Hypothesis Testing

Single Sample Proportion

Testing a Population Proportion

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- A random sample is selected from a binomial population (*success or failure*)

- The sample size n is large
  - Will be met if np and $n(1-p)$ are each at least 15

Test Statistic and Standard Error

The sample proportion, $\hat{p}$ is given by

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with sample standard error

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Test Statistic and Critical Value Method

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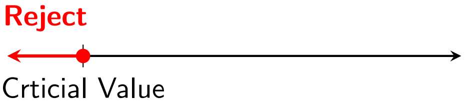
Significance Level	**Critical Value**
$\alpha = 0.01$	± 2.576
$\alpha = 0.05$	± 1.96
$\alpha = 0.10$	± 1.645

Test Statistic and Critical Value Method

Left-tailed



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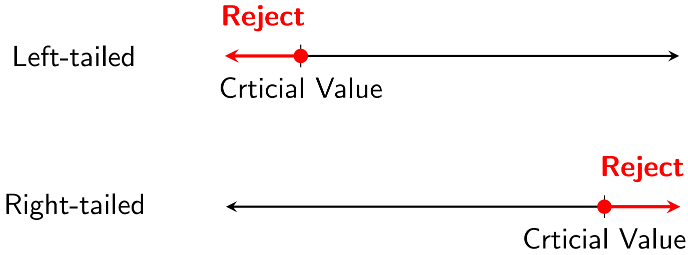

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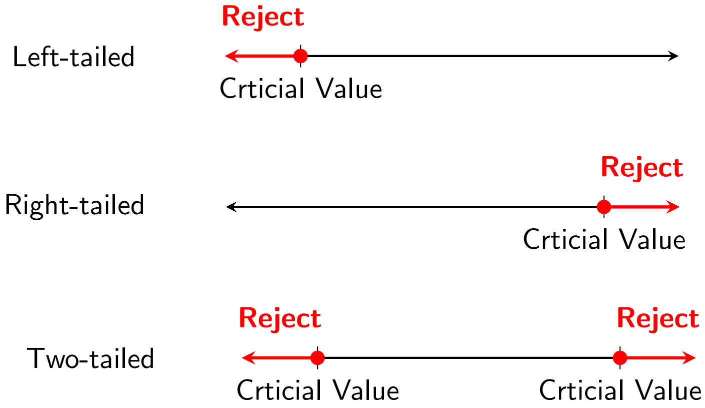

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If our p-value is lower than α, then our results are **statistically significant**, and would not likely occur by chance if the null hypothesis were true.

Reminder, if the p-value $< \alpha$, we reject the null hypothesis.

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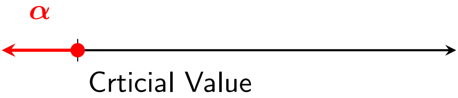

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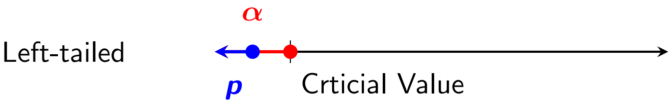

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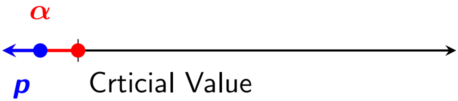
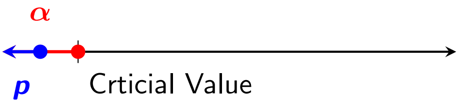


Reject null

p-Value Method

Left-tailed



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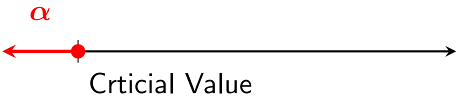

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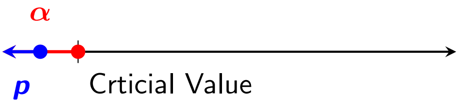

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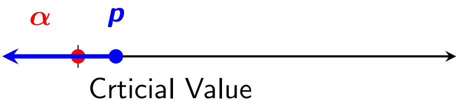

