Ellipses

Objectives

1. Identify the center, vertices, and foci of an ellipse.
2. Write the equation of an ellipse in standard form.
3. Solve application problems involving ellipses.

Intro

An ellipse is the set of points such that the sum of their distances from 2 fixed points (called foci) is constant.

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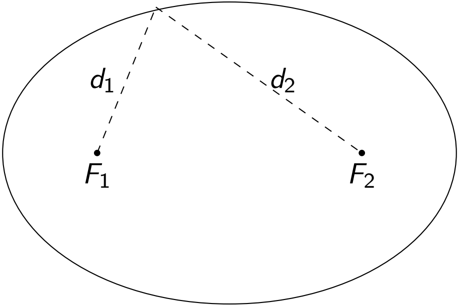

$$d_1 + d_2 = \text{constant}$$

Intro

Ellipses will typically either appear wider or taller based on their equation.

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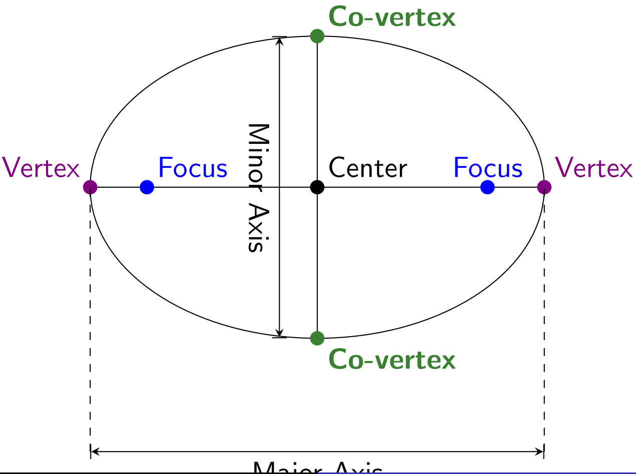

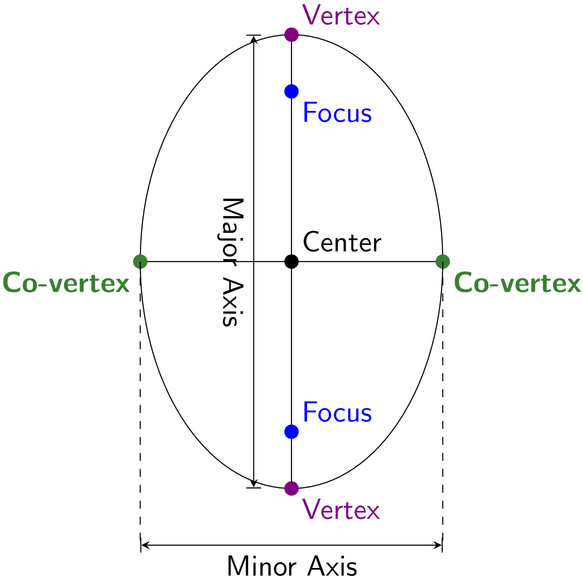

Vertex
Focus
Center
Co-vertex
Co-vertex
Major Axis
Focus
Vertex
Minor Axis

Intro

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Each vertex (*pl: vertices*) also lies on the major axis, and is a units away from the center.

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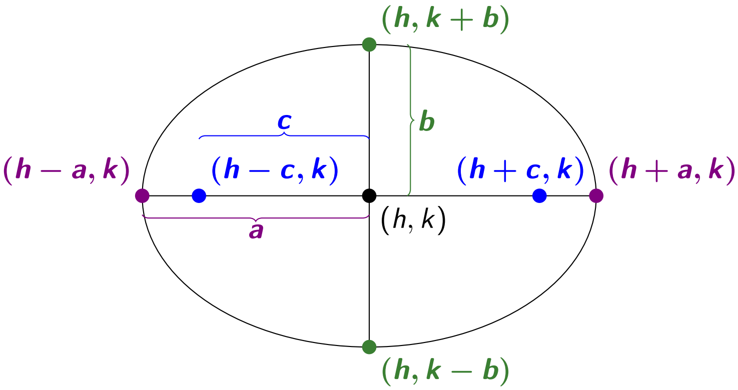
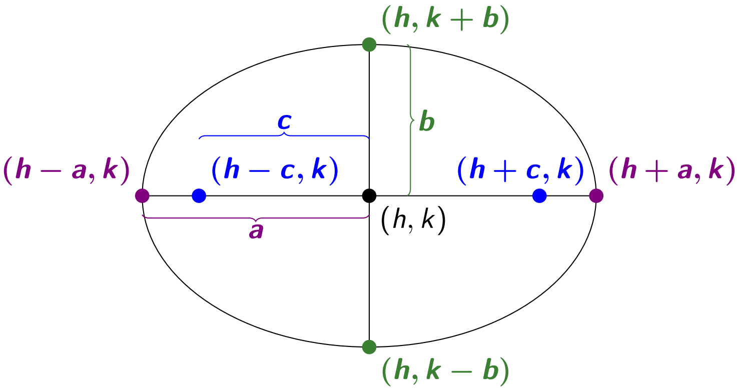
Each focal point (*pl: foci*) is c units from the center. The foci are on the major axis.

