Normal Distribution in the Real World

Objectives

1. Find the area under a normal curve for an observed value
2. Find observed values given area under the normal curve

Normal Distribution Revisited

The equation for calculating area when working with z scores is

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

with mean $\mu = 0$ and standard deviation $\sigma = 1$.

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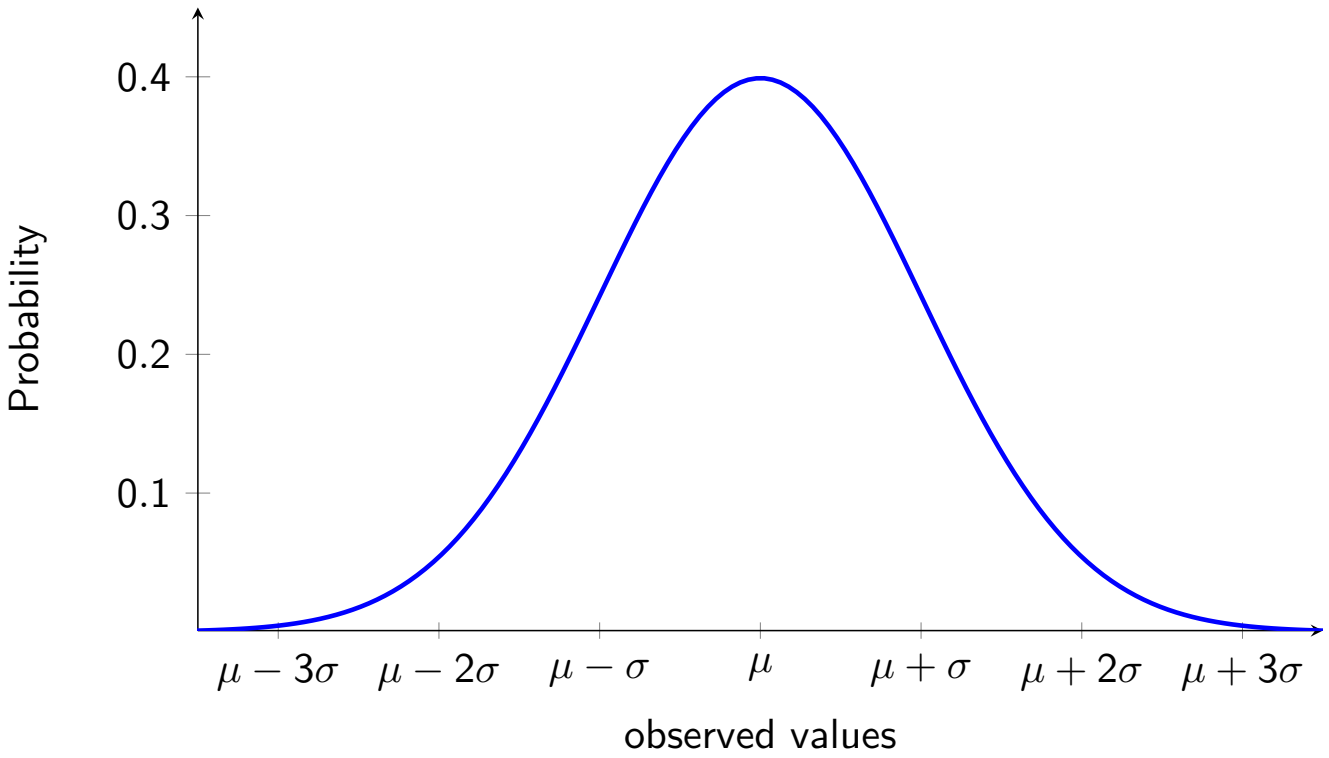
Using observed values, and the z score formula $z = \frac{x-\mu}{\sigma}$, we get the following equation in terms of an observed value, x:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2/2\sigma^2}$$

Observed Value Graph



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Go with what you feel most comfortable with.

Example 1

(a) An air conditioning manufacturer says their machines have a mean cooling capacity of 10,000 BTUs. Due to variations that arise in manufacturing, there is a standard deviation of 400 BTUs.

What is the probability of selecting an air conditioning unit that is 11,000 BTUs or less?