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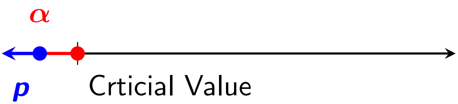

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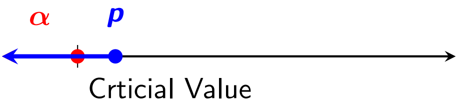

Do Not Reject Null

Confidence Interval Method

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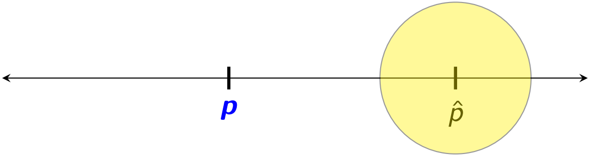
For one-sided testing, use the critical value z_{α}.

Confidence Interval Method

Reject Null



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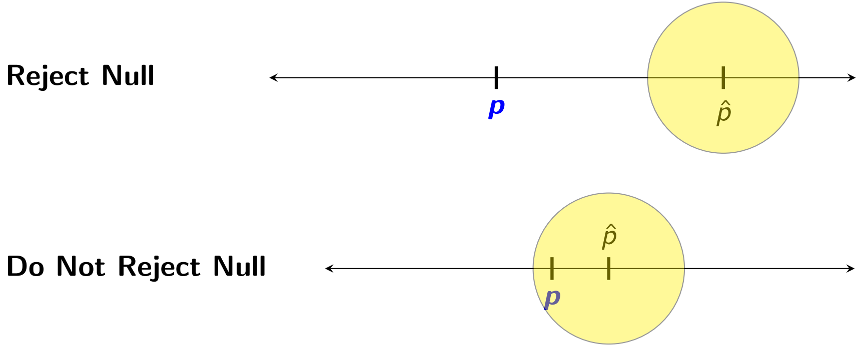

Example 1

According to a recent study, 72% of Americans favor starting school after Labor Day. You believe the percentage is greater than 72%, so you obtain a sample of 400 Americans and find that 75% of them favor starting school after Labor Day.

At the $\alpha = 0.05$ significance level, test the claim.

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$H_0: \; p = 0.72$
$H_A: \; p > 0.72$

Example 1

Test statistic: $z = 1.336$ (Critical Value $= 1.645$)
p-value $= 0.0907$
95% lower bound: 0.7144
(we are 95% confident the population proportion is 71.44% or higher)

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Do not reject null hypothesis

> *At the 5% significance level, we do not have sufficient evidence to reject the null hypothesis and conclude that our sample does not support the claim that the mean percentage of Americans who favor starting school after Labor Day is greater than 72%.*

Example 2

According to a recent poll, 40% of people would vote for Bryan Bain to be President of the United States (never going to happen). You want to test the claim (at the $\alpha = 0.05$ significance level) that the percent of people who would vote for Bryan Bain is not 40%, so you obtain a random sample of 700 voters and find that 45% would vote for Bryan Bain.

Example 2

According to a recent poll, 40% of people would vote for Bryan Bain to be President of the United States (never going to happen). You want to test the claim (at the $\alpha = 0.05$ significance level) that the percent of people who would vote for Bryan Bain is not 40%, so you obtain a random sample of 700 voters and find that 45% would vote for Bryan Bain.

$H_0: \ p = 0.40$
$H_A: \ p \neq 0.40$

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Test statistic: $z = 2.700$ (Critical value $= 1.96$)
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> *At the 5% significance level, we have sufficient evidence to reject the null hypothesis that the percentage of voters that would vote for Bryan Bain is 40% and conclude that our sample supports the claim that the percentage may be different than 40.*

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Confidence Interval $= (0.4131, 0.4869)$

Reject the null hypothesis.

> *At the 5% significance level, we have sufficient evidence to reject the null hypothesis that the percentage of voters that would vote for Bryan Bain is 40% and conclude that our sample supports the claim that the percentage may be different than 40.*

*On a personal note, take a look at any US President when they begin their first term and then see what they look like when they leave office. *That* is why I don't want to be President.