Inverse Trig Functions

Objectives

1 Find the values of inverse trig functions

2 Find the values of the compositions of inverse trig functions

Write an algebraic expression for the compositions of inverse trig functions

Trig Functions and the Horizontal Line Test

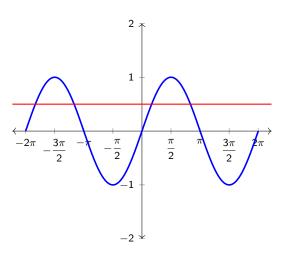
Recall that for a function to have an inverse, any horizontal line can only hit the function <u>at most once</u>.

Trig Functions and the Horizontal Line Test

Recall that for a function to have an inverse, any horizontal line can only hit the function at most once.

This creates a problem with periodic functions such as $y = \sin x$.

Graph of $y = \sin x$ and $y = \frac{1}{2}$

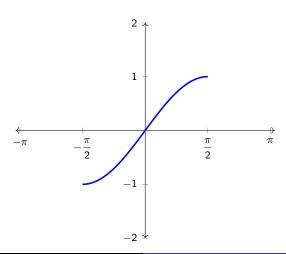


Passing the Horizontal Line Test

If we limit the graph to a smaller domain, then the Horizontal Line Test will demonstrate the function $y = \sin x$ will have an inverse:

Passing the Horizontal Line Test

If we limit the graph to a smaller domain, then the Horizontal Line Test will demonstrate the function $y = \sin x$ will have an inverse:



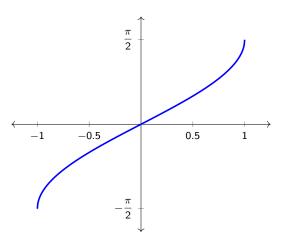
Graph of $y = \sin^{-1} x$

The inverse of
$$y = \sin x$$
 is $y = \sin^{-1} x$ or $y = \arcsin x$

Graph of $y = \sin^{-1} x$

The inverse of $y = \sin x$ is

$$y = \sin^{-1} x$$
 or $y = \arcsin x$



Inverse Function Domain (Input) Range (Output)

Inverse Function	Domain (Input)	Range (Output)
$y = \sin^{-1} x$	[-1, 1]	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Inverse Function	Domain (Input)	Range (Output)
$y = \sin^{-1} x$	[-1,1]	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[-1,1]	$[0,\pi]$

Inverse Function	Domain (Input)	Range (Output)
$y = \sin^{-1} x$	[-1,1]	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[-1,1]	$[0,\pi]$
$y = \tan^{-1} x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

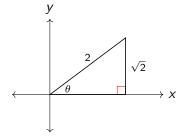
Inverse Function	Domain (Input)	Range (Output)
$y = \sin^{-1} x$	[-1, 1]	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[-1,1]	$[0,\pi]$
$y = \tan^{-1} x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$y = \csc^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-rac{\pi}{2},0 ight)\cup\left(0,rac{\pi}{2} ight]$

Inverse Function	Domain (Input)	Range (Output)
$y = \sin^{-1} x$	[-1, 1]	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[-1, 1]	$[0,\pi]$
$y = \tan^{-1} x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$y = \csc^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$
$y = \sec^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

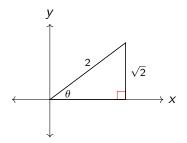
Inverse Function	Domain (Input)	Range (Output)
$y = \sin^{-1} x$	[-1, 1]	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[-1,1]	$[0,\pi]$
$y = \tan^{-1} x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$y = \csc^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$
$y = \sec^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[0,\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2},\pi\right]$
$y = \cot^{-1} x$	$(-\infty,\infty)$	$(0,\pi)$

(a)
$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

(a)
$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

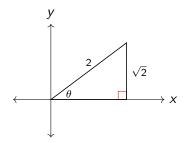


(a)
$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$



$$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

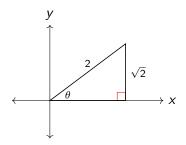
(a)
$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$



$$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\theta = 45^{\circ}$$

(a)
$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$



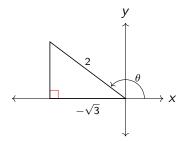
$$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\theta = 45^{\circ}$$

