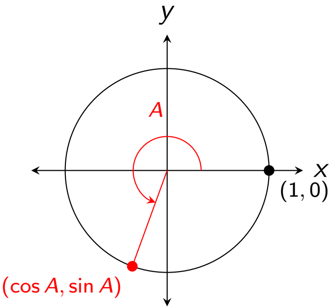
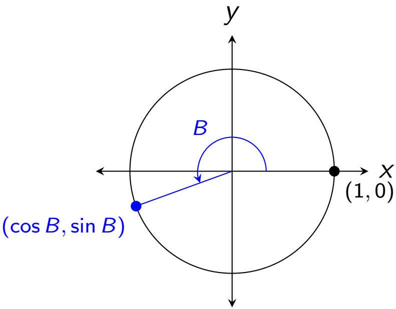
Angle Sum and Difference Identities

Objectives

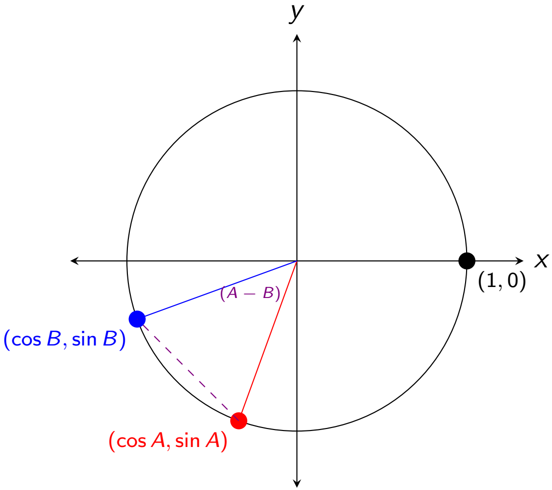
1. Solve problems using angle sum and difference identities

$\cos(A - B)$





$\cos(A - B)$



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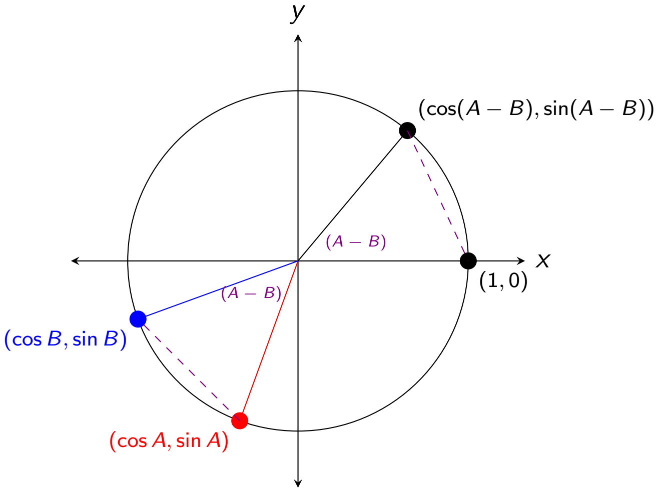

Distance Between $(\cos A, \sin A)$ and $(\cos B, \sin B)$

$$\sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2}$$

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$$= \sqrt{\cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B}$$

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$$= \sqrt{\cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B}$$

$$= \sqrt{1 - 2\cos A \cos B - 2\sin A \sin B + 1}$$

Distance Between $(\cos A, \sin A)$ and $(\cos B, \sin B)$

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$$= \sqrt{1 - 2\cos A \cos B - 2\sin A \sin B + 1}$$

$$= \sqrt{2 - 2\cos A \cos B - 2\sin A \sin B}$$

Distance between $(\cos(A-B), \sin(A-B))$ and $(1,0)$

$$\sqrt{(\cos(A-B)-1)^2+(\sin(A-B)-0)^2}$$

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$$= \sqrt{\cos^2(A-B)-2\cos(A-B)+1+\sin^2(A-B)}$$

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$$= \sqrt{\cos^2(A-B) - 2\cos(A-B) + 1 + \sin^2(A-B)}$$

$$= \sqrt{1 - 2\cos(A-B) + 1}$$

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$$= \sqrt{\cos^2(A-B)-2\cos(A-B)+1+\sin^2(A-B)}$$

$$= \sqrt{1-2\cos(A-B)+1}$$

$$= \sqrt{2-2\cos(A-B)}$$

Equal distances

$$\sqrt{2 - 2\cos(A - B)} = \sqrt{2 - 2\cos A \cos B - 2\sin A \sin B}$$

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$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

Example 1

Find the *exact* value of $\cos 15^\circ$.

