Rational, Radical, and Power Functions P-Set

Given each rational function

- Determine the domain
- Use a table of values to describe what happens as x approaches each restricted value listed above.

1.
$$f(x) = \frac{3x-12}{x-2}$$

2.
$$f(x) = \frac{x^2-1}{x+1}$$

3.
$$f(x) = \frac{x^2 + 2x + 1}{x^2 + 4x + 3}$$

Determine the equations for any vertical asymptote(s) and/or exact coordinates of any holes in the graph.

4.
$$f(x) = \frac{2x-16}{x-5}$$

5.
$$g(x) = \frac{12x^3}{3x}$$

6.
$$h(x) = \frac{4x^2 + 8x + 3}{2x^2 - x - 6}$$

A slightly banked highway corner will safely accommodate traffic at a speed given by the model

$$f(x) = \frac{29}{20}\sqrt{x}$$

where x represents the radius of the corner in feet and f(x) represents the speed at which traffic can safely travel in mph.

- 7. Evaluate and interpret f(300).
- 8. If the highway planners expect the traffic to travel at a speed of 45 mph, what radius should the corner be?

In a forest fire tower, the distance that an observer is able to see into the forest is related to the height of the fire tower via the function

$$g(x) = \frac{7}{5}\sqrt{x}$$

where x represents the height of the tower in feet and g(x) represents the distance that the observer can see in miles.

- 9. Evaluate and interpret g(70).
- 10. If the observer is required to see 20 miles into the forest, how high must the tower be?

The number of retired workers in the US receiving Social Security benefits can be modeled by

$$f(x) = 19.14x^{0.112}$$
 $1 \le x \le 20$

where x represents the number of years since 1979 and f(x) represents the number of retired workers in millions receiving Social Security benefits.

- 11. Evaluate and interpret f(19)
- 12. Find the average rate of change between 1995 and 1996.

Key:

1. $x \neq 2$: $(-\infty, 2) \cup (2, \infty)$; as $x \to 2$, $f(x) \to \infty$ and $f(x) \to -\infty$

2. $x \neq -1$: $(-\infty, -1) \cup (-1, \infty)$; as $x \to -1, f(x) \to -2$

3. $x \neq -3, -1$: $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$

• As $x \to -3$: $f(x) \to -\infty$ and $f(x) \to \infty$

• As $x \to -1$: $f(x) \to 0$

4. V.A. x = 5

5. Hole (0,0)

6. V.A. x = 2; Hole $\left(-\frac{3}{2}, \frac{4}{7}\right)$

7. $f(300) \approx 25.11$; At a radius of 300 ft, traffic can safely travel at speeds of about 25 mph.

8. About 963.14 ft.

9. $g(70) \approx 11.71$; A 70 ft tower will allow about 11.71 miles of visibility.

10. About 204.08 ft.

11. $f(19) \approx 26.62$; in 1998, about 26.62 million retired workers received Social Security benefits.

12. About 17.79 million retired workers.