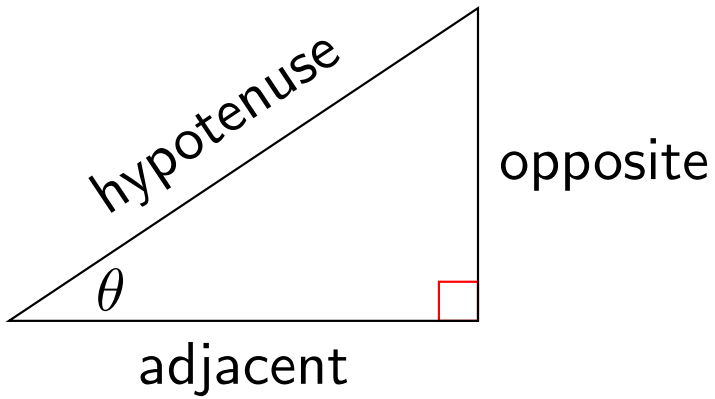
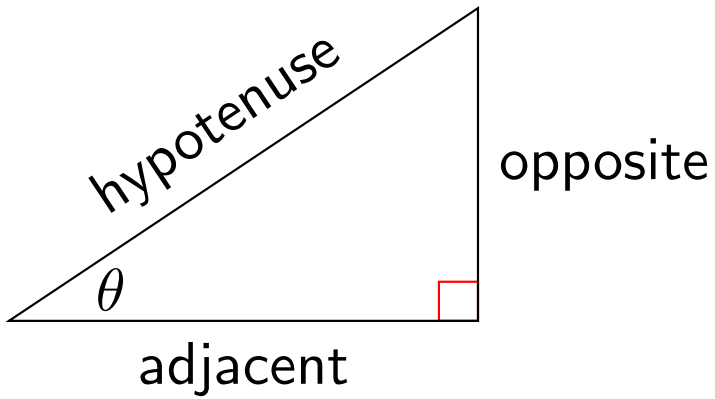
# Right Triangle Trigonometry

# Table of Contents

# The 3 Main Trig Ratios

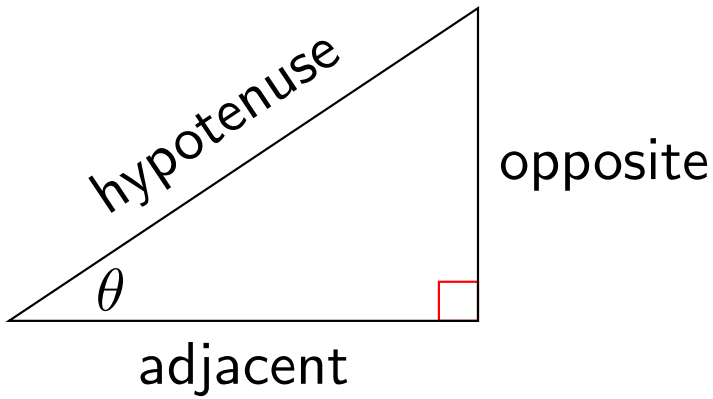


# The 3 Main Trig Ratios



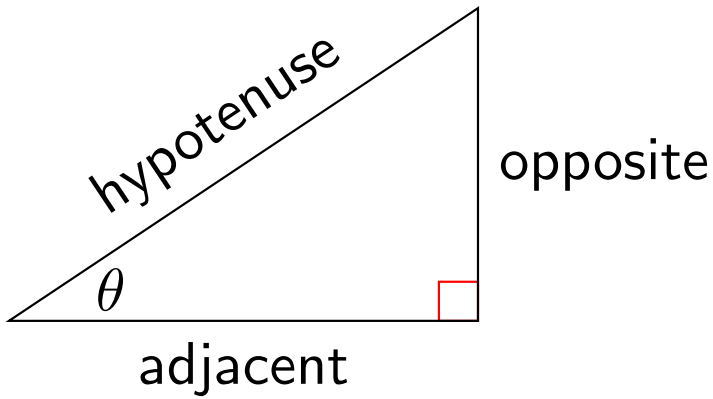

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

# The 3 Main Trig Ratios



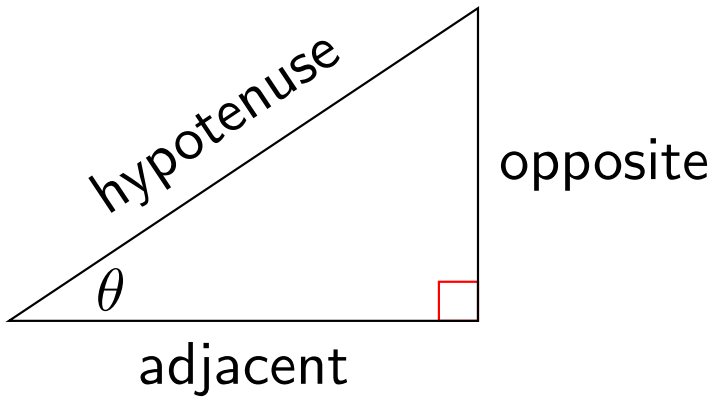

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

# The 3 Main Trig Ratios




$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

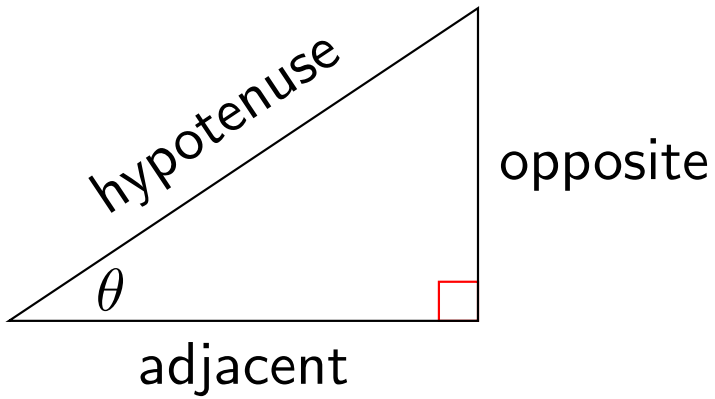
# The 3 Main Trig Ratios



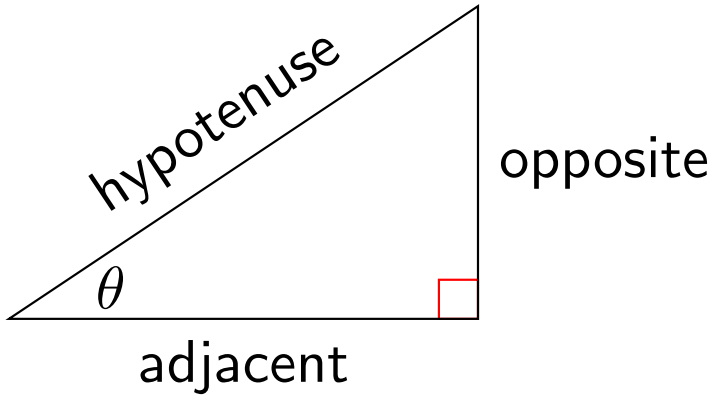

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

We usually remember this as SOH-CAH-TOA.