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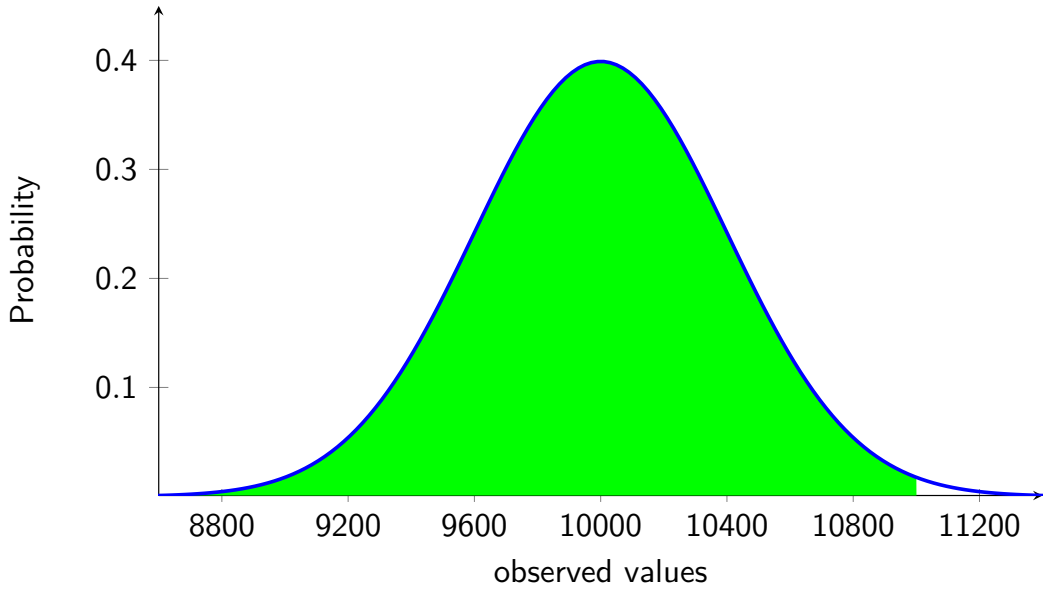

Example 1a

We have $x = 11,000$, $\mu = 10,000$, and $\sigma = 400$

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There is about a 99.38% chance of selecting an air conditioning unit that is 11,000 BTUs or less.

Example 1

(b) What is the probability that you select an air conditioning unit between 9,500 and 10,600 BTUs?

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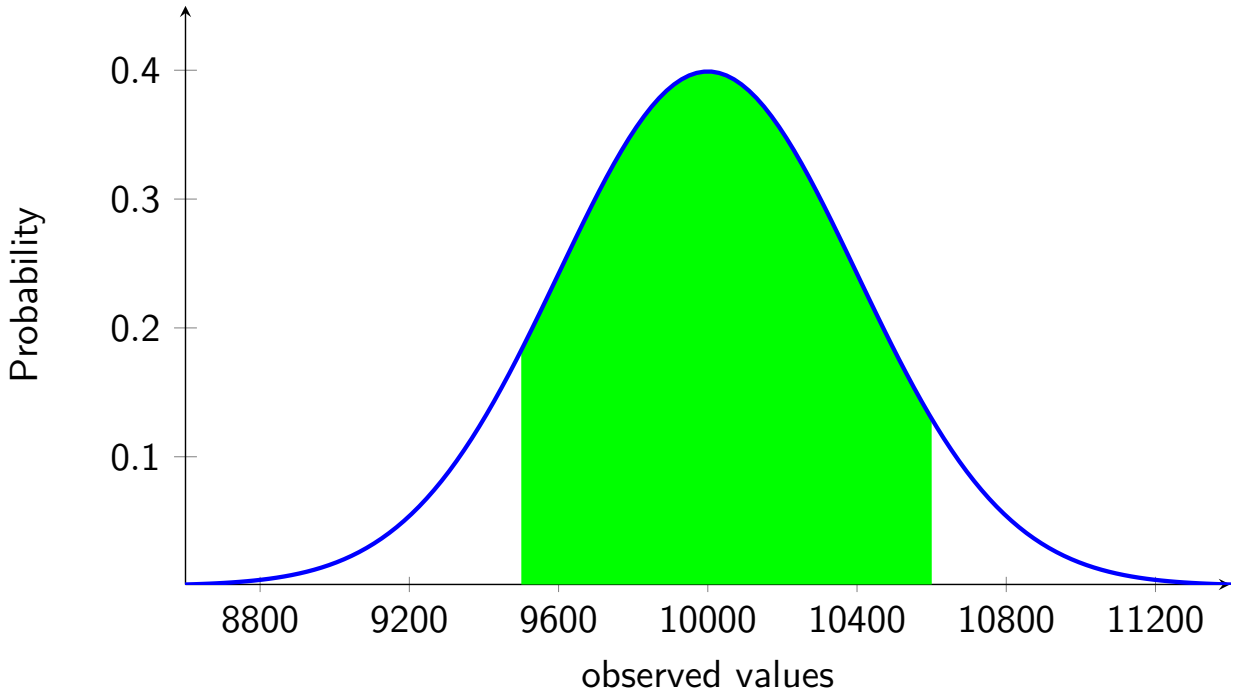

