Measures of Center

Objectives

1 Calculate the mean, median, and mode of a dataset

2 Calculate the weighted mean of a dataset

- 3 Approximate the mean for a grouped dataset
- Determine if a dataset appears skewed left, skewed right, or normal

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In some instances, this can give us a good value to expect from that dataset.

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When most people use the term *average*, they are referring to the mean.

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- The mean of a data set uses every value, unless the mean is a *trimmed mean*.
- One extreme value (called an outlier) can change the value of the mean drastically.

Mean Formula

Sample mean: $\overline{\mathbf{x}}$ Population mean: μ

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$$\bar{\mathbf{x}}$$
, or μ , $=\frac{\sum x_i}{n} = \frac{1}{n} \sum x_i$

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The median is denoted by \tilde{x} , however, some technologies uses Med to denote it.

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The mode is the only measure of center that is applicable to qualitative data.

The dataset below represents the number of complaints I receive each week about my teaching.

Calculate the mean, median, and mode of the number of complaints.

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Median number of complaints per week is 4.5

Mode number of complaints per week are 4 and 7.

The next week, I received 400 complaints. Re-calculate the mean, median, and mode now.

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California has a mean class size of 20.9 students per teacher and Alaska has a mean of 16.8 students per teacher.

If we combine the two states, we might find the mean number of students per teacher to be 18.85 (0.5 * (20.9 \pm 16.8)) but is this result correct? Why or why not?

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It is not correct because California has a much higher population that Alaska. We would have to find what is known as the **weighted mean**.

Weighted Mean

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- Add those results.
- **1** Then divide that by the total of the weights.

You've recently completed a semester. Determine the semester's GPA (A = 4pts, B = 3pts, etc).

Course	Grade	Credit Hours
Statistics	А	4
Advanced Chris Farley	Α	3
Airplane! Quotes	В	5
Obnoxious Examples	С	3

Grade	Credit Hours	TOTALS
4	4	16
4	3	12
3	5	15
2	3	6
	15	49

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Weighted Mean: 49/15 = 3.267

In a statistics course, tests count for 60% of the final grade, homework for 20% and midterm and final exams are 10% each. Suppose you've earned an 87% average on tests, 94% average on homeworks and a 77% average on the exams.

What is your overall percentage?

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Assessment	Your Scores	Grade Weights	TOTALS
Tests	0.87	0.60	0.522
Homework	0.94	0.20	0.188
Exams	0.77	0.20	0.154

Grade Weights	TOTALS
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Grade: 0.864/1 = 86.4%

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We will use similar techniques like we did in finding the weighted mean, however we will have to use the class midpoints as our observed values.

(I swear this is a true story) On one very hungry day of mine, I ordered and consumed 19 sushi rolls from an all-you-can-eat sushi restaurant. This, as you might guess, is not typical (at least for me). The table below indicates the frequencies of sushi rolls I typically eat.

Number of Rolls	Frequency
1 – 5	4
6 – 10	17
11 – 15	12

Estimate the mean number of rolls consumed.

Class Midpoint	Frequency	TOTALS
3	4	12
8	17	136
13	12	156
	33	304

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13	12	156
	33	304

Mean: $304/33 \approx 9.2$ rolls per visit.

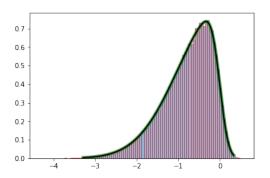
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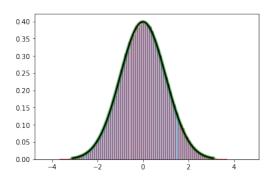
When normally distributed (bell-shaped), the mean, median, and mode are (roughly) equal. However, some data sets my be *skewed* (remember, skewness refers to the tail).

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If the mean is (significantly) less than the median, the data is skewed left (or negatively skewed).



If the mean is (approximately) equal to the median, the data is **normal** (no skewness)



If the mean is (significantly) greater than the median, the data is **skewed right** (or **positively skewed**).

