Standard Normal Distribution

Objectives

1. Find the area under a normal curve given z score(s)
2. Find the z scores corresponding to a given area

Continuous Distributions

Continuous Probability Distribution

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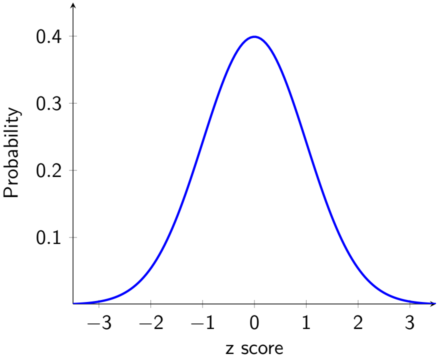
In this section, we are going to discuss the **standard normal distribution**, whose histogram resembles a bell-shaped curve.

Equation and Graph of Standard Normal Distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$



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- The graph is symmetric about the mean
- **The area under the curve represents the probability of obtaining a z score in that area.**
- The total area under the curve equals 1

Example 1

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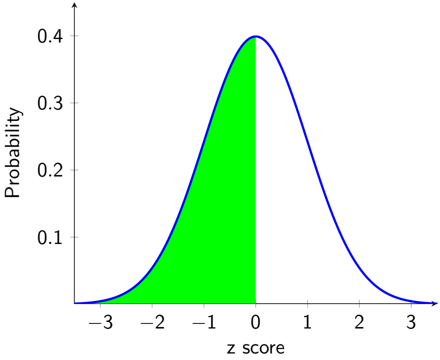

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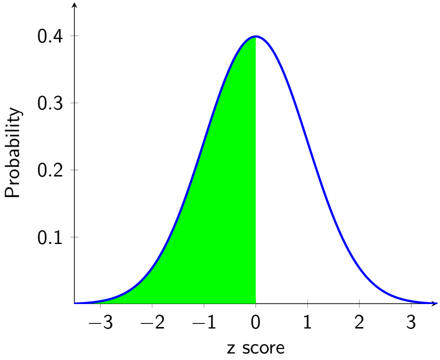

0.5

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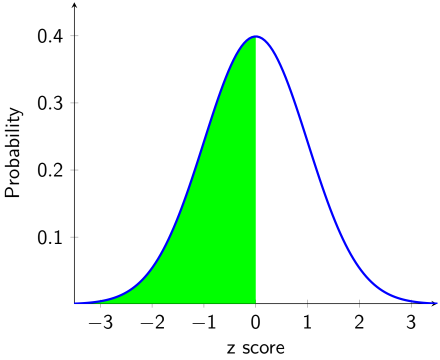
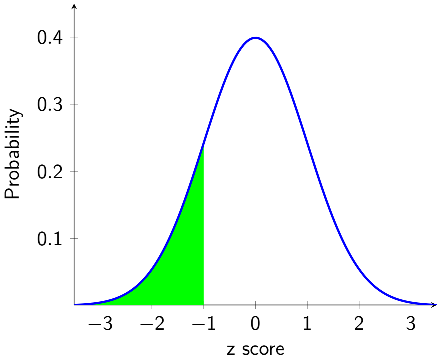


0.5

$P(z \leq 0) = 0.5$

Example 1

(b) $z = -1$



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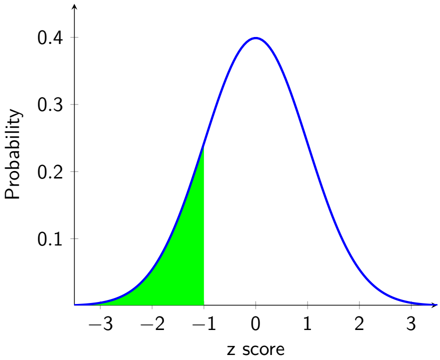