# Reciprocals for SOH-CAH-TOA

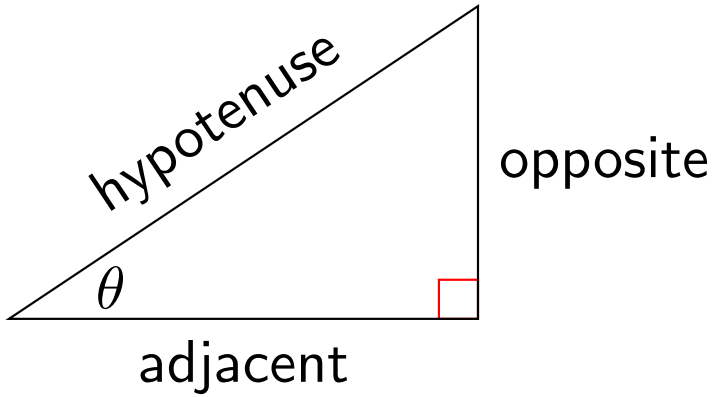


# Reciprocals for SOH-CAH-TOA



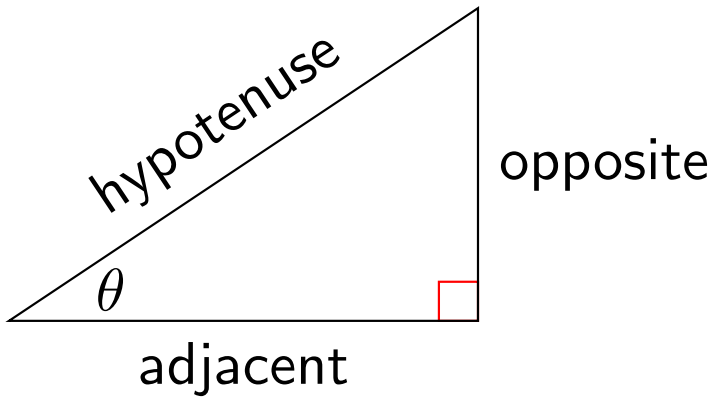

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

# Reciprocals for SOH-CAH-TOA



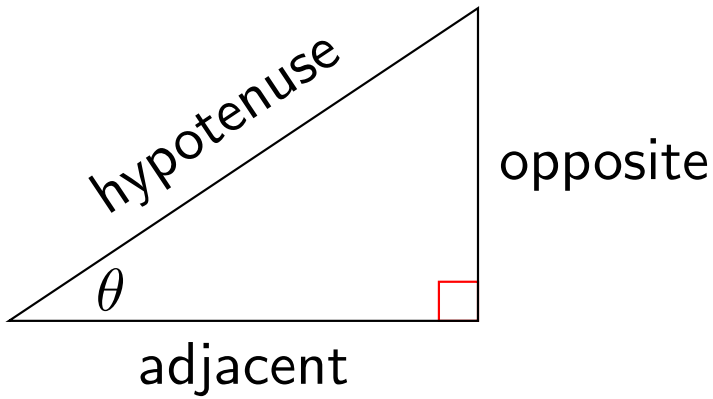

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

# Reciprocals for SOH-CAH-TOA




$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

# Reciprocals for SOH-CAH-TOA



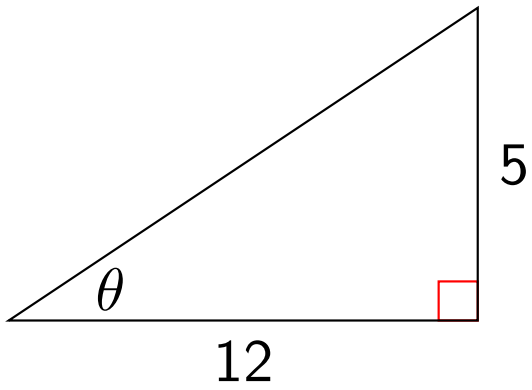

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Sometimes you may need to use the Pythagorean Theorem, $a^2 + b^2 = c^2$, in order to find any missing sides.

# Example 1

Find the value of each of the six trig functions of $\theta$.

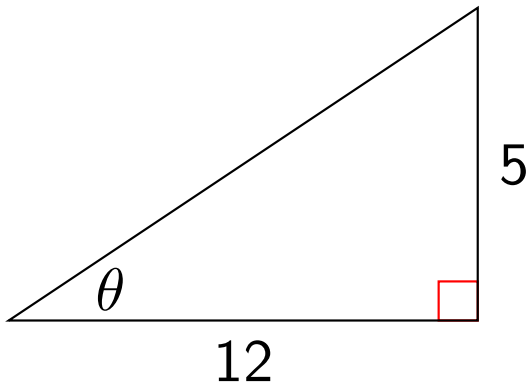
(a)



# Example 1

Find the value of each of the six trig functions of $\theta$.

(a)

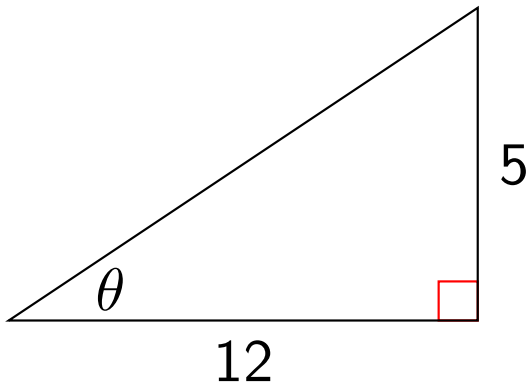



$5^2 + 12^2 = c^2$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(a)



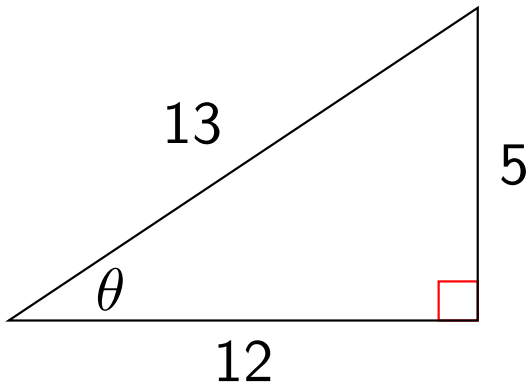

$$5^2 + 12^2 = c^2$$

$$c = \sqrt{169} = 13$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(a)



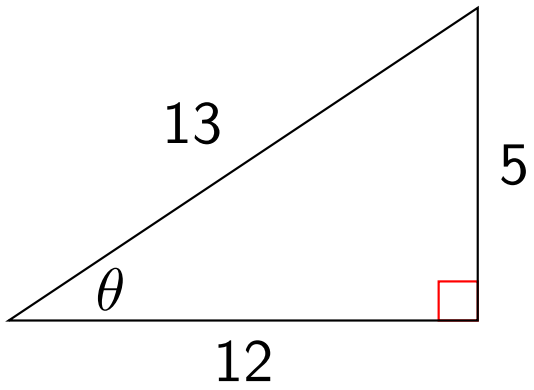

$$5^2 + 12^2 = c^2$$

$$c = \sqrt{169} = 13$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(a)




$$\sin\theta = \frac{5}{13}$$

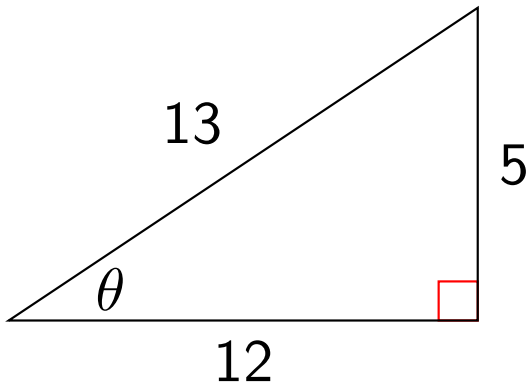
$$5^2 + 12^2 = c^2$$

$$c = \sqrt{169} = 13$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(a)




$$\sin\theta = \frac{5}{13}$$

$$\cos\theta = \frac{12}{13}$$

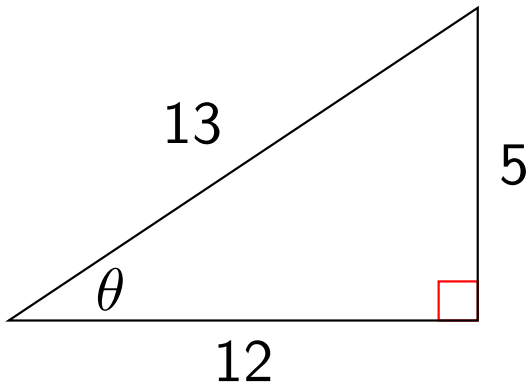
$$5^2 + 12^2 = c^2$$

$$c = \sqrt{169} = 13$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(a)




$$\sin\theta = \frac{5}{13}$$

$$\cos\theta = \frac{12}{13}$$

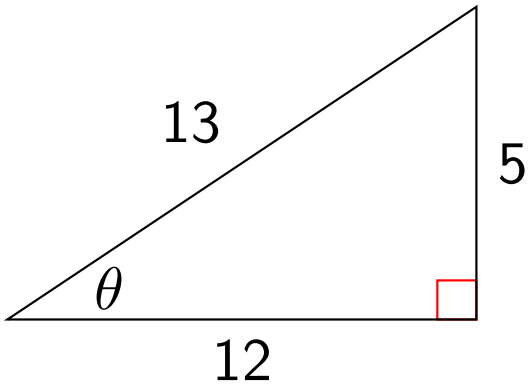
$$5^2 + 12^2 = c^2$$

$$\tan\theta = \frac{5}{12}$$

$$c = \sqrt{169} = 13$$

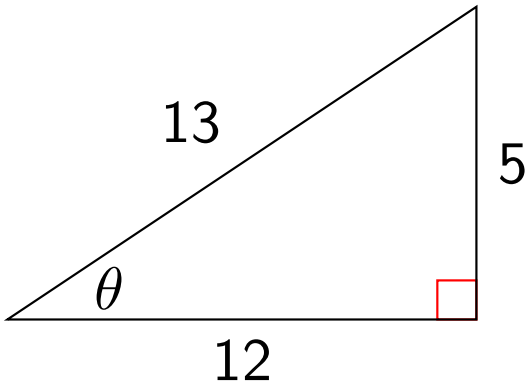
# Example 1

(a)