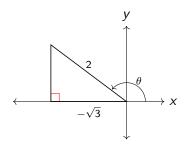
$$\theta = \frac{\pi}{4}$$

(b)
$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

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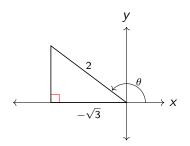


(b)
$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$



$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

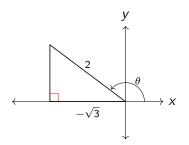
(b)
$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$



$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

$$\theta = -150^{\circ}$$

(b)
$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$



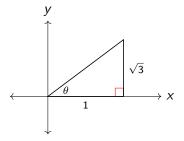
$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

$$\theta = -150^{\circ}$$

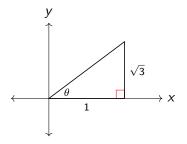
$$\theta = -\frac{5\pi}{6}$$

(c)
$$\arctan\left(\sqrt{3}\right)$$

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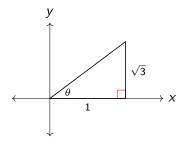


(c)
$$\arctan\left(\sqrt{3}\right)$$



$$\theta = \arctan\left(\sqrt{3}\right)$$

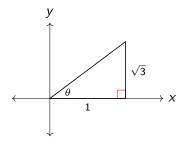
(c)
$$\arctan\left(\sqrt{3}\right)$$



$$\theta = \arctan\left(\sqrt{3}\right)$$

$$\theta = 60^{\circ}$$

(c)
$$\arctan\left(\sqrt{3}\right)$$



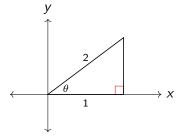
$$\theta = \arctan\left(\sqrt{3}\right)$$

$$\theta = 60^{\circ}$$

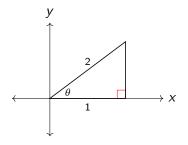
$$\theta = \frac{\pi}{3}$$

(d)
$$\sec^{-1}(2)$$

(d)
$$\sec^{-1}(2)$$

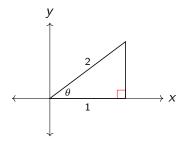


(d)
$$\sec^{-1}(2)$$



$$\theta = \sec^{-1}(2)$$

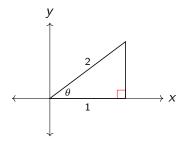
(d)
$$\sec^{-1}(2)$$



$$\theta = \sec^{-1}(2)$$

$$\theta = 60^{\circ}$$

(d)
$$\sec^{-1}(2)$$



$$\theta = \sec^{-1}(2)$$

$$\theta = 60^{\circ}$$

$$\theta = \frac{\pi}{3}$$

Objectives

1 Find the values of inverse trig functions

2 Find the values of the compositions of inverse trig functions

Write an algebraic expression for the compositions of inverse trig functions

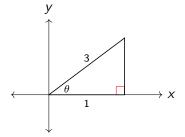
Compositions of Inverse Trig Functions

When dealing with problems like these, it helps to sketch a right triangle, much like in the previous example.

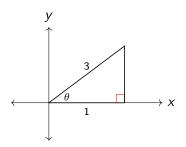
You may need to use the Pythagorean Theorem.

(a)
$$\cos\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$$

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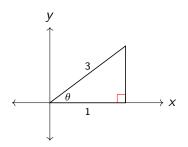


(a)
$$\cos\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$$



$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

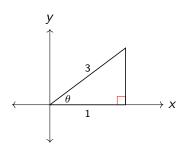
(a)
$$\cos\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$$



$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

$$\cos \theta = \frac{x}{r}$$

(a)
$$\cos\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$$



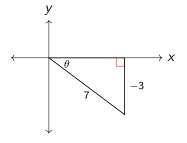
$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

$$\cos\theta = \frac{x}{r}$$

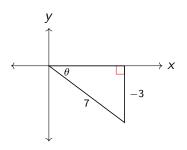
$$\cos \theta = \frac{1}{3}$$

(b)
$$\sin\left(\arcsin\left(-\frac{3}{7}\right)\right)$$

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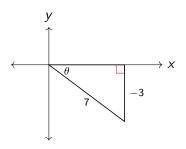