Each vertex (*pl: vertices*) also lies on the major axis, and is a units away from the center.

Each co-vertex (*pl: co-vertices*) lies on the minor axis, which is perpendicular to the major axis. The co-vertices are each b units away from the center.

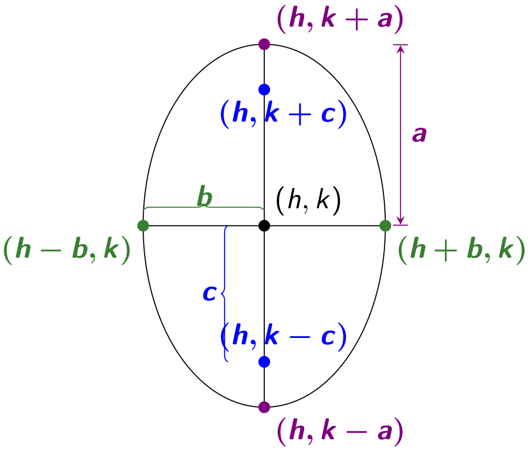

$(h, k+b)$
b
c
$(h-a, k)$
$(h-c, k)$
$(h+c, k)$
$(h+a, k)$
(h, k)
a
$(h, k-b)$

Intro



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Intro



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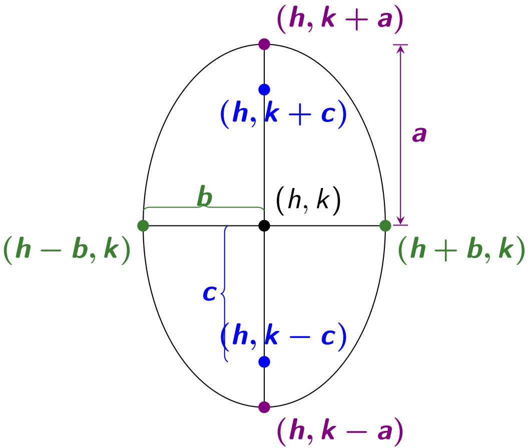


$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

Intro

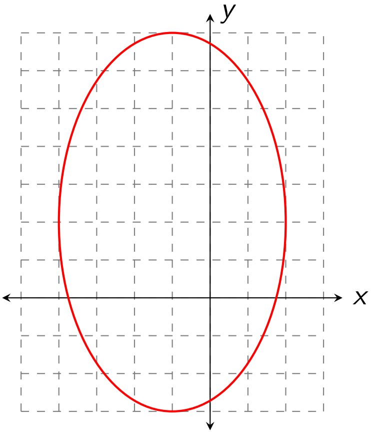
Note: In both cases, $c^2 = a^2 - b^2$, and $a > b$.

Example 1

Find the center, the lines containing the major and minor axes, the vertices, the endpoints of the minor axis (co-vertices), and the foci for

$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{25} = 1$$

Example 1 $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{25} = 1$



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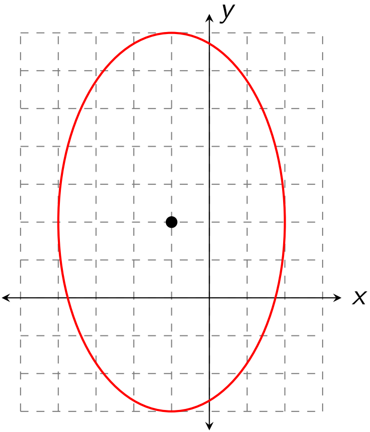
Center: $(-1, 2)$