Example 1b

We have $x_1 = 9,500$, $x_2 = 10,600$, $\mu = 10,000$, and $\sigma = 400$

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Example 1b

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We are finding the area *between* $x = 9,500$ and $x = 10,600$:

$$x \approx 0.8275$$

Example 1b

We have $x_1 = 9,500$, $x_2 = 10,600$, $\mu = 10,000$, and $\sigma = 400$

We are finding the area *between* $x = 9,500$ and $x = 10,600$:

$$x \approx 0.8275$$

There is about an 82.75% chance of selecting an air conditioning unit between 9,500 and 10,600 BTUs.

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Finding Values as the Inverse

Once again, this is just the inverse of what we did in the last example.

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Fortunately, most statistical technology has the ability to solve problems like these without the need to convert back from z scores first.

Example 2

(a) An air conditioning manufacturer says their machines have a mean cooling capacity of 10,000 BTUs. Due to variations that arise in manufacturing, there is a standard deviation of 400 BTUs.

How many BTUs must an air conditioning unit have to be in the 90th percentile?

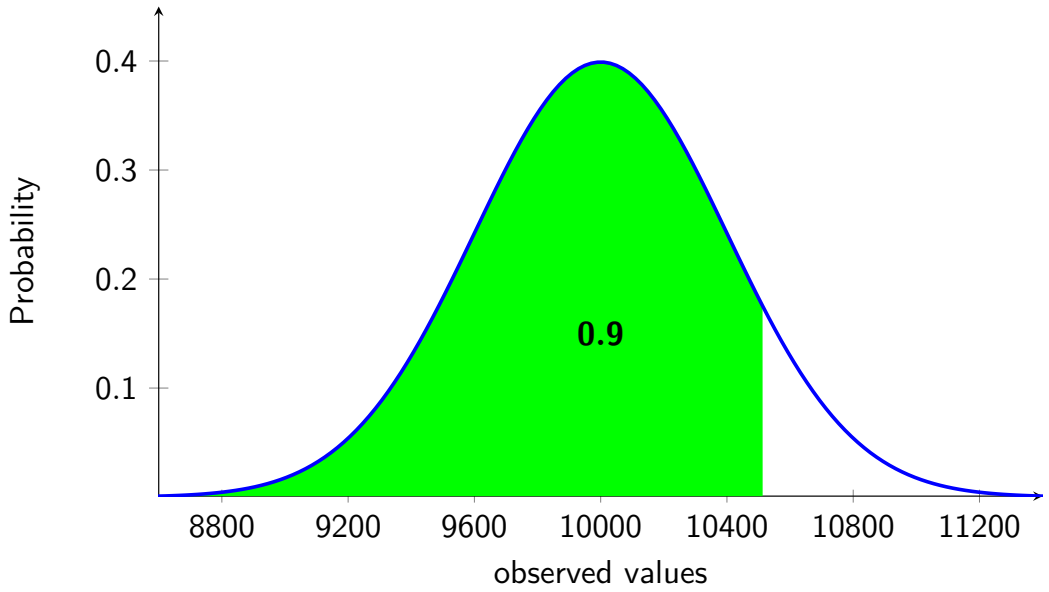
Example 2

(a) An air conditioning manufacturer says their machines have a mean cooling capacity of 10,000 BTUs. Due to variations that arise in manufacturing, there is a standard deviation of 400 BTUs.

