# Hyperbolas

# Intro

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A hyperbola is the set of points such that the difference of their distances from 2 fixed points (called foci) is constant.

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Whereas ellipses could appear taller or wider, hyperbolas will open up and down, or left and right.

A key difference, however, is that hyperbolas will open left/right if the sign in front of $x$ is positive, and will open up/down if the sign in front of $y$ is positive; regardless of the values of $a$ and $b$.

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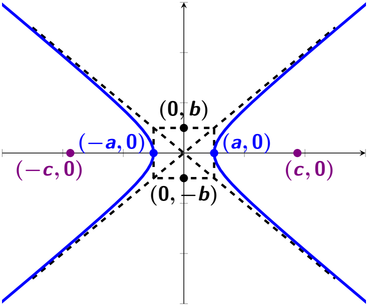

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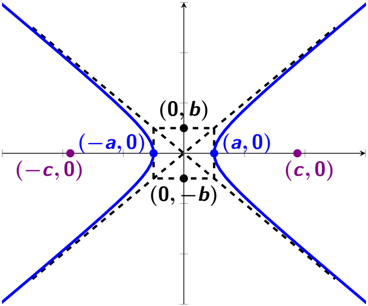


**Equation** $$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

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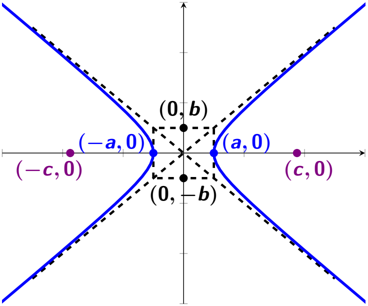


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**Vertices** $(h \pm a, k)$

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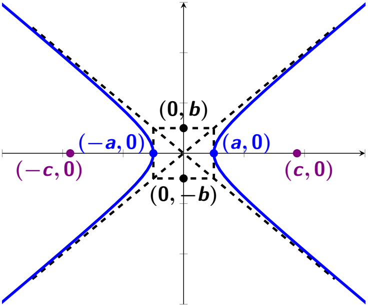


**Equation** $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

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**Foci** $(h \pm c, k)$

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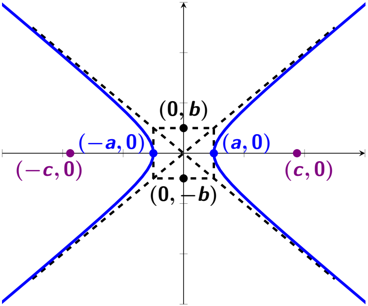

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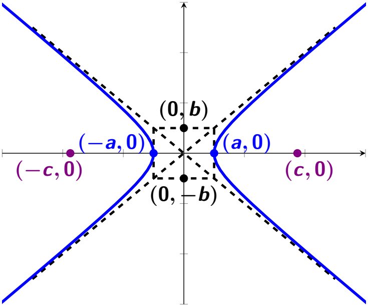


**x-Axis** Transverse Axis

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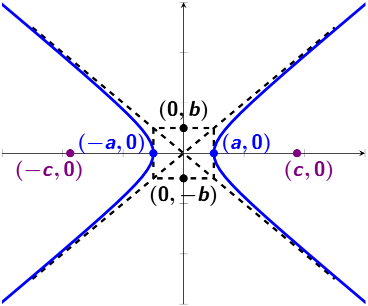


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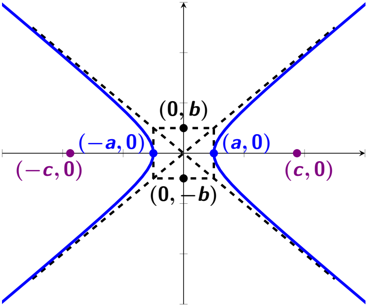