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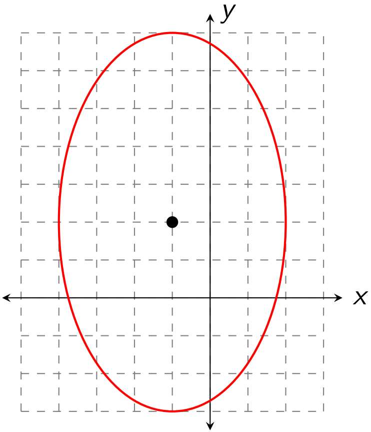

Center: $(-1, 2)$

Vertices: $a^2 = 25 \longrightarrow a = \pm 5$

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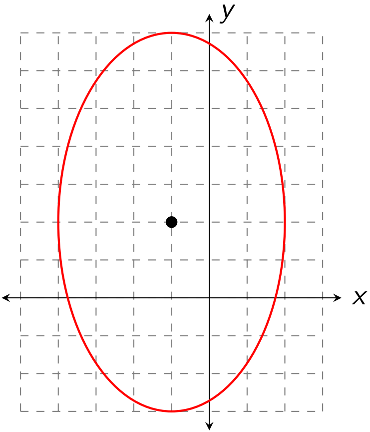

Center: $(-1, 2)$

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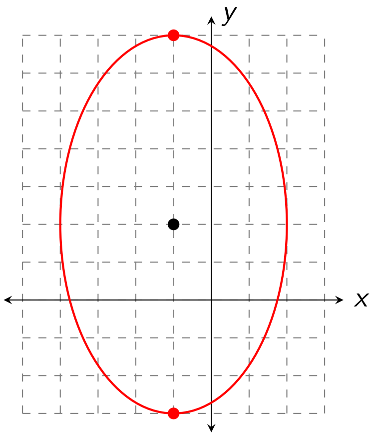

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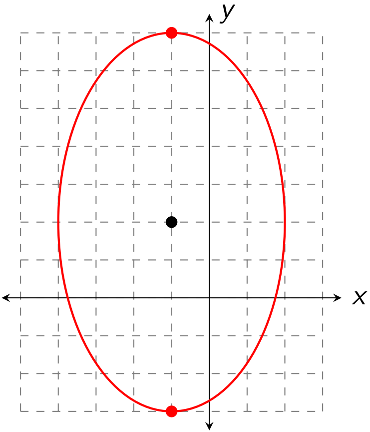

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Vertices: $a^2 = 25 \longrightarrow a = \pm 5$
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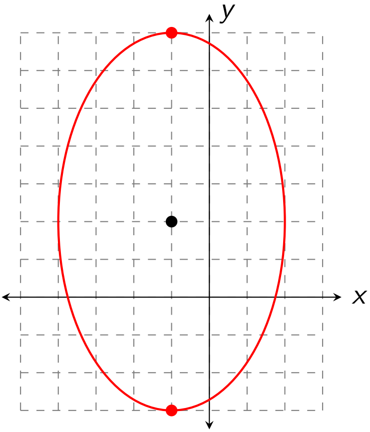

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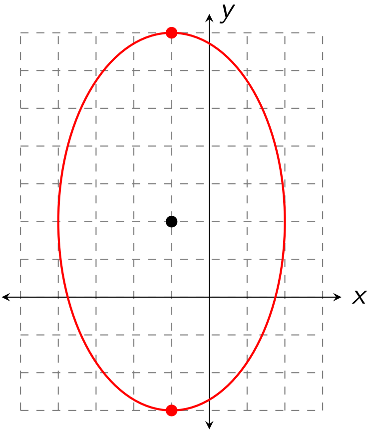

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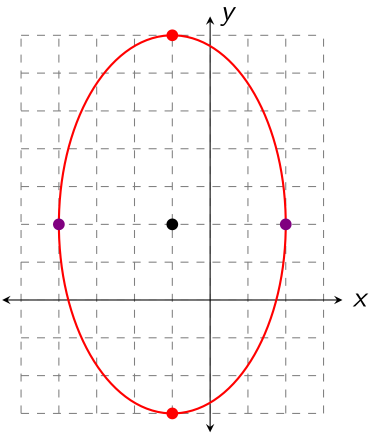

