Other Probability Distributions

1. Solve problems involving geometric probability distributions

2. Solve problems involving Poisson probability distributions

Binomial vs. Geometric Distributions

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- There are a fixed number of n repeated independent trials
- Each trial's outcome is either a success or failure
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One of these outcomes is HHHTHHHTHH.

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To put it another way, a geometric distribution has the following property:

$$\underbrace{FFF\cdots F}_{x-1 \text{ failures}} S$$

Geometric Distributions

The probability of obtaining our first success after x binomial experiments is given by

$$P(X = x) = (1 - p)^{x-1} \cdot p$$

Example 1

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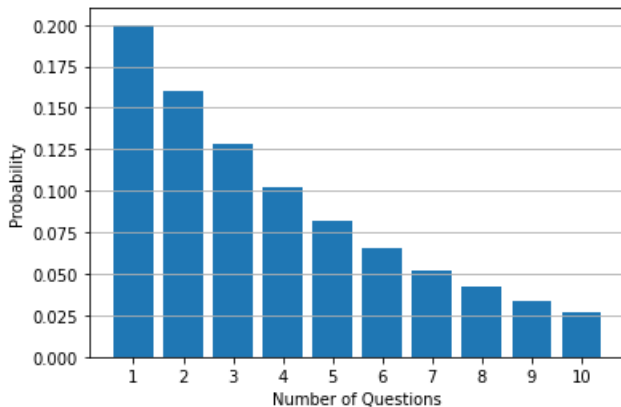$$P(X = 4) = (1 - 0.2)^3(0.2)$$

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The student has about an 10.24% chance of first guessing correctly on the 4th question.

Bar Graph of Example 1

Mean and Standard Deviation of Geometric Distributions

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From which the standard deviation, σ is

$$\sigma = \sqrt{\frac{1 - p}{p^2}} = \frac{\sqrt{1 - p}}{p}$$

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We would expect the student to answer 5 questions before guessing one correctly.

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The standard deviation is about 4.47 questions.

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- The probability of the event occurring is the same for each interval.
- The number of occurrences in one interval is independent of the number of occurrences in other intervals.

Poisson Formula

$$P(X = x) = \frac{\mu^x \cdot e^{-\mu}}{x!}$$

where μ is the mean number of occurrences of the event over the

intervals and $e \approx 2.71828$

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There is about a 15.63% chance that 5 dogs will be adopted in a given month.

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There is about a 23.81% chance that 2 or less dogs will be adopted in a given month.

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$$\approx 1 - 0.2381$$

$$\approx 0.7619$$

There is about a 76.19% chance that at least 3 dogs will be adopted in a given month.

Mean and Standard Deviation of Poisson Distributions

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Since the standard deviation is the square root of variance, it follows that the standard deviation of a Poisson distribution is also the square root of the mean:

$$\sigma = \sqrt{\mu}$$