Linear Regression

Objectives

1. Determine and interpret the linear correlation coefficient
2. Determine the linear regression equation

Linear Correlation Coefficient

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In this section, we will examine the correlation type the way it is done in the real world: calculating the linear correlation coefficient (r).

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- $r > 0$: positive linear correlation
- $r = 0$: no linear correlation
- $r < 0$: negative linear correlation

Linear Correlation Coefficient

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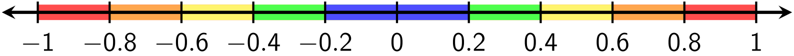
The closer r is to 1 (or -1), the more the data points "fall in line"

The closer r is to 0, the more the data points resemble a "cloud"

Interpreting r



None
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Moderate
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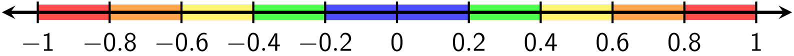

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Note: These interpretations are not universal.

Example 1

Find and interpret the linear correlation coefficient, r, for each.

(a)

x	y
7.6	19.1
9.2	22.9
3.3	10.3
1.1	6.6
3.7	10.6
3.9	11.3
4.6	12.9
2.3	8.6
5.1	15.2
5.3	15.1
2.5	13
3.4	11.2
3.1	10.6
1.7	6.8
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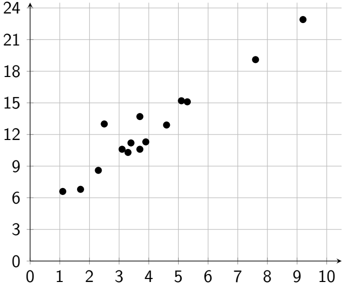

Example 1

(b)

x	y
7.6	11.0
9.2	3.6
6.3	8.9
1.1	14.9
6.7	8.1
3.9	12.0
4.6	9.4
2.3	10.3
5.1	11.4
5.3	12.4
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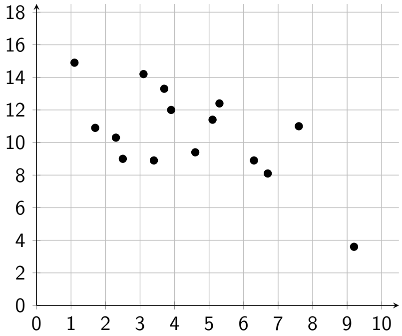

Example 1

(c)

x	y
6.9	3.4
7.7	4.5
0.9	9.8
3.4	1.5
8.9	3.3
5.7	8.9
3.1	8.4
2.2	8.1
4.5	6.8
4.1	0.5
5.0	0.4
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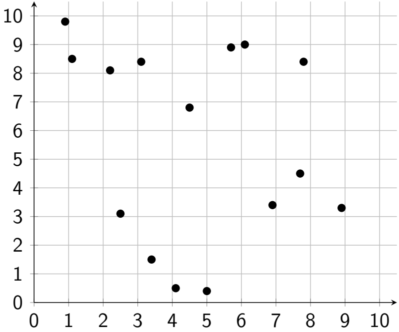

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While determining the linear correlation coefficient is valuable, it is also helpful to be able to predict data values not contained in the data set.

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To do this, we can create the **least squares regression equation**, (also called the *line of best fit*) which will **minimize** the total squared distance each data point is from the line:

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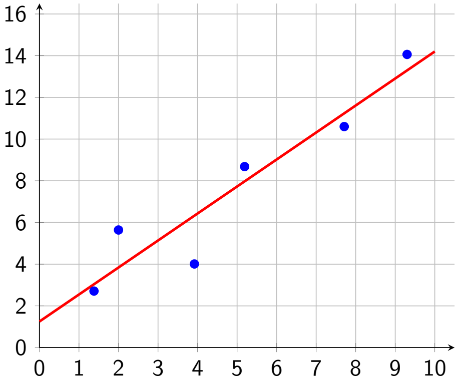
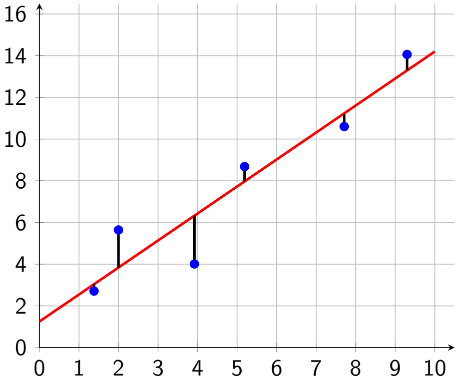
$$\hat{y} = mx + b$$

Line of Best Fit



Line of Best Fit



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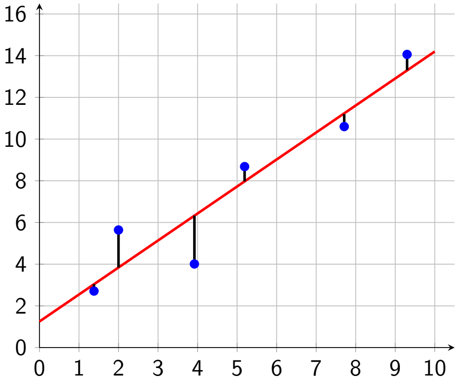

The black lines are **residuals**

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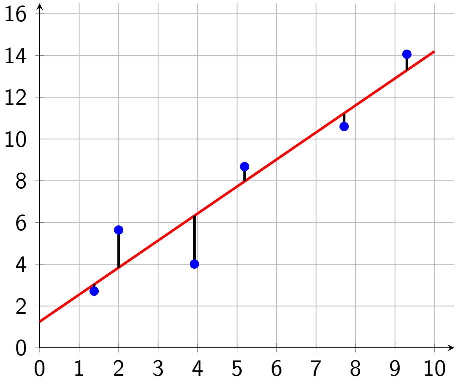

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Like deviations from the mean, the sum of the residuals is 0

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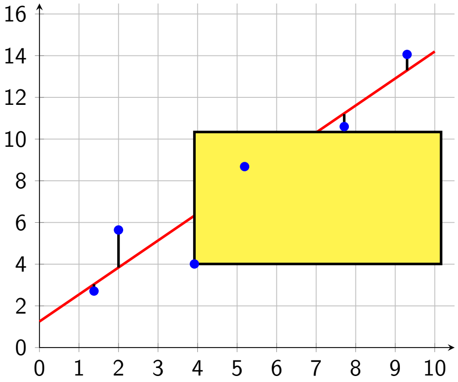

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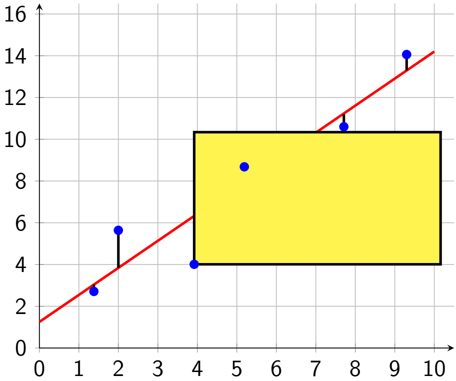

The line of best fit minimizes the sum of the areas of the squares