Other Probability Distributions

Objectives

1. Solve problems involving geometric probability distributions

2. Solve problems involving Poisson probability distributions

Binomial vs. Geometric Distributions

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- There are a fixed number of n repeated independent trials
- Each trial's outcome is either a success or failure
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One of these outcomes is TTHTHHHTTH.

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To put it another way, a geometric distribution has the following property:

$$\underbrace{FFF\cdots F}_{x-1 \text{ failures}}S$$

Geometric Distributions

The probability of obtaining our first success after x binomial experiments is given by

$$P(X = x) = (1 - p)^{x-1} \cdot p$$

Example 1

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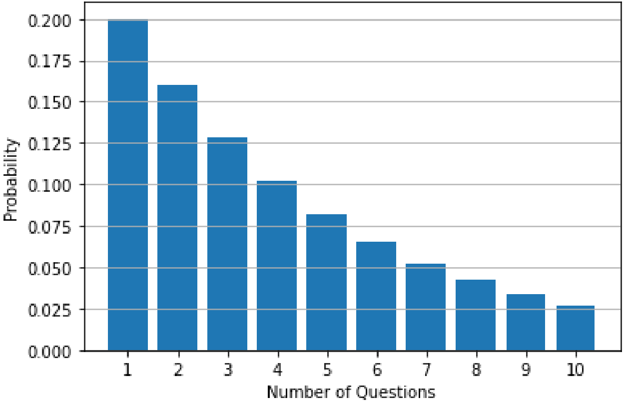
The student has about an 10.24% chance of first guessing correctly on the 4th question.

Bar Graph of Example 1



Mean and Standard Deviation of Geometric Distributions

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From which the standard deviation, σ is

$$\sigma = \sqrt{\frac{1-p}{p^2}} = \frac{\sqrt{1-p}}{p}$$

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We would expect the student to answer 5 questions before guessing one correctly.

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The standard deviation is about 4.47 questions.

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- The experiment consists of counting the number of times, x, an event occurs in a given interval.
- The probability of the event occurring is the same for each interval.
- The number of occurrences in one interval is independent of the number of occurrences in other intervals.

Poisson Formula

$$P(X = x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$$

where μ is the mean number of occurrences of the event over the

intervals and $e \approx 2.71828$

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There is about a 15.63% chance that 5 dogs will be adopted in a given month.

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There is about a 23.81% chance that 2 or less dogs will be adopted in a given month.

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&\approx 0.7619
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There is about a 76.19% chance that at least 3 dogs will be adopted in a given month.

Mean and Standard Deviation of Poisson Distributions

The **mean** of a Poisson distribution is given to us as μ, which also happens to be the variance as well.

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The **mean** of a Poisson distribution is given to us as μ, which also happens to be the variance as well.

Since the standard deviation is the square root of variance, it follows that the standard deviation of a Poisson distribution is also the square root of the mean:

$$\sigma = \sqrt{\mu}$$