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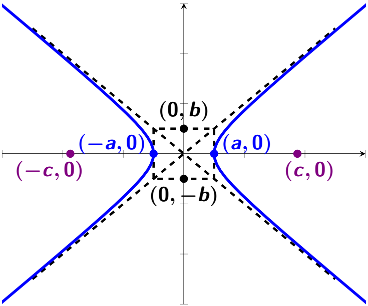
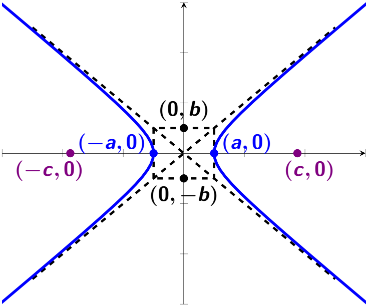



$$\boldsymbol{c^2} \quad = a^2 + b^2$$

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$$\textbf{Asymptotes} \quad y = \pm\frac{b}{a}(x-h)+k$$

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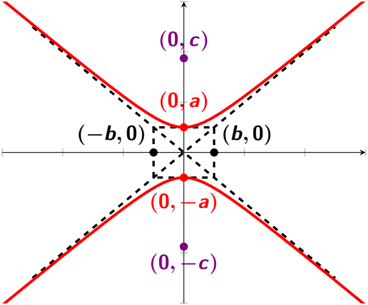

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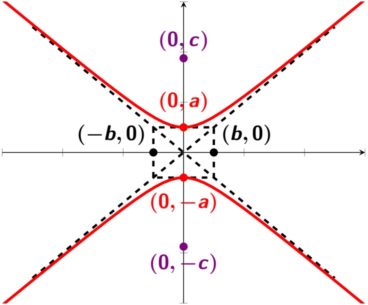


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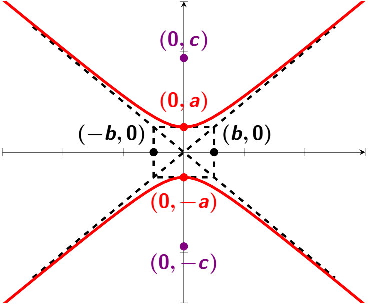


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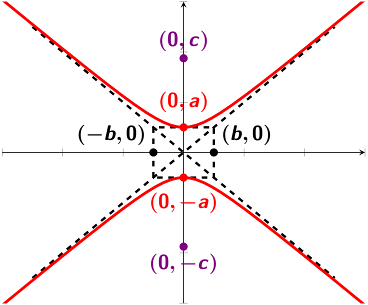


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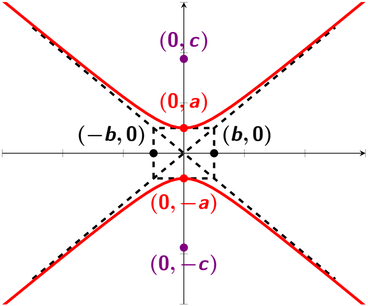

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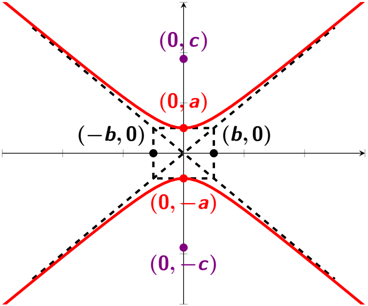


**x-Axis** Conjugate Axis

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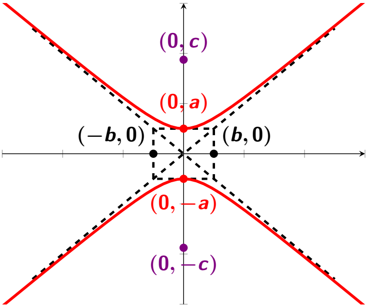


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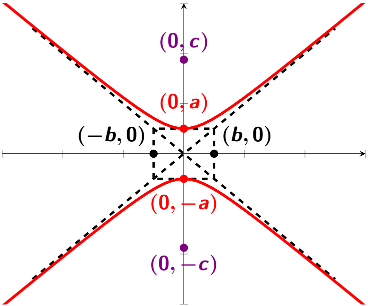

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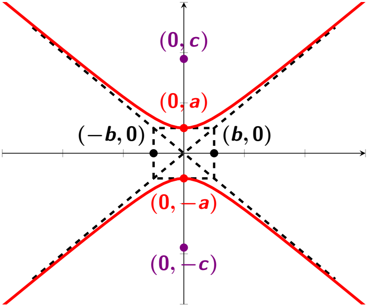


