Graphs of Polar Equations

Objectives

1. Graph polar equations

2. Find the intersection of polar equations

Example 1

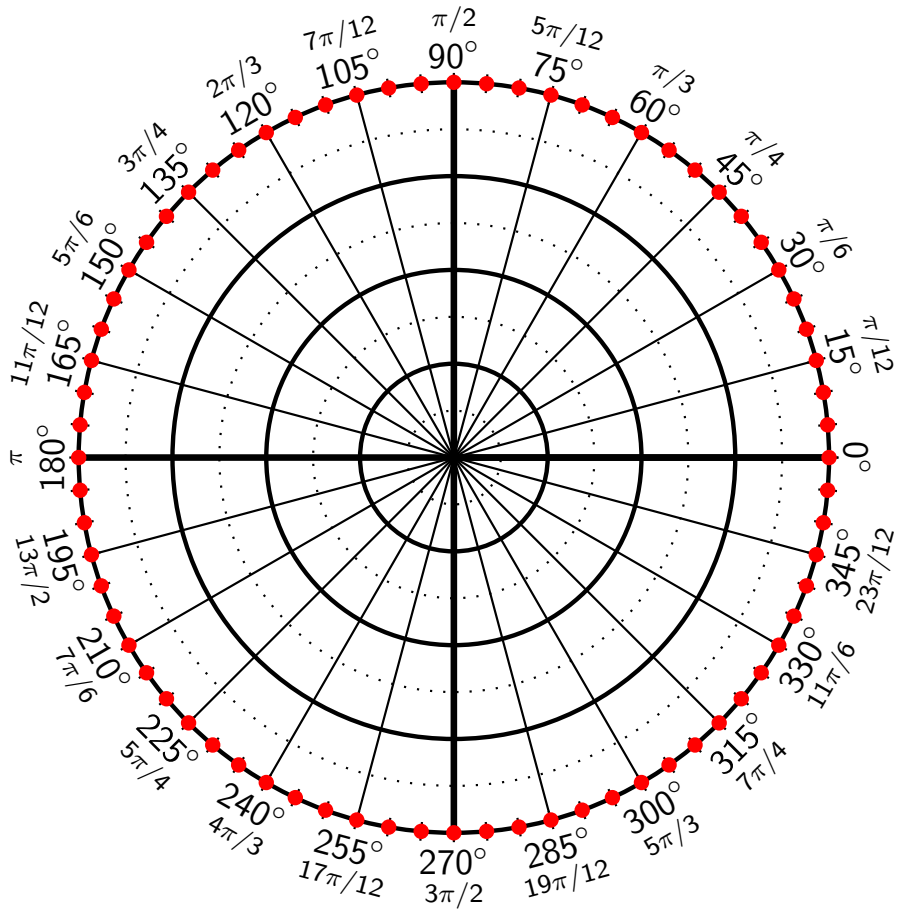
Graph each and comment on the graph.

(a) $r = 4$

Example 1

Graph each and comment on the graph.

