

Probability: AND

Objectives

- 1 Calculate probabilities using the Multiplication Rule
- 2 Find probabilities of independent events
- 3 Find conditional probabilities
- 4 Find probabilities of dependent events

Example 1

You flip a coin and then roll a single die. What is the probability that you flip heads **and** roll a 5?

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Sample space:

	1	2	3	4	5	6
Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

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You flip a coin and then roll a single die. What is the probability that you flip heads **and** roll a 5?

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Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

$$P(\text{heads and 5}) = \frac{1}{12}$$

Multiplication Rule

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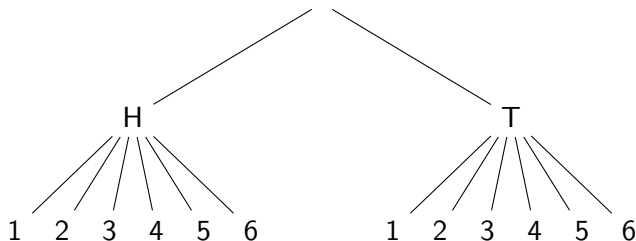
The probability of rolling a 5 was $\frac{1}{6}$

Multiplication Rule

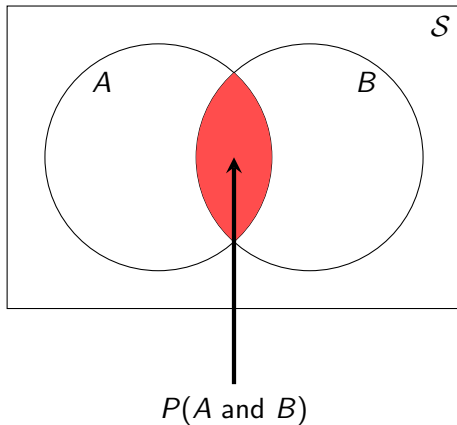
If $P(A)$ is the probability of event A occurring, and $P(B)$ is the probability of event B occurring, then

$$P(A \text{ and } B) = P(A) \times P(B)$$

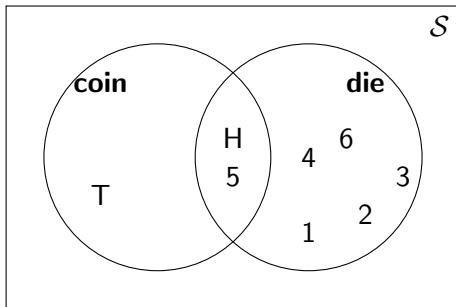
Tree Diagram



Venn Diagram – AND



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Mutually Exclusive Events

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Two events are **mutually exclusive** if they can not happen together. In other words,

$$P(A \text{ and } B) = 0$$

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When selecting items from a collection, independent events often contain selections made with replacement.

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A jar contains 10 blue, 12 black, and 15 red marbles.

(a) What is the probability of selecting a black marble, putting it back, and then selecting a blue marble?

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(a) What is the probability of selecting a black marble, putting it back, and then selecting a blue marble?

$$\begin{aligned}P(\text{black and blue}) &= \frac{12}{37} \times \frac{10}{37} \\&= \frac{120}{1,369}\end{aligned}$$

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A jar contains 10 blue, 12 black, and 15 red marbles.

(b) What is the probability of selecting a red marble, putting it back, and then selecting another red marble?

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$$P(\text{red and red}) = \frac{15}{37} \times \frac{15}{37}$$

Example 2

A jar contains 10 blue, 12 black, and 15 red marbles.

(b) What is the probability of selecting a red marble, putting it back, and then selecting another red marble?

$$\begin{aligned}P(\text{red and red}) &= \frac{15}{37} \times \frac{15}{37} \\&= \frac{225}{1,369}\end{aligned}$$

Example 3

A certain blood test can determine the presence of a bloodborne pathogen 97% of the time (that is, if 100 people have the pathogen, the test will confirm true for 97 of them). If 4 people with the pathogen are given the test, find the probability that the test is accurate for all of them.

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$$P(\text{accurate for all}) = P(\text{first}) \cdot P(\text{second}) \cdot P(\text{third}) \cdot P(\text{fourth})$$

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$$\begin{aligned}P(\text{accurate for all}) &= P(\text{first}) \cdot P(\text{second}) \cdot P(\text{third}) \cdot P(\text{fourth}) \\&= (0.97)(0.97)(0.97)(0.97)\end{aligned}$$

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There is about an 88.53% chance the test is accurate for all four people.

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With conditional probability, the denominator will often be the total of something that follows the words “if”, “suppose”, or “given that”.

Example 4

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	3–5	6–10	Over 10	Total
Import	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a domestic car is randomly selected, what is the probability that it is 6–10 years old?

Example 4

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	3–5	6–10	Over 10	Total
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Total	72	44	23	61	200

(a) If a domestic car is randomly selected, what is the probability that it is 6–10 years old?

$$P(6-10 \text{ years old} \mid \text{it is a domestic car}) = \frac{11}{100}$$

Example 4

	0-2	3-5	6-10	Over 10	Total
Import	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(b) What is the probability of selecting a domestic car given that the car is 6-10 years old?

Example 4

	0–2	3–5	6–10	Over 10	Total
Import	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(b) What is the probability of selecting a domestic car given that the car is 6–10 years old?

$$P(\text{domestic car} \mid \text{it is 6–10 years old}) = \frac{11}{23}$$

Example 4

	0-2	3-5	6-10	Over 10	Total
Import	37	21	12	30	100
Domestic	35	23	11	31	100
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(c) Suppose a new car is selected, what is the probability that it is an import?

