Standard Normal Distribution

Objectives

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2 Find the z scores corresponding to a given area

Continuous Distributions

Continuous Probability Distribution

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Continuous Distributions

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> A **continuous probability distribution** is a probability distribution in which the observations are continuous variables.

In this section, we are going to discuss the **standard normal distribution**, whose histogram resembles a bell-shaped curve.

Equation and Graph of Standard Normal Distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$
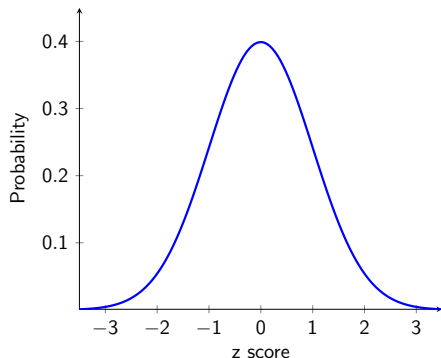
Properties of the Standard Normal Distribution

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- The mean is 0 and the standard deviation is 1
- The graph is symmetric about the mean
- **The area under the curve represents the probability of obtaining a z score in that area.**
- The total area under the curve equals 1

Example 1

Find the area under the curve to the left of each of the following z scores.

(a) $z = 0$

Example 1

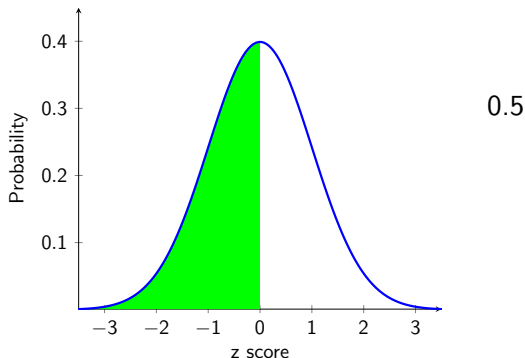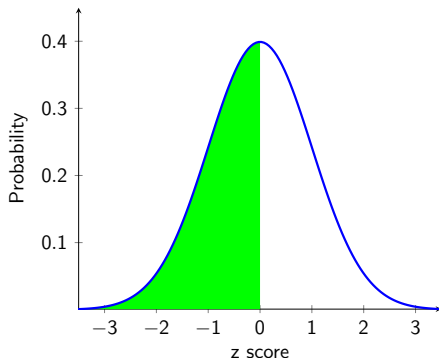Find the area under the curve to the left of each of the following z scores.
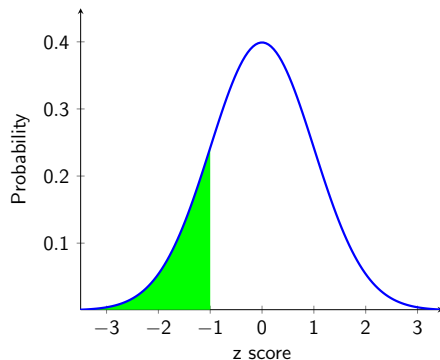
(a) $z = 0$

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Find the area under the curve to the left of each of the following z scores.

(a) $z = 0$

0.5
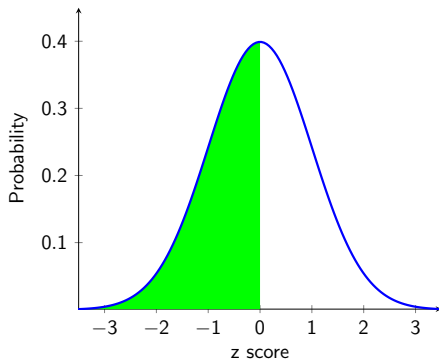
Example 1
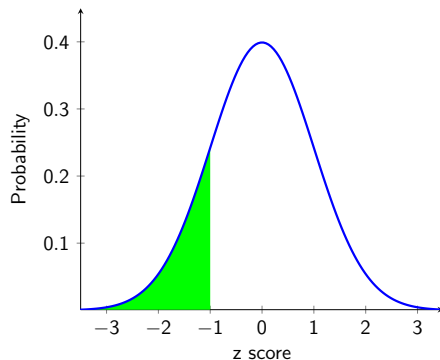
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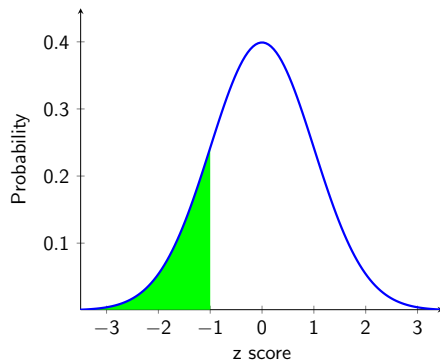$P(z \leq 0) = 0.5$

