

# Discrete Probability Distributions

# Objectives

- 1 Create a probability distribution
- 2 Determine the expected value of a probability distribution
- 3 Determine the variance and standard deviation of a probability distribution

# Probability Distributions

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- The sum of all probabilities in a distribution equals 1

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Familiar Characteristics:

- $0 \leq \text{each probability} \leq 1$
- The sum of all probabilities in a distribution equals 1
- $P(A \text{ or } B) = P(A) + P(B)$



# Probability Distribution of Rolling 2 Dice

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>1</b>	2	3	4	5	6	7
<b>2</b>	3	4	5	6	7	8
<b>3</b>	4	5	6	7	8	9
<b>4</b>	5	6	7	8	9	10
<b>5</b>	6	7	8	9	10	11
<b>6</b>	7	8	9	10	11	12

We can create a probability distribution of the sums of rolling two dice.

# Probability Distribution of Rolling 2 Dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We can create a probability distribution of the sums of rolling two dice.

We use the notation  $P(X = x)$  where  $X$  is our **random variable** and  $x$  represents the outcomes, such as 2, 3, 4, ..., 12.

# Probability Distribution of Rolling 2 Dice

$x$	$P(X = x)$
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

# Probability Distribution of Rolling 2 Dice

$x$	$P(X = x)$
2	$1/36$
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

# Probability Distribution of Rolling 2 Dice

$x$	$P(\mathbf{X} = x)$
2	$1/36$
3	$1/18$
4	
5	
6	
7	
8	
9	
10	
11	
12	

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11	
12	

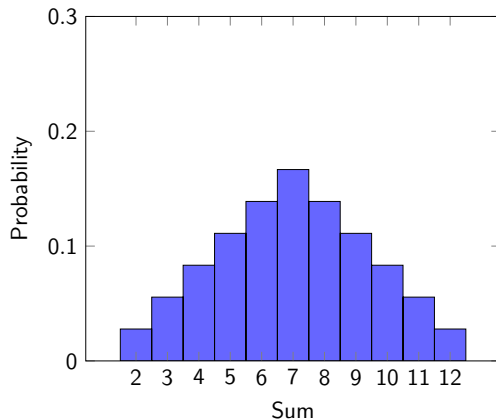
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10	$1/12$
11	$1/18$
12	

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6	$5/36$
7	$1/6$
8	$5/36$
9	$1/9$
10	$1/12$
11	$1/18$
12	$1/36$

# Probability Histogram of Rolling 2 Dice



## Example 1

- (a) Create a probability distribution for flipping a coin three times, where  $X$  represents the number of times heads is flipped.



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The sample space is HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT

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$x$	$P(X = x)$
0	
1	
2	
3	

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The sample space is HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT

$x$	$P(X = x)$
0	1/8
1	
2	
3	

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$x$	$P(X = x)$
0	$1/8$
1	$1/4$
2	
3	

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The sample space is HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT

$x$	$P(X = x)$
0	$1/8$
1	$1/4$
2	$1/4$
3	

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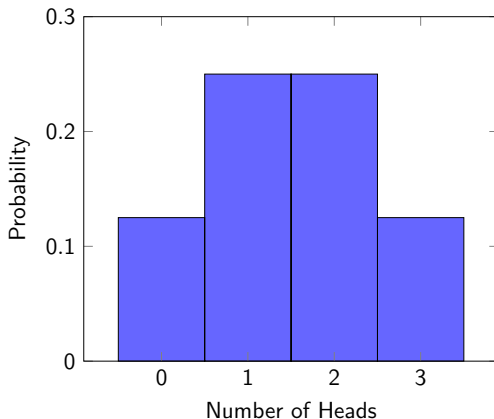
$x$	$P(X = x)$
0	1/8
1	1/4
2	1/4
3	1/8

## Example 1

(b) Create a probability distribution histogram for the number of times heads appears when flipping a coin 3 times.

