Linear Regression

Objectives

1. Determine and interpret the linear correlation coefficient
2. Determine the linear regression equation
3. Determine and Interpret the Coefficient of Determination

Linear Correlation Coefficient

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In this section, we will examine the correlation type the way it is done in the real world: calculating the linear correlation coefficient (r).

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- $r > 0$: positive linear correlation
- $r = 0$: no linear correlation
- $r < 0$: negative linear correlation

Linear Correlation Coefficient

$$r = \frac{\sum(x-\overline{x})(y-\overline{y})}{\sqrt{\sum(x-\overline{x})^2 \cdot \sum(y-\overline{y})^2}}$$

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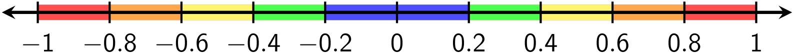
The closer r is to 1 (or -1), the more the data points “fall in line”

The closer r is to 0, the more the data points resemble a “cloud”

Interpreting r



None

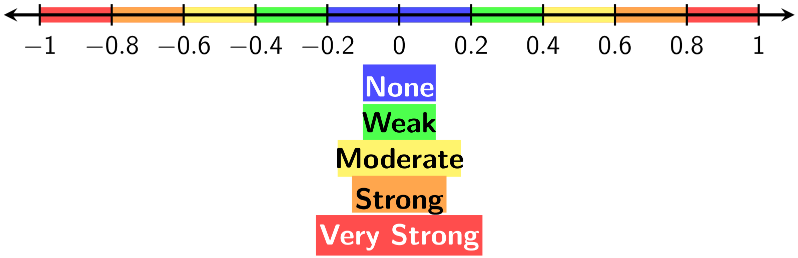
Weak

Moderate

Strong

Very Strong

Interpreting r



Note: These interpretations are not universal.

Example 1

Find and interpret the linear correlation coefficient, r, for each.

(a)

x	y
7.6	19.1
9.2	22.9
3.3	10.3
1.1	6.6
3.7	10.6
3.9	11.3
4.6	12.9
2.3	8.6
5.1	15.2
5.3	15.1
2.5	13
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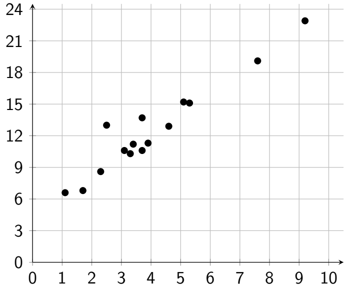

Example 1

(b)

x	y
7.6	11.0
9.2	3.6
6.3	8.9
1.1	14.9
6.7	8.1
3.9	12.0
4.6	9.4
2.3	10.3
5.1	11.4
5.3	12.4
2.5	9.0
3.4	8.9
3.1	14.2
1.7	10.9
3.7	13.3

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5.1	11.4
5.3	12.4
2.5	9.0
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$r \approx -0.6273$

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6.7	8.1
3.9	12.0
4.6	9.4
2.3	10.3
5.1	11.4
5.3	12.4
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3.4	8.9
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5.1	11.4
5.3	12.4
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3.4	8.9
3.1	14.2
1.7	10.9
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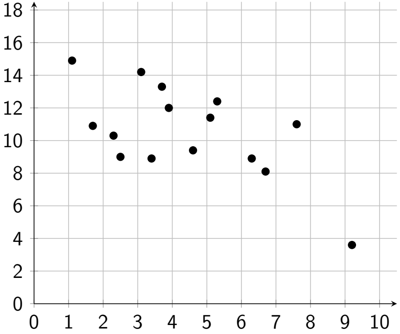
Strong negative linear correlation



Example 1

(c)

x	y
6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
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3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
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$r \approx -0.2218$

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6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
7.8	8.4
2.5	3.1
6.1	9.0
1.1	8.5

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Weak negative correlation

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6.9	3.4
7.7	4.5
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3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
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2.5	3.1
6.1	9.0
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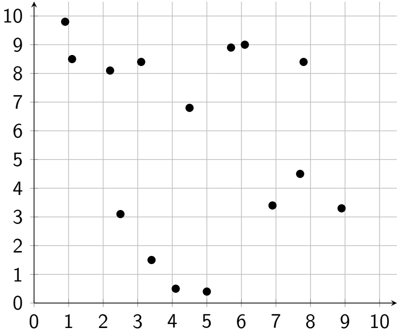
Weak negative correlation



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To do this, we can create the **least squares regression equation**, (also called the *line of best fit*) which will **minimize** the total squared distance each data point is from the line:

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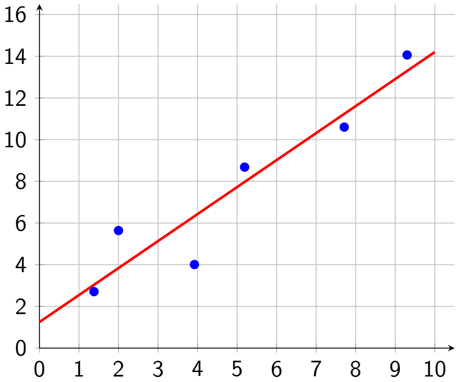
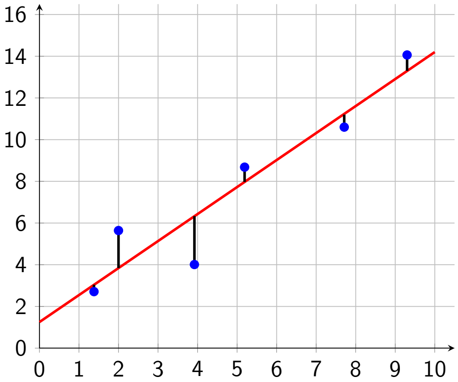
$$\hat{y} = mx + b$$

