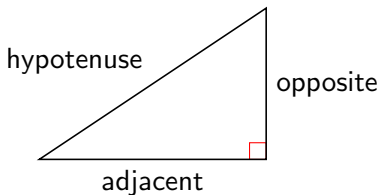


# Trig Functions of Any Angle

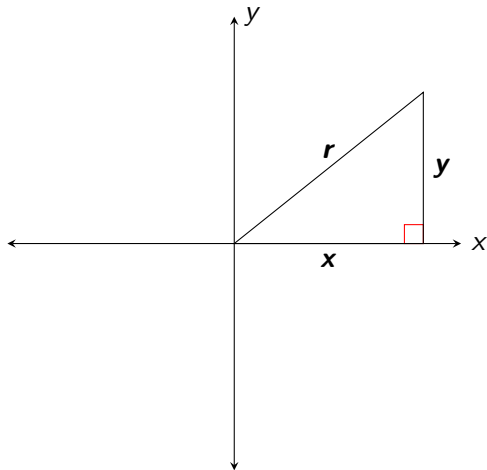
# From Right Triangle Trig to the Coordinate Plane

In this section, adjacent, opposite and hypotenuse



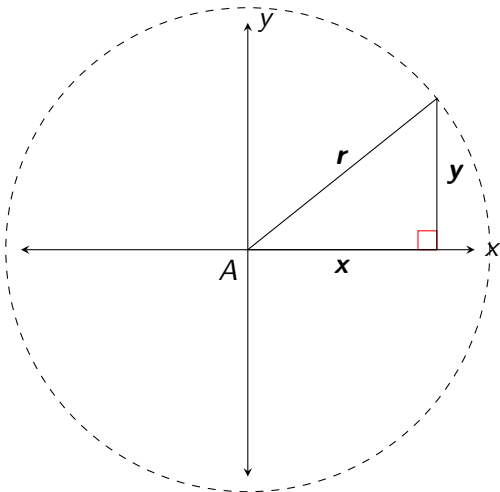
# From Right Triangle Trig to the Coordinate Plane

become



# From Right Triangle Trig to the Coordinate Plane

where  $r$  represents the radius of the circle shown below.



# From Right Triangle Trig to the Coordinate Plane

The biggest difference between this section and the last is that in this section, trig ratios can be positive, negative, zero, or undefined, since  $x$  and  $y$  can each be positive, negative, or zero.

# Objectives

- 1 Calculate the 6 Trig Ratios for a Point in the Coordinate Plane
- 2 Find the Exact Values of the Trig Ratios of Special Angles in the Coordinate Plane

# The 6 Trig Ratios

In the coordinate plane, the six trig ratios become

$$\sin A = \frac{y}{r}$$

$$\cos A = \frac{x}{r}$$

$$\tan A = \frac{y}{x}$$

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$$\begin{array}{l|l} \sin A = \frac{y}{r} & \csc A = \frac{r}{y} \\ \cos A = \frac{x}{r} & \sec A = \frac{r}{x} \\ \tan A = \frac{y}{x} & \cot A = \frac{x}{y} \end{array}$$



## Example 1a

For the given point, find the exact values of the 6 trig functions for the angle drawn in standard form.

(a)  $(3, -4)$

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## Example 1b

$$(b) \quad (-8, -15)$$

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## Example 1c

(c)  $(-1, 5)$

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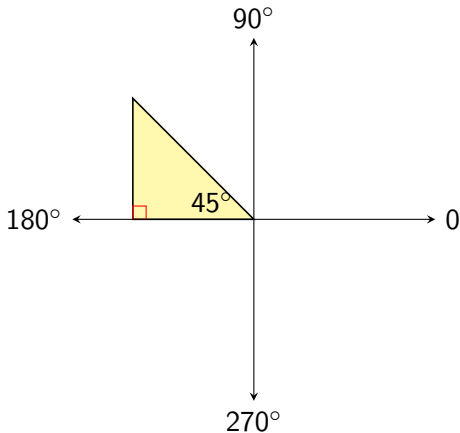
$$\tan A = \frac{5}{-1} = -5$$

$$\cot A = \frac{-1}{5}$$

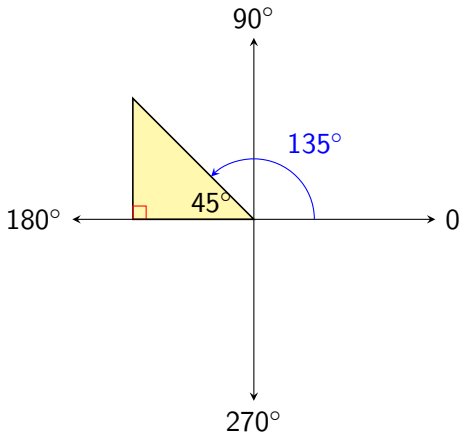
# Objectives

- 1 Calculate the 6 Trig Ratios for a Point in the Coordinate Plane
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If we put a 45-45-90 triangle in the second quadrant of the coordinate plane, it would resemble the following:

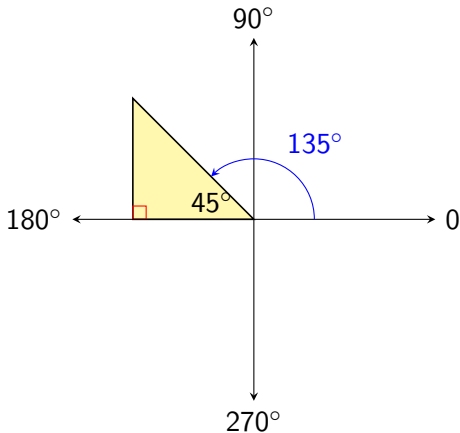


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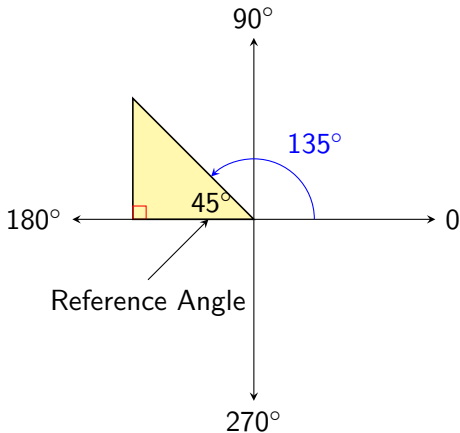
The hypotenuse,  $r$ , would have rotated  $180^\circ - 45^\circ = 135^\circ$ .

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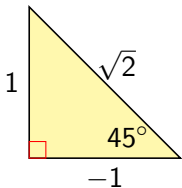
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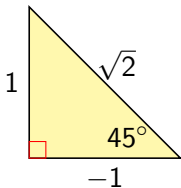
The hypotenuse,  $r$ , would have rotated  $180^\circ - 45^\circ = 135^\circ$ .



In the second quadrant,  $x$ -coordinates are negative and  $y$ -coordinates are positive ( $r$  is always positive). Thus, the values would be



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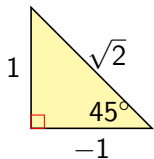


So  $\sin 135^\circ = \frac{\sqrt{2}}{2}$ ,  $\cos 135^\circ = -\frac{\sqrt{2}}{2}$ , and  $\tan 135^\circ = -1$ .

## Example 2a

Find the exact values of the 6 trig functions for each of the following.

(a)  $135^\circ = \frac{3\pi}{4}$



$$\sin 135^\circ = \frac{\sqrt{2}}{2}$$

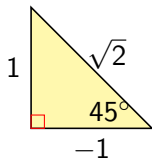
$$\cos 135^\circ = -\frac{\sqrt{2}}{2}$$

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$$\csc 135^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

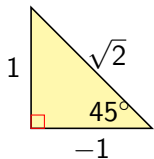
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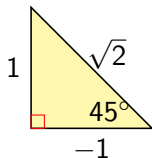
$$\sec 135^\circ = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

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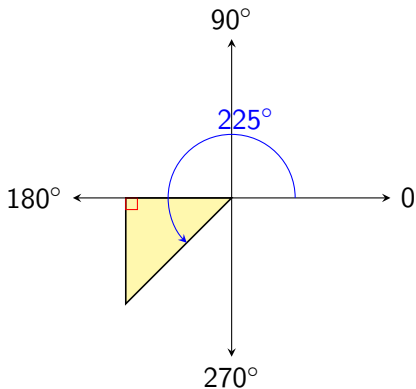
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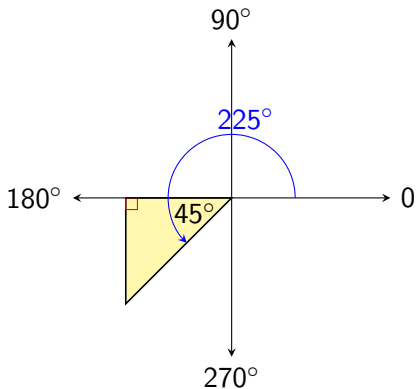
## Example 2b

(b)  $225^\circ = \frac{5\pi}{4}$



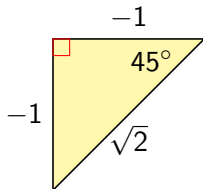
## Example 2b

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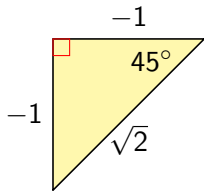




## Example 2b

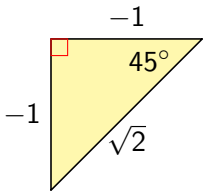


## Example 2b



$$\sin 225^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

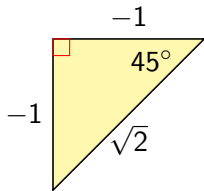
## Example 2b



$$\sin 225^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos 225^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

## Example 2b

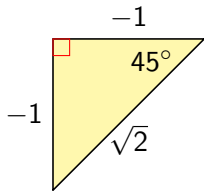


$$\sin 225^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos 225^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\tan 225^\circ = \frac{-1}{-1} = 1$$

## Example 2b



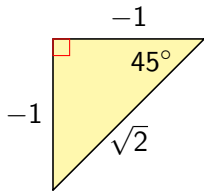
$$\sin 225^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\csc 225^\circ = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

