Polar Form of Complex Numbers

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The Complex Plane

The complex plane is very similar to the Cartesian (x, y) plane from algebra.

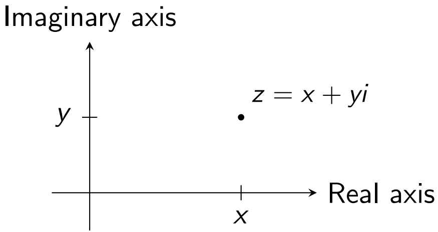
The point (x, y) in the Cartesian plane is represented by

$$z = x + yi$$

in the complex plane.

x is called the *real part* and y is called the *imaginary part*.

The Complex Plane



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Modulus of a Complex Number

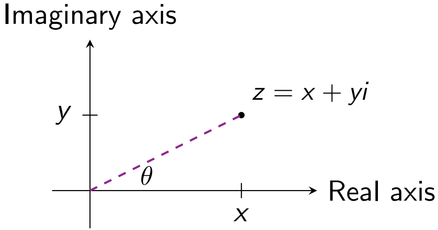
The modulus of a complex number is the absolute value of it:

$$|z| = |x + yi| = \sqrt{x^2 + y^2}$$

and it denotes the *distance* the point is to the origin.

Argument of a Complex Number

If we connect out point to the origin, it models an angle drawn in standard position.



Argument of a Complex Number

If z is in polar form, the total angle rotated θ is the argument of z.

The set of all arguments of z is denoted $\arg(z)$. (These can be found by using co-terminal angles).

If $z \neq 0$ and $-\pi < \theta \leq \pi$, then θ is the principal argument of z, and is written $\theta = \text{Arg}(z)$

We can get the angle rotated via reference angles using

$$\theta' = \tan^{-1}\left(\frac{y}{x}\right)$$

Example 1a.

Plot each complex number, find its modulus, real and imaginary parts, $\arg(z)$ and $\text{Arg}(z)$.

(a) $z = \sqrt{3} - i$

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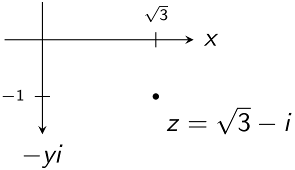
Plot each complex number, find its modulus, real and imaginary parts, $\arg(z)$ and $\mathrm{Arg}(z)$.

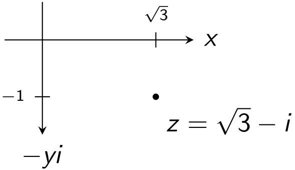
(a) $z = \sqrt{3} - i$

The real part of z is $\sqrt{3}$ and the imaginary part of z is -1.

Example 1a.



Example 1a.



The modulus is

$$|z| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$$

Example 1a.

Tip: Find the principal argument of z, denoted $\operatorname{Arg}(z)$, before finding the general solution, $\arg(z)$.

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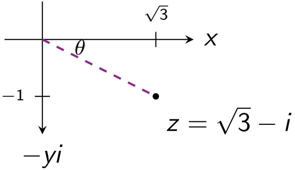
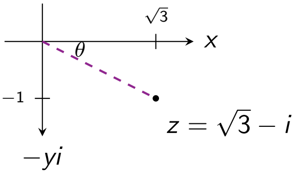


Example 1a.

Tip: Find the principal argument of z, denoted $\mathrm{Arg}(z)$, before finding the general solution, $\arg(z)$.



$$\mathrm{Arg}(z) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -30^\circ = -\frac{\pi}{6}$$

Example 1a.

To find $\arg(z)$, use co-terminal angles with the answer you got for $\operatorname{Arg}(z)$ to get

$$\arg(z) = -\frac{\pi}{6} + 2\pi k$$

where k is an integer.

Example 1b.

(b) $z = -2 + 4i$

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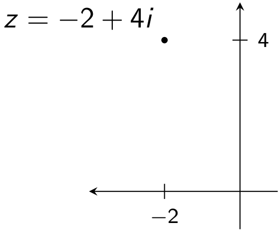
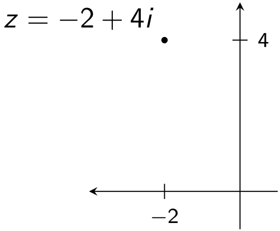
The real part of z is -2 and the imaginary part of z is 4.

