Hypothesis Testing

Two Sample Means

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Thus, our null hypotheses will be $\mu_X = \mu_Y$.

Objectives

1. Perform hypothesis test on the mean for two dependent samples

2. Perform hypothesis test on the mean for two independent samples

Dependent Samples

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This part is sometimes known as a **paired t test** or a **matched pairs test**.

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s represents the sample standard deviation of the differences, and there are $n-1$ degrees of freedom.

Example 1

A medication is given to patients in an attempt to lower their LDL cholesterol. The tables below list the levels. At the 5% significance level, test the claim that the medicine is effective in lowering LDL cholesterol.

Before	**After**
95	91
109	107
127	129
131	125
117	110
135	120
103	97
98	101
111	107

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> *At the 5% significance level we reject the null hypothesis that there is no difference in LDL cholesterol levels and conclude that our sample suggests the medication may be effective in lowering LDL cholesterol levels.*

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In other words, there is no "before-and-after" relationship between our samples, and our samples don't even have to be the same sizes.

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However, p-values obtained through pooled tests can be significantly off if the population variances are, in fact, *not* equal.

As such, the examples in this section will assume that the population variances are *not equal*.

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Test statistic is given as

$$t = \frac{\overline{d}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Degrees of Freedom

The degrees of freedom is found by calculating

$$\mathrm{df} = \frac{(V_1 + V_2)^2}{\dfrac{V_1^2}{n_1 - 1} + \dfrac{V_2^2}{n_2 - 1}}$$

where

$$V_1 = \frac{s_1^2}{n_1} \text{ and } V_2 = \frac{s_2^2}{n_2}$$

Note: Round the degrees of freedom down to the nearest integer.

For Pools and Giggles

If working with pooled variances,

$$t = \frac{(\overline{X} - \overline{Y}) - (\mu_X - \mu_Y)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$\mathrm{df} = n_1 + n_2 - 2$$

Example 2

A local store claims that the waiting time for its customers to be served is the lowest in the area. A competitor's store checks the waiting times at both. The sample statistics are listed below. At the $\alpha = 0.05$ significance level, test the local store's hypothesis.

Local	Competitor
$n_x = 15$	$n_y = 12$
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$H_0 : \mu_x = \mu_y \longrightarrow \mu_x - \mu_y = 0$

$H_A : \mu_x < \mu_y \longrightarrow \mu_x - \mu_y < 0$

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Test statistic: $t = -1.7303$ Critical value: -1.7088
p-value $= 0.0480$ $\alpha = 0.05$
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> *At the 5% significance level, we have sufficient evidence to reject the claim that the mean wait times are equal and conclude that our evidence supports the claim that the mean wait times at the local store are less than the competitor.*

Example 3

Yellowfin tuna were caught from two different areas. The weights of each are listed.

Area 1	Area 2
30	48
67	52
49	53
39	54
80	54
35	51
53	55
41	49
	49

Test the claim the mean population weights are different at the $\alpha = 0.05$ significance level.

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Do not reject null hypothesis

> *At the 5% significance level we do not have sufficient evidence to reject the hypothesis that the mean weights of the yellowfin tuna are different and conclude that our evidence supports the claim that the mean weights are equal.*