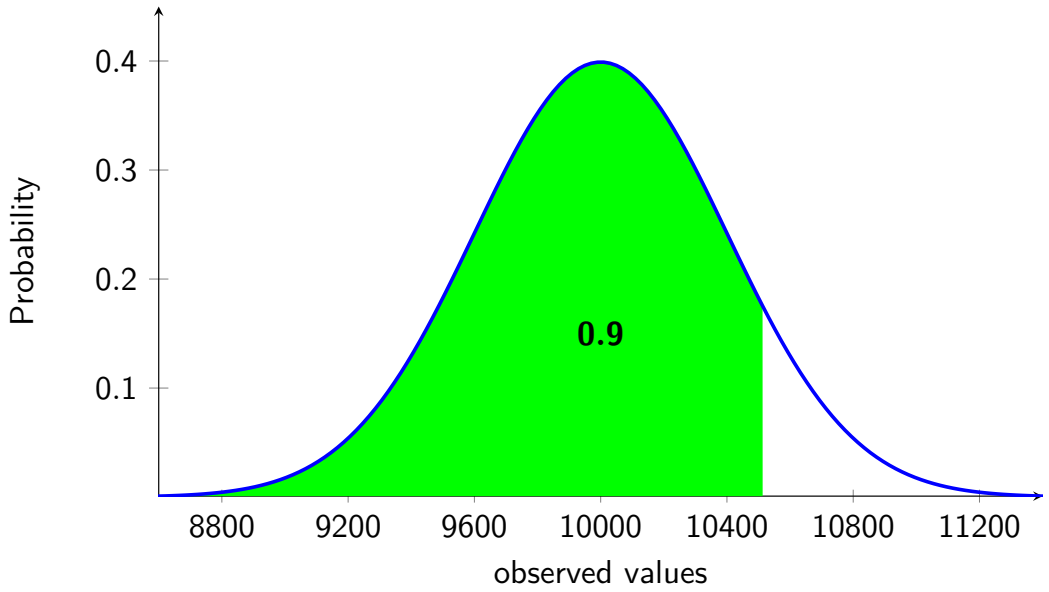
How many BTUs must an air conditioning unit have to be in the 90th percentile?

The 90th percentile means that 90% of the other units have less BTUs than the value we are finding.

Example 2a

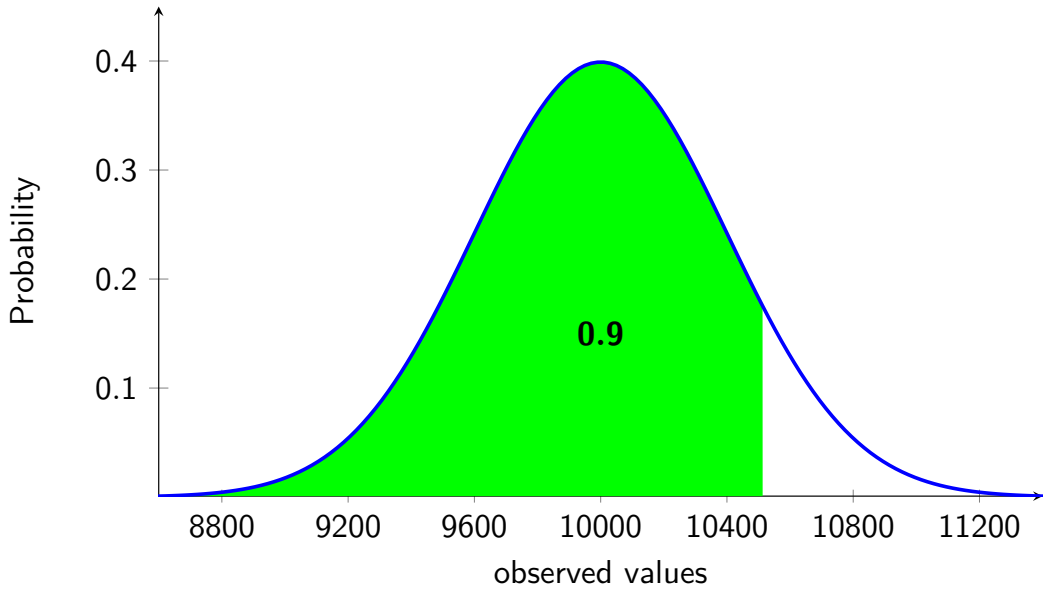

Probability
0.4
0.3
0.2
0.1
0.9
8800
9200
9600
10000
10400
10800
11200
observed values

Example 2a



$$x \approx 10,512.62$$

Example 2a



$$x \approx 10,512.62$$

An air conditioning unit with about 10,512 BTUs is in the 90th percentile.