Example 1b.



Example 1b.



The modulus is

$$|z| = \sqrt{(-2)^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$$

Example 1b.

Arg(z) is

$$\tan^{-1}\left(\frac{4}{-2}\right) = \tan^{-1}(-2) \approx -63.4^\circ$$

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Since z is in the 2nd quadrant, we can put a 63.4° reference angle (*Reminder*: $63.4^\circ \approx \tan^{-1}(2)$) in quadrant II, so the total angle rotated is about 116.6°, or $\pi - \tan^{-1}(2)$ to be exact.

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Thus, $\text{Arg}(z) = \pi - \tan^{-1}(2)$,

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Thus, $\text{Arg}(z) = \pi - \tan^{-1}(2)$,

and

$\arg(z) = \left(\pi - \tan^{-1}(2)\right) + 2\pi k$

Example 1c.

(c) $z = 3i$

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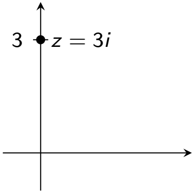
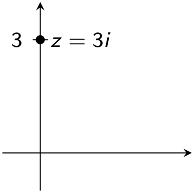
The real part of z is 0 and the imaginary part of z is 3.

Example 1c.



Example 1c.



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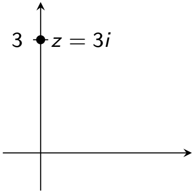

The modulus is 3.

$$\mathrm{Arg}(z) = \frac{\pi}{2}$$

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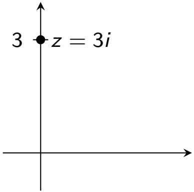

The modulus is 3.

$$\mathrm{Arg}(z) = \frac{\pi}{2}$$

$$\arg(z) = \frac{\pi}{2} + 2\pi k$$

Example 1d.

(d) $z = -117$

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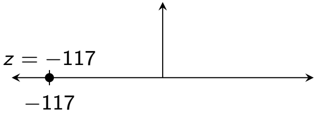
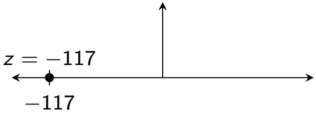
(d) $\quad z = -117$

The real part of z is -117 and the imaginary part of z is 0.

Example 1d.

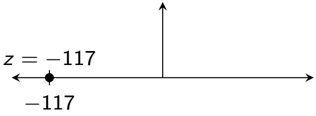


Example 1d.



The modulus of z is 117.

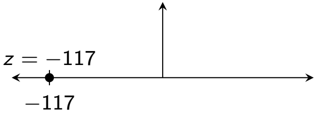
Example 1d.



The modulus of z is 117.

$$\operatorname{Arg}(z) = \pi$$

Example 1d.



The modulus of z is 117.

$$\text{Arg}(z) = \pi$$

$$\arg(z) = \pi + 2\pi k$$

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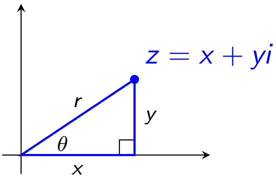
Write Complex Numbers in Polar Form

Writing polar form of complex numbers is a lot like writing rectangular coordinates as polar coordinates.



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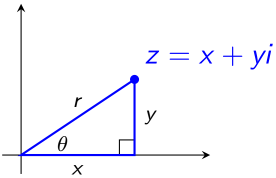

Using the fact that $x = r\cos\theta$ and $y = r\sin\theta$, we get

$$z = x + yi$$

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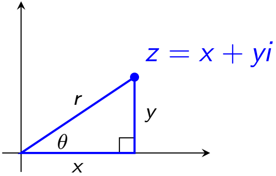


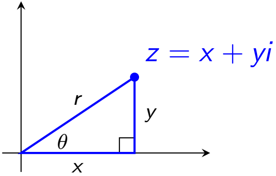
Using the fact that $x = r\cos\theta$ and $y = r\sin\theta$, we get

$$z = x + yi$$

$$= r\cos\theta + (r\sin\theta)i$$

Write Complex Numbers in Polar Form

Writing polar form of complex numbers is a lot like writing rectangular coordinates as polar coordinates.



