Hypothesis Testing

Two Sample Means

Moving from One Sample to Two

In this section, we are going to analyze the difference of means between two samples of data.

Moving from One Sample to Two

In this section, we are going to analyze the difference of means between two samples of data.

In particular, we will examine whether there is a difference in means that is statistically significant, assuming there is no difference in means of each sample's population.

Moving from One Sample to Two

In this section, we are going to analyze the difference of means between two samples of data.

In particular, we will examine whether there is a difference in means that is statistically significant, assuming there is no difference in means of each sample's population.

To put it mathematically, if we denote our first sample as X, and our second sample as Y, then we will want to know if

$$\mu_X - \mu_Y = 0$$

Moving from One Sample to Two

In this section, we are going to analyze the difference of means between two samples of data.

In particular, we will examine whether there is a difference in means that is statistically significant, assuming there is no difference in means of each sample's population.

To put it mathematically, if we denote our first sample as X, and our second sample as Y, then we will want to know if

$$\mu_X - \mu_Y = 0 \quad \text{or} \quad \mu_X = \mu_Y$$

Moving from One Sample to Two

In this section, we are going to analyze the difference of means between two samples of data.

In particular, we will examine whether there is a difference in means that is statistically significant, assuming there is no difference in means of each sample's population.

To put it mathematically, if we denote our first sample as X, and our second sample as Y, then we will want to know if

$$\mu_X - \mu_Y = 0 \quad \text{or} \quad \mu_X = \mu_Y$$

Thus, our null hypotheses will be $\mu_X = \mu_Y$.

1. Perform hypothesis test on the mean for two dependent samples

2. Perform hypothesis test on the mean for two independent samples

Dependent Samples

Recall from probability that two events are **dependent** if the chance of the second event happening is affected by the first event happening.

Dependent Samples

Recall from probability that two events are **dependent** if the chance of the second event happening is affected by the first event happening.

For this part of the section, we will look at examples with data that is paired together, often times having somewhat of a "before-and-after" theme; much like with experiments.

Dependent Samples

Recall from probability that two events are **dependent** if the chance of the second event happening is affected by the first event happening.

For this part of the section, we will look at examples with data that is paired together, often times having somewhat of a "before-and-after" theme; much like with experiments.

This part is sometimes known as a **paired t test** or a **matched pairs test**.

Nuts and Bolts of a Paired t Test

We will assume the distribution in the differences of sample X and sample Y are normal.

Nuts and Bolts of a Paired t Test

We will assume the distribution in the differences of sample X and sample Y are normal.

The test statistic is

$$t = \frac{\overline{d} - 0}{s_d/\sqrt{n}}$$

Nuts and Bolts of a Paired t Test

We will assume the distribution in the differences of sample X and sample Y are normal.

The test statistic is

$$t = \frac{\overline{d} - 0}{s_d/\sqrt{n}} = \frac{\overline{d}}{s_d/\sqrt{n}}$$

Nuts and Bolts of a Paired t Test

We will assume the distribution in the differences of sample X and sample Y are normal.

The test statistic is

$$t = \frac{\overline{d} - 0}{s_d/\sqrt{n}} = \frac{\overline{d}}{s_d/\sqrt{n}}$$

$\overline{d}$ represents the mean differences between the samples.

$$\overline{d} = \overline{X - Y}$$

Nuts and Bolts of a Paired t Test

We will assume the distribution in the differences of sample X and sample Y are normal.

The test statistic is

$$t = \frac{\overline{d} - 0}{s_d/\sqrt{n}} = \frac{\overline{d}}{s_d/\sqrt{n}}$$

$\overline{d}$ represents the mean differences between the samples.

$$\overline{d} = \overline{X - Y}$$

The population difference, μ_d will be 0 for hypothesis testing.

Nuts and Bolts of a Paired t Test

We will assume the distribution in the differences of sample X and sample Y are normal.

The test statistic is

$$t = \frac{\overline{d} - 0}{s_d/\sqrt{n}} = \frac{\overline{d}}{s_d/\sqrt{n}}$$

$\overline{d}$ represents the mean differences between the samples.

$$\overline{d} = \overline{X - Y}$$

The population difference, μ_d will be 0 for hypothesis testing.

s represents the sample standard deviation of the differences, and there are $n - 1$ degrees of freedom.

Example 1

A medication is given to patients in an attempt to lower their LDL cholesterol. The tables below list the levels. At the 5% significance level, test the claim that the medicine is effective in lowering LDL cholesterol.