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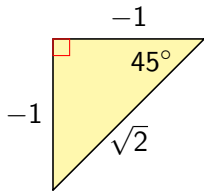
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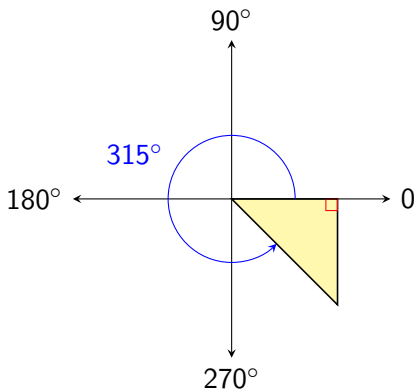
$$\sec 225^\circ = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

$$\tan 225^\circ = \frac{-1}{-1} = 1$$

$$\cot 225^\circ = \frac{-1}{-1} = 1$$

## Example 2c

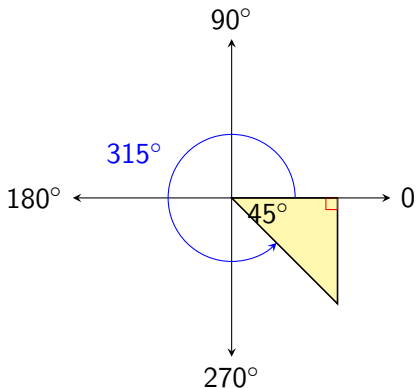
(c)  $315^\circ = \frac{7\pi}{4}$



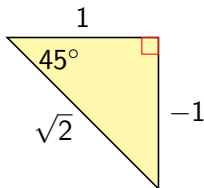


## Example 2c

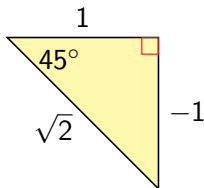
(c)  $315^\circ = \frac{7\pi}{4}$



## Example 2c

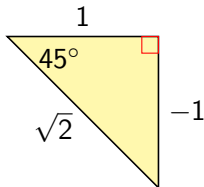


## Example 2c



$$\sin 315^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

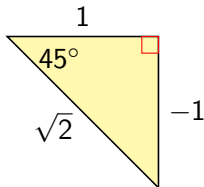
## Example 2c



$$\sin 315^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos 315^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

## Example 2c

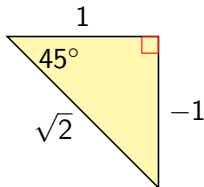


$$\sin 315^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos 315^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 315^\circ = \frac{-1}{1} = -1$$

## Example 2c



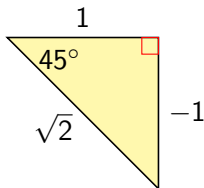
$$\sin 315^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

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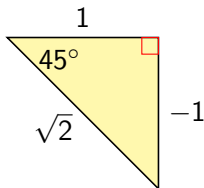
$$\csc 315^\circ = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

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## Example 2c



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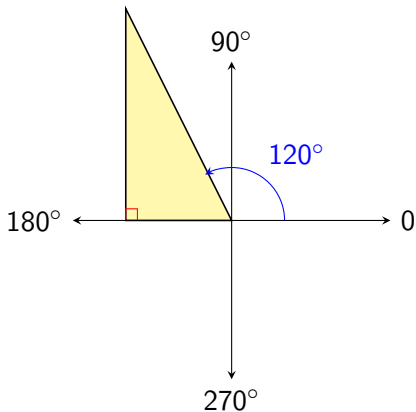
$$\tan 315^\circ = \frac{-1}{1} = -1$$

$$\cot 315^\circ = \frac{1}{-1} = -1$$



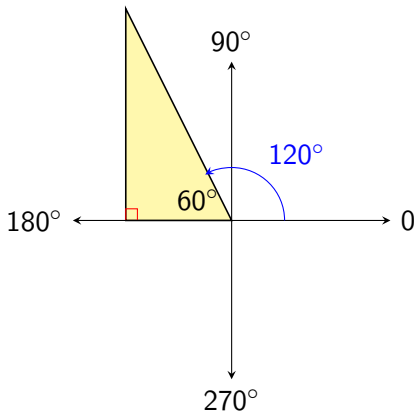
## Example 2d

(d)  $120^\circ = \frac{2\pi}{3}$

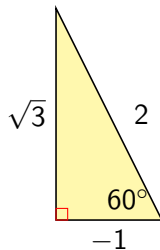


## Example 2d

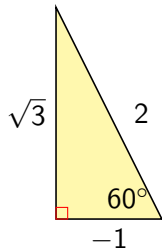
(d)  $120^\circ = \frac{2\pi}{3}$



## Example 2d

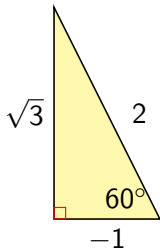


## Example 2d



$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

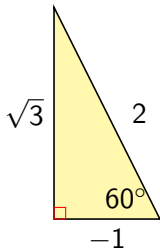
## Example 2d



$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 120^\circ = \frac{-1}{2}$$

