Linear Regression

Objectives

1. Determine and interpret the linear correlation coefficient
2. Determine the linear regression equation
3. Determine and Interpret the Coefficient of Determination

Linear Correlation Coefficient

In the previous section, we examined correlation types (positive, negative, or none) with the help of the means of the explanatory (x) and response variables (y).

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In this section, we will examine the correlation type the way it is done in the real world: calculating the linear correlation coefficient (r).

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- $r > 0$: positive linear correlation
- $r = 0$: no linear correlation
- $r < 0$: negative linear correlation

Linear Correlation Coefficient

$$r = \frac{\sum(x - \overline{x})(y - \overline{y})}{\sqrt{\sum(x - \overline{x})^2 \cdot \sum(y - \overline{y})^2}}$$

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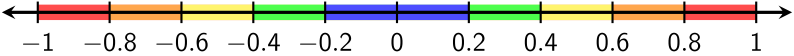
The closer r is to 1 (or -1), the more the data points “fall in line”

The closer r is to 0, the more the data points resemble a “cloud”

Interpreting r



None

Weak

Moderate

Strong

Very Strong

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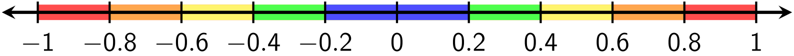

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Note: These interpretations are not universal.

Example 1

Find and interpret the linear correlation coefficient, r, for each.

(a)

x	y
7.6	19.1
9.2	22.9
3.3	10.3
1.1	6.6
3.7	10.6
3.9	11.3
4.6	12.9
2.3	8.6
5.1	15.2
5.3	15.1
2.5	13
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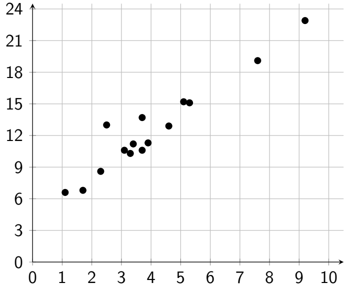

Example 1

(b)

x	y
7.6	11.0
9.2	3.6
6.3	8.9
1.1	14.9
6.7	8.1
3.9	12.0
4.6	9.4
2.3	10.3
5.1	11.4
5.3	12.4
2.5	9.0
3.4	8.9
3.1	14.2
1.7	10.9
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5.1	11.4
5.3	12.4
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5.1	11.4
5.3	12.4
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5.1	11.4
5.3	12.4
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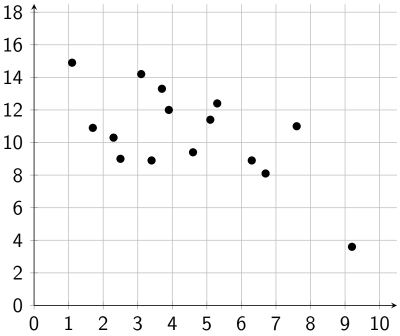

Example 1

(c)

x	y
6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
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2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
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x	y
6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
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6.9	3.4
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3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
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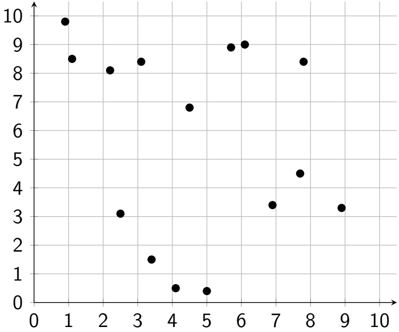

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To do this, we can create the **least squares regression equation**, (also called the *line of best fit*) which will **minimize** the total squared distance each data point is from the line:

