

# Trig Equations

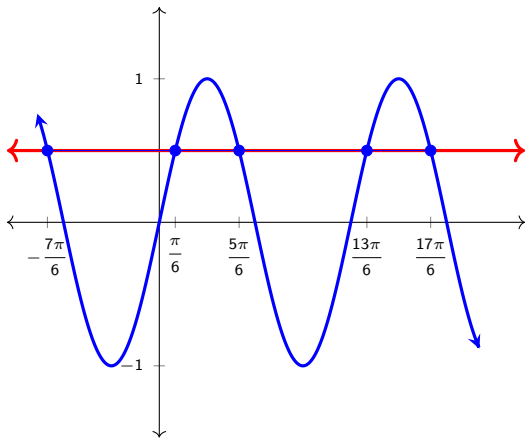
# Objectives

- 1 Solve trigonometric equations

# Trig Equations

A **trigonometric equation** is one that contains a trig function with variable, such as  $\sin x = \frac{1}{2}$ .

# Trig Equations



# General Form of Solutions

The **general form** of this solution is

$$x = \frac{\pi}{6} + 2\pi n \quad \text{or} \quad x = \frac{5\pi}{6} + 2\pi n$$

where  $2\pi$  is the **period** of the sine function.

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where  $2\pi$  is the **period** of the sine function.

Since there are an infinite number of solutions, we will usually confine our answers to be between 0 and  $2\pi$ .

# How to Solve a Trig Equation

- 1 Get the trig function by itself, if possible.
- 2 Solve for the variable using inverse trig.

## Example 1

Solve each of the following in the interval  $[0, 2\pi)$ .

(a)  $\cos(2x) = -\frac{\sqrt{3}}{2}$



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$$(a) \quad \cos(2x) = -\frac{\sqrt{3}}{2}$$

$$2x = 150^\circ + 360n$$

$$2x = 210 + 360n$$

## Example 1

Solve each of the following in the interval  $[0, 2\pi)$ .

$$(a) \quad \cos(2x) = -\frac{\sqrt{3}}{2}$$

$$2x = 150^\circ + 360n$$

$$x = 75^\circ + 180n$$

$$2x = 210 + 360n$$

$$x = 105^\circ + 180n$$

## Example 1

Solve each of the following in the interval  $[0, 2\pi)$ .

$$(a) \quad \cos(2x) = -\frac{\sqrt{3}}{2}$$

$$2x = 150^\circ + 360n$$

$$x = 75^\circ + 180n$$

$$x = 75^\circ, 255^\circ$$

$$2x = 210 + 360n$$

$$x = 105^\circ + 180n$$

$$x = 105^\circ, 285^\circ$$

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Solve each of the following in the interval  $[0, 2\pi)$ .

$$(a) \quad \cos(2x) = -\frac{\sqrt{3}}{2}$$

$$2x = 150^\circ + 360n$$

$$2x = 210 + 360n$$

$$x = 75^\circ + 180n$$

$$x = 105^\circ + 180n$$

$$x = 75^\circ, 255^\circ$$

$$x = 105^\circ, 285^\circ$$

$$x = \left\{ \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12} \right\}$$

## Example 1

$$(b) \quad \csc\left(\frac{1}{3}x - \pi\right) = \sqrt{2}$$

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$$\frac{1}{3}x - 180^\circ = 45^\circ + 360n$$

$$\frac{1}{3}x - 180^\circ = 135^\circ + 360n$$

## Example 1

$$(b) \quad \csc\left(\frac{1}{3}x - \pi\right) = \sqrt{2}$$

$$\frac{1}{3}x - 180^\circ = 45^\circ + 360n$$

$$\frac{1}{3}x = 225^\circ + 360n$$

$$\frac{1}{3}x - 180^\circ = 135^\circ + 360n$$

$$\frac{1}{3}x = 315^\circ + 360n$$

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$$(b) \quad \csc\left(\frac{1}{3}x - \pi\right) = \sqrt{2}$$

$$\frac{1}{3}x - 180^\circ = 45^\circ + 360n$$

$$\frac{1}{3}x = 225^\circ + 360n$$

$$x = 675^\circ + 1080n$$

$$\frac{1}{3}x - 180^\circ = 135^\circ + 360n$$

$$\frac{1}{3}x = 315^\circ + 360n$$

$$x = 945^\circ + 1080n$$



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$$\frac{1}{3}x - 180^\circ = 45^\circ + 360n$$

