# Confidence Intervals

# Objectives

1. Determine confidence intervals for population mean
2. Determine confidence intervals for population proportion
3. Determine the necessary sample size

# How Close Are We to the Population Mean?

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So, for our samples, how confident are we that they contain the population mean?

That is where confidence intervals come into play.

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The **confidence level**, or **level of confidence**, is the percentage of the number of times our confidence intervals will contain the population parameter.

Typical confidence levels are 90%, 95%, 98%, and 99%.

# Confidence Interval Setup

A confidence interval for a population parameter is in the form

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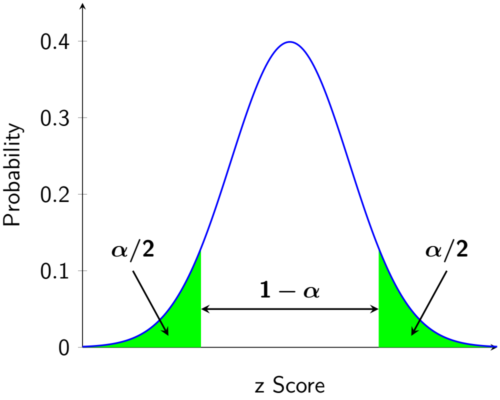
The margin of error is in the form

$$\text{critical value} \times \text{standard error}$$

# Critical Values

Critical values are typically in the form $z_{\alpha/2}$ where



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*Note:* If $\sigma$ is unknown, you can use the sample standard deviation, $s$, when the sample size is large enough ($n \geq 30$).

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A 95% confidence interval for the population mean waiting time is 3.353 to 3.647 minutes.

# Error Bars

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However, be advised that some graphs use standard deviation for their error bars.

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A 98% confidence interval for the mean population price per gallon is \$2.41 to \$2.49

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A 98% confidence interval for the mean population price per gallon is \$2.44 to \$2.46

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- There is a ____ % chance that the population mean is (*whatever the sample mean is*).

# Objectives

1. Determine confidence intervals for population mean
2. Determine confidence intervals for population proportion
3. Determine the necessary sample size

# Mean and Standard Error

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Our confidence interval for the population proportion is

$$p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

# Example 3

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&= 0.521 \pm 0.034 \\
&= (0.487,\ 0.555)
\end{aligned}
$$

A 90% confidence interval for the population proportion of voters who favor the renovation is 48.7 to 55.5%

# Example 4

The results of a sample of 40 students who took a pass/fail statistics exam are shown below. Construct a 98% confidence interval for the population proportion of students who passed the exam.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Pass | Pass | Fail | Pass | Pass | Pass | Fail | Pass |
| Fail | Pass | Pass | Pass | Fail | Pass | Pass | Pass |
| Pass | Fail | Fail | Pass | Pass | Fail | Pass | Pass |
| Pass | Pass | Fail | Fail | Pass | Pass | Pass | Fail |

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$$= 0.75 \pm 0.070$$

$$= (0.68,\ 0.82)$$

A 98% confidence interval for the population proportion of students who passed the exam is 68 to 82%.

# Objectives

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# Margin of Error for Population Mean

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$$\sqrt{n} = \frac{\sigma \cdot z_{\alpha/2}}{E}$$

$$n = \left(\frac{\sigma \cdot z_{\alpha/2}}{E}\right)^2$$

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$$n = 138.2976 \to 139$$

We would need a sample of at least 139.

# Sample Size for Population Proportion

If we solve

$$E = z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

for $n$, we get

$$n = p(1-p)\left(\frac{z_{\alpha/2}}{E}\right)^2$$

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$$n = 0.521(0.479)\left(\frac{1.96}{0.04}\right)^2$$

$$n = 599.191159 \to 600$$

We would need a sample size of at least 600.