Sampling Distributions

Objectives

1. Obtain a sampling distribution of sample means
2. Determine the mean and standard error of a sampling distribution
3. Understand the Central Limit Theorem
4. Determine the mean and standard error for sampling distribution of proportions

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For instance, let's say we have the following population prices of laptops: \$1000, \$1200, \$1600, and \$2000.

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Note: We will sample with replacement. Differences in sampling with and without replacement become negligible as sample sizes increase.

Example 1

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Sample	Sample Mean	Sample	Sample Mean
1000, 1000	1000	1600, 1000	1300
1000, 1200	1100	1600, 1200	1400
1000, 1600	1300	1600, 1600	1600
1000, 2000	1500	1600, 2000	1800
1200, 1000	1100	2000, 1000	1500
1200, 1200	1200	2000, 1200	1600
1200, 1600	1400	2000, 1600	1800
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Example 2

Create a histogram of the sample means from Example 1.

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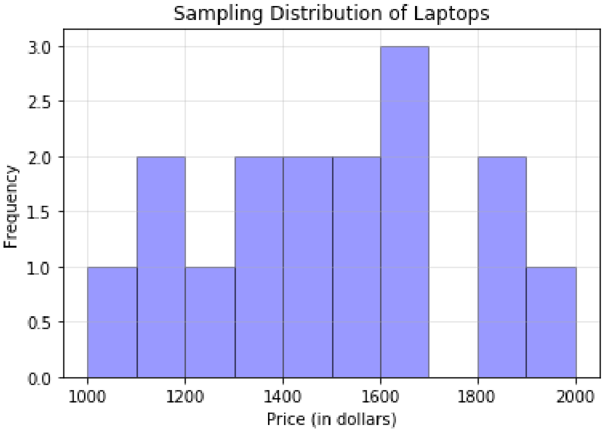

Example 3

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Std. Dev $\approx \$271.57$

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$$\sigma \approx \frac{\sigma_{\overline{x}}}{\sqrt{n}}$$

where $\frac{\sigma}{\sqrt{n}}$ is called the **standard error of the mean**.

Example 4

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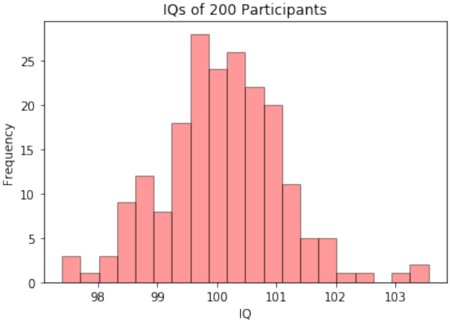

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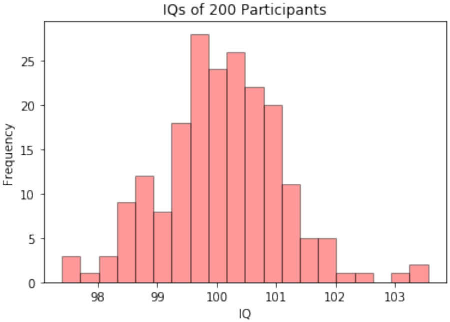

What are the approximate mean and standard error of the sample?

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$$\approx 1.13$$

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Sample Means of Any Distribution

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It turns out that the distributions of sample means for <u>any</u> population will be normal as our sample sizes increase.

Central Limit Theorem

As the sample size increases, the distribution of sample means becomes normal with a mean of μ and standard deviation $\frac{\sigma}{\sqrt{n}}$ (the standard error).

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(a) What is the probability that an individual has an IQ greater than 102?

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There is about a 45.03% chance that an individual has an IQ greater than 102.

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The standard error of the mean is $\frac{16}{\sqrt{50}} \approx 2.263$.

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There is about an 18.84% chance the mean IQ of a sample of 50 people is greater than 102.

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We could also calculate the probability using the z score approach with:

$$z = \frac{x - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

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There is about a 0.00386% chance that the mean IQ of a sample of 1000 people is greater than 102.

Reflections on Example 5

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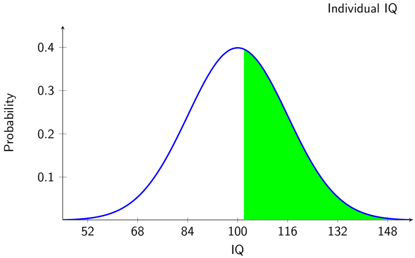
Or, to put it another way, you are now examining a greater variety of people, so you should expect to have subjects in your samples that have IQs below 102.



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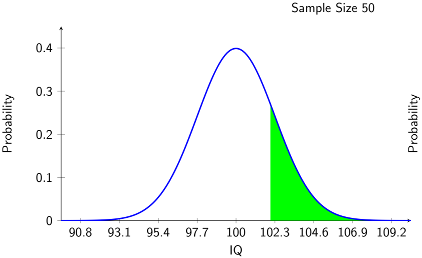

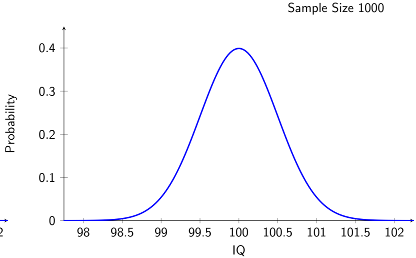

The area of the first shaded region is about 0.1884. The (non-noticeable) shaded area of the second region is about 0.0000386.

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The mean of the sample proportions ($\hat{p}$) targets the population proportion, $p = 0.62$.

The standard error is about $1/10$ of the population standard deviation, which again is due to dividing the population standard deviation by $\sqrt{n}$, or in this case $\sqrt{100}$.

Mean and Standard Error of Sampling Distribution of Proportions

Recall that the mean and standard deviation of a binomial probability distribution are $\mu = np$ and $\sigma = \sqrt{np(1-p)}$, where p is the probability of success and n is the sample size.

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$$\sigma_{\hat{p}} = \frac{\sqrt{np(1-p)}}{n} = \sqrt{\frac{p(1-p)}{n}}$$

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In a sample of 400, there is about a 99.38% chance the medicine is between 75 and 90% effective.