Example 1

(b) $z = -1$

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0.1587

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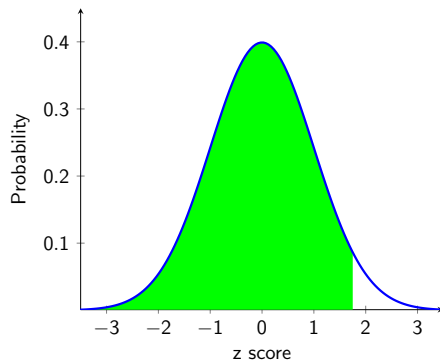
$P(z \leq -1) = 0.1587$

Example 1

(c) $z = 1.75$
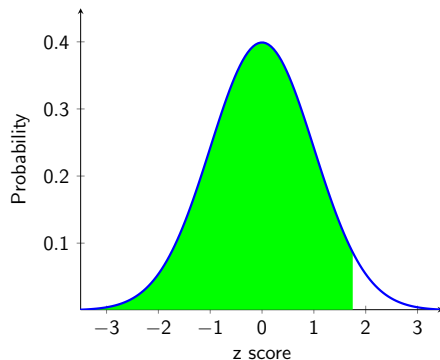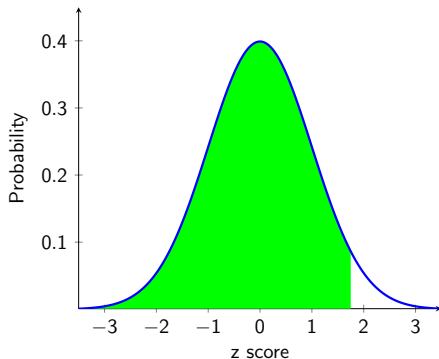
Example 1

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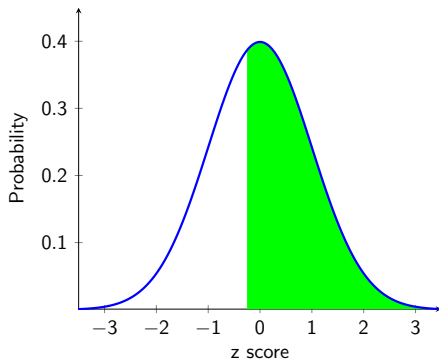
$P(z \leq 1.75) = 0.9599$

Example 2

Find the area under the curve to the right of each of the following z scores.

(a) $z = -0.25$

Example 2

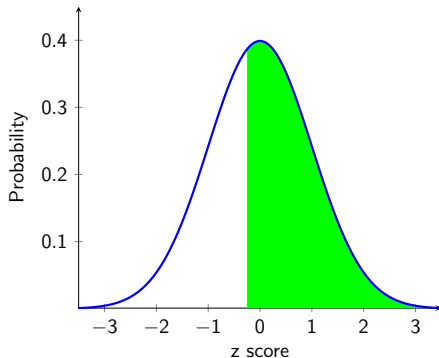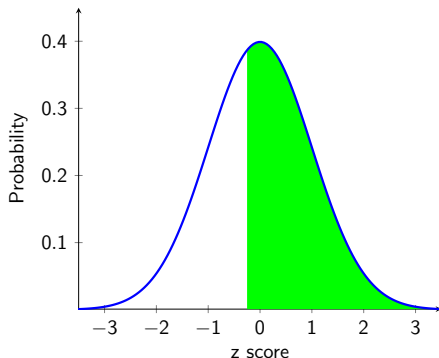Find the area under the curve to the right of each of the following z scores.

