Limits and Asymptotes P-Set

Estimate the following limits. Use the symbols $-\infty$ and ∞ where applicable.

1. $f(x) = \dfrac{1}{x^3}$ $\quad \lim_{x \to 0^-} f(x)$

x	-0.1	-0.01	-0.001	-0.0001	0
$f(x)$					

2. $f(x) = \dfrac{1}{(x-1)^2}$ $\quad \lim_{x \to 1^+} f(x)$

x	1	1.0001	1.001	1.01	1.1
$f(x)$					

3. $f(x) = \dfrac{x+2}{x^2 - x - 6}$ $\quad \lim_{x \to 3} f(x)$

x	2.9	2.99	2.999	2.9999	3	3.0001	3.001	3.01	3.1
$f(x)$									

State the domain of each rational function. Then find the limit *algebraically*.

4. $\lim_{x \to -2} \dfrac{x+2}{x^2 - x - 6}$
5. $\lim_{x \to 3} \dfrac{x+2}{x^2 - x - 6}$
6. $\lim_{x \to 3} \dfrac{x-3}{x^2 - 9}$
7. $\lim_{x \to 1} \dfrac{x-1}{x^2 - 1}$

The cost $C(x)$ in thousands of dollars of removing x% of a city's pollution discharged into a lake is given by

$$C(x) = \frac{223x}{100 - x}$$

8. Determine a reasonable domain for $C(x)$
9. Evaluate and interpret $C(40)$
10. Determine and interpret $\lim_{x \to 100^-} C(x)$

Pharmacological studies have determined that the amount of medication present in the body is a function of the amount given and how much time has elapsed since the medication was administered. For a certain medication, the amount present $A(t)$, in mL, can be approximated by the function

$$A(t) = 3.5e^{-0.3t} \quad t \geq 0$$

where t is the number of hours since the medication was administered.

11. Determine and interpret $A(0)$
12. Determine and interpret $\lim_{t \to \infty} A(t)$

Evaluate each of the following.

13. $\lim_{x \to -\infty} \dfrac{2x+5}{x-1}$
14. $\lim_{x \to \infty} \dfrac{3x^2 - x + 2}{2x^2 + x - 5}$
15. $\lim_{x \to -\infty} \dfrac{3x^2 - 2x + 5}{2x^3 + x^2 - 2x + 3}$

The average cost of producing x units of a product is given by

$$AC(x) = \frac{15,325 + 7.11x}{x}$$

16. Determine and interpret $\lim_{x \to \infty} AC(x)$

Key:

1. $-\infty$
2. ∞
3. Does not exist
4. Domain: $x \neq -2, 3$; $-\frac{1}{5}$
5. Domain: $x \neq -2, 3$; Does not exist
6. Domain: $x \neq -3, 3$; $\frac{1}{6}$
7. Domain: $x \neq -1, 1$; $\frac{1}{2}$
8. $0 \leq x < 100$
9. $C(40) \approx 148.7$; removing 40% of the lake's pollution will cost about \$148,700
10. $\lim_{x \to 100^-} C(x) = \infty$; The costs increase indefinitely as the percent of pollution removed increases towards 100%
11. $A(0) = 3.5$; 3.5 mL of medication were administered
12. $\lim_{t \to \infty} A(t) = 0$; as time progresses, the amount of medication in the system approaches 0 mL
13. 2
14. $\frac{3}{2}$
15. 0
16. $\lim_{x \to \infty} AC(x) = 7.11$; as the number of units produced increases, the average cost per unit approaches \$7.11