Vectors

Table of Contents

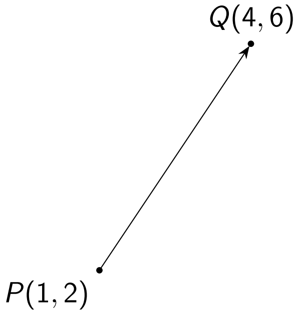
Intro

A **vector** is a mathematical object with both magnitude (length) and direction.

It is represented geometrically by a directed line segment.

Intro

The following shows an example of a vector $\vec{v} = \overrightarrow{PQ}$.



P is the initial point and Q is the terminal point.

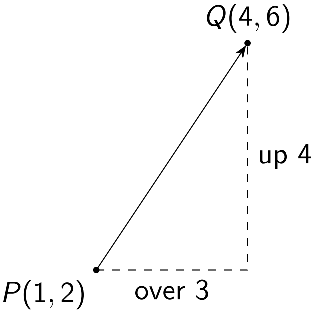
Magnitude

The magnitude of a vector (denoted $\|\vec{v}\|$ or $|\vec{v}|$) is the distance between P and Q.

Think Pythagorean Theorem.

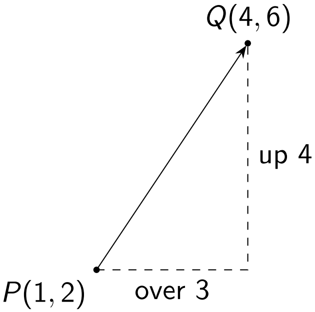
Component Form

If we connect the point P to point Q we get the following diagram:



Component Form

If we connect the point P to point Q we get the following diagram:



This can be represented by $\vec{v} = \langle 3, 4 \rangle$

Component Form

To find the component form for $P(x_0, y_0)$ and $Q(x_1, y_1)$, just take the difference in coordinates:

$$\vec{v} = \overrightarrow{PQ} = \langle x_1 - x_0, y_1 - y_0 \rangle$$

Example 1a

Find the component form and exact magnitude of $\overrightarrow{AB}$ for each.

(a) $A(2, -1) \quad B(-5, 3)$

Example 1a

Find the component form and exact magnitude of $\overrightarrow{AB}$ for each.

(a) $A(2, -1) \quad B(-5, 3)$

$$= \langle -5 - 2, 3 - (-1) \rangle$$

Example 1a

Find the component form and exact magnitude of $\overrightarrow{AB}$ for each.

(a) $A(2,-1) \quad B(-5,3)$

$$= \langle -5-2, 3-(-1) \rangle$$

$$= \langle -7, 4 \rangle$$

Example 1a

Find the component form and exact magnitude of $\overrightarrow{AB}$ for each.

(a) $A(2,-1) \quad B(-5,3)$

$$= \langle -5-2, 3-(-1) \rangle$$

$$= \langle -7, 4 \rangle$$

$$\|\overrightarrow{AB}\| = \sqrt{7^2+4^2}$$

Example 1a

Find the component form and exact magnitude of $\overrightarrow{AB}$ for each.

(a) $A(2,-1)$ $B(-5,3)$

$$= \langle -5-2, 3-(-1)\rangle$$

$$= \langle -7, 4\rangle$$

$$\|\overrightarrow{AB}\| = \sqrt{7^2+4^2}$$

$$= \sqrt{49+16}$$

Example 1a

Find the component form and exact magnitude of $\overrightarrow{AB}$ for each.

(a) $A(2, -1)$ $B(-5, 3)$

$$= \langle -5 - 2, 3 - (-1) \rangle$$

$$= \langle -7, 4 \rangle$$

$$\|\overrightarrow{AB}\| = \sqrt{7^2 + 4^2}$$

$$= \sqrt{49 + 16}$$

$$= \sqrt{65}$$

Example 1b

(b) $A(-3, 2) \quad B(0, 1)$

Example 1b

(b) $A(-3,2)$ $\quad B(0,1)$

$$= \langle 0-(-3), 1-2 \rangle$$

Example 1b

(b) $A(-3,2) \quad B(0,1)$

$$= \langle 0-(-3), 1-2 \rangle$$

$$= \langle 3, -1 \rangle$$

Example 1b

(b) $A(-3,2) \quad B(0,1)$

$$= \langle 0-(-3), 1-2\rangle$$

$$= \langle 3, -1\rangle$$

$$\|\overrightarrow{AB}\| = \sqrt{3^2+1^2}$$

Example 1b

(b) $A(-3,2) \quad B(0,1)$

$$
\begin{aligned}
&= \langle 0-(-3), 1-2\rangle \\
&= \langle 3, -1\rangle \\
\|\overrightarrow{AB}\| &= \sqrt{3^2+1^2} \\
&= \sqrt{10}
\end{aligned}
$$

