Logarithmic Functions

Summary

1. The inverse of the exponential function $y = b^x$ is $y = \log_b(x)$ with $b > 0$ and $b \neq 1$
2. $\log_{10}(x) = \log(x)$ and $\log_e(x) = \ln x$
3. The domain of the logarithmic function is $(0, \infty)$

Recall that an **inverse function** allows you to get your input back from your output of a function.

The **inverse** of the exponential function $f(x) = b^x$ is the **logarithmic function**.

It is denoted $f^{-1}(x) = \log_b x$

$$\log_b(y) = x$$

$$b^x = y$$

Common Logarithm

The **common logarithm** of a real number x is $\log_{10} x$ and is usually written

$$\log x$$

Natural Logarithm

The **natural logarithm** of a real number x is $\log_e x$ and is usually written

$$\ln x$$

Example 1. Re-write each as a logarithmic expression.

(a) $2^5 = 32$

(b) $3^4 = 81$

(c) $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

(d) $7^{-2} = \frac{1}{49}$

(e) $e^x = 5$

Properties of Logarithms

Exponents and logarithms have similar properties where $x > 0$ and $y > 0$

Property	**Exponents**	**Logarithms**
Product	$b^x \cdot b^y = b^{x+y}$	$\log_b(xy) = \log_b(x) + \log_b(y)$
Quotient	$\frac{b^x}{b^y} = b^{x-y}$	$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
Power	$(b^x)^y = b^{xy}$	$\log_b(x)^y = y \cdot \log_b(x)$
Equality	$b^x = b^y \iff x = y$	$\log_b(x) = \log_b(y) \iff x = y$
Zero Power	$b^0 = 1$	$\log_b(1) = 0$
1st Power	$b^1 = b$	$\log_b(b) = 1$

Example 2. Use the properties of logarithms to rewrite each of the following.

(a) $\log(6 \cdot 17)$

(b) $\log_5\left(\frac{10}{3}\right)$

(c) $\log_6\left(\sqrt{11}\right)$

Solving Exponential Equations

Solving an exponential equation involves

- Isolating the exponential base
- Taking the logarithm of both sides

Example 3. \$20,000 is deposited and compounded continuously ($A = Pe^{rt}$) at a rate of 6.5%.

(a) How many years until there is \$25,000 in the account?

(b) How many years until the account value doubles?