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## Example 2

The distribution below represents the percentage of households that have  $x$  dogs according to a recent study.

$x$	$P(X = x)$
0	44%
1	27%
2	18%
3 or more	11%

How many households have at least 1 dog?

## Example 2

The distribution below represents the percentage of households that have  $x$  dogs according to a recent study.

$x$	$P(X = x)$
0	44%
1	27%
2	18%
3 or more	11%

How many households have at least 1 dog?

Using the Complement Rule:  $100\% - 44\% = 56\%$

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# Expected Value

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The **expected value** of a probability distribution is the outcome we would expect to happen if the experiment was performed a very large number of times.

In other words, it is a **weighted mean** of the distribution of outcomes:

$$E(X) = \sum (x \cdot P(x))$$

## Example 3

Determine the expected value of rolling two dice.

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$x$	$P(X = x)$	$x \cdot P(x)$	$x$	$P(X = x)$	$x \cdot P(x)$
2	$1/36$	$1/18$	7	$1/6$	$7/6$
3	$1/18$	$1/6$	8	$5/36$	$10/9$
4	$1/12$	$1/3$	9	$1/9$	1
5	$1/9$	$5/9$	10	$1/12$	$5/6$
6	$5/36$	$5/6$	11	$1/18$	$11/18$
			12	$1/36$	$1/3$

## Example 3

The expected value is the sum of all of the entries in the last column:



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$$\frac{1}{18} + \frac{1}{6} + \frac{1}{3} + \cdots + \frac{11}{18} + \frac{1}{3} = 7$$

Calculating the weighted mean of our distribution, the expected value of rolling two dice is 7.

## Example 4

The distribution below represents the percentage of households that have  $x$  dogs according to a recent study.

$x$	$P(X = x)$
0	44%
1	27%
2	18%
3	11%

What is the expected number of dogs per household?

## Example 4

The distribution below represents the percentage of households that have  $x$  dogs according to a recent study.

$x$	$P(X = x)$
0	44%
1	27%
2	18%
3	11%

What is the expected number of dogs per household?

$$0(0.44) + 1(0.27) + 2(0.18) + 3(0.11)$$

## Example 4

The distribution below represents the percentage of households that have  $x$  dogs according to a recent study.

$x$	$P(X = x)$
0	44%
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What is the expected number of dogs per household?

$$0(0.44) + 1(0.27) + 2(0.18) + 3(0.11) = 0.96$$

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0	44%
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What is the expected number of dogs per household?

$$0(0.44) + 1(0.27) + 2(0.18) + 3(0.11) = 0.96$$

There is about 1 dog per household.

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# Variance and Standard Deviation

Previously, we saw that variance, which was the average squared deviation the data values are from the mean, was

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Similarly, the variance for probability distributions is given as

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from which the **standard deviation** is

$$\sigma = \sqrt{\sum ((x - \mu)^2 \cdot P(x))}$$

# Variance and Standard Deviation Alternate Definition

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$$2: \quad (2 - 7)^2 \cdot \frac{1}{36} = \frac{25}{36}$$

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$$2: \quad (2 - 7)^2 \cdot \frac{1}{36} = \frac{25}{36}$$

$$3: \quad (3 - 7)^2 \cdot \frac{1}{18} = \frac{8}{9}$$

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What is the standard deviation of rolling two dice?

From a previous example, we have  $\mu = E(x) = 7$ .

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$\vdots$



## Example 5

What is the standard deviation of rolling two dice?

From a previous example, we have  $\mu = E(x) = 7$ .

$$2: \quad (2 - 7)^2 \cdot \frac{1}{36} = \frac{25}{36}$$

$$3: \quad (3 - 7)^2 \cdot \frac{1}{18} = \frac{8}{9}$$

$\vdots$

$$12: \quad (12 - 7)^2 \cdot \frac{1}{36} = \frac{25}{36}$$

## Example 5

Adding up all of the results and then taking the square root, we get

$$\sigma \approx 2.415$$