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Find the *exact* value of $\cos 15^\circ$.

$$\begin{aligned}
\cos 15^\circ &= \cos(45^\circ - 30^\circ) \\
&= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ \\
&= \frac{\sqrt{2}}{2} \quad \cdot \quad \frac{\sqrt{3}}{2} \quad + \quad \frac{\sqrt{2}}{2} \quad \cdot \quad \frac{1}{2}
\end{aligned}$$

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Find the *exact* value of $\cos 15^\circ$.

$$\begin{aligned}\cos 15^\circ &= \cos(45^\circ - 30^\circ)\\ &= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ\\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\end{aligned}$$

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\cos 15^\circ &= \cos(45^\circ - 30^\circ) \\
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&= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\
&= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\
&= \frac{\sqrt{6} + \sqrt{2}}{4}
\end{aligned}$$

Example 2

Verify $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

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$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos\frac{\pi}{2} \cdot \cos\theta + \sin\frac{\pi}{2} \cdot \sin\theta$$

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Verify $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos\frac{\pi}{2} \cdot \cos\theta + \sin\frac{\pi}{2} \cdot \sin\theta$$

$$= 0\cos\theta + 1\sin\theta$$

Example 2

Verify $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$$\begin{aligned}
\cos\left(\frac{\pi}{2} - \theta\right) &= \cos\frac{\pi}{2} \cdot \cos\theta + \sin\frac{\pi}{2} \cdot \sin\theta \\
&= 0\cos\theta + 1\sin\theta \\
&= \sin\theta
\end{aligned}$$

Example 3

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\cos(-\theta) &= \cos(0 - \theta) \\
&= \cos 0 \cdot \cos\theta + \sin 0 \cdot \sin\theta \\
&= 1\cos\theta + 0\sin\theta \\
&= \cos\theta
\end{aligned}$$

Additional Angle Sum and Difference Identities

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$**\text{Note:} \quad \tan(A \pm B) = \frac{\sin(A \pm B)}{\cos(A \pm B)}$$

Example 4

If α is a Quadrant II angle with $\sin\alpha = \frac{5}{13}$ and β is a Quadrant III angle with $\tan\beta = 2$, find each of the following.

(a) $\sin(\alpha - \beta)$

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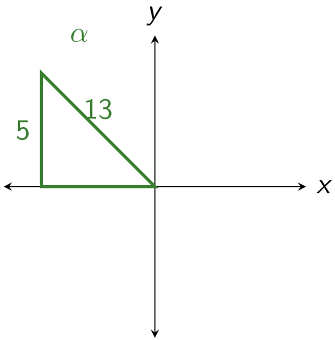

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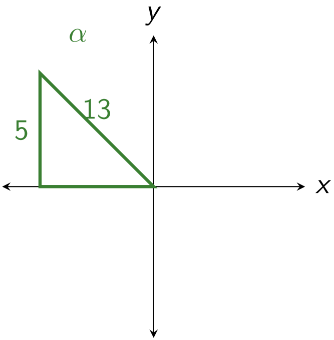

$$x^2 + 5^2 = 13^2$$

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If α is a Quadrant II angle with $\sin\alpha = \frac{5}{13}$ and β is a Quadrant III angle with $\tan\beta = 2$, find each of the following.

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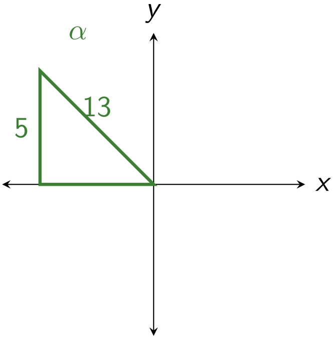

$$x^2 + 5^2 = 13^2$$
$$x = -12$$

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If α is a Quadrant II angle with $\sin\alpha = \frac{5}{13}$ and β is a Quadrant III angle with $\tan\beta = 2$, find each of the following.

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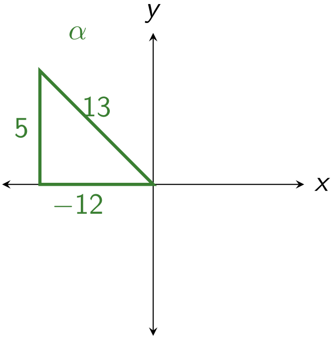

$$x^2 + 5^2 = 13^2$$
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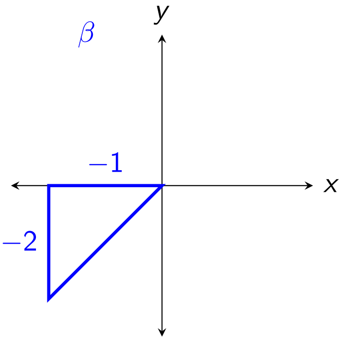
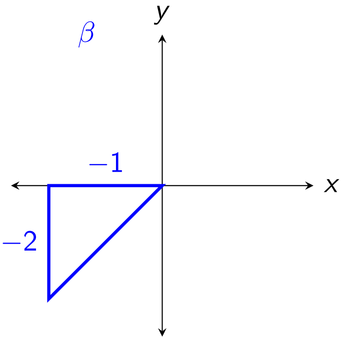
Example 4

