

# Graphs of Sine and Cosine

# Intro

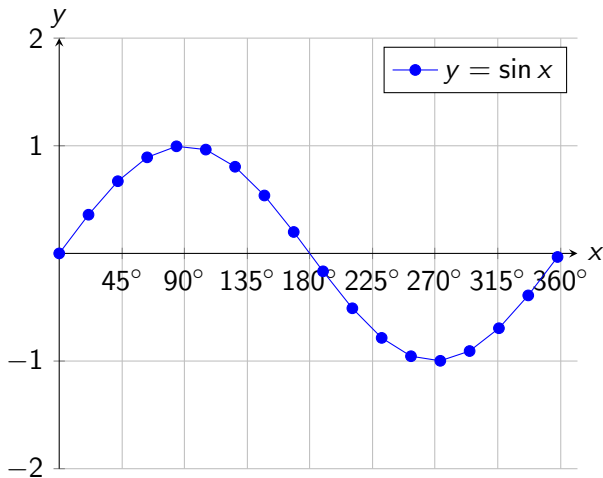
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# Intro

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If we let the  $x$ -coordinates represent the measure of the angle and let the  $y$ -coordinates represent the value of the ratio, we can plot the graph of the sine function.

# Sine Graph



# Table of Contents

- 1 Find the Amplitude of a Sine or Cosine Function
- 2 Determine the Period of a Sine or Cosine Function
- 3 Determine the Vertical Shift of a Sine or Cosine Function
- 4 Determine Phase Shifts of a Sine or Cosine Function

# Amplitude

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In the graph previously, the maximum of the graph is 1. The minimum of the graph is  $-1$ . Thus, the amplitude of  $y = \sin x$  is

$$\frac{1}{2} (1 - (-1)) = 1$$

If we want to change the amplitude, we need to **stretch the graph vertically**. Recall that vertical stretches involve multiplying the function by a positive value (negative values reflect the graph across the  $x$ -axis).



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For functions in the form

$$y = A \sin x \quad \text{or} \quad y = B \cos x$$

the amplitude is  $|A|$ .

## Example 1

Determine the amplitude of each of the following.

(a)  $y = 3 \sin x$

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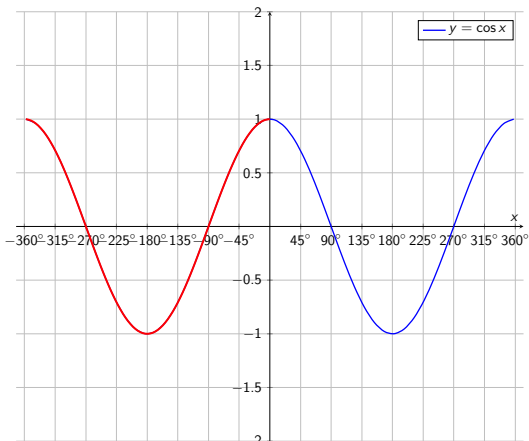
$$\text{Amplitude} = |-2| = 2$$

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# Period

The graphs of sine and cosine are **periodic**, in that the pattern repeats itself infinitely in both directions. The graph of  $y = \cos x$  is shown below.



# Period

Notice that the part of the graph in purple is a copy of the part of the graph in blue. This is because the cosine function is **periodic**.

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Each of the following are ways in which you can think of the period of the graph of a function:

- How long until the graph starts to repeat the same values in the same order.
- What is the least amount of the graph you would need to copy to paste it before and after the copy?
- If the units along the  $x$ -axis were length, period would be the wavelength from science class.



# Changing the Period

We can adjust the period of the sine and cosine functions by multiplying the input values,  $x$ , by a positive number other than 1.

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Thus, the equation for adjusting the period of sine and cosine functions is

$$y = \sin(Bx) \quad \text{and} \quad y = \cos(Bx)$$

## Example 2

Use a graphing utility to determine the period of each of the following.

(a)  $y = \sin(2x)$

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Period =  $180^\circ$

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Use a graphing utility to determine the period of each of the following.

(a)  $y = \sin(2x)$

Period =  $180^\circ$

(b)  $y = \cos(3x)$

Period =  $120^\circ$

## Example 2

(c)  $y = \sin\left(\frac{1}{2}x\right)$

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$$(c) \quad y = \sin\left(\frac{1}{2}x\right)$$

$$\text{Period} = 720^\circ$$



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$$\text{Period} = 720^\circ$$

$$(d) \quad y = \cos\left(\frac{1}{4}x\right)$$

## Example 2

$$(c) \quad y = \sin\left(\frac{1}{2}x\right)$$

$$\text{Period} = 720^\circ$$

$$(d) \quad y = \cos\left(\frac{1}{4}x\right)$$

$$\text{Period} = 1440^\circ$$

# Period

With  $y = \sin(Bx)$  and  $y = \cos(Bx)$ , looking at the graphs, it would seem that the different values of  $B$  affect the graphs in different ways.

