Dot Product and Projection

Table of Contents

Dot Product

For vectors $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$, the dot product of $\vec{v}$ and $\vec{w}$ is given as

$$\vec{v} \cdot \vec{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle$$

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$$\begin{aligned} \vec{v} \cdot \vec{w} &= \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle \\ &= v_1 w_1 + v_2 w_2 \end{aligned}$$

Example 1

Find $\vec{v} \cdot \vec{w}$ if $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = \langle 1, -2 \rangle$.

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$$\begin{aligned}
\vec{v} \cdot \vec{w} &= \langle 3, 4 \rangle \cdot \langle 1, -2 \rangle \\
&= 3(1) + 4(-2) \\
&= -5
\end{aligned}$$

Dot Product Properties

Because the dot product produces a scalar, it is often called the scalar product.

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$k\,(\vec{v}) \cdot \vec{w} = k\,(\vec{v} \cdot \vec{w}) = \vec{v} \cdot (k\vec{w})$	Scalar Property
$\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$	Relation to Magnitude

Example 2

Show that $\|\vec{v} - \vec{w}\|^2 = \|\vec{v}\|^2 - 2\,(\vec{v} \cdot \vec{w}) + \|\vec{w}\|^2$

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Show that $\|\vec{v} - \vec{w}\|^2 = \|\vec{v}\|^2 - 2\left(\vec{v} \cdot \vec{w}\right) + \|\vec{w}\|^2$

$$\begin{aligned} \|\vec{v} - \vec{w}\| &= \left(\vec{v} - \vec{w}\right) \cdot \left(\vec{v} - \vec{w}\right) \\ &= \vec{v} \cdot \left(\vec{v} - \vec{w}\right) - \vec{w} \cdot \left(\vec{v} - \vec{w}\right) \end{aligned}$$

Example 2

Show that $\|\vec{v}-\vec{w}\|^2 = \|\vec{v}\|^2 - 2\left(\vec{v}\cdot\vec{w}\right) + \|\vec{w}\|^2$

$$
\begin{aligned}
\|\vec{v}-\vec{w}\| &= (\vec{v}-\vec{w})\cdot(\vec{v}-\vec{w}) \\
&= \vec{v}\cdot(\vec{v}-\vec{w}) - \vec{w}\cdot(\vec{v}-\vec{w}) \\
&= \vec{v}\cdot\vec{v} - \vec{v}\cdot\vec{w} - \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{w}
\end{aligned}
$$

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$$\begin{aligned} \|\vec{v} - \vec{w}\| &= (\vec{v} - \vec{w}) \cdot (\vec{v} - \vec{w}) \\ &= \vec{v} \cdot (\vec{v} - \vec{w}) - \vec{w} \cdot (\vec{v} - \vec{w}) \\ &= \vec{v} \cdot \vec{v} - \vec{v} \cdot \vec{w} - \vec{v} \cdot \vec{w} + \vec{w} \cdot \vec{w} \\ &= \vec{v} \cdot \vec{v} - 2(\vec{v} \cdot \vec{w}) + \vec{w} \cdot \vec{w} \end{aligned}$$

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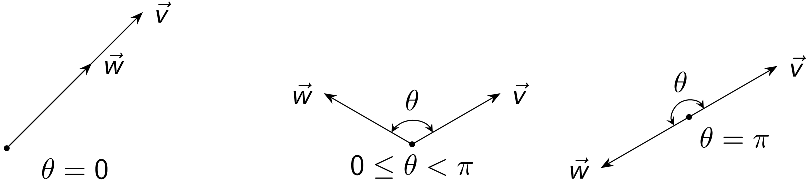
\|\vec{v}-\vec{w}\| &= (\vec{v}-\vec{w})\cdot(\vec{v}-\vec{w}) \\
&= \vec{v}\cdot(\vec{v}-\vec{w}) - \vec{w}\cdot(\vec{v}-\vec{w}) \\
&= \vec{v}\cdot\vec{v} - \vec{v}\cdot\vec{w} - \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{w} \\
&= \vec{v}\cdot\vec{v} - 2(\vec{v}\cdot\vec{w}) + \vec{w}\cdot\vec{w} \\
&= \|v\|^2 - 2(\vec{v}\cdot\vec{w}) + \|w\|^2
\end{aligned}$$

Table of Contents

Angles Between Vectors

If we draw $\vec{v}$ and $\vec{w}$ with the same initial point, then the angle θ between them is illustrated below:



