Linear Regression

Objectives

1. Determine and interpret the linear correlation coefficient
2. Determine the linear regression equation
3. Determine and Interpret the Coefficient of Determination

Linear Correlation Coefficient

In the previous section, we examined correlation types (positive, negative, or none) with the help of the means of the explanatory (x) and response variables (y).

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In this section, we will examine the correlation type the way it is done in the real world: calculating the linear correlation coefficient (r).

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- $r > 0$: positive linear correlation
- $r = 0$: no linear correlation
- $r < 0$: negative linear correlation

Linear Correlation Coefficient

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \cdot \sum(y - \bar{y})^2}}$$

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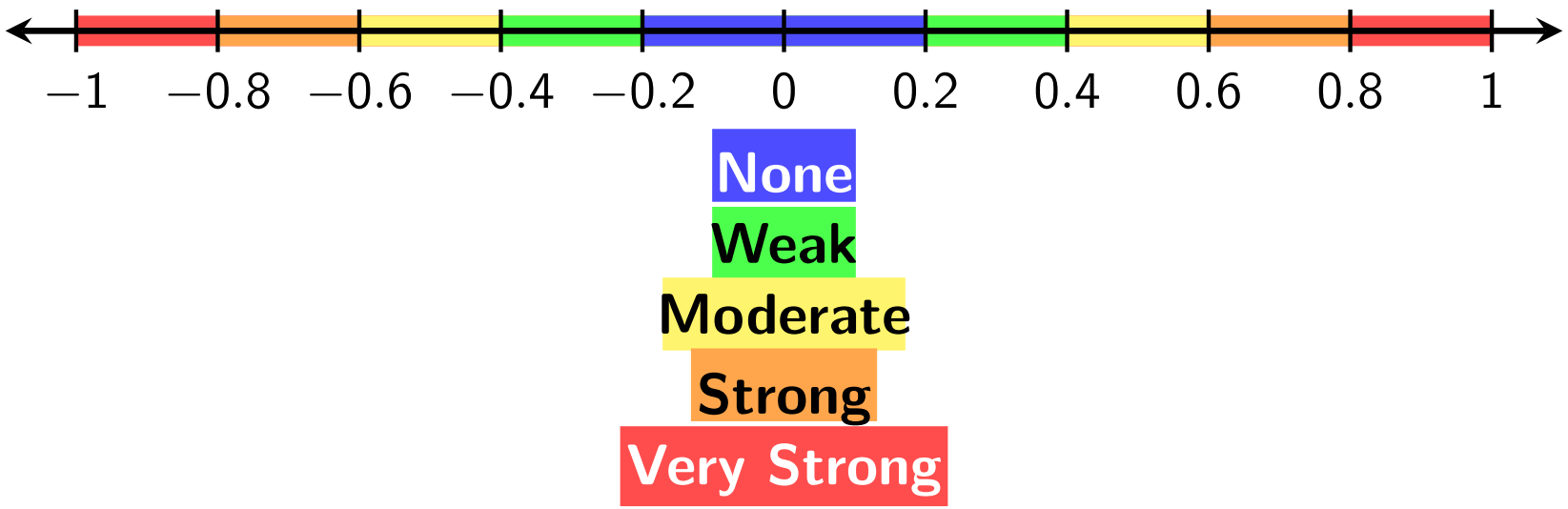
The closer r is to 1 (or -1), the more the data points "fall in line"

The closer r is to 0, the more the data points resemble a "cloud"

Interpreting r



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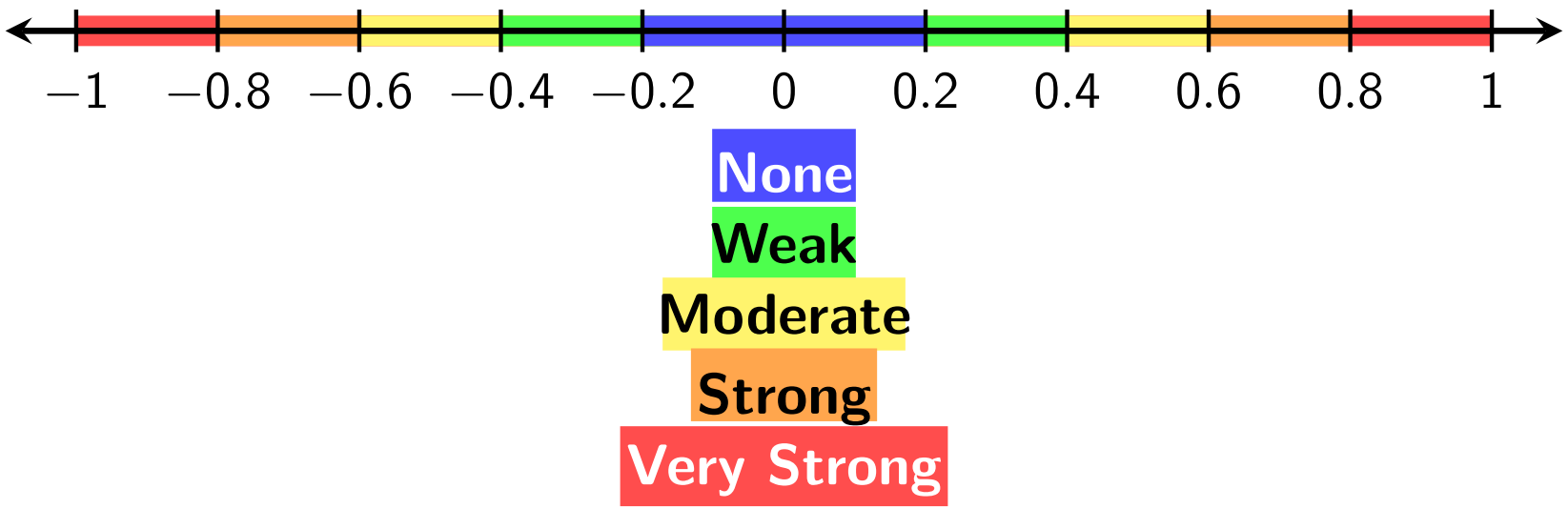

