Probability: AND

Objectives

1. Calculate probabilities using the Multiplication Rule
2. Find probabilities of independent events
3. Find conditional probabilities
4. Find probabilities of dependent events

Example 1

You flip a coin and then roll a single die. What is the probability that you flip heads **and** roll a 5?

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Sample space:

	1	**2**	**3**	**4**	**5**	**6**
Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

Example 1

You flip a coin and then roll a single die. What is the probability that you flip heads **and** roll a 5?

Sample space:

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Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

$$P(\text{heads and } 5) = \frac{1}{12}$$

Multiplication Rule

In the previous example, the probability of flipping heads was $\frac{1}{2}$

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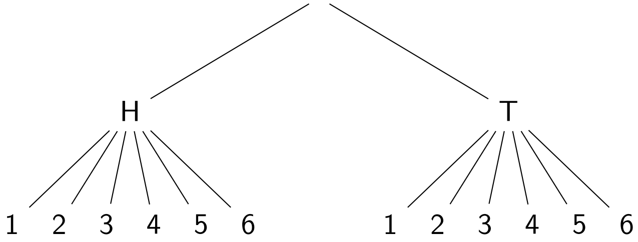
Multiplication Rule

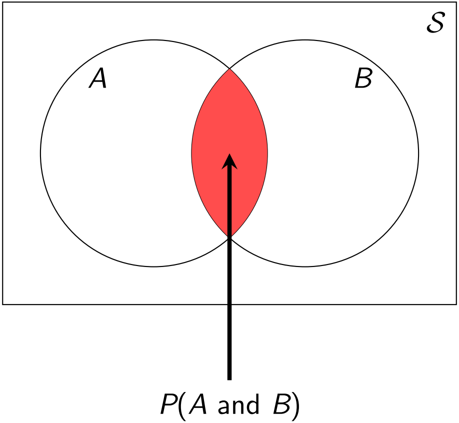
If $P(A)$ is the probability of event A occurring, and $P(B)$ is the probability of event B occurring, then

$$P(A \text{ and } B) = P(A) \times P(B)$$

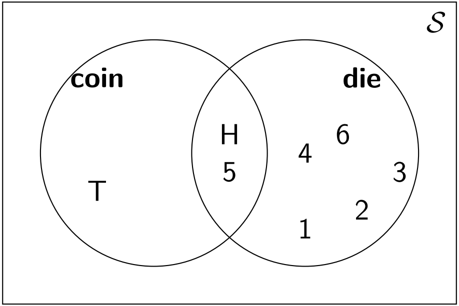
Tree Diagram



Venn Diagram – AND



Venn Diagram – AND



Mutually Exclusive Events

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Two events are **mutually exclusive** if they can not happen together. In other words,

$$P(A \text{ and } B) = 0$$

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When selecting items from a collection, independent events often contain selections made with replacement.

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A jar contains 10 blue, 12 black, and 15 red marbles.

(a) What is the probability of selecting a black marble, putting it back, and then selecting a blue marble?

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$$P(\text{black and blue}) = \frac{12}{37} \times \frac{10}{37}$$

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(a) What is the probability of selecting a black marble, putting it back, and then selecting a blue marble?

$$\begin{aligned} P(\text{black and blue}) &= \frac{12}{37} \times \frac{10}{37} \\ &= \frac{120}{1,369} \end{aligned}$$

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A jar contains 10 blue, 12 black, and 15 red marbles.

(b) What is the probability of selecting a red marble, putting it back, and then selecting another red marble?

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$$P(\text{red and red}) = \frac{15}{37} \times \frac{15}{37}$$

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A jar contains 10 blue, 12 black, and 15 red marbles.

(b) What is the probability of selecting a red marble, putting it back, and then selecting another red marble?

$$
\begin{aligned}
P(\text{red and red}) &= \frac{15}{37} \times \frac{15}{37} \\
&= \frac{225}{1,369}
\end{aligned}
$$

Example 3

A certain blood test can determine the presence of a bloodborne pathogen 97% of the time (that is, if 100 people have the pathogen, the test will confirm true for 97 of them). If 4 people with the pathogen are given the test, find the probability that the test is accurate for all of them.

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$$P(\text{accurate for all}) = P(\text{first}) \cdot P(\text{second}) \cdot P(\text{third}) \cdot P(\text{fourth})$$

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\begin{aligned}
P(\text{accurate for all}) &= P(\text{first}) \cdot P(\text{second}) \cdot P(\text{third}) \cdot P(\text{fourth}) \\
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There is about an 88.53% chance the test is accurate for all four people.

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With conditional probability, the denominator will often be the total of something that follows the words "if", "suppose", or "given that".

Example 4

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a domestic car is randomly selected, what is the probability that it is 6–10 years old?

Example 4

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

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$$P(\text{6–10 years old given that it is a domestic car}) = \frac{12}{100}$$

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(a) If a domestic car is randomly selected, what is the probability that it is 6–10 years old?

$$P(\text{6–10 years old given that it is a domestic car}) = \frac{12}{100}$$

$$= \frac{3}{25}$$

Example 4

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(b) What is the probability of selecting a domestic car given that the car is 6–10 years old?