Before	After
95	91
109	107
127	129
131	125
117	110
135	120
103	97
98	101
111	107

Example 1

$H_0: \mu_{\text{Before}} = \mu_{\text{After}}$

$H_A: \mu_{\text{Before}} < \mu_{\text{After}}$

Example 1

$H_0 : \mu_{\text{Before}} = \mu_{\text{After}}$

$H_A : \mu_{\text{Before}} < \mu_{\text{After}}$

- $t = 2.446$

Example 1

$H_0 : \mu_{\text{Before}} = \mu_{\text{After}}$

$H_A : \mu_{\text{Before}} < \mu_{\text{After}}$

- $t = 2.446$ (critical value $= 1.96$)

Example 1

$H_0: \mu_{\text{Before}} = \mu_{\text{After}}$

$H_A: \mu_{\text{Before}} < \mu_{\text{After}}$

- $t = 2.446$ (critical value $= 1.96$)

- $p = 0.0201$

Example 1

$H_0 : \mu_{\text{Before}} = \mu_{\text{After}}$

$H_A : \mu_{\text{Before}} < \mu_{\text{After}}$

- $t = 2.446$ (critical value $= 1.96$)

- $p = 0.0201$ ($\alpha = 0.05$)

Example 1

$H_0: \mu_{\text{Before}} = \mu_{\text{After}}$

$H_A: \mu_{\text{Before}} < \mu_{\text{After}}$

- $t = 2.446$ (critical value $= 1.96$)

- $p = 0.0201$ ($\alpha = 0.05$)

- 95% confidence interval: (0.2478, 8.4189)

Example 1

$H_0 : \mu_{\text{Before}} = \mu_{\text{After}}$

$H_A : \mu_{\text{Before}} < \mu_{\text{After}}$

- $t = 2.446$ (critical value $= 1.96$)

- $p = 0.0201$ ($\alpha = 0.05$)

- 95% confidence interval: $(0.2478, 8.4189)$ does not contain $\mu_d = 0$

Example 1

Reject the null hypothesis.

Example 1

Reject the null hypothesis.

At the 5% significance level we reject the null hypothesis that there is no difference in LDL cholesterol levels and conclude that our sample suggests the medication may be effective in lowering LDL cholesterol levels.

1. Perform hypothesis test on the mean for two dependent samples

2. Perform hypothesis test on the mean for two independent samples

Independent Samples

For this part, our samples will be *independent* of one another.

Independent Samples

For this part, our samples will be *independent* of one another.

In other words, there is no "before-and-after" relationship between our samples, and our samples don't even have to be the same sizes.

To Pool or Not To Pool

Since our independent samples don't even have to have the same sample size, we will have to know about assumptions we can make about their *variances*.

To Pool or Not To Pool

Since our independent samples don't even have to have the same sample size, we will have to know about assumptions we can make about their *variances*.

If we operate under the assumption that the two populations have equal variances, $\sigma_X = \sigma_Y$, then we use a **pooled** (i.e. *weighted*) estimate of the common variance.

To Pool or Not To Pool

Since our independent samples don't even have to have the same sample size, we will have to know about assumptions we can make about their *variances*.

If we operate under the assumption that the two populations have equal variances, $\sigma_X = \sigma_Y$, then we use a **pooled** (i.e. *weighted*) estimate of the common variance.

However, *p*-values obtained through pooled tests can be significantly off if the population variances are, in fact, *not* equal.

To Pool or Not To Pool

Since our independent samples don't even have to have the same sample size, we will have to know about assumptions we can make about their *variances*.

If we operate under the assumption that the two populations have equal variances, $\sigma_X = \sigma_Y$, then we use a **pooled** (i.e. *weighted*) estimate of the common variance.

However, *p*-values obtained through pooled tests can be significantly off if the population variances are, in fact, *not* equal.

As such, the examples in this section will assume that the population variances are *not equal*.

Nuts and Bolts

Assumptions

Nuts and Bolts

Assumptions

- Two samples are randomly chosen and independent of each other.

Nuts and Bolts

Assumptions

- Two samples are randomly chosen and independent of each other.
- Population distributions are approximately normal, OR

Nuts and Bolts

Assumptions

- Two samples are randomly chosen and independent of each other.
- Population distributions are approximately normal, OR
- Sample size are large enough ($n_1 \geq 30$ and $n_2 \geq 30$)

Nuts and Bolts

Assumptions

- Two samples are randomly chosen and independent of each other.
- Population distributions are approximately normal, OR
- Sample size are large enough ($n_1 \geq 30$ and $n_2 \geq 30$)

Test statistic is given as

Nuts and Bolts

Assumptions

- Two samples are randomly chosen and independent of each other.
- Population distributions are approximately normal, OR
- Sample size are large enough ($n_1 \geq 30$ and $n_2 \geq 30$)

Test statistic is given as

$$t = \frac{\overline{d}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Degrees of Freedom

The degrees of freedom is found by calculating

$$\text{df} = \frac{(V_1 + V_2)^2}{\dfrac{V_1^2}{n_1 - 1} + \dfrac{V_2^2}{n_2 - 1}}$$

where

$$V_1 = \frac{s_1^2}{n_1} \text{ and } V_2 = \frac{s_2^2}{n_2}$$

Note: Round the degrees of freedom down to the nearest integer.

For Pools and Giggles

If working with pooled variances,

$$t = \frac{\overline{d}}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$\mathrm{df} = n_1 + n_2 - 2$$