

Normal Distribution in the Real World

Objectives

- 1 Find the area under a normal curve for an observed value
- 2 Find observed values given area under the normal curve

Normal Distribution Revisited

The equation for calculating area when working with z scores is

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

with mean $\mu = 0$ and standard deviation $\sigma = 1$.

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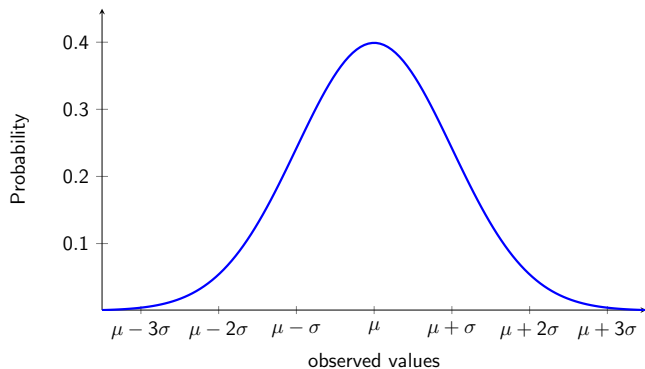
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Using observed values, and the z score formula $z = \frac{x-\mu}{\sigma}$, we get the following equation in terms of an observed value, x :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2/2\sigma^2}$$

Observed Value Graph



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Go with what you feel most comfortable with.

Example 1

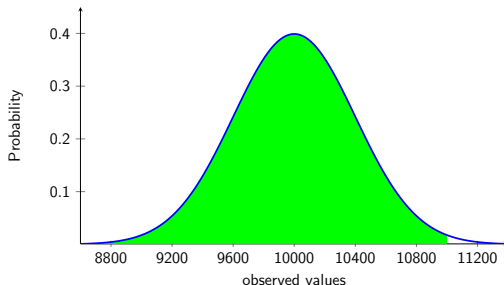
(a) An air conditioning manufacturer says their machines have a mean cooling capacity of 10,000 BTUs. Due to variations that arise in manufacturing, there is a standard deviation of 400 BTUs.

What is the probability of selecting an air conditioning unit that is 11,000 BTUs or less?

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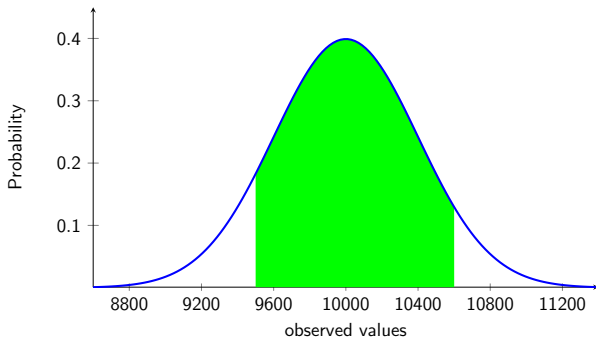
There is about a 99.38% chance of selecting an air conditioning unit that is 11,000 BTUs or less.

Example 1

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We are finding the area *between* $x = 9,500$ and $x = 10,600$:

$$x \approx 0.8275$$

There is about an 82.75% chance of selecting an air conditioning unit between 9,500 and 10,600 BTUs.

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Finding Values as the Inverse

Once again, this is just the inverse of what we did in the last example.

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Fortunately, most statistical technology has the ability to solve problems like these without the need to convert back from z scores first.

Example 2

(a) An air conditioning manufacturer says their machines have a mean cooling capacity of 10,000 BTUs. Due to variations that arise in manufacturing, there is a standard deviation of 400 BTUs.

How many BTUs must an air conditioning unit have to be in the 90th percentile?

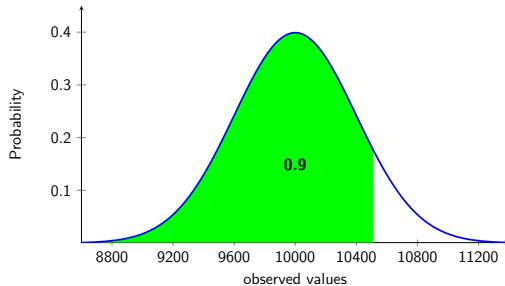
Example 2

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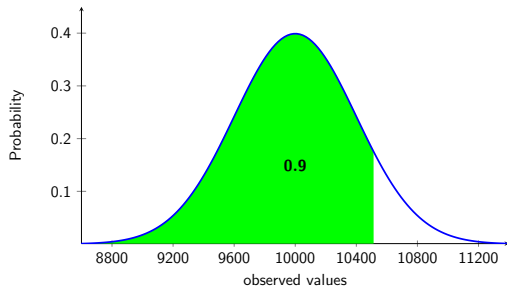
How many BTUs must an air conditioning unit have to be in the 90th percentile?

The 90th percentile means that 90% of the other units have less BTUs than the value we are finding.

Example 2a

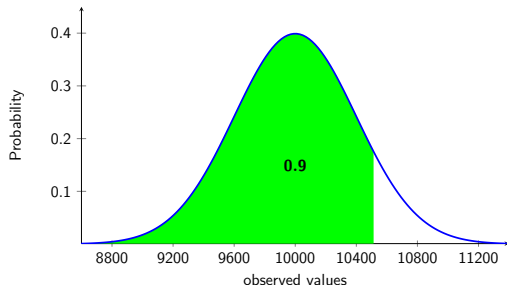


Example 2a



$$x \approx 10,512.62$$

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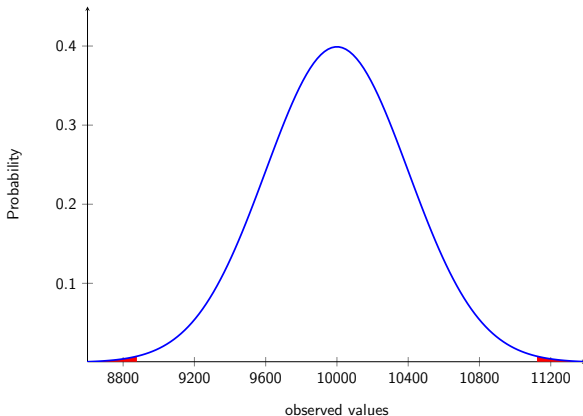
An air conditioning unit with about 10,512 BTUs is in the 90th percentile.

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(b) An air conditioning unit will not be shipped if it is in either the bottom 0.25% or top 0.25% of BTUs. What are those BTU cutoff values?

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An air conditioning unit will not be shipped if it is outside 8,877 and 11,123 BTUs.