Note: These interpretations are not universal.

Example 1

Find and interpret the linear correlation coefficient, r, for each.

(a)

x	y
7.6	19.1
9.2	22.9
3.3	10.3
1.1	6.6
3.7	10.6
3.9	11.3
4.6	12.9
2.3	8.6
5.1	15.2
5.3	15.1
2.5	13
3.4	11.2
3.1	10.6
1.7	6.8
3.7	13.7

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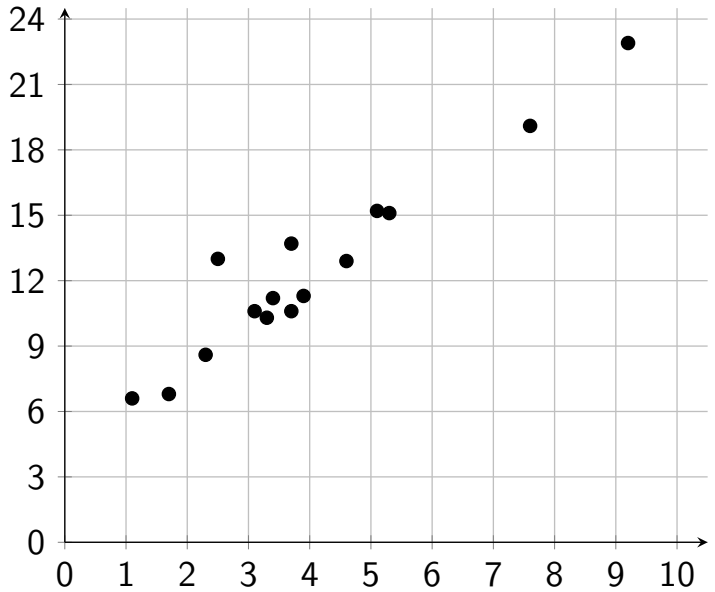

Example 1

(b)

x	y
7.6	11.0
9.2	3.6
6.3	8.9
1.1	14.9
6.7	8.1
3.9	12.0
4.6	9.4
2.3	10.3
5.1	11.4
5.3	12.4
2.5	9.0
3.4	8.9
3.1	14.2
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5.1	11.4
5.3	12.4
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(b)

x	y
7.6	11.0
9.2	3.6
6.3	8.9
1.1	14.9
6.7	8.1
3.9	12.0
4.6	9.4
2.3	10.3
5.1	11.4
5.3	12.4
2.5	9.0
3.4	8.9
3.1	14.2
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x	y
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9.2	3.6
6.3	8.9
1.1	14.9
6.7	8.1
3.9	12.0
4.6	9.4
2.3	10.3
5.1	11.4
5.3	12.4
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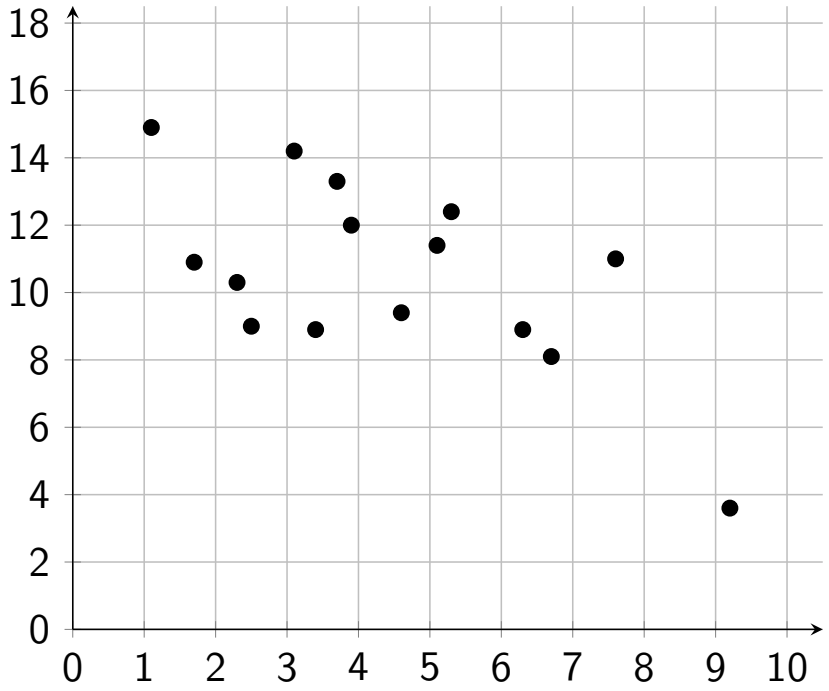