0.1587

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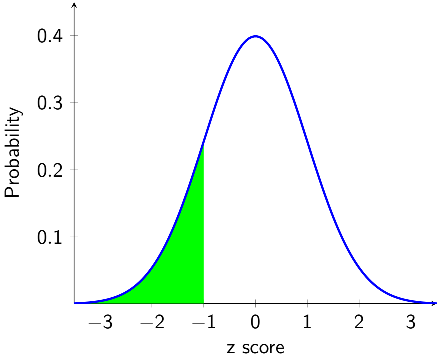

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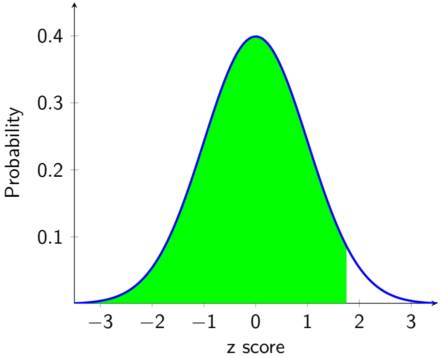
Example 1

(c) $z = 1.75$



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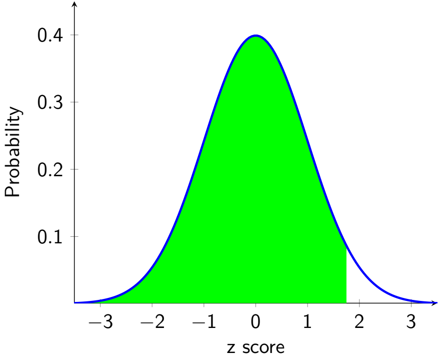

0.9599

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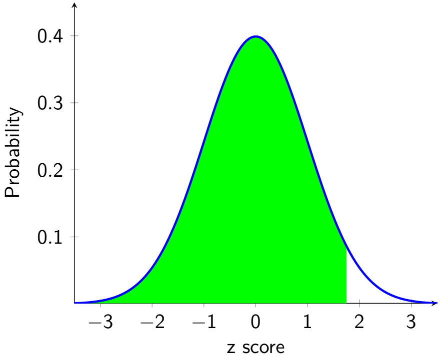

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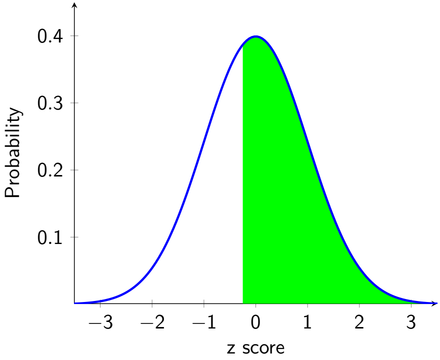

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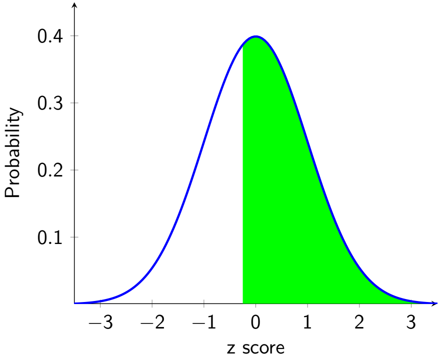

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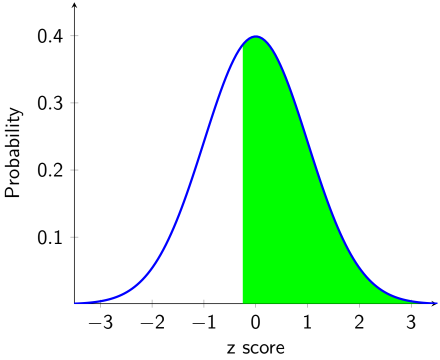

