Derivatives of Constants, Powers, and Sums

Summary

1.

Derivative of a Constant Function

The derivative of a constant is 0.

Example 1. Find $\frac{dy}{dx}$ for each.

(a) $f(x) = 7$

(b) $g(x) = \sqrt[3]{2}$

Power Rule

If $f(x) = x^n$, where n is any real number, then $f'(x) = nx^{n-1}$.

For radicals, remember that $\sqrt[r]{x^p} = x^{p/r}$.

Example 2. Find $\frac{dy}{dx}$ for each.

(a) $f(x) = x^4$

(b) $g(x) = x^{1.32}$

(c) $y = \sqrt{x}$

(d) $g(x) = \frac{1}{x^3}$

Constant Multiple Rule

- For $f(x) = 2x^3$, $f'(x) = 6x^2$
- For $f(x) = 3x^5$, $f'(x) = 15x^4$
- For $f(x) = -2x^4$, $f'(x) = -8x^3$

For $f(x) = c \cdot x^n$, $f'(x) = n \cdot c \cdot x^{n-1}$

Example 3. Differentiate each.

(a) $g(x) = 1.2x^5$

(b) $y = \frac{1}{7x^3}$

(c) $f(x) = \frac{2}{3}\sqrt[5]{x}$

Sum and Difference Rules

To differentiate a sum/difference of 2 (or more functions), differentiate each function separately and add/subtract the results.

Example 4. Differentiate each.

(a) $f(x) = 3x^2 + 2x - 1$

(b) $g(x) = \frac{1}{2}x^3 - \frac{3}{2}x^{-1}$

(c) $y = 3x^4 + 2\sqrt{x} - \frac{2}{x^2}$

Applications

Example 5. A coconut falls from a tree that is 75 feet tall.
Its height above the ground after t seconds is given by

$$s(t) = 75 - 16t^2$$

where $s(t)$ is measured in feet and is the **position function**.

(a) The **velocity function**, $v(t)$, is $s'(t)$. Determine $v(t)$.

(b) Compute and interpret $s(2)$ and $v(2)$.

(c) When does the coconut hit the ground?

Example 6. A refrigerator company determines the total cost function for producing fridges is

$$C(x) = 2x^2 + 15x + 1500$$

where x is the weekly production of fridges and $C(x)$ is the total cost (in dollars).
The revenue function is given as

$$R(x) = -0.3x^2 + 460x$$

where $R(x)$ is in dollars.

(a) Determine $P(x)$ and $P'(x)$.

(b) Evaluate and interpret $P(20)$ and $P'(20)$.