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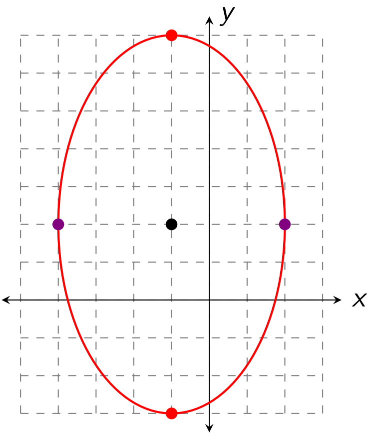

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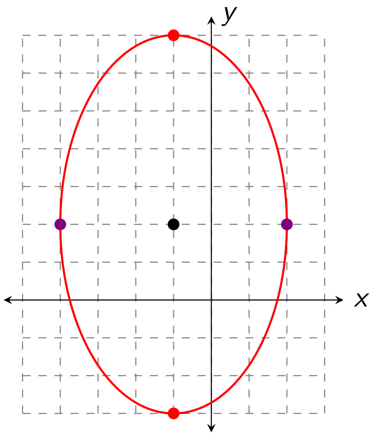

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Foci: $c^2 = 25 - 9 \longrightarrow c = \pm 4$

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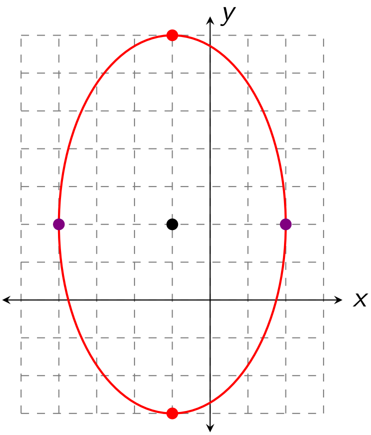

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Co-Vertices: $b^2 = 9 \longrightarrow b = \pm 3$
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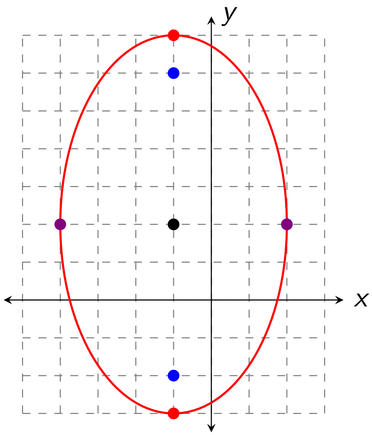

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Vertices: $a^2 = 25 \longrightarrow a = \pm 5$
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Minor Axis Line: $y = 2$

Foci: $c^2 = 25 - 9 \longrightarrow c = \pm 4$
Foci: $(-1, 2 \pm 4)$
Foci: $(-1, -2)$ and $(-1, 6)$

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1. Identify the center, vertices, and foci of an ellipse.
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Example 2

Find the equation of the ellipse with foci $(2, 1)$ and $(4, 1)$ and vertex $(0, 1)$.

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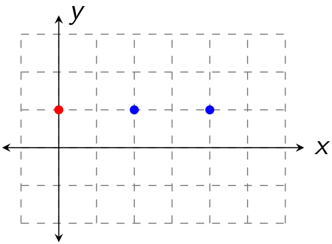

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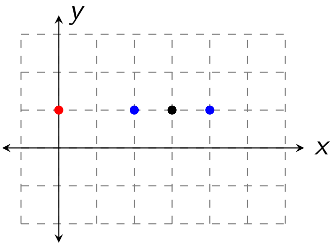
$$\text{Center: } \left(\frac{2+4}{2}, \frac{1+1}{2}\right) = (3, 1)$$



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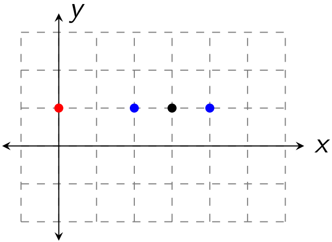

Center: $\left(\dfrac{2+4}{2}, \dfrac{1+1}{2}\right) = (3, 1)$

$c = 1, \quad a = 3$

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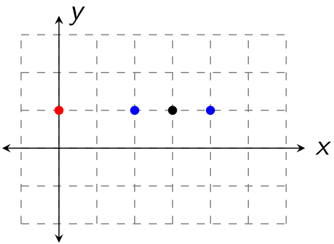