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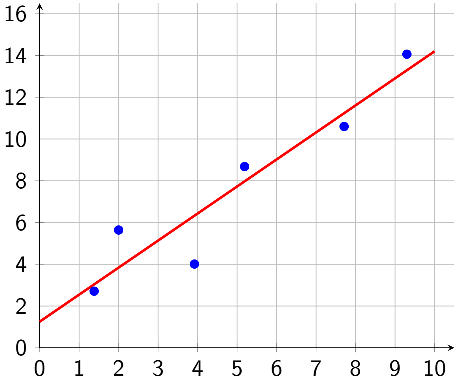
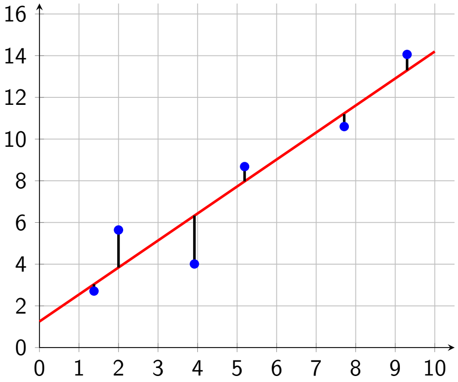
$$\hat{y} = mx + b$$

Line of Best Fit



Least Squares Regression Equation



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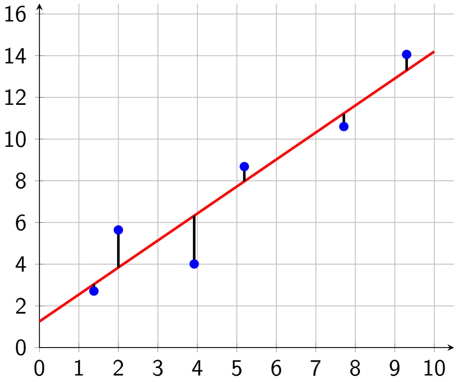
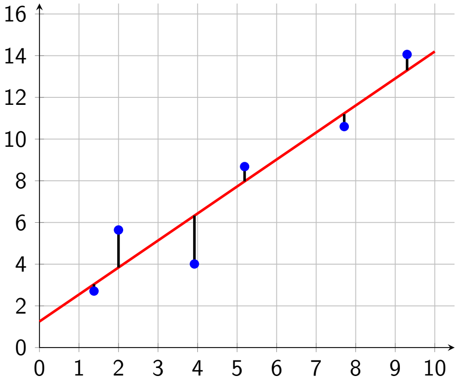


The black lines are **residuals**.

Least Squares Regression Equation



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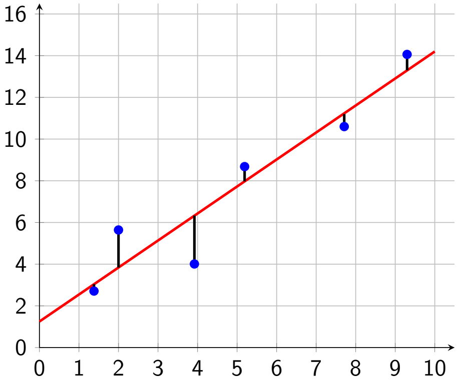

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So we need to square the deviations so the negatives don't cancel the positives.

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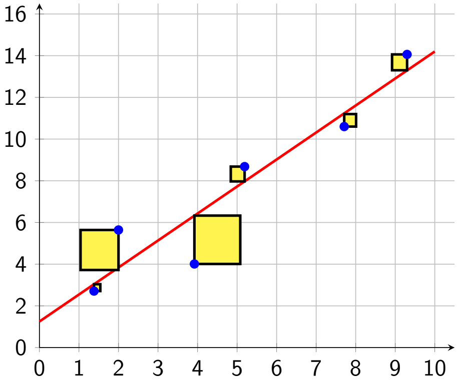

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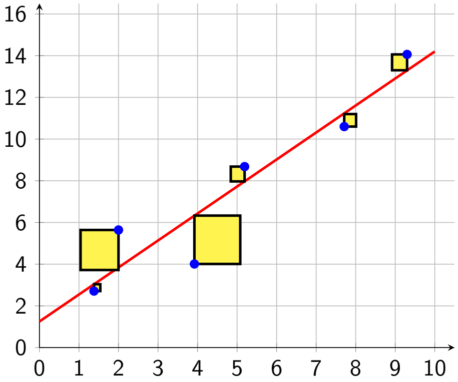

The line of best fit minimizes the sum of the areas of the squares.

