Hypothesis Testing

Two Sample Means

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Thus, our null hypotheses will be $\mu_X = \mu_Y$.

Objectives

1. Perform hypothesis test on the mean for two dependent samples

2. Perform hypothesis test on the mean for two independent samples

Dependent Samples

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This part is sometimes known as a **paired t test** or a **matched pairs test**.

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s represents the sample standard deviation of the differences, and there are $n - 1$ degrees of freedom.

Example 1

A medication is given to patients in an attempt to lower their LDL cholesterol. The tables below list the levels. At the 5% significance level, test the claim that the medicine is effective in lowering LDL cholesterol.

Before	**After**
95	91
109	107
127	129
131	125
117	110
135	120
103	97
98	101
111	107

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- 95% Lower Bound: 1.0388 (will not contain $\mu_d = 0$)

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> *At the 5% significance level we reject the null hypothesis that there is no difference in LDL cholesterol levels and conclude that our sample suggests the medication may be effective in lowering LDL cholesterol levels.*

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Independent Samples

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In other words, there is no "before-and-after" relationship between our samples, and our samples don't even have to be the same sizes.

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Since our independent samples don't even have to have the same sample size, we will have to know about assumptions we can make about their *variances*.

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However, p-values obtained through pooled tests can be significantly off if the population variances are, in fact, *not* equal.

As such, the examples in this section will assume that the population variances are *not equal*.

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$$t = \frac{\overline{d}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Degrees of Freedom

The degrees of freedom is found by calculating

$$\text{df} = \frac{(V_1 + V_2)^2}{\frac{V_1^2}{n_1 - 1} + \frac{V_2^2}{n_2 - 1}}$$

where

$$V_1 = \frac{s_1^2}{n_1} \text{ and } V_2 = \frac{s_2^2}{n_2}$$

Note: Round the degrees of freedom down to the nearest integer.

For Pools and Giggles

If working with pooled variances,

$$t = \frac{(\overline{X} - \overline{Y}) - (\mu_X - \mu_Y)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$\text{df} = n_1 + n_2 - 2$$

Example 2

A local store claims that the waiting time for its customers to be served is the lowest in the area. A competitor's store checks the waiting times at both. The sample statistics are listed below. At the $\alpha = 0.05$ significance level, test the local store's hypothesis.

Local	Competitor
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$H_0 : \mu_x = \mu_y \longrightarrow \mu_x - \mu_y = 0$

$H_A : \mu_x < \mu_y \longrightarrow \mu_x - \mu_y < 0$

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Test statistic: $t = -1.730$ Critical value: -1.711

p-value $= 0.0482$

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