## Example 2d

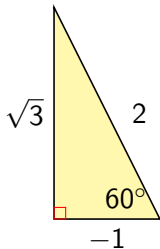


$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 120^\circ = \frac{-1}{2}$$

$$\tan 120^\circ = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

## Example 2d



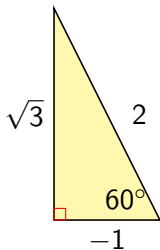
$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

$$\csc 120^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos 120^\circ = \frac{-1}{2}$$

$$\tan 120^\circ = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

## Example 2d



$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

$$\csc 120^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

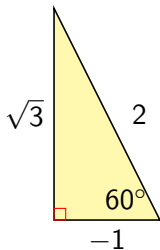
$$\cos 120^\circ = \frac{-1}{2}$$

$$\sec 120^\circ = \frac{2}{-1} = -2$$

$$\tan 120^\circ = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$



## Example 2d



$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 120^\circ = \frac{-1}{2}$$

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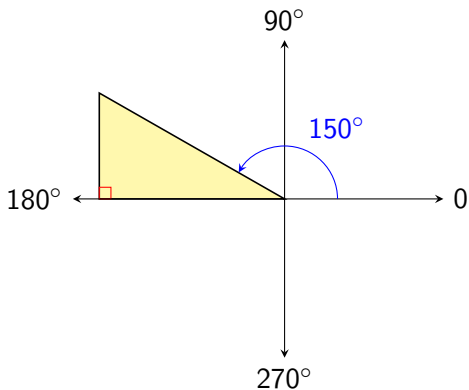
$$\csc 120^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec 120^\circ = \frac{2}{-1} = -2$$

$$\cot 120^\circ = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

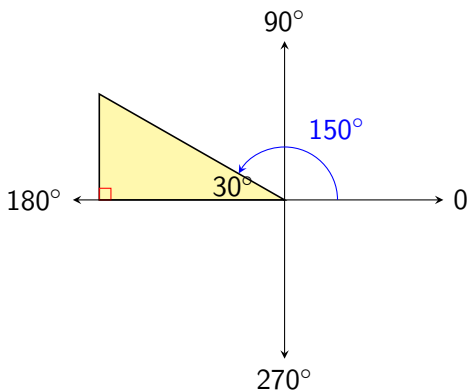
## Example 2e

(e)  $150^\circ = \frac{5\pi}{6}$

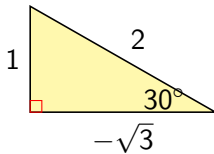


## Example 2e

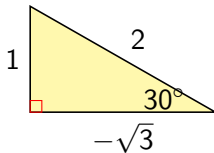
(e)  $150^\circ = \frac{5\pi}{6}$



## Example 2e

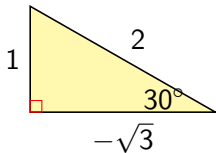


## Example 2e



$$\sin 150^\circ = \frac{1}{2}$$

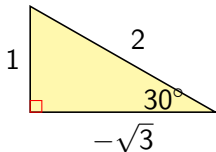
## Example 2e



$$\sin 150^\circ = \frac{1}{2}$$

$$\cos 150^\circ = \frac{-\sqrt{3}}{2}$$

## Example 2e

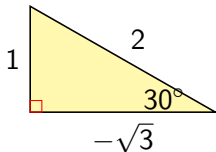


$$\sin 150^\circ = \frac{1}{2}$$

$$\cos 150^\circ = \frac{-\sqrt{3}}{2}$$

$$\tan 150^\circ = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

## Example 2e



$$\sin 150^\circ = \frac{1}{2}$$

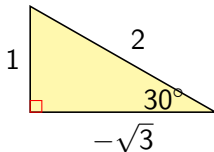
$$\csc 150^\circ = \frac{2}{1} = 2$$

$$\cos 150^\circ = \frac{-\sqrt{3}}{2}$$

$$\tan 150^\circ = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$



## Example 2e



$$\sin 150^\circ = \frac{1}{2}$$

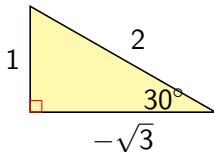
$$\csc 150^\circ = \frac{2}{1} = 2$$

$$\cos 150^\circ = \frac{-\sqrt{3}}{2}$$

$$\sec 150^\circ = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

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## Example 2e



$$\sin 150^\circ = \frac{1}{2}$$

$$\csc 150^\circ = \frac{2}{1} = 2$$

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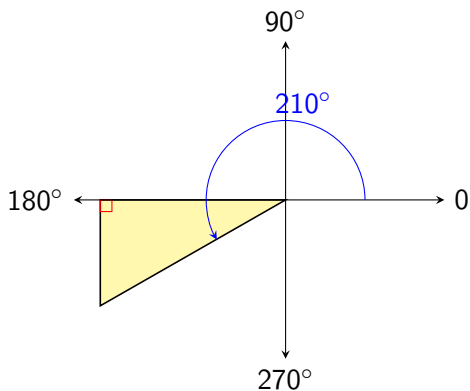
$$\sec 150^\circ = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\tan 150^\circ = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\cot 150^\circ = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