0.5987

$P(z \geq -0.25) = 0.5987$

Example 2

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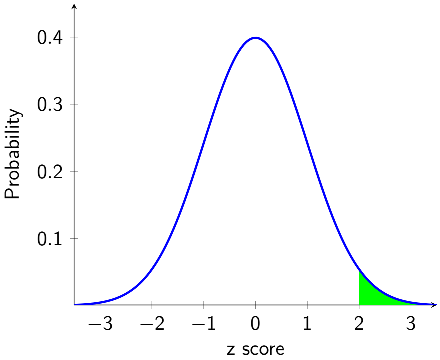

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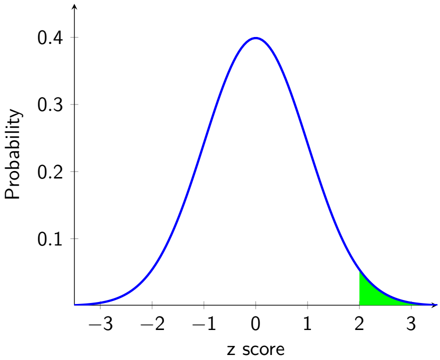

0.0228

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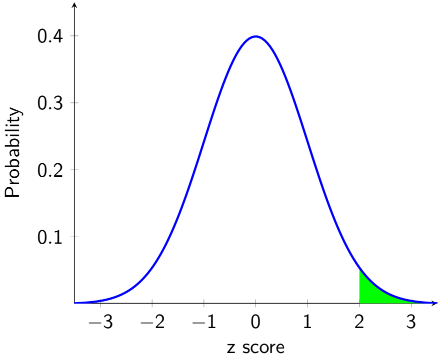

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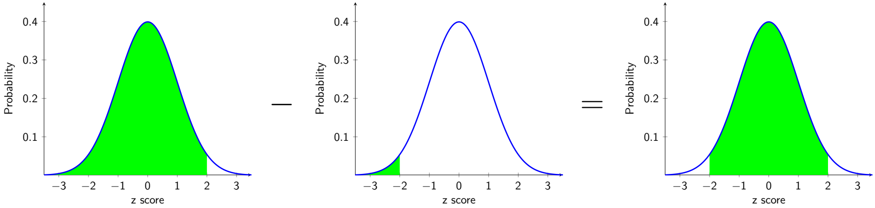
$P(z \geq 2) = 0.0228$

Finding the Area Between Two z Scores



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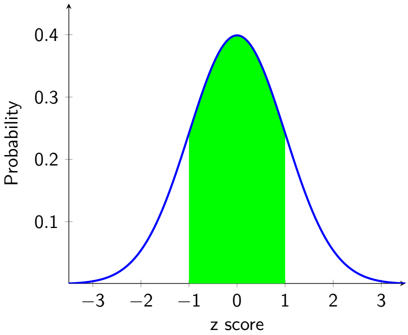

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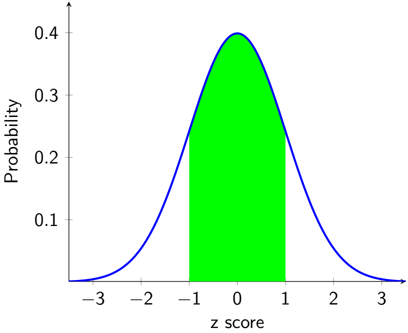

$$P(-1 \leq z \leq 1) = 0.6827$$

Example 3

(b) $z = -2$ and $z = 2$

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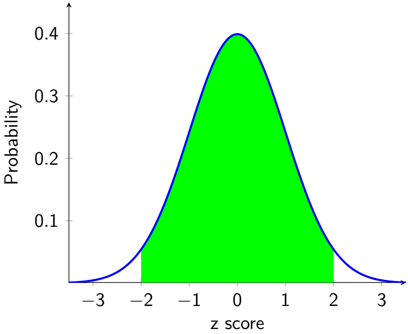