Table of Contents

Operation Properties of Vectors

The component form of vectors allows us to justify the following properties for $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$:

Operation Properties of Vectors

The component form of vectors allows us to justify the following properties for $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$:

- $\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$

Operation Properties of Vectors

The component form of vectors allows us to justify the following properties for $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$:

- $\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$
- $\vec{v} - \vec{w} = \langle v_1 - w_1, v_2 - w_2 \rangle$

Operation Properties of Vectors

The component form of vectors allows us to justify the following properties for $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$:

- $\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$
- $\vec{v} - \vec{w} = \langle v_1 - w_1, v_2 - w_2 \rangle$
- $\vec{v} + \vec{w} = \vec{w} + \vec{v}$ (Commutative Prop.)

Operation Properties of Vectors

The component form of vectors allows us to justify the following properties for $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$:

- $\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$
- $\vec{v} - \vec{w} = \langle v_1 - w_1, v_2 - w_2 \rangle$
- $\vec{v} + \vec{w} = \vec{w} + \vec{v}$ (Commutative Prop.)
- $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (Associative Prop.)

Operation Properties of Vectors

The component form of vectors allows us to justify the following properties for $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$:

- $\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$
- $\vec{v} - \vec{w} = \langle v_1 - w_1, v_2 - w_2 \rangle$
- $\vec{v} + \vec{w} = \vec{w} + \vec{v}$ (Commutative Prop.)
- $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (Associative Prop.)
- $\vec{v} + \vec{0} = \vec{v}$ (Identity Property)

Operation Properties of Vectors

The component form of vectors allows us to justify the following properties for $\vec{v} = \langle v_1, v_2 \rangle$ and $\vec{w} = \langle w_1, w_2 \rangle$:

- $\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2 \rangle$
- $\vec{v} - \vec{w} = \langle v_1 - w_1, v_2 - w_2 \rangle$
- $\vec{v} + \vec{w} = \vec{w} + \vec{v}$ (Commutative Prop.)
- $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (Associative Prop.)
- $\vec{v} + \vec{0} = \vec{v}$ (Identity Property)
- $\vec{v} + (-\vec{v}) = 0$ (Inverse Prop.)

Example 2

Let $\vec{v} = \langle 3, 4 \rangle$ and suppose $w = \overrightarrow{PQ}$ where $P(-3, 7)$ and $Q(-2, 5)$. Find $\vec{v} + \vec{w}$ and interpret the result geometrically.

Example 2

Let $\vec{v} = \langle 3, 4 \rangle$ and suppose $w = \overrightarrow{PQ}$ where $P(-3, 7)$ and $Q(-2, 5)$. Find $\vec{v} + \vec{w}$ and interpret the result geometrically.

$$\vec{w} = \langle -2 - (-3), 5 - 7 \rangle$$

Example 2

Let $\vec{v} = \langle 3, 4 \rangle$ and suppose $w = \overrightarrow{PQ}$ where $P(-3, 7)$ and $Q(-2, 5)$. Find $\vec{v} + \vec{w}$ and interpret the result geometrically.

$$\vec{w} = \langle -2 - (-3), 5 - 7 \rangle$$

$$= \langle 1, -2 \rangle$$

Example 2

Let $\vec{v} = \langle 3, 4 \rangle$ and suppose $w = \overrightarrow{PQ}$ where $P(-3, 7)$ and $Q(-2, 5)$. Find $\vec{v} + \vec{w}$ and interpret the result geometrically.