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Example 2

Find the area under the curve to the right of each of the following z scores.

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0.5987

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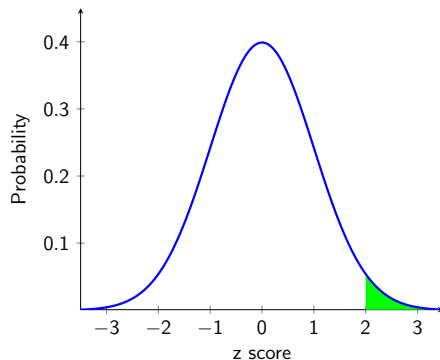
(a) $z = -0.25$

0.5987

$P(z \geq -0.25) = 0.5987$

Example 2

(b) $z = 2$
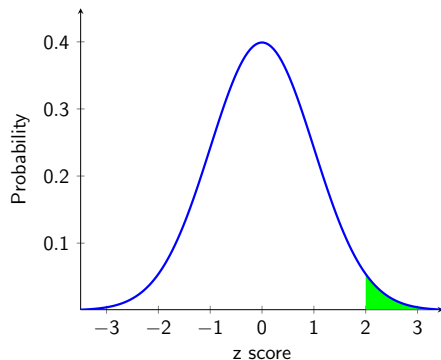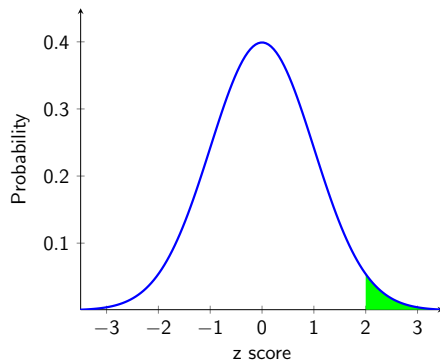
Example 2

(b) $z = 2$

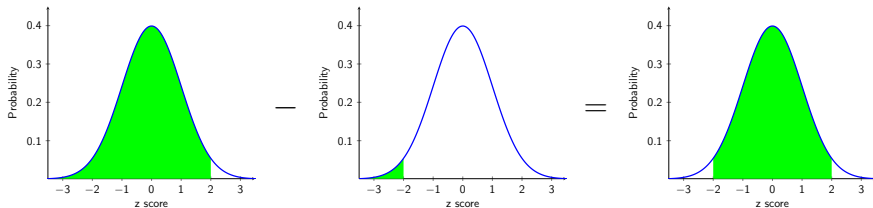# Example 2

(b) $z = 2$

0.0228

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$P(z \geq 2) = 0.0228$

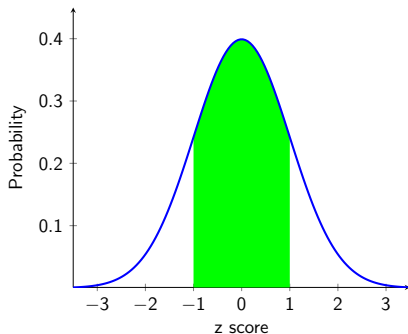# Finding the Area Between Two z Scores

Example 3

Find the area under the curve between the given z scores.

(a) $z = -1$ and $z = 1$

Example 3

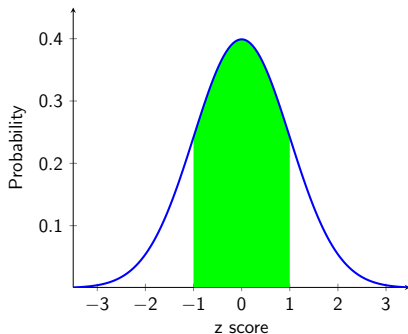Find the area under the curve between the given z scores.
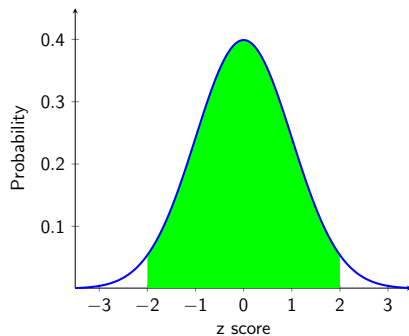
(a) $z = -1$ and $z = 1$

Example 3

Find the area under the curve between the given z scores.