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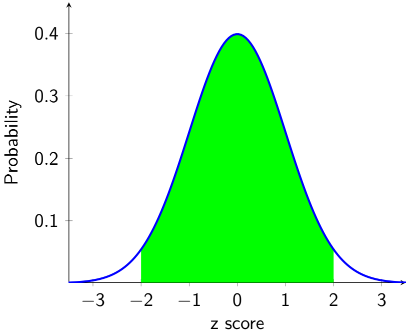

$$P(-2 \leq z \leq 2) = 0.9545$$

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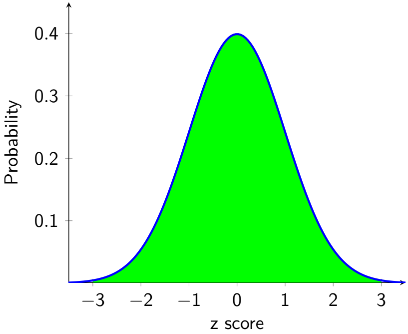

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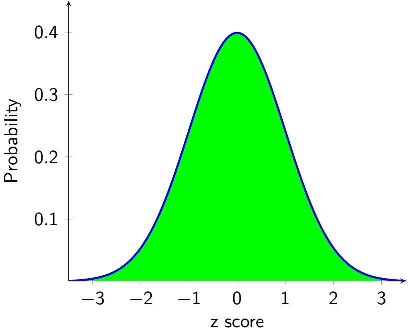

$$P(-3 \leq z \leq 3) = 0.9973$$

Empirical Rule

For normally distributed data, the Empirical (sometimes called 68-95-99.7 Rule) states that

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- About 68% of the data will lie within 1 standard deviation of the mean.
- About 95% of the data will lie within 2 standard deviations of the mean.
- About 99.7% of the data will lie within 3 standard deviations of the mean.

Total Outside Two z Scores

We can use the **complement rule** to find the total area under the normal curve *not between* two z scores:

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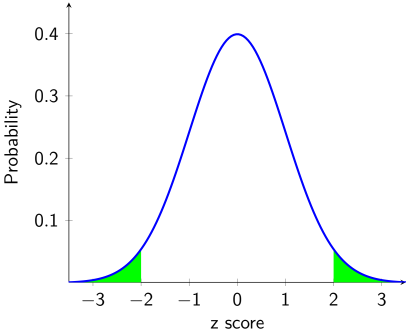

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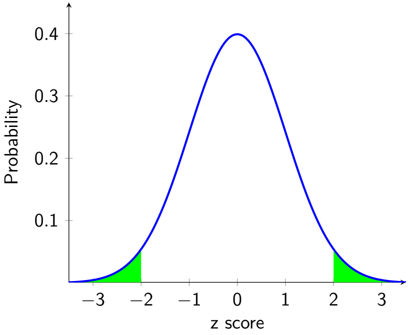

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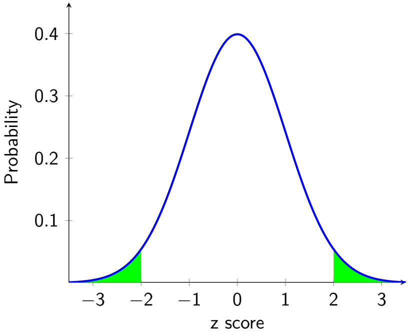

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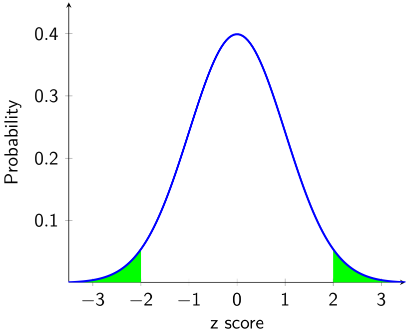

$$P(|z| \geq 2) =$$

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$$\approx 0.0455$$

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Finding z Score for a Given Area

This is just working backwards from the our previous examples.

