Other Probability Distributions

Objectives

1. Solve problems involving geometric probability distributions
2. Solve problems involving hypergeometric probability distributions

Binomial vs. Geometric Distributions

With binomial distributions, we had the following conditions:

- There are a fixed number of n repeated independent trials
- Each trial's outcome is either a success or failure
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One of these outcomes is TTHTHHHTTH.

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$$\underbrace{FFF\cdots F}_{x-1 \text{ failures}}S$$

Geometric Distributions

The probability of obtaining our first success after x binomial experiments is given by

$$P(X = x) = (1 - p)^{x-1} \cdot p$$

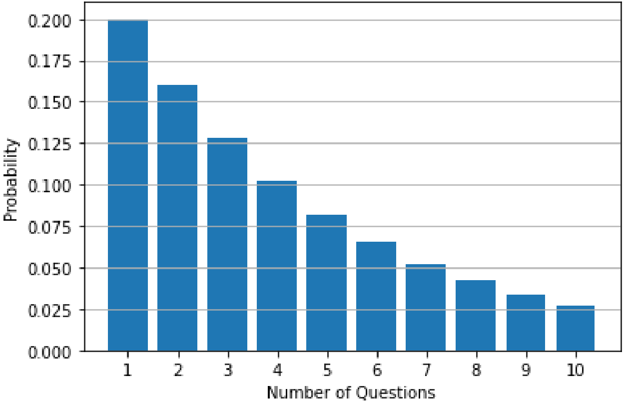
Example 1

A student is given a 10-question multiple choice test in which each question has 5 possible answers. What is the probability that the first question the student guesses correctly on the 4th question?

FFFS

$$\begin{aligned} P(X = 4) &= (1 - 0.2)^4 (0.2) \\ &= 0.08192 \end{aligned}$$

Bar Graph of Example 1



Mean and Standard Deviation of Geometric Distributions

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From which the standard deviation, σ is

$$\sigma = \sqrt{\frac{1-p}{p^2}} = \frac{\sqrt{1-p}}{p}$$

Example 2

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We would expect the student to answer 5 questions before guessing one correctly.

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The standard deviation is about 4.47 questions.

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