Indefinite Integrals

Summary

1.

The WorkShop Company determines the marginal cost function for their designer suspenders is

$$MC(x) = C'(x) = 2x \quad [0, 10]$$

where x is the number of designer suspenders produced in thousands and $MC(x)$ is the marginal cost in thousands of dollars.

- Given this information, how can we find the cost function?
- We need to find a function $C(x)$ such that

$$C'(x) = MC(x)$$

- What function, when we take its derivative, gives us $2x$?
- That function is the **antiderivative** of the function.

  Antidifferentiation is the inverse of differentiation.

Function	Derivative	Rule
$f(x) = x^n$	$f'(x) = n \cdot x^{n-1}$	Power Rule
$f(x) = k$	$f'(x) = 0$	Constant Rule
$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$	Sum and Difference Rules

What function has a derivative of $2x$?
Other functions with derivative of $2x$:

- $x^2 + 3$
- $x^2 - 1.7$
- $x^2 + \frac{1}{5}$

If C is any real number, then $x^2 + C$ is the antiderivative of $2x$.
C is called an **arbitrary constant**.

Example 1. Determine if the function F is the general antiderivative of the function f.

(a) $F(x) = \frac{2}{3}x^{3/2} + 4x + C$; $f(x) = \sqrt{x} + 4$

(b) $f(x) = 2x^4 - x + C$; $f(x) = \frac{2}{3}x^3 - 1$

Another way to represent the general antiderivative of a function f is by the **indefinite integral**

$$\int f(x)\,dx$$

Note that, from Example 1a, $\int \left(\sqrt{x} + 4\right)\,dx = \frac{2}{3}x^{3/2} + 4x + C$

`Power Rule for Integration`
For $n \neq -1$,

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

Example 2. Determine the following indefinite integrals.

(a) $\int x^8 \, dx$

(b) $\int \sqrt[4]{x} \, dx$

(c) $\int \frac{1}{x^5} \, dx$

`Constant Rule for Integration`
If k is a real number, then

$$\int k \, dx = kx + C$$

`Sum and Difference Rules for Integration`

$$\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

Example 3. Determine each indefinite integral.

(a) $\int \left(x^2 + 3\right) \, dx$

(b) $\int \left(\sqrt[3]{x} + 5\right) \, dx$

`Coefficient Rule for Integration`

$$\int c \cdot f(x) \, dx = c \cdot \int f(x) \, dx$$

Example 4. Determine $\int \left(-2t^3 + 3t + 5\right) \, dt$