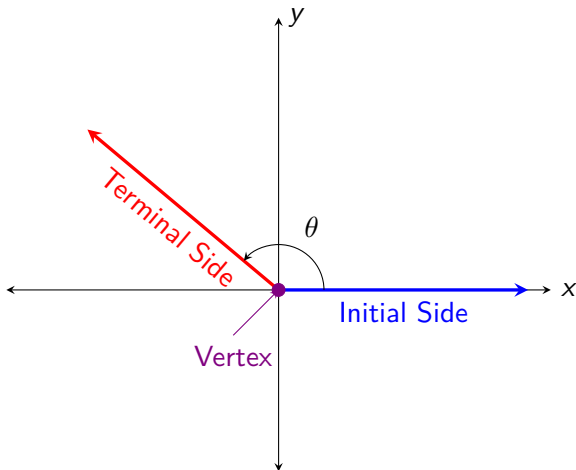


Angles and Radian Measure

Angles in Standard Position



Angles in Standard Position

An angle is in **standard position** if

Angles in Standard Position

An angle is in **standard position** if

- Vertex is at the origin.

Angles in Standard Position

An angle is in **standard position** if

- Vertex is at the origin.
- Initial side runs along positive x -axis.

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Positive angles open **counter-clockwise** and negative angles open **clockwise**.

Angles in Standard Position

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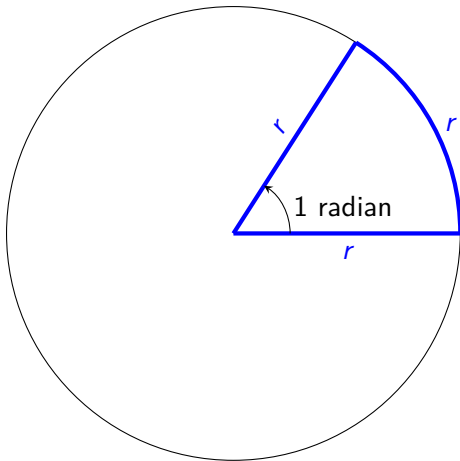
- Vertex is at the origin.
- Initial side runs along positive x -axis.

Positive angles open **counter-clockwise** and negative angles open **clockwise**.

A **quadrantal angle** is one whose terminal side lies on an axis. In other words, it's a multiple of 90° .

Radians

One **radian** is the measure of the central angle of a circle in which the radius equals the length of the intercepted arc.



Radian Interpretation

In other words, its when the length of your slice of pizza's crust is equal to the radius of the pizza.

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Relationship Between Radians and Degrees

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One rotation is the circumference of the circle ($2\pi r$) and also 360° :

$$360^\circ = 2\pi \text{ radians}$$

$$180^\circ = \pi \text{ radians}$$

$$\frac{180^\circ}{\pi} = 1 \text{ radian}$$

Convert Degrees to Radians and Radians to Degrees

- To convert degrees to radians, multiply by $\frac{\pi}{180^\circ}$.

Convert Degrees to Radians and Radians to Degrees

- To convert degrees to radians, multiply by $\frac{\pi}{180^\circ}$.
- To convert radians to degrees, multiply by $\frac{180^\circ}{\pi}$.

Example 1

Convert each angle to radians.

(a) 30°

Example 1

Convert each angle to radians.

(a) 30°

$$30^\circ$$

Example 1

Convert each angle to radians.

(a) 30°

$$30^\circ \rightarrow 30^\circ \left(\frac{\pi}{180^\circ} \right)$$

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$$30^\circ \rightarrow 30^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$= \frac{30\pi}{180}$$

Example 1

Convert each angle to radians.

(a) 30°

$$30^\circ \rightarrow 30^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$= \frac{30\pi}{180}$$

$$= \frac{\pi}{6}$$

Example 1

Convert each angle to radians.

(b) 90°

Example 1

Convert each angle to radians.

(b) 90°

$$90^\circ$$

Example 1

Convert each angle to radians.

