Derivatives of Products and Quotients

Summary

1.

Product Rule

Given $f(x) \cdot g(x)$, the derivative is $f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

Example 1. Find the derivative of each.

(a) $3x^3(x^4 + 2)$

(b) $(2x^2 + 4x + 5)(5x - 4)$

(c) $\sqrt{x}\left(3x^3 - 4x^2 + 8x\right)$

Example 2. Extensive market research has determined that for the next 5 years the price of a certain mountain bike is predicted to vary according to $p(t) = 300 - 30x + 7.5t^2$, where t is time in years and $p(t)$ is the price in dollars.

The number of mountain bikes sold each year is expected to follow $q(t) = 3000 + 90t - 15t^2$, where $q(t)$ is the number sold and t is time in years.

(a) Determine $R(t)$ and $R'(t)$

(b) Compute and interpret $R'(1)$

(c) Compute and interpret $R'(4)$

Quotient Rule

Given $\dfrac{f(x)}{g(x)}$, the derivative is $\dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$, where $g(x) \neq 0$.

Example 3. Find $\dfrac{dy}{dx}$ for each.

(a) $y = \dfrac{x+3}{x-2}$

(b) $y = \dfrac{x^4 - 3x}{x^2 + 1}$

(c) $y = \dfrac{5\sqrt{x} - 6}{x + 1}$

Example 4. Researchers have determined through experimentation that the percent concentration of a certain medication can be approximated by

$$p(t) = \frac{200t}{2t^{+}5} - 4 \quad [0.25, 20]$$

where t is the time in hours after administering the medication and $p(t)$ is the percent concentration.

(a) Evaluate and interpret $p'(1)$

(b) Evaluate and interpret $p'(6)$