Probability: AND

Summary

1. Typically, the word and indicates multiplication.

2. In general, $P(A \text{ and } B = P(A) \times P(B|A)$.

Example 1. You flip a coin and then roll a single die. What is the probability that you flip heads **and** roll a 5?

Multiplication Rule

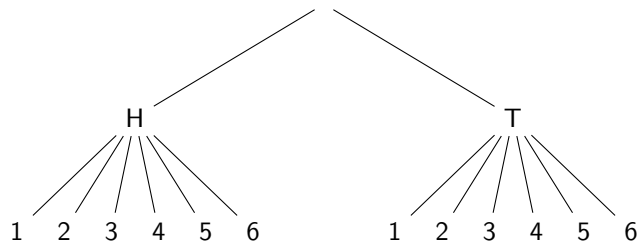In the previous example, the probability of flipping heads was $\frac{1}{2}$

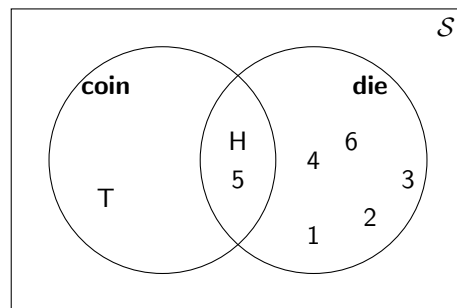The probability of rolling a 5 was $\frac{1}{6}$
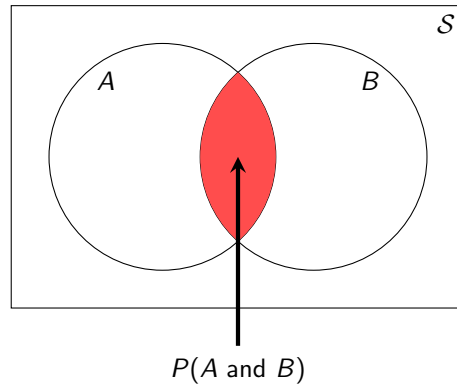
Multiplication Rule

If $P(A)$ is the probability of event A occurring, and $P(B)$ is the probability of event B occurring, then
$$P(A \text{ and } B) = P(A) \times P(B)$$

Tree Diagrams

Venn Diagrams: AND

$$P(A \text{ and } B)$$

Mutually Exclusive Events

Two events are **mutually exclusive** if they can not happen together. In other words,

$$P(A \text{ and } B) = 0$$

Independent Events

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Independent Events

Two events are **independent** if the outcome of the second is not affected by the first happening.

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$$P(A \text{ and } B) = P(A) \times P(B)$$

When selecting items from a collection, independent events often contain selections made **with replacement**.

Example 2. A jar contains 10 blue, 12 black, and 15 red marbles.

(a) What is the probability of selecting a black marble, putting it back, and then selecting a blue marble?

(b) What is the probability of selecting a red marble, putting it back, and then selecting another red marble?

Example 3. A certain blood test can determine the presence of a bloodborne pathogen 97% of the time (that is, if 100 people have the pathogen, the test will confirm true for 97 of them). If 4 people with the pathogen are given the test, find the probability that the test is accurate for all of them.

Conditional Probability

The notation for the probability that event B occurs given that event A has occurred is

$$P(B \mid A)$$

With conditional probability, the denominator will often be the total of something that follows the words "if", "suppose", or "given that".

Example 4. The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	3–5	6–10	Over 10	Total
Import	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a domestic car is randomly selected, what is the probability that it is 6–10 years old?

(b) What is the probability of selecting a domestic car given that the car is 6–10 years old?

(c) Suppose a new car is selected, what is the probability that it is an import?

Dependent Events

Dependent events utilize conditional probability:

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

Note: When selecting items from a collection, dependent events often contain selections made **without replacement**.

Example 5. You are dealt a card from a standard deck and then you are dealt another (without replacement). Find each probability.

(a) The first card is an ace and the second card is a ten.

(b) The first card is an ace and the second card is an ace.

Conditional Probability Revisited

The formula for dependent events

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

leads us to the following formula for conditional probability:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

When finding AND probabilities using tabular data

- Look for the intersection of a row and column.

- Two rows (likewise, two columns) will never intersect, so their probabilities are *mutually exclusive*.

Example 6. The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	3–5	6–10	Over 10	Total
Import	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a car is randomly selected, what is the probability that it is a 6–10 year old import?

(b) If a car is randomly selected, what is the probability that it is a domestic car that is 0–2 years old?

(c) If a car is randomly selected, what is the probability that it is 0–2 years and 6–10 years old?