(a) $r = 4$

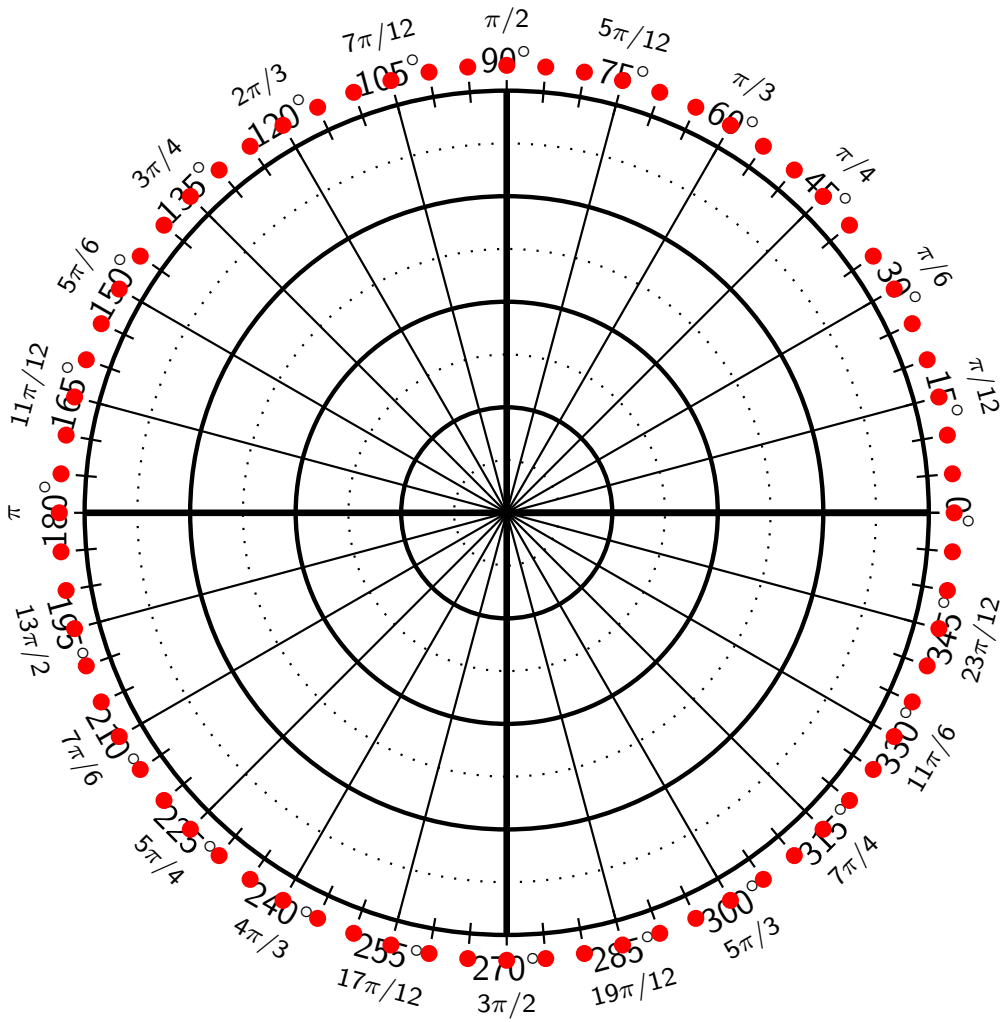


Example 1

(b) $r = -3\sqrt{2}$

Example 1

(b) $r = -3\sqrt{2}$

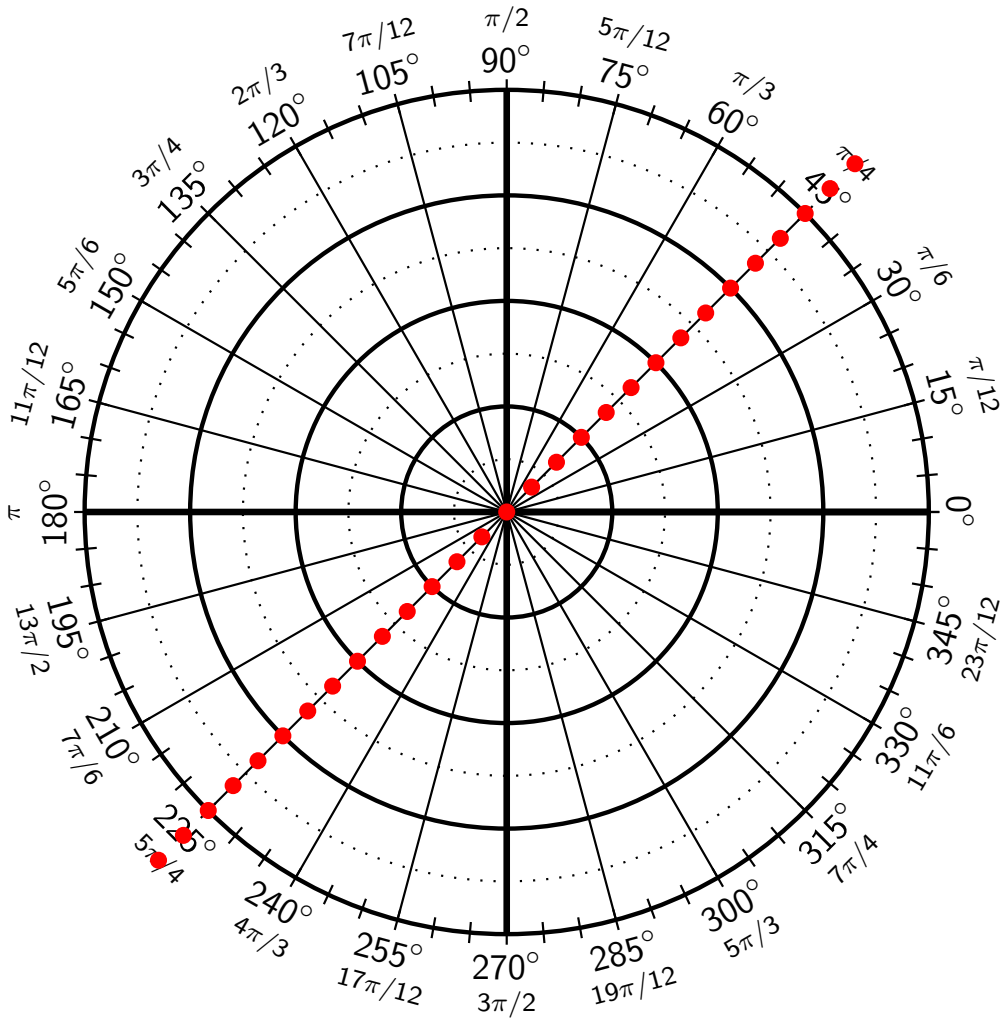


Example 1

(c) $\theta = \dfrac{5\pi}{4}$

Example 1

(c) $\theta = \dfrac{5\pi}{4}$

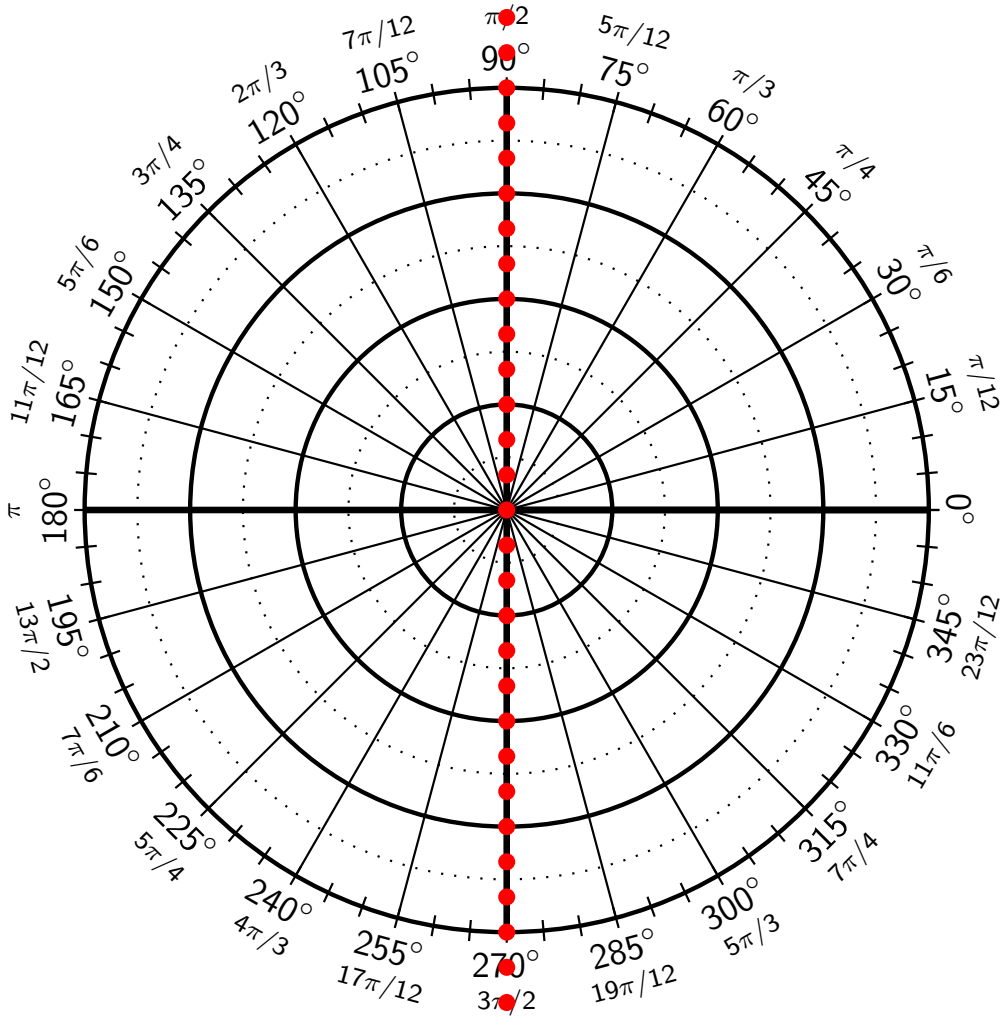


Example 1

(d) $\theta = -\dfrac{3\pi}{2}$

Example 1

(d) $\theta = -\dfrac{3\pi}{2}$



$r = \#$ and $\theta = \#$

- $r = \#$ is a circle with center at origin and radius $=$ that number.

$r = \#$ and $\theta = \#$

- $r = \#$ is a circle with center at origin and radius $=$ that number.
- $\theta = \#$ is a line through origin with slope $=$ tangent of that number.

Example 2