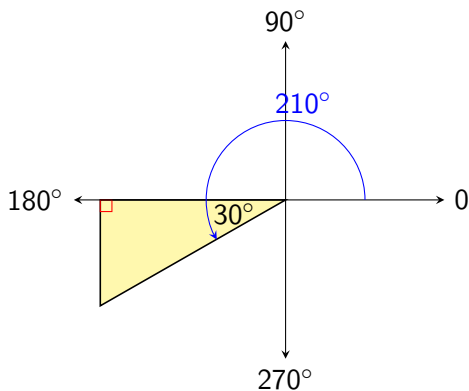
## Example 2f

(a)  $210^\circ = \frac{7\pi}{6}$

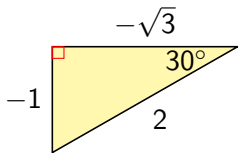


## Example 2f

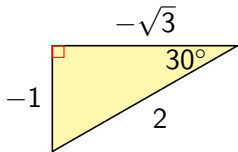
(a)  $210^\circ = \frac{7\pi}{6}$



## Example 2f

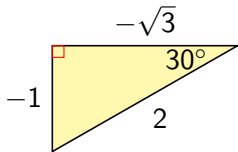


## Example 2f



$$\sin 210^\circ = \frac{-1}{2}$$

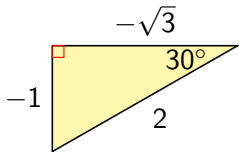
## Example 2f



$$\sin 210^\circ = \frac{-1}{2}$$

$$\cos 210^\circ = \frac{-\sqrt{3}}{2}$$

## Example 2f



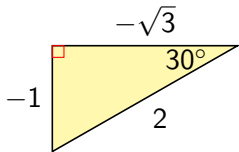
$$\sin 210^\circ = \frac{-1}{2}$$

$$\cos 210^\circ = \frac{-\sqrt{3}}{2}$$

$$\tan 210^\circ = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$$



## Example 2f



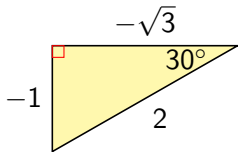
$$\sin 210^\circ = \frac{-1}{2}$$

$$\csc 210^\circ = \frac{2}{-1} = -2$$

$$\cos 210^\circ = \frac{-\sqrt{3}}{2}$$

$$\tan 210^\circ = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$$

## Example 2f



$$\sin 210^\circ = \frac{-1}{2}$$

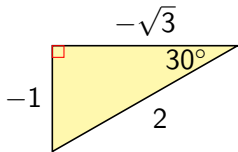
$$\csc 210^\circ = \frac{2}{-1} = -2$$

$$\cos 210^\circ = \frac{-\sqrt{3}}{2}$$

$$\sec 210^\circ = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

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## Example 2f



$$\sin 210^\circ = \frac{-1}{2}$$

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$$\cot 210^\circ = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

# Quadrantal Angles

A **quadrantal angle** is an angle whose terminal side lies on an axis.

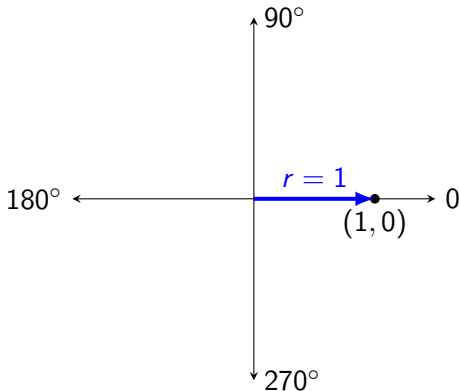
# Quadrantal Angles

A **quadrantal angle** is an angle whose terminal side lies on an axis.

Finding the values of the trig functions for quadrantal angles can be found by using any point on the axes.

# Quadrantal Angles

For simplicity, we will use combinations of 0s and 1s. Below is the point we will use for  $0^\circ$ .



# Quadrantal Angles

So, for  $0^\circ$ , we have  $x = 1$ ,  $y = 0$ , and  $r = 1$ . Thus

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# Quadrantal Angles

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$$\sin 0 = \frac{0}{1} = 0$$

$$\csc 0 = \frac{1}{0} = \text{undefined}$$

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# Quadrantal Angles

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$$\sec 0 = \frac{1}{1} = 1$$

$$\tan 0 = \frac{0}{1} = 0$$

$$\cot 0 = \frac{1}{0} = \text{undefined}$$

## Example 3a

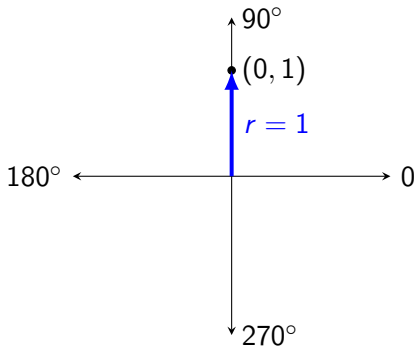
Find the exact values of each of the 6 trig functions for the following angles.

(a)  $90^\circ = \frac{\pi}{2}$

## Example 3a

Find the exact values of each of the 6 trig functions for the following angles.