Dot Product and Angles Between Vectors

Geometrically, the dot product between two vectors is

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos\theta$$

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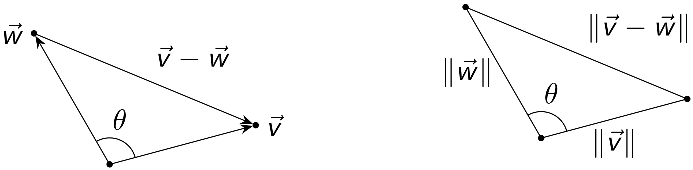

Dot Product and Angles Between Vectors

By the Law of Cosines, $\|\vec{v}-\vec{w}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2\|\vec{v}\|\|\vec{w}\|\cos\theta$, and by Example 2, $\|\vec{v}-\vec{w}\|^2 = \|\vec{v}\|^2 - 2\,(\vec{v}\cdot\vec{w}) + \|\vec{w}\|^2$.

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Equating these and solving for $\vec{v}\cdot\vec{w}$ gives us $\vec{v}\cdot\vec{w} = \|\vec{v}\|\|\vec{w}\|\cos\theta$, from which

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\,\|\vec{w}\|}$$

Dot Product and Angles Between Vectors

By the Law of Cosines, $\|\vec{v} - \vec{w}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2\|\vec{v}\|\|\vec{w}\| \cos\theta$, and by Example 2, $\|\vec{v} - \vec{w}\|^2 = \|\vec{v}\|^2 - 2(\vec{v} \cdot \vec{w}) + \|\vec{w}\|^2$.

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$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \, \|\vec{w}\|}$$

To find the angle between two vectors, solve for θ:

$$\theta = \cos^{-1}\left(\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \, \|\vec{w}\|}\right) = \cos^{-1}(\hat{v} \cdot \hat{w})$$

Example 3a

Find the angle between each of the following.

(a) $\vec{v} = \langle 3, -3\sqrt{3} \rangle$ and $\vec{w} = \langle -\sqrt{3}, 1 \rangle$

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$$\vec{v} \cdot \vec{w} = \langle 3, -3\sqrt{3} \rangle \cdot \langle -\sqrt{3}, 1 \rangle$$

$$= -3\sqrt{3} - 3\sqrt{3} = -6\sqrt{3}$$

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$$\|\vec{v}\| = \sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{36}$$

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$$\|\vec{w}\| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4}$$

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$$= -3\sqrt{3} - 3\sqrt{3} = -6\sqrt{3}$$

$$\|\vec{v}\| = \sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{36}$$

$$\|\vec{w}\| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4}$$

$$\theta = \arccos\left(\frac{-6\sqrt{3}}{\sqrt{36 \times 4}} \right) = \frac{5\pi}{6}$$

Example 3b

(b) $\vec{v} = \langle 2, 2 \rangle$ and $\vec{w} = \langle 5, -5 \rangle$

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$$\vec{v} \cdot \vec{w} = 2(5) + 2(-5) = 0$$

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(b) $\vec{v} = \langle 2, 2 \rangle$ and $\vec{w} = \langle 5, -5 \rangle$

$$\vec{v} \cdot \vec{w} = 2(5) + 2(-5) = 0$$

$$\|\vec{v}\| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

Example 3b

(b) $\vec{v} = \langle 2, 2 \rangle$ and $\vec{w} = \langle 5, -5 \rangle$

$$\vec{v} \cdot \vec{w} = 2(5) + 2(-5) = 0$$

$$\|\vec{v}\| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$\|\vec{w}\| = \sqrt{5^2 + 5^2} = \sqrt{50}$$

Example 3b

(b) $\vec{v} = \langle 2, 2 \rangle$ and $\vec{w} = \langle 5, -5 \rangle$

$$\vec{v} \cdot \vec{w} = 2(5) + 2(-5) = 0$$

$$\|\vec{v}\| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$\|\vec{w}\| = \sqrt{5^2 + 5^2} = \sqrt{50}$$

