Measures of Center

Objectives

1. Calculate the mean, median, and mode of a dataset
2. Calculate the weighted mean of a dataset
3. Approximate the mean for a grouped dataset
4. Determine if a dataset appears skewed left, skewed right, or normal

Measures of Center

In this section, we will look at finding a central location for a dataset.

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In some instances, this can give us a good value to expect from that dataset.

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When most people use the term *average*, they are referring to the mean.

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- The mean of a data set uses every value, unless the mean is a *trimmed mean*.
- One extreme value (called an **outlier**) can change the value of the mean drastically.

Mean Formula

Sample mean: $\overline{x}$ Population mean: μ

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$$\overline{x}, \text{ or } \mu, = \frac{\sum x_i}{n} = \frac{1}{n} \sum x_i$$

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The median is denoted by $\tilde{x}$, however, some technologies uses `Med` to denote it.

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The mode is the only measure of center that is applicable to qualitative data.

Example 1

The dataset below represents the number of complaints I receive each week about my teaching.

8, 2, 3, 7, 4, 4, 1, 9, 7, 5

Calculate the mean, median, and mode of the number of complaints.

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Mode number of complaints per week are 4 and 7.

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Mode: 4 and 7 complaints per week.

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Example 3

California has a mean class size of 20.9 students per teacher and Alaska has a mean of 16.8 students per teacher.

If we combine the two states, we might find the mean number of students per teacher to be 18.85 (0.5 * (20.9 + 16.8)) but is this result correct? Why or why not?

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If we combine the two states, we might find the mean number of students per teacher to be 18.85 (0.5 * (20.9 + 16.8)) but is this result correct? Why or why not?

It is not correct because California has a much higher population that Alaska. We would have to find what is known as the **weighted mean**.

Weighted Mean

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In other words,

1. Multiply each data value by its corresponding weight.
2. Add those results.
3. Then divide that by the total of the weights.

Example 4

You've recently completed a semester. Determine the semester's GPA (A = 4pts, B = 3pts, etc).

Course	**Grade**	**Credit Hours**
Statistics	A	4
Advanced Chris Farley	A	3
Airplane! Quotes	B	5
Obnoxious Examples	C	3

Example 4

Grade	Credit Hours	TOTALS
4	4	16
4	3	12
3	5	15
2	3	6
	15	49

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Weighted Mean: $49/15 = 3.267$

Example 5

In a statistics course, tests count for 60% of the final grade, homework for 20% and midterm and final exams are 10% each. Suppose you've earned an 87% average on tests, 94% average on homeworks and a 77% average on the exams.

What is your overall percentage?

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What is your overall percentage?

Assessment	**Your Scores**	**Grade Weights**	**TOTALS**
Tests	0.87	0.60	0.522
Homework	0.94	0.20	0.188
Exams	0.77	0.20	0.154

Example 5

Grade Weights	TOTALS
0.60	0.522
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Grade: $0.864/1 = 86.4\%$

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We will use similar techniques like we did in finding the weighted mean, however we will have to use the class midpoints as our observed values.

Example 6

(I swear this is a true story) On one very hungry day of mine, I ordered and consumed 19 sushi rolls from an all-you-can-eat sushi restaurant. This, as you might guess, is not typical (at least for me). The table below indicates the frequencies of sushi rolls I typically eat.

Number of Rolls	**Frequency**
1 – 5	4
6 – 10	17
11 – 15	12

Estimate the mean number of rolls consumed.

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Class Midpoint	Frequency	TOTALS
3	4	12
8	17	136
13	12	156
	33	304

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	33	304

Mean: $304/33 \approx 9.2$ rolls per visit.

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When normally distributed (bell-shaped), the mean, median, and mode are (roughly) equal. However, some data sets my be *skewed* (remember, skewness refers to the tail).

