

Introduction to Probability

Objectives

- 1 Determine the probability of an event
- 2 Find the experimental probability of an event

Sample Space

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Common sample spaces:

- *Flipping a coin*: Heads, Tails
- *Rolling a single die*: 1, 2, 3, 4, 5, 6
- *Drawing a card from a standard deck*: Ace of spades, ace of hearts, . . . , king of diamonds

Venn Diagrams

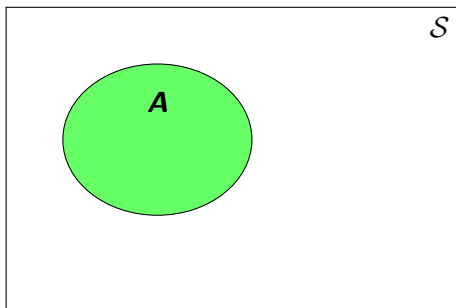
Venn Diagrams

A **Venn diagram** is a visualization of events and sample spaces.

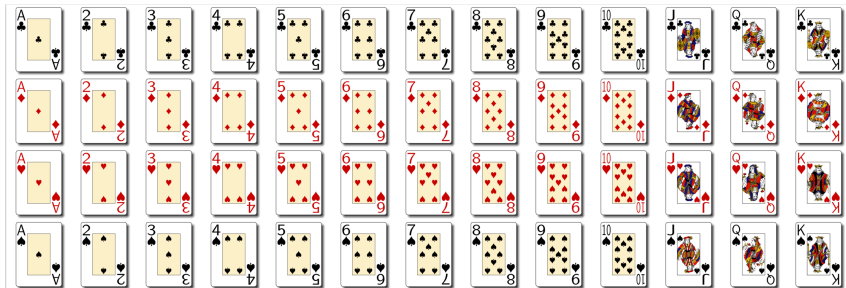
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Playing Cards



Probability

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$$\text{Probability} = \frac{\text{number of ways the event can occur}}{\text{total outcomes in sample space}}$$

Example 1

Determine the probability of each event.

- (a) Flipping a coin and landing on heads

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$$P(\text{heads}) = \frac{1}{2}$$

Example 1

(b) Rolling a number less than 3 on a single die.

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2 outcomes: 1, 2

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2 outcomes: 1, 2 6 outcomes in sample space

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$$P(\text{rolling less than 3 on a single die}) = \frac{2}{6} = \frac{1}{3}$$

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Thus, there are 12 total face cards: jack of clubs, jack of diamonds, ..., king of spades. There are 52 total cards

$$P(\text{drawing a face card}) = \frac{12}{52}$$

$$P(\text{drawing a face card}) = \frac{3}{13}$$

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(d) The number of students in each class at a college is shown in the table below.

Freshmen	Sophomore	Junior	Senior
1670	2017	2975	3026

Find the probability that a randomly selected student is a sophomore.

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$$P(\text{sophomore}) = \frac{2017}{9688}$$

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(e) The 36 possible sums from rolling 2 dice are shown below.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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$$\begin{aligned}P(\text{sum of } 7) &= \frac{6}{36} \\&= \frac{1}{6}\end{aligned}$$

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Types of Probability

In the previous examples, each outcome had an equal chance of being selected. This is referred to as **classical probability**.

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There are two other types of probability: experimental and subjective.

Experimental and Subjective Probabilities

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Experimental probability is probability based on events that have actually occurred.

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Subjective Probability

Subjective probability is probability based on opinion.

Example 2

Flip a coin 10 times and state the experimental probability of landing on tails.

Law of Large Numbers

As the number of events increases, the experimental probability of an event will approach the classical (a.k.a. *theoretical*) probability.

Rules of Probability

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 - A probability of 1 is a *certain event*.
- The sum of all possible probabilities of a sample space must equal 1.