Discrete Probability Distributions

Objectives

1. Create a probability distribution
2. Determine the expected value of a probability distribution
3. Determine the variance and standard deviation of a probability distribution

Probability Distributions

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Familiar Characteristics:

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Familiar Characteristics:

- $0 \leq$ each probability ≤ 1

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- The sum of all probabilities in a distribution equals 1

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A **probability distribution** is a listing of each outcome of a probability experiment with their probabilities.

A **discrete probability distribution** is one in which the outcomes of each experiment are discrete (countable) values.

Familiar Characteristics:

- $0 \leq$ each probability ≤ 1
- The sum of all probabilities in a distribution equals 1
- $P(A \text{ or } B) = P(A) + P(B)$

Probability Distribution of Rolling 2 Dice

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We can create a probability distribution of the sums of rolling two dice.

Probability Distribution of Rolling 2 Dice

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5	6	7	8	9	10	11
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We can create a probability distribution of the sums of rolling two dice.

We use the notation $P(X = x)$ where X is our **random variable** and x represents the outcomes, such as 2, 3, 4, ..., 12.

Probability Distribution of Rolling 2 Dice

x	$P(X = x)$
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Probability Distribution of Rolling 2 Dice

x	$\boldsymbol{P(X = x)}$
2	$^1/_{36}$
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Probability Distribution of Rolling 2 Dice

x	$\boldsymbol{P(X = x)}$
2	$^1/_{36}$
3	$^1/_{18}$
4	
5	
6	
7	
8	
9	
10	
11	
12	

Probability Distribution of Rolling 2 Dice

x	$P(X = x)$
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6	$5/36$
7	
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9	
10	
11	
12	

Probability Distribution of Rolling 2 Dice

x	$P(X = x)$
2	$^1/_{36}$
3	$^1/_{18}$
4	$^1/_{12}$
5	$^1/_9$
6	$^5/_{36}$
7	$^1/_6$
8	
9	
10	
11	
12	

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2	$1/36$
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6	$5/36$
7	$1/6$
8	$5/36$
9	
10	
11	
12	

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8	$^5/_{36}$
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11	
12	

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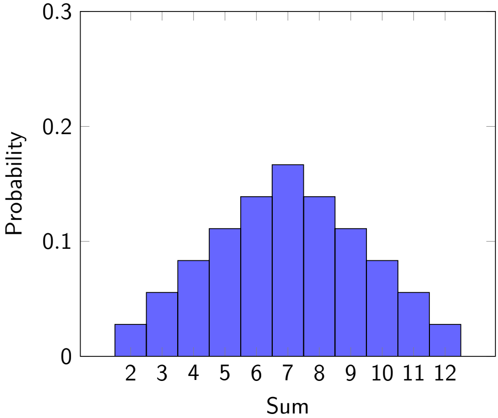
4	$^1/_{12}$
5	$^1/_9$
6	$^5/_{36}$
7	$^1/_6$
8	$^5/_{36}$
9	$^1/_9$
10	$^1/_{12}$
11	$^1/_{18}$
12	

Probability Distribution of Rolling 2 Dice

x	$P(X = x)$
2	$^1/_{36}$
3	$^1/_{18}$
4	$^1/_{12}$
5	$^1/_{9}$
6	$^5/_{36}$
7	$^1/_{6}$
8	$^5/_{36}$
9	$^1/_{9}$
10	$^1/_{12}$
11	$^1/_{18}$
12	$^1/_{36}$

Probability Histogram of Rolling 2 Dice



Example 1

(a) Create a probability distribution for flipping a coin three times, where X represents the number of times heads is flipped.

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$\boldsymbol{x}$	$\boldsymbol{P(X=x)}$
0	
1	
2	
3	

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The sample space is HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT

$\boldsymbol{x}$	$\boldsymbol{P(X = x)}$
0	$1/8$
1	
2	
3	

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The sample space is HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT

$\boldsymbol{x}$	$\boldsymbol{P(X = x)}$
0	$1/8$
1	$3/8$
2	
3	

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1	3/8
2	3/8
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$\boldsymbol{x}$	$\boldsymbol{P(X = x)}$
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2	$3/8$
3	$1/8$

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(b) Create a probability distribution histogram for the number of times heads appears when flipping a coin 3 times.