(b)
$$\sin\left(\arcsin\left(-\frac{3}{7}\right)\right)$$



$$\theta = \arcsin\left(-\frac{3}{7}\right)$$

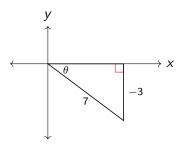
(b)
$$\sin\left(\arcsin\left(-\frac{3}{7}\right)\right)$$



$$\theta = \arcsin\left(-\frac{3}{7}\right)$$

$$\sin\theta = \frac{y}{r}$$

(b)
$$\sin\left(\arcsin\left(-\frac{3}{7}\right)\right)$$



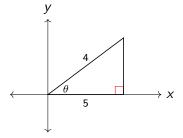
$$\theta = \arcsin\left(-\frac{3}{7}\right)$$

$$\sin\theta = \frac{y}{r}$$

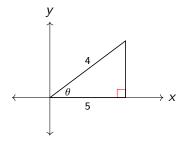
$$\sin heta = -rac{3}{7}$$

(c)
$$\cos\left(\arccos\left(\frac{5}{4}\right)\right)$$

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$$\cos\left(\arccos\left(\frac{5}{4}\right)\right)$$

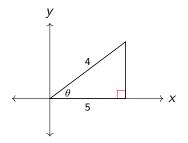


(c)
$$\cos\left(\arccos\left(\frac{5}{4}\right)\right)$$



$$\theta = \arccos\left(\frac{5}{4}\right)$$

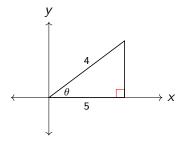
(c)
$$\cos\left(\arccos\left(\frac{5}{4}\right)\right)$$



$$\theta = \arccos\left(\frac{5}{4}\right)$$

Domain error.

(c)
$$\cos\left(\arccos\left(\frac{5}{4}\right)\right)$$



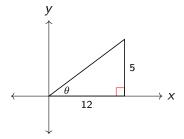
$$\theta = \arccos\left(\frac{5}{4}\right)$$

Domain error.

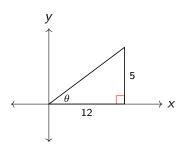
Ø

(d)
$$\cos\left(\arctan\left(\frac{5}{12}\right)\right)$$

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$$\cos\left(\arctan\left(\frac{5}{12}\right)\right)$$

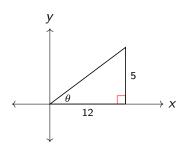


(d)
$$\cos\left(\arctan\left(\frac{5}{12}\right)\right)$$



$$\theta = \arctan\left(\frac{5}{12}\right)$$

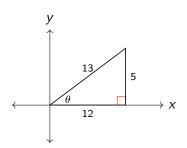
(d)
$$\cos\left(\arctan\left(\frac{5}{12}\right)\right)$$



$$\theta = \arctan\left(\frac{5}{12}\right)$$

$$12^2 + 5^2 = r^2$$

(d)
$$\cos\left(\arctan\left(\frac{5}{12}\right)\right)$$

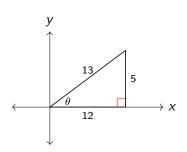


$$\theta = \arctan\left(\frac{5}{12}\right)$$

$$12^2 + 5^2 = r^2$$

$$r = \sqrt{169} = 13$$

(d)
$$\cos\left(\arctan\left(\frac{5}{12}\right)\right)$$



$$heta=\arctan\left(rac{5}{12}
ight)$$

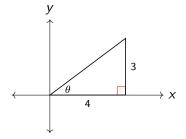
$$12^2+5^2=r^2$$

$$r=\sqrt{169}=13$$

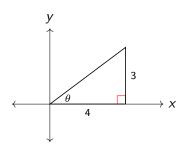
$$\cos \theta = \frac{12}{13}$$

(e) $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$

(e) $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$

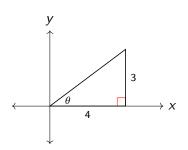


(e) $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$



$$\theta = \arctan\left(\frac{3}{4}\right)$$

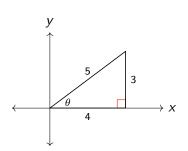
(e)
$$\sin\left(\arctan\left(\frac{3}{4}\right)\right)$$