$$\vec{w} = \langle -2 - (-3), 5 - 7 \rangle$$

$$= \langle 1, -2 \rangle$$

$$\vec{v} + \vec{w} = \langle 3 + 1, 4 - 2 \rangle$$

Example 2

Let $\vec{v} = \langle 3, 4 \rangle$ and suppose $w = \overrightarrow{PQ}$ where $P(-3, 7)$ and $Q(-2, 5)$. Find $\vec{v} + \vec{w}$ and interpret the result geometrically.

$$\vec{w} = \langle -2 - (-3), 5 - 7 \rangle$$

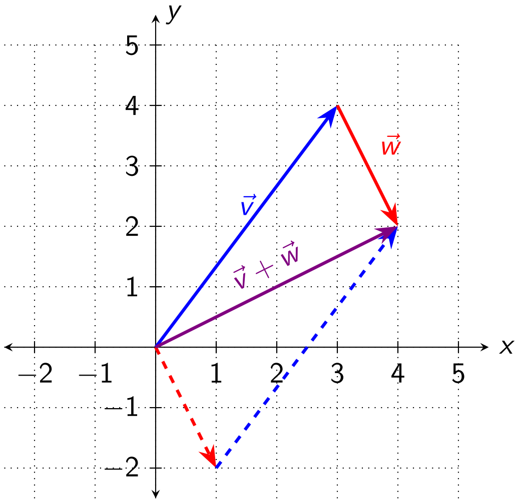
$$= \langle 1, -2 \rangle$$

$$\vec{v} + \vec{w} = \langle 3 + 1, 4 - 2 \rangle$$

$$= \langle 4, 2 \rangle$$

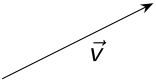
Example 2

Geometrically, $\vec{v} + \vec{w}$ is as follows:



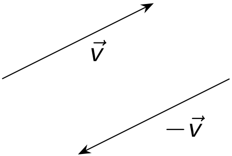
Scalar Multiplication

If we multiply our vector by a real number (a scalar), we get a new vector.



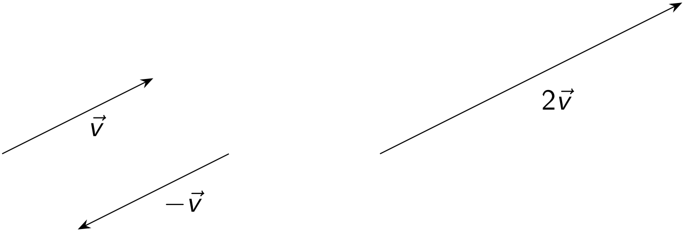
Scalar Multiplication

If we multiply our vector by a real number (a scalar), we get a new vector.



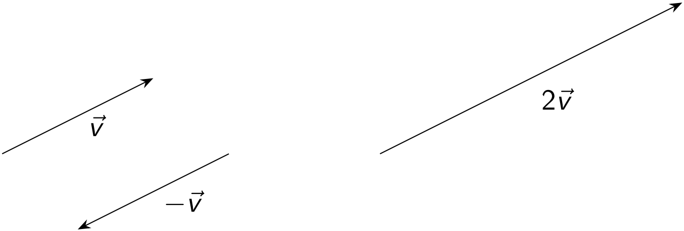
Scalar Multiplication

If we multiply our vector by a real number (a scalar), we get a new vector.



Scalar Multiplication

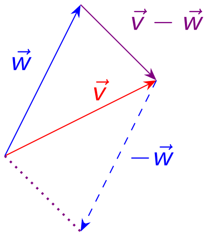
If we multiply our vector by a real number (a scalar), we get a new vector.



Thus, $k\vec{v} = k\langle v_1, v_2\rangle = \langle kv_1, kv_2\rangle$ for all real numbers k.

Vector Subtraction

Vector subtraction $\vec{v} - \vec{w}$ can be thought of as $\vec{v} + (-\vec{w})$ and is illustrated below:



Scalar Properties

- $(kr)\vec{v} = k(r\vec{v})$ (Associative Prop.)

Scalar Properties

- $(kr)\vec{v} = k(r\vec{v})$ (Associative Prop.)
- $1\vec{v} = \vec{v}$ (Identity Prop.)

Scalar Properties

- $(kr)\vec{v} = k(r\vec{v})$ (Associative Prop.)
- $1\vec{v} = \vec{v}$ (Identity Prop.)
- $-\vec{v} = -1\vec{v}$ (Additive Inverse Prop.)