$$\frac{1}{3}x = 225^\circ + 360n$$

$$x = 675^\circ + 1080n$$

$$\frac{1}{3}x - 180^\circ = 135^\circ + 360n$$

$$\frac{1}{3}x = 315^\circ + 360n$$

$$x = 945^\circ + 1080n$$

No angles between 0 and  $2\pi$

## Example 1

$$(c) \quad \cot(3x) = 0$$

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$$(c) \quad \cot(3x) = 0$$

$$3x = 90^\circ + 180n$$

$$3x = 270^\circ + 180n$$

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$$3x = 90^\circ + 180n$$

$$x = 30^\circ + 60n$$

$$3x = 270^\circ + 180n$$

$$x = 90^\circ + 60n$$

## Example 1

$$(c) \quad \cot(3x) = 0$$

$$3x = 90^\circ + 180n$$

$$x = 30^\circ + 60n$$

$$x = 30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ, 330^\circ$$

$$3x = 270^\circ + 180n$$

$$x = 90^\circ + 60n$$

$$x = 90^\circ, 150^\circ, \dots$$

## Example 1

$$(c) \quad \cot(3x) = 0$$

$$3x = 90^\circ + 180n$$

$$3x = 270^\circ + 180n$$

$$x = 30^\circ + 60n$$

$$x = 90^\circ + 60n$$

$$x = 30^\circ, 90^\circ, 150^\circ, 210^\circ, 270^\circ, 330^\circ$$

$$x = 90^\circ, 150^\circ, \dots$$

$$x = \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\}$$

## Example 1

(d)  $\sec^2 x = 4$

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$$\sec x = 2$$

$$x = 60^\circ, 300^\circ$$

$$\sec x = -2$$

$$x = 120^\circ, 240^\circ$$

## Example 1

$$(d) \quad \sec^2 x = 4$$

$$\sqrt{\sec^2 x} = \pm\sqrt{4}$$

$$\sec x = 2$$

$$x = 60^\circ, 300^\circ$$

$$\sec x = -2$$

$$x = 120^\circ, 240^\circ$$

$$x = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

## Example 1

$$(e) \quad \tan\left(\frac{x}{2}\right) = -3$$

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$$x = 2 \tan^{-1}(-3) + 360n \qquad (2 \arctan(-3) \approx -286^\circ)$$

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$$x = 2 \tan^{-1}(-3) + 360^\circ$$

$$x = 2 \tan^{-1}(-3) + 2\pi$$



# Using Algebraic Techniques and Trig Identities

The following examples make use of trig identities and algebraic techniques to solve the equations.

## Example 2

Solve each in the interval  $[0, 2\pi)$

(a)  $3 \sin^3 x = \sin^2 x$

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$$3 \sin^3 x - \sin^2 x = 0$$

$$\sin^2 x (3 \sin x - 1) = 0$$

## Example 2

Solve each in the interval  $[0, 2\pi)$

$$(a) \quad 3 \sin^3 x = \sin^2 x$$

$$3 \sin^3 x - \sin^2 x = 0$$

$$\sin^2 x (3 \sin x - 1) = 0$$

$$\sin^2 x = 0$$

$$3 \sin x - 1 = 0$$

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Solve each in the interval  $[0, 2\pi)$

(a)  $3 \sin^3 x = \sin^2 x$

$$3 \sin^3 x - \sin^2 x = 0$$

$$\sin^2 x (3 \sin x - 1) = 0$$

$$\sin^2 x = 0$$

$$\sin x = 0$$

$$3 \sin x - 1 = 0$$

$$\sin x = \frac{1}{3}$$

## Example 2

Solve each in the interval  $[0, 2\pi)$

(a)  $3 \sin^3 x = \sin^2 x$

$$3 \sin^3 x - \sin^2 x = 0$$

$$\sin^2 x (3 \sin x - 1) = 0$$

$$\sin^2 x = 0$$

$$\sin x = 0$$

$$x = 0, 180^\circ$$

$$3 \sin x - 1 = 0$$

$$\sin x = \frac{1}{3}$$

$$x \approx 19.471^\circ, 160.529^\circ$$

## Example 2

Solve each in the interval  $[0, 2\pi)$

$$(a) \quad 3 \sin^3 x = \sin^2 x$$

$$3 \sin^3 x - \sin^2 x = 0$$

$$\sin^2 x (3 \sin x - 1) = 0$$

$$\sin^2 x = 0$$

$$3 \sin x - 1 = 0$$

$$\sin x = 0$$

$$\sin x = \frac{1}{3}$$

$$x = 0, 180^\circ$$

$$x \approx 19.471^\circ, 160.529^\circ$$

$$x = \left\{ 0, \arcsin\left(\frac{1}{3}\right), \pi - \arcsin\left(\frac{1}{3}\right), \pi \right\}$$



## Example 2

$$(b) \quad \sec^2 x = \tan x + 3$$

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$$\tan^2 x + 1 = \tan x + 3$$

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$$(b) \quad \sec^2 x = \tan x + 3$$