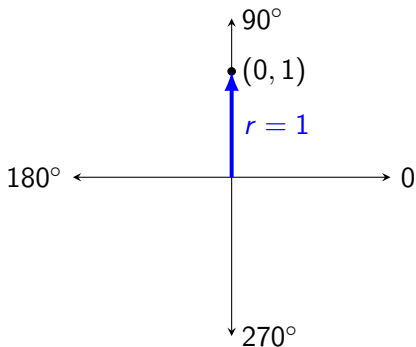
(a)  $90^\circ = \frac{\pi}{2}$



## Example 3a

Find the exact values of each of the 6 trig functions for the following angles.

(a)  $90^\circ = \frac{\pi}{2}$



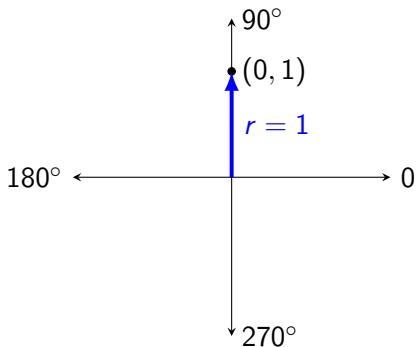
$$\sin 90^\circ = \frac{1}{1} = 1$$



## Example 3a

Find the exact values of each of the 6 trig functions for the following angles.

(a)  $90^\circ = \frac{\pi}{2}$



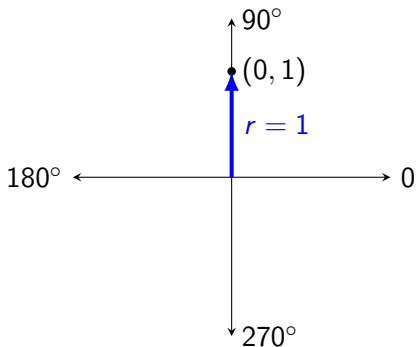
$$\sin 90^\circ = \frac{1}{1} = 1$$

$$\cos 90^\circ = \frac{0}{1} = 0$$

## Example 3a

Find the exact values of each of the 6 trig functions for the following angles.

