Hypothesis Testing

or: How I Learned to Stop Worrying and Love Inferential Statistics

Objectives

1. State the null and alternative hypothesis
2. Understand errors and interpret p-value
3. Perform a hypothesis test of the population mean with known population standard deviation

What is Hypothesis Testing?

Hypothesis testing is the process of testing a claim about a (population) parameter by obtaining a sample and analyzing the results against that parameter.

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The **null hypothesis** is the original claim about the parameter, denoted H_0, and usually is stated in terms of equality.

Alternative Hypothesis

The **alternative hypothesis**, denoted H_A, is the new claim that is made against the null hypothesis.

Types of Hypothesis Testing

- Left-tailed

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- Left-tailed
  - H_A: parameter $< H_0$

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Example 1

Determine the null and alternative hypotheses for each.

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- Fail to reject the null hypothesis

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> *... we have sufficient evidence to reject [the null hypothesis].*

Rejecting the null hypothesis is like a jury declaring a defendant guilty.

Note: There is still a chance that the defendant is innocent, but the evidence is strong enough to bring a guilty verdict.

Failing to Reject the Null Hypothesis

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> *... we do not have sufficient evidence to reject [the null hypothesis].*

Failing to reject the null hypothesis is like a jury declaring a defendant not guilty.

Note: A declaration of not guilty is not the same as a declaration of innocence (which is never handed down in a court of law). It just means that there is not sufficient evidence to declare guilt, *but the defendant could still actually be guilty*.

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In hypothesis testing, α is called the **level of significance**.

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If we make it harder to put an innocent person in jail, we make it tougher to return a guilty verdict. This will also have the affect of increasing the number of actual guilty defendants who are let go.

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If we make it harder to put an innocent person in jail, we make it tougher to return a guilty verdict. This will also have the affect of increasing the number of actual guilty defendants who are let go.

The **power** of a test is given as $1 - P(\beta)$

Errors Summary

$\boldsymbol{H_0}$	Reject H_0	Fail to reject H_0
H_0 True	Type I error	Correct decision
H_0 False	Correct decision	Type II error

Defendant	Declare Guilty	Declare Not Guilty
Actually Innocent	Type I error	Correct decision
Actually Guilty	Correct decision	Type II error

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The **p-value** is the probability of obtaining a sample as extreme as the one obtained **under the assumption that the null hypothesis is true.**

If our *p*-value is less than a given acceptable value (α), then our sample was not likely to occur by chance *assuming the null hypothesis is true*, so we have sufficient evidence to reject the null hypothesis.

Example 2

(a) A new car's mpg is listed as 33. You want to know if the mpg is not 33, so you perform a hypothesis test at the 5% level of significance. Your sample shows a mean mpg of 37, which has a probability of 2.2% of happening.

Based on your findings, should you reject or fail to reject the null hypothesis that $\mu = 33$?

Example 2

(a) A new car's mpg is listed as 33. You want to know if the mpg is not 33, so you perform a hypothesis test at the 5% level of significance. Your sample shows a mean mpg of 37, which has a probability of 2.2% of happening.

Based on your findings, should you reject or fail to reject the null hypothesis that $\mu = 33$?

If the null hypothesis is true, then the probability we would obtain a sample mean as extreme (or more) than 37 mpg is 2.2%.

Example 2

(a) A new car's mpg is listed as 33. You want to know if the mpg is not 33, so you perform a hypothesis test at the 5% level of significance. Your sample shows a mean mpg of 37, which has a probability of 2.2% of happening.

Based on your findings, should you reject or fail to reject the null hypothesis that $\mu = 33$?

If the null hypothesis is true, then the probability we would obtain a sample mean as extreme (or more) than 37 mpg is 2.2%.

Since our *p*-value is 2.2%, which is less than our significance level of 5%, we will reject the null hypothesis.

Example 2a

Final answer:

At the 5% significance level, we have sufficient evidence to reject the claim that the mean mpg is 33, and conclude that our evidence shows that the mean mpg differs from 33.