Scalar Properties

- $(kr)\vec{v} = k(r\vec{v})$ (Associative Prop.)
- $1\vec{v} = \vec{v}$ (Identity Prop.)
- $-\vec{v} = -1\vec{v}$ (Additive Inverse Prop.)
- $(k + r)\vec{v} = k\vec{v} + r\vec{v}$ (Distributive Prop. over Scalar Addition)

Scalar Properties

- $(kr)\vec{v} = k(r\vec{v})$ (Associative Prop.)
- $1\vec{v} = \vec{v}$ (Identity Prop.)
- $-\vec{v} = -1\vec{v}$ (Additive Inverse Prop.)
- $(k+r)\vec{v} = k\vec{v} + r\vec{v}$ (Distributive Prop. over Scalar Addition)
- $k(\vec{v} + \vec{w}) = k\vec{v} + k\vec{w}$ (Distributive Prop. over Vector Addition)

Scalar Properties

- $(kr)\vec{v} = k(r\vec{v})$ (Associative Prop.)
- $1\vec{v} = \vec{v}$ (Identity Prop.)
- $-\vec{v} = -1\vec{v}$ (Additive Inverse Prop.)
- $(k+r)\vec{v} = k\vec{v} + r\vec{v}$ (Distributive Prop. over Scalar Addition)
- $k(\vec{v} + \vec{w}) = k\vec{v} + k\vec{w}$ (Distributive Prop. over Vector Addition)
- $k\vec{v} = 0$ if and only if $k = 0$ or $\vec{v} = 0$ (Zero Product Prop.)

Table of Contents

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2 \rangle) = \vec{0}$ for $\vec{v}$.

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2 \rangle) = \vec{0}$ for $\vec{v}$.

$$5\vec{v} - 2(\vec{v} + \langle 1, -2 \rangle) = \langle 0, 0 \rangle$$

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2 \rangle) = \vec{0}$ for $\vec{v}$.

$$5\vec{v} - 2(\vec{v} + \langle 1, -2 \rangle) = \langle 0, 0 \rangle$$

$$5\langle x, y \rangle - 2\left(\langle x, y \rangle + \langle 1, -2 \rangle\right) = \langle 0, 0 \rangle$$

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \vec{0}$ for $\vec{v}$.

$$5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \langle 0, 0\rangle$$

$$5\langle x, y\rangle - 2\left(\langle x, y\rangle + \langle 1, -2\rangle\right) = \langle 0, 0\rangle$$

$$\langle 5x, 5y\rangle - 2\langle x + 1, y - 2\rangle = \langle 0, 0\rangle$$

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \vec{0}$ for $\vec{v}$.

$$5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \langle 0, 0\rangle$$

$$5\langle x, y\rangle - 2\left(\langle x, y\rangle + \langle 1, -2\rangle\right) = \langle 0, 0\rangle$$

$$\langle 5x, 5y\rangle - 2\langle x + 1, y - 2\rangle = \langle 0, 0\rangle$$

$$\langle 5x, 5y\rangle - \langle 2x + 2, 2y - 4\rangle = \langle 0, 0\rangle$$

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \vec{0}$ for $\vec{v}$.

$$5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \langle 0, 0\rangle$$

$$5\langle x, y\rangle - 2\left(\langle x, y\rangle + \langle 1, -2\rangle\right) = \langle 0, 0\rangle$$

$$\langle 5x, 5y\rangle - 2\langle x + 1, y - 2\rangle = \langle 0, 0\rangle$$

$$\langle 5x, 5y\rangle - \langle 2x + 2, 2y - 4\rangle = \langle 0, 0\rangle$$

$$\langle 3x - 2, 3y + 4\rangle = \langle 0, 0\rangle$$

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \vec{0}$ for $\vec{v}$.