Graph each of the following and comment on the graph.

(a) $r = 6\cos\theta$

Example 2

Graph each of the following and comment on the graph.

(a) $r = 6\cos\theta$

- Circle

Example 2

Graph each of the following and comment on the graph.

(a) $r = 6\cos\theta$

- Circle
- Center at $(3, 0)$

Example 2

Graph each of the following and comment on the graph.

(a) $r = 6\cos\theta$

- Circle
- Center at $(3, 0)$
- Diameter $= 6$

Example 2

(b) $r = 4 - 2\sin\theta$

Example 2

(b) $r = 4 - 2\sin\theta$

- Limaçon

Example 2

(b) $r = 4 - 2\sin\theta$

- Limaçon
- x-intercepts at $(\pm 4, 0)$

Example 2

(b) $r = 4 - 2\sin\theta$

- Limaçon
- x-intercepts at $(\pm 4, 0)$
- y-intercepts at $(0, 2)$ and $(0, -6)$

Example 2

(c) $r = 2 + 4\cos\theta$

Example 2

(c) $r = 2 + 4\cos\theta$

- Limaçon

Example 2

(c) $r = 2 + 4\cos\theta$

- Limaçon
- x-intercepts at $(0, 0)$, $(2, 0)$ and $(6, 0)$

Example 2

(c) $r = 2 + 4\cos\theta$

- Limaçon
- x-intercepts at $(0, 0)$, $(2, 0)$ and $(6, 0)$
- y-intercepts at $(0, \pm 2)$

Example 2

(c) $r = 2 + 4\cos\theta$

- Limaçon
- x-intercepts at $(0, 0)$, $(2, 0)$ and $(6, 0)$
- y-intercepts at $(0, \pm 2)$
- Inner-loop diameter $= 2$

Example 2

(c) $r = 2 + 4\cos\theta$

- Limaçon
- x-intercepts at $(0, 0)$, $(2, 0)$ and $(6, 0)$
- y-intercepts at $(0, \pm 2)$
- Inner-loop diameter $= 2$
- Outer-loop diameter $= 6$