(b) 90°

$$90^\circ \rightarrow 90^\circ \left(\frac{\pi}{180^\circ} \right)$$

Example 1

Convert each angle to radians.

(b) 90°

$$90^\circ \rightarrow 90^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$= \frac{90\pi}{180}$$

Example 1

Convert each angle to radians.

(b) 90°

$$90^\circ \rightarrow 90^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$= \frac{90\pi}{180}$$

$$= \frac{\pi}{2}$$

Example 1

Convert each angle to radians.

(c) -135°

Example 1

Convert each angle to radians.

(c) -135°

$$-135^\circ$$

Example 1

Convert each angle to radians.

(c) -135°

$$-135^\circ \rightarrow -135^\circ \left(\frac{\pi}{180^\circ} \right)$$

Example 1

Convert each angle to radians.

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$$-135^\circ \rightarrow -135^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$= \frac{-135\pi}{180}$$

Example 1

Convert each angle to radians.

(c) -135°

$$-135^\circ \rightarrow -135^\circ \left(\frac{\pi}{180^\circ} \right)$$

$$= \frac{-135\pi}{180}$$

$$= \frac{-3\pi}{4}$$

Example 2

Convert each angle to degrees.

(a) $\frac{\pi}{3}$

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Convert each angle to degrees.

(a) $\frac{\pi}{3}$

$$\frac{\pi}{3}$$

Example 2

Convert each angle to degrees.

(a) $\frac{\pi}{3}$

$$\frac{\pi}{3} \rightarrow \frac{\pi}{3} \left(\frac{180^\circ}{\pi} \right)$$

Example 2

Convert each angle to degrees.

(a) $\frac{\pi}{3}$

$$\begin{aligned}\frac{\pi}{3} &\rightarrow \frac{\pi}{3} \left(\frac{180^\circ}{\pi} \right) \\ &= \frac{180^\circ}{3}\end{aligned}$$

Example 2

Convert each angle to degrees.

(a) $\frac{\pi}{3}$

$$\frac{\pi}{3} \rightarrow \frac{\pi}{3} \left(\frac{180^\circ}{\pi} \right)$$

$$= \frac{180^\circ}{3}$$

$$= 60^\circ$$

Example 2

Convert each angle to degrees.

(b) $-\frac{5\pi}{4}$

Example 2

Convert each angle to degrees.

(b) $-\frac{5\pi}{4}$

$$-\frac{5\pi}{4}$$

Example 2

Convert each angle to degrees.

(b) $-\frac{5\pi}{4}$

$$-\frac{5\pi}{4} \rightarrow -\frac{5\pi}{4} \left(\frac{180^\circ}{\pi} \right)$$

Example 2

Convert each angle to degrees.

(b) $-\frac{5\pi}{4}$

$$\begin{aligned}-\frac{5\pi}{4} &\rightarrow -\frac{5\pi}{4} \left(\frac{180^\circ}{\pi} \right) \\ &= -\frac{900^\circ}{4}\end{aligned}$$

Example 2

Convert each angle to degrees.

$$(b) \quad -\frac{5\pi}{4}$$

$$-\frac{5\pi}{4} \rightarrow -\frac{5\pi}{4} \left(\frac{180^\circ}{\pi} \right)$$

$$= -\frac{900^\circ}{4}$$

$$= -225^\circ$$

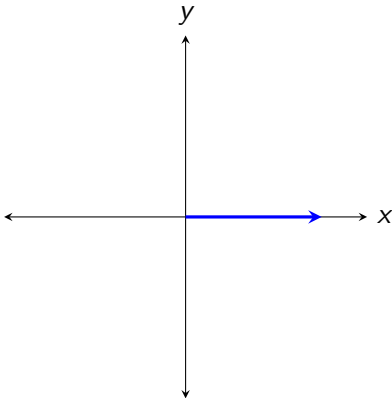
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Example 3

Draw and label each angle in standard position.