$$5\vec{v} - 2(\vec{v} + \langle 1, -2\rangle) = \langle 0, 0\rangle$$

$$5\langle x, y\rangle - 2\left(\langle x, y\rangle + \langle 1, -2\rangle\right) = \langle 0, 0\rangle$$

$$\langle 5x, 5y\rangle - 2\langle x + 1, y - 2\rangle = \langle 0, 0\rangle$$

$$\langle 5x, 5y\rangle - \langle 2x + 2, 2y - 4\rangle = \langle 0, 0\rangle$$

$$\langle 3x - 2, 3y + 4\rangle = \langle 0, 0\rangle$$

$$3x - 2 = 0 \quad 3y + 4 = 0$$

Example 3

Solve $5\vec{v} - 2(\vec{v} + \langle 1, -2 \rangle) = \vec{0}$ for $\vec{v}$.

$$5\vec{v} - 2(\vec{v} + \langle 1, -2 \rangle) = \langle 0, 0 \rangle$$

$$5\langle x, y \rangle - 2\left(\langle x, y \rangle + \langle 1, -2 \rangle\right) = \langle 0, 0 \rangle$$

$$\langle 5x, 5y \rangle - 2\langle x + 1, y - 2 \rangle = \langle 0, 0 \rangle$$

$$\langle 5x, 5y \rangle - \langle 2x + 2, 2y - 4 \rangle = \langle 0, 0 \rangle$$

$$\langle 3x - 2, 3y + 4 \rangle = \langle 0, 0 \rangle$$

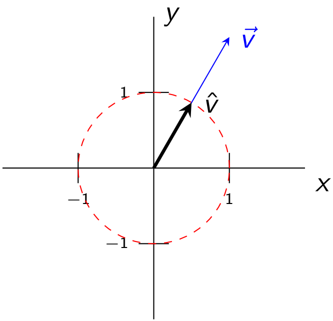
$$3x - 2 = 0 \quad 3y + 4 = 0$$

$$x = \frac{2}{3} \quad y = -\frac{4}{3} \longrightarrow \left\langle \frac{2}{3}, -\frac{4}{3} \right\rangle$$

Table of Contents

Unit Vectors

A unit vector, $\hat{v}$, is a vector that has a magnitude of 1.



Notice the unit vector $\hat{v}$ is parallel to $\vec{v}$.

Unit Vectors

We get $\hat{v}$ by dividing the magnitude of $\vec{v}$ by itself:

$$\hat{v} = \frac{\vec{v}}{\|v\|}$$

Unit Vectors

We get $\hat{v}$ by dividing the magnitude of $\vec{v}$ by itself:

$$\hat{v} = \frac{\vec{v}}{\|v\|}$$

Since we are dividing the vector (the hypotenuse) by the magnitude, we also divide the x and y components as well:

$$\hat{v} = \left\langle \frac{x}{\|v\|}, \frac{y}{\|v\|} \right\rangle$$

Example 4a

Given $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = \langle 1, -2 \rangle$, find each of the following.

(a) $\hat{v}$

Example 4a

Given $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = \langle 1, -2 \rangle$, find each of the following.

(a) $\hat{v}$

$$|\vec{v}| = \sqrt{3^2 + 4^2}$$

Example 4a

Given $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = \langle 1, -2 \rangle$, find each of the following.

(a) $\hat{v}$

$$\begin{aligned} |\vec{v}| &= \sqrt{3^2 + 4^2} \\ &= 5 \end{aligned}$$

Example 4a

Given $\vec{v} = \langle 3, 4 \rangle$ and $\vec{w} = \langle 1, -2 \rangle$, find each of the following.

(a) $\hat{v}$

$$\begin{aligned} |\vec{v}| &= \sqrt{3^2 + 4^2} \\ &= 5 \\ \hat{v} &= \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle \end{aligned}$$

Example 4b

(b) $\|\vec{v}\| - 2\|\vec{w}\|$

Example 4b

(b) $\|\vec{v}\| - 2\|\vec{w}\|$

$$\|\vec{v}\| = 5$$

Example 4b

(b) $\|\vec{v}\| - 2\|\vec{w}\|$

$$\|\vec{v}\| = 5$$
$$\|\vec{w}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

Example 4b

(b) $\|\vec{v}\| - 2\|\vec{w}\|$

$$\begin{aligned} \|\vec{v}\| &= 5 \\ \|\vec{w}\| &= \sqrt{1^2 + 2^2} = \sqrt{5} \\ \|\vec{v}\| - 2\|\vec{w}\| &= 5 - 2\sqrt{5} \end{aligned}$$