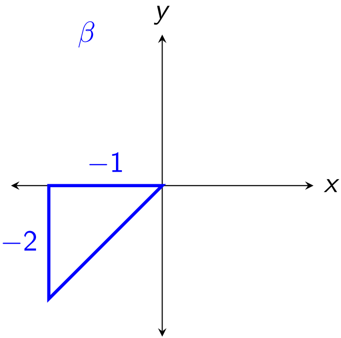


Example 4



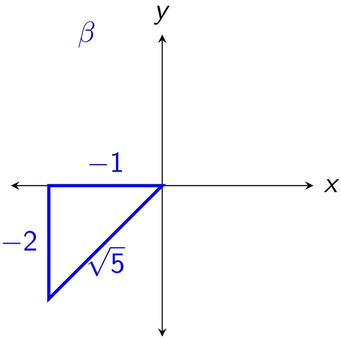
$$1^2 + 2^2 = r^2$$

Example 4



$$1^2 + 2^2 = r^2$$
$$r = \sqrt{5}$$

Example 4



$$1^2 + 2^2 = r^2$$
$$r = \sqrt{5}$$

Example 4

α	β
$x = -12$	$x = -1$
$y = 5$	$y = -2$
$r = 13$	$r = \sqrt{5}$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(a) $\sin(\alpha - \beta)$

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(a) $\sin(\alpha - \beta)$

$$\sin(\alpha - \beta) = (\sin \alpha)(\cos \beta) - (\sin \beta)(\cos \alpha)$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(a) $\sin(\alpha - \beta)$

$$\begin{aligned}\sin(\alpha - \beta) &= (\sin\alpha)(\cos\beta) - (\sin\beta)(\cos\alpha) \\ &= \left(\frac{5}{13}\right)\left(\frac{-1}{\sqrt{5}}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{-12}{13}\right)\end{aligned}$$

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Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(a) $\sin(\alpha - \beta)$

$$
\begin{aligned}
\sin(\alpha - \beta) &= (\sin\alpha)(\cos\beta) - (\sin\beta)(\cos\alpha) \\
&= \left(\frac{5}{13}\right)\left(\frac{-1}{\sqrt{5}}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{-12}{13}\right) \\
&= \left(\frac{5}{13}\right)\left(\frac{-\sqrt{5}}{5}\right) - \left(\frac{-2\sqrt{5}}{5}\right)\left(\frac{-12}{13}\right) \\
&= \frac{-5\sqrt{5}}{65} - \frac{24\sqrt{5}}{65}
\end{aligned}
$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(a) $\sin(\alpha - \beta)$

$$\begin{aligned}
\sin(\alpha - \beta) &= (\sin\alpha)(\cos\beta) - (\sin\beta)(\cos\alpha) \\
&= \left(\frac{5}{13}\right)\left(\frac{-1}{\sqrt{5}}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{-12}{13}\right) \\
&= \left(\frac{5}{13}\right)\left(\frac{-\sqrt{5}}{5}\right) - \left(\frac{-2\sqrt{5}}{5}\right)\left(\frac{-12}{13}\right) \\
&= \frac{-5\sqrt{5}}{65} - \frac{24\sqrt{5}}{65} \\
&= \frac{-29\sqrt{5}}{65}
\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(b) $\sin(\alpha + \beta)$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(b) $\sin(\alpha + \beta)$

$$\sin(\alpha + \beta) = -\frac{5\sqrt{5}}{65} + \frac{24\sqrt{5}}{65}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(b) $\sin(\alpha + \beta)$

$$\sin(\alpha+\beta) = -\frac{5\sqrt{5}}{65} + \frac{24\sqrt{5}}{65}$$

$$= \frac{19\sqrt{5}}{65}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(c) $\cos(\alpha - \beta)$

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(c) $\cos(\alpha - \beta)$

$$\cos(\alpha - \beta) = (\cos\alpha)(\cos\beta) + (\sin\alpha)(\sin\beta)$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(c) $\cos(\alpha - \beta)$

$$\begin{aligned}\cos(\alpha - \beta) &= (\cos\alpha)(\cos\beta) + (\sin\alpha)(\sin\beta) \\ &= \left(\frac{-12}{13}\right)\left(\frac{-\sqrt{5}}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{-2\sqrt{5}}{5}\right)\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

