Rotation of Axes

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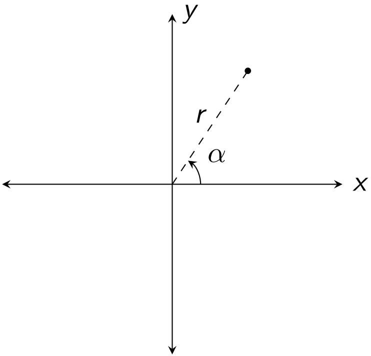
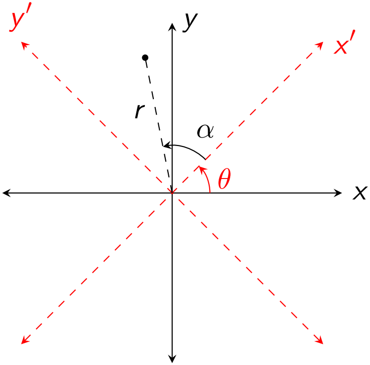
Intro

Normally, in the coordinate plane, each point has both an x- and y-coordinate; polar representation: (r, α).



The point makes an angle α with the x-axis.

Now, suppose we rotate the axes through the origin at an angle of θ:



Rotate the Axes

From the x'- and y' axes perspective, the point (x', y') has polar coordinates $(r\cos\alpha, r\sin\alpha)$.

Rotate the Axes

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From the x- and y-axes perspective, the point has polar coordinates:

$$x = r\cos(\theta + \alpha) \quad \text{and} \quad y = r\sin(\theta + \alpha)$$

Derivation

Expanding each of these using the angle sum identities for cosine and sine gives us

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$$\begin{aligned} x &= r\cos(\theta + \alpha) \\ &= r(\cos\theta)(\cos\alpha) - r(\sin\theta)(\sin\alpha) \end{aligned}$$

Derivation

Expanding each of these using the angle sum identities for cosine and sine gives us

$$\begin{aligned} x &= r\cos(\theta+\alpha) \\ &= r(\cos\theta)(\cos\alpha) - r(\sin\theta)(\sin\alpha) \\ &= x'\cos\theta - y'\sin\theta \quad (\text{since } x' = r\cos\alpha \text{ and } y' = r\sin\alpha) \end{aligned}$$

Derivation

Expanding each of these using the angle sum identities for cosine and sine gives us

$$
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\end{aligned}
$$

and

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$$y = r\sin(\theta+\alpha)$$

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$$\begin{aligned} y &= r\sin(\theta+\alpha) \\ &= r(\sin\theta)(\cos\alpha) + r(\sin\alpha)(\cos\theta) \end{aligned}$$

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and

$$\begin{aligned} y &= r\sin(\theta+\alpha) \\ &= r(\sin\theta)(\cos\alpha) + r(\sin\alpha)(\cos\theta) \\ &= x'\sin\theta + y'\cos\theta \quad (\text{since } x' = r\cos\alpha \text{ and } y' = r\sin\alpha) \end{aligned}$$

Derivation

So

$$\begin{cases} x &= x' \cos\theta - y' \sin\theta \\ y &= x' \sin\theta + y' \cos\theta \end{cases}$$

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$$\begin{cases} x &= x' \cos\theta - y' \sin\theta \\ y &= x' \sin\theta + y' \cos\theta \end{cases} \quad \text{and} \quad \begin{cases} x' &= x \cos\theta + y \sin\theta \\ y' &= -x \sin\theta + y \cos\theta \end{cases}$$

Note: The x' and y' cases can be found by replacing θ with $-\theta$:

$$x' = r\cos(\alpha - \theta) \text{ and } y' = r\sin(\alpha - \theta)$$

Derivation

Also Note: The matrix representations of (x, y) and (x', y') are below:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Example 1

Suppose the x- and y-axes are both rotated counter-clockwise through the angle $\theta = \dfrac{\pi}{3}$ to produce the x'- and y'-axes, respectively.

(a) Let $P(x, y) = (2, -4)$ and find $P(x', y')$.

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$$x' = 2\cos\left(\frac{\pi}{3}\right) + (-4)\sin\left(\frac{\pi}{3}\right) \qquad y' = -2\sin\left(\frac{\pi}{3}\right) + (-4)\cos\left(\frac{\pi}{3}\right)$$

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$x' = 2\cos\left(\dfrac{\pi}{3}\right) + (-4)\sin\left(\dfrac{\pi}{3}\right)$ $\qquad$ $y' = -2\sin\left(\dfrac{\pi}{3}\right) + (-4)\cos\left(\dfrac{\pi}{3}\right)$

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$$x' = 1 - 2\sqrt{3} \qquad y' = -\sqrt{3} - 2$$

$$P(x', y') = \left(1 - 2\sqrt{3}, -2 - \sqrt{3}\right)$$

Example 1

(b) Convert the equation $21x^2 + 10xy\sqrt{3} + 31y^2 = 144$ to an equation in x' and y'.