# Example 1

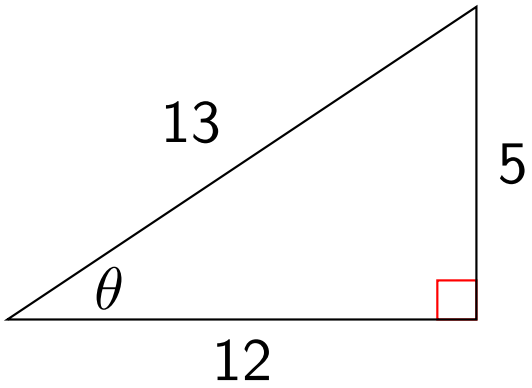
(a)




$$\csc\theta = \frac{13}{5}$$

# Example 1

(a)

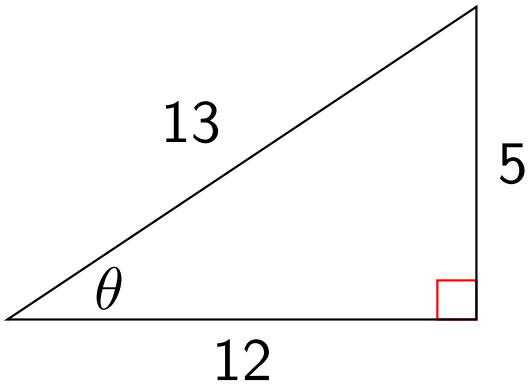



$$\csc\theta = \frac{13}{5}$$

$$\sec\theta = \frac{13}{12}$$

# Example 1

(a)




$$\csc\theta = \frac{13}{5}$$

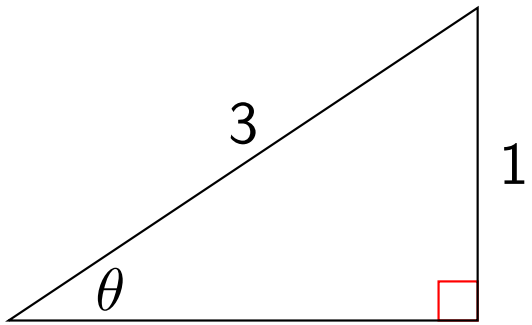
$$\sec\theta = \frac{13}{12}$$

$$\cot\theta = \frac{12}{5}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

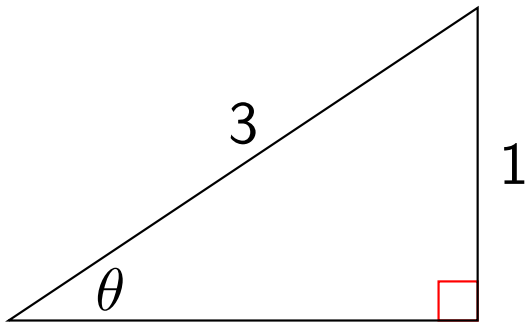
(b)



# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)

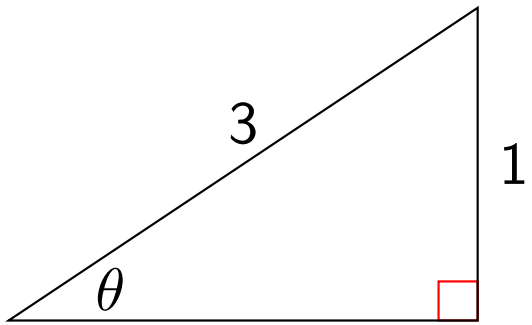



$$a^2 + 1^2 = 3^2$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)



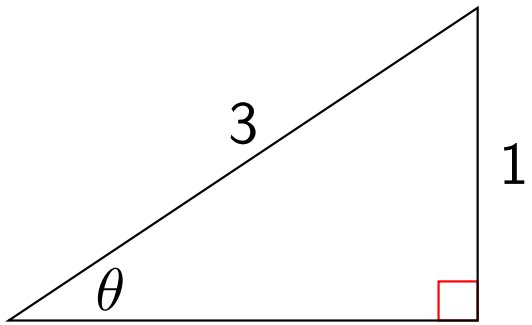

$$a^2 + 1^2 = 3^2$$

$$a^2 = 8$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$a^2 + 1^2 = 3^2$$

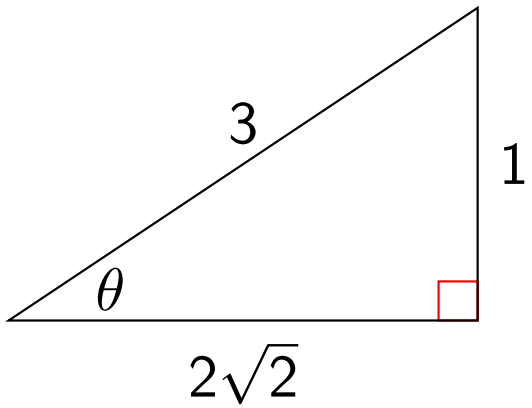
$$a^2 = 8$$

$$a = \sqrt{8} = 2\sqrt{2}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$a^2 + 1^2 = 3^2$$

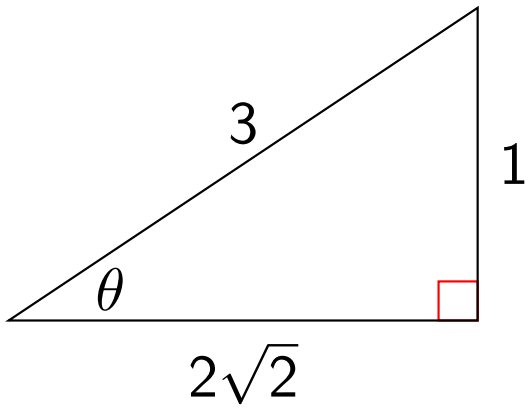
$$a^2 = 8$$

$$a = \sqrt{8} = 2\sqrt{2}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$\sin\theta = \frac{1}{3}$$

$$a^2 + 1^2 = 3^2$$

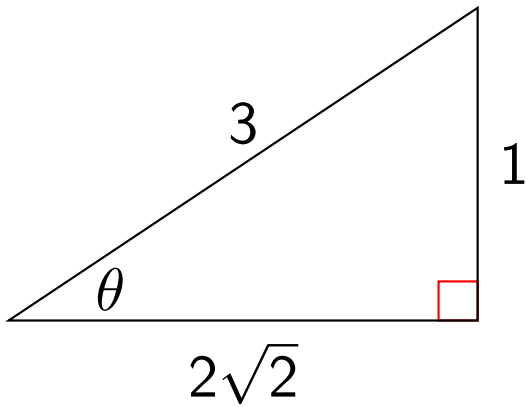
$$a^2 = 8$$

$$a = \sqrt{8} = 2\sqrt{2}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$\sin\theta = \frac{1}{3}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

$$a^2 + 1^2 = 3^2$$

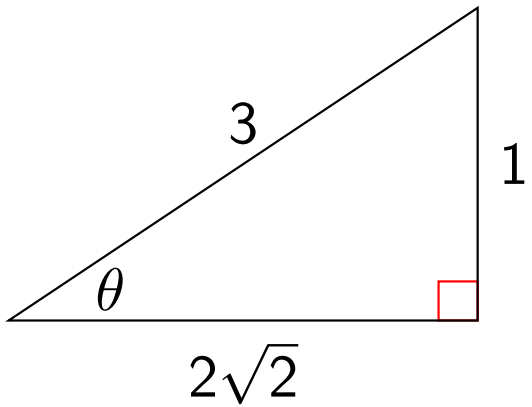
$$a^2 = 8$$

$$a = \sqrt{8} = 2\sqrt{2}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$\sin\theta = \frac{1}{3}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

$$a^2 + 1^2 = 3^2$$

$$a^2 = 8$$

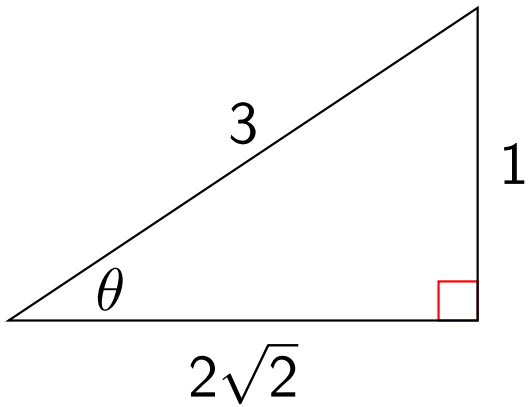
$$a = \sqrt{8} = 2\sqrt{2}$$

$$\tan\theta = \frac{1}{2\sqrt{2}}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$\sin\theta = \frac{1}{3}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

