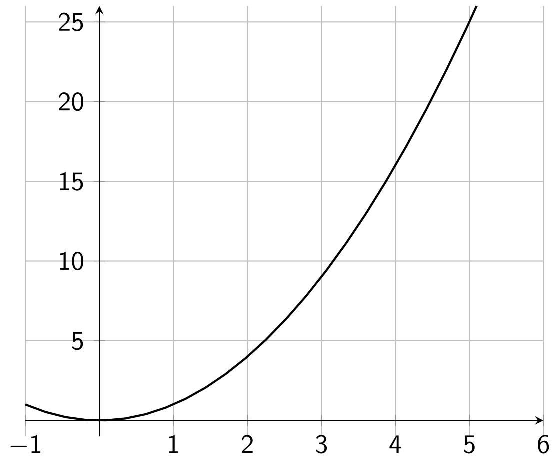
We will work this out in the first example.

Example 1. Find the area under the curve $f(x) = x^2$, above the x-axis, and between $x = 1$ and $x = 5$ as follows:

(a) Using 4 rectangles of equal width



(b) Using 8 rectangles of equal width



Using the Right Endpoints

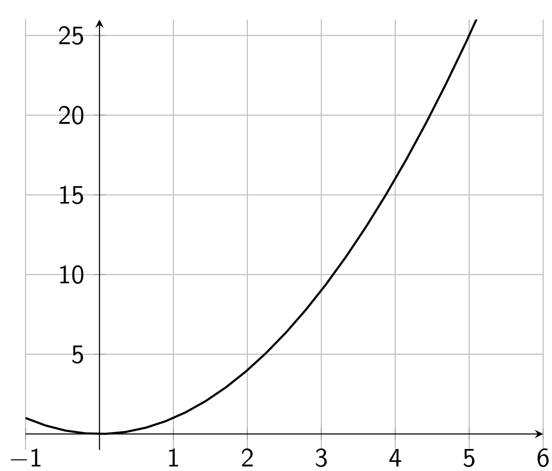
In the previous example, we approximated the area under the curve $f(x) = x^2$ from $x = 1$ to $x = 5$ by using rectangles.

The heights of the rectangles were determined by $f(1)$, $f(2)$, $f(3)$, and $f(4)$.

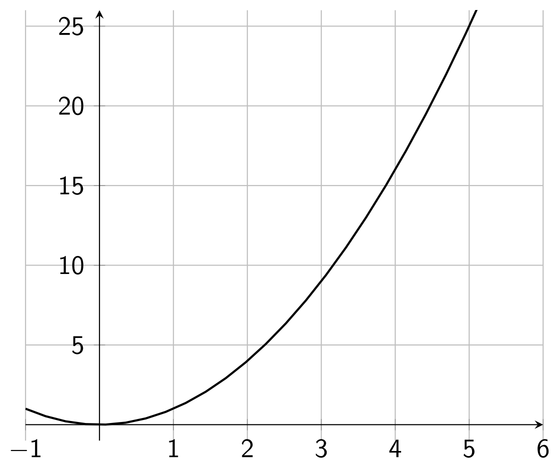
These were the **left endpoints** of the rectangles. However, we can also use the right endpoints.

Example 2. Using right endpoints, find the area under the curve $f(x) = x^2$, above the x-axis, and between $x = 1$ and $x = 5$ as follows:

(a) Using 4 rectangles of equal width



(b) Using 8 rectangles of equal width



You can use any point on the width of each rectangle.

Most of the time it will be the left or right endpoint, or the center of each rectangle.

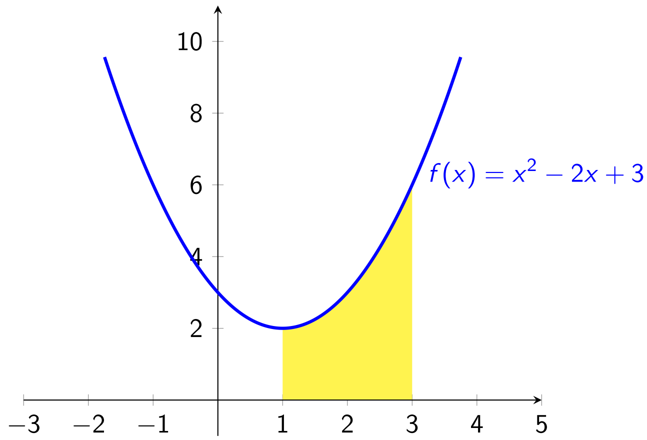
The Definite Integral

As the number of rectangles on a closed interval $[a, b]$ increases indefinitely, the width of each rectangle becomes smaller and smaller.

This will give us better and better approximations to the area under the curve.

$$\begin{aligned}\text{Area under curve} &= \text{Add up: (each rectangle height} \times \text{each rectangle width)}\\ &= \text{Add up: } f\,(\text{each } x \text{ value}) \times \text{change in } x \text{ value}\\ &= \text{Add up: } f(x) \times \text{very small change in } x \text{ value}\\ &= \int_a^b f(x)\,\mathrm{d}x\end{aligned}$$

Example 3. Write the definite integral to represent the shaded area of the following:



$f(x) = x^2 - 2x + 3$
10
8
6
4
2
−3
−2
−1
1
2
3
4
5