Slope and y-intercept

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$$m = r\left(\frac{\sigma_y}{\sigma_x}\right)$$

and

$$b = \overline{y} - m\left(\overline{x}\right)$$

Example 2

Find the least squares regression equation for the following dataset.

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7.6	19.1
9.2	22.9
3.3	10.3
1.1	6.6
3.7	10.6
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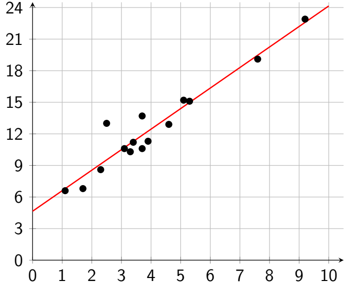

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The predicted value when $x = 6$ is $y = 16.35$

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Suppose we actually obtain a datapoint and realize that the actual value of y when $x = 6$ is 16, not the predicted 16.35.

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We could then add that observation to our dataset and use it to create a better linear regression equation.

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Since 11 is outside of the x values in our dataset, finding its y-coordinate is called **extrapolation**.

$$\begin{aligned}\hat{y} &= 1.95x + 4.65 \\ &= 1.95(11) + 4.65 \\ &= 26.1\end{aligned}$$

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The predicted value when $x = 11$ is $y = 26.1$.

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The rest may be due to such things as lurking variables.

Since $-1 \leq r \leq 1$, $0 \leq r^2 \leq 1$

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Note: If r (and hence r^2) is close to 0, then the linear regression equation is not going to be a good predictor. In this case, use $\hat{y} = \bar{y}$ as the linear predictor equation.

Example 4

Determine and interpret the value of r^2 given the paw width (in inches) and dog's weight (in pounds) below.

Paw	Weight
1.32	10
1.67	13
2.76	25
3.98	65
2.97	39
2.49	19
3.84	85
4.04	104
4.52	158
3.18	65
3.29	58
3.63	101
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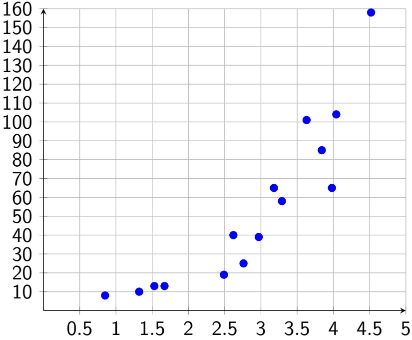

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$$r^2 \approx 0.8049$$

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$$r^2 \approx 0.8049$$

The prediction error used to predict y is about 80.5% smaller than using $\overline{y}$ as a predictor.

Other Types of Regression

Since we only get values of r for *linear* regression, we will not calculate a linear correlation coefficient for a data set that does not appear linear.

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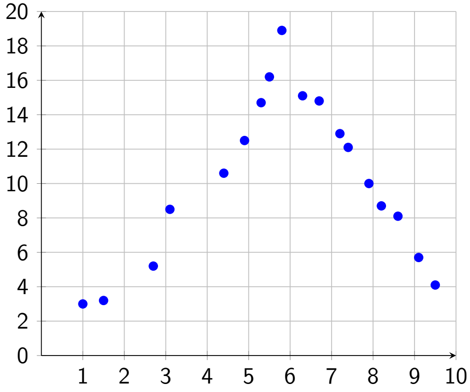

Other Types of Regression

We can still calculate r^2 using the more-traditional formula than was presented in this section; although you would likely want to let technology calculate it for you.