$$a^2 + 1^2 = 3^2$$

$$a^2 = 8$$

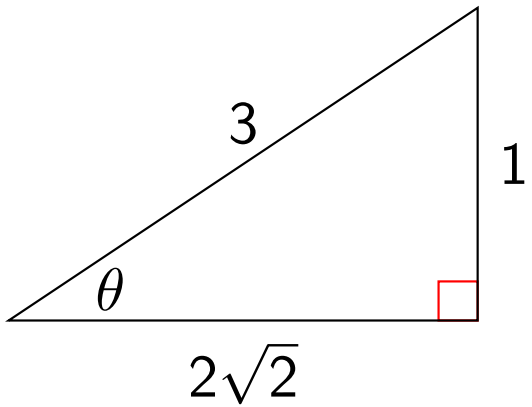
$$a = \sqrt{8} = 2\sqrt{2}$$

$$\tan\theta = \frac{1}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$\sin\theta = \frac{1}{3}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

$$a^2 + 1^2 = 3^2$$

$$a^2 = 8$$

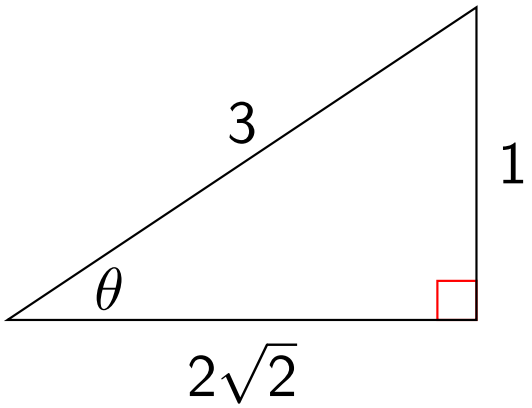
$$a = \sqrt{8} = 2\sqrt{2}$$

$$\tan\theta = \frac{1}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{4}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

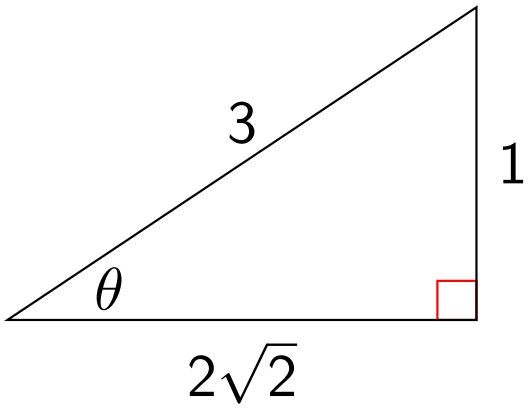
(b)



# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)

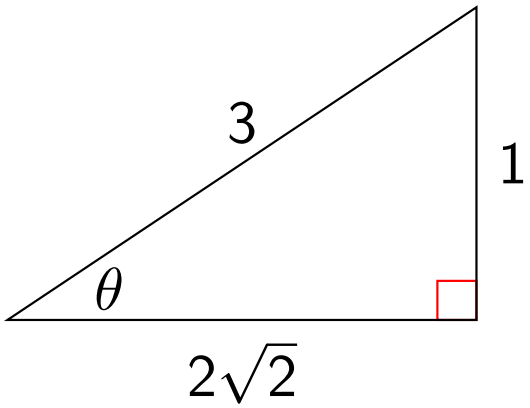



$$\csc\theta = \frac{3}{1} = 3$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)



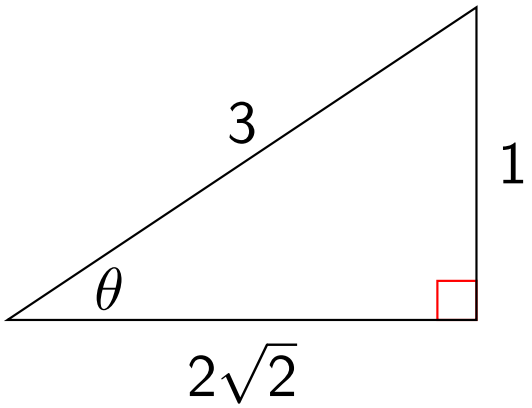

$$\csc\theta = \frac{3}{1} = 3$$

$$\sec\theta = \frac{3}{2\sqrt{2}}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)



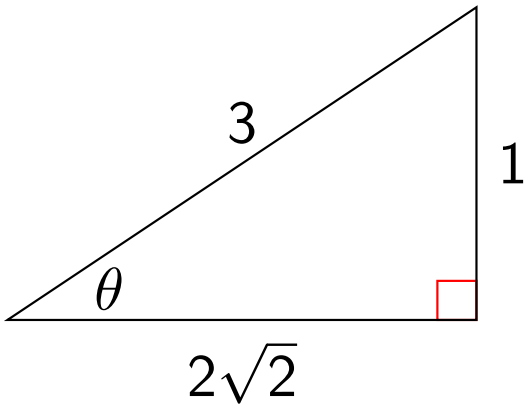

$$\csc\theta = \frac{3}{1} = 3$$

$$\sec\theta = \frac{3}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)



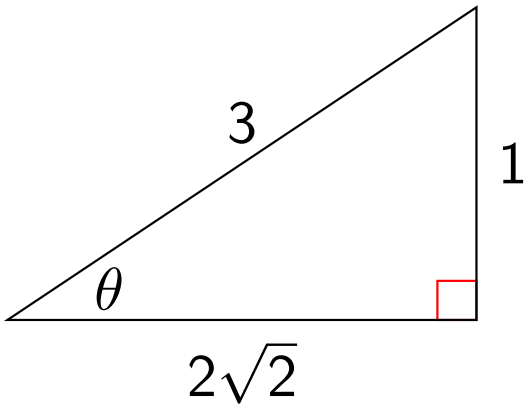

$$\csc\theta = \frac{3}{1} = 3$$

$$\sec\theta = \frac{3}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{3\sqrt{2}}{4}$$

# Example 1

Find the value of each of the six trig functions of $\theta$.

(b)




$$\csc\theta = \frac{3}{1} = 3$$

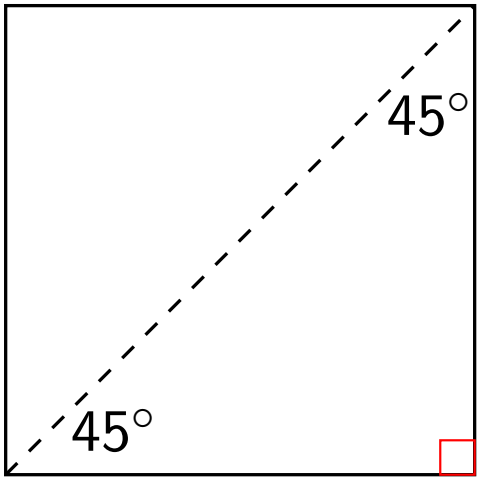
$$\sec\theta = \frac{3}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

# Table of Contents

# 45-45-90 Triangles

45-45-90 triangles (also known as *isosceles right triangles*) can be created by drawing a diagonal across a square:



# 45-45-90 Triangles

Since each side of a square is the same length, we can use whatever length we want. For simplicity, we will use a length of 1.

# 45-45-90 Triangles

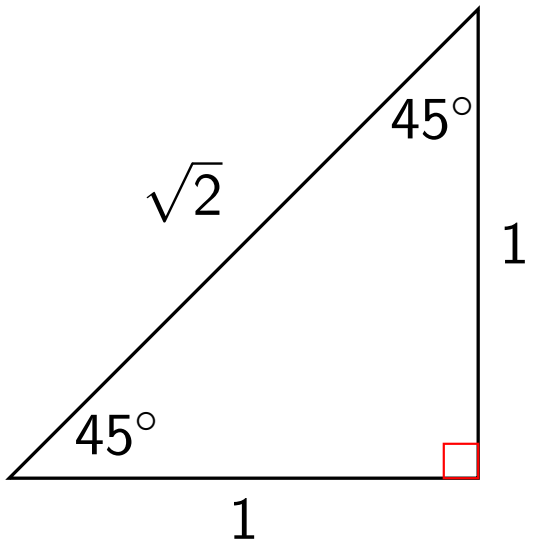
Since each side of a square is the same length, we can use whatever length we want. For simplicity, we will use a length of 1.