Using the fact that $x = r\cos\theta$ and $y = r\sin\theta$, we get

$$\begin{aligned} z &= x + yi \\ &= r\cos\theta + (r\sin\theta)i \\ &= r(\cos\theta + i\sin\theta) \end{aligned}$$

Write Complex Numbers in Polar Form

$r(\cos\theta + i\sin\theta)$ is often abbreviated $r\,\mathrm{cis}\,\theta$.

Example 2a.

Write each in polar form.

(a) $z = \sqrt{3} - i$

Example 2a.

Write each in polar form.

(a) $z = \sqrt{3} - i$

From Example 1a, $r = 2$, and $\theta = -\frac{\pi}{6}$.

Thus, $z = 2\,\text{cis}\left(-\frac{\pi}{6}\right)$

Example 2b.

Write each in polar form.

(b) $z = -2 + 4i$

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Write each in polar form.

(b) $z = -2 + 4i$

From Example 1b, $r = 2\sqrt{5}$, and $\theta = \pi - \tan^{-1}(2)$.

Thus, $z = \left(2\sqrt{5}\right) \operatorname{cis}\left(\pi - \tan^{-1}(2)\right)$

Example 2c.

Write each in polar form.

(c) $z = 3i$

Example 2c.

Write each in polar form.

(c) $z = 3i$

From Example 1c, $r = 3$, and $\theta = \frac{\pi}{2}$.

Thus, $z = 3\,\text{cis}\left(\frac{\pi}{2}\right)$

Example 2d.

Write each in polar form.

(d) $z = -117$

Example 2d.

Write each in polar form.

(d) $z = -117$

From Example 1d, $r = 117$, and $\theta = \pi$.

Thus, $z = 117\,\text{cis}\,(\pi)$

Write Complex Polar Numbers in Rectangular Form

Going backwards, expand and evaluate cis and distribute the modulus r.

Example 3a.

Write each of the following in rectangular form.

(a) $4\,\text{cis}\left(\frac{2\pi}{3}\right)$

Example 3a.

Write each of the following in rectangular form.

(a) $4\,\text{cis}\left(\frac{2\pi}{3}\right)$

$$4\,\text{cis}\left(\frac{2\pi}{3}\right) = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$$

Example 3a.

Write each of the following in rectangular form.

(a) $4\,\mathrm{cis}\left(\frac{2\pi}{3}\right)$

$$4\,\mathrm{cis}\left(\frac{2\pi}{3}\right) = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$$

$$= 4\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

Example 3a.

Write each of the following in rectangular form.

(a) $4\,\text{cis}\left(\frac{2\pi}{3}\right)$

$$
\begin{aligned}
4\,\text{cis}\left(\frac{2\pi}{3}\right) &= 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right) \\
&= 4\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
&= -2 + 2i\sqrt{3}
\end{aligned}
$$

Example 3b.

(b) $\quad 2\,\text{cis}\left(-\frac{3\pi}{4}\right)$

Example 3b.

(b) $2\,\text{cis}\left(-\frac{3\pi}{4}\right)$

$$2\,\text{cis}\left(-\frac{3\pi}{4}\right) = 2\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$$

Example 3b.

(b) $2\,\mathrm{cis}\left(-\frac{3\pi}{4}\right)$

$$2\,\mathrm{cis}\left(-\frac{3\pi}{4}\right) = 2\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$$

$$= 2\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$

Example 3b.

(b) $2\,\text{cis}\left(-\frac{3\pi}{4}\right)$

$$\begin{aligned} 2\,\text{cis}\left(-\frac{3\pi}{4}\right) &= 2\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right) \\ &= 2\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) \\ &= -\sqrt{2} - i\sqrt{2} \end{aligned}$$

Example 3c.

(c) $\quad 3\,\text{cis}(0)$

Example 3c.

(c) $3\operatorname{cis}(0)$

$$3\operatorname{cis}(0) = 3(\cos(0) + i\sin(0))$$

Example 3c.

(c) $3\,\mathrm{cis}(0)$

$$3\,\mathrm{cis}(0) = 3(\cos(0) + i\sin(0))$$

$$= 3(1 + 0) = 3$$

Example 3d.

(d) $\quad \text{cis}\left(\frac{\pi}{2}\right)$

Example 3d.

(d) $\operatorname{cis}\left(\frac{\pi}{2}\right)$

$$\operatorname{cis}\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$$

Example 3d.

