Sum to Product and Product to Sum Identities

Objectives

1. Write a product of trig functions as a sum of trig functions.

2. Write a sum of trig functions as a product of trig functions.

Product to Sum Identities

$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ +\cos(A-B) &= \cos A \cos B + \sin A \sin B \end{aligned}$$

$$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$$

$$\frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right) = \cos A \cos B$$

Other Product to Sum Identities

$$\sin A \sin B = \frac{1}{2}\left(\cos(A - B) - \cos(A + B)\right)$$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A - B) + \sin(A + B)\right)$$

Example 1

Write $\cos(2\theta)\cos(6\theta)$ as a sum

Example 1

Write $\cos(2\theta)\cos(6\theta)$ as a sum

$$\cos(2\theta)\cos(6\theta) = \frac{1}{2}\left(\cos(2\theta + 6\theta) + \cos(2\theta - 6\theta)\right)$$

Example 1

Write $\cos(2\theta)\cos(6\theta)$ as a sum

$$\cos(2\theta)\cos(6\theta) = \frac{1}{2}\left(\cos(2\theta + 6\theta) + \cos(2\theta - 6\theta)\right)$$

$$= \frac{1}{2}\left(\cos(8\theta) + \cos(-4\theta)\right)$$

Example 1

Write $\cos(2\theta)\cos(6\theta)$ as a sum

$$\cos(2\theta)\cos(6\theta) = \frac{1}{2}\left(\cos(2\theta + 6\theta) + \cos(2\theta - 6\theta)\right)$$

$$= \frac{1}{2}\left(\cos(8\theta) + \cos(-4\theta)\right)$$

$$= \frac{1}{2}\left(\cos(8\theta) + \cos(4\theta)\right)$$

Objectives

1. Write a product of trig functions as a sum of trig functions.

2. Write a sum of trig functions as a product of trig functions.

Sum to Product Identities

For $\cos(A+B)+\cos(A-B)=2\cos A\cos B$, if we let $x=A+B$ and $y=A-B$, we get

$$x+y=2A \quad \text{and} \quad x-y=2B$$

from which

$$A=\frac{x+y}{2} \quad \text{and} \quad B=\frac{x-y}{2}$$

$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

Other Sum to Product Identities

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\sin x \pm \sin y = 2 \sin\left(\frac{x \pm y}{2}\right) \cos\left(\frac{x \mp y}{2}\right)$$

Example 2

Write $\sin\theta - \sin(3\theta)$ as a product.

Example 2

Write $\sin\theta - \sin(3\theta)$ as a product.

$$\sin\theta - \sin(3\theta) = 2\sin\left(\frac{\theta - 3\theta}{2}\right)\cos\left(\frac{\theta + 3\theta}{2}\right)$$

Example 2

Write $\sin\theta - \sin(3\theta)$ as a product.

$$\sin\theta - \sin(3\theta) = 2\sin\left(\frac{\theta - 3\theta}{2}\right)\cos\left(\frac{\theta + 3\theta}{2}\right)$$

$$= 2\sin\left(\frac{-2\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)$$

Example 2

Write $\sin\theta - \sin(3\theta)$ as a product.

$$\sin\theta - \sin(3\theta) = 2\sin\left(\frac{\theta - 3\theta}{2}\right)\cos\left(\frac{\theta + 3\theta}{2}\right)$$

$$= 2\sin\left(\frac{-2\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)$$

$$= 2\sin(-\theta)\cos(2\theta)$$

Example 2

Write $\sin\theta - \sin(3\theta)$ as a product.

$$\sin\theta - \sin(3\theta) = 2\sin\left(\frac{\theta - 3\theta}{2}\right)\cos\left(\frac{\theta + 3\theta}{2}\right)$$

$$= 2\sin\left(\frac{-2\theta}{2}\right)\cos\left(\frac{4\theta}{2}\right)$$

$$= 2\sin(-\theta)\cos(2\theta)$$

$$= -2\sin\theta\cos(2\theta)$$