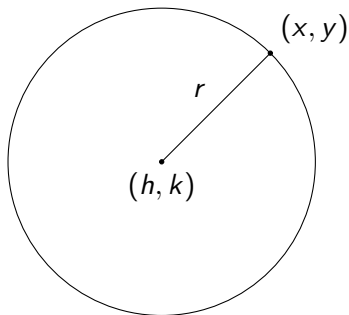


Circles

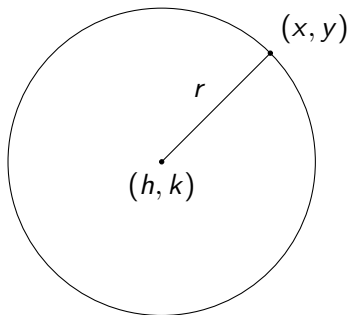
Objectives

- 1 Write the Standard Form of the Equation of a Circle.
- 2 Find the Center and Radius of a Circle.

A **circle** is the set of all points (x, y) in the plane whose distance (the *radius*) from a fixed point (the *center*) is constant.



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The equation can be found by incorporating the distance formula (Pythagorean Theorem):

$$r^2 = (x - h)^2 + (y - k)^2$$

Example 1

Write the standard form of the equation of a circle with center $(-2, 3)$ and radius 5.

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$$(x - (-2))^2 + (y - 3)^2 = 5^2$$

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$$(x + 2)^2 + (y - 3)^2 = 25$$

Example 2

Write the standard form of the equation of a circle which has $(-1, 3)$ and $(2, 4)$ as the endpoints of the diameter.

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$$h = \frac{1}{2} \quad k = \frac{7}{2}$$

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$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{10}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{5}{2}$$

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Find the center and radius of $(x + 2)^2 + (y - 1)^2 = 4$.

$$h = -2 \quad k = 1$$

Center: $(-2, 1)$

$$r^2 = 4$$

$$r = 2$$

Radius: 2

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These are your h and k , respectively.
 - Use x 's as the variable when finding the vertex for the y 's parabola.
- 3 Add the absolute value of the y -coordinates of the vertices to the right side.

Example 4a

Find the center and radius of each.

(a) $3x^2 - 6x + 3y^2 + 4y - 4 = 0$

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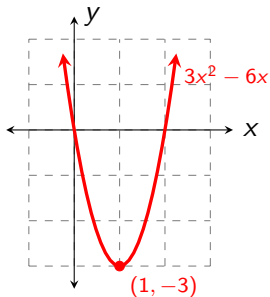
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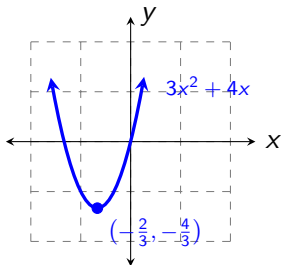
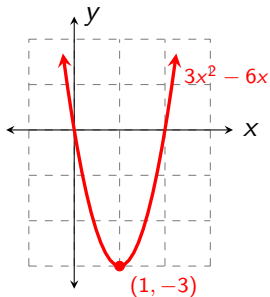


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$$3(x-1)^2 + 3\left(y + \frac{2}{3}\right)^2 = 4 + |-3| + \left|-\frac{4}{3}\right|$$

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$$3(x-1)^2 + 3\left(y + \frac{2}{3}\right)^2 = \frac{25}{3}$$

$$(x-1)^2 + \left(y + \frac{2}{3}\right)^2 = \frac{25}{9}$$

Example 4a

$$3(x-1)^2 + 3\left(y + \frac{2}{3}\right)^2 = 4 + |-3| + \left|-\frac{4}{3}\right|$$

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Center: $\left(1, -\frac{2}{3}\right)$

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$$(x-1)^2 + \left(y + \frac{2}{3}\right)^2 = \frac{25}{9}$$

Center: $\left(1, -\frac{2}{3}\right)$ Radius: $\sqrt{\frac{25}{9}} = \frac{5}{3}$

Example 4b

$$(b) \quad 2x^2 + 5x + 2y^2 - 8y + 1 = 0$$

Example 4b

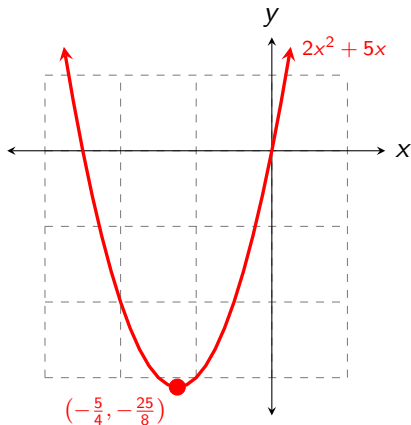
$$(b) \quad 2x^2 + 5x + 2y^2 - 8y + 1 = 0$$

$$2x^2 + 5x + 2y^2 - 8y = -1$$

Example 4b

$$(b) \quad 2x^2 + 5x + 2y^2 - 8y + 1 = 0$$

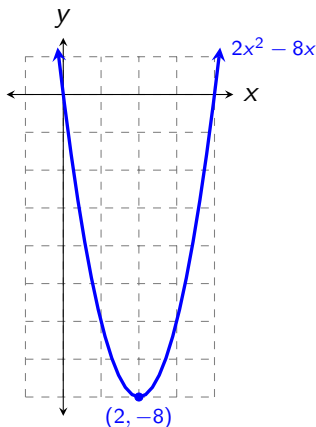
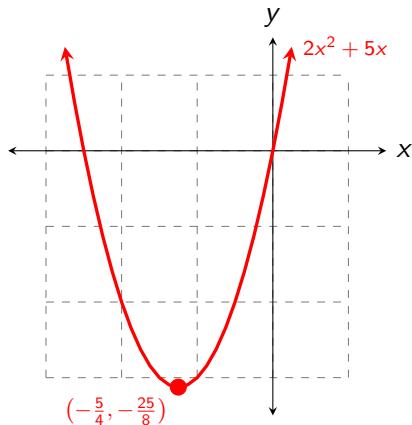
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Example 4b

$$(b) \quad 2x^2 + 5x + 2y^2 - 8y + 1 = 0$$

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Example 4b

$$2\left(x + \frac{5}{4}\right)^2 + 2(y - 2)^2 = -1 + \left| -\frac{25}{8} \right| + | -8 |$$

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$$\left(x + \frac{5}{4}\right)^2 + (y - 2)^2 = \frac{81}{16}$$

Example 4b

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Center: $\left(-\frac{5}{4}, 2\right)$

Example 4b

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$$2\left(x + \frac{5}{4}\right)^2 + 2(y - 2)^2 = \frac{81}{8}$$

$$\left(x + \frac{5}{4}\right)^2 + (y - 2)^2 = \frac{81}{16}$$

Center: $\left(-\frac{5}{4}, 2\right)$ Radius: $\sqrt{\frac{81}{16}} = \frac{9}{4}$