Speed Counting

Objectives

1. Use the Fundamental Counting Rule
2. Understand factorial notation
3. Find permutations of objects
4. Find combinations of objects
5. Find probabilities using counting techniques

Example 1

For a special at a restaurant, you can choose between 3 appetizers, 4 entrees, and 2 desserts. If you select one item from each category (appetizer, entree, and dessert), how many different meals can you create?

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With appetizer A: ADH, ADI, AEH, AEI, AFH, AFI, AGH, AGI

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With appetizer B: BDH, BDI, BEH, BEI, BFH, BFI, BGH, BGI

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For a total of 24 possible different meals.

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This can be generalized to multiple events, such as those in example 1: $3 \times 4 \times 2 = 24$

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$$9 \times 8 \times 7 \times \cdots \times 2 \times 1 = 362,880 \text{ unique lineups}$$

Factorial Notation

Rather than write out all the numbers from 9 to 1 and then multiplying them, mathematicians created **factorial notation** to expedite the process.

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In general, for a positive integer n,

$$n! = n(n-1)(n-2) \cdot \cdots \cdot 3(2)(1)$$

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with $0! = 1$

Factorial Growth

Factorial values grow very quickly:

$$\begin{aligned}
2! &= 2(1) && = 2\\
3! &= 3(2)(1) && = 6\\
4! &= 4(3)(2)(1) && = 24\\
5! &= 5(4)(3)(2)(1) && = 120\\
6! &= 6(5)(4)(3)(2)(1) && = 720\\
7! &= 7(6)(5)(4)(3)(2)(1) && = 5,040
\end{aligned}$$

Example 3

How many ways are there to arrange 5 books on a shelf?

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$$5! = 120 \text{ different arrangements}$$

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Five people are competing for three prizes: \$1,000, \$500, and \$100. How many different ways can the prizes be awarded?

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Using the Fundamental Counting Rule:

$$5 \times 4 \times 3 = 60 \text{ different ways}$$

Takeaways from Example 4

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So is there an easy way to do this if that's the case?

Yes, and that is where **permutations** come into play.

Example 5

How many ways are there to award gold, silver, and bronze medals to 8 contestants?

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Gold	Silver	Bronze

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8	7	

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8	7	6

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Gold	Silver	Bronze
8	7	6

Using the Fundamental Counting Rule:

$$8 \times 7 \times 6 = 336 \text{ different ways}$$

Example 5

$$8 \times 7 \times 6 \times \cdots \times 2 \times 1$$

Example 5

$$\frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

Example 5

$$8 \times 7 \times 6 = 336$$

Permutations

If there are n items available and we take r at a time, then the total number of permutations is given by

$$\frac{n!}{(n-r)!}$$

with $n \geq r$

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With permutations, the order in which an item is selected matters.

Knowing to Use Permutations

Permutations

- Offering various prizes

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Permutations

- Offering various prizes
- Running a race

Knowing to Use Permutations

Permutations

- Offering various prizes
- Running a race
- Assigning officer positions

Example 6

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