Exponential Functions

Summary

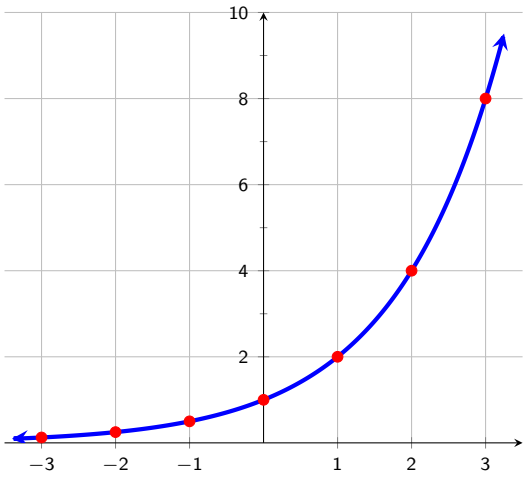
1. Exponential functions are in the form $f(x) = b^x$, where $b > 0$ and $b \neq 1$.
2. The value of b determines if the function is exponential growth or exponential decay.
3. Exponential functions will have a horizontal asymptote in one direction.
4. A special base is base $e \approx 2.718282$.

Exponential Function

An **exponential function** is a function in which each successive output value is obtained by **multiplying** the previous one by a constant value.

One example of an exponential function is the **doubling function**

$$f(x) = 2^x$$



x	$f(x)$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

As the values of $x \to -\infty$, $\quad 2^{\text{very big negative number}} \to 0$

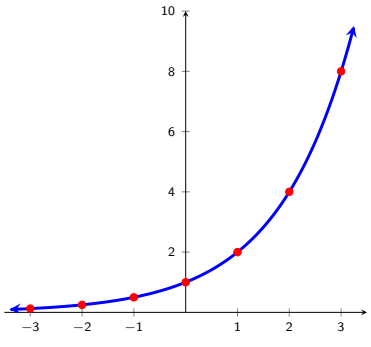
For instance, $2^{-50} \approx 0.0000000000000008882$

As the values of $x \to \infty$, $\quad 2^{\text{very big positive number}} \to \infty$

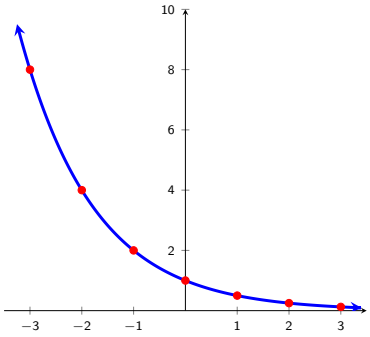
For instance, $2^{50} = 1,125,899,906,842,624$

A function in the form $f(x) = b^x$ where b is a fixed real number, $b > 0, b \neq 1$ is called an **exponential function of base b**

$f(x) = 2^x$



$g(x) = \left(\frac{1}{2}\right)^x$



Properties of Exponential Functions

For $f(x) = b^x$:

- Domain is $(-\infty, \infty)$ and the range is $(0, \infty)$
- $(0, 1)$ is on the graph of f and $y = 0$ is a horizontal asymptote.

Example 1. The value of a car can be modeled by $V(x) = 25\left(\frac{4}{5}\right)^x$, where $x \geq 0$ is the age of the car in years and $V(x)$ is the value in thousands of dollars.

(a) Find and interpret $V(0)$

(b) Find and interpret the horizontal asymptote of the graph of $V(x)$.

Interest

Simple Interest, I

- Paid out at *one moment in time*.
- P dollars deposited into an account
- Interest rate r (convert percent to decimal)
- Time t

$$\text{Interest} = \text{Principal} \cdot \text{rate} \cdot \text{time}$$
$$I = Prt$$

Compound Interest, A

- Paid out on a *regular basis*.
- k times per year

$$A = P\left(1 + \frac{r}{k}\right)^{kt}$$

Example 2. Suppose $20,000 is deposited into a retirement account that yields an interest rate of 6.5% compounded quarterly.

(a) How much will be in the account after 3 years?

(b) How much total interest was earned after 3 years?

(c) How much will be in the account after 40 years?

Special Base e

Of all possible bases for exponential functions,
the most common is the irrational base e (**natural base**).

Example 3. Evaluate $f(x) = \left(1 + \frac{1}{x}\right)^x$ for very large values of x.

The **exponential function** has the form

$$f(x) = a \cdot e^{bx}$$

where a and b are real numbers.

Continuous Compounded Interest

- Limits the amount of interest earned through compounding.
- Called *continuous compounded interest*
- $A = Pe^{rt}$

Example 4. This time \$20,000 is deposited at 6.5% compounded continuously.

(a) How much will be in the account after 3 years?

(b) How much total interest was earned after 3 years?

(c) How much will be in the account after 40 years?

(d) What is the average rate of change from 3 to 40 years?