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	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(b) What is the probability of selecting a domestic car given that the car is 6–10 years old?

$$P(\text{domestic car given that it is 6–10 years old}) = \frac{11}{23}$$

Example 4

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
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(c) Suppose a new car is selected, what is the probability that it is a foreign car?

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	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
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(c) Suppose a new car is selected, what is the probability that it is a foreign car?

$$P(\text{foreign car given that it is 0–2 years old}) = \frac{37}{72}$$

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$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

Note: When selecting items from a collection, dependent events often contain selections made without replacement.

Example 5

You are dealt a card from a standard deck and then you are dealt another (without replacement). Find the probability that

(a) The first card is an ace and the second card is a ten.

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(a) The first card is an ace and the second card is a ten.

$$\begin{aligned} P(\text{ace and ten}) &= P(\text{ace}) \times P(\text{ten}) \\ &= \frac{4}{52} \times \frac{4}{51} \end{aligned}$$

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You are dealt a card from a standard deck and then you are dealt another (without replacement). Find the probability that

(a) The first card is an ace and the second card is a ten.

$$
\begin{aligned}
P(\text{ace and ten}) &= P(\text{ace}) \times P(\text{ten}) \\
&= \frac{4}{52} \times \frac{4}{51} \\
&= \frac{4}{663} \\
&\approx 0.603\%
\end{aligned}
$$

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(b) The first card is an ace and the second card is an ace.

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$$= \frac{4}{52} \times \frac{3}{51}$$

Example 5

(b) The first card is an ace and the second card is an ace.

$$
\begin{aligned}
P(\text{ace and ace}) &= P(\text{ace}) \times P(\text{ace}) \\
&= \frac{4}{52} \times \frac{3}{51} \\
&= \frac{1}{221} \\
&\approx 0.452\%
\end{aligned}
$$

Conditional Probability Revisited

The formula for dependent events

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

leads us to the following formula for conditional probability:

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leads us to the following formula for conditional probability:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Tabular Data

When finding `AND` probabilities using tabular data, look for the intersection of a row and column.

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When finding `AND` probabilities using tabular data, look for the intersection of a row and column.

Two rows (likewise, two columns) will never intersect, so their probabilities are *mutually exclusive*.

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The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	**3–5**	**6–10**	**Over 10**	**Total**
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(a) If a car is randomly selected, what is the probability that it is a 6–10 year old foreign car?

Example 6

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
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$$P(\text{6–10 years old and foreign}) = \frac{12}{200}$$

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Foreign	37	21	12	30	100
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Total	72	44	23	61	200

(a) If a car is randomly selected, what is the probability that it is a 6–10 year old foreign car?

$$P(\text{6–10 years old and foreign}) = \frac{12}{200}$$

$$= \frac{3}{50}$$

Example 6

The reasoning behind our answer for Example 6a:

$$P(\text{6–10 and foreign}) = P(\text{6–10}) \times P(\text{foreign} \mid \text{6–10 years old})$$

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The reasoning behind our answer for Example 6a:

$$\begin{aligned} P(6\text{–}10 \text{ and foreign}) &= P(6\text{–}10) \times P(\text{foreign} \mid 6\text{–}10 \text{ years old}) \\ &= \frac{23}{200} \times \frac{12}{23} \end{aligned}$$

Example 6

The reasoning behind our answer for Example 6a:

$$\begin{aligned} P(\text{6–10 and foreign}) &= P(\text{6–10}) \times P(\text{foreign} \mid \text{6–10 years old}) \\ &= \frac{23}{200} \times \frac{12}{23} \\ &= \frac{12}{200} \end{aligned}$$

Example 6

The reasoning behind our answer for Example 6a:

$$
\begin{aligned}
P(\text{6–10 and foreign}) &= P(\text{6–10}) \times P(\text{foreign} \mid \text{6–10 years old}) \\
&= \frac{23}{200} \times \frac{12}{23} \\
&= \frac{12}{200} \\
&= \frac{3}{50}
\end{aligned}
$$

Example 6

	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
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(b) If a car is randomly selected, what is the probability that it is a domestic car that is 0–2 years old?

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Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
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(b) If a car is randomly selected, what is the probability that it is a domestic car that is 0–2 years old?

$$P(\text{0–2 years old and domestic}) = \frac{35}{200}$$

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	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
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(b) If a car is randomly selected, what is the probability that it is a domestic car that is 0–2 years old?

$$
\begin{aligned}
P(\text{0–2 years old and domestic}) &= \frac{35}{200} \\
&= \frac{7}{40}
\end{aligned}
$$

Example 6

	0–2	3–5	6–10	Over 10	Total
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(c) If a car is randomly selected, what is the probability that it is 0–2 years and 6–10 years old?

$$P(\text{0–2 years and 6–10 years old}) = \frac{0}{200}$$

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	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
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(c) If a car is randomly selected, what is the probability that it is 0–2 years and 6–10 years old?

$$P(\text{0–2 years and 6–10 years old}) = \frac{0}{200}$$

$$= 0$$