(d) $\operatorname{cis}\left(\frac{\pi}{2}\right)$

$$\operatorname{cis}\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$$

$$= 0 + 1i = i$$

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Arithmetic Operations on Complex Numbers in Polar Form

The rules for multiplying, dividing, and finding powers of complex numbers are the same as those for exponents.

Arithmetic Operations on Complex Numbers in Polar Form

The rules for multiplying, dividing, and finding powers of complex numbers are the same as those for exponents.

Rule	**Exponents**	**Complex Numbers**
Product	$3x^{15} \cdot 4x^7 = 12x^{22}$	$3\operatorname{cis} 15^\circ \cdot 4\operatorname{cis} 7^\circ = 12\operatorname{cis} 22^\circ$
Quotient	$\dfrac{3x^{15}}{4x^7} = \dfrac{3}{4}x^8$	$\dfrac{3\operatorname{cis} 15^\circ}{4\operatorname{cis} 7^\circ} = \dfrac{3}{4}\operatorname{cis} 8^\circ$
Power	$(3x^{15})^2 = 9x^{30}$	$(3\operatorname{cis} 15^\circ)^2 = 9\operatorname{cis} 30^\circ$

The Power Rule is known as DeMoivre's Theorem

Example 4a

Let $z = 2\sqrt{3} + 2i$ and $w = -1 + i\sqrt{3}$. Find each of the following and write your answers in rectangular form.

(a) zw

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Let $z = 2\sqrt{3} + 2i$ and $w = -1 + i\sqrt{3}$. Find each of the following and write your answers in rectangular form.

(a) zw

In polar form, $z = 4\,\mathrm{cis}\,\dfrac{\pi}{6}$ and $w = 2\,\mathrm{cis}\,\dfrac{2\pi}{3}$.

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Let $z = 2\sqrt{3} + 2i$ and $w = -1 + i\sqrt{3}$. Find each of the following and write your answers in rectangular form.

(a) zw

In polar form, $z = 4\,\mathrm{cis}\,\dfrac{\pi}{6}$ and $w = 2\,\mathrm{cis}\,\dfrac{2\pi}{3}$.

Thus, $zw = \left(4\,\mathrm{cis}\,\dfrac{\pi}{6}\right)\left(2\,\mathrm{cis}\,\dfrac{2\pi}{3}\right) = 8\,\mathrm{cis}\,\dfrac{5\pi}{6}$

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Let $z = 2\sqrt{3} + 2i$ and $w = -1 + i\sqrt{3}$. Find each of the following and write your answers in rectangular form.

(a) zw

In polar form, $z = 4\operatorname{cis}\frac{\pi}{6}$ and $w = 2\operatorname{cis}\frac{2\pi}{3}$.

Thus, $zw = \left(4\operatorname{cis}\frac{\pi}{6}\right)\left(2\operatorname{cis}\frac{2\pi}{3}\right) = 8\operatorname{cis}\frac{5\pi}{6}$

In rectangular form, $8\operatorname{cis}\frac{5\pi}{6} = -4\sqrt{3} + 4i$

Example 4b

(b) w^5

Example 4b

(b) w^5

$$w^5 = \left(2 \operatorname{cis} \frac{2\pi}{3}\right)^5 = 32 \operatorname{cis} \frac{10\pi}{3}$$

Example 4b

(b) w^5

$$w^5 = \left(2\,\mathrm{cis}\,\frac{2\pi}{3}\right)^5 = 32\,\mathrm{cis}\,\frac{10\pi}{3}$$

$$32\,\mathrm{cis}\,\frac{10\pi}{3} = 32\,\mathrm{cis}\,\frac{4\pi}{3} = -16 - 16i\sqrt{3}$$

Example 4c

(c) $\dfrac{z}{w}$

Example 4c

(c) $\dfrac{z}{w}$

$$\frac{z}{w} = \frac{4\,\mathrm{cis}\,\frac{\pi}{6}}{2\,\mathrm{cis}\,\frac{2\pi}{3}}$$

Example 4c

(c) $\dfrac{z}{w}$

$$\frac{z}{w} = \frac{4 \operatorname{cis} \frac{\pi}{6}}{2 \operatorname{cis} \frac{2\pi}{3}}$$

$$= 2 \operatorname{cis} \left(\frac{\pi}{6} - \frac{2\pi}{3} \right)$$

Example 4c

(c) $\dfrac{z}{w}$

$$\frac{z}{w} = \frac{4\operatorname{cis}\frac{\pi}{6}}{2\operatorname{cis}\frac{2\pi}{3}}$$

$$= 2\operatorname{cis}\left(\frac{\pi}{6} - \frac{2\pi}{3}\right)$$

$$= 2\operatorname{cis}\left(-\frac{\pi}{2}\right) = -2i$$

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Find Roots of Complex Numbers

Recall that $\sqrt[n]{x} = x^{1/n}$.