$$\theta = \arccos\left(\frac{0}{\sqrt{8 \times 50}}\right) = \frac{\pi}{2}$$

Example 3c

(c) $\vec{v} = \langle 3, -4 \rangle$ and $\vec{w} = \langle 2, 1 \rangle$

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Example 3c

(c) $\vec{v} = \langle 3, -4 \rangle$ and $\vec{w} = \langle 2, 1 \rangle$

$$\vec{v} \cdot \vec{w} = 3(2) + (-4)(1) = 2$$

$$\|\vec{v}\| = \sqrt{3^2 + 4^2} = \sqrt{25}$$

Example 3c

(c) $\vec{v} = \langle 3, -4 \rangle$ and $\vec{w} = \langle 2, 1 \rangle$

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Example 3c

(c) $\vec{v} = \langle 3, -4 \rangle$ and $\vec{w} = \langle 2, 1 \rangle$

$$\vec{v} \cdot \vec{w} = 3(2) + (-4)(1) = 2$$

$$\|\vec{v}\| = \sqrt{3^2 + 4^2} = \sqrt{25}$$

$$\|\vec{w}\| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\theta = \arccos\left(\frac{2}{\sqrt{25 \times 5}}\right) \approx 80^\circ$$

Table of Contents

Orthogonal Projection

Two vectors are **orthogonal** if they meet at a right angle.

If two vectors are orthogonal, then their dot product is 0.

Orthogonal Projection

The orthogonal projection of $\vec{v}$ onto $\vec{w}$ is a new vector $\vec{p}$ that is parallel to $\vec{w}$ and has a magnitude of $\|\vec{v}\| \cos\theta$.

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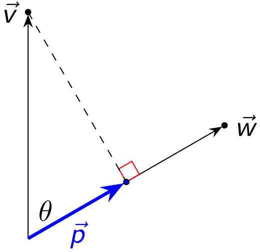
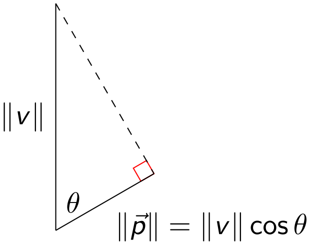
The orthogonal projection of $\vec{v}$ onto $\vec{w}$ is a new vector $\vec{p}$ that is parallel to $\vec{w}$ and has a magnitude of $\|\vec{v}\| \cos\theta$.

$\vec{p}$ can be thought of as the "shadow" $\vec{v}$ casts over $\vec{w}$:





Orthogonal Projection

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Recall that when finding a unit vector for a given vector, that unit vector has a magnitude of 1 and is parallel to the original vector.

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Thus, we can multiply the magnitude of $\vec{p}$ by a unit vector for $\vec{w}$.

$$\|\vec{p}\|\,\hat{w}$$

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However, that would likely change the magnitude of $\vec{p}$.

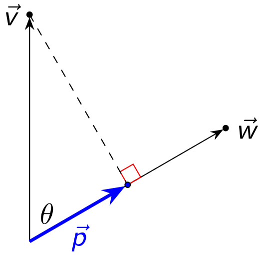
Recall that when finding a unit vector for a given vector, that unit vector has a magnitude of 1 and is parallel to the original vector.

Thus, we can multiply the magnitude of $\vec{p}$ by a unit vector for $\vec{w}$.

$$\|\vec{p}\| \, \hat{w}$$

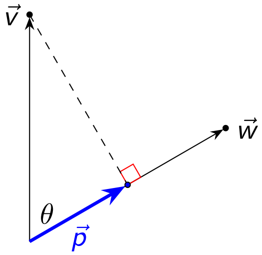
This will guarantee that $\vec{w}$ is scaled to $\vec{p}$ and give us the projection of $\vec{v}$ onto $\vec{w}$.

Orthogonal Projection



Orthogonal Projection

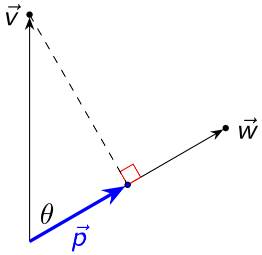
$$\text{proj}_{\vec{w}}\vec{v} = \|\color{blue}{\vec{p}}\| \; \color{red}{\hat{w}}$$



Orthogonal Projection

$$\text{proj}_{\vec{w}}\vec{v} = \|\vec{p}\|\ \hat{w}$$

$$= (\|v\| \cos\theta) \left(\frac{\vec{w}}{\|w\|}\right)$$



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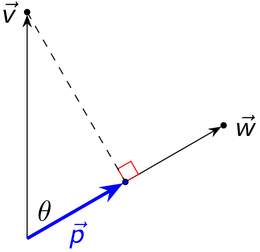
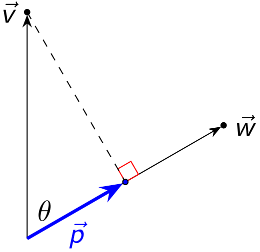


$$\begin{aligned}
\mathrm{proj}_{\vec{w}}\vec{v} &= \|\vec{p}\|\ \hat{w} \\
&= (\|v\|\cos\theta)\left(\frac{\vec{w}}{\|w\|}\right) \\
&= \|v\|\left(\frac{v \cdot w}{\|v\|\ \|w\|}\right)\left(\frac{\vec{w}}{\|w\|}\right)
\end{aligned}$$