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$$3^2 - b^2 = 1^2$$

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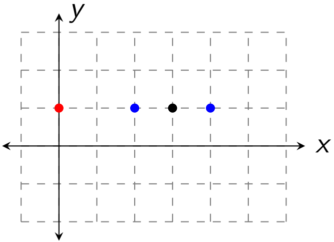

$$\text{Center: } \left(\frac{2+4}{2}, \frac{1+1}{2}\right) = (3, 1)$$

$$c = 1, \quad a = 3$$

$$3^2 - b^2 = 1^2$$

$$9 - b^2 = 1$$

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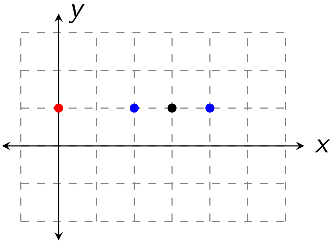
Find the equation of the ellipse with foci $(2, 1)$ and $(4, 1)$ and vertex $(0, 1)$.



Center: $\left(\frac{2+4}{2}, \frac{1+1}{2}\right) = (3, 1)$

$c = 1, \quad a = 3$

$$3^2 - b^2 = 1^2$$
$$9 - b^2 = 1$$
$$-b^2 = -8$$

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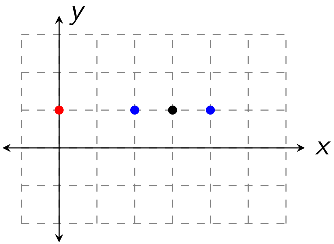
Find the equation of the ellipse with foci $(2, 1)$ and $(4, 1)$ and vertex $(0, 1)$.



Center: $\left(\frac{2+4}{2}, \frac{1+1}{2}\right) = (3, 1)$

$c = 1, \quad a = 3$

$$
\begin{aligned}
3^2 - b^2 &= 1^2 \\
9 - b^2 &= 1 \\
-b^2 &= -8 \\
b^2 &= 8
\end{aligned}
$$

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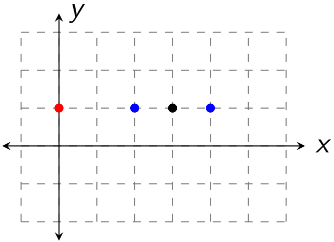
Find the equation of the ellipse with foci $(2,1)$ and $(4,1)$ and vertex $(0,1)$.



Center: $\left(\frac{2+4}{2}, \frac{1+1}{2}\right) = (3,1)$

$c = 1, \quad a = 3$

$$\begin{aligned} 3^2 - b^2 &= 1^2 \\ 9 - b^2 &= 1 \\ -b^2 &= -8 \\ b^2 &= 8 \end{aligned}$$

$$\frac{(x-3)^2}{9} + \frac{(y-1)^2}{8} = 1$$

Writing the Equation of an Ellipse

Writing the equation of an ellipse in standard form is a lot like that for circles.

Example 3

Find the center, the lines containing the major and minor axes, the vertices, the endpoints of the minor axis, and the foci for
$x^2 - 2x + 4y^2 + 24y + 33 = 0$

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Vertex: $(1, -1)$ Vertex: $(-3, -36)$

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Vertex: $(1, -1)$ Vertex: $(-3, -36)$

$$(x-1)^2 + 4(y+3)^2 = -33 + |-1| + |-36|$$

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Find the center, the lines containing the major and minor axes, the vertices, the endpoints of the minor axis, and the foci for $x^2 - 2x + 4y^2 + 24y + 33 = 0$

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$$(x-1)^2 + 4(y+3)^2 = -33 + |-1| + |-36|$$

$$(x-1)^2 + 4(y+3)^2 = 4$$

Example 3

Find the center, the lines containing the major and minor axes, the vertices, the endpoints of the minor axis, and the foci for $x^2 - 2x + 4y^2 + 24y + 33 = 0$

