Normal Probability Distribution

Summary

1. The normal probability distribution is a common probability distribution often called a "bell curve."
2. Normal probability distribution is a very common one in statistics.

The Standard Normal Distribution

A **standard normal distribution** is a continuous probability distribution, utilizing z-scores, with the following properties:

- The graph is bell-shaped
- The mean is 0
- The standard deviation is 1
- The total area under the graph is equal to 1.
- There is a correspondence between area and probability (relative frequency). Some probabilities can be found by identifying the corresponding areas in the graph.
  - For instance, the area to the left of $z = 0.24$ is denoted $P(z < 0.24)$
  - The area to the right of $z = 2.12$ is $P(z > 2.12)$
  - The area between $z = -1.41$ and $z = 1.55$ is $P(-1.41 < z < 1.55)$
  - The area of any particular z-score is 0; *i.e.* $P(z = 1)$ is 0
- The curve cannot fall below the x-axis.

Find Area/Probability Given Z-Score(s)

Pro Tip:
Sketch a curve when finding probabilities of normal distributions.

Example 1. Find each probability/area.

(a) $P(z < 0.24)$ (b) $P(z > 2.12)$ (c) $P(-1.41 < z < 1.55)$

Find Z-Score For a Given Area/Probability

Area is *cumulative* and will typically be coming in from the **left** on the graph.

Still a good idea to sketch the graphs.

Example 2. Find the z-score that corresponds to an area of 0.4216 to the left. Round your answer to 2 decimal places.

Example 3. Find the z-score that corresponds to the 90th percentile.

Example 4. Find the z-scores that correspond to each.

(a) Middle 90%

(b) Middle 95%

(c) Middle 99%

Applied Normal Probability Distribution

- We could find z-score of an observed value if we know mean and standard deviation.
- Technology allows us to avoid this process.

Example 5. A classic example of normal probability distribution is IQ scores. Most IQ tests have a mean of 100 and a standard deviation of 15. Find each probability.

(a) $P(\text{IQ} > 100)$ (b) $P(\text{IQ} \geq 100)$ (c) $P(\text{IQ} < 110)$

(d) $P(95 < \text{IQ} < 125)$ (e) $P(\text{IQ} > 135)$ (f) $P(\text{IQ} < 80)$

"People who boast about their IQ are losers."

– Stephen Hawking

Find Specific Value(s) For a Given Area/Probability

Example 6. A study was done to test the reaction time of subjects under poor lighting to simulate evening-time driving.

Subjects were to press a buzzer as soon as they saw something appear on the side of their screen.

The mean reaction time was 0.85 seconds with a standard deviation of 0.18 seconds.

Find the times corresponding to each percentile score.

(a) 25th percentile (b) 50th percentile (c) 95th percentile