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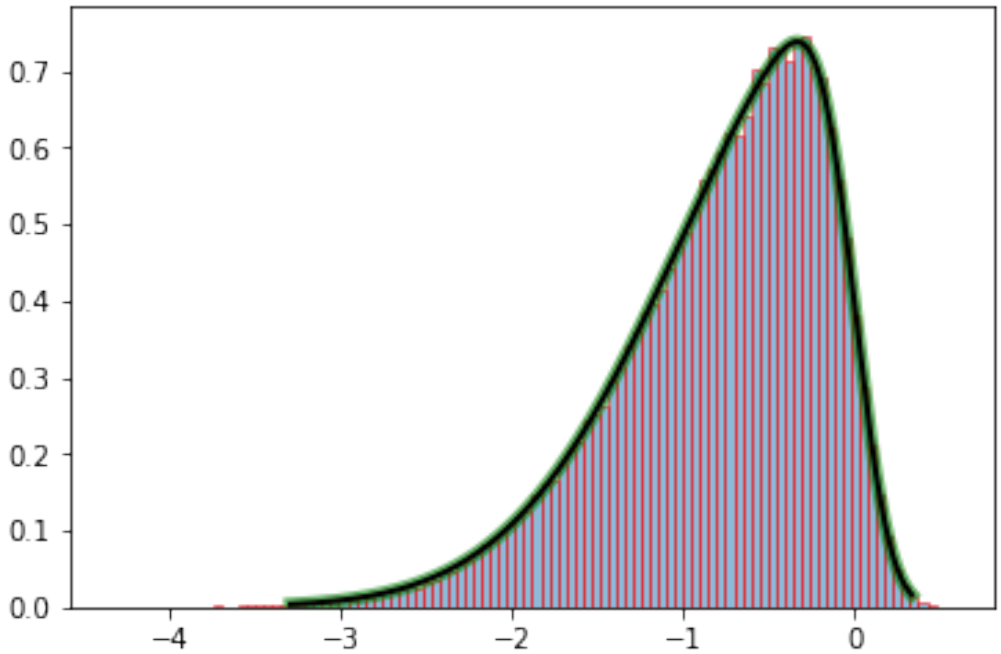
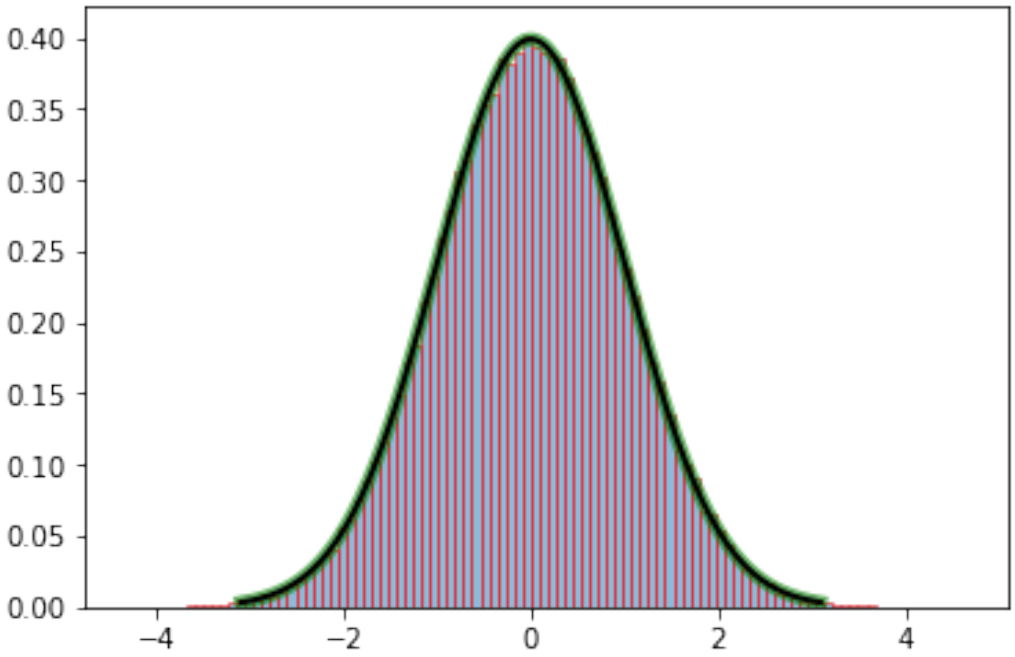
If the mean is (significantly) less than the median, the data is **skewed left** (or **negatively skewed**).



Skewness

If the mean is (approximately) equal to the median, the data is **normal** (no skewness)



Skewness

If the mean is (significantly) greater than the median, the data is **skewed right** (or **positively skewed**).