

Sampling Distributions

Objectives

- 1 Obtain a sampling distribution of sample means
- 2 Determine the mean and standard error of a sampling distribution
- 3 Understand the Central Limit Theorem
- 4 Determine the mean and standard error for sampling distribution of proportions

Sampling Distributions

In statistics, we take samples in order to gather information regarding a population.

Sampling Distributions

In statistics, we take samples in order to gather information regarding a population.

For instance, let's say we have the following population prices of laptops: \$1000, \$1200, \$1600, and \$2000.

Sampling Distributions

In statistics, we take samples in order to gather information regarding a population.

For instance, let's say we have the following population prices of laptops: \$1000, \$1200, \$1600, and \$2000.

We can take a simple random sample of, say 2 computers, and create a frequency distribution of each sample.

Sampling Distributions

In statistics, we take samples in order to gather information regarding a population.

For instance, let's say we have the following population prices of laptops: \$1000, \$1200, \$1600, and \$2000.

We can take a simple random sample of, say 2 computers, and create a frequency distribution of each sample.

Note: We will sample with replacement. Differences in sampling with and without replacement become negligent as sample sizes increase.

Example 1

Obtain a sampling distribution, taking 2 at a time, of the laptop prices \$1000, \$1200, \$1600, and \$2000. Then find the mean of each sample.

Example 1

Obtain a sampling distribution, taking 2 at a time, of the laptop prices \$1000, \$1200, \$1600, and \$2000. Then find the mean of each sample.

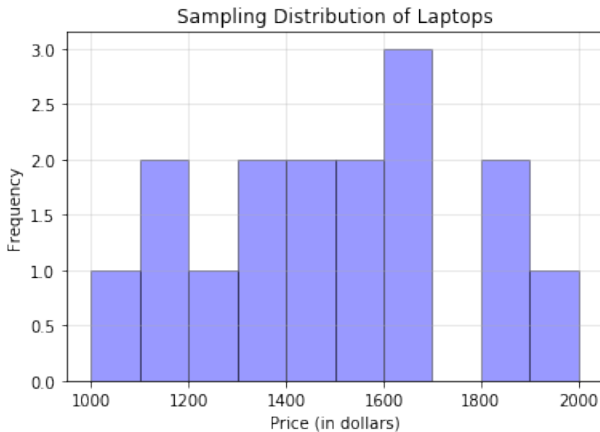
Sample	Sample Mean	Sample	Sample Mean
1000, 1000	1000	1600, 1000	1300
1000, 1200	1100	1600, 1200	1400
1000, 1600	1300	1600, 1600	1600
1000, 2000	1500	1600, 2000	1800
1200, 1000	1100	2000, 1000	1500
1200, 1200	1200	2000, 1200	1600
1200, 1600	1400	2000, 1600	1800
1200, 2000	1600	2000, 2000	2000

Example 2

Create a histogram of the sample means from Example 1.

Example 2

Create a histogram of the sample means from Example 1.



Example 3

Determine the mean and standard deviation of the sample means.

Example 3

Determine the mean and standard deviation of the sample means.

Mean = \$1450

Example 3

Determine the mean and standard deviation of the sample means.

Mean = \$1450

Std. Dev \approx \$271.57

Objectives

- 1 Obtain a sampling distribution of sample means
- 2 Determine the mean and standard error of a sampling distribution
- 3 Understand the Central Limit Theorem
- 4 Determine the mean and standard error for sampling distribution of proportions

Larger Population and Sample Sizes

What happens if we have a much larger population and take a larger sample size, such as 100?

Larger Population and Sample Sizes

What happens if we have a much larger population and take a larger sample size, such as 100?

Notice that the mean of the sample means targets the population mean.

Larger Population and Sample Sizes

What happens if we have a much larger population and take a larger sample size, such as 100?

Notice that the mean of the sample means targets the population mean.

$$\mu \approx \mu_{\bar{x}}$$

Larger Population and Sample Sizes

What happens if we have a much larger population and take a larger sample size, such as 100?

Notice that the mean of the sample means targets the population mean.

$$\mu \approx \mu_{\bar{x}}$$

Also, the standard deviation is around 1/10 the population standard deviation.

Larger Population and Sample Sizes

What happens if we have a much larger population and take a larger sample size, such as 100?

Notice that the mean of the sample means targets the population mean.

$$\mu \approx \mu_{\bar{x}}$$

Also, the standard deviation is around 1/10 the population standard deviation.

$$\sigma \approx \frac{\sigma_{\bar{x}}}{\sqrt{n}}$$

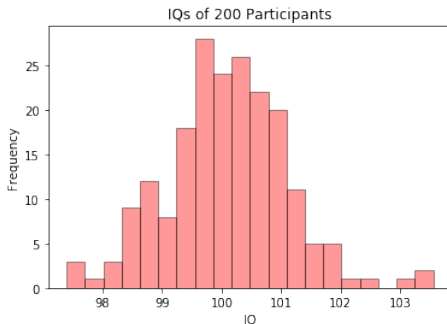
where $\frac{\sigma}{\sqrt{n}}$ is called the **standard error of the mean**.