The diagonal of the square can be found by using Pythagorean Theorem:

# 45-45-90 Triangles

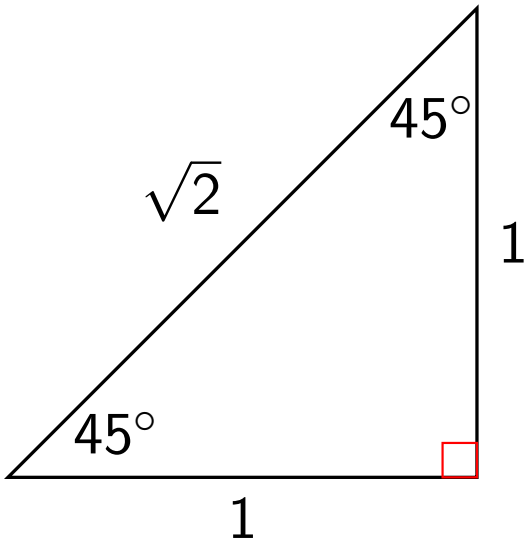
Since each side of a square is the same length, we can use whatever length we want. For simplicity, we will use a length of 1.

The diagonal of the square can be found by using Pythagorean Theorem:



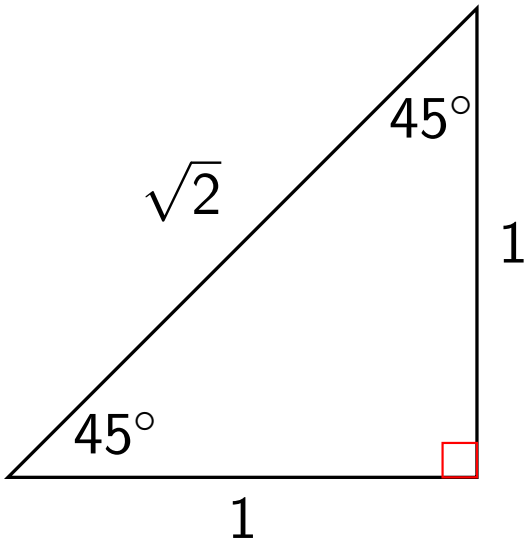
# Example 2

Find the exact values of the six trig ratios for $45°$.



# Example 2

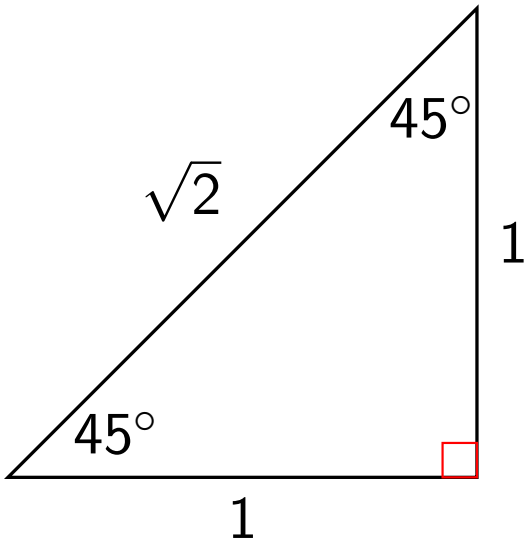
Find the exact values of the six trig ratios for $45°$.




$$\sin 45° = \frac{1}{\sqrt{2}}$$

# Example 2

Find the exact values of the six trig ratios for $45°$.

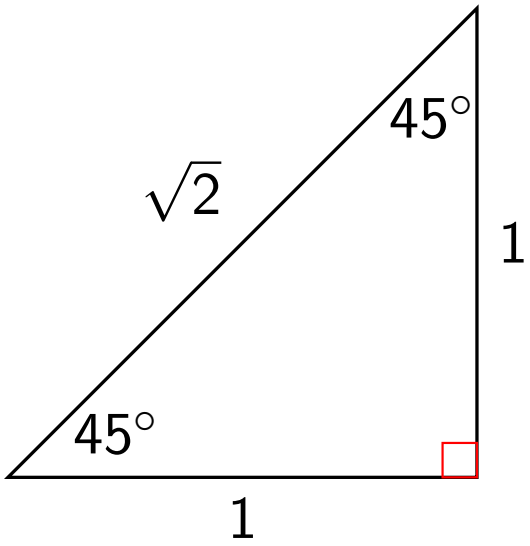



$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

# Example 2

Find the exact values of the six trig ratios for $45°$.



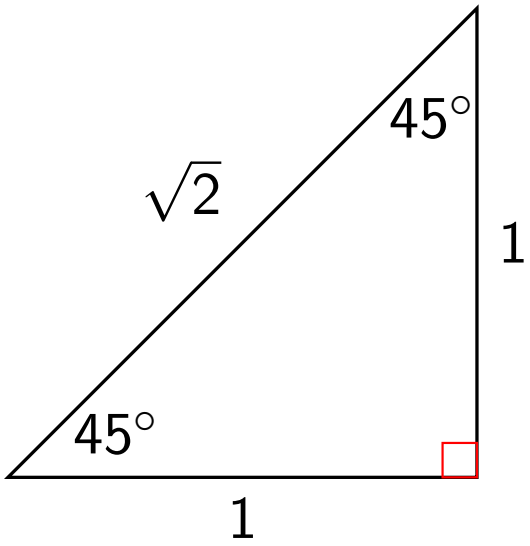

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

# Example 2

Find the exact values of the six trig ratios for $45^\circ$.



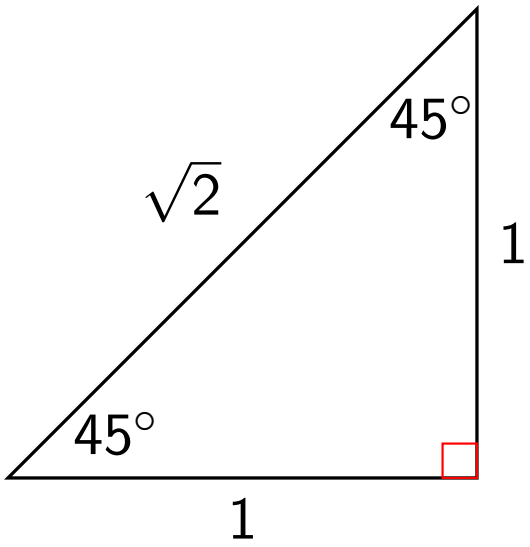

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

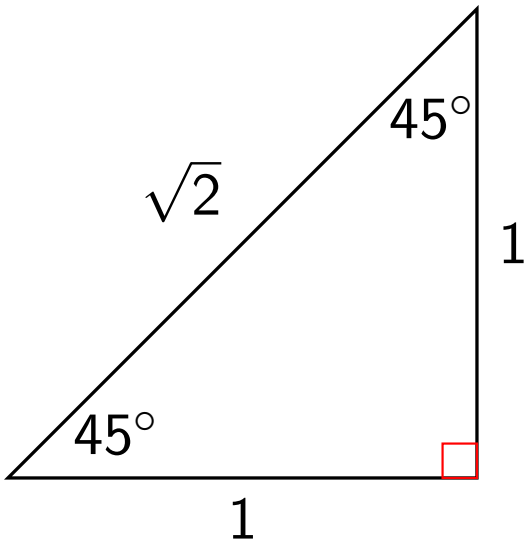
$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$