We can reverse DeMoivre's Theorem to get roots of complex numbers.

Find Roots of Complex Numbers

Recall that $\sqrt[n]{x} = x^{1/n}$.

We can reverse DeMoivre's Theorem to get roots of complex numbers.

Given w and z are complex numbers, if there is a natural number n such that $w^n = z$, then w is an nth root of z.

Find Roots of Complex Numbers

Let $z \neq 0$ be a complex numbrer with polar form $z = r \operatorname{cis} \theta$. For each natural number n, z has n distinct nth roots (denoted by $w_0, w_1, \ldots, w_{n-1}$, given by

$$w_k = \sqrt[n]{r} \operatorname{cis} \left(\frac{\theta + 2\pi k}{n} \right)$$

Find Roots of Complex Numbers

Let $z \neq 0$ be a complex numbrer with polar form $z = r \operatorname{cis} \theta$. For each natural number n, z has n distinct nth roots (denoted by $w_0, w_1, \ldots, w_{n-1}$, given by

$$w_k = \sqrt[n]{r} \operatorname{cis} \left(\frac{\theta + 2\pi k}{n} \right)$$

Tip: You can still work in degrees and convert your answers to radians.

Example 5a

Find each of the following.

(a) both square roots of $z = -2 + 2i\sqrt{3}$

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Find each of the following.

(a) both square roots of $z = -2 + 2i\sqrt{3}$

$$z = 4 \operatorname{cis} \frac{2\pi}{3} \text{ (or } 4 \operatorname{cis} 120^\circ\text{)}$$

Example 5a

Find each of the following.

(a) both square roots of $z = -2 + 2i\sqrt{3}$

$$z = 4\,\text{cis}\,\frac{2\pi}{3}\ (\text{or } 4\,\text{cis}\,120^\circ)$$
$$= 4^{1/2}\left(\cos\frac{120^\circ + 360k}{2} + i\sin\frac{120^\circ + 360k}{2}\right)$$

Example 5a

Find each of the following.

(a) both square roots of $z = -2 + 2i\sqrt{3}$

$$
\begin{aligned}
z &= 4\,\text{cis}\,\frac{2\pi}{3}\ (\text{or } 4\,\text{cis}\,120^\circ) \\
&= 4^{1/2}\left(\cos\frac{120^\circ + 360k}{2} + i\sin\frac{120^\circ + 360k}{2}\right) \\
&= 2\left(\cos\left(60^\circ + 180k\right) + i\sin\left(60^\circ + 180k\right)\right)
\end{aligned}
$$

Example 5a

Find each of the following.

(a) both square roots of $z = -2 + 2i\sqrt{3}$

$$\begin{aligned} z &= 4 \operatorname{cis} \frac{2\pi}{3} \text{ (or } 4 \operatorname{cis} 120^\circ\text{)} \\ &= 4^{1/2} \left(\cos \frac{120^\circ + 360k}{2} + i \sin \frac{120^\circ + 360k}{2} \right) \\ &= 2 \left(\cos \left(60^\circ + 180k \right) + i \sin \left(60^\circ + 180k \right) \right) \\ &= 2 \operatorname{cis} 60^\circ, 2 \operatorname{cis} 240^\circ \end{aligned}$$

Example 5a

Find each of the following.