(a)  $90^\circ = \frac{\pi}{2}$

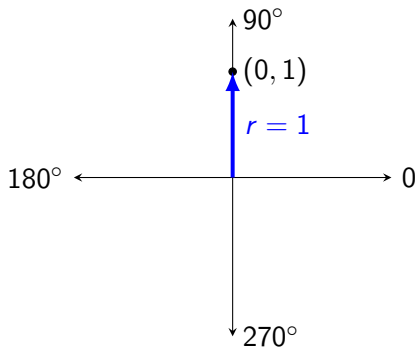


$$\sin 90^\circ = \frac{1}{1} = 1$$

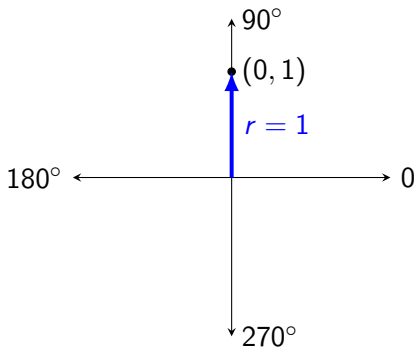
$$\cos 90^\circ = \frac{0}{1} = 0$$

$$\tan 90^\circ = \frac{1}{0} = \text{undefined}$$

## Example 3a

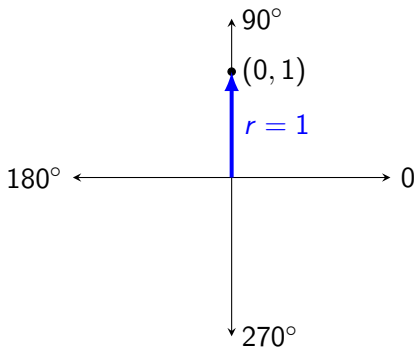


## Example 3a



$$\csc 90^\circ = \frac{1}{1} = 1$$

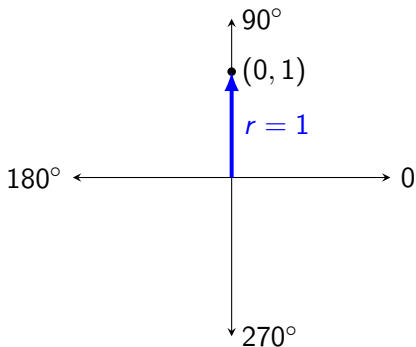
## Example 3a



$$\csc 90^\circ = \frac{1}{1} = 1$$

$$\sec 90^\circ = \frac{1}{0} = \text{undefined}$$

## Example 3a



$$\csc 90^\circ = \frac{1}{1} = 1$$

$$\sec 90^\circ = \frac{1}{0} = \text{undefined}$$

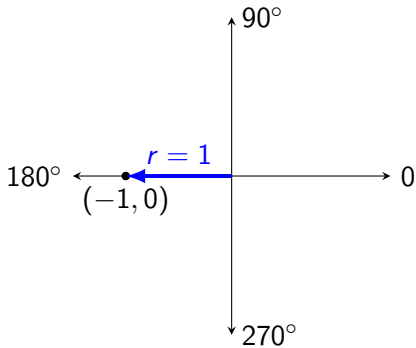
$$\cot 90^\circ = \frac{0}{1} = 0$$

## Example 3b

$$(b) \quad 180^\circ = \pi$$

## Example 3b

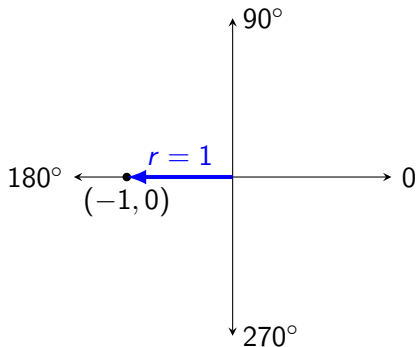
(b)  $180^\circ = \pi$





## Example 3b

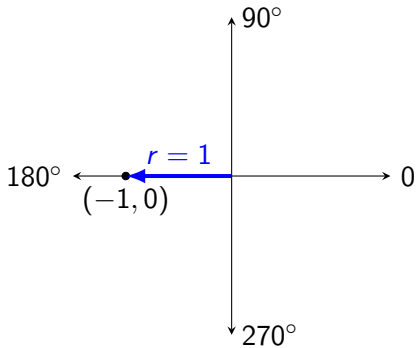
(b)  $180^\circ = \pi$



$$\sin 180^\circ = \frac{0}{1} = 0$$

## Example 3b

(b)  $180^\circ = \pi$

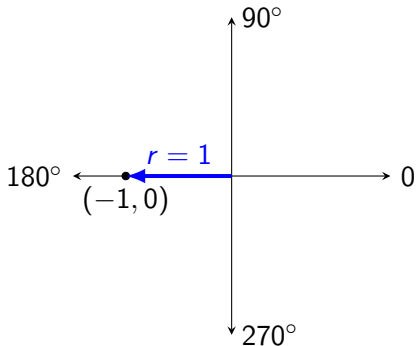


$$\sin 180^\circ = \frac{0}{1} = 0$$

$$\cos 180^\circ = \frac{-1}{1} = -1$$

## Example 3b

(b)  $180^\circ = \pi$

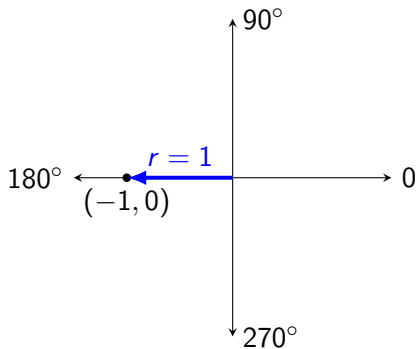


$$\sin 180^\circ = \frac{0}{1} = 0$$

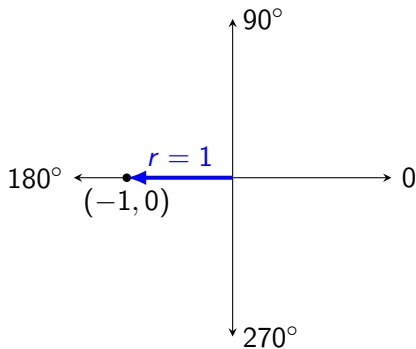
$$\cos 180^\circ = \frac{-1}{1} = -1$$

$$\tan 180^\circ = \frac{0}{-1} = 0$$

## Example 3b

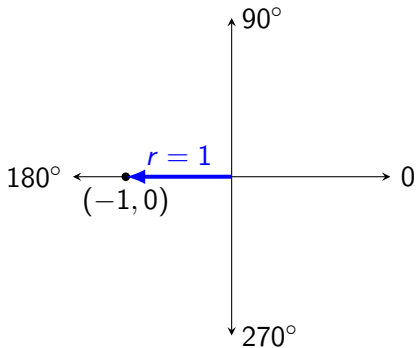


## Example 3b



$$\csc 180^\circ = \frac{1}{0} = \text{undefined}$$

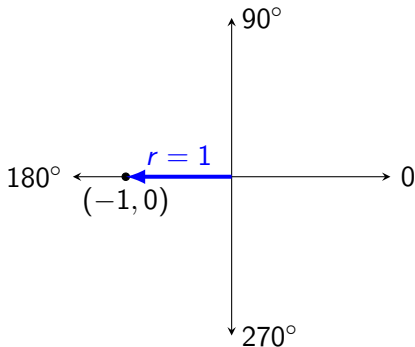
## Example 3b



$$\csc 180^\circ = \frac{1}{0} = \text{undefined}$$

$$\sec 180^\circ = \frac{1}{-1} = -1$$

## Example 3b



$$\csc 180^\circ = \frac{1}{0} = \text{undefined}$$

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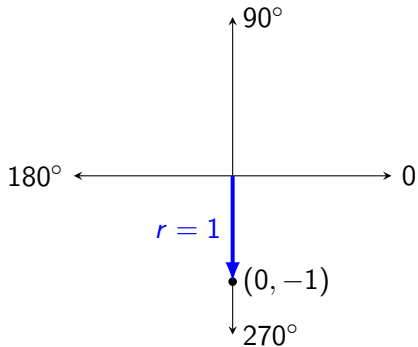
## Example 3c

