Dot Product and Projection

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Dot Product

For vectors $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$, the dot product of $\vec{v}$ and $\vec{w}$ is given as

$$\vec{v} \cdot \vec{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle$$

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$$\begin{aligned} \vec{v} \cdot \vec{w} &= \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle \\ &= v_1 w_1 + v_2 w_2 \end{aligned}$$

Example 1

Find $\vec{v} \cdot \vec{w}$ if $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = \langle 1, -2 \rangle$.

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$$\begin{aligned} \vec{v} \cdot \vec{w} &= \langle 3, 4 \rangle \cdot \langle 1, -2 \rangle \\ &= 3(1) + 4(-2) \\ &= -5 \end{aligned}$$

Dot Product Properties

Because the dot product produces a scalar, it is often called the scalar product.

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$\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$	Relation to Magnitude

Example 2

Show that $\|\vec{v} - \vec{w}\|^2 = \|\vec{v}\|^2 - 2\left(\vec{v} \cdot \vec{w}\right) + \|\vec{w}\|^2$

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$$\begin{aligned} \|\vec{v} - \vec{w}\|^2 &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) \\ &= \vec{v} \cdot (\vec{v} - \vec{w}) - \vec{w} \cdot (\vec{v} - \vec{w}) \end{aligned}$$

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\|\vec{v} - \vec{w}\|^2 &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) \\
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&= \vec{v} \cdot \vec{v} - \vec{v} \cdot \vec{w} - \vec{v} \cdot \vec{w} + \vec{w} \cdot \vec{w}
\end{aligned}$$

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&= \vec{v} \cdot \vec{v} - 2(\vec{v} \cdot \vec{w}) + \vec{w} \cdot \vec{w}
\end{aligned}$$

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\|\vec{v}-\vec{w}\|^2 &= (\vec{v}-\vec{w})\cdot(\vec{v}-\vec{w}) \\
&= \vec{v}\cdot(\vec{v}-\vec{w}) - \vec{w}\cdot(\vec{v}-\vec{w}) \\
&= \vec{v}\cdot\vec{v} - \vec{v}\cdot\vec{w} - \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{w} \\
&= \vec{v}\cdot\vec{v} - 2(\vec{v}\cdot\vec{w}) + \vec{w}\cdot\vec{w} \\
&= \|v\|^2 - 2(\vec{v}\cdot\vec{w}) + \|w\|^2
\end{aligned}$$

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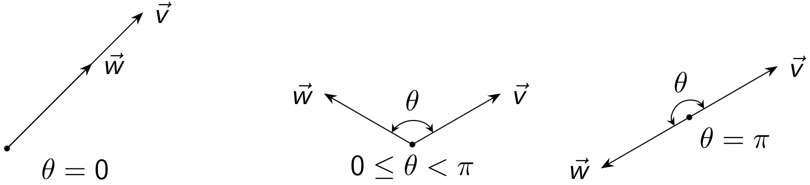
Angles Between Vectors

If we draw $\vec{v}$ and $\vec{w}$ with the same initial point, then the angle θ between them is illustrated below:



Dot Product and Angles Between Vectors

Geometrically, the dot product between two vectors is

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos\theta$$

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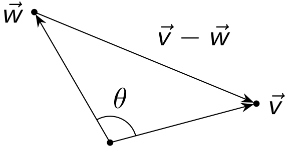

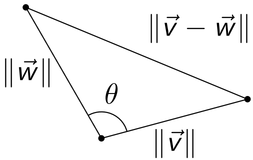

Dot Product and Angles Between Vectors

By the Law of Cosines, $\|\vec{v} - \vec{w}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2\|\vec{v}\|\|\vec{w}\| \cos\theta$, and by Example 2, $\|\vec{v} - \vec{w}\|^2 = \|\vec{v}\|^2 - 2(\vec{v} \cdot \vec{w}) + \|\vec{w}\|^2$.

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Equating these and solving for $\vec{v}\cdot\vec{w}$ gives us $\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos\theta$, from which

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|}$$

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By the Law of Cosines, $\|\vec{v}-\vec{w}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2\|\vec{v}\|\|\vec{w}\|\cos\theta$, and by Example 2, $\|\vec{v}-\vec{w}\|^2 = \|\vec{v}\|^2 - 2\left(\vec{v}\cdot\vec{w}\right) + \|\vec{w}\|^2$.

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$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|}$$

To find the angle between two vectors, solve for θ:

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|}\right) = \cos^{-1}\left(\hat{v}\cdot\hat{w}\right)$$

Example 3a

Find the angle between each of the following.