Example 1

(c)

x	y
6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
1.1	8.5

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6.9	3.4
7.7	4.5
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3.4	1.5
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5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
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(c)

x	y
6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
1.1	8.5

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x	y
6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
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2.5	3.1
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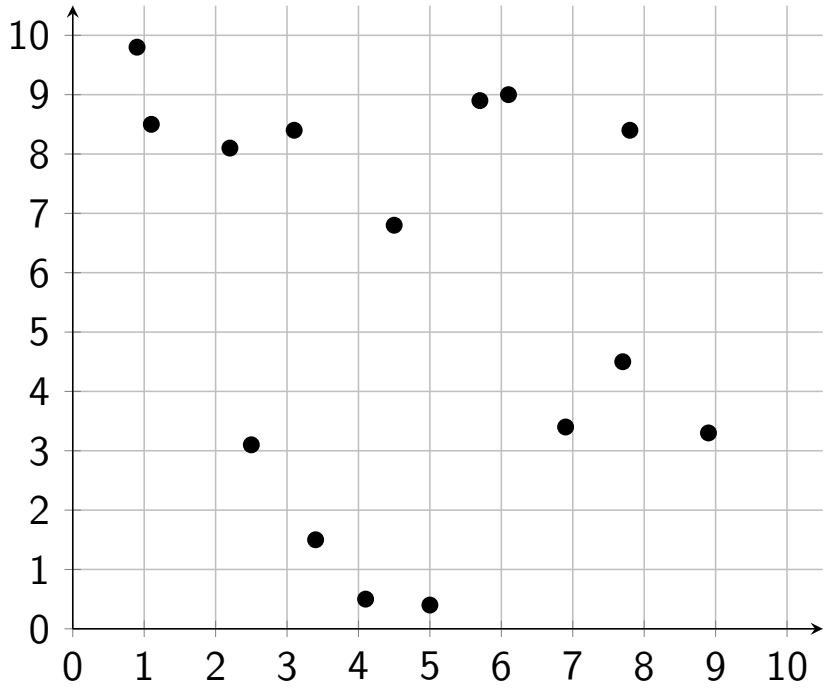

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While determining the linear correlation coefficient is valuable, it is also helpful to be able to predict data values not contained in the data set.