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Keep in mind that most (not all) technology will ask you for the area to the *left* of the needed z score.

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Find the z score which has the given area to the left of it. Round to 3 decimal places.

(a) 0.05

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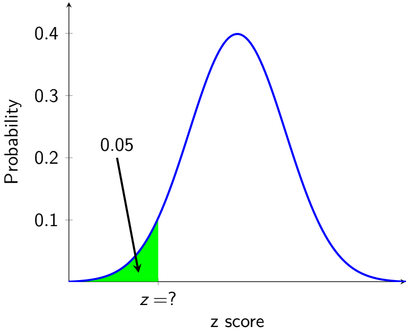

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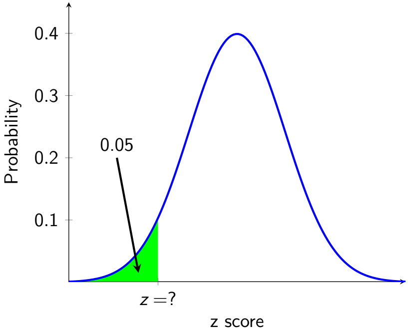

$z \approx -1.645$

Area to the Left as a Percent

Since the area to the left of $z = -1.645$ in Example 4a is about 0.05, we would say that a z score of -1.645 is in the 5th percentile.

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(b) What z score is in the 90th percentile?

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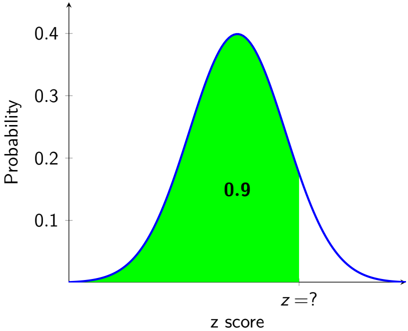

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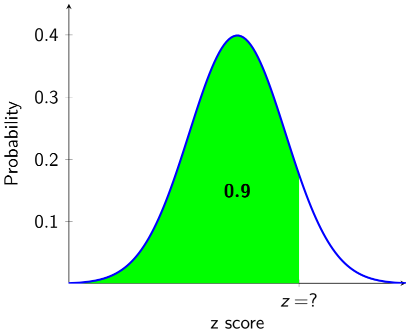

$z \approx 1.282$

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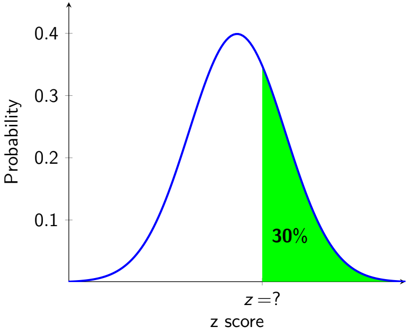

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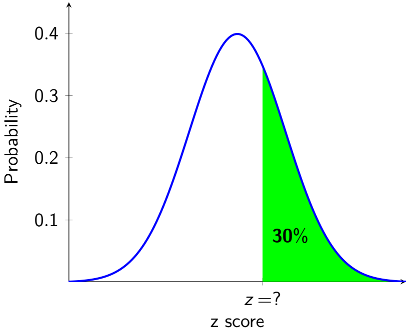

$z \approx 0.524$

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(d) Area to the right is 0.5

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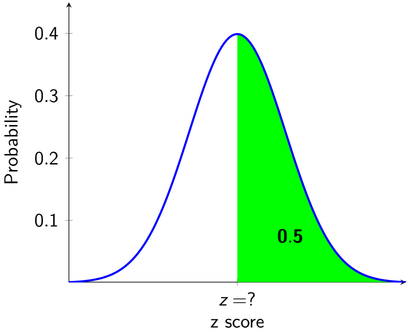