Example 2

(b) A hospital says the mean wait time for patients to see a doctor is 3.5 minutes. You want to know if the mean wait time is more than 3.5 minutes, so you perform a hypothesis test at the 10% level of significance. Your sample shoes a mean wait time of 3.8 minutes, which has a probability of 10.5%.

Based on your findings, should you reject or fail to reject the null hypothesis that $\mu = 3.5$?

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(b) A hospital says the mean wait time for patients to see a doctor is 3.5 minutes. You want to know if the mean wait time is more than 3.5 minutes, so you perform a hypothesis test at the 10% level of significance. Your sample shoes a mean wait time of 3.8 minutes, which has a probability of 10.5%.

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(b) A hospital says the mean wait time for patients to see a doctor is 3.5 minutes. You want to know if the mean wait time is more than 3.5 minutes, so you perform a hypothesis test at the 10% level of significance. Your sample shoes a mean wait time of 3.8 minutes, which has a probability of 10.5%.

Based on your findings, should you reject or fail to reject the null hypothesis that $\mu = 3.5$?

If the null hypothesis is true, then the probability we would obtain a sample mean as extreme (or more) than 3.5 minutes is 10.5%.

Since our *p*-value is 10.5%, which is greater than our significance level of 10%, we will fail to reject the null hypothesis.

Example 2b

Final answer:

> *At the 10% significance level, we do not have sufficient evidence to reject the claim that the mean wait time is 3.5 minutes, and thus there is not enough evidence to conclude that the mean wait time is more than 3.5 minutes.*

Example 2

(c) A company claims their program will increase your grade in statistics class by 10%. You think it might not be that much, so you perform a hypothesis test at the 1% significance level. You obtain a sample with a grade increase of 7%, which has a probability of 0.8% of occurring.

Based on your findings, should you reject or fail to reject the null hypothesis?

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Based on your findings, should you reject or fail to reject the null hypothesis?

If the null hypothesis is true, then the probability we would obtain a sample mean increase as extreme (or more) than 7% is 0.8%.

Since our p-value of 0.8% is less than our significance level of 1%, we reject the null hypothesis.

Example 2c

Final answer:

At the 1% significance level, we have sufficient evidence to reject the claim that the mean grade increase is 10%, and conclude that our evidence shows that the mean grade increase is less than 10%.

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5. State your conclusion

Hypothesis Testing Methods

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Hypothesis Test for Population Mean

We will look at some examples of hypothesis testing of the population mean when the population standard deviation is known.

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We will also assume that a simple random sample (or $n \geq 30$) is obtained and the population is normally distributed.

Example 3

A pharmaceutical company claims that the mean time for relief from its headache medicine is 3 minutes. A sample of 50 pills is obtained and the sample's mean is 3.1 minutes. Assuming $\sigma = 0.35$ minutes, test the claim that the mean remedy time is different than 3 minutes at the $\alpha = 0.05$ significance level.

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$H_0 : \mu = 3$ minutes

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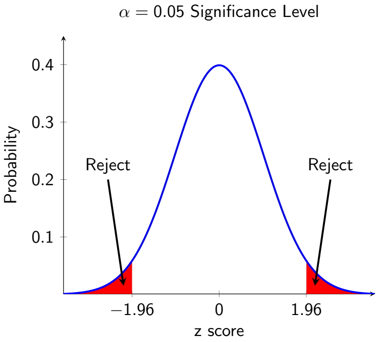
We will first examine the test statistic and critical value approach.