$$\theta = \arctan\left(\frac{3}{4}\right)$$

$$3^2 + 4^2 = r^2$$

(e)
$$\sin\left(\arctan\left(\frac{3}{4}\right)\right)$$

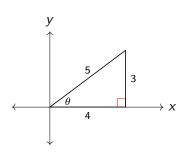


$$heta= \arctan\left(rac{3}{4}
ight)$$

$$3^2 + 4^2 = r^2$$

$$r=\sqrt{25}=5$$

(e)
$$\sin\left(\arctan\left(\frac{3}{4}\right)\right)$$



$$heta=\arctan\left(rac{3}{4}
ight)$$

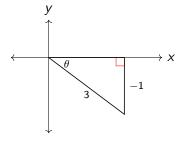
$$3^2 + 4^2 = r^2$$

$$r=\sqrt{25}=5$$

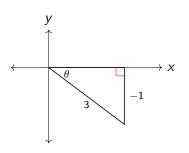
$$\sin\theta = \frac{3}{5}$$

(f)
$$\cot\left(\arcsin\left(-\frac{1}{3}\right)\right)$$

$$(\mathsf{f}) \quad \cot\left(\arcsin\left(-\frac{1}{3}\right)\right)$$

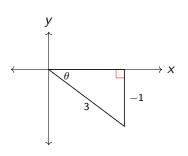


$$\mathsf{(f)} \quad \cot\left(\arcsin\left(-\frac{1}{3}\right)\right)$$



$$\theta = \arcsin\left(-\frac{1}{3}\right)$$

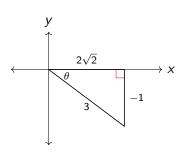
$$\text{(f)} \quad \cot\left(\arcsin\left(-\frac{1}{3}\right)\right)$$



$$\theta = \arcsin\left(-\frac{1}{3}\right)$$

$$x^2 + 1^2 = 3^2$$

$$\text{(f)} \quad \cot\left(\arcsin\left(-\frac{1}{3}\right)\right)$$

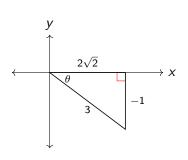


$$\theta = \arcsin\left(-\frac{1}{3}\right)$$

$$x^2 + 1^2 = 3^2$$

$$x = \sqrt{8} = 2\sqrt{2}$$

(f)
$$\cot\left(\arcsin\left(-\frac{1}{3}\right)\right)$$



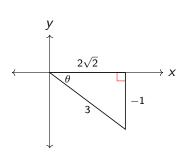
$$\theta = \arcsin\left(-\frac{1}{3}\right)$$

$$x^2 + 1^2 = 3^2$$

$$x = \sqrt{8} = 2\sqrt{2}$$

$$\cot \theta = \frac{x}{y}$$

(f)
$$\cot \left(\arcsin \left(-\frac{1}{3} \right) \right)$$



$$\theta = \arcsin\left(-\frac{1}{3}\right)$$

$$x^2 + 1^2 = 3^2$$

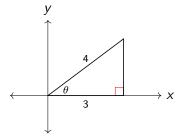
$$x = \sqrt{8} = 2\sqrt{2}$$

$$\cot \theta = \frac{x}{y}$$

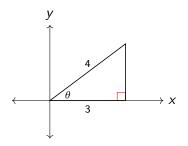
$$\cot \theta = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$$

(g)
$$\sec\left(\arccos\left(\frac{3}{4}\right)\right)$$

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$$\sec\left(\arccos\left(\frac{3}{4}\right)\right)$$

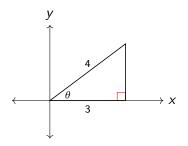


(g)
$$\sec\left(\arccos\left(\frac{3}{4}\right)\right)$$



$$\theta = \arccos\left(\frac{3}{4}\right)$$

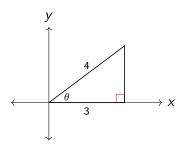
(g)
$$\sec\left(\arccos\left(\frac{3}{4}\right)\right)$$



$$\theta = \arccos\left(\frac{3}{4}\right)$$

$$\sec \theta = \frac{r}{x}$$

(g)
$$\sec\left(\arccos\left(\frac{3}{4}\right)\right)$$



$$\theta = \arccos\left(\frac{3}{4}\right)$$

$$\sec \theta = \frac{r}{x}$$

$$\sec \theta = \frac{4}{3}$$

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2 Find the values of the compositions of inverse trig functions