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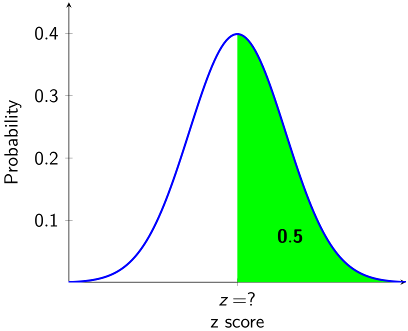

$z = 0$

Example 5

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(a) 50%

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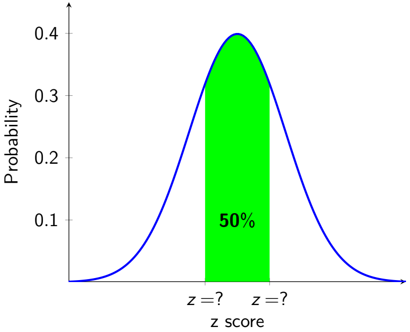

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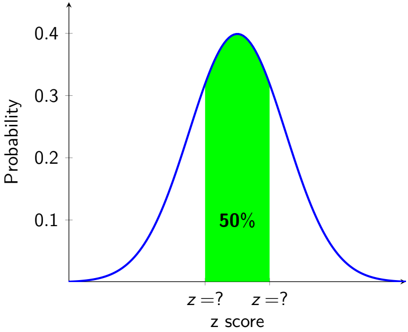

$z = \pm 0.674$

Example 5

(b) The middle 95%

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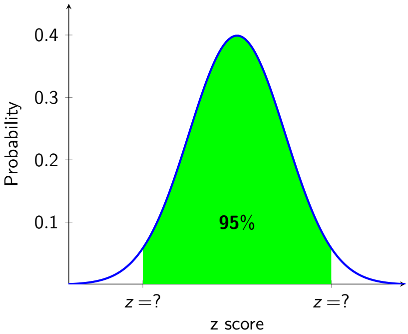

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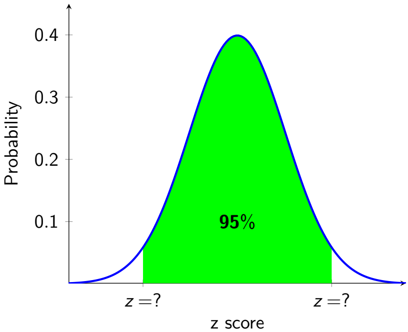

$$z = \pm 1.960$$

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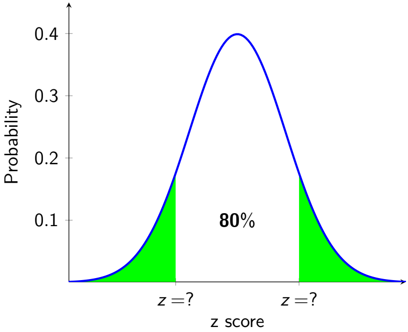

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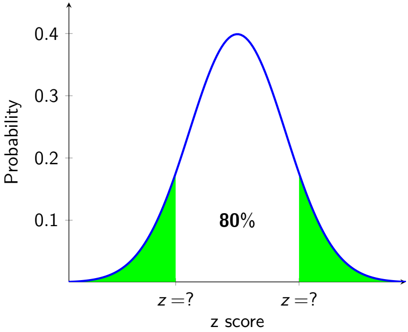

$z = \pm 1.282$

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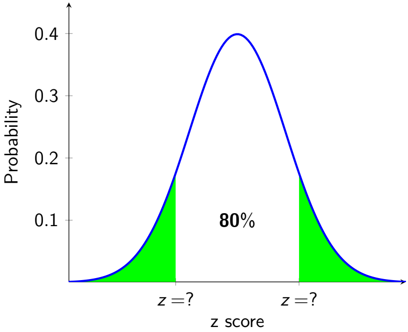

$z = \pm 1.282$

Note:

The notation for the above problem is $z_{0.1}$