Probability: OR

Objectives

1. Calculate probabilities using the Addition Rule
2. Calculate the complement of an event
3. Calculate "at least one" probabilities
4. Calculate the odds of an event

AND vs. OR

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In this section, we will focus on the word *or*, which will mean adding probabilities.

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The Addition Rule

In the previous example, the events “rolling a 4” and “rolling a 5” were *mutually exclusive*.

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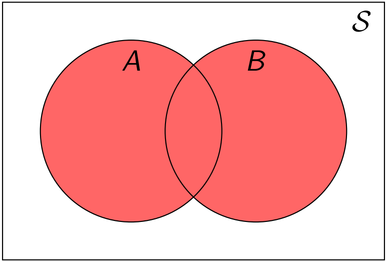
To find the `OR` probability of two mutually exclusive events, use the Addition Rule:

$$P(A \text{ or } B) = P(A) + P(B)$$

Venn Diagram – OR



$P(A \text{ or } B)$

Example 2

The table below lists the types and numbers of cars sold at Lemon Autos along with their ages. Find each probability.

	0–2	**3–5**	**6–10**	**Over 10**	**Total**
Import	37	21	12	30	100
Domestic	35	23	11	31	100
Total	72	44	23	61	200

(a) If a car is randomly selected, what is the probability that the car is 0–2 years old or over 10 years old?

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$$\begin{aligned} P(\text{0–2 or over } 10) &= P(0-2) + P(\text{over } 10) \\ &= \frac{72}{200} + \frac{61}{200} \\ &= \frac{133}{200} \end{aligned}$$

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(b) If a car is randomly selected, what is the probability that the car is 3–5 years old or a domestic car?

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(b) If a car is randomly selected, what is the probability that the car is 3–5 years old or a domestic car?

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$$\begin{aligned} P(\text{3–5 or domestic}) &= P(3-5) + P(\text{domestic}) \\ &= \frac{44}{200} + \frac{100}{200} \\ &= \frac{144}{200} \end{aligned}$$

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$$\begin{aligned} P(\text{3–5 or domestic}) &= P(3-5) + P(\text{domestic}) \\ &= \frac{44}{200} + \frac{100}{200} \\ &= \frac{144}{200} \\ &= \frac{18}{25} \end{aligned}$$

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$$\begin{aligned} P(3\text{–}5 \text{ or domestic}) &= P(3-5) + P(\text{domestic}) \\ &= \frac{44}{200} + \frac{100}{200} \\ &= \frac{144}{200} \\ &= \frac{18}{25} \quad \ldots \text{or is it?} \end{aligned}$$

Example 2 $P(3-5 \text{ or domestic})$

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There are 23 cars that are counted twice: once as a 3–5 year old car and again as a domestic car.

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There are 23 cars that are counted twice: once as a 3–5 year old car and again as a domestic car.

So, we need to subtract 23 cars from our original total of 144

$$P(3-5 \text{ years old or domestic}) = \frac{121}{200}$$

General Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

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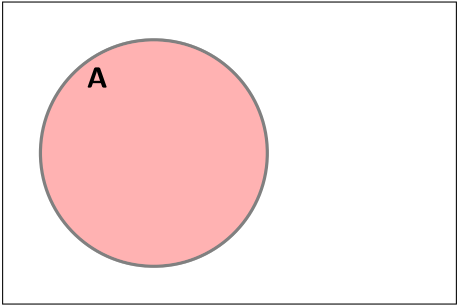

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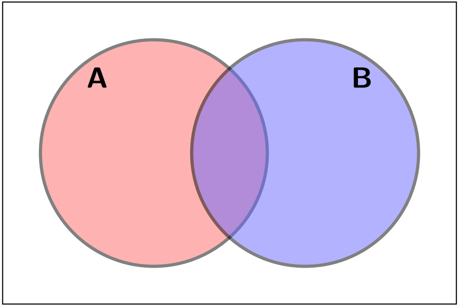

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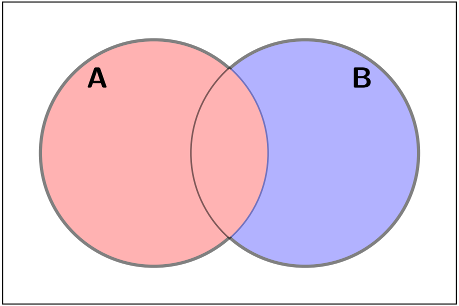

Venn Diagram of Example 2b

	0–2	3–5	6–10	Over 10	Total
Import	37	21	12	30	100
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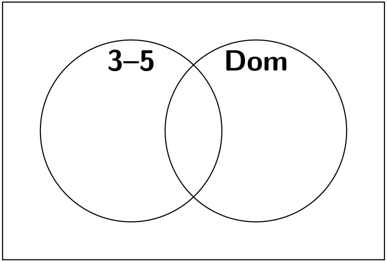

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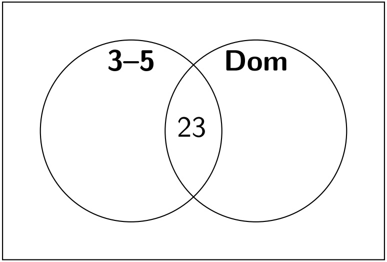

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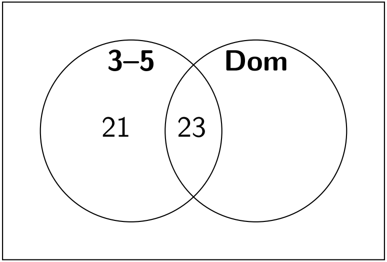