Example 4c

(c) $\|\vec{v} - 2\vec{w}\|$

Example 4c

(c) $\|\vec{v} - 2\vec{w}\|$

$$\|\vec{v} - 2\vec{w}\| = \|\langle 3, 4\rangle - 2\langle 1, -2\rangle\|$$

Example 4c

(c) $\|\vec{v} - 2\vec{w}\|$

$$\begin{aligned}\|\vec{v} - 2\vec{w}\| &= \|\langle 3, 4\rangle - 2\langle 1, -2\rangle\| \\ &= \|\langle 3, 4\rangle - \langle 2, -4\rangle\|\end{aligned}$$

Example 4c

(c) $\|\vec{v} - 2\vec{w}\|$

$$\begin{aligned} \|\vec{v} - 2\vec{w}\| &= \|\langle 3, 4\rangle - 2\langle 1, -2\rangle\| \\ &= \|\langle 3, 4\rangle - \langle 2, -4\rangle\| \\ &= \|\langle 1, 8\rangle\| \end{aligned}$$

Example 4c

(c) $\|\vec{v} - 2\vec{w}\|$

$$\begin{aligned}
\|\vec{v} - 2\vec{w}\| &= \|\langle 3, 4\rangle - 2\langle 1, -2\rangle\| \\
&= \|\langle 3, 4\rangle - \langle 2, -4\rangle\| \\
&= \|\langle 1, 8\rangle\| \\
&= \sqrt{1^2 + 8^2} = \sqrt{65}
\end{aligned}$$

Example 4d

(d) $\|\hat{w}\|$

Example 4d

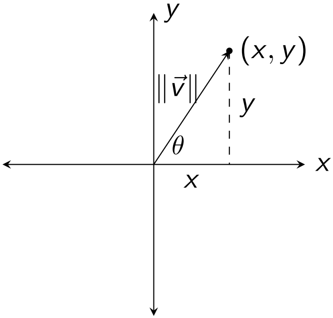
(d) $\|\hat{w}\|$

$$\|\hat{w}\| = 1 \quad \text{(magnitude of any unit vector is 1)}$$

Table of Contents

Magnitude and Direction

If we place a vector $\langle x, y \rangle$ in the coordinate plane, we can put the initial point at the origin and the terminal point at (x, y).



Magnitude and Direction

From Trig Functions of Any Angle (or Polar Coordinates),

$$x = \|\vec{v}\| \cos\theta \text{ and } y = \|\vec{v}\| \sin\theta$$

.

Magnitude and Direction

From Trig Functions of Any Angle (or Polar Coordinates),

$$x = \|\vec{v}\| \cos\theta \text{ and } y = \|\vec{v}\| \sin\theta$$

. Thus,

$$\langle x, y\rangle = \langle \|\vec{v}\| \cos\theta, \|\vec{v}\| \sin\theta\rangle = \|\vec{v}\| \langle \cos\theta, \sin\theta\rangle$$

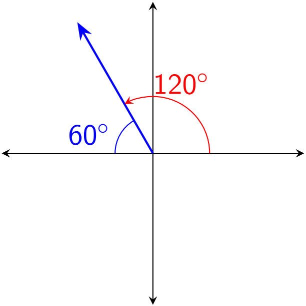
Example 5

Find the component form of the vector with $\|\vec{v}\| = 5$, with $\vec{v}$ in Quadrant II and makes a 60° angle with the negative x-axis.

Example 5

Find the component form of the vector with $\|\vec{v}\| = 5$, with $\vec{v}$ in Quadrant II and makes a 60° angle with the negative x-axis.

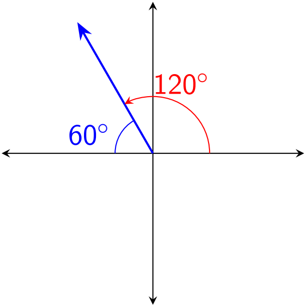
If the angle makes a 60° with the negative x-axis in quadrant II, then the total amount rotated must be $180^\circ - 60^\circ = 120^\circ$.



Example 5

Find the component form of the vector with $\|\vec{v}\| = 5$, with $\vec{v}$ in Quadrant II and makes a 60° angle with the negative x-axis.

