Graphs of Tangent, Cotangent, Secant, and Cosecant

Objectives

1. Determine the amplitude, period, phase shift, and vertical shift of the tangent and cotangent graphs.
2. Determine the amplitude, period, phase shift, and vertical shift of the secant and cosecant graphs.

Tangent and Cotangent Graphs

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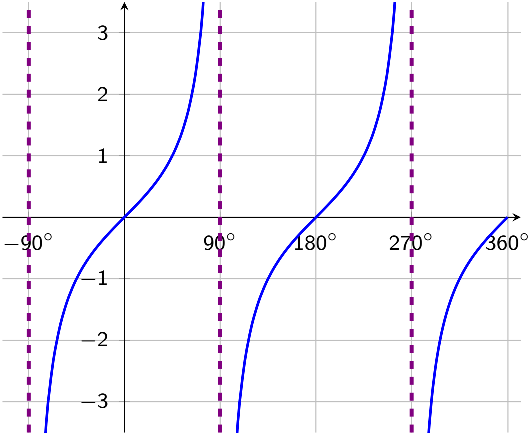
Since x- and y-coordinates can be positive, negative, or zero, the graphs of tangent and cotangent functions pose some interesting behavior; in particular, when the x-coordinate is 0.

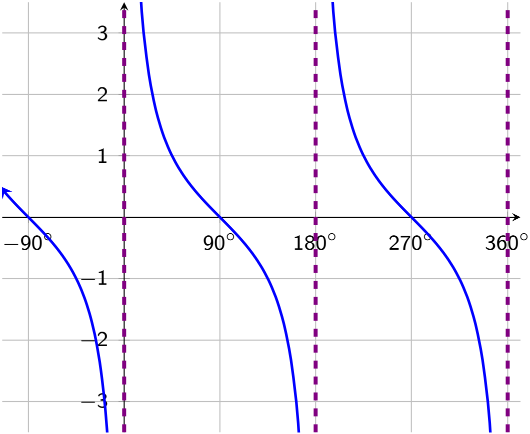
Vertical Asymptotes

A vertical asymptote is a vertical line that the graph will get infinitely close to, but never cross.

Tangent Graph



Cotangent Graphs



Amplitude?

The graphs of tangent and cotangent functions do not stop going up or down. Thus, they have neither a maximum point nor a minimum point.

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The graphs of tangent and cotangent functions do not stop going up or down. Thus, they have neither a maximum point nor a minimum point.

In other words, *tangents and cotangents have no amplitude*.

Period of Tangent and Cotangent

Tangents and cotangents complete one full cycle between asymptotes. Notice for each graph, that period is 180°, or π radians.

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Like the graphs of sine and cosine, multiplying the inputs, x, by a positive value other than 1 will affect the period of the graphs for tangent and cotangent.

Instead of dividing 360° (or 2π) by that value, for tangent and cotangent divide 180° (or π radians).

Shifts

Determining phase shift and vertical shift follow the same procedures as that for sine and cosine.

Example 1

Determine the amplitude, period, phase shift, and vertical shift of each of the following.

(a) $y = 2\tan(x - 45^\circ)$

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Period: $\dfrac{180^\circ}{1} = 180^\circ$

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Vertical Shift: 0 (or none)

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Phase Shift: 0 (or none)

Vertical Shift: Up 1

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(c) $y = 1.5\tan(2x + 120^\circ) - 5$

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(c) $\quad y = 1.5\tan(2x + 120^\circ) - 5$

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Period: $\dfrac{180^\circ}{2} = 90^\circ$

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Phase Shift: 60° left

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Phase Shift: 60° left

Vertical Shift: 5 down

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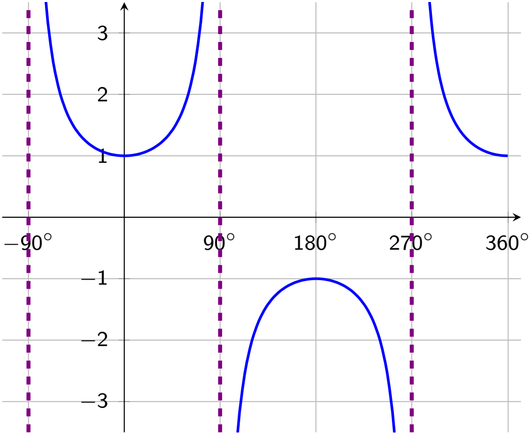
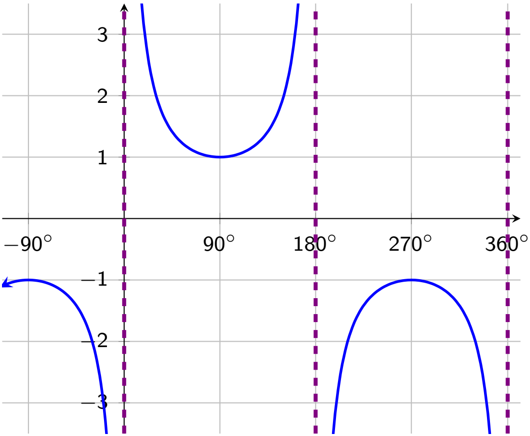
Secant Graph