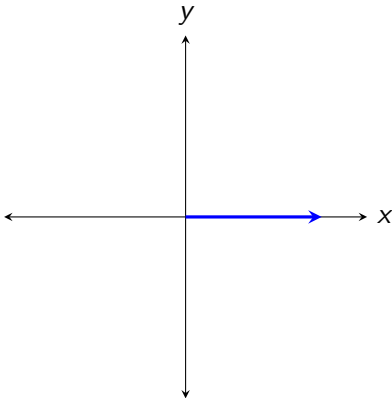
(a) $\alpha = \frac{\pi}{6}$



Example 3

Draw and label each angle in standard position.

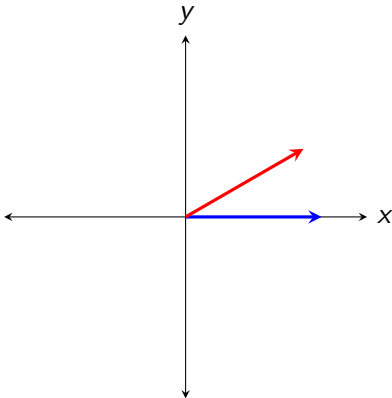
(a) $\alpha = \frac{\pi}{6}$ $\alpha = 30^\circ$



Example 3

Draw and label each angle in standard position.

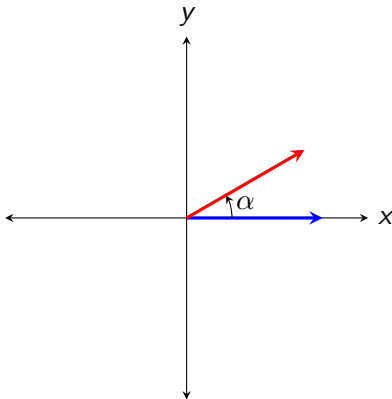
(a) $\alpha = \frac{\pi}{6}$ $\alpha = 30^\circ$



Example 3

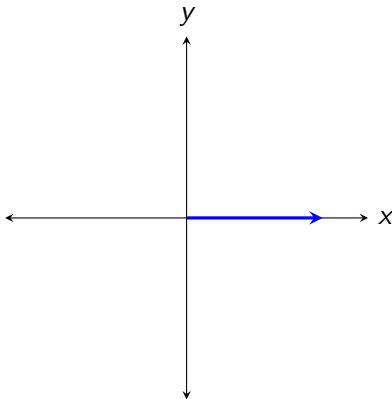
Draw and label each angle in standard position.