(a) $z = -1$ and $z = 1$

$P(-1 \leq z \leq 1) = 0.6827$
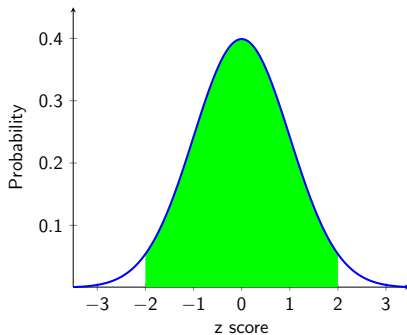
Example 3

(b) $z = -2$ and $z = 2$

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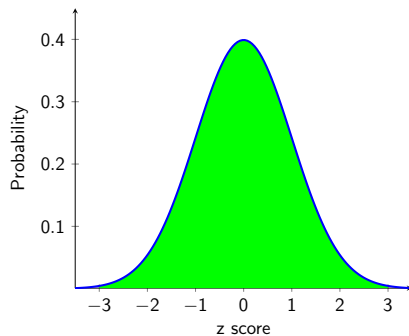
(b) $z = -2$ and $z = 2$

$P(-2 \leq z \leq 2) = 0.9545$

Example 3

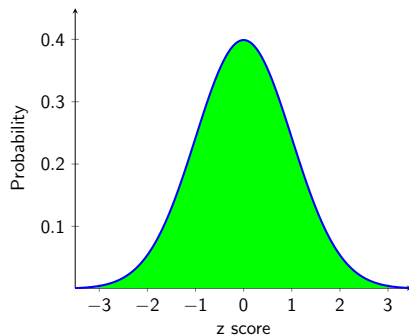(c) $z = -3$ and $z = 3$

Example 3

(c) $z = -3$ and $z = 3$

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$P(-3 \leq z \leq 3) = 0.9973$

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For normally distributed data, the Empirical (sometimes called 68-95-99.7 Rule) states that

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- About 68% of the data will lie within 1 standard deviation of the mean.
- About 95% of the data will lie within 2 standard deviations of the mean.
- About 99.7% of the data will lie within 3 standard deviations of the mean.
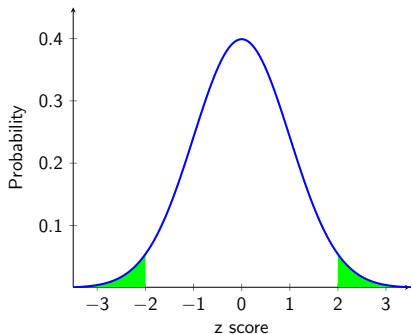
Total Outside Two z Scores

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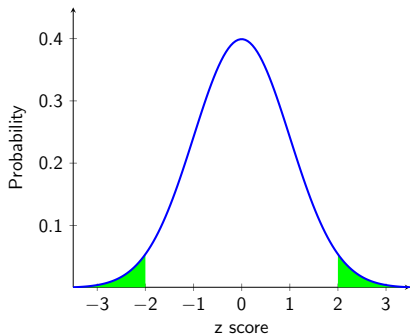
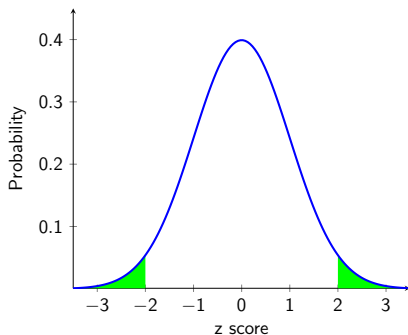# Total Outside Two z Scores

We can use the **complement rule** to find the total area under the normal curve *not between* two z scores:

$P(|z| \geq 2) =$
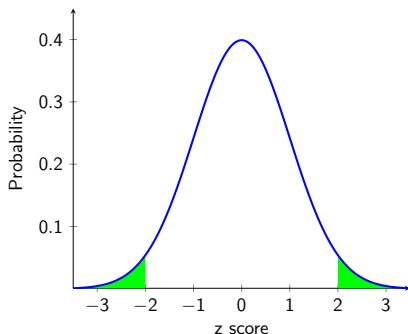
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$$= 1 - P(|z| \leq 2)$$

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$$P(|z| \geq 2) =$$

$$= 1 - P(|z| \leq 2)$$

$$\approx 0.0455$$

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Finding z Score for a Given Area

This is just working backwards from the our previous examples.