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	0-2	3-5	6-10	Over 10	Total
Import	37	21	12	30	100
Domestic	35	23	11	31	100
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(c) Suppose a new car is selected, what is the probability that it is an import?

$$P(\text{import} \mid \text{it is 0-2 years old}) = \frac{37}{72}$$

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Dependent events utilize conditional probability:

$$P(A \text{ and } B) = P(A) \times P(B | A)$$

Note: When selecting items from a collection, dependent events often contain selections made without replacement.

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You are dealt a card from a standard deck and then you are dealt another (without replacement). Find the probability that

- (a) The first card is an ace and the second card is a ten.

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You are dealt a card from a standard deck and then you are dealt another (without replacement). Find the probability that

- (a) The first card is an ace and the second card is a ten.

$$\begin{aligned}P(\text{ace and ten}) &= P(\text{ace}) \times P(\text{ten} \mid \text{ace}) \\&= \frac{4}{52} \times \frac{4}{51}\end{aligned}$$

Example 5

You are dealt a card from a standard deck and then you are dealt another (without replacement). Find the probability that

- (a) The first card is an ace and the second card is a ten.

$$P(\text{ace and ten}) = P(\text{ace}) \times P(\text{ten} \mid \text{ace})$$

$$= \frac{4}{52} \times \frac{4}{51}$$

$$= \frac{4}{663}$$

$$\approx 0.603\%$$

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- (b) The first card is an ace and the second card is an ace.

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$$= \frac{4}{52} \times \frac{3}{51}$$

Example 5

(b) The first card is an ace and the second card is an ace.

$$P(\text{ace and ace}) = P(\text{ace}) \times P(\text{ace} \mid \text{ace})$$

$$= \frac{4}{52} \times \frac{3}{51}$$

$$= \frac{1}{221}$$

$$\approx 0.452\%$$

Conditional Probability Revisited

The formula for dependent events

$$P(A \text{ and } B) = P(A) \times P(B | A)$$

leads us to the following formula for conditional probability:

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$$P(A \text{ and } B) = P(A) \times P(B | A)$$

leads us to the following formula for conditional probability:

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$$

Tabular Data

When finding AND probabilities using tabular data, look for the intersection of a row and column.

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When finding AND probabilities using tabular data, look for the intersection of a row and column.

Two rows (likewise, two columns) will never intersect, so their probabilities are *mutually exclusive*.

Example 6

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a car is randomly selected, what is the probability that it is a 6–10 year old foreign car?

Example 6

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a car is randomly selected, what is the probability that it is a 6–10 year old foreign car?

$$P(6-10 \text{ years old and foreign}) = \frac{12}{200}$$

Example 6

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a car is randomly selected, what is the probability that it is a 6–10 year old foreign car?

$$\begin{aligned}P(6-10 \text{ years old and foreign}) &= \frac{12}{200} \\&= \frac{3}{50}\end{aligned}$$

Example 6

The reasoning behind our answer for Example 6a:

$$P(6-10 \text{ and foreign}) = P(6-10) \times P(\text{foreign} \mid 6-10 \text{ years old})$$

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The reasoning behind our answer for Example 6a:

$$\begin{aligned}P(6-10 \text{ and foreign}) &= P(6-10) \times P(\text{foreign} \mid 6-10 \text{ years old}) \\&= \frac{23}{200} \times \frac{12}{23}\end{aligned}$$

Example 6

The reasoning behind our answer for Example 6a:

$$\begin{aligned}P(6-10 \text{ and foreign}) &= P(6-10) \times P(\text{foreign} \mid 6-10 \text{ years old}) \\&= \frac{23}{200} \times \frac{12}{23} \\&= \frac{12}{200}\end{aligned}$$

Example 6

The reasoning behind our answer for Example 6a:

$$\begin{aligned}P(6-10 \text{ and foreign}) &= P(6-10) \times P(\text{foreign} \mid 6-10 \text{ years old}) \\&= \frac{23}{200} \times \frac{12}{23} \\&= \frac{12}{200} \\&= \frac{3}{50}\end{aligned}$$

Example 6

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
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(b) If a car is randomly selected, what is the probability that it is a domestic car that is 0–2 years old?

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	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(b) If a car is randomly selected, what is the probability that it is a domestic car that is 0–2 years old?

$$P(0\text{--}2 \text{ years old and domestic}) = \frac{35}{200}$$

Example 6

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(b) If a car is randomly selected, what is the probability that it is a domestic car that is 0–2 years old?

$$\begin{aligned}P(0-2 \text{ years old and domestic}) &= \frac{35}{200} \\&= \frac{7}{40}\end{aligned}$$

Example 6

	0-2	3-5	6-10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(c) If a car is randomly selected, what is the probability that it is 0-2 years and 6-10 years old?

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	0-2	3-5	6-10	Over 10	Total
Foreign	37	21	12	30	100
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(c) If a car is randomly selected, what is the probability that it is 0-2 years and 6-10 years old?

$$P(0-2 \text{ years and } 6-10 \text{ years old}) = \frac{0}{200}$$

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	0-2	3-5	6-10	Over 10	Total
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(c) If a car is randomly selected, what is the probability that it is 0-2 years and 6-10 years old?

$$\begin{aligned}P(0-2 \text{ years and } 6-10 \text{ years old}) &= \frac{0}{200} \\&= 0\end{aligned}$$