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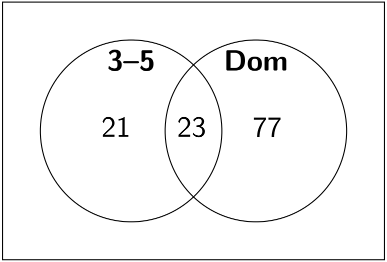

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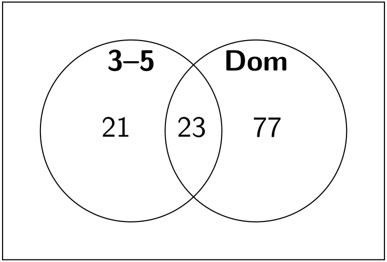

$$23 + 21 + 77 = 121$$

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$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

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&= \frac{12}{52} + \frac{26}{52} - \frac{6}{52}
\end{aligned}
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&= \frac{32}{52} \\
&= \frac{8}{13}
\end{aligned}
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$$P(A') = 1 - P(A)$$

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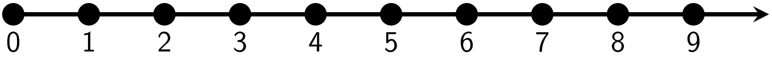
"At Least One" Probability



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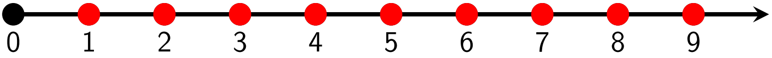

At least one means 1 or 2 or 3 or 4 or ...

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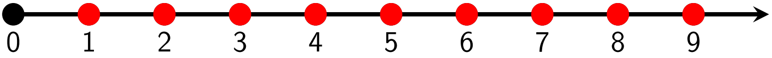

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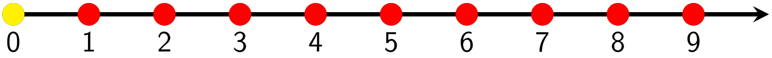

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The complement of *at least one* is **none**.

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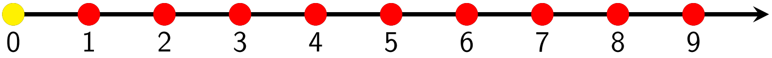

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Rather than add all of the probabilities of those events up, we can instead find the complement of *at least one*.

The complement of *at least one* is **none**.

In general, the complement of *at least n events* is $n-1$ events or less.

Example 4

Two dice are rolled. What is the probability of rolling a sum of at least 4?

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(\text{at least } 4) = 1 - P(\text{less than } 4)$$

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3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\begin{aligned} P(\text{at least } 4) &= 1 - P(\text{less than } 4) \\ &= 1 - P(2 \text{ or } 3) \end{aligned}$$

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3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\begin{aligned} P(\text{at least } 4) &= 1 - P(\text{less than } 4) \\ &= 1 - P(2 \text{ or } 3) \\ &= 1 - \frac{3}{36} \end{aligned}$$

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5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\begin{aligned} P(\text{at least } 4) &= 1 - P(\text{less than } 4) \\ &= 1 - P(2 \text{ or } 3) \\ &= 1 - \frac{3}{36} = \frac{33}{36} \end{aligned}$$

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5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\begin{aligned} P(\text{at least } 4) &= 1 - P(\text{less than } 4) \\ &= 1 - P(2 \text{ or } 3) \\ &= 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12} \end{aligned}$$

Example 5

A certain blood test can determine the presence of a bloodborne pathogen 97% of the time (that is, if 100 people have the pathogen, the test will confirm true for 97 of them). If 4 people with the pathogen are given the test, find the probability that the test is accurate for at least one of them.

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$P(\text{at least 1 accurate}) = 1 - P(\text{none are accurate})$

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$$\begin{aligned} P(\text{at least 1 accurate}) &= 1 - P(\text{none are accurate}) \\ &= 1 - P(\text{1st inaccurate}) \times P(\text{2nd inaccurate}) \cdots \end{aligned}$$

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$$\begin{aligned} P(\text{at least 1 accurate}) &= 1 - P(\text{none are accurate}) \\ &= 1 - P(\text{1st inaccurate}) \times P(\text{2nd inaccurate}) \cdots \\ &= 1 - (0.03)^4 \end{aligned}$$

Example 5

A certain blood test can determine the presence of a bloodborne pathogen 97% of the time (that is, if 100 people have the pathogen, the test will confirm true for 97 of them). If 4 people with the pathogen are given the test, find the probability that the test is accurate for at least one of them.

$$\begin{aligned} P(\text{at least 1 accurate}) &= 1 - P(\text{none are accurate}) \\ &= 1 - P(\text{1st inaccurate}) \times P(\text{2nd inaccurate}) \cdots \\ &= 1 - (0.03)^4 \\ &= 0.99999919 \end{aligned}$$

Objectives

1. Calculate probabilities using the Addition Rule
2. Calculate the complement of an event
3. Calculate "at least one" probabilities
4. Calculate the odds of an event

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Note: Typically when odds are listed, they are the odds against.

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$$\frac{\text{yellow}}{\text{red}} = \frac{5}{3} \longrightarrow \text{total marbles} = 8$$

$$P(\text{red marble}) = \frac{3}{8}$$