Example 3 – Test Statistic and Critical Value

In this approach, we will compute the z test statistic and see if it falls in the acceptance or rejection region:



Example 3 – Test Statistic and Critical Value

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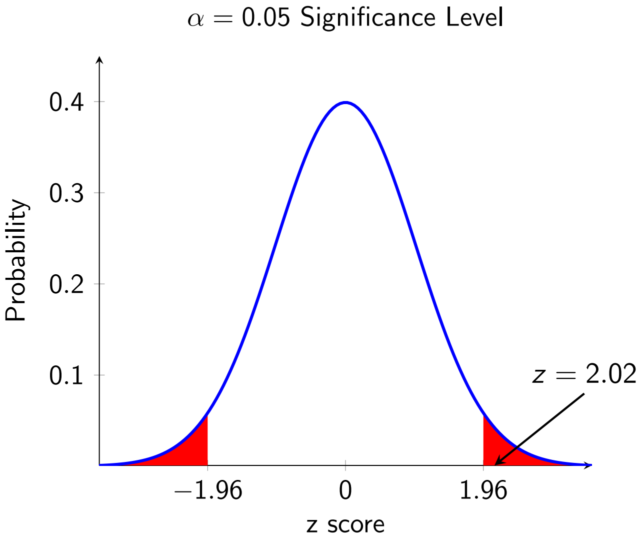

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At the 5% level of significance, we have sufficient evidence to reject the null hypothesis that the mean remedy time is 3 minutes and conclude that there is evidence to prove the mean remedy time is different.

Example 3 – p-value

We can obtain the p-value for our sample by computing the total area associated with our z test statistic.

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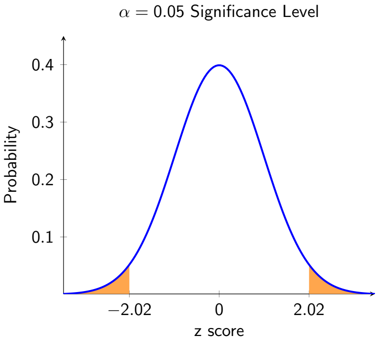

The p-value is approximately 0.0434

Example 3 – p-value

Our p-value of 0.0434, or 4.34%, means that assuming the mean remedy time of 3 minutes is true, we would get results as extreme (or more) as ours about 4.34% of the time.

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Since this value is less than the $\alpha = 0.05$ significance level, our results are statistically significant, and we thus reject the null hypothesis.

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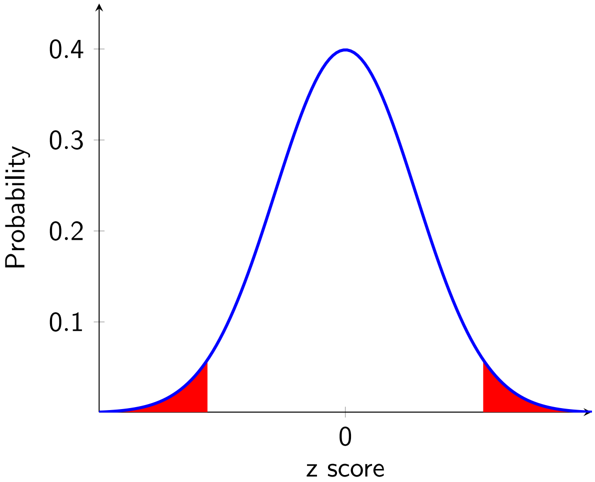
> *At the 5% level of significance, we have sufficient evidence to reject the null hypothesis that the mean remedy time is 3 minutes and conclude that there is evidence to prove the mean remedy time is different.*

p-value Visual Interpretation



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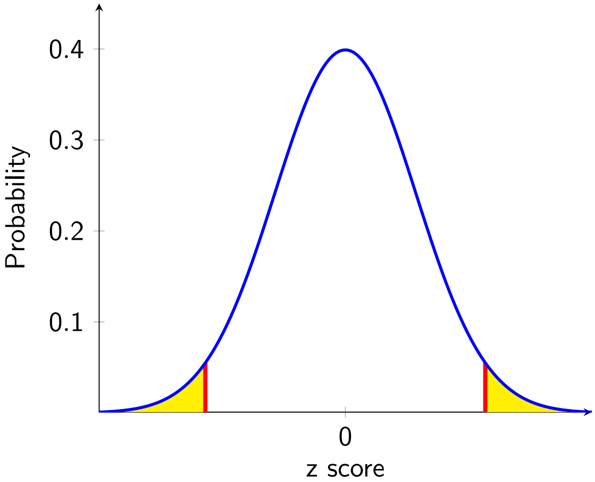

Example 3 – Confidence Interval

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As such, if our confidence interval contains the (alleged) population mean of 3 minutes, then we will not reject the null hypothesis.

