# Confidence Intervals

# Objectives

1. Determine confidence intervals for population mean
2. Determine the confidence interval for Student's t-Distribution
3. Determine confidence intervals for population proportion
4. Determine the necessary sample size

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That is where confidence intervals come into play.

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Typical confidence levels are 90%, 95%, 98%, and 99%.

# Confidence Interval Setup

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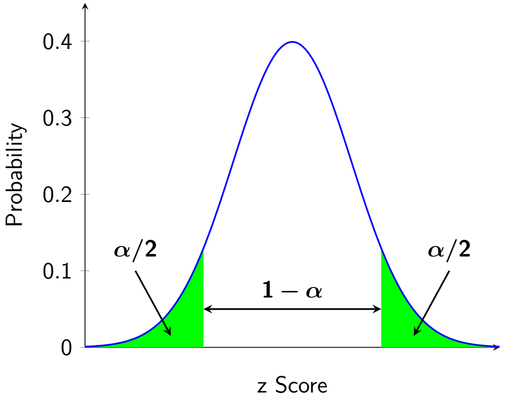
The margin of error is in the form

$$\text{critical value} \times \text{standard error}$$

# Critical Values

Critical values are typically in the form $z_{\alpha/2}$ where



# Common Critical Values of $z_{\alpha/2}$

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*Note:* If $\sigma$ is unknown, you can use the sample standard deviation, $s$, when the sample size is large enough ($n \geq 30$).

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A 95% confidence interval for the population mean waiting time is 3.353 to 3.647 minutes.

# Error Bars

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However, be advised that some graphs use standard deviation for their error bars.

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A 98% confidence interval for the mean population price per gallon is \$2.41 to \$2.49

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A 98% confidence interval for the mean population price per gallon is \$2.44 to \$2.46

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- There is a ____ % chance that the population mean is (*whatever the sample mean is*).

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