$$\tan^2 x + 1 = \tan x + 3$$

$$\tan^2 x - \tan x - 2 = 0$$

## Example 2

$$(b) \quad \sec^2 x = \tan x + 3$$

$$\tan^2 x + 1 = \tan x + 3$$

$$\tan^2 x - \tan x - 2 = 0$$

$$(\tan x - 2)(\tan x + 1) = 0$$

## Example 2

$$(b) \quad \sec^2 x = \tan x + 3$$

$$\tan^2 x + 1 = \tan x + 3$$

$$\tan^2 x - \tan x - 2 = 0$$

$$(\tan x - 2)(\tan x + 1) = 0$$

$$\tan x - 2 = 0$$

$$\tan x + 1 = 0$$

## Example 2

$$(b) \quad \sec^2 x = \tan x + 3$$

$$\tan^2 x + 1 = \tan x + 3$$

$$\tan^2 x - \tan x - 2 = 0$$

$$(\tan x - 2)(\tan x + 1) = 0$$

$$\tan x - 2 = 0$$

$$\tan x = 2$$

$$\tan x + 1 = 0$$

$$\tan x = -1$$

## Example 2

$$(b) \quad \sec^2 x = \tan x + 3$$

$$\tan^2 x + 1 = \tan x + 3$$

$$\tan^2 x - \tan x - 2 = 0$$

$$(\tan x - 2)(\tan x + 1) = 0$$

$$\tan x - 2 = 0$$

$$\tan x = 2$$

$$x \approx 63.5^\circ, 243.5^\circ$$

$$\tan x + 1 = 0$$

$$\tan x = -1$$

$$x = 135^\circ, 315^\circ$$

## Example 2

$$(b) \quad \sec^2 x = \tan x + 3$$

$$\tan^2 x + 1 = \tan x + 3$$

$$\tan^2 x - \tan x - 2 = 0$$

$$(\tan x - 2)(\tan x + 1) = 0$$

$$\tan x - 2 = 0$$

$$\tan x = 2$$

$$x \approx 63.5^\circ, 243.5^\circ$$

$$\tan x + 1 = 0$$

$$\tan x = -1$$

$$x = 135^\circ, 315^\circ$$

$$x = \left\{ \arctan(2), \frac{3\pi}{4}, \pi + \arctan(2), \frac{7\pi}{4} \right\}$$



## Example 2

$$(c) \quad \cos(2x) = 3 \cos x - 2$$

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$$(c) \quad \cos(2x) = 3 \cos x - 2$$

$$2 \cos^2 x - 1 = 3 \cos x - 2$$

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$$(c) \quad \cos(2x) = 3 \cos x - 2$$

$$2 \cos^2 x - 1 = 3 \cos x - 2$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

## Example 2

$$(c) \quad \cos(2x) = 3 \cos x - 2$$

$$2 \cos^2 x - 1 = 3 \cos x - 2$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

## Example 2

$$(c) \quad \cos(2x) = 3 \cos x - 2$$

$$2 \cos^2 x - 1 = 3 \cos x - 2$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x - 1 = 0$$

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$$(c) \quad \cos(2x) = 3 \cos x - 2$$

$$2 \cos^2 x - 1 = 3 \cos x - 2$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

## Example 2

$$(c) \quad \cos(2x) = 3 \cos x - 2$$

$$2 \cos^2 x - 1 = 3 \cos x - 2$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ, 300^\circ$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$x = 0$$

## Example 2

$$(c) \quad \cos(2x) = 3 \cos x - 2$$

$$2 \cos^2 x - 1 = 3 \cos x - 2$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ, 300^\circ$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$x = 0$$

$$x = \left\{ 0, \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$



## Example 2

$$(d) \quad \sin(2x) = \sqrt{3} \cos x$$

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$$(d) \quad \sin(2x) = \sqrt{3} \cos x$$

$$2 \sin x \cos x = \sqrt{3} \cos x$$

$$2 \sin x \cos x - \sqrt{3} \cos x = 0$$

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$$2 \sin x \cos x = \sqrt{3} \cos x$$

$$2 \sin x \cos x - \sqrt{3} \cos x = 0$$

$$\cos x(2 \sin x - \sqrt{3}) = 0$$

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$$\cos x(2 \sin x - \sqrt{3}) = 0$$

$$\cos x = 0$$

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$$2 \sin x \cos x = \sqrt{3} \cos x$$

$$2 \sin x \cos x - \sqrt{3} \cos x = 0$$

$$\cos x(2 \sin x - \sqrt{3}) = 0$$

$$\cos x = 0$$

$$x = 90^\circ, 270^\circ$$

$$2 \sin x - \sqrt{3} = 0$$

$$2 \sin x = \sqrt{3}$$

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$$(d) \quad \sin(2x) = \sqrt{3} \cos x$$

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$$2 \sin x \cos x - \sqrt{3} \cos x = 0$$

$$\cos x(2 \sin x - \sqrt{3}) = 0$$

$$\cos x = 0$$

$$x = 90^\circ, 270^\circ$$

$$2 \sin x - \sqrt{3} = 0$$

$$2 \sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

## Example 2

$$(d) \quad \sin(2x) = \sqrt{3} \cos x$$

$$2 \sin x \cos x = \sqrt{3} \cos x$$

$$2 \sin x \cos x - \sqrt{3} \cos x = 0$$

$$\cos x(2 \sin x - \sqrt{3}) = 0$$

$$\cos x = 0$$

$$x = 90^\circ, 270^\circ$$

$$2 \sin x - \sqrt{3} = 0$$

$$2 \sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = 60^\circ, 120^\circ$$



## Example 2

$$x = 60^\circ, 90^\circ, 120^\circ, 270^\circ$$

## Example 2

$$x = 60^\circ, 90^\circ, 120^\circ, 270^\circ$$

$$x = \left\{ \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2} \right\}$$