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$$3.1 \pm 1.96\left(0.35/\sqrt{50}\right)$$

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Since our confidence interval does not contain the population mean of 3, we reject the null hypothesis.

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Our sample's 95% confidence interval is

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Since our confidence interval does not contain the population mean of 3, we reject the null hypothesis.

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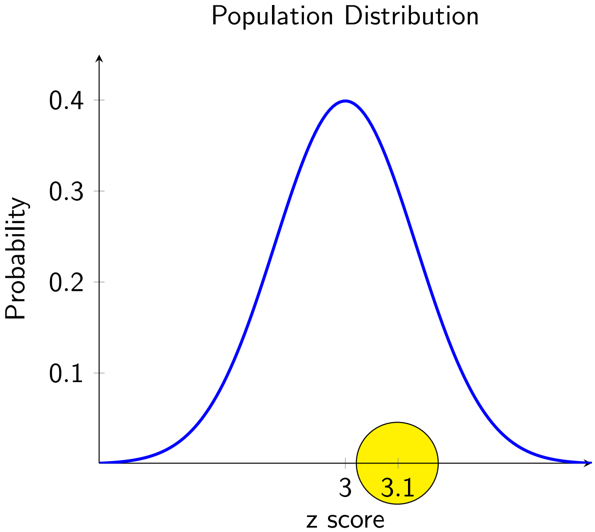
Example 3 – Confidence Interval Visual Interpretation



Example 4

A company claims their program will increase your grade in statistics class by at least 10%. You think it might not be that much, so you obtain a sample of 40 students and find that the sample mean increase in grades for the class is 9%.

Assuming $\sigma = 3.75\%$ and the population data is normally distributed, test the claim that the mean percent increase in score is less than 10% at the $\alpha = 0.02$ significance level.

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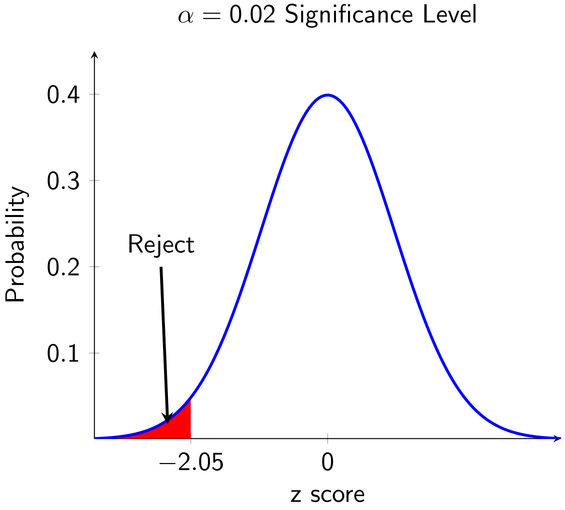
$H_0 : \mu = 0.10$ increase

$H_A : \mu < 0.10$ increase

Example 4



$\alpha = 0.02$ Significance Level
Probability
0.4
0.3
0.2
0.1
Reject
-2.05
0
z score

Example 4

Test statistic: $z = -1.687 > -2.054$ (outside rejection region)

p-value: $0.046 > 0.02$

Confidence Interval: $(0.099, 0.101)$, which contains $\mu = 0.10$

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Test statistic: $z = -1.687 > -2.054$ (outside rejection region)

p-value: $0.046 > 0.02$

Confidence Interval: $(0.099, 0.101)$, which contains $\mu = 0.10$

All 3 lead to same conclusion: Fail to reject the null hypothesis.

> *At the 2% significance level, we do not have sufficient evidence to reject the claim that the mean increase in score is 10%, and thus there is not enough evidence to conclude that the mean increase in score is less than 10%.*