The Chain Rule

Summary

1.

The chain rule is used when taking the derivative of a **composition of functions.**
If $h(x) = f(g(x))$, then

$$h'(x) = f'(g(x)) \cdot g'(x)$$

Example 1. Find the derivative of each.

(a) $f(x) = (x-5)^3$ (b) $f(x) = \left(5x^3 + 3x\right)^4$ (c) $y = \left(x^2+1\right)^{15}$

We can use the chain rule to find derivatives of radical and rational functions.

Example 2. Find the derivative of each.

(a) $f(x) = \sqrt[3]{2x-4}$

(b) $y = \dfrac{5}{(2x-3)^2}$

Applications

Example 3. An actuary has determined that the number of people surviving over the duration of a century can be modeled by

$$f(x) = 400\sqrt{100-x} \quad [0, 100]$$

where x represents the age of the person in years and $f(x)$ represents the number of people surviving. Evaluate and interpret $f'(70)$.

Example 4. A company has assumed that the price-demand function for their spark plug is

$$p(x) = \frac{125}{\sqrt{2x+5}} \quad [0, 20]$$

where x represents the number of spark plugs manufactured in hundreds and $p(x)$ is the price of the spark plug.

(a) Determine and interpret $p'(20)$

(b) Determine the revenue function $R(x)$

(c) Determine and interpret $R'(20)$