

Hypothesis Testing

Single Sample Proportion

Testing a Population Proportion

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Assumptions

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- The sample size n is large
 - Will be met if np and $n(1 - p)$ are each at least 15

Test Statistic and Standard Error

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with sample standard error

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Test Statistic and Critical Value Method

The test statistic of the population proportion, z , can be found by calculating

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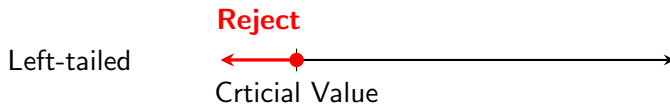
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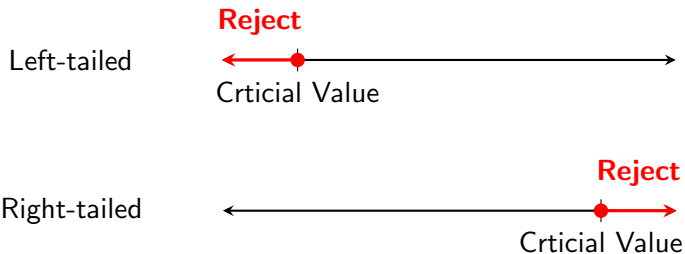
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Significance Level	Critical Value
$\alpha = 0.01$	± 2.576
$\alpha = 0.05$	± 1.96
$\alpha = 0.10$	± 1.645

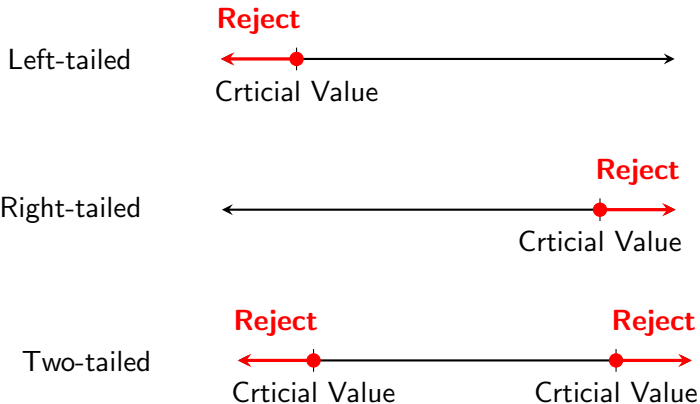
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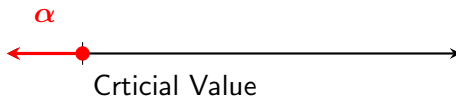
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Reminder, if the p -value $< \alpha$, we reject the null hypothesis.

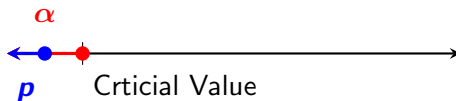
p -Value Method

Left-tailed



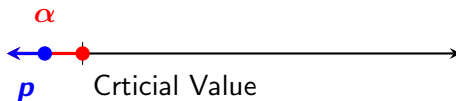
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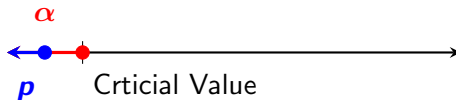
Left-tailed



Reject null

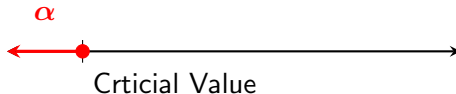
p -Value Method

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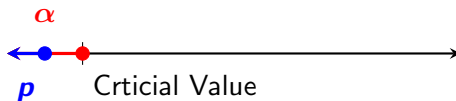
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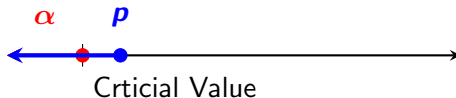
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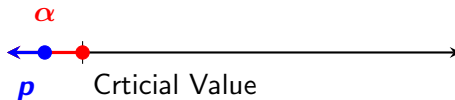
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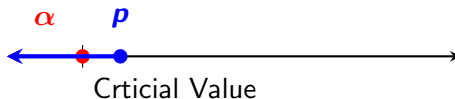
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Do Not Reject Null

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point estimate \pm margin of error

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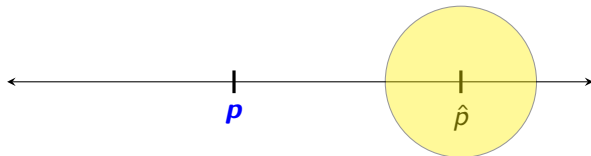
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$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

If our confidence interval contains the (claimed) population proportion, we do not reject the null hypothesis.

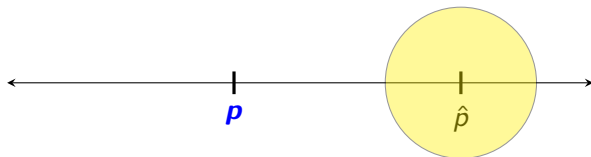
Confidence Interval Method

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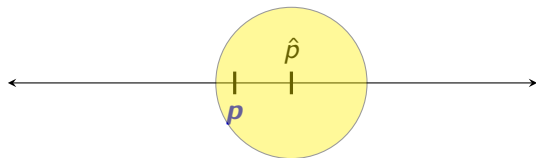


Confidence Interval Method

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Do Not Reject Null



Example 1

According to a recent study, 72% of Americans favor starting school after Labor Day. You believe the percentage is greater than 72%, so you obtain a sample of 400 Americans and find that 75% of them favor starting school after Labor Day.

At the $\alpha = 0.05$ significance level, test your claim.

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$$H_0 : p = 0.72$$

$$H_A : p > 0.72$$

Example 1

Test statistic: $z = 1.336$ (Critical Value = 1.645)

$p\text{-value} = 0.0907 > 0.05$

Confidence Interval = (0.7076, 0.7924); contains $p = 0.72$

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Do not reject null hypothesis

At the 5% significance level, we do not have sufficient evidence to reject the null hypothesis and conclude that our sample does not support the claim that the mean percentage of Americans who favor starting school after Labor Day is greater than 72%.

Example 2

According to a recent poll, 40% of people would vote for Bryan Bain to be President of the United States (never going to happen). You want to test the claim (at the $\alpha = 0.05$ significance level) that the percent of people who would vote for Bryan Bain is not 40%, so you obtain a random sample of 700 voters and find that 45% would vote for Bryan Bain.

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$$H_0 : p = 0.40$$

$$H_A : p \neq 0.40$$

Example 2

Test statistic: $z = 2.700 > 1.96$

$p\text{-value} = 0.0069 < 0.05$

Confidence Interval = $(0.4131, 0.4869)$; does not contain $p = 0.40$

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Reject the null hypothesis.

At the 5% significance level, we have sufficient evidence to reject the null hypothesis that the percentage of voters that would vote for Bryan Bain is 40% and conclude that our sample supports the claim that the percentage may be different than 40.

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At the 5% significance level, we have sufficient evidence to reject the null hypothesis that the percentage of voters that would vote for Bryan Bain is 40% and conclude that our sample supports the claim that the percentage may be different than 40.

*On a personal note, take a look at any US President when they begin their first term and then see what they look like when they leave office. *That* is why I don't want to be President.