# Binomial Probability Distributions

# Objectives

1. Calculate probabilities of binomial distributions
2. Calculate the mean, variance, and standard deviation of a binomial distribution

# Binomial Probability Experiment

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- There are a fixed number of $n$ repeated independent trials
- Each trial's outcome is either a success or failure
- The probability of success, $p$, never changes

# Flipping a Coin

Number of heads when flipping a coin 3 times:

| $x$ | Outcomes | $P(X = x)$ |
|---|---|---|
| 0 | TTT | $1/8$ |
| 1 | HTT THT TTH | $3/8$ |
| 2 | HHT HTH THH | $3/8$ |
| 3 | HHH | $1/8$ |

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Thus, combinations play a role in binomial probability distributions.

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It is more likely you will obtain 4 heads from 5 flips than 8 heads from 10 flips.

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$$\approx 0.00551$$

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$$P(x \geq 6) = 1 - P(x < 6)$$
$$\approx 0.00637$$

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