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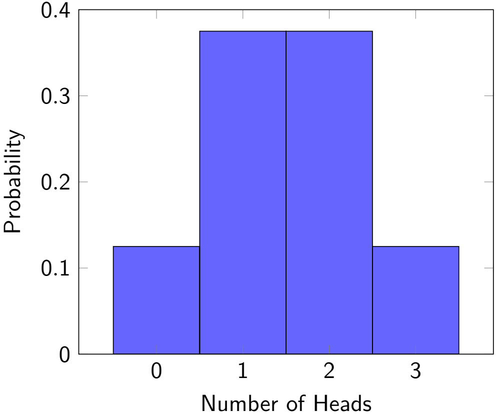

Example 2

The distribution below represents the percentage of households that have x dogs according to a recent study.

$\boldsymbol{x}$	$\boldsymbol{P(X = x)}$
0	44%
1	27%
2	18%
3 or more	11%

How many households have at least 1 dog?

Example 2

The distribution below represents the percentage of households that have x dogs according to a recent study.

x	$P(X = x)$
0	44%
1	27%
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How many households have at least 1 dog?

Using the Complement Rule: $100\% - 44\% = 56\%$

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The **expected value** of a probability distribution is the outcome we would expect to happen if the experiment was performed a very large number of times.

In other words, it is a **weighted mean** of the distribution of outcomes:

$$E(X) = \sum (x \cdot P(x))$$

Example 3

Determine the expected value of rolling two dice.

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x	$\boldsymbol{P(X=x)}$	$x \cdot P(x)$
2	$1/36$	$1/18$
3	$1/18$	$1/6$
4	$1/12$	$1/3$
5	$1/9$	$5/9$
6	$5/36$	$5/6$

x	$\boldsymbol{P(X=x)}$	$x \cdot P(x)$
7	$1/6$	$7/6$
8	$5/36$	$10/9$
9	$1/9$	1
10	$1/12$	$5/6$
11	$1/18$	$11/18$
12	$1/36$	$1/3$

Example 3

The expected value is the sum of all of the entries in the last column:

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$$\frac{1}{18} + \frac{1}{6} + \frac{1}{3} + \cdots + \frac{11}{18} + \frac{1}{3} = 7$$

Calculating the weighted mean of our distribution, the expected value of rolling two dice is 7.

Example 4

The distribution below represents the percentage of households that have x dogs according to a recent study.

$\boldsymbol{x}$	$\boldsymbol{P(X = x)}$
0	44%
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2	18%
3	11%

What is the expected number of dogs per household?

Example 4

The distribution below represents the percentage of households that have x dogs according to a recent study.

$\boldsymbol{x}$	$\boldsymbol{P(X = x)}$
0	44%
1	27%
2	18%
3	11%

What is the expected number of dogs per household?

$0(0.44) + 1(0.27) + 2(0.18) + 3(0.11)$

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$\boldsymbol{x}$	$\boldsymbol{P(X = x)}$
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$$0(0.44) + 1(0.27) + 2(0.18) + 3(0.11) = 0.96$$

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$0(0.44) + 1(0.27) + 2(0.18) + 3(0.11) = 0.96$

There is about 1 dog per household.

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Variance and Standard Deviation

Previously, we saw that variance, which was the average squared deviation the data values are from the mean, was

$$\frac{\sum (x - \mu)^2}{n}$$

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Similarly, the variance for probability distributions is given as

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from which the **standard deviation** is

$$\sigma = \sqrt{\sum ((x-\mu)^2 \cdot P(x))}$$

Variance and Standard Deviation Alternate Definition

$$\sigma^2 = \sum \left((x - E(x))^2 \cdot P(x) \right)$$

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2: $(2-7)^2 \cdot \frac{1}{36} = \frac{25}{36}$

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$\vdots$

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From a previous example, we have $\mu = E(x) = 7$.

2: $(2-7)^2 \cdot \frac{1}{36} = \frac{25}{36}$

3: $(3-7)^2 \cdot \frac{1}{18} = \frac{8}{9}$

$\vdots$

12: $(12-7)^2 \cdot \frac{1}{36} = \frac{25}{36}$

Example 5

Adding up all of the results and then taking the square root, we get

$$\sigma \approx 2.415$$