(a) $\alpha = \frac{\pi}{6}$ $\alpha = 30^\circ$



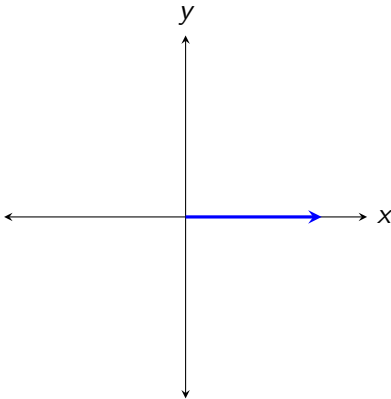
Example 3

(b) $\beta = -\frac{4\pi}{3}$



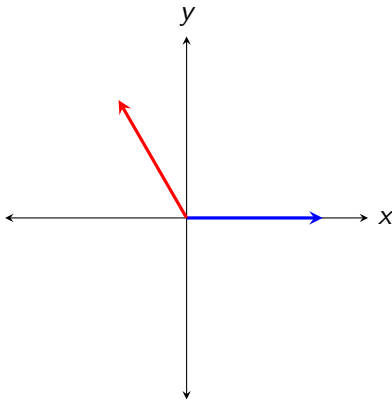
Example 3

$$(b) \quad \beta = -\frac{4\pi}{3} \quad \beta = -240^\circ$$



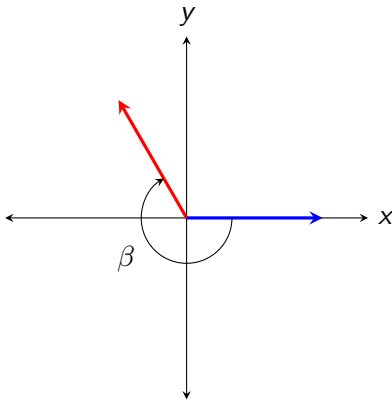
Example 3

$$(b) \quad \beta = -\frac{4\pi}{3} \quad \beta = -240^\circ$$



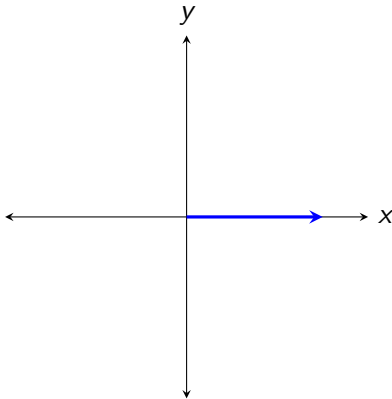
Example 3

$$(b) \quad \beta = -\frac{4\pi}{3} \quad \beta = -240^\circ$$



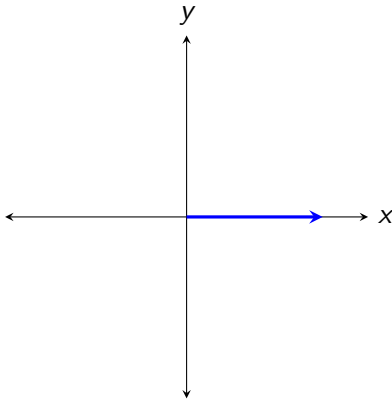
Example 3

(c) $\gamma = -\frac{9\pi}{4}$



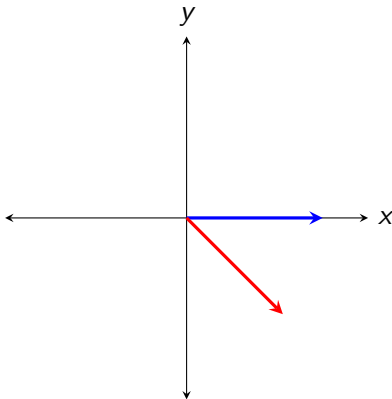
Example 3

$$(c) \quad \gamma = -\frac{9\pi}{4} \quad \gamma = -405^\circ$$



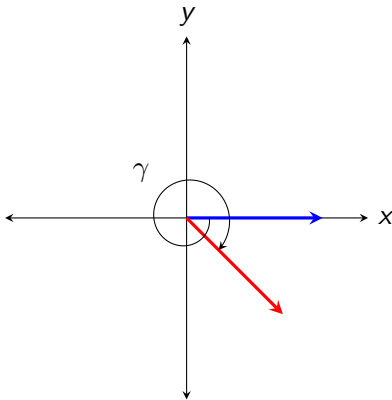
Example 3

$$(c) \quad \gamma = -\frac{9\pi}{4} \quad \gamma = -405^\circ$$



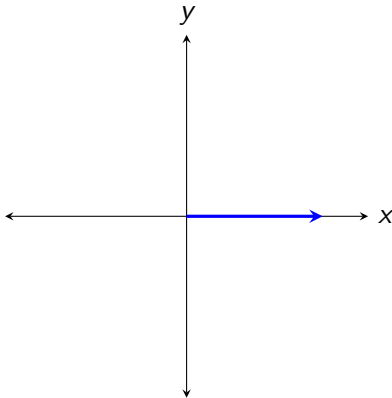
Example 3

$$(c) \quad \gamma = -\frac{9\pi}{4} \quad \gamma = -405^\circ$$



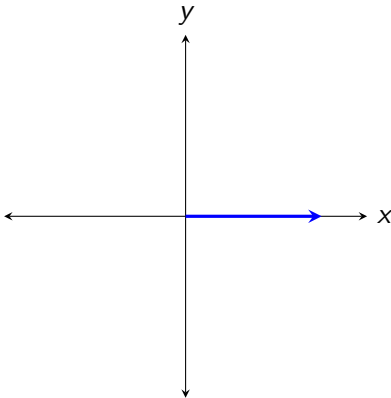
Example 3

(d) $\delta = -\frac{5\pi}{2}$



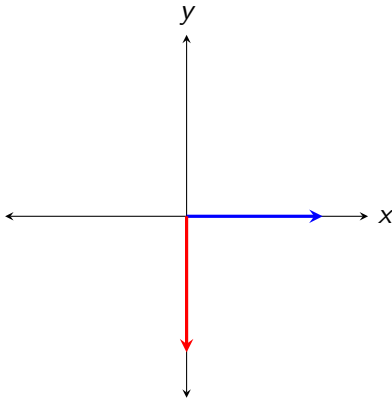
Example 3

$$(d) \quad \delta = -\frac{5\pi}{2} \quad \delta = -450^\circ$$



Example 3

$$(d) \quad \delta = -\frac{5\pi}{2} \quad \delta = -450^\circ$$



Example 3

$$(d) \quad \delta = -\frac{5\pi}{2} \quad \delta = -450^\circ$$

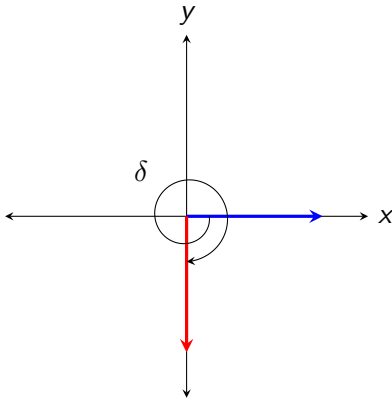


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Coterminal Angles

Two angles that have the same initial and terminal side are **coterminal angles**.