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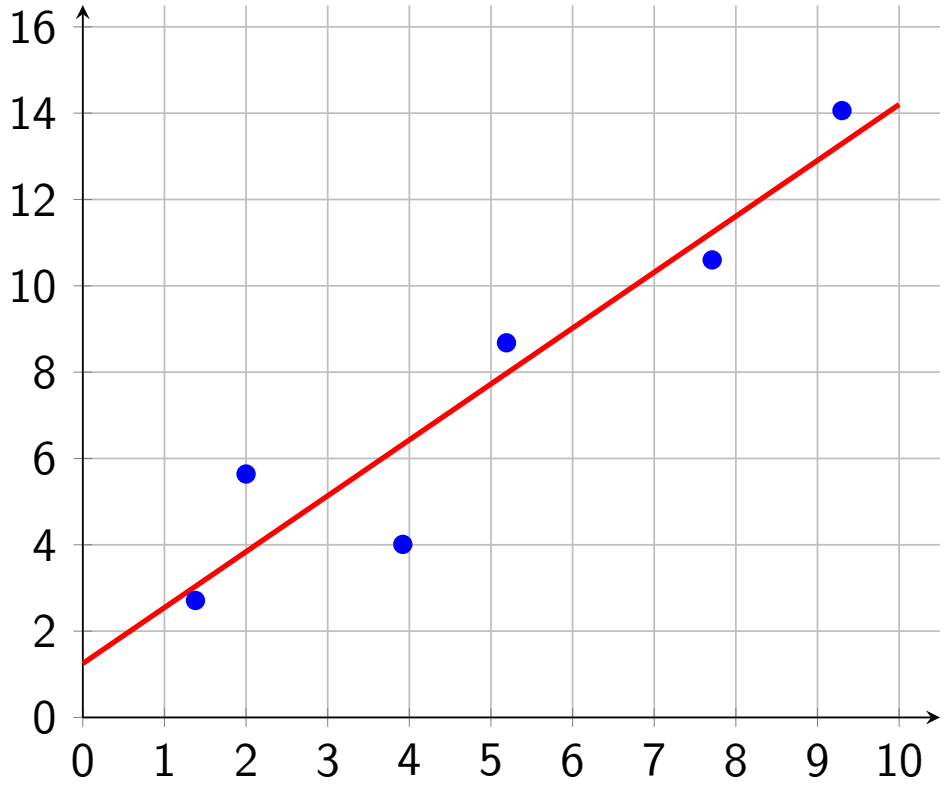
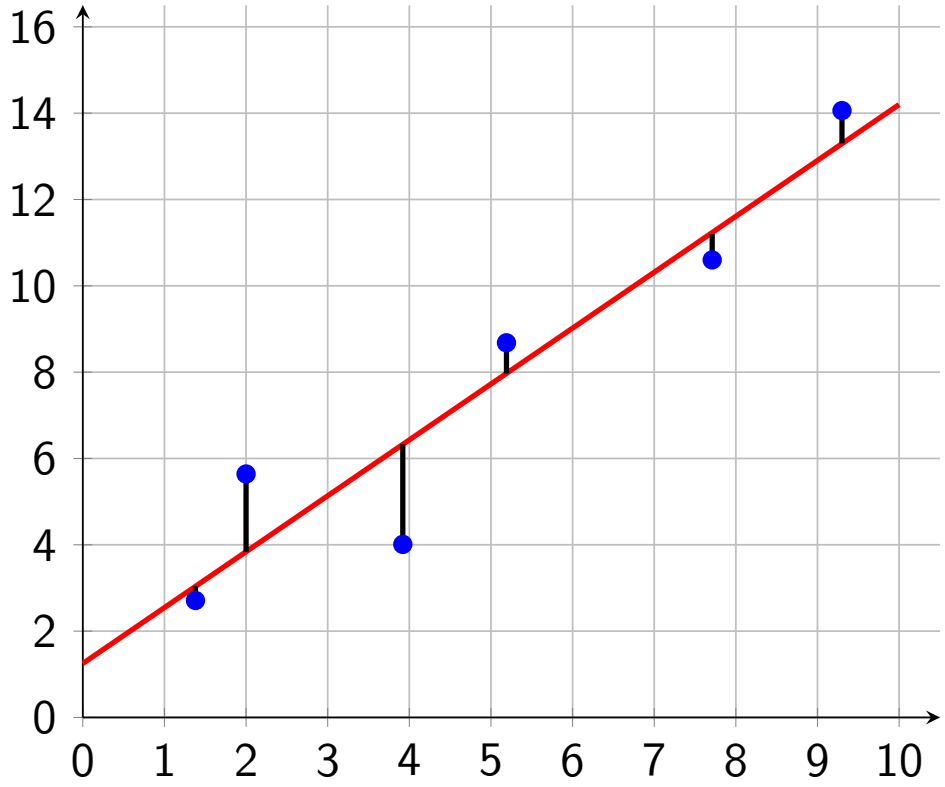
$$\hat{y} = mx + b$$

Line of Best Fit



Least Squares Regression Equation



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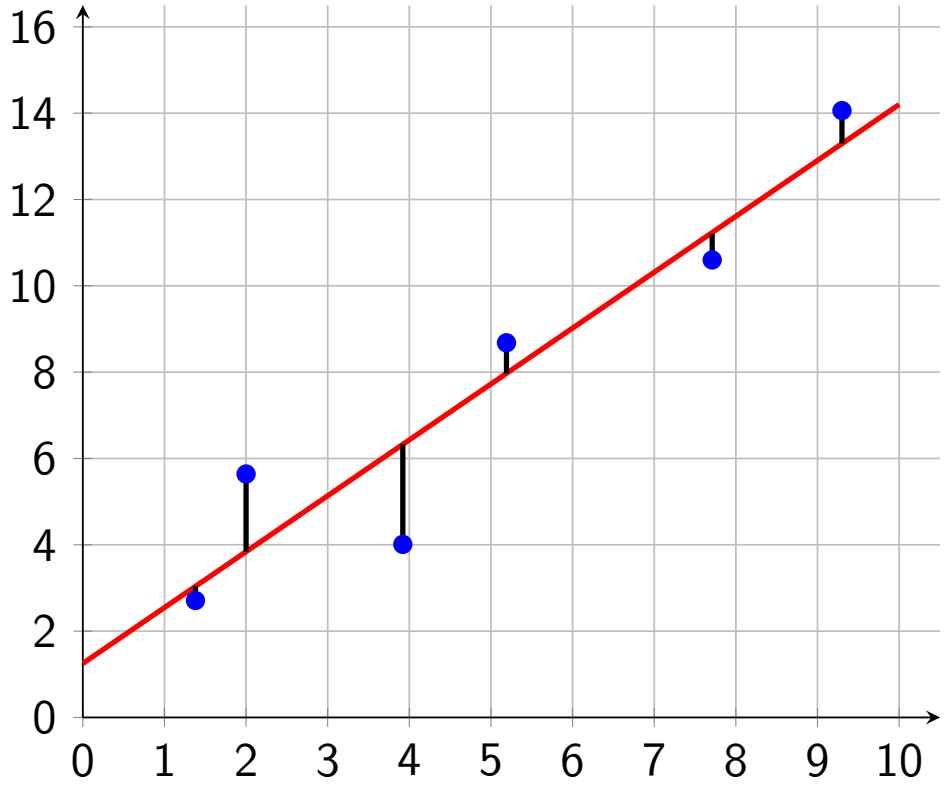