$$x = x' \cos\left(\frac{\pi}{3}\right) - y' \sin\left(\frac{\pi}{3}\right)$$

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$$21x^2 = \frac{21(x')^2}{4} - \frac{21(x')(y')\sqrt{3}}{2} + \frac{63(y')^2}{4}$$

Example 1 $\quad 21x^2 + 10xy\sqrt{3} + 31y^2 = 144$

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$$31y^2 = \frac{93(x')^2}{4} + \frac{31(x')(y')\sqrt{3}}{2} + \frac{31(y')^2}{4}$$

Example 1 $\quad 21x^2 + 10xy\sqrt{3} + 31y^2 = 144$

$$xy = \left(\frac{1}{2}x' - \frac{\sqrt{3}}{2}y'\right)\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)$$

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$$xy = \frac{(x')^2\sqrt{3}}{4} - \frac{(x')(y')}{2} - \frac{(y')^2\sqrt{3}}{4}$$

$$10xy\sqrt{3} = \frac{30(x')^2}{4} - \frac{10(x')(y')\sqrt{3}}{2} - \frac{30(y')^2}{4}$$

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$$
\begin{array}{ccccccc}
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+ & \dfrac{93(x')^2}{4} & + & \dfrac{31(x')(y')\sqrt{3}}{2} & + & \dfrac{31(y')^2}{4} \\
\hline
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\hline
 & 36(x')^2 & & & + & 16(y')^2 & = 144
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\hline
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\end{array}$$

$$\frac{(x')^2}{4} + \frac{(y')^2}{9} = 1$$

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Eliminating the xy-Term

Given an equation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $B \neq 0$, there exists an angle θ such that if we rotate the equation counter-clockwise by θ, the Bxy term will be eliminated.

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Substituting $x' \cos\theta - y' \sin\theta$ and $x' \sin\theta + y' \cos\theta$ for x and y, respectively, into $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, we get

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$$\begin{aligned} A(x' \cos\theta - y' \sin\theta)^2 + B(x' \cos\theta - y' \sin\theta)(x' \sin\theta + y' \cos\theta) \\ + C(x' \sin\theta + y' \cos\theta)^2 + D(x' \cos\theta - y' \sin\theta) \\ + E(x' \sin\theta + y' \cos\theta) + F = 0 \end{aligned}$$

Eliminating the xy-Term

Doing algebra, we get the following coefficient for $x'y'$:

$$= 2(C - A)\sin\theta\cos\theta + B(\cos^2\theta - \sin^2\theta)$$

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If we set this equal to 0, we get $(A - C)\sin(2\theta) = B\cos(2\theta)$ from which

$$\cot(2\theta) = \frac{A - C}{B}$$

Example 2

Find the smallest angle of rotation in order to rewrite each of the following without the xy-term.

(a) $5x^2 + 26xy + 5y^2 - 16x\sqrt{2} + 16y\sqrt{2} - 104 = 0$

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$$\theta = 45^\circ = \frac{\pi}{4}$$

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$$\theta \approx 36.9^\circ$$

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Given that $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is a non-degenerate conic section:

- If $B^2 - 4AC > 0$, then the graph is a hyperbola.

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Given that $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is a non-degenerate conic section:

- If $B^2 - 4AC > 0$, then the graph is a hyperbola.
- If $B^2 - 4AC = 0$, then the graph is a parabola.
- If $B^2 - 4AC < 0$, then the graph is an ellipse or circle.

Example 3

Classify each of the following.

(a) $21x^2 + 10xy\sqrt{3} + 31y^2 = 144$

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Classify each of the following.

(a) $21x^2 + 10xy\sqrt{3} + 31y^2 = 144$

$A = 21 \quad B = 10\sqrt{3} \quad C = 31$

$B^2 - 4AC = (10\sqrt{3})^2 - 4(21)(31) = -2304$

Equation is an ellipse.

Example 3

Classify each of the following.

(b) $5x^2 + 26xy + 5y^2 - 16x\sqrt{2} + 16y\sqrt{2} - 104 = 0$

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(b) $5x^2 + 26xy + 5y^2 - 16x\sqrt{2} + 16y\sqrt{2} - 104 = 0$

$A = 5 \quad B = 26 \quad C = 5$

Example 3

Classify each of the following.

(b) $5x^2 + 26xy + 5y^2 - 16x\sqrt{2} + 16y\sqrt{2} - 104 = 0$

$A = 5 \quad B = 26 \quad C = 5$

$B^2 - 4AC = 26^2 - 4(5)(5) = 576$

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Classify each of the following.

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$B^2 - 4AC = 26^2 - 4(5)(5) = 576$

Equation is a hyperbola.

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Classify each of the following.

(c) $16x^2 + 24xy + 9y^2 + 15x - 20y = 0$

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Equation is a parabola.