$$x^2 - 2x + 4y^2 + 24y = -33$$

Vertex: $(1, -1)$ Vertex: $(-3, -36)$

$$(x-1)^2 + 4(y+3)^2 = -33 + |-1| + |-36|$$

$$(x-1)^2 + 4(y+3)^2 = 4$$

$$\frac{(x-1)^2}{4} + \frac{(y+3)^2}{1} = 1$$

Example 3 $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{1} = 1$

Center: $(1, -3)$

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$$a^2 = 4$$

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Center: $(1, -3)$

$$a^2 = 4$$
$$a = \pm 2$$

Example 3 $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{1} = 1$

Center: $(1, -3)$

$$\begin{aligned} a^2 &= 4 \\ a &= \pm 2 \end{aligned}$$

Vertices: $(1 \pm 2, -3)$

Example 3 $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{1} = 1$

Center: $(1, -3)$

$$a^2 = 4$$
$$a = \pm 2$$

Vertices: $(1 \pm 2, -3)$

Vertices: $(-1, -3)$ and $(3, -3)$

Example 3 $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{1} = 1$

Center: $(1, -3)$

$$a^2 = 4$$
$$a = \pm 2$$

Vertices: $(1 \pm 2, -3)$

Vertices: $(-1, -3)$ and $(3, -3)$

Major axis: $y = -3$

Example 3

Co-Vertices: $b^2 = 1 \longrightarrow b = \pm 1$

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Co-Vertices: $(1, -3 \pm 1) \longrightarrow (1, -4)$ and $(1, -2)$

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Co-Vertices: $(1, -3 \pm 1) \longrightarrow (1, -4)$ and $(1, -2)$

Minor axis: $x = 1$

Example 3

Co-Vertices: $b^2 = 1 \longrightarrow b = \pm 1$

Co-Vertices: $(1, -3 \pm 1) \longrightarrow (1, -4)$ and $(1, -2)$

Minor axis: $x = 1$

Foci:

Example 3

Co-Vertices: $b^2 = 1 \longrightarrow b = \pm 1$

Co-Vertices: $(1, -3 \pm 1) \longrightarrow (1, -4)$ and $(1, -2)$

Minor axis: $x = 1$

Foci:

$$c^2 = 4 - 1$$

Example 3

Co-Vertices: $b^2 = 1 \longrightarrow b = \pm 1$

Co-Vertices: $(1, -3 \pm 1) \longrightarrow (1, -4)$ and $(1, -2)$

Minor axis: $x = 1$

Foci:

$$c^2 = 4 - 1$$
$$c = \pm\sqrt{3}$$

Example 3

Co-Vertices: $b^2 = 1 \longrightarrow b = \pm 1$

Co-Vertices: $(1, -3 \pm 1) \longrightarrow (1, -4)$ and $(1, -2)$

Minor axis: $x = 1$

Foci:

$$c^2 = 4 - 1$$
$$c = \pm\sqrt{3}$$

Foci: $(1 \pm \sqrt{3}, -3)$

Eccentricity

The “roundness” of an ellipse is measured by its eccentricity. It is a ratio and is denoted by e.

$$e = \frac{\text{Distance from center to focus}}{\text{Distance from center to vertex}}$$

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The "roundness" of an ellipse is measured by its eccentricity. It is a ratio and is denoted by e.

$$e = \frac{\text{Distance from center to focus}}{\text{Distance from center to vertex}} = \frac{c}{a}$$

Example 4

Find the equation of the ellipse with vertices $(\pm 5, 0)$ and eccentricity $e = \frac{1}{4}$.

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Center: $(0, 0)$

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Center: $(0, 0)$

$$\frac{1}{4} = \frac{c}{a}$$

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Find the equation of the ellipse with vertices $(\pm 5, 0)$ and eccentricity $e = \frac{1}{4}$.

Center: $(0, 0)$

$$\frac{1}{4} = \frac{c}{a}$$

$$\frac{1}{4} = \frac{c}{5}$$

Example 4

Find the equation of the ellipse with vertices $(\pm 5, 0)$ and eccentricity $e = \frac{1}{4}$.

Center: $(0, 0)$

$$\frac{1}{4} = \frac{c}{a}$$

$$\frac{1}{4} = \frac{c}{5}$$

$$4c = 5$$

Example 4

Find the equation of the ellipse with vertices $(\pm 5, 0)$ and eccentricity $e = \frac{1}{4}$.