The black lines are **residuals**.

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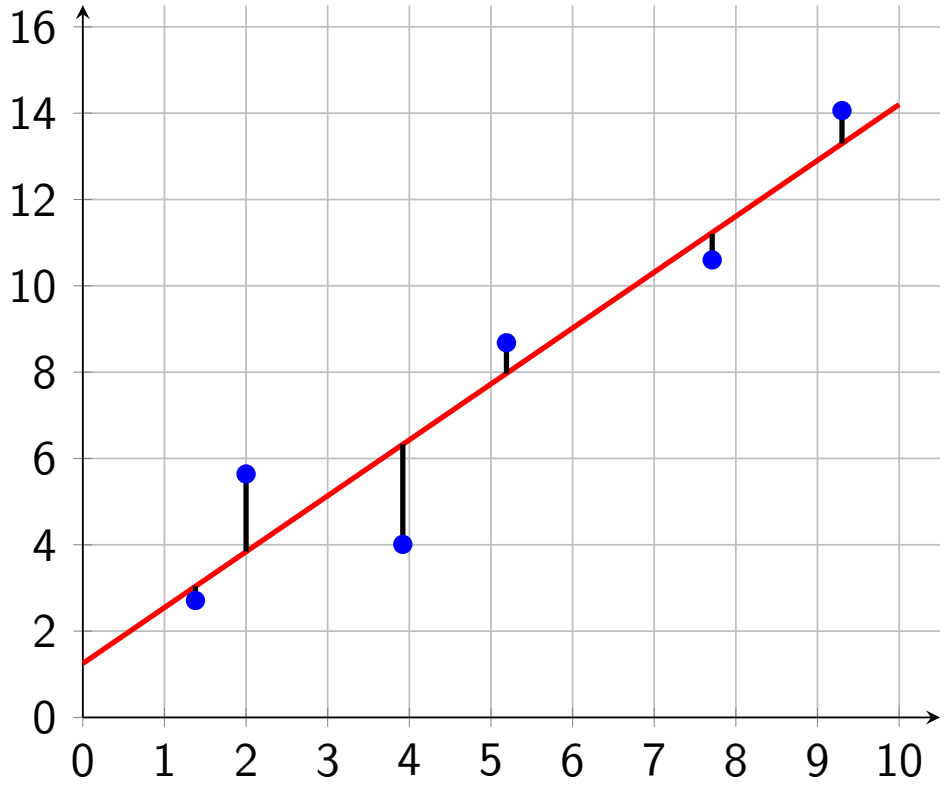

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Like deviations from the mean, the sum of the residuals is 0.