Example 4

IQ scores are normally distributed with a mean of 100 and standard deviation of 16. A sample of 200 participants had their IQs measured.

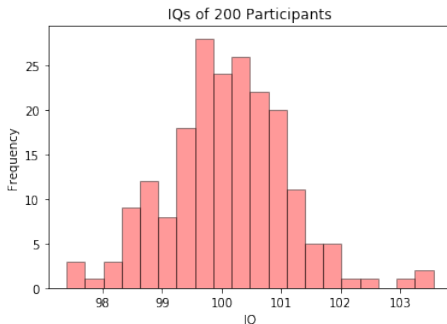
Example 4

IQ scores are normally distributed with a mean of 100 and standard deviation of 16. A sample of 200 participants had their IQs measured.



Example 4

IQ scores are normally distributed with a mean of 100 and standard deviation of 16. A sample of 200 participants had their IQs measured.



What are the approximate mean and standard error of the sample?

Example 4

The mean of the sample will target the population mean of 100.

Example 4

The mean of the sample will target the population mean of 100.

The standard error is

Example 4

The mean of the sample will target the population mean of 100.

The standard error is

$$\frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{200}}$$

Example 4

The mean of the sample will target the population mean of 100.

The standard error is

$$\frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{200}}$$
$$\approx 1.13$$

Objectives

- 1 Obtain a sampling distribution of sample means
- 2 Determine the mean and standard error of a sampling distribution
- 3 Understand the Central Limit Theorem
- 4 Determine the mean and standard error for sampling distribution of proportions

Sample Means of Any Distribution

We can create a histogram of a population, but how will our histograms of sample means look for various populations?

Sample Means of Any Distribution

We can create a histogram of a population, but how will our histograms of sample means look for various populations?

It turns out that the distributions of sample means for any population will be normal as our sample sizes increase.

Central Limit Theorem

As the sample size increases, the distribution of sample means becomes normal with a mean of μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ (the standard error).

Example 5

IQ scores are normally distributed with a mean of 100 and standard deviation of 16.

(a) What is the probability that an individual has an IQ greater than 102?

Example 5

IQ scores are normally distributed with a mean of 100 and standard deviation of 16.

(a) What is the probability that an individual has an IQ greater than 102?

$$P(x > 102) \approx 0.4503$$

Example 5

IQ scores are normally distributed with a mean of 100 and standard deviation of 16.

(a) What is the probability that an individual has an IQ greater than 102?

$$P(x > 102) \approx 0.4503$$

There is about a 45.03% chance that an individual has an IQ greater than 102.

Example 5

IQ scores are normally distributed with a mean of 100 and standard deviation of 16.

(a) What is the probability that an individual has an IQ greater than 102?

$$P(x > 102) \approx 0.4503$$

There is about a 45.03% chance that an individual has an IQ greater than 102.

We could also calculate the probability using the z score:

$$z = \frac{x - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Example 5

(b) What is the probability that in a sample of 50 people, the mean IQ is greater than 102?

Example 5

(b) What is the probability that in a sample of 50 people, the mean IQ is greater than 102?

The standard error of the mean is $\frac{16}{\sqrt{50}} \approx 2.263$.

Example 5

(b) What is the probability that in a sample of 50 people, the mean IQ is greater than 102?

The standard error of the mean is $\frac{16}{\sqrt{50}} \approx 2.263$.

$$P(\bar{x} > 102) \approx 0.1884$$

Example 5

(b) What is the probability that in a sample of 50 people, the mean IQ is greater than 102?

The standard error of the mean is $\frac{16}{\sqrt{50}} \approx 2.263$.

$$P(\bar{x} > 102) \approx 0.1884$$

There is about an 18.84% chance the mean IQ of a sample of 50 people is greater than 102.

Example 5

(c) What is the probability that in a sample of 1000 people, the mean IQ is greater than 102?

Example 5

(c) What is the probability that in a sample of 1000 people, the mean IQ is greater than 102?

The standard error of the mean is $\frac{16}{\sqrt{1000}} \approx 0.5051$.

Example 5

(c) What is the probability that in a sample of 1000 people, the mean IQ is greater than 102?

The standard error of the mean is $\frac{16}{\sqrt{1000}} \approx 0.5051$.

$$P(\bar{x} > 102) \approx 0.0000386$$

Example 5

(c) What is the probability that in a sample of 1000 people, the mean IQ is greater than 102?

The standard error of the mean is $\frac{16}{\sqrt{1000}} \approx 0.5051$.

$$P(\bar{x} > 102) \approx 0.0000386$$

There is about a 0.00386% chance that the mean IQ of a sample of 1000 people is greater than 102.