Write an algebraic expression for the compositions of inverse trig functions

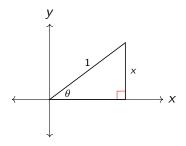
Algebraic Inverse Trig

In calculus, inverse trigonometric functions will be expressed algebraically (i.e. in terms of x).

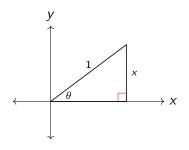
Set up a right triangle and use the Pythagorean Theorem to write the missing side in terms of x.

(a)
$$\cos\left(\sin^{-1}x\right)$$

(a)
$$\cos(\sin^{-1}x)$$

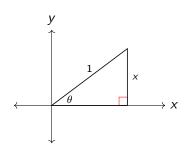


(a)
$$\cos(\sin^{-1}x)$$



$$\theta = \arcsin(x)$$

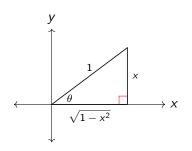
(a)
$$\cos(\sin^{-1}x)$$



$$\theta = \arcsin(x)$$

$$a^2 + x^2 = 1^2$$

(a)
$$\cos(\sin^{-1}x)$$

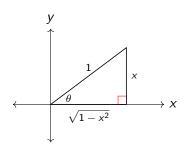


$$\theta = \arcsin(x)$$

$$a^2 + x^2 = 1^2$$

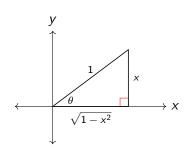
$$a = \sqrt{1 - x^2}$$

(a)
$$\cos(\sin^{-1}x)$$



$$heta = \arcsin(x)$$
 $a^2 + x^2 = 1^2$
 $a = \sqrt{1 - x^2}$
 $\cos \theta = \frac{\text{adj}}{2}$

(a)
$$\cos(\sin^{-1}x)$$



$$\theta = \arcsin(x)$$

$$a^{2} + x^{2} = 1^{2}$$

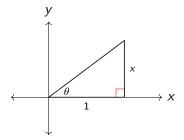
$$a = \sqrt{1 - x^{2}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

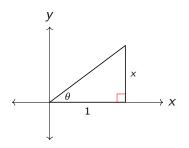
$$\cos \theta = \frac{\sqrt{1 - x^{2}}}{1 - x^{2}}$$

(b) $\sec\left(\tan^{-1}x\right)$

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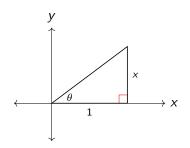


(b)
$$\sec(\tan^{-1}x)$$



$$\theta = \arctan(x)$$

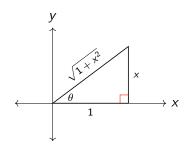
(b)
$$\sec(\tan^{-1}x)$$



$$\theta = \arctan(x)$$

$$1^2 + x^2 = c^2$$

(b)
$$\sec(\tan^{-1}x)$$

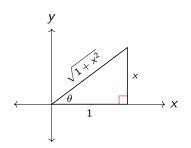


$$\theta = \arctan(x)$$

$$1^2 + x^2 = c^2$$

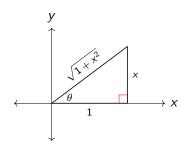
$$c=\sqrt{1+x^2}$$

(b)
$$\sec(\tan^{-1}x)$$



$$heta=\arctan\left(x
ight)$$
 $1^2+x^2=c^2$ $c=\sqrt{1+x^2}$ $\sec heta=rac{\mathsf{hyp}}{\mathsf{adj}}$