(a) $\vec{v} = \langle 3, -3\sqrt{3} \rangle$ and $\vec{w} = \langle -\sqrt{3}, 1 \rangle$

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$$= -3\sqrt{3} - 3\sqrt{3} = -6\sqrt{3}$$

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$$\|\vec{v}\| = \sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{36}$$

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$$\|\vec{v}\| = \sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{36}$$

$$\|\vec{w}\| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4}$$

$$\theta = \arccos\left(\frac{-6\sqrt{3}}{\sqrt{36 \times 4}}\right) = \frac{5\pi}{6}$$

Example 3b

(b) $\vec{v} = \langle 2, 2 \rangle$ and $\vec{w} = \langle 5, -5 \rangle$

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$$\vec{v} \cdot \vec{w} = 2(5) + 2(-5) = 0$$

$$\|\vec{v}\| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

Example 3b

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$$\vec{v} \cdot \vec{w} = 2(5) + 2(-5) = 0$$

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$$\|\vec{w}\| = \sqrt{5^2 + 5^2} = \sqrt{50}$$

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(b) $\vec{v} = \langle 2, 2\rangle$ and $\vec{w} = \langle 5, -5\rangle$

$$\vec{v} \cdot \vec{w} = 2(5) + 2(-5) = 0$$

$$\|\vec{v}\| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$\|\vec{w}\| = \sqrt{5^2 + 5^2} = \sqrt{50}$$

$$\theta = \arccos\left(\frac{0}{\sqrt{8 \times 50}}\right) = \frac{\pi}{2}$$

Example 3c

(c) $\vec{v} = \langle 3, -4 \rangle$ and $\vec{w} = \langle 2, 1 \rangle$

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Example 3c

(c) $\vec{v} = \langle 3, -4 \rangle$ and $\vec{w} = \langle 2, 1 \rangle$

$$\vec{v} \cdot \vec{w} = 3(2) + (-4)(1) = 2$$

$$\|\vec{v}\| = \sqrt{3^2 + 4^2} = \sqrt{25}$$

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(c) $\vec{v} = \langle 3, -4 \rangle$ and $\vec{w} = \langle 2, 1 \rangle$

$$\vec{v} \cdot \vec{w} = 3(2) + (-4)(1) = 2$$

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$$\|\vec{w}\| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\theta = \arccos\left(\frac{2}{\sqrt{25 \times 5}}\right) \approx 80^\circ$$

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Orthogonal Projection

Two vectors are **orthogonal** if they meet at a right angle.

If two vectors are orthogonal, then their dot product is 0.

Orthogonal Projection

The orthogonal projection of $\vec{v}$ onto $\vec{w}$ is a new vector $\vec{p}$ that is parallel to $\vec{w}$ and has a magnitude of $\|\vec{v}\| \cos\theta$.

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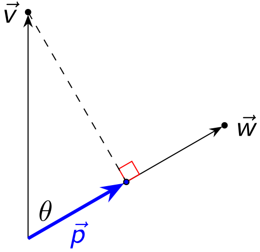
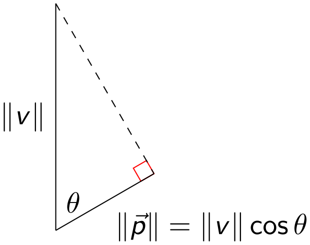
$\vec{p}$ can be thought of as the "shadow" $\vec{v}$ casts over $\vec{w}$:





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Thus, we can multiply the magnitude of $\vec{p}$ by a unit vector for $\vec{w}$.

$$\|\vec{p}\|\,\hat{w}$$

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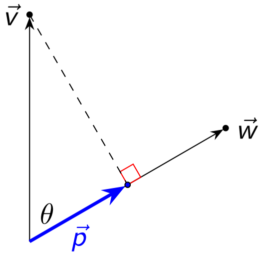
Thus, we can multiply the magnitude of $\vec{p}$ by a unit vector for $\vec{w}$.

$$\|\vec{p}\|\,\hat{w}$$

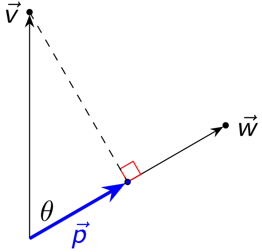
This will guarantee that $\vec{w}$ is scaled to $\vec{p}$ and give us the projection of $\vec{v}$ onto $\vec{w}$.