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(b) An air conditioning unit will not be shipped if it is in either the bottom 0.25% or top 0.25% of BTUs. What are those BTU cutoff values?

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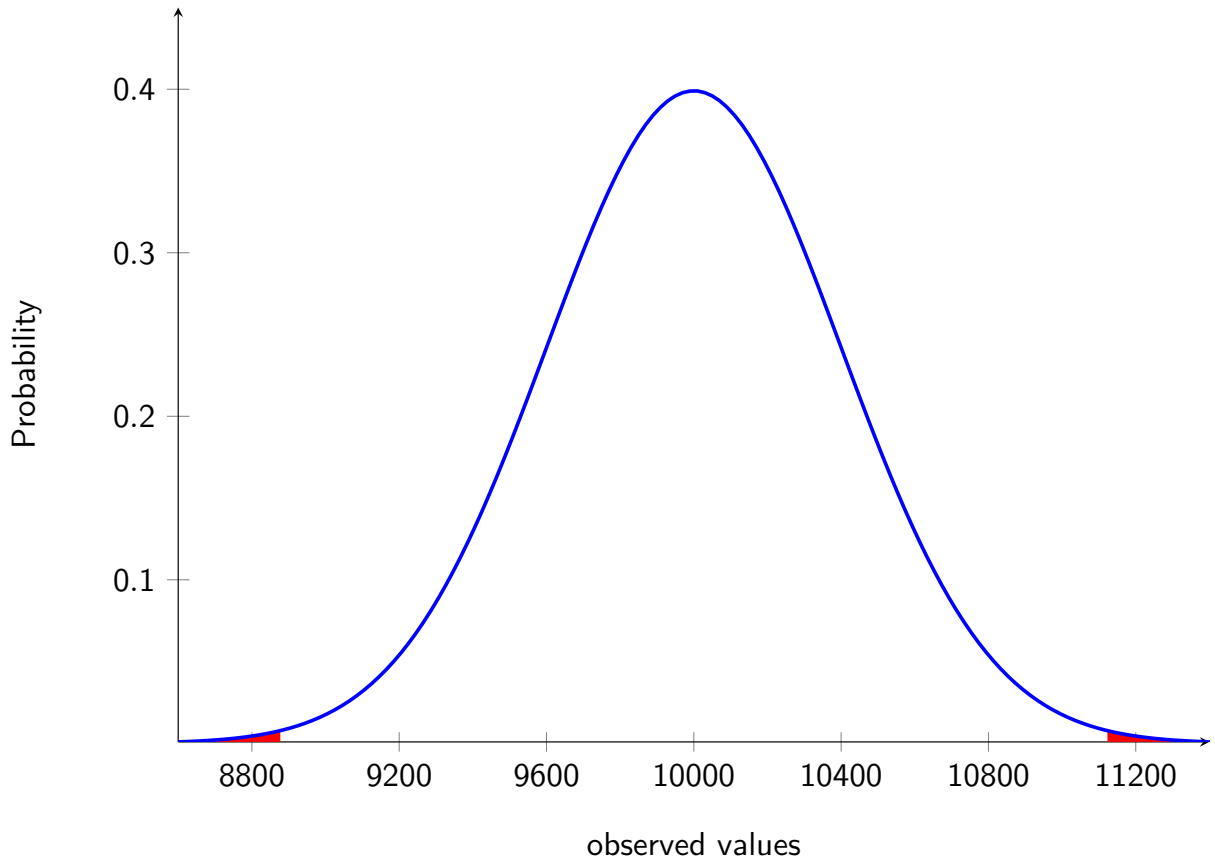

Example 2b

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An air conditioning unit will not be shipped if it is between around 8,877 and 11,123 BTUs.