# Speed Counting

# Objectives

1. Use the Fundamental Counting Rule
2. Understand factorial notation
3. Find permutations of objects
4. Find combinations of objects
5. Find probabilities using counting techniques

# Example 1

For a special at a restaurant, you can choose between 3 appetizers, 4 entrees, and 2 desserts. If you select one item from each category (appetizer, entree, and dessert), how many different meals can you create?

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For a total of 24 possible different meals.

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This can be generalized to multiple events, such as those in example 1: $3 \times 4 \times 2 = 24$

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$$9 \times 8 \times 7 \times \cdots \times 2 \times 1$$

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$$9 \times 8 \times 7 \times \cdots \times 2 \times 1 = 362,880 \text{ unique lineups}$$

# Factorial Notation

Rather than write out all the numbers from 9 to 1 and then multiplying them, mathematicians created **factorial notation** to expedite the process.

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In general, for a positive integer $n$,

$$n! = n(n-1)(n-2) \cdot \cdots \cdot 3(2)(1)$$

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In general, for a positive integer $n$,

$$n! = n(n-1)(n-2)\cdot \cdots \cdot 3(2)(1)$$

with $0! = 1$

# Factorial Growth

Factorial values grow very quickly:

$$\begin{aligned}
2! &= 2(1) && = 2\\
3! &= 3(2)(1) && = 6\\
4! &= 4(3)(2)(1) && = 24\\
5! &= 5(4)(3)(2)(1) && = 120\\
6! &= 6(5)(4)(3)(2)(1) && = 720\\
7! &= 7(6)(5)(4)(3)(2)(1) && = 5,040
\end{aligned}$$

# Example 3

How many ways are there to arrange 5 books on a shelf?

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$$5! = 120 \text{ different arrangements}$$

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|---|---|---|
| 5 | 4 | |

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|---|---|---|
| 5 | 4 | 3 |

Using the Fundamental Counting Rule:

$$5 \times 4 \times 3 = 60 \text{ different ways}$$

# Takeaways from Example 4

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If we had 10,000 contestants and 75 prizes, we would have a lot of multiplying to do.

So is there an easy way to do this if that's the case?

Yes, and that is where **permutations** come into play.

# Example 5

How many ways are there to award gold, silver, and bronze medals to 8 contestants?

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Using the Fundamental Counting Rule:

$$8 \times 7 \times 6 = 336 \text{ different ways}$$

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