Confidence Intervals

Objectives

1. Determine confidence intervals for population mean
2. Determine confidence intervals for population proportion
3. Determine the necessary sample size
4. Determine a one-sided confidence interval

How Close Are We to the Population Mean?

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That is where confidence intervals come into play.

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Typical confidence levels are 90%, 95%, 98%, and 99%.

Confidence Interval Setup

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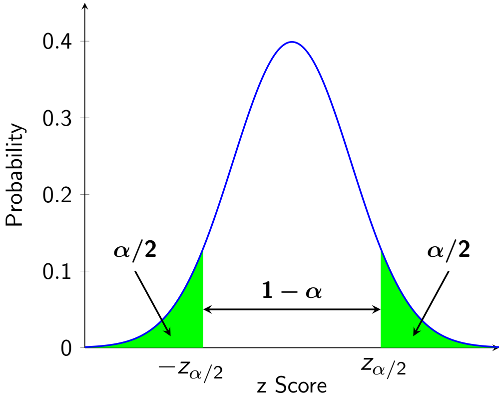
A **point estimate** is a value based on our sample data that represents a reasonable value of the population parameter.

The margin of error is in the form

$$\text{critical value} \times \text{standard error}$$

Critical Values

Critical values are typically in the form $z_{\alpha/2}$ where



Common Critical Values of $z_{\alpha/2}$

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Notice the higher the confidence level, the further away from $z = 0$ the critical value is.

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Note: If σ is unknown, you can use the sample standard deviation, s, when the sample size is large enough ($n \geq 30$).

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A 95% confidence interval for the population mean waiting time is 3.353 to 3.647 minutes.

Error Bars

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However, be advised that some graphs use standard deviation for their error bars.

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A 98% confidence interval for the mean population price per gallon is \$2.41 to \$2.49

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A 98% confidence interval for the mean population price per gallon is \$2.42 to \$2.48

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Increasing the sample size decreases the standard error.

By doing so, we decrease the width of our confidence interval, while still keeping the confidence level the same.

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- There is a ____ % chance that the population mean is (*whatever the sample mean is*).

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Mean and Standard Error

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Our confidence interval for the population proportion is

$$p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

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A 90% confidence interval for the population proportion of voters who favor the renovation is 48.7 to 55.5%

Example 4

The results of a sample of 40 students who took a pass/fail statistics exam are shown below. Construct a 98% confidence interval for the population proportion of students who passed the exam.

Pass	Pass	Fail	Pass	Pass	Pass	Fail	Pass
Fail	Pass	Pass	Pass	Fail	Pass	Pass	Pass
Pass	Fail	Fail	Pass	Pass	Fail	Pass	Pass
Pass	Pass	Fail	Fail	Pass	Pass	Pass	Fail

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A 98% confidence interval for the population proportion of students who passed the exam is 59 to 91%.

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1. Determine confidence intervals for population mean
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Margin of Error for Population Mean

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$$\sqrt{n} = \frac{\sigma \cdot z_{\alpha/2}}{E}$$

$$n = \left(\frac{\sigma \cdot z_{\alpha/2}}{E}\right)^2$$

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$$n = 138.2976 \to 139$$

We would need a sample of at least 139.

Sample Size for Population Proportion

If we solve

$$E = z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

for n, we get

$$n = p(1-p)\left(\frac{z_{\alpha/2}}{E}\right)^2$$

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$$n = 0.521(0.479)\left(\frac{1.96}{0.04}\right)^2$$

$$n = 599.191159 \to 600$$

We would need a sample size of at least 600.

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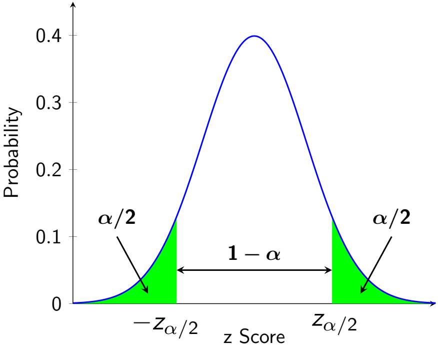
Two-Sided vs. One-Sided

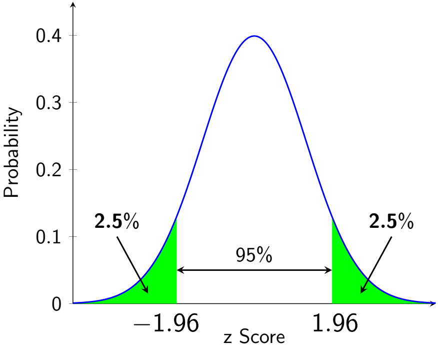
With a two-sided confidence interval (what we've discussed so far), the total area outside of our interval is *alpha*:



Notice the endpoints of our interval are $\pm z_{\alpha/2}$.

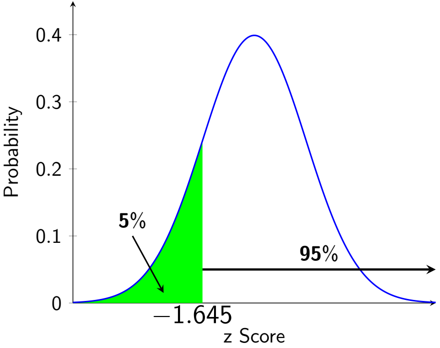
Two-Sided vs. One-Sided

For example, constructing a 95% confidence interval will mean that a total of 5% will be outside of our interval:



Two-Sided vs. One-Sided

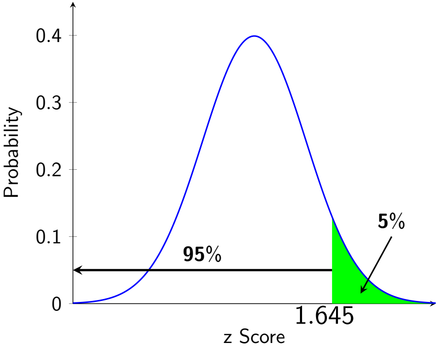
For a one-sided interval, in order to have the same area outside of our interval, our critical values will need to change:



The value $z = -1.645$ is a **lower bound** on the one-sided 95% confidence interval.

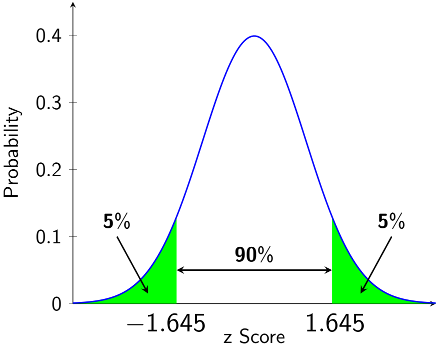
Two-Sided vs. One-Sided

The value $z = 1.645$ is an **upper bound** on the one-sided 95% confidence interval going the other way (to the left).



Two-Sided vs. One-Sided

We can also get those values of $z = -1.645$ and $z = 1.645$ by constructing a 90% confidence interval:



Two-Sided vs. One-Sided

For left- or right-tailed hypothesis testing, some statistical software will report the one-sided interval, others will only report the usual two-sided interval with an adjusted value of *alpha*:

Critical Values

Two-Tailed	One-Tailed
$z_{\alpha/2}$	z_{α}