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Two angles that have the same initial and terminal side are **coterminal angles**.

To find coterminal angles, add (or subtract) multiples of 360° (or 2π radians).

Example 4

Find a coterminal angle between 0° and 360° (or 0 and 2π radians) for each.

(a) 400°

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Find a coterminal angle between 0° and 360° (or 0 and 2π radians) for each.

(a) 400°

400°

Example 4

Find a coterminal angle between 0° and 360° (or 0 and 2π radians) for each.

(a) 400°

$$400^\circ \rightarrow 400^\circ - 360^\circ$$

Example 4

Find a coterminal angle between 0° and 360° (or 0 and 2π radians) for each.

(a) 400°

$$\begin{aligned} 400^\circ &\rightarrow 400^\circ - 360^\circ \\ &= 40^\circ \end{aligned}$$

Example 4

$$(b) \quad -\frac{4\pi}{3}$$

Example 4

$$(b) \quad -\frac{4\pi}{3}$$

$$-\frac{4\pi}{3}$$

Example 4

$$(b) \quad -\frac{4\pi}{3}$$

$$-\frac{4\pi}{3} \rightarrow -\frac{4\pi}{3} + 2\pi$$

Example 4

$$(b) \quad -\frac{4\pi}{3}$$

$$-\frac{4\pi}{3} \rightarrow -\frac{4\pi}{3} + 2\pi$$

$$= \frac{2\pi}{3}$$

Example 4

(c) $\frac{9\pi}{4}$

Example 4

$$(c) \quad \frac{9\pi}{4}$$

$$\frac{9\pi}{4}$$

Example 4

$$(c) \quad \frac{9\pi}{4}$$

$$\frac{9\pi}{4} \rightarrow \frac{9\pi}{4} - 2\pi$$

Example 4

$$(c) \quad \frac{9\pi}{4}$$

$$\begin{aligned}\frac{9\pi}{4} &\rightarrow \frac{9\pi}{4} - 2\pi \\ &= \frac{\pi}{4}\end{aligned}$$