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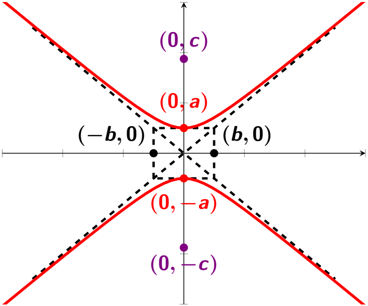


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# Objectives

# Example 1

Find the center, vertices, foci, and equations of the asymptotes for the hyperbola

$$\frac{(x-2)^2}{4} - \frac{y^2}{25} = 1$$

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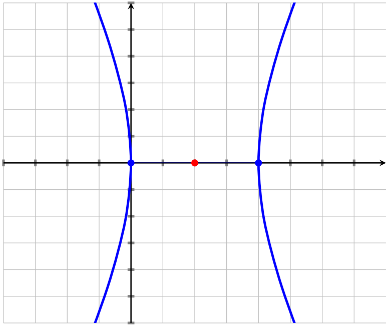

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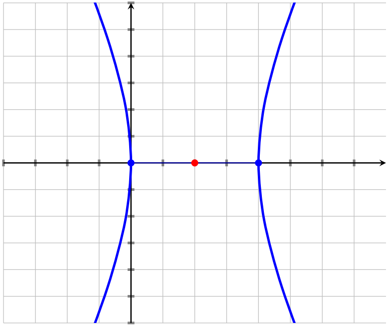

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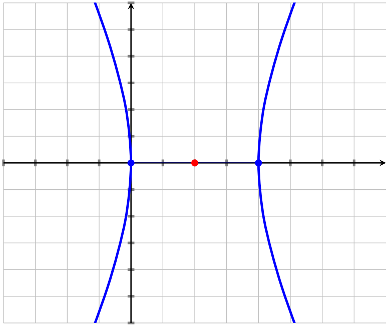

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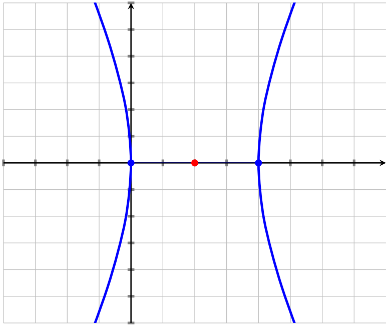

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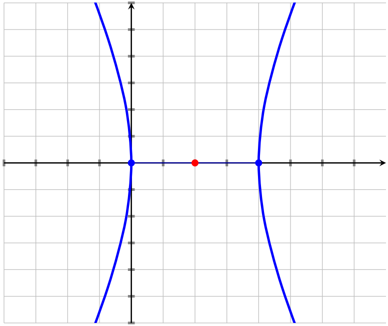

Center: $(2, 0)$

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Vertices: $(2 \pm 2, 0)$

Vertices: $(0, 0)$ and $(4, 0)$

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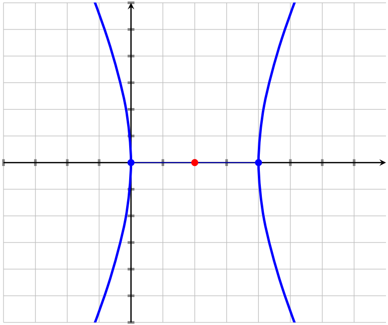

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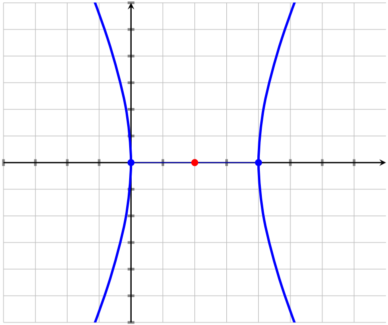

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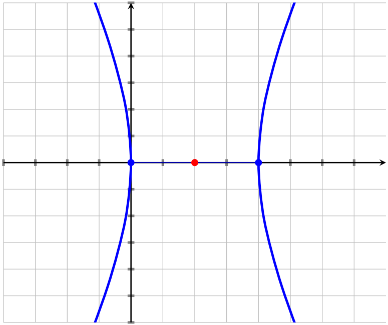