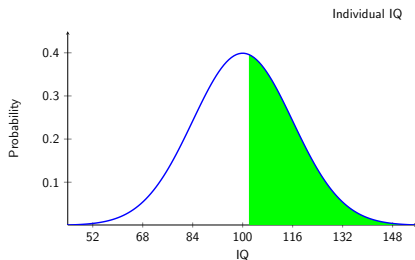
Reflections on Example 5

By increasing the sample size, we decreased the probability. This is due to the original standard deviation now being divided by a larger number (which will decrease its overall value).

Reflections on Example 5

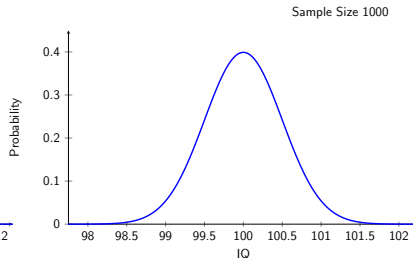
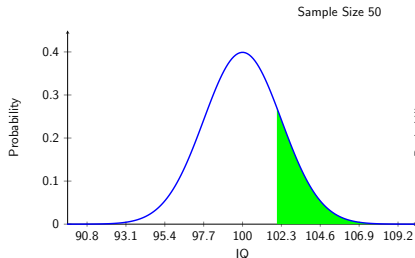
By increasing the sample size, we decreased the probability. This is due to the original standard deviation now being divided by a larger number (which will decrease its overall value).

Or, to put it another way, you are now examining a greater variety of people, so you should expect to have subjects in your samples that have IQs below 102.



The area of the shaded region is about 0.4503

Reflections on Example 5



The area of the first shaded region is about 0.1884. The (non-noticeable) shaded area of the second region is about 0.0000386.

Objectives

- 1 Obtain a sampling distribution of sample means
- 2 Determine the mean and standard error of a sampling distribution
- 3 Understand the Central Limit Theorem
- 4 Determine the mean and standard error for sampling distribution of proportions

Example 6

In an election, the incumbent received 62% of votes cast. Obtain a sampling distribution using a sample size of 100 and comment on the mean and standard error of the distribution.

Example 6

In an election, the incumbent received 62% of votes cast. Obtain a sampling distribution using a sample size of 100 and comment on the mean and standard error of the distribution.

The mean of the sample proportions (\hat{p}) targets the population proportion, $p = 0.62$.

Example 6

In an election, the incumbent received 62% of votes cast. Obtain a sampling distribution using a sample size of 100 and comment on the mean and standard error of the distribution.

The mean of the sample proportions (\hat{p}) targets the population proportion, $p = 0.62$.

The standard error is about 1/10 of the population standard deviation, which again is due to dividing the population standard deviation by \sqrt{n} , or in this case $\sqrt{100}$.

Mean and Standard Error of Sampling Distribution of Proportions

The mean of sample proportions targets the population proportion (just like the mean of sample means target the population mean).

Mean and Standard Error of Sampling Distribution of Proportions

The mean of sample proportions targets the population proportion (just like the mean of sample means target the population mean).

$$\hat{p} = p$$

Mean and Standard Error of Sampling Distribution of Proportions

The mean of sample proportions targets the population proportion (just like the mean of sample means target the population mean).

$$\hat{p} = p$$

The standard error targets the population standard deviation divided by $\sqrt{\text{sample size}}$

Mean and Standard Error of Sampling Distribution of Proportions

The mean of sample proportions targets the population proportion (just like the mean of sample means target the population mean).

$$\hat{p} = p$$

The standard error targets the population standard deviation divided by $\sqrt{\text{sample size}}$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Example 7

A headache medicine claims to be 80% effective with a standard deviation of 10%. What is the probability that in a sample of 15 people, the medicine is between 75 to 90% effective?

Example 7

A headache medicine claims to be 80% effective with a standard deviation of 10%. What is the probability that in a sample of 15 people, the medicine is between 75 to 90% effective?

$$p = 0.80$$

Example 7

A headache medicine claims to be 80% effective with a standard deviation of 10%. What is the probability that in a sample of 15 people, the medicine is between 75 to 90% effective?

$$p = 0.80$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.1)(0.9)}{15}} \approx 0.0775$$

Example 7

A headache medicine claims to be 80% effective with a standard deviation of 10%. What is the probability that in a sample of 15 people, the medicine is between 75 to 90% effective?

$$p = 0.80$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.1)(0.9)}{15}} \approx 0.0775$$

$$P(0.75 \leq x \leq 0.9) \approx 0.6421$$

Example 7

A headache medicine claims to be 80% effective with a standard deviation of 10%. What is the probability that in a sample of 15 people, the medicine is between 75 to 90% effective?

$$p = 0.80$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.1)(0.9)}{15}} \approx 0.0775$$

$$P(0.75 \leq x \leq 0.9) \approx 0.6421$$

In a sample of 15, there is about a 64.21% chance the medicine is between 75 and 90% effective.