Rational, Radical, and Power Functions

Summary

1. The denominator of a rational function can never equal 0.

2. Each x-value that causes this is either a vertical asymptote or a hole in the graph.

3. You can't take even roots of negative numbers.

Rational Functions

Rational functions have the form $y = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$.

Example 1. Given the rational function $f(x) = \frac{2x}{x-3}$

(a) Determine the domain.

(b) Complete the tables below and describe what happens as x approaches 3.

x	$f(x)$
2	
2.9	
2.99	
2.999	
2.9999	

x	$f(x)$
4	
3.1	
3.01	
3.001	
3.0001	

Notice how the output values get out of control:

$$f(x) \to -\infty \quad \text{or} \quad f(x) \to \infty$$

Since x can't equal 3, the graph of $\frac{2x}{x-3}$ will never cross the vertical line $x = 3$.

This line is called a **vertical asymptote**.

At each of the values that cause the denominator to $= 0$,
there will be **only one of two things there**:

- A vertical asymptote

- Or a hole in the graph

How to Tell Your Asymptote From a Hole In the Graph:

1. **Factor** both numerator and denominator ****completely**** and then **simplify that**.

2. For **each domain issue** in the underlined denominator, evaluate the **simplified function** at those x-values:

   - If you still get a 0 in the denominator (typically an error message) you have a **vertical asymptote**.
   - Otherwise, there is a hole in the graph there (y-coordinate is that output value)

Example 2. Determine the equations for any vertical asymptote(s) and/or exact coordinates of any holes in the graph.

(a) $\frac{x^2-4}{x^2-x-2}$

(b) $\frac{x+5}{x^2+8x+15}$

Radical Functions

- *Can't* take square roots (or even roots in general) of negative numbers: $\sqrt{} \geq 0$

- *Can* take cube roots (or odd roots in general) of any number.

Example 3. A sapling of a type of tree grows according to the equation

$$g(x) = 10\sqrt{x} + 0.75$$

where x is the number of years since planting and $g(x)$ is the height of the tree.

(a) How tall will it be on its 20th birthday?

(b) How long will it take to be 30 feet tall?

Power Functions

- Has form $f(x) = a \cdot x^b$

- If a is positive:

    - Opens downward if $0 < b < 1$
    - Opens upward if $b > 1$

Example 4. The amount of money spent (in billions) on R&D each year is given by

$$f(x) = 19.66x^{0.74} \quad 5 \leq x \leq 21$$

where x is the number of years since 2000.

(a) Evaluate and interpret $f(12)$

(b) Find the average rate of change for the amount spent between 2008 and 2012.