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$$\begin{aligned}
\cos(\alpha - \beta) &= (\cos\alpha)(\cos\beta) + (\sin\alpha)(\sin\beta) \\
&= \left(\frac{-12}{13}\right)\left(\frac{-\sqrt{5}}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{-2\sqrt{5}}{5}\right) \\
&= \frac{12\sqrt{5}}{65} + \frac{-10\sqrt{5}}{65}
\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

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$$\begin{aligned}\cos(\alpha - \beta) &= (\cos\alpha)(\cos\beta) + (\sin\alpha)(\sin\beta) \\ &= \left(\frac{-12}{13}\right)\left(\frac{-\sqrt{5}}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{-2\sqrt{5}}{5}\right) \\ &= \frac{12\sqrt{5}}{65} + \frac{-10\sqrt{5}}{65}\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(c) $\cos(\alpha - \beta)$

$$\begin{aligned}
\cos(\alpha - \beta) &= (\cos\alpha)(\cos\beta) + (\sin\alpha)(\sin\beta) \\
&= \left(\frac{-12}{13}\right)\left(\frac{-\sqrt{5}}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{-2\sqrt{5}}{5}\right) \\
&= \frac{12\sqrt{5}}{65} + \frac{-10\sqrt{5}}{65} \\
&= \frac{2\sqrt{5}}{65}
\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(d) $\cos(\alpha + \beta)$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(d) $\cos(\alpha + \beta)$

$$\cos(\alpha + \beta) = \frac{12\sqrt{5}}{65} - \frac{-10\sqrt{5}}{65}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(d) $\cos(\alpha + \beta)$

$$\begin{aligned} \cos(\alpha + \beta) &= \frac{12\sqrt{5}}{65} - \frac{-10\sqrt{5}}{65} \\ &= \frac{22\sqrt{5}}{65} \end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + (\tan\alpha)(\tan\beta)}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + (\tan\alpha)(\tan\beta)}$$

$$= \frac{\frac{-5}{12} - 2}{1 + \left(\frac{-5}{12}\right)(2)}$$

(e) $\tan(\alpha - \beta)$

$$\begin{aligned}
\tan(\alpha - \beta) &= \frac{\tan\alpha - \tan\beta}{1 + (\tan\alpha)(\tan\beta)} \\
&= \frac{\frac{-5}{12} - 2}{1 + \left(\frac{-5}{12}\right)(2)} \\
&= \frac{\frac{-29}{12}}{\frac{2}{12}}
\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

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\tan(\alpha - \beta) &= \frac{\tan\alpha - \tan\beta}{1 + (\tan\alpha)(\tan\beta)} \\
&= \frac{\frac{-5}{12} - 2}{1 + \left(\frac{-5}{12}\right)(2)} \\
&= \frac{\frac{-29}{12}}{\frac{2}{12}} \left(\frac{12}{12}\right)
\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

$$\begin{aligned}
\tan(\alpha-\beta) &= \frac{\tan\alpha - \tan\beta}{1 + (\tan\alpha)(\tan\beta)} \\
&= \frac{\frac{-5}{12} - 2}{1 + \left(\frac{-5}{12}\right)(2)} \\
&= \frac{\frac{-29}{12}}{\frac{2}{12}}\left(\frac{12}{12}\right) \\
&= \frac{-29}{2}
\end{aligned}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

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(e) $\tan(\alpha - \beta)$

$$\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

$$\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$$

$$= \frac{\frac{-29\sqrt{5}}{65}}{\frac{2\sqrt{5}}{65}}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

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$$\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$$

$$= \frac{\frac{-29\sqrt{5}}{65}}{\frac{2\sqrt{5}}{65}} \left(\frac{65}{65}\right)$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

(e) $\tan(\alpha - \beta)$

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$= \frac{\frac{-29\sqrt{5}}{65}}{\frac{2\sqrt{5}}{65}}\left(\frac{65}{65}\right)$$

$$= \frac{-29\sqrt{5}}{2\sqrt{5}}$$

Example 4 $x_\alpha = -12, y_\alpha = 5, r_\alpha = 13 \quad x_\beta = -1, y_\beta = -2, r_\beta = \sqrt{5}$

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$$\begin{aligned}
\tan(\alpha-\beta) &= \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} \\
&= \frac{\frac{-29\sqrt{5}}{65}}{\frac{2\sqrt{5}}{65}}\left(\frac{65}{65}\right) \\
&= \frac{-29\sqrt{5}}{2\sqrt{5}} \\
&= \frac{-29}{2}
\end{aligned}$$