Derivatives of Exponential Functions

Summary

1.

The derivative of $f(x) = e^x$ is $f'(x) = e^x$.

The function $f(x) = e^x$ *is its own derivative.*

Example 1. Find the derivative of each.

(a) $y = 7e^x$

(b) $y = x^4 \cdot e^x$

(c) $g(x) = \dfrac{x^2}{e^x}$

(d) $f(x) = \ln(e^x)$

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Chain Rule for the Exponential Function
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If $f(x)$ is differentiable, then the derivative of $h(x) = e^{f(x)}$ is $h'(x) = e^{f(x)} \cdot f'(x)$

Example 2. Find $\dfrac{dy}{dx}$ for each.

(a) $y = e^{3x-3}$

(b) $y = e^{6x-0.5x^6}$

(c) $g(x) = e^{1-\ln(x)}$

(d) $f(x) = \ln\left(e^{3x} - 8\right)$

Applications

The world's population during the 20th century closely follows the equation

$$p(t) = 1.419e^{0.014t} \quad [0, 100]$$

where t represents the number of years since 1900 and $p(t)$ represents the world's population (in billions).

Evaluate and interpret $p'(10)$ and compare it to $p'(99)$.