Center: $(0, 0)$

$$\frac{1}{4} = \frac{c}{a}$$

$$\frac{1}{4} = \frac{c}{5}$$

$$4c = 5$$

$$c = \frac{5}{4}$$

Example 4 $a = 5, c = \frac{5}{4}$

$$\left(\frac{5}{4}\right)^2 = 5^2 - b^2$$

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$$b^2 = \frac{375}{16}$$

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$$\frac{x^2}{25} + \frac{y^2}{\frac{375}{16}} = 1$$

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$$\frac{x^2}{25} + \frac{y^2}{\frac{375}{16}}\left(\frac{16}{16}\right) = 1$$

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$$\frac{x^2}{25} + \frac{y^2}{\frac{375}{16}}\left(\frac{16}{16}\right) = 1$$

$$\frac{x^2}{25} + \frac{16y^2}{375} = 1$$

Objectives

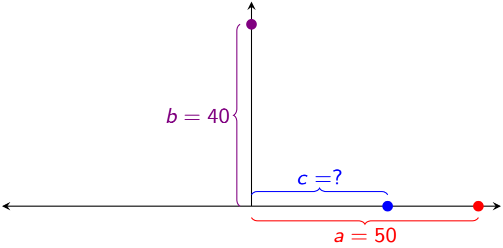
1. Identify the center, vertices, and foci of an ellipse.
2. Write the equation of an ellipse in standard form.
3. Solve application problems involving ellipses.

Example 5

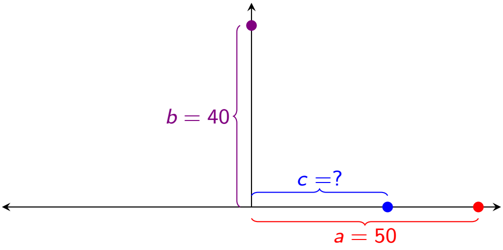
Jamie and Jason want to exchange secrets (terrible secrets) from across a crowded whispering gallery. A whispering gallery is a room which, in cross section, is half of an ellipse. If the room is 40 feet high at the center and 100 feet wide at the floor, how far from the outer wall should each of them stand so that they will be positioned at the foci of the ellipse?

Example 5

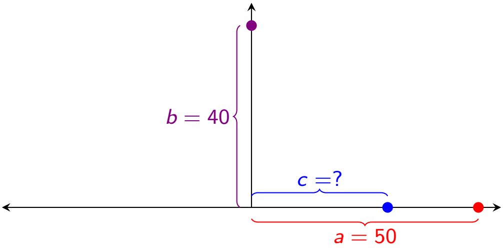
Jamie and Jason want to exchange secrets (terrible secrets) from across a crowded whispering gallery. A whispering gallery is a room which, in cross section, is half of an ellipse. If the room is 40 feet high at the center and 100 feet wide at the floor, how far from the outer wall should each of them stand so that they will be positioned at the foci of the ellipse?



Example 5

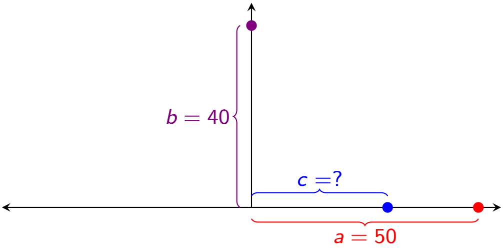


Example 5



$$c^2 = 50^2 - 40^2$$

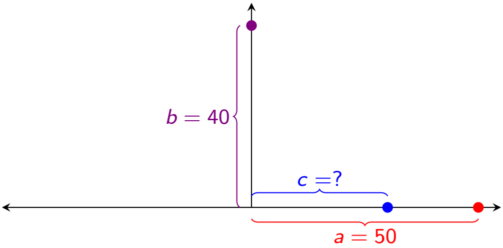
Example 5



$$c^2 = 50^2 - 40^2$$

$$c^2 = 900$$

Example 5

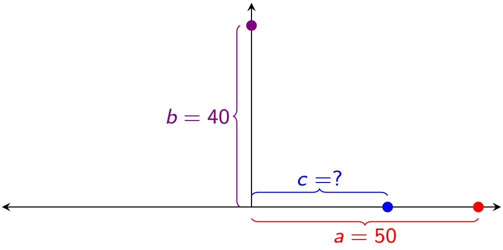


$$c^2 = 50^2 - 40^2$$

$$c^2 = 900$$

$$c = \sqrt{900}$$

Example 5



$$c^2 = 50^2 - 40^2$$

$$c^2 = 900$$

$$c = \sqrt{900}$$

$$c = 30 \text{ feet}$$