Line of Best Fit



Least Squares Regression Equation



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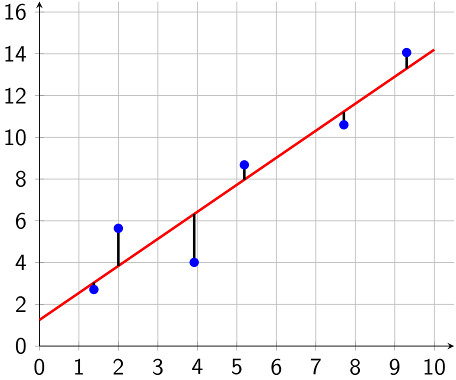

The black lines are **residuals**.

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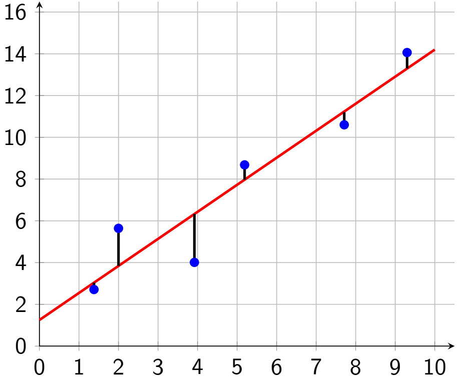

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Like deviations from the mean, the sum of the residuals is 0.

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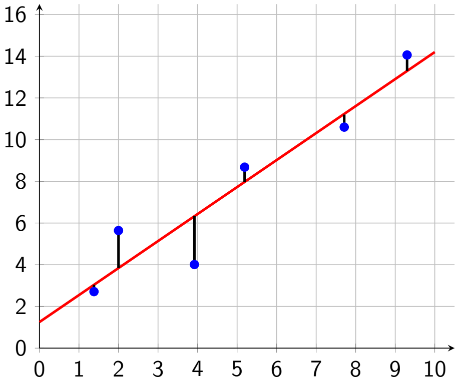

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Like deviations from the mean, the sum of the residuals is 0.

So we need to square the deviations so the negatives don't cancel the positives.

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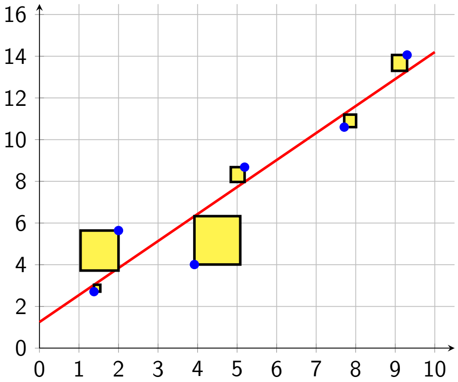

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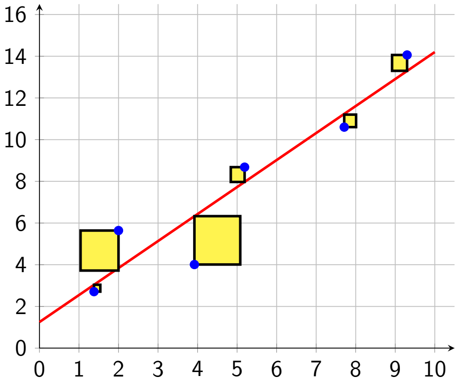

The line of best fit minimizes the sum of the areas of the squares.

Slope and y-intercept

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$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

and

$$b = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

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Find the least squares regression equation for the following dataset.

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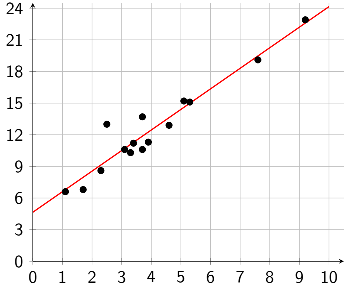

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Given the regression equation $\hat{y} = 1.95x + 4.65$, predict the values of the following response variables for each explanatory variable.

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$$\begin{aligned} \hat{y} &= 1.95x + 4.65 \\ &= 1.95(6) + 4.65. \\ &= 16.35 \end{aligned}$$

The predicted value when $x = 6$ is $y = 16.35$

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We could then add that observation to our dataset and use it to create a better linear regression equation.

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$$= 1.95(11) + 4.65$$

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Since 11 is outside of the x values in our dataset, finding its y-coordinate is called **extrapolation**.

$$\begin{aligned} \hat{y} &= 1.95x + 4.65 \\ &= 1.95(11) + 4.65 \\ &= 26.1 \end{aligned}$$

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The predicted value when $x = 11$ is $y = 26.1$.

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