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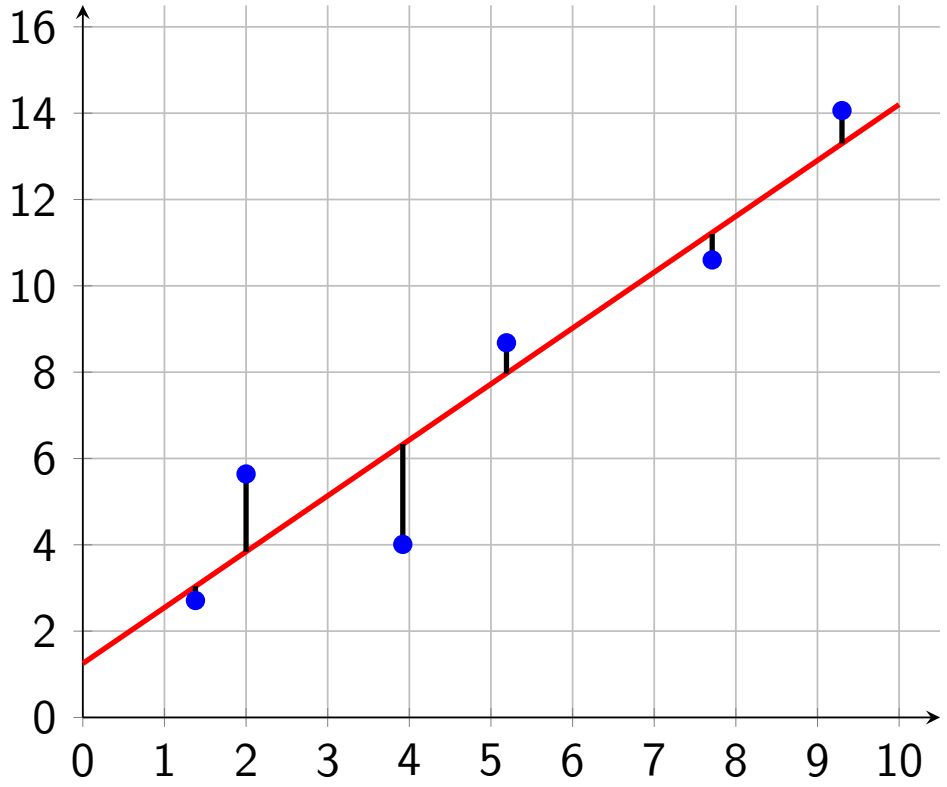

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So we need to square the deviations so the negatives don't cancel the positives.

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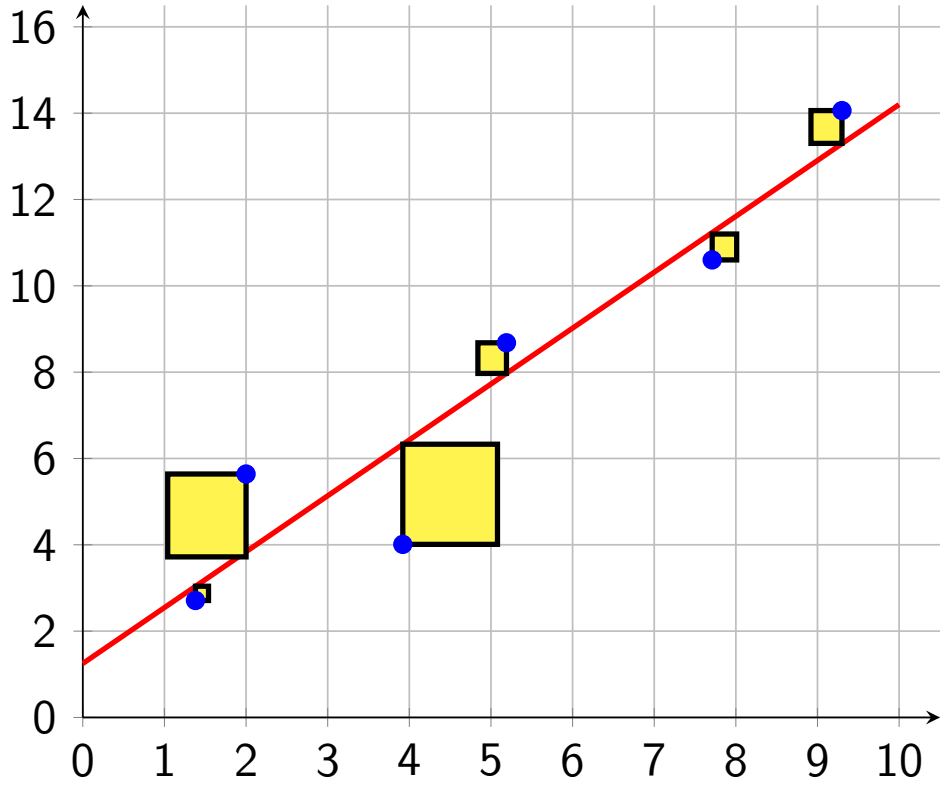