# Example 2

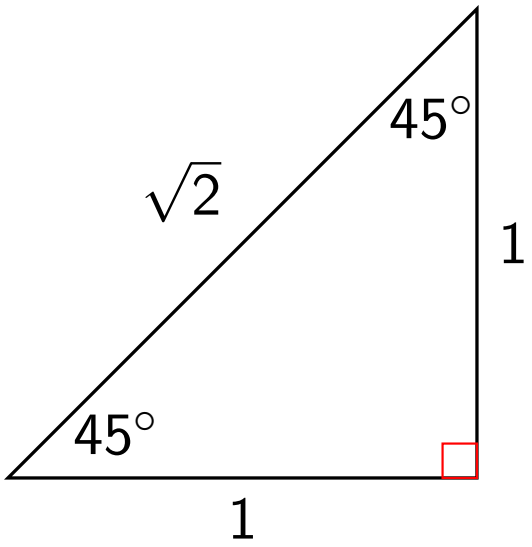


# Example 2




$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

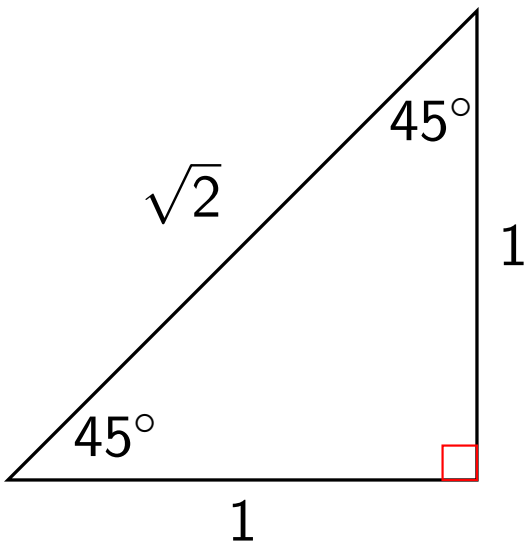
# Example 2




$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

# Example 2

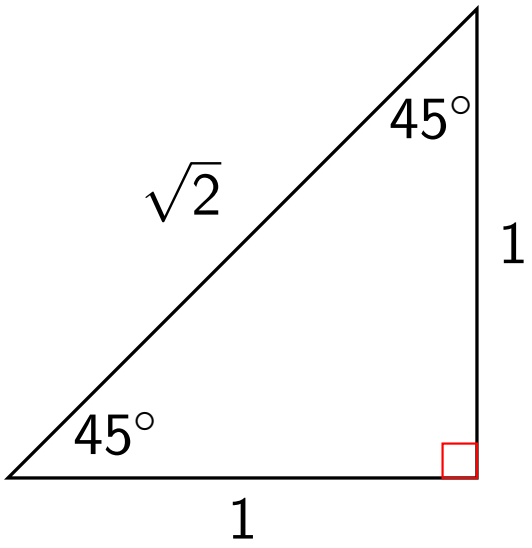



$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45^\circ = \frac{1}{1} = 1$$

# Example 2




$$\csc 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

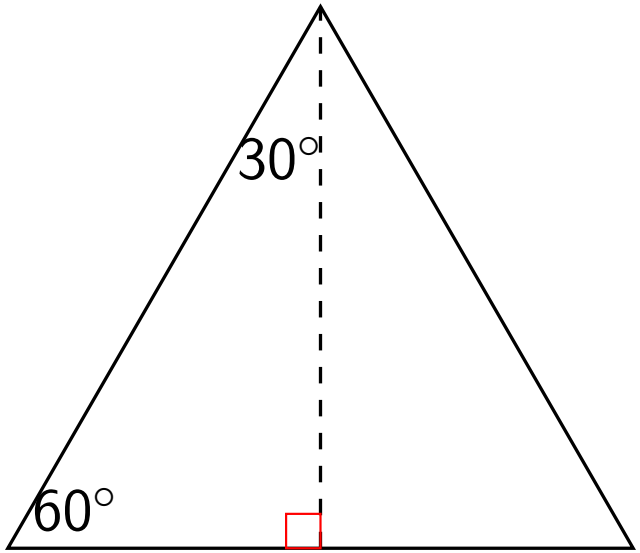
$$\sec 45^\circ = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45^\circ = \frac{1}{1} = 1$$

*Note*: Your answers from the above example will be the same if you replace $45^\circ$ with $\frac{\pi}{4}$.

# 30-60-90 Triangles

We can create a 30-60-90 triangle by drawing an altitude in an equilateral triangle.



# 30-60-90 Triangles

Recall that the altitude of an equilateral triangle bisects one of the sides.

# 30-60-90 Triangles

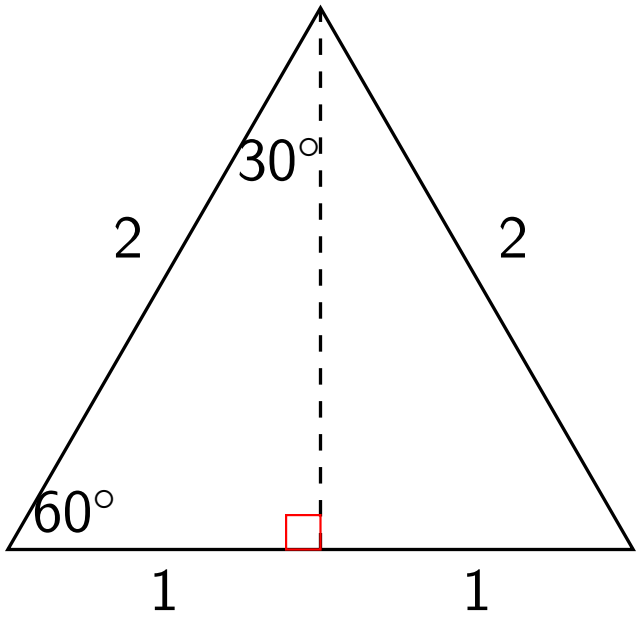
Recall that the altitude of an equilateral triangle bisects one of the sides.

Rather than use a length of 1 for the sides of the equilateral triangle, we will use a length of 2 (if only to avoid using fractions).

# 30-60-90 Triangles

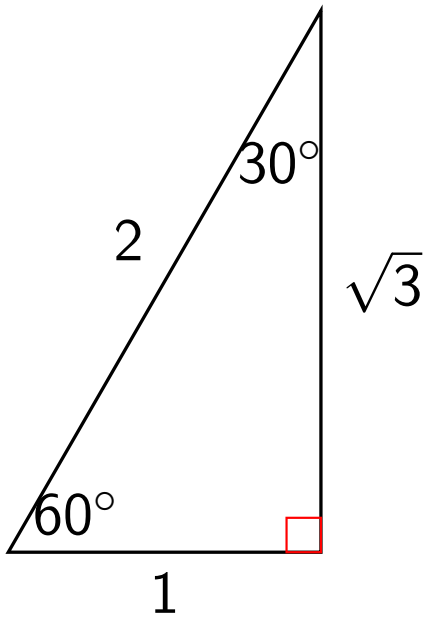
Recall that the altitude of an equilateral triangle bisects one of the sides.

Rather than use a length of 1 for the sides of the equilateral triangle, we will use a length of 2 (if only to avoid using fractions).



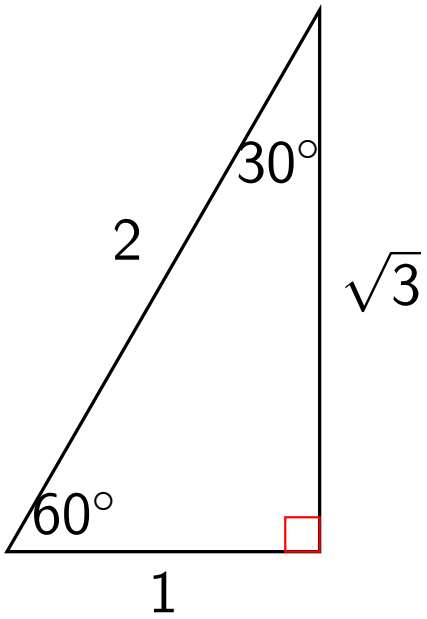
# 30-60-90 Triangles

We can use the Pythagorean Theorem to find the length of the altitude, $\sqrt{3}$:



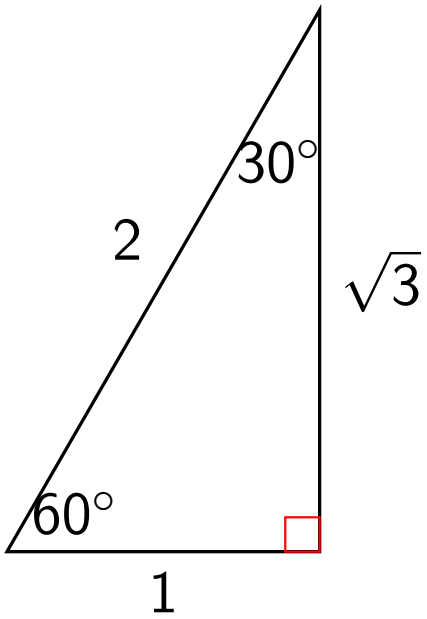
# Example 3

Find the exact values of the six trig ratios for $60°$.