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Keep in mind that most (not all) technology will ask you for the area to the *left* of the needed z score.
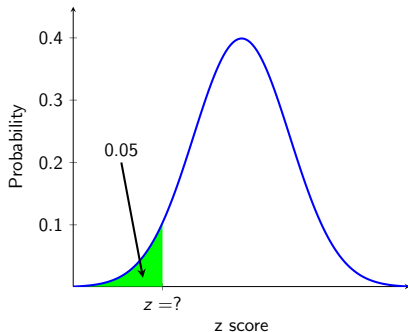
Example 4

Find the z score which has the given area to the left of it. Round to 3 decimal places.

(a)   0.05

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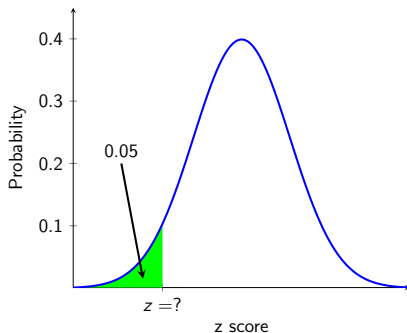
(a)   0.05

$z \approx -1.645$

Area to the Left as a Percent

Since the area to the left of $z = -1.645$ in Example 4a is about 0.05, we would say that a z score of -1.645 is in the 5th percentile.

Example 4
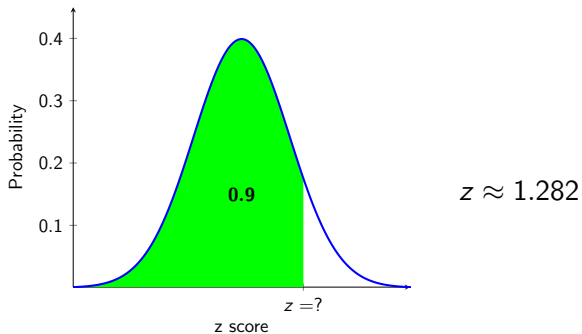
(b)   What z score is in the 90th percentile?

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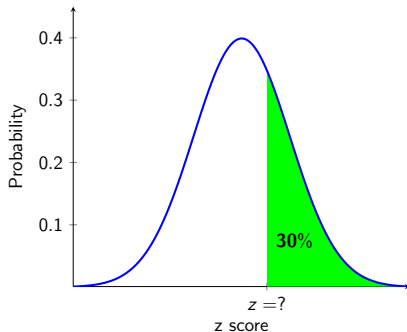

$z \approx 1.282$

Example 4

Find the z score which has the given area to the *right* of it. Round to 3 decimal places.

(c)   0.3

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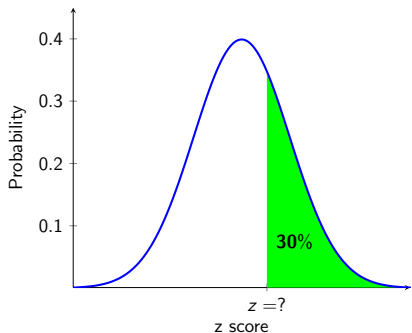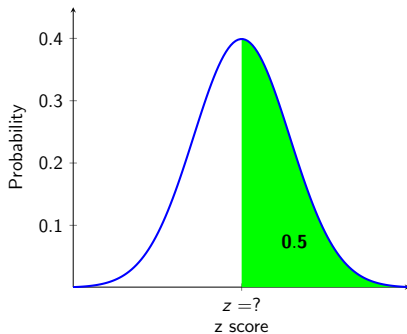
(c)   0.3
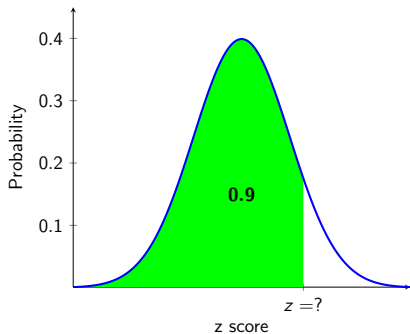
Example 4

Find the z score which has the given area to the *right* of it. Round to 3 decimal places.