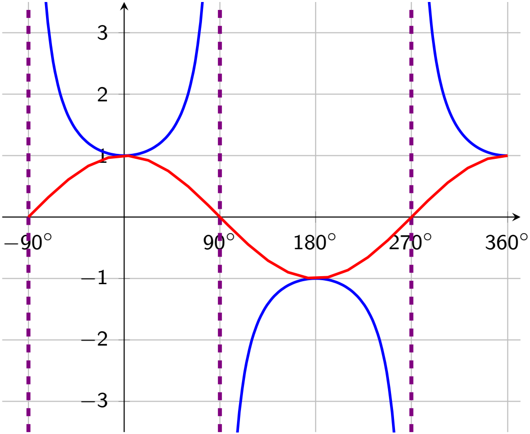


Cosecant Graph



Relationship to Sine and Cosine

If we graph $y = \cos x$ in the same plane as $y = \sec x$, we see some interesting features:



Cosine and Secant

Notice that when $\cos x$ is at a maximum, we get a "smile" on the secant graph, and when $\cos x$ is at a minimum, we get a "frown" on the secant graph.

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Also, whenever $y = \cos x$ crosses the x-axis, there is a vertical asymptote for $y = \sec x$ (why?)

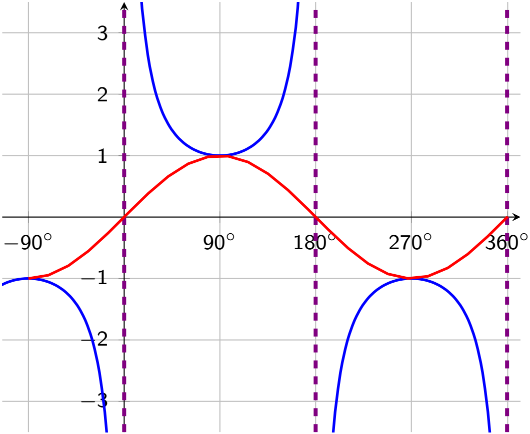
Cosine and Secant

Notice that when $\cos x$ is at a maximum, we get a "smile" on the secant graph, and when $\cos x$ is at a minimum, we get a "frown" on the secant graph.

Also, whenever $y = \cos x$ crosses the x-axis, there is a vertical asymptote for $y = \sec x$ (why?)

The same logic applies with $y = \csc x$ and $y = \sin x$.

Sine and Cosecant



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Multiplying the inputs by a positive value other than 1 changes the period.

Phase shifts and vertical shifts are calculated in the same way as the other four trig functions.

Example 2

the amplitude, period, phase shift, and vertical shift for each of the following.

(a) $y = 2\sec(x - 45^\circ)$

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Period: $\dfrac{360^\circ}{1} = 360^\circ$

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Vertical Shift: 0 (or none)

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Amplitude: None

Period: $\dfrac{360^\circ}{1} = 360^\circ$

Phase Shift: None

Example 2

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Amplitude: None

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Phase Shift: None

Vertical Shift: Up 1

Example 3

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Amplitude: None

Period: $\dfrac{360^\circ}{2} = 180^\circ$

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Phase Shift: 60° left

Vertical Shift: 5 down

Summary

	Amplitude	Period	Phase Shift	Vertical Shift
$y = A\tan(Bx - C) + D$	None	$\frac{180^\circ}{B}$ or $\frac{\pi}{B}$	$\frac{C}{B}$	D
$y = A\cot(Bx - C) + D$	None	$\frac{180^\circ}{B}$ or $\frac{\pi}{B}$	$\frac{C}{B}$	D
$y = A\sec(Bx - C) + D$	None	$\frac{360^\circ}{B}$ or $\frac{2\pi}{B}$	$\frac{C}{B}$	D
$y = A\csc(Bx - C) + D$	None	$\frac{360^\circ}{B}$ or $\frac{2\pi}{B}$	$\frac{C}{B}$	D