$$(c) \quad 270^\circ = \frac{3\pi}{2}$$



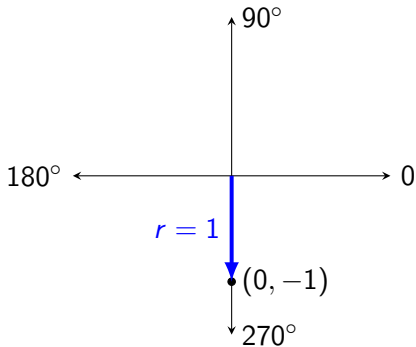
## Example 3c

(c)  $270^\circ = \frac{3\pi}{2}$



## Example 3c

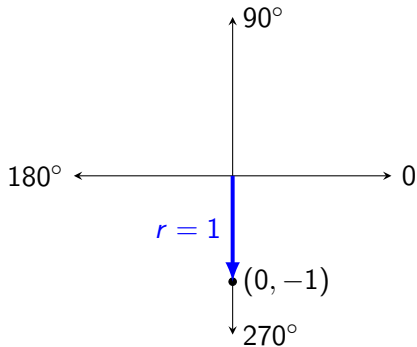
(c)  $270^\circ = \frac{3\pi}{2}$



$$\sin 270^\circ = \frac{-1}{1} = -1$$

## Example 3c

(c)  $270^\circ = \frac{3\pi}{2}$

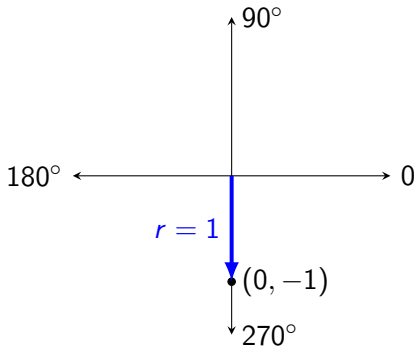


$$\sin 270^\circ = \frac{-1}{1} = -1$$

$$\cos 270^\circ = \frac{0}{1} = 0$$

## Example 3c

(c)  $270^\circ = \frac{3\pi}{2}$

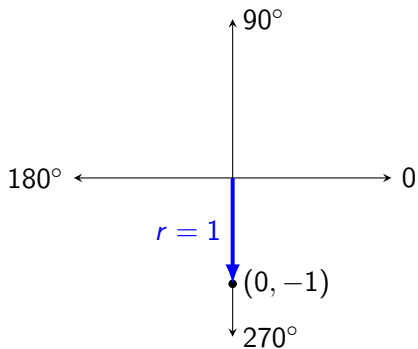


$$\sin 270^\circ = \frac{-1}{1} = -1$$

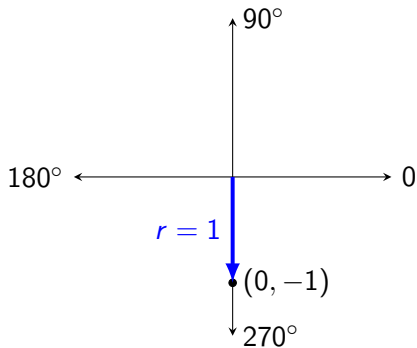
$$\cos 270^\circ = \frac{0}{1} = 0$$

$$\tan 270^\circ = \frac{-1}{0} = \text{undefined}$$

## Example 3c

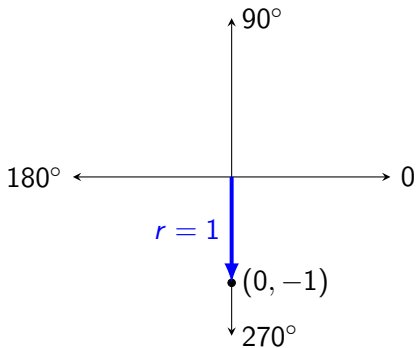


## Example 3c



$$\csc 270^\circ = \frac{1}{-1} = -1$$

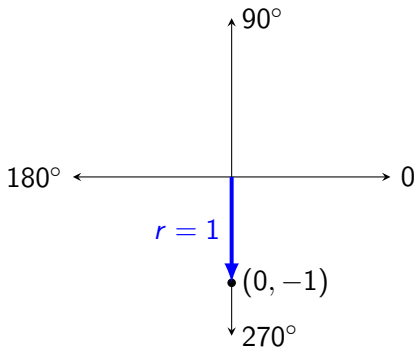
## Example 3c



$$\csc 270^\circ = \frac{1}{-1} = -1$$

$$\sec 270^\circ = \frac{1}{0} = \text{undefined}$$

## Example 3c



$$\csc 270^\circ = \frac{1}{-1} = -1$$

$$\sec 270^\circ = \frac{1}{0} = \text{undefined}$$

$$\cot 270^\circ = \frac{0}{-1} = 0$$



## Angles Not Between $0$ and $360^\circ$ or $0$ and $2\pi$

For angles not within 1 standard rotation, use **coterminal angles** to bring the angle within 1 rotation and then apply the rules of the previous notes.

For instance,  $\cos\left(\frac{9\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$