Orthogonal Projection



Orthogonal Projection

$$\text{proj}_{\vec{w}}\vec{v} = \|\vec{p}\|\ \hat{w}$$



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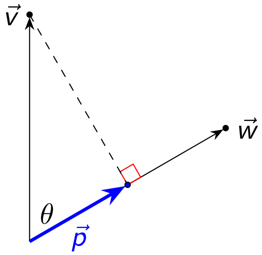

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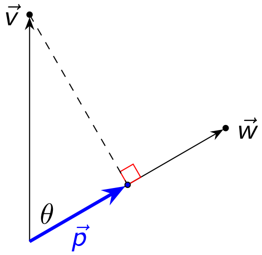

$$\mathrm{proj}_{\vec{w}}\,\vec{v} = \|\vec{p}\|\ \hat{w}$$

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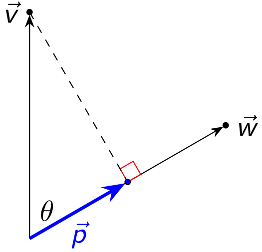
$$= \|v\|\left(\frac{v \cdot w}{\|v\|\ \|w\|}\right)\left(\frac{\vec{w}}{\|w\|}\right)$$

Orthogonal Projection



$$
\begin{aligned}
\operatorname{proj}_{\vec{w}} \vec{v} &= \|\vec{p}\|\ \hat{w} \\
&= (\|v\| \cos\theta) \left(\frac{\vec{w}}{\|w\|} \right) \\
&= \|v\| \left(\frac{v \cdot w}{\|v\|\ \|w\|} \right) \left(\frac{\vec{w}}{\|w\|} \right) \\
&= \left(\frac{v \cdot w}{\|w\|\ \|w\|} \right) \vec{w}
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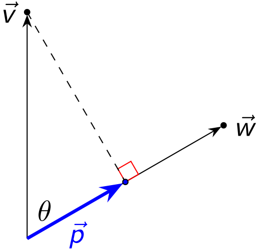
$$

Orthogonal Projection



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\operatorname{proj}_{\vec{w}} \vec{v} &= \|\vec{p}\| \, \hat{w} \\
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&= \left(\frac{v \cdot w}{\|w\| \, \|w\|} \right) \vec{w} \\
\operatorname{proj}_{\vec{w}} \vec{v} &= \left(\frac{v \cdot w}{\|w\|^2} \right) \vec{w}
\end{aligned}$$

Example 4a

Let $\vec{v} = \langle 1, 8 \rangle$ and $\vec{w} = \langle -1, 2 \rangle$. Find each of the following.

(a) $\text{proj}_{\vec{w}}\vec{v}$

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$$\text{proj}_{\vec{w}} \vec{v} = \left(\frac{v \cdot w}{\|w\|^2} \right) \vec{w}$$

$$v \cdot w = 1(-1) + 8(2) = 15$$

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$$v \cdot w = 1(-1) + 8(2) = 15 \qquad \|\vec{w}\|^2 = \left(\sqrt{1^2 + 2^2} \right)^2 = 5$$

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$$\text{proj}_{\vec{w}} \vec{v} = \frac{15}{5} \langle -1, 2 \rangle$$

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$$= 3\langle -1, 2 \rangle$$

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$$v \cdot w = 1(-1) + 8(2) = 15 \qquad \|\vec{w}\|^2 = \left(\sqrt{1^2 + 2^2} \right)^2 = 5$$

$$\begin{aligned} \text{proj}_{\vec{w}} \vec{v} &= \frac{15}{5} \langle -1, 2 \rangle \\ &= 3\langle -1, 2 \rangle \\ &= \langle -3, 6 \rangle \end{aligned}$$

Example 4b

Let $\vec{v} = \langle 1, 8 \rangle$ and $\vec{w} = \langle -1, 2 \rangle$. Find each of the following.

(b) $\text{proj}_{\vec{v}}\vec{w}$

Example 4b

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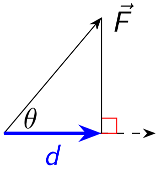

$$W = Fd = \|F\| \, \|d\| \cos\theta = F \cdot d$$

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Mr. Bain exerts a force of 230 pounds to pull a stack of rocks a distance of 50 ft. over level ground. If the rope makes a 30° angle with the horizontal, how much work did Mr. Bain do pulling the rocks? Assume Mr. Bain exerts the force of 230 pounds at a 30° angle for the duration of the 50 feet.

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