Operations and Compositions of Functions

Summary

1. We can add, subtract, multiply, and divide outputs of functions just like we can with real numbers.
2. Revenue = Products sold × unit price per product.
3. Profit = Revenue − Cost.
4. The break-even point is when revenue = cost.
5. Function compositions involve plugging one function into another.

Operations on Functions

For functions $f(x)$ and $g(x)$:

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(f \cdot g)(x) = f(x) \cdot g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided that $g(x) \neq 0$

Example 1. Let $f(x) = x^2 + x - 3$ and $g(x) = 4x + 2$. Evaluate the following.

(a) $(f+g)(1)$ (b) $(f-g)(1)$ (c) $(f \cdot g)(1)$ (d) $\left(\frac{f}{g}\right)(1)$

Functions of Business

The **price-demand function** p gives the price $p(x)$ at which people buy exactly x units of a product.
The cost $C(x)$ of producing x units of a product is given by the cost function

$$C(x) = (\text{variable costs}) \cdot (\text{units produced}) + (\text{fixed costs})$$

The revenue $R(x)$ of producing x units is

$$\begin{aligned} R(x) &= (\text{quantity sold}) \cdot (\text{unit price}) \\ &= x \cdot p(x) \end{aligned}$$

Example 2. An online streaming service knows from past sales records that the weekly number of units demanded x of their content is shown below:

Demand x	**Price $p(x)$**
10	\$25
30	\$20

(a) Find the linear price function p, in slope-intercept form.

(b) Determine the revenue function, $R(x)$.

(c) Use a graphing utility to find the maximum point of the revenue function.

The profit is given by

$$\text{Profit} = \text{revenue} - \text{cost}$$
$$P(x) = R(x) - C(x)$$

Example 3. A company makes designer ties. It determines the weekly cost and revenue functions, in dollars, for producing and selling x designer ties are

$$C(x) = 30x + 50 \quad \text{and} \quad R(x) = 90x - x^2$$

(a) Determine the profit function P.

(b) Find the maximum point of the profit function.

The **break-even point** is when revenue equals cost: $R(x) = C(x)$.

Example 4. Suppose a company's revenue and cost functions are $R(x) = 25x - 0.25x^2$ and $C(x) = 2x + 5$. Find and interpret the break-even point.

Compositions of Functions

We can also use the output of one function as input into another.

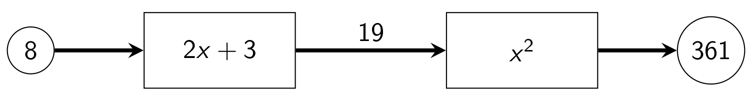
The **composition of a function f and g**, denoted $(f \circ g)(x)$, is

$$(f \circ g)(x) = f(g(x))$$

where we plug $g(x)$ into the variable for $f(x)$.

The diagram below shows the result of $f(g(8))$, in which $g(x) = 2x + 3$ and $f(x) = x^2$.

1. Evaluate $g(8)$ to get $2(8) + 3$, or 19.
2. Evaluate $f(19)$ to get 19^2, or 361.



Example 5. Suppose $f(x) = \frac{3}{x}$ and $g(x) = 2x - 1$. Evaluate each.

(a) $f(g(6))$

(b) $g(f(6))$

We will later use compositions with the Chain Rule.

An important part of that will be **decomposing** a composite function into simpler functions.

Example 6. Determine functions f and g for each composition such that $f(g(x)) = h(x)$.

(a) $h(x) = \sqrt{(x+5)^3}$

(b) $h(x) = (5x^3 + 6)^2$