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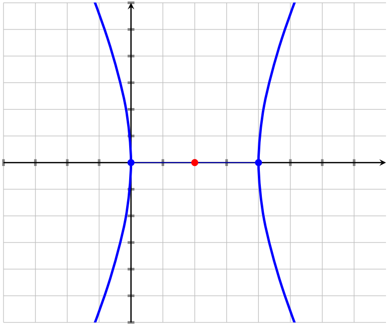

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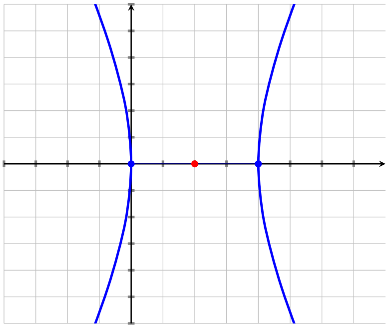

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Foci: $(2 \pm \sqrt{29}, 0)$

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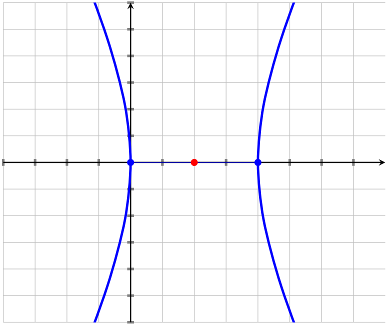

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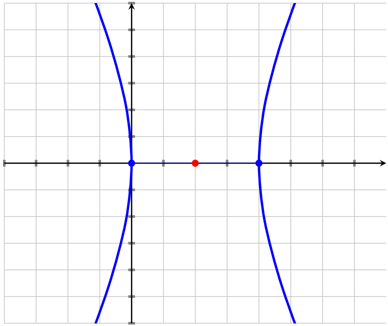

Asymptotes: $y - 0 = \pm\frac{5}{2}(x - 2)$

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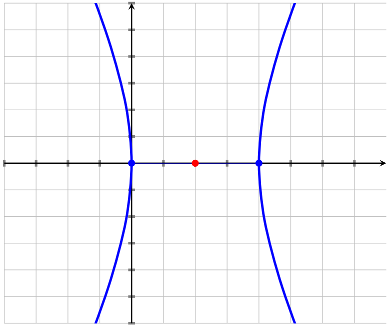

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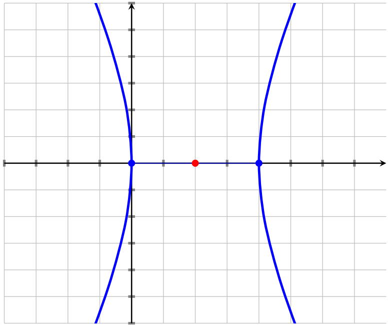

Asymptotes: $y - 0 = \pm\frac{5}{2}(x-2)$

Asymptotes: $y = \pm\frac{5}{2}(x-2)$

$y = \frac{5}{2}x - 5$ and $y = -\frac{5}{2}x + 5$

# Objectives

# Writing Equations in Standard Form

Writing the equation of a hyperbola in standard form uses a similar approach to that for ellipses. *However, the negative term will actually be subtracted from the right side.*

# Example 2a

Find the center, vertices, foci, and equations of the asymptotes for each hyperbola.

(a) $9x^2 + 162x - 16y^2 + 64y + 89 = 0$

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Vertex: $(-9, -729)$ Vertex: $(2, 64)$

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$$9(x+9)^2 - 16(y-2)^2 = -89 + |-729| - |64|$$

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$$9(x+9)^2 - 16(y-2)^2 = 576$$

$$\frac{(x+9)^2}{64} - \frac{(y-2)^2}{36} = 1$$

# Example 2a

ex2a.png

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Vertices: $(-9 \pm 8, 2)$

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Center: $(-9, 2)$

$$a^2 = 64$$

$$a = \pm 8$$

Vertices: $(-9 \pm 8, 2)$

Vertices: $(-17, 2)$ and $(-1, 2)$

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Foci: $(-19, 2)$ and $(1, 2)$

# Example 2a $\frac{(x+9)^2}{64} - \frac{(y-2)^2}{36} = 1$

$a = 8 \quad b = 6 \quad$ Center: $(-9, 2)$

$$y = \pm\frac{6}{8}(x+9) + 2$$

$$y = \pm\frac{3}{4}(x+9) + 2$$

# Example 2b

Find the center, vertices, foci, and equations of the asymptotes for each hyperbola.