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Notice in each case, our answers are whatever  $360^\circ/B$  equals.

# Period

With  $y = \sin(Bx)$  and  $y = \cos(Bx)$ , looking at the graphs, it would seem that the different values of  $B$  affect the graphs in different ways.

Notice in each case, our answers are whatever  $360^\circ/B$  equals.

Therefore, the period of the graph of the sine and cosine functions is

$$\frac{360^\circ}{B} \quad \text{or} \quad \frac{2\pi}{B}$$

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# Vertical Shifts

Recall from transforming functions that we shift functions vertically by adding or subtracting a value *from the function itself*.

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Recall from transforming functions that we shift functions vertically by adding or subtracting a value *from the function itself*.

For sine and cosine functions, the vertical shift becomes

$$y = \sin x + D \quad \text{and} \quad y = \cos x + D$$



## Example 3

Determine the vertical shift for each of the following.

(a)  $y = \sin x + 3$

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Vertical shift = Up 3 units

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Determine the vertical shift for each of the following.

(a)  $y = \sin x + 3$

Vertical shift = Up 3 units

(b)  $y = \cos x - 1$

Vertical shift = Down 1 unit

## Example 3

(c)  $y = 2 \sin x - 4$

## Example 3

(c)  $y = 2 \sin x - 4$

Vertical shift = Down 4 units

## Example 3

(c)  $y = 2 \sin x - 4$

Vertical shift = Down 4 units

(d)  $y = -0.5 \cos x$

## Example 3

(c)  $y = 2 \sin x - 4$

Vertical shift = Down 4 units

(d)  $y = -0.5 \cos x$

Vertical shift = None (or 0 units)



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# Phase Shifts

In trigonometry, phase shifts are the periodic functions' version of **horizontal shifts**.

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In trigonometry, phase shifts are the periodic functions' version of **horizontal shifts**.

For sine and cosine functions, phase shifts typically resemble the following:

$$y = \sin(Bx - C) \quad \text{and} \quad y = \cos(Bx - C)$$

# Phase Shifts

To find the value of the phase shift, set  $Bx - C = 0$  and solve.

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This value will be how far from the origin your graph shifts:

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- Positive answer: shifts right

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To find the value of the phase shift, set  $Bx - C = 0$  and solve.

This value will be how far from the origin your graph shifts:

- Positive answer: shifts right
- Negative answer: shifts left

## Example 4a

Determine the phase shift of each of the following. Don't forget to indicate direction.

(a)  $y = \sin(x - 30^\circ)$



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$$x = 30$$

## Example 4a

Determine the phase shift of each of the following. Don't forget to indicate direction.

(a)  $y = \sin(x - 30^\circ)$

$$x - 30 = 0$$

$$x = 30$$

The graph is shifted  $30^\circ$  to the right.

## Example 4b

$$(b) \quad y = \cos(x + 135^\circ)$$

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$$x + 135 = 0$$

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$$(b) \quad y = \cos(x + 135^\circ)$$

$$x + 135 = 0$$

$$x = -135$$

## Example 4b

$$(b) \quad y = \cos(x + 135^\circ)$$

$$x + 135 = 0$$

$$x = -135$$

The graph is shifted  $135^\circ$  to the left

## Example 4c

$$(c) \quad y = \sin(2x + 90^\circ)$$



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$$(c) \quad y = \sin(2x + 90^\circ)$$

$$2x + 90 = 0$$

## Example 4c

$$(c) \quad y = \sin(2x + 90^\circ)$$

$$2x + 90 = 0$$

$$2x = -90$$

## Example 4c

$$(c) \quad y = \sin(2x + 90^\circ)$$

$$2x + 90 = 0$$

$$2x = -90$$

$$x = -45$$

## Example 4c

$$(c) \quad y = \sin(2x + 90^\circ)$$

$$2x + 90 = 0$$

$$2x = -90$$

$$x = -45$$

The graph is shifted  $45^\circ$  to the left

## Example 4d

(d)  $y = \cos(3x - 270^\circ)$

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$$(d) \quad y = \cos(3x - 270^\circ)$$

$$3x - 270 = 0$$

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$$3x - 270 = 0$$

$$3x = 270$$

## Example 4d

$$(d) \quad y = \cos(3x - 270^\circ)$$

$$3x - 270 = 0$$

$$3x = 270$$

$$x = 90$$



## Example 4d

$$(d) \quad y = \cos(3x - 270^\circ)$$

$$3x - 270 = 0$$

$$3x = 270$$

$$x = 90$$

The graph is shifted  $90^\circ$  to the right

# Summary

For  $y = A \sin(Bx - C) + D$  or  $y = A \cos(Bx - C) + D$ :

Amplitude	Period	Phase Shift	Vertical Shift
$ A $	$\frac{360^\circ}{B}$ or $\frac{2\pi}{B}$	$\frac{C}{B}$	$D$