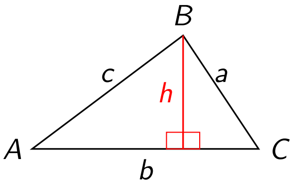
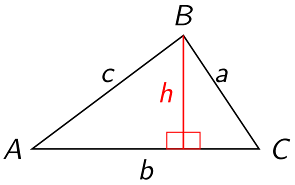
Area of Triangles

Derivation of Area Formula



Derivation of Area Formula

$$\text{Area} = \frac{1}{2} b h$$



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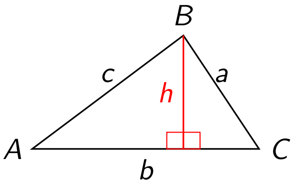

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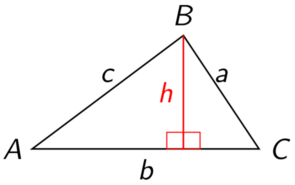

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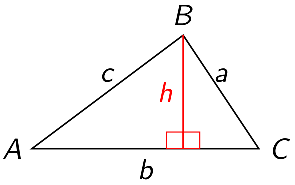

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$$\text{Area} = \frac{1}{2} bc \sin A$$

Objectives

1. Find the area of a SAS triangle
2. Find the area of an ASA or AAS triangle
3. Find the area of a SSS triangle

Find the Area of a SAS Triangle

The previous formula can be extended to find the area of a SAS triangle given any 2 side lengths and the angle measure between them:

$$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

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$$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

In other words,

$$\text{Area} = \frac{1}{2} \times \text{product of 2 sides} \times \text{sine of angle between them}$$

Example 1

Find the area of each triangle. Round your answers to 2 decimal places.

$A = 62^\circ, b = 10, c = 24$

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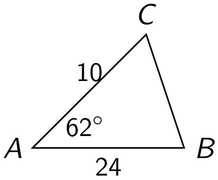

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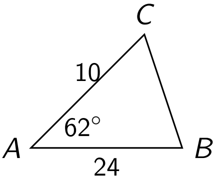

$$\text{Area} = \frac{1}{2}(10)(24)\sin 62^\circ$$

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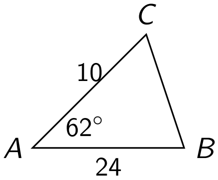

$$\text{Area} = \frac{1}{2}(10)(24)\sin 62^\circ$$

$$\approx 105.95 \text{ sq. units}$$

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1. Find the area of a SAS triangle
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Needing Law of Sines

Sometimes, you must use the Law of Sines to get enough information to find the area.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 2

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(a) $a = 5, B = 149^\circ, C = 16^\circ$

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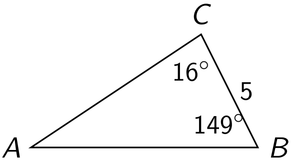

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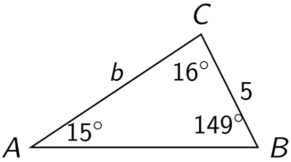

$$\frac{\sin 15}{5} = \frac{\sin 149}{b}$$

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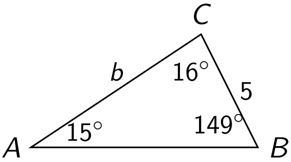

$$\frac{\sin 15}{5} = \frac{\sin 149}{b}$$

$$b \sin 15 = 5 \sin 149$$

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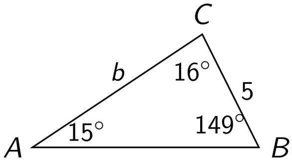
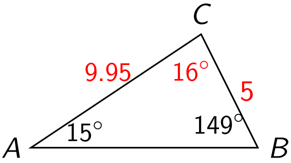


$$\frac{\sin 15}{5} = \frac{\sin 149}{b}$$

$$b \sin 15 = 5 \sin 149$$

$$b = \frac{5 \sin 149}{\sin 15} \approx 9.95$$

Example 2



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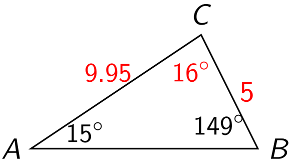

$$\text{Area} = \frac{1}{2} \times 9.95 \times 5 \times \sin 16^\circ \approx 6.86 \text{ sq. units}$$