If the angle makes a 60° with the negative x-axis in quadrant II, then the total amount rotated must be $180^\circ - 60^\circ = 120^\circ$.

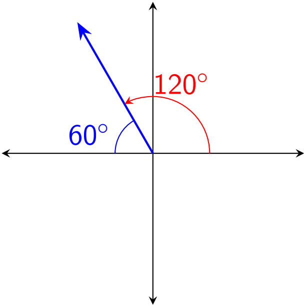


$$\vec{v} = 5\langle \cos 120^\circ, \sin 120^\circ \rangle$$

Example 5

Find the component form of the vector with $\|\vec{v}\| = 5$, with $\vec{v}$ in Quadrant II and makes a 60° angle with the negative x-axis.

If the angle makes a 60° with the negative x-axis in quadrant II, then the total amount rotated must be $180^\circ - 60^\circ = 120^\circ$.



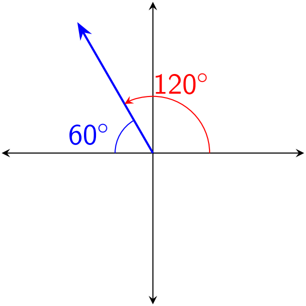
$$\vec{v} = 5\langle \cos 120^\circ, \sin 120^\circ \rangle$$

$$= 5 \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

Example 5

Find the component form of the vector with $\|\vec{v}\| = 5$, with $\vec{v}$ in Quadrant II and makes a 60° angle with the negative x-axis.

If the angle makes a 60° with the negative x-axis in quadrant II, then the total amount rotated must be $180^\circ - 60^\circ = 120^\circ$.



$$\vec{v} = 5\langle \cos 120^\circ, \sin 120^\circ \rangle$$

$$= 5\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$$

$$= \left\langle -\frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle$$

Example 6

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ $(0 \leq \theta < 2\pi)$ and write in $\|\vec{v}\|\langle \cos\theta, \sin\theta \rangle$ form.

Example 6

For $\vec{v} = \langle 3, -3\sqrt{3}\rangle$, find $\|\vec{v}\|$ and θ $(0 \le \theta < 2\pi)$ and write in $\|\vec{v}\|\langle \cos\theta, \sin\theta\rangle$ form.

$$\|\vec{v}\| = \sqrt{3^2 + (3\sqrt{3})^2}$$

Example 6

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ $(0 \leq \theta < 2\pi)$ and write in $\|\vec{v}\|\langle \cos\theta, \sin\theta \rangle$ form.

$$\|\vec{v}\| = \sqrt{3^2 + (3\sqrt{3})^2}$$

$$= \sqrt{36} = 6$$

Example 6

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ $(0 \le \theta < 2\pi)$ and write in $\|\vec{v}\| \langle \cos\theta, \sin\theta \rangle$ form.

$$\theta' = \tan^{-1} \left| \frac{-3\sqrt{3}}{3} \right|$$

Example 6

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ $(0 \leq \theta < 2\pi)$ and write in $\|\vec{v}\| \langle \cos\theta, \sin\theta \rangle$ form.

$$\theta' = \tan^{-1}\left|\frac{-3\sqrt{3}}{3}\right|$$

$$\theta' = \frac{\pi}{3}$$

Example 6

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ $(0 \leq \theta < 2\pi)$ and write in $\|\vec{v}\| \langle \cos\theta, \sin\theta \rangle$ form.

$$\theta' = \tan^{-1} \left| \frac{-3\sqrt{3}}{3} \right|$$

$$\theta' = \frac{\pi}{3}$$

$$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

Example 6

For $\vec{v} = \langle 3, -3\sqrt{3} \rangle$, find $\|\vec{v}\|$ and θ $(0 \leq \theta < 2\pi)$ and write in $\|\vec{v}\| \langle \cos\theta, \sin\theta \rangle$ form.

$$\theta' = \tan^{-1}\left|\frac{-3\sqrt{3}}{3}\right|$$

$$\theta' = \frac{\pi}{3}$$

$$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$6\left\langle \cos\left(\tfrac{5\pi}{3}\right), \sin\left(\tfrac{5\pi}{3}\right)\right\rangle$$