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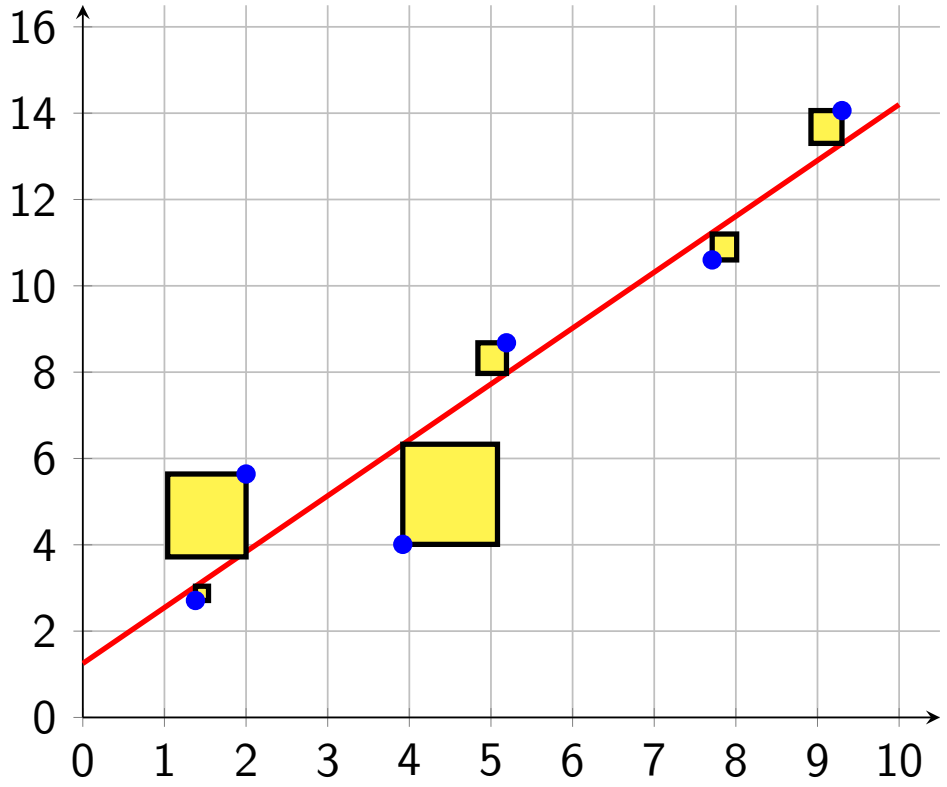

The line of best fit minimizes the sum of the areas of the squares.

Slope and y-intercept

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$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

and

$$b = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

Example 2

Find the least squares regression equation for the following dataset.

x	y
7.6	19.1
9.2	22.9
3.3	10.3
1.1	6.6
3.7	10.6
3.9	11.3
4.6	12.9
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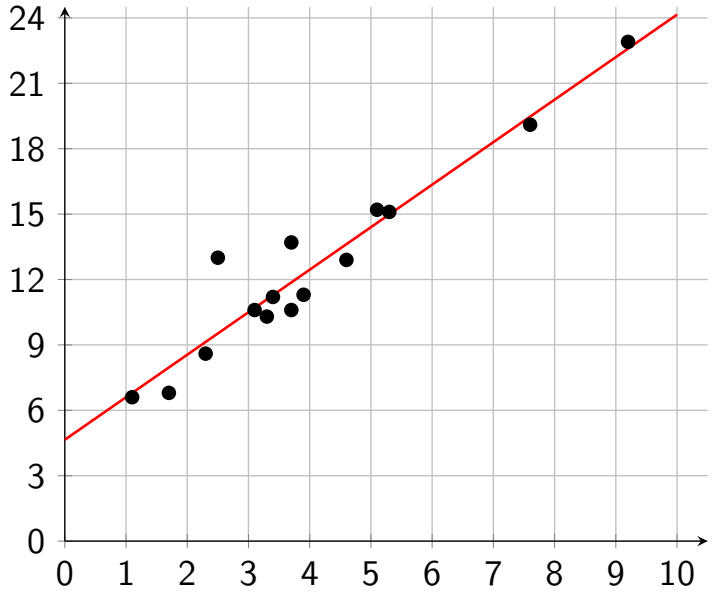

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The predicted value when $x = 6$ is $y = 16.35$

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We could then add that observation to our dataset and use it to create a better linear regression equation.

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$$\begin{aligned}\hat{y} &= 1.95x + 4.65 \\ &= 1.95(11) + 4.65 \\ &= 26.1\end{aligned}$$

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The value r^2 is called the **coefficient of determination** and it tells us what percentage of the variability in the response (y) variable is due to the variability in the explanatory variable (x). The rest may be due to such things as lurking variables.

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Since $-1 \leq r \leq 1$, $0 \leq r^2 \leq 1$

Note: If r (and hence r^2) is close to 0, then the linear regression equation is not going to be a good predictor. In this case, use $\hat{y} = \bar{y}$ as the linear predictor equation.

Example 4

Determine and interpret the value of r^2 given the paw width (in inches) and dog's weight (in pounds) below.