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(b) $c = 7, B = 132^\circ, A = 28^\circ$

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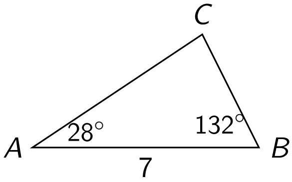

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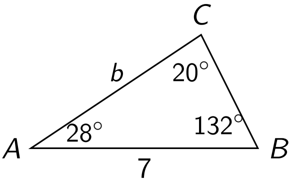

$$\frac{\sin 20}{7} = \frac{\sin 132}{b}$$

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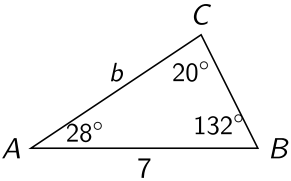

$$\frac{\sin 20}{7} = \frac{\sin 132}{b}$$

$$b \sin 20 = 7 \sin 132$$

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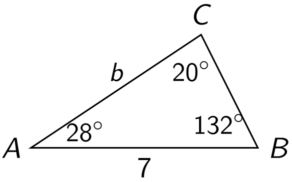

$$\frac{\sin 20}{7} = \frac{\sin 132}{b}$$

$$b \sin 20 = 7 \sin 132$$

$$b = \frac{7 \sin 132}{\sin 20} \approx 15.21$$

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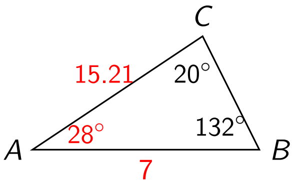

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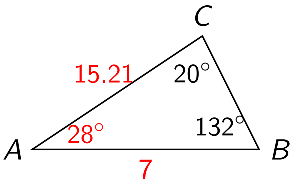

$$\text{Area} = \frac{1}{2} \times 15.21 \times 7 \times \sin 28^\circ \approx 24.99 \text{ sq. units}$$

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Find the Area of a SSS Triangle

While it is OK to use the area formulas that we have been using (after using Law of Cosines to find an angle measure) there is a quicker way to find the area of a SSS triangle by using Heron's Formula.

Heron's Formula

Let s be the semi-perimeter of $\triangle ABC$:

$$s = \frac{1}{2}(a + b + c)$$

Then

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Example 3

Find the area of each triangle. Round your answers to 2 decimal places.

(a) $a = 11, b = 13, c = 12$

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$$\text{Area} = \sqrt{18(18 - 11)(18 - 13)(18 - 12)}$$

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Find the area of each triangle. Round your answers to 2 decimal places.

(a) $a = 11, b = 13, c = 12$

$$s = \frac{1}{2}(11 + 13 + 12) = 18$$

$$\text{Area} = \sqrt{18(18-11)(18-13)(18-12)}$$

$$\approx 61.48 \text{ sq. units}$$

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(b) $a = 8, b = 10, c = 15$

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$$s = \frac{1}{2}(8 + 10 + 15) = 16.5$$

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(b) $a = 8, b = 10, c = 15$

$$s = \frac{1}{2}(8 + 10 + 15) = 16.5$$

$$\text{Area} = \sqrt{16.5(16.5 - 8)(16.5 - 10)(16.5 - 15)}$$

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(b) $a = 8, b = 10, c = 15$

$$s = \frac{1}{2}(8 + 10 + 15) = 16.5$$

$$\text{Area} = \sqrt{16.5(16.5 - 8)(16.5 - 10)(16.5 - 15)}$$

$$\approx 36.98 \text{ sq. units}$$