# Example 3

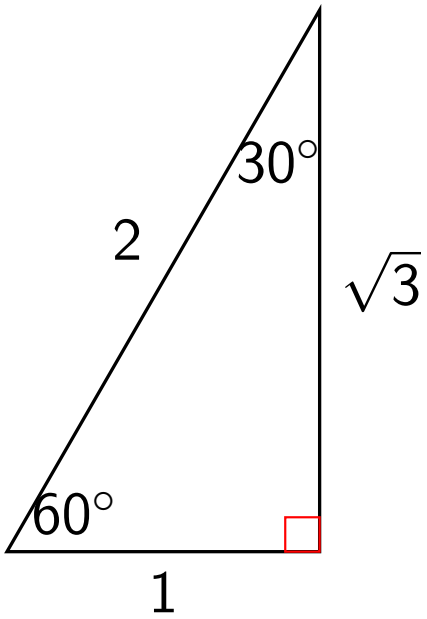
Find the exact values of the six trig ratios for $60°$.




$$\sin 60° = \frac{\sqrt{3}}{2}$$

# Example 3

Find the exact values of the six trig ratios for $60°$.

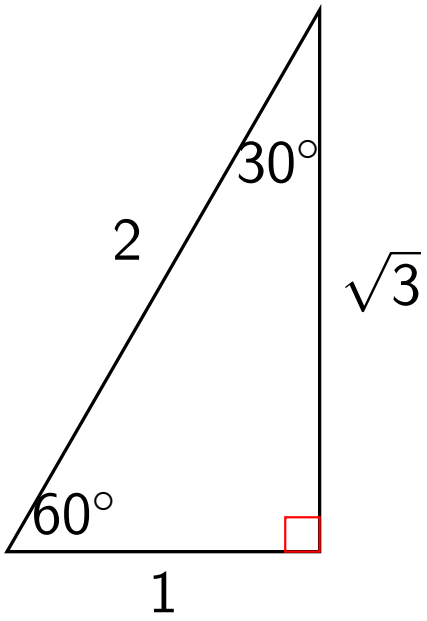



$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

# Example 3

Find the exact values of the six trig ratios for $60°$.

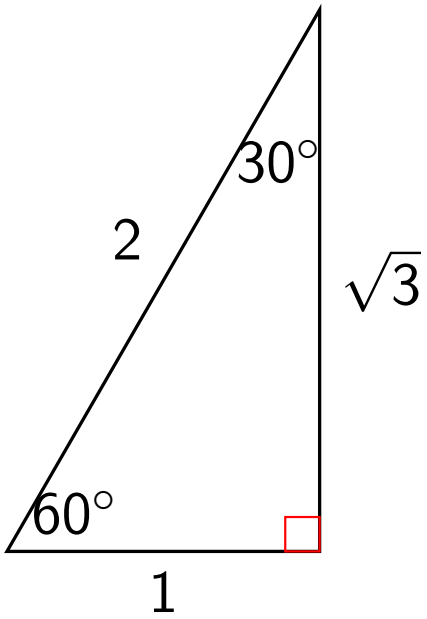



$$\sin 60° = \frac{\sqrt{3}}{2}$$

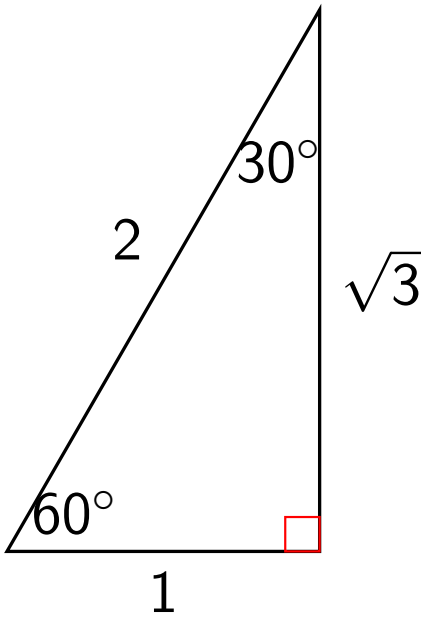
$$\cos 60° = \frac{1}{2}$$

$$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$$

# Example 3

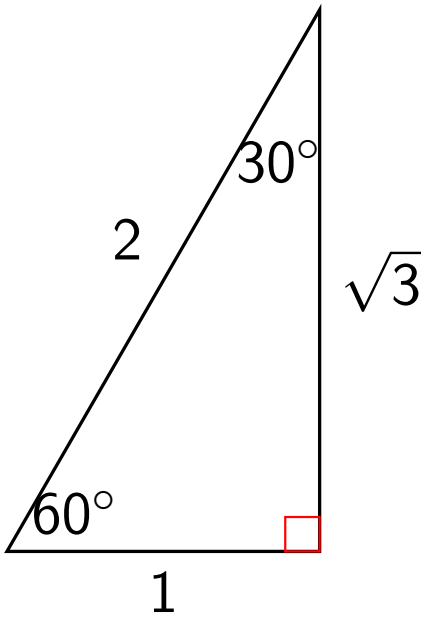


# Example 3




$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

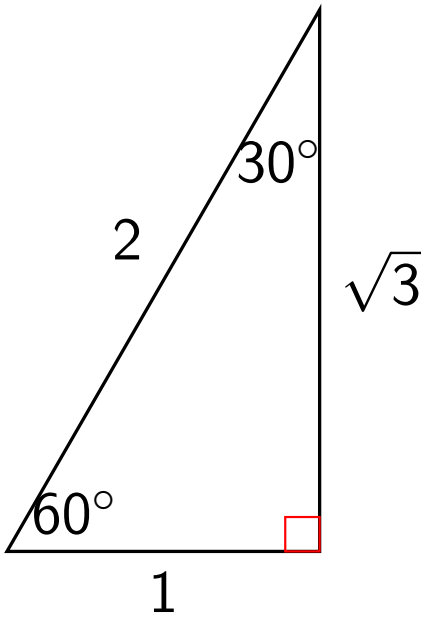
# Example 3




$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

# Example 3

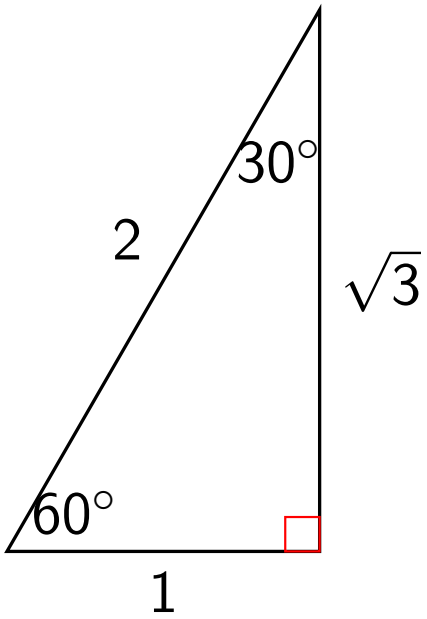



$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec 60^\circ = \frac{2}{1} = 2$$

# Example 3



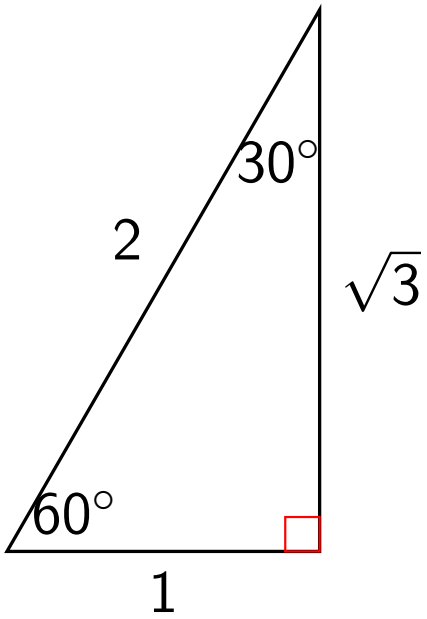

$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec 60^\circ = \frac{2}{1} = 2$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}}$$

# Example 3




$$\csc 60^\circ = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

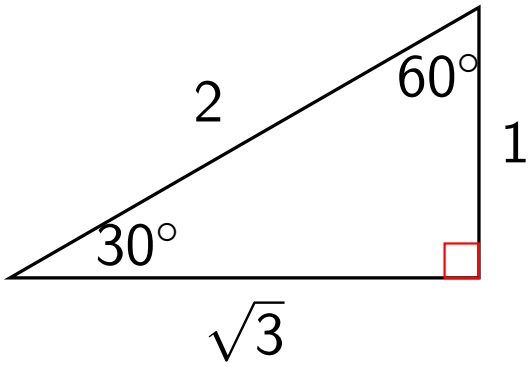
$$\sec 60^\circ = \frac{2}{1} = 2$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

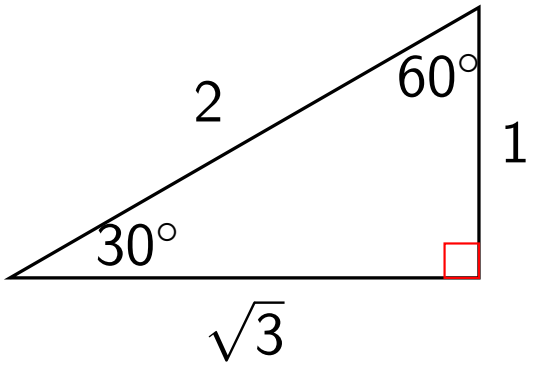
# Example 4

Find the exact values of the six trig ratios for $30°$.



