Speed Counting

Summary

1. The concepts in this section are based on the Fundamental Counting Rule.
2. $n! = n(n-1)(n-2)\cdots(3)(2)(1)$ and $0! = 1$.
3. With permutations, selection order matters; with combinations, it does not.

Fundamental Counting Rule

Example 1. For a special at a restaurant, you can choose between 3 appetizers, 4 entrees, and 2 desserts. If you select one item from each category (appetizer, entree, and dessert), how many different meals can you create?

- If event A can occur in a different ways and event B can occur in b different ways, then the total number of ways both events can occur is ab ways.
- Can be generalized to multiple events, such as those in example 1: $3 \times 4 \times 2 = 24$

Factorial Notation

Example 2. A baseball lineup consists of 9 players. How many different lineups using all 9 players on a team exist?

- Mathematicians created **factorial notation** to expedite the process.
- $9! = 9 \times 8 \times 7 \times \cdots \times 3 \times 2 \times 1 = 362,880$
- In general, for a positive integer n,

$$n! = n(n-1)(n-2) \cdot \cdots \cdot 3(2)(1)$$

  with $0! = 1$

Example 3. How many ways are there to arrange 5 books on a shelf?

Permutations

Example 4. Five people are competing for three prizes: \$1,000, \$500, and \$100. How many different ways can the prizes be awarded?

Example 5. How many ways are there to award gold, silver, and bronze medals to 8 contestants?

If there are n items available and we take r at a time, then the total number of permutations is given by

$$_nP_r = \frac{n!}{(n-r)!}$$

with $n \geq r$

With permutations, the order in which an item is selected matters.

- Offering various prizes
- Running a race
- Assigning officer positions
- Combination locks and passwords

Example 6. How many ways are there of selecting a president, vice president, secretary, and treasurer out of a pool of 10 candidates?

Combinations

- For permutations, order selection matters, so

  ABC, ACB, BAC, BCA, CAB, and CBA

  were all different.

- For combinations, order selection *does not* matter, so

  ABC, ACB, BAC, BCA, CAB, and CBA

  are all the same.

- Notice that there are 6, or 3!, arrangements of the letters A, B, and C.

If we have n items available and we take r at a time **without regard to order of selection**, then the total number of possible combinations are

$$_nC_r = \frac{n!}{r!(n-r)}! = \binom{n}{r}$$

Combinations

- Awarding equal prizes
- Combinations (not the lock though)
- Committees

Example 7. Five people are competing for three equal prizes. How many ways can the prizes be awarded?

Example 8. A committee of 5 is to be formed from a pool of 12 potential candidates. How many committees are possible?

Example 9. A committee of 5 is to be formed from a pool of 12 potential candidates. The committee is to be made up of 3 managers and 2 accountants. If there are 8 managers and 4 accountants available, how many committees can be formed?