Paw	Weight
1.32	10
1.67	13
2.76	25
3.98	65
2.97	39
2.49	19
3.84	85
4.04	104
4.52	158
3.18	65
3.29	58
3.63	101
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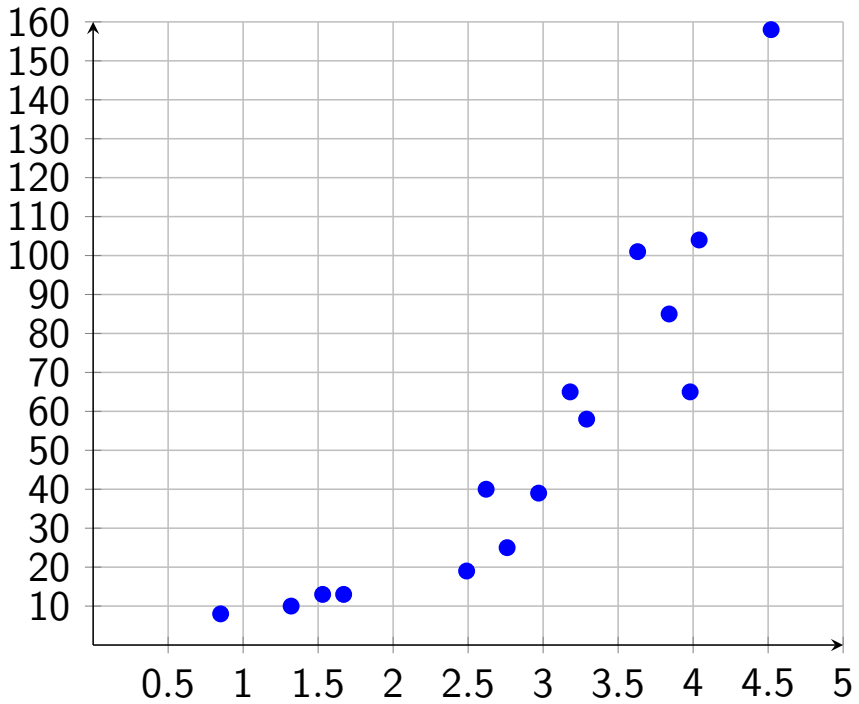

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$$r^2 \approx 0.8049$$

About 80.49% of the variation in dog weights is explained by the size of the dog's paw.

Example 4

Paw	Weight
1.32	10
1.67	13
2.76	25
3.98	65
2.97	39
2.49	19
3.84	85
4.04	104
4.52	158
3.18	65
3.29	58
3.63	101
0.85	8
4.62	157
1.53	13

$$r^2 \approx 0.8049$$

About 80.49% of the variation in dog weights is explained by the size of the dog's paw.

The remaining 19.5% can be attributed to other factors such as diet, genetics, exercise, etc.

Other Types of Regression

Since we only get values of r for *linear* regression, we will not calculate a linear correlation coefficient for a data set that does not appear linear.

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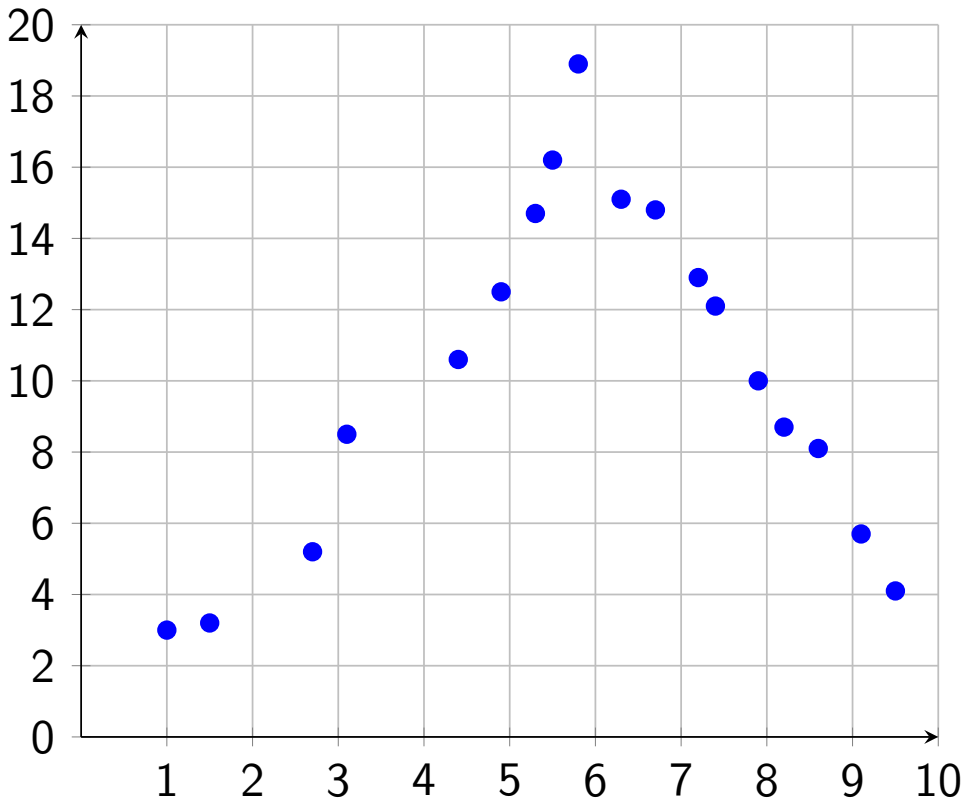

Other Types of Regression

We can still calculate r^2 using the more-traditional formula than was presented in this section; although you would likely want to let technology calculate it for you.