Orthogonal Projection



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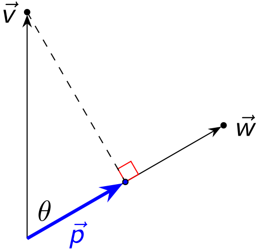
Orthogonal Projection



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$$= \left(\frac{v \cdot w}{\|w\|\ \|w\|}\right)\vec{w}$$

$$\mathrm{proj}_{\vec{w}}\vec{v} = \left(\frac{v \cdot w}{\|w\|^2}\right)\vec{w}$$

Example 4a

Let $\vec{v} = \langle 1, 8 \rangle$ and $\vec{w} = \langle -1, 2 \rangle$. Find each of the following.

(a) $\text{proj}_{\vec{w}} \vec{v}$

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$$\text{proj}_{\vec{w}} \vec{v} = \left(\frac{v \cdot w}{\|w\|^2} \right) \vec{w}$$

$$v \cdot w = 1(-1) + 8(2) = 15$$

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$$\text{proj}_{\vec{w}}\vec{v} = \left(\frac{v \cdot w}{\|w\|^2} \right) \vec{w}$$

$$v \cdot w = 1(-1) + 8(2) = 15 \qquad \|\vec{w}\|^2 = \left(\sqrt{1^2 + 2^2} \right)^2 = 5$$

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$$v \cdot w = 1(-1) + 8(2) = 15 \qquad \|\vec{w}\|^2 = \left(\sqrt{1^2 + 2^2} \right)^2 = 5$$

$$\text{proj}_{\vec{w}} \vec{v} = \frac{15}{5} \langle -1, 2 \rangle$$

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$$\text{proj}_{\vec{w}} \vec{v} = \frac{15}{5} \langle -1, 2 \rangle$$

$$= 3\langle -1, 2 \rangle$$

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Let $\vec{v} = \langle 1, 8 \rangle$ and $\vec{w} = \langle -1, 2 \rangle$. Find each of the following.

(a) $\text{proj}_{\vec{w}}\vec{v}$

$$\text{proj}_{\vec{w}}\vec{v} = \left(\frac{v \cdot w}{\|w\|^2}\right)\vec{w}$$

$$v \cdot w = 1(-1) + 8(2) = 15 \qquad \|\vec{w}\|^2 = \left(\sqrt{1^2 + 2^2}\right)^2 = 5$$

$$\begin{aligned} \text{proj}_{\vec{w}}\vec{v} &= \frac{15}{5}\langle -1, 2 \rangle \\ &= 3\langle -1, 2 \rangle \\ &= \langle -3, 6 \rangle \end{aligned}$$

Example 4b

Let $\vec{v} = \langle 1, 8 \rangle$ and $\vec{w} = \langle -1, 2 \rangle$. Find each of the following.

(b) $\text{proj}_{\vec{v}}\vec{w}$

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Let $\vec{v} = \langle 1, 8 \rangle$ and $\vec{w} = \langle -1, 2 \rangle$. Find each of the following.

(b) $\text{proj}_{\vec{v}}\vec{w}$

$$\text{proj}_{\vec{v}}\vec{w} = \left(\frac{w \cdot v}{\|v\|^2} \right) \vec{v}$$

Example 4b

Let $\vec{v} = \langle 1, 8 \rangle$ and $\vec{w} = \langle -1, 2 \rangle$. Find each of the following.

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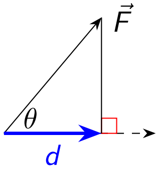

$$W = Fd = \|F\| \, \|d\| \cos\theta = F \cdot d$$

Example 5

Mr. Bain exerts a force of 230 pounds to pull a stack of rocks a distance of 50 ft. over level ground. If the rope makes a 30° angle with the horizontal, how much work did Mr. Bain do pulling the rocks? Assume Mr. Bain exerts the force of 230 pounds at a 30° angle for the duration of the 50 feet.

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