Derivatives

Summary

1. The derivative $\left(\text{often denoted } y',\ f'(x),\ \text{ or } \dfrac{dy}{dx}\right)$ is a major component of calculus.
2. The derivative of a function is the limit as h approaches 0 of the difference quotient:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Average Rate of Change and Limits

Recall that the average rate of change of a function in an interval is the slope of the line connecting the points at those interval values:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

But what happens as the difference between x_2 and x_1 approaches 0?

In other words, what is $\lim\limits_{\Delta x \to 0}$ or what is $\lim\limits_{x_2 \to x_1}$?

Example 1. Find the average rate of change over each interval for the function $f(x) = x^2 + 1$.

(a) $[3, 3.001]$

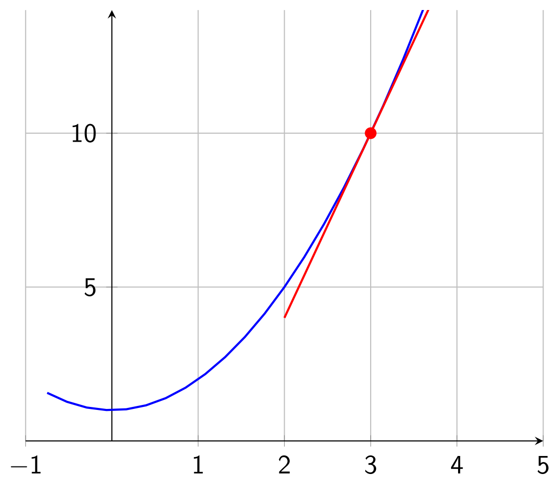
(b) $[3, 3.0001]$

(c) $[3, 3.00001]$

(d) What is $\lim\limits_{\Delta x \to 0}$?

The answer in Example 1d is the **derivative** of $f(x) = x^2 + 1$ at the point $x = 3$.

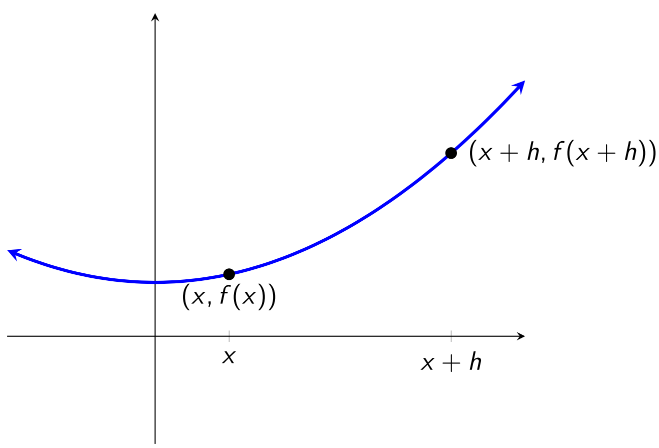
It will tell us the slope of the line tangent to the curve at $x = 3$.



But what if we want to find the derivative of a function not just at one value of x, but for <u>all</u> values of x in the function's domain?

Interpreting Derivatives as a Limit of the Difference Quotient

What is the slope of the line connecting the 2 points below?



The **difference quotient** of a function is defined as

$$\frac{f(x+h)-f(x)}{h}$$

This is a 3-step process:

1. Evaluate $f(x+h)$; a function composition
2. Subtract the given function, $f(x)$
3. Divide previous result by h and simplify

Example 2. Find the difference quotient of each.

(a) $f(x) = x^2$

(b) $f(x) = 5x^2$

(c) $f(x) = x^2 - 2x$

(d) $f(x) = 7$

The **derivative** is the limit of the difference quotient as h approaches 0.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Example 3. Find the derivative of each.

(a) $f(x) = x$

(b) $f(x) = x^2$

(c) $f(x) = 5x^2$

(d) $f(x) = x^2 - 2x$

(e) $f(x) = 7$

Other Common Notations for Derivatives: $\dfrac{dy}{dx}$ and y'

When you evaluate a derivative at an x-value, you are finding the slope of the line tangent to the curve there.

This is also called finding the *instantaneous rate of change*.

Example 4. Find each of the following.

(a) $f'(5)$ from Example **??**(a).

(b) $f'(0)$ from Example **??**(b).

(c) $f'(-9)$ from Example **??**(c).