(c)   0.3

$z \approx 0.524$

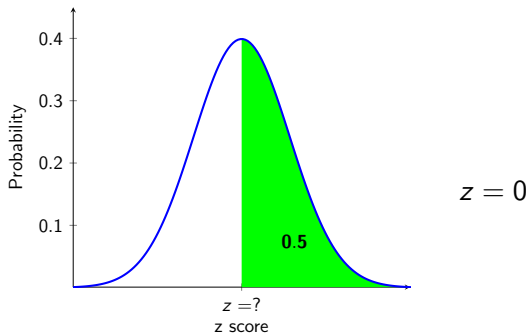## Example 4

(d) Area to the right is 0.5

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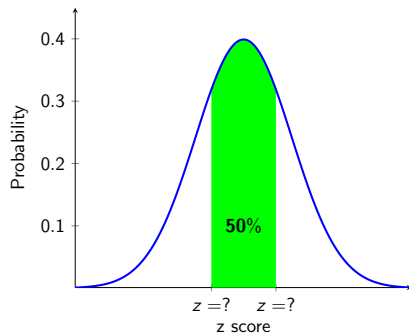

$z = 0$

Example 5

Find the z scores that separate the given middle percent of the data from the other values.

(a) 50%

Example 5

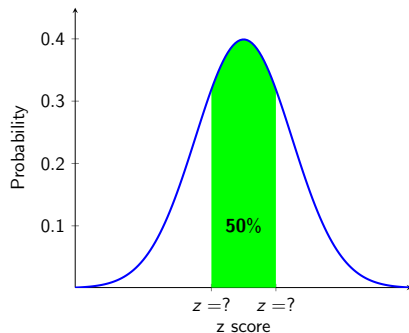Find the z scores that separate the given middle percent of the data from the other values.
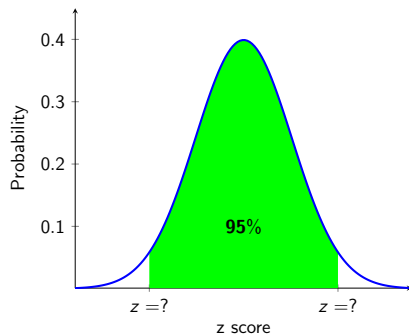
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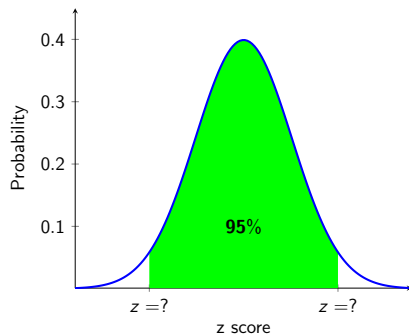(a) 50%

$z = \pm 0.674$

Example 5

(b) The middle 95%

Example 5

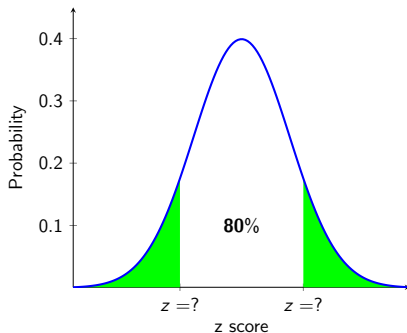(b)   The middle 95%

Example 5
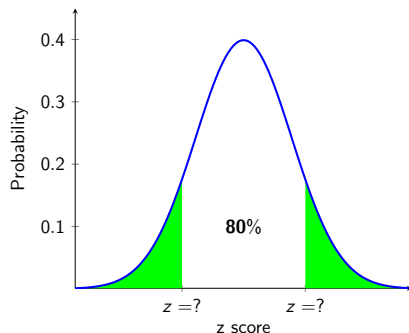
(b) The middle 95%

$z = \pm 1.960$

Example 5

(c)   Find the z scores that separate the middle from the remaining 20%

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$z = \pm 1.282$