Hypothesis Testing

Single Sample Mean

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William Sealy Gosset, under the pseudonym *Student*, created a hypothesis test for the population mean when the population standard deviation is unknown.

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- Sample was obtained randomly

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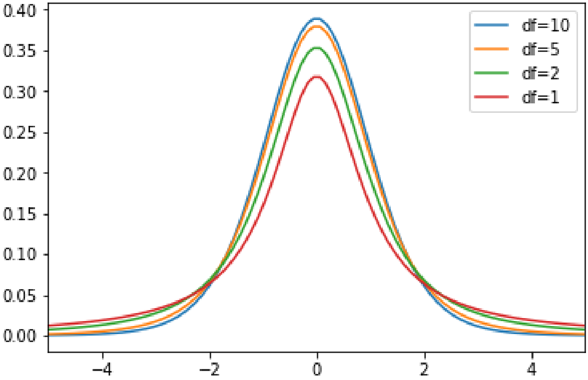

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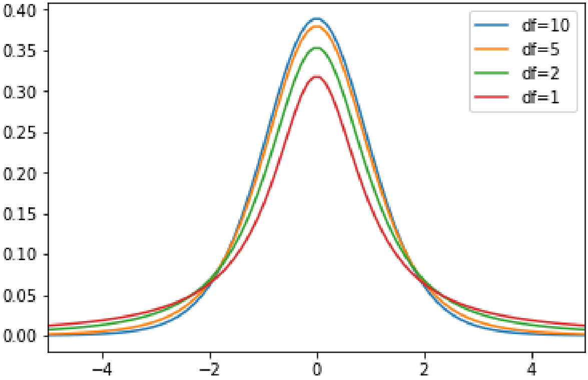

As the degrees of freedom grow, the t distribution becomes more normal.

Summary Stats

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Confidence intervals for t distribution are given by

$$\overline{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

where the degrees of freedom help determine the value of $t_{\alpha/2}$

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In the grand scheme of things, it's more valuable to be able to *interpret those results*.

Example 1

A car company claims one model of their SUV gets 40 mpg. You want to test the claim that the mean mpg of this model of SUV is less than 40 mpg, so you obtain a random sample of 65 of these cars and find the sample mean is 39 mpg with a standard deviation of 5.1 mpg. Test your claim at the $\alpha = 0.05$ significance level.

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$H_0: \mu = 40$ mpg
$H_A: \mu < 40$ mpg

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Do not reject the null hypothesis.

> *At the 95% confidence level, we do not have sufficient evidence to reject the claim that the mean mpg of this model SUV is 40 mpg, and conclude that our sample did not give us reason to believe the mean mpg might be less than 40.*

Example 2

A company claims it can boost your statistics grade by 10%. You want to test the claim that the mean grade increase is not 10%, so you randomly sample 20 statistics students who used the program and recorded their percent grade increase from one test to the next. Test your claim at the $\alpha = 0.1$ level of significance.

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12	7	8	11	15
4	13	9	8	11
9	3	5	8	12

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$H_0 : \mu = 10$
$H_A : \mu \neq 10$

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90% confidence interval: (7.5027, 9.8973)

Reject the null hypothesis

> *At the 90% confidence level, we have sufficient evidence to reject the claim that the mean increase in score is 10% and conclude that our sample gives us reason to believe the mean increase in score is not 10%.*