Example 2

(d) $r = 5\sin(2\theta)$

Example 2

(d) $r = 5\sin(2\theta)$

- Rose

Example 2

(d) $r = 5\sin(2\theta)$

- Rose
- Radius $= 5$

Example 2

(d) $r = 5\sin(2\theta)$

- Rose
- Radius $= 5$
- 4 petals

Example 2

(d) $r = 5\sin(2\theta)$

- Rose
- Radius $= 5$
- 4 petals
- No petals on an axis

Example 2

(e) $r^2 = 16\cos(2\theta)$

Example 2

(e) $r^2 = 16\cos(2\theta)$

$$r = \pm 4\sqrt{\cos(2\theta)}$$

Example 2

(e) $r^2 = 16\cos(2\theta)$

$$r = \pm 4\sqrt{\cos(2\theta)}$$

- Lemniscate

Example 2

(e) $r^2 = 16\cos(2\theta)$

$$r = \pm 4\sqrt{\cos(2\theta)}$$

- Lemniscate
- x-intercepts at $(0, 0)$ and $(\pm 4, 0)$

Example 2

(e) $r^2 = 16\cos(2\theta)$

$$r = \pm 4\sqrt{\cos(2\theta)}$$

- Lemniscate
- x-intercepts at $(0, 0)$ and $(\pm 4, 0)$
- y-intercept at $(0, 0)$

Objectives

1. Graph polar equations

2. Find the intersection of polar equations

Finding intersections of polar equations

Solving these will use elements of solving trigonometric equations.

Finding intersections of polar equations

Solving these will use elements of solving trigonometric equations.

Use graphing capabilities to check for intersection at the pole (origin).

Example 2

Find all exact points of intersection for each pair of equations.

(a) $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

Example 2

Find all exact points of intersection for each pair of equations.

(a) $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

Graphs intersect at pole.

Example 2

Find all exact points of intersection for each pair of equations.

(a) $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

Graphs intersect at pole.

$$2\sin\theta = 2 - 2\sin\theta$$

Example 2

Find all exact points of intersection for each pair of equations.

(a) $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

Graphs intersect at pole.

$$2\sin\theta = 2 - 2\sin\theta$$

$$4\sin\theta = 2$$

Example 2

Find all exact points of intersection for each pair of equations.

(a) $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

Graphs intersect at pole.

$$2\sin\theta = 2 - 2\sin\theta$$

$$4\sin\theta = 2$$

$$\sin\theta = \frac{1}{2}$$

Example 2

Find all exact points of intersection for each pair of equations.

(a) $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

Graphs intersect at pole.

$$2\sin\theta = 2 - 2\sin\theta$$

$$4\sin\theta = 2$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Example 2 $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Example 2 $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$r = 2\sin\left(\frac{\pi}{6}\right) \qquad r = 2\sin\left(\frac{5\pi}{6}\right)$$

Example 2 $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$r = 2\sin\left(\frac{\pi}{6}\right) \qquad r = 2\sin\left(\frac{5\pi}{6}\right)$$

$$r = 1 \qquad r = 1$$

Example 2 $r = 2\sin\theta$ and $r = 2 - 2\sin\theta$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$r = 2\sin\left(\frac{\pi}{6}\right) \qquad r = 2\sin\left(\frac{5\pi}{6}\right)$$

$$r = 1 \qquad r = 1$$

$$\text{pole}, \quad \left(1, \frac{\pi}{6}\right), \quad \left(1, \frac{5\pi}{6}\right)$$

Example 2

(b) $r = 2$ and $r = 3\cos\theta$

Example 2

(b) $r = 2$ and $r = 3\cos\theta$

$$3\cos\theta = 2$$

Example 2

(b) $r = 2$ and $r = 3\cos\theta$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

Example 2

(b) $r = 2$ and $r = 3\cos\theta$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \arccos\left(\frac{2}{3}\right)$$

Example 2

(b) $r = 2$ and $r = 3\cos\theta$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \arccos\left(\frac{2}{3}\right) \approx 48.2^\circ$$

Example 2

(b) $r = 2$ and $r = 3\cos\theta$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \arccos\left(\frac{2}{3}\right) \approx 48.2^\circ$$

$(2, 48.2^\circ)$ $(2, 311.8^\circ)$

Example 2

(b) $r = 2$ and $r = 3\cos\theta$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \arccos\left(\frac{2}{3}\right) \approx 48.2^\circ$$