# Example 4

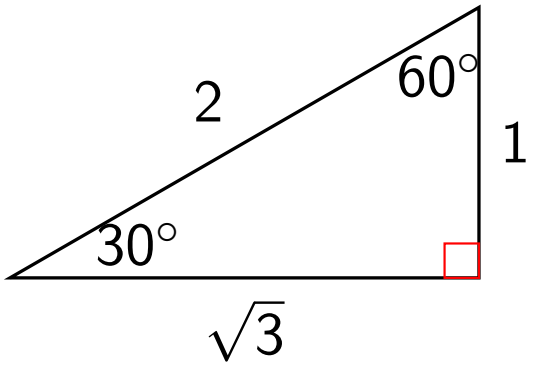
Find the exact values of the six trig ratios for $30°$.




$$\sin 30° = \frac{1}{2}$$

# Example 4

Find the exact values of the six trig ratios for $30°$.

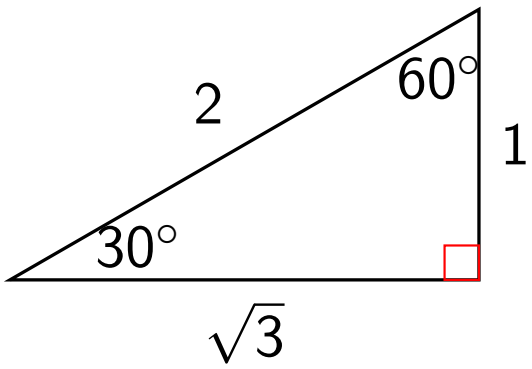



$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

# Example 4

Find the exact values of the six trig ratios for $30^\circ$.

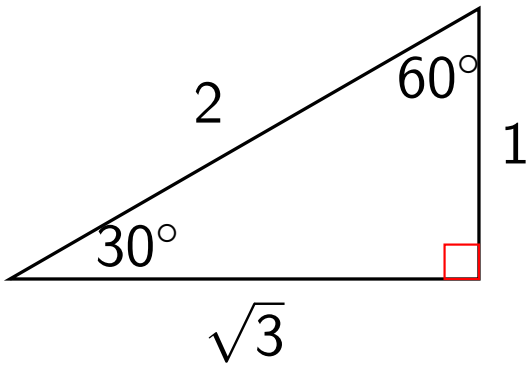



$$\sin 30^\circ = \frac{1}{2}$$

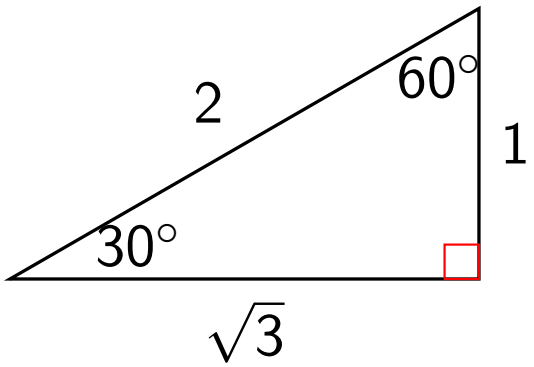
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

# Example 4

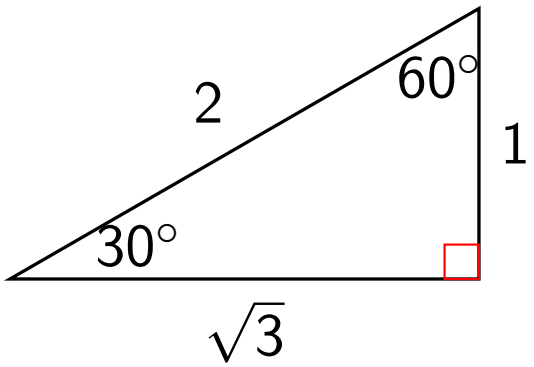


# Example 4




$$\csc 30^\circ = \frac{2}{1} = 2$$

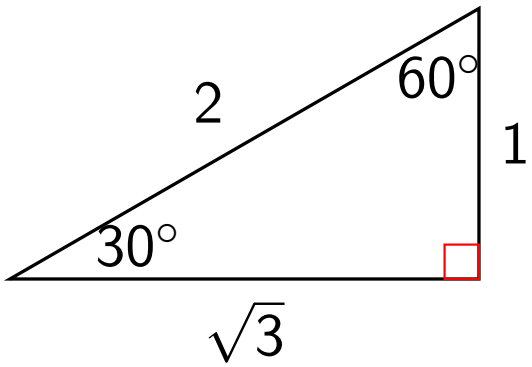
# Example 4




$$\csc 30^\circ = \frac{2}{1} = 2$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

# Example 4




$$\csc 30^\circ = \frac{2}{1} = 2$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot 30^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

# Cofunctions

Notice how $\sin 30^\circ = \cos 60^\circ$, $\tan 30^\circ = \cot 60^\circ$, etc. This is because these ratios are cofunctions.

# Cofunctions

Notice how $\sin 30^\circ = \cos 60^\circ$, $\tan 30^\circ = \cot 60^\circ$, etc. This is because these ratios are cofunctions.

Any pair of trig functions $f$ and $g$ for which

$$f(\theta) = g\left(90^\circ - \theta\right)$$

and vice versa are **cofunctions**.

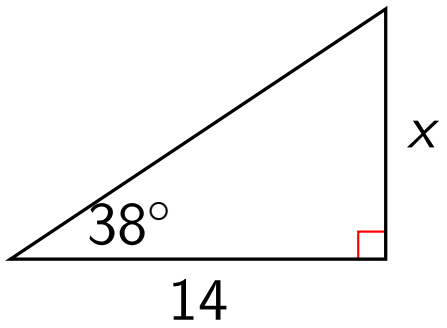
# Table of Contents

# Finding Missing Sides

You can use SOH-CAH-TOA and your calculator to find missing sides in right triangles.

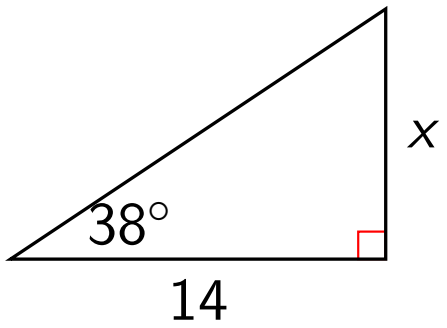
# Example 5

Find the value of $x$. Round your answer to 2 decimal places.



# Example 5

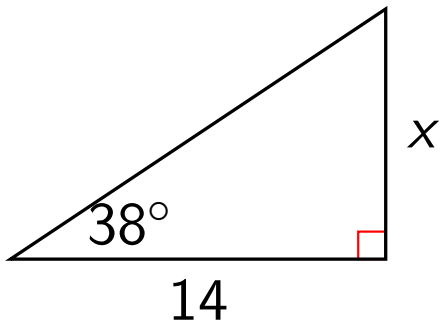
Find the value of $x$. Round your answer to 2 decimal places.




$$\tan 38^\circ = \frac{x}{14}$$

# Example 5

Find the value of $x$. Round your answer to 2 decimal places.

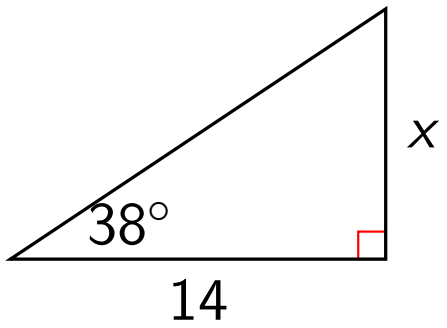



$$\tan 38^\circ = \frac{x}{14}$$

$$0.7813 = \frac{x}{14}$$

# Example 5

Find the value of $x$. Round your answer to 2 decimal places.




$$\tan 38^\circ = \frac{x}{14}$$

$$0.7813 = \frac{x}{14}$$

$$x = 14(0.7813) \approx 10.94$$