(a) $9y^2 - x^2 - 6x = 10$

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Find the center, vertices, foci, and equations of the asymptotes for each hyperbola.

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Vertex: $(0, 0)$ Vertex: $(-3, 9)$

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$$9(y-0)^2 - (x-3)^2 = 1$$

$$\frac{(y-0)^2}{\frac{1}{9}} - \frac{(x-3)^2}{1} = 1$$

# Example 2b

ex2b.png

# Example 2b $\frac{(y-0)^2}{\frac{1}{9}} - \frac{(x-3)^2}{1} = 1$

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Vertices: $\left(3, 0 \pm \frac{1}{3}\right)$

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## Example 2b $\frac{(y-0)^2}{\frac{1}{9}} - \frac{(x-3)^2}{1} = 1$

$$a = \frac{1}{3} \quad b = 1 \quad \text{Center: } (3, 0)$$

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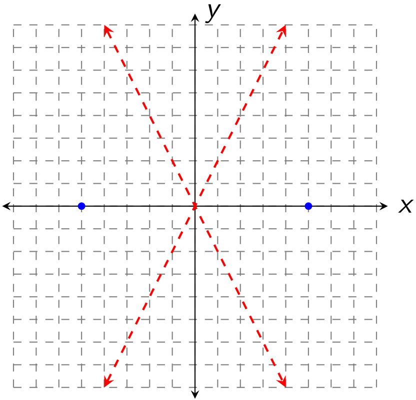

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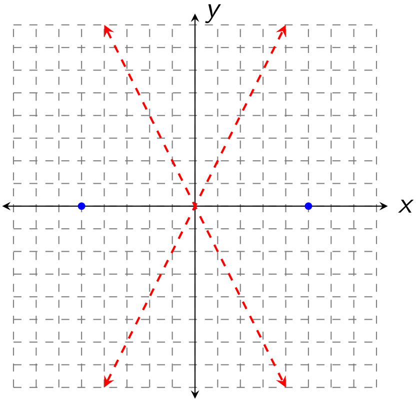


$a = 5$

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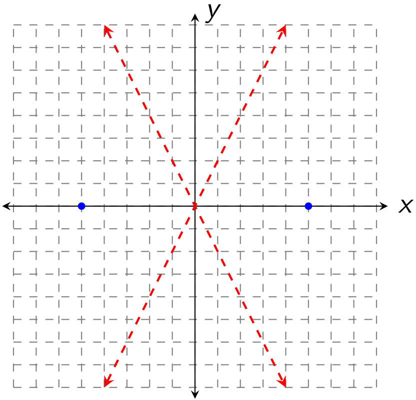


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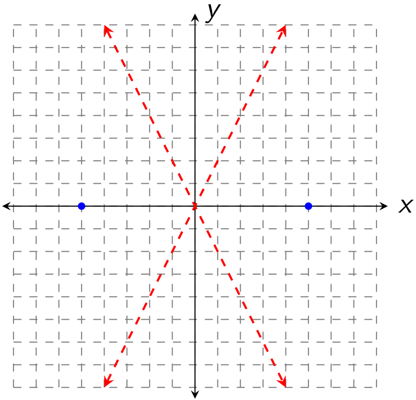


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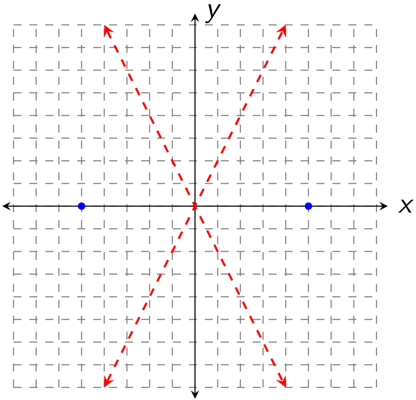


$$a = 5$$

$$\frac{b}{5} = 2$$

$$b = 10$$

$$\frac{x^2}{25} - \frac{y^2}{100} = 1$$