$(2, 48.2^\circ)$ $(2, 311.8^\circ)$

$\left(2, \arccos\left(\frac{2}{3}\right)\right)$ $\left(2, 2\pi - \arccos\left(\frac{2}{3}\right)\right)$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = 3$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = 3$$

$$\cos(2\theta) = \frac{1}{2}$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = 3$$

$$\cos(2\theta) = \frac{1}{2}$$

$$2\theta = 60^\circ + 360n \qquad 2\theta = 300^\circ + 360n$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = 3$$

$$\cos(2\theta) = \frac{1}{2}$$

$$2\theta = 60^\circ + 360n \qquad 2\theta = 300^\circ + 360n$$

$$\theta = 30^\circ + 180n \qquad \theta = 150^\circ + 180n$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = 3$$

$$\cos(2\theta) = \frac{1}{2}$$

$$2\theta = 60^\circ + 360n \qquad 2\theta = 300^\circ + 360n$$

$$\theta = 30^\circ + 180n \qquad \theta = 150^\circ + 180n$$

$$\theta = 30^\circ, 210^\circ \qquad \theta = 150^\circ, 330^\circ$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = 3$$

$$\cos(2\theta) = \frac{1}{2}$$

$$2\theta = 60^\circ + 360n \qquad 2\theta = 300^\circ + 360n$$

$$\theta = 30^\circ + 180n \qquad \theta = 150^\circ + 180n$$

$$\theta = 30^\circ, 210^\circ \qquad \theta = 150^\circ, 330^\circ$$

$$(3, 30^\circ), \quad (3, 150^\circ), \quad (3, 210^\circ), \quad (3, 330^\circ)$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = -3$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = -3$$

$$\cos(2\theta) = -\frac{1}{2}$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = -3$$

$$\cos(2\theta) = -\frac{1}{2}$$

$$2\theta = 120^\circ + 360n \qquad 2\theta = 240^\circ + 360n$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = -3$$

$$\cos(2\theta) = -\frac{1}{2}$$

$$2\theta = 120^\circ + 360n \qquad 2\theta = 240^\circ + 360n$$

$$\theta = 60^\circ + 180n \qquad \theta = 120^\circ + 180n$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = -3$$

$$\cos(2\theta) = -\frac{1}{2}$$

$$2\theta = 120^\circ + 360n \qquad 2\theta = 240^\circ + 360n$$

$$\theta = 60^\circ + 180n \qquad \theta = 120^\circ + 180n$$

$$\theta = 60^\circ, 240^\circ \qquad \theta = 120^\circ, 300^\circ$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$$6\cos(2\theta) = -3$$

$$\cos(2\theta) = -\frac{1}{2}$$

$$2\theta = 120^\circ + 360n \qquad 2\theta = 240^\circ + 360n$$

$$\theta = 60^\circ + 180n \qquad \theta = 120^\circ + 180n$$

$$\theta = 60^\circ,\ 240^\circ \qquad \theta = 120^\circ,\ 300^\circ$$

$$(-3, 60^\circ), \quad (-3, 120^\circ), \quad (-3, 240^\circ), \quad (-3, 300^\circ)$$

Example 2

(c) $r = 3$ and $r = 6\cos(2\theta)$

$(3, 30^\circ)$, $(3, 150^\circ)$, $(3, 210^\circ)$, $(3, 330^\circ)$

$\left(3, \frac{\pi}{6}\right)$, $\left(3, \frac{5\pi}{6}\right)$, $\left(3, \frac{7\pi}{6}\right)$, $\left(3, \frac{11\pi}{6}\right)$

$(-3, 60^\circ)$, $(-3, 120^\circ)$, $(-3, 240^\circ)$, $(-3, 300^\circ)$

$\left(-3, \frac{\pi}{3}\right)$, $\left(-3, \frac{2\pi}{3}\right)$, $\left(-3, \frac{4\pi}{3}\right)$, $\left(-3, \frac{5\pi}{3}\right)$