Example 4

(d) -785°

Example 4

(d) -785°

$$-785^\circ$$

Example 4

(d) -785°

$$-785^\circ \rightarrow -785^\circ + 1080^\circ$$

Example 4

(d) -785°

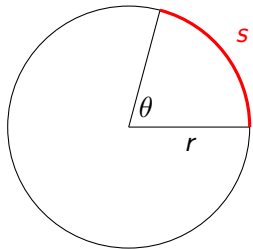
$$-785^\circ \rightarrow -785^\circ + 1080^\circ$$

$$= 295^\circ$$

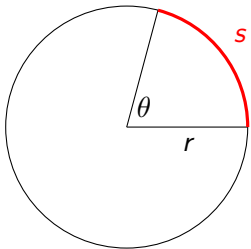
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Arc Length

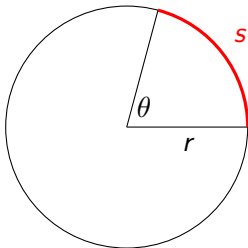


Arc Length



$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

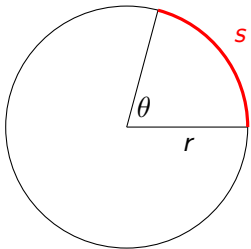
Arc Length



$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

$$2\pi s = 2\pi r\theta$$

Arc Length

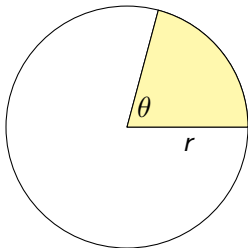


$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

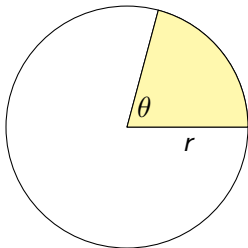
$$2\pi s = 2\pi r\theta$$

$$s = r\theta$$

Sector Area

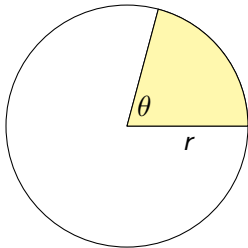


Sector Area



$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

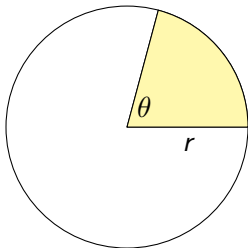
Sector Area



$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$2\pi A = \theta \pi r^2$$

Sector Area



$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$2\pi A = \theta \pi r^2$$

$$A = \frac{1}{2} \theta r^2$$

Example 5

Find the exact arc length and sector area of the circle with

$$r = 5 \text{ ft}; \theta = \frac{\pi}{2}$$

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Arc length:

Example 5

Find the exact arc length and sector area of the circle with

$$r = 5 \text{ ft}; \theta = \frac{\pi}{2}$$

Arc length:

$$s = r\theta$$

Example 5

Find the exact arc length and sector area of the circle with

$$r = 5 \text{ ft}; \theta = \frac{\pi}{2}$$

Arc length:

$$s = r\theta$$

$$s = 5 \left(\frac{\pi}{2} \right)$$

Example 5

Find the exact arc length and sector area of the circle with

$$r = 5 \text{ ft}; \theta = \frac{\pi}{2}$$

Arc length:

$$s = r\theta$$

$$s = 5 \left(\frac{\pi}{2} \right)$$

$$s = \frac{5\pi}{2} \text{ ft}$$

Example 5

Sector Area:

Example 5

Sector Area:

$$A = \frac{1}{2}\theta r^2$$

Example 5

Sector Area:

$$A = \frac{1}{2}\theta r^2$$

$$A = \frac{1}{2} \left(\frac{\pi}{2} \right) (5^2)$$

Example 5

Sector Area:

$$A = \frac{1}{2}\theta r^2$$

$$A = \frac{1}{2} \left(\frac{\pi}{2} \right) (5^2)$$

$$A = \frac{25\pi}{4} \text{ ft}^2$$