(b)
$$\sec(\tan^{-1}x)$$



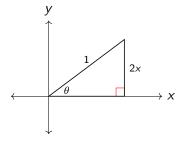
$$\theta = \arctan(x)$$

$$1^2 + x^2 = c^2$$

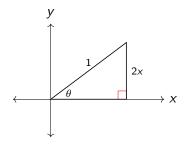
$$c=\sqrt{1+x^2}$$

$$\sec \theta = \frac{\mathsf{hyp}}{\mathsf{adj}}$$

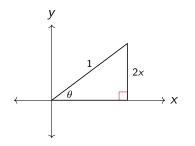
$$\sec \theta = \frac{\sqrt{1+x^2}}{1} = \sqrt{1+x^2}$$



(c)
$$\tan \left(\sin^{-1}(2x)\right)$$

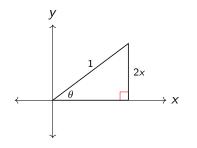


$$\theta = \arcsin(2x)$$



$$\theta = \arcsin(2x)$$

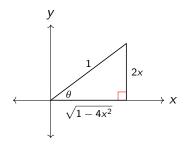
$$a^2 + (2x)^2 = 1^2$$



$$\theta = \arcsin(2x)$$

$$a^2 + (2x)^2 = 1^2$$

$$a^2 + 4x^2 = 1$$

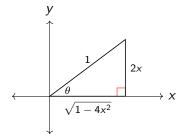


$$\theta = \arcsin(2x)$$

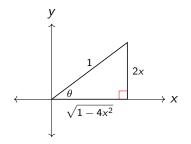
$$a^2 + (2x)^2 = 1^2$$

$$a^2 + 4x^2 = 1$$

$$a=\sqrt{1-4x^2}$$

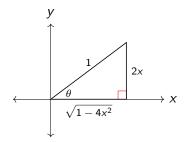


(c)
$$\tan\left(\sin^{-1}(2x)\right)$$



$$\tan \theta = \frac{\mathsf{opp}}{\mathsf{adj}}$$

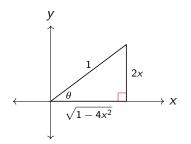
(c)
$$\tan (\sin^{-1}(2x))$$



$$\tan \theta = \frac{\mathsf{opp}}{\mathsf{adj}}$$

$$\tan \theta = \frac{2x}{\sqrt{1 - 4x^2}}$$

(c)
$$\tan (\sin^{-1}(2x))$$



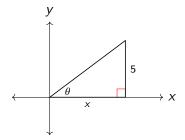
$$\tan \theta = \frac{\mathsf{opp}}{\mathsf{adj}}$$

$$\tan\theta = \frac{2x}{\sqrt{1 - 4x^2}}$$

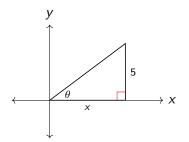
$$\tan\theta = \frac{2x\sqrt{1-4x^2}}{1-4x^2}$$

$$\mathsf{(d)} \quad \sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$

(d)
$$\sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$

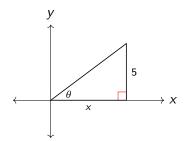


(d)
$$\sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$



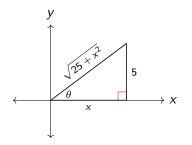
$$\theta = \cot^{-1}\left(\frac{x}{5}\right)$$

(d)
$$\sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$



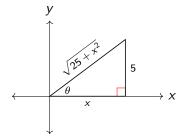
$$\theta = \cot^{-1}\left(\frac{x}{5}\right)$$
$$5^2 + x^2 = c^2$$

(d)
$$\sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$

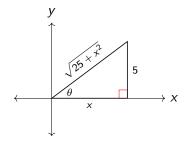


$$\theta = \cot^{-1}\left(\frac{x}{5}\right)$$
$$5^{2} + x^{2} = c^{2}$$
$$c = \sqrt{25 + x^{2}}$$

(d)
$$\sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$

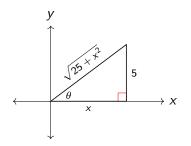


(d)
$$\sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$



$$\sec heta = rac{\mathsf{hyp}}{\mathsf{adj}}$$

(d)
$$\sin\left(\cot^{-1}\left(\frac{x}{5}\right)\right)$$



$$\sec \theta = \frac{\mathsf{hyp}}{\mathsf{adj}}$$

$$\sec \theta = \frac{\sqrt{25 + x^2}}{x}$$