Hypothesis Testing

Regression

Slope of a Line

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which we can use to predict data values both inside and outside our dataset.

Also, recall that the coefficient of determination, r^2, tells us the prediction error decrease we get by using the linear regression equation, as opposed to the mean of the y-coordinates.

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We can then test whether or not the linear regression model we obtained contributes to predicting values of y.

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To put it mathematically:

$$H_0 : m = 0$$

Our alternative hypothesis will be one of the forms

Left-Tailed	**Right-Tailed**	**Two-Tailed**
$m < 0$	$m > 0$	$m \neq 0$

Test Statistic

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Our test statistic, t, is thus

$$t = \frac{\hat{m}}{s/\sqrt{SS_{xx}}}$$

where $\hat{m}$ is the slope of our regression equation.

p-Value

The area under the curve either to the left, right, or outside of our test statistic(s), i.e. the p-value, can be found by taking $n - 2$ degrees of freedom with that test statistic.

Confidence Intervals

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Remember, for one-sided confidence intervals, use t_α instead of $t_{\alpha/2}$.

Residuals Review

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When performing hypothesis testing regarding the slope of the regression equation, we operate under the following assumptions:

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Assumptions:

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- The variance of all values of ϵ is constant for all values of x.
- The distribution of ϵ is normal.
- Values of ϵ associated with any two values of y are independent.