Probability: AND

Objectives

1. Calculate probabilities using the Multiplication Rule
2. Find probabilities of independent events
3. Find conditional probabilities
4. Find probabilities of dependent events

Example 1

You flip a coin and then roll a single die. What is the probability that you flip heads **and** roll a 5?

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Sample space:

	1	**2**	**3**	**4**	**5**	**6**
Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

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Tails	T1	T2	T3	T4	T5	T6

$$P(\text{heads and } 5) = \frac{1}{12}$$

Multiplication Rule

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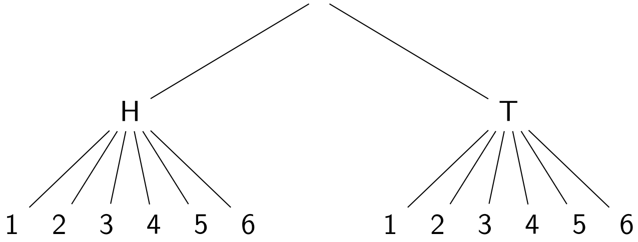
Multiplication Rule

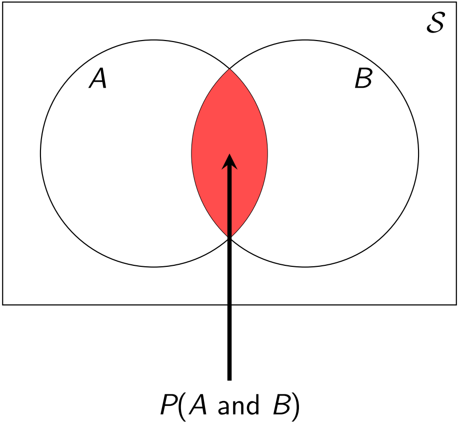
If $P(A)$ is the probability of event A occurring, and $P(B)$ is the probability of event B occurring, then

$$P(A \text{ and } B) = P(A) \times P(B)$$

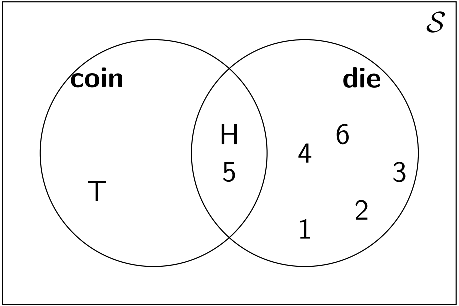
Tree Diagram



Venn Diagram – AND



Venn Diagram – AND



Mutually Exclusive Events

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Two events are **mutually exclusive** if they can not happen together. In other words,

$$P(A \text{ and } B) = 0$$

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When selecting items from a collection, independent events often contain selections made with replacement.

Example 2

A jar contains 10 blue, 12 black, and 15 red marbles.

(a) What is the probability of selecting a black marble, putting it back, and then selecting a blue marble?

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(a) What is the probability of selecting a black marble, putting it back, and then selecting a blue marble?

$$
\begin{aligned}
P(\text{black and blue}) &= \frac{12}{37} \times \frac{10}{37} \\
&= \frac{120}{1,369}
\end{aligned}
$$

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(b) What is the probability of selecting a red marble, putting it back, and then selecting another red marble?

$$P(\text{red and red}) = \frac{15}{37} \times \frac{15}{37}$$

$$= \frac{225}{1,369}$$

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With conditional probability, the denominator will often by the total of something that follows the words "if", "suppose that", or "given that".

Example 3

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a domestic car is randomly selected, what is the probability that it is 6–10 years old?

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$$P(\text{6–10 years old given that it is a domestic car}) = \frac{12}{100}$$

$$= \frac{3}{25}$$

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Foreign	37	21	12	30	100
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(b) What is the probability of selecting a domestic car given that the car is 6–10 years old?

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	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
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(b) What is the probability of selecting a domestic car given that the car is 6–10 years old?

$$P(\text{domestic car given that it is 6–10 years old}) = \frac{11}{23}$$

Example 3

	0–2	3–5	6–10	Over 10	Total
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(c) Suppose that a new car is selected, what is the probability that it is a foreign car?

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	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Foreign	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(c) Suppose that a new car is selected, what is the probability that it is a foreign car?

$$P(\text{foreign car given that it is 0–2 years old}) = \frac{37}{72}$$

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