

# Sampling Distributions

# Objectives

- 1 Obtain a sampling distribution of sample means
- 2 Determine the mean and standard error of a sampling distribution
- 3 Understand the Central Limit Theorem
- 4 Determine the mean and standard error for sampling distribution of proportions

# Sampling Distributions

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We can take a simple random sample of, say 2 computers, and create a frequency distribution of each sample.

*Note:* We will sample with replacement. Differences in sampling with and without replacement become negligible as sample sizes increase.

## Example 1

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Sample	Sample Mean	Sample	Sample Mean
1000, 1000	1000	1600, 1000	1300
1000, 1200	1100	1600, 1200	1400
1000, 1600	1300	1600, 1600	1600
1000, 2000	1500	1600, 2000	1800
1200, 1000	1100	2000, 1000	1500
1200, 1200	1200	2000, 1200	1600
1200, 1600	1400	2000, 1600	1800
1200, 2000	1600	2000, 2000	2000

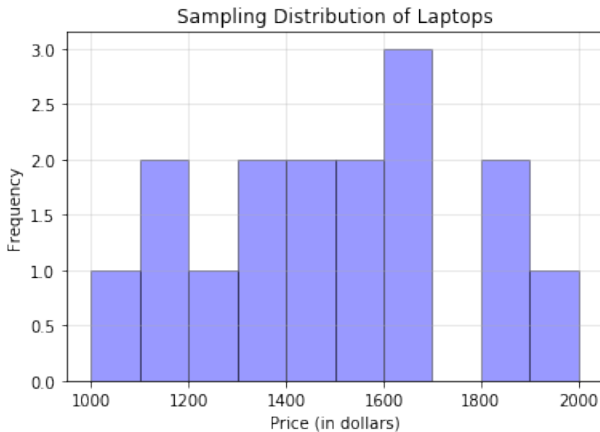


## Example 2

Create a histogram of the sample means from Example 1.

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## Example 3

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Std. Dev  $\approx$  \$271.57

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$$\sigma \approx \frac{\sigma_{\bar{x}}}{\sqrt{n}}$$

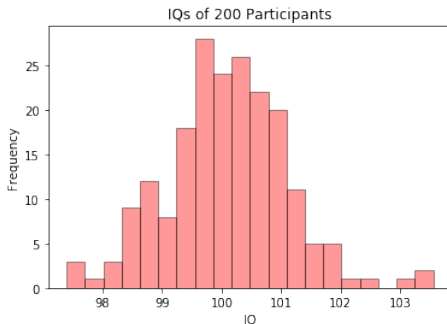
where  $\frac{\sigma}{\sqrt{n}}$  is called the **standard error of the mean**.

## Example 4

IQ scores are normally distributed with a mean of 100 and standard deviation of 16. A sample of 200 participants had their IQs measured.

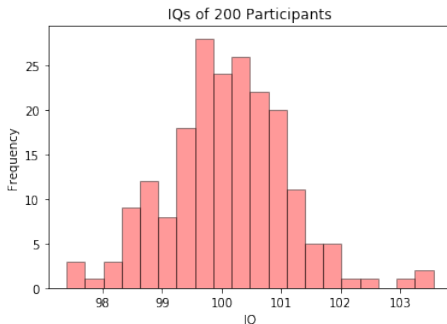
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What are the approximate mean and standard error of the sample?

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$$\approx 1.13$$

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It turns out that the distributions of sample means for any population will be normal as our sample sizes increase.

## Central Limit Theorem

As the sample size increases, the distribution of sample means becomes normal with a mean of  $\mu$  and standard deviation  $\frac{\sigma}{\sqrt{n}}$  (the standard error).

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We could also calculate the probability using the z score:

$$z = \frac{x - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

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The standard error of the mean is  $\frac{16}{\sqrt{50}} \approx 2.263$ .

$$P(\bar{x} > 102) \approx 0.1884$$

There is about an 18.84% chance the mean IQ of a sample of 50 people is greater than 102.

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$$P(\bar{x} > 102) \approx 0.0000386$$

There is about a 0.00386% chance that the mean IQ of a sample of 1000 people is greater than 102.

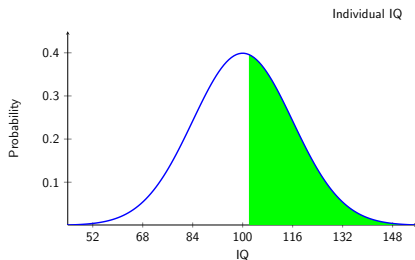
## Reflections on Example 5

By increasing the sample size, we decreased the probability. This is due to the original standard deviation now being divided by a larger number (which will decrease its overall value).

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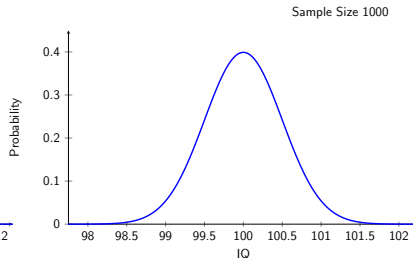
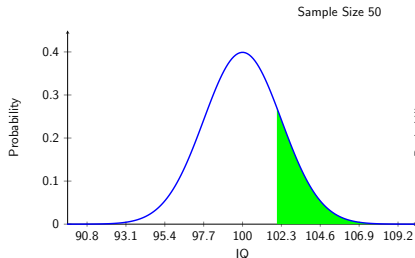
By increasing the sample size, we decreased the probability. This is due to the original standard deviation now being divided by a larger number (which will decrease its overall value).

Or, to put it another way, you are now examining a greater variety of people, so you should expect to have subjects in your samples that have IQs below 102.



The area of the shaded region is about 0.4503

# Reflections on Example 5



The area of the first shaded region is about 0.1884. The (non-noticeable) shaded area of the second region is about 0.0000386.

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The mean of the sample proportions ( $\hat{p}$ ) targets the population proportion,  $p = 0.62$ .

The standard error is about 1/10 of the population standard deviation, which again is due to dividing the population standard deviation by  $\sqrt{n}$ , or in this case  $\sqrt{100}$ .

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$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

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In a sample of 15, there is about a 64.21% chance the medicine is between 75 and 90% effective.