Binomial Probability Distributions

Objectives

1. Calculate probabilities of binomial distributions
2. Calculate the mean, variance, and standard deviation of a binomial distribution

Binomial Probability Experiment

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- Each trial's outcome is either a success or failure
- The probability of success, p, never changes

Flipping a Coin

Number of heads when flipping a coin 3 times:

x	Outcomes	$P(X = x)$
0	TTT	$1/8$
1	HTT THT TTH	$3/8$
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Thus, combinations play a role in binomial probability distributions.

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It is more likely you will obtain 4 heads from 5 flips than 8 heads from 10 flips.

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$$\approx 0.00637$$

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$$\mu = E(x) = np$$

Variance and Standard Deviation of a Binomial Distribution

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The student can expect to get 2 questions correct out of 10

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The standard deviation is about 1.265 questions.

Standard Deviation Interpretation

If we perform this same experiment several times, around 95% of the time, the number of correct guesses will be within 2 standard deviations of the mean (expected value).

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So, around 95% of the time, the student will guess between 0 and around 4.5 questions correctly.