(a) both square roots of $z = -2 + 2i\sqrt{3}$

$$
\begin{aligned}
z &= 4\operatorname{cis}\frac{2\pi}{3} \text{ (or } 4\operatorname{cis}120^\circ\text{)} \\
&= 4^{1/2}\left(\cos\frac{120^\circ + 360k}{2} + i\sin\frac{120^\circ + 360k}{2}\right) \\
&= 2\left(\cos\left(60^\circ + 180k\right) + i\sin\left(60^\circ + 180k\right)\right) \\
&= 2\operatorname{cis}60^\circ, 2\operatorname{cis}240^\circ \\
&= 1 + i\sqrt{3}, \quad -2 + 2i\sqrt{3}
\end{aligned}
$$

Example 5b

(b) the four fourth roots of $z = -16$

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$$z = 16\,\mathrm{cis}\,180^\circ$$

Example 5b

(b) the four fourth roots of $z = -16$

$$\begin{aligned} z &= 16 \operatorname{cis} 180^\circ \\ &= 16^{1/4} \operatorname{cis}\left(\frac{180^\circ + 360k}{4}\right) \end{aligned}$$

Example 5b

(b) the four fourth roots of $z = -16$

$$\begin{aligned} z &= 16 \operatorname{cis} 180^\circ \\ &= 16^{1/4} \operatorname{cis}\left(\frac{180^\circ + 360k}{4}\right) \\ &= 2 \operatorname{cis}(45^\circ + 90k) \end{aligned}$$

Example 5b

(b) the four fourth roots of $z = -16$

$$\begin{aligned} z &= 16 \operatorname{cis} 180^\circ \\ &= 16^{1/4} \operatorname{cis}\left(\frac{180^\circ + 360k}{4}\right) \\ &= 2 \operatorname{cis}(45^\circ + 90k) \end{aligned}$$

$2 \operatorname{cis} 45^\circ, 2 \operatorname{cis} 135^\circ, 2 \operatorname{cis} 225^\circ, 2 \operatorname{cis} 315^\circ$

Example 5b

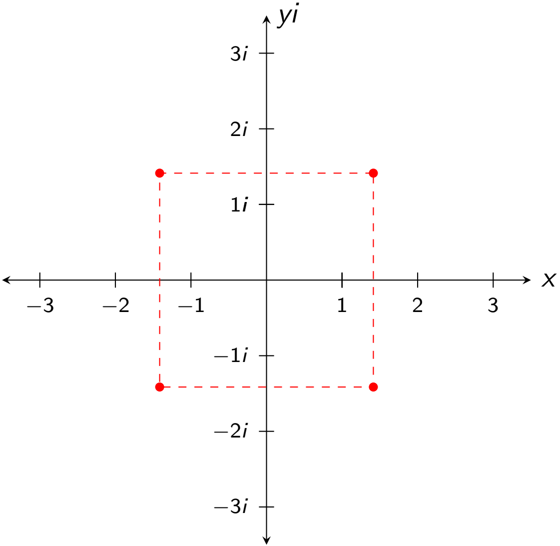
(b) the four fourth roots of $z = -16$

$$\begin{aligned} z &= 16 \operatorname{cis} 180^\circ \\ &= 16^{1/4} \operatorname{cis}\left(\frac{180^\circ + 360k}{4}\right) \\ &= 2 \operatorname{cis}(45^\circ + 90k) \end{aligned}$$

$2 \operatorname{cis} 45^\circ, 2 \operatorname{cis} 135^\circ, 2 \operatorname{cis} 225^\circ, 2 \operatorname{cis} 315^\circ$

$\sqrt{2} + i\sqrt{2}, -\sqrt{2} + i\sqrt{2}, -\sqrt{2} - i\sqrt{2}, \sqrt{2} - i\sqrt{2}$

Visualization of Solutions



Example 5c

(c) the three cube roots of $z = \sqrt{2} + i\sqrt{2}$

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$$z = 2\,\text{cis}\,(45^\circ)$$

Example 5c

(c) the three cube roots of $z = \sqrt{2} + i\sqrt{2}$

$$\begin{aligned} z &= 2\,\mathrm{cis}\,(45^\circ) \\ &= 2^{1/3}\left(\mathrm{cis}\left(\frac{45^\circ + 360k}{3}\right)\right) \end{aligned}$$

Example 5c

(c) the three cube roots of $z = \sqrt{2} + i\sqrt{2}$

$$\begin{aligned} z &= 2\,\text{cis}\,(45^\circ) \\ &= 2^{1/3}\left(\text{cis}\left(\frac{45^\circ + 360k}{3}\right)\right) \\ &= \sqrt[3]{2}\,\text{cis}\,(15^\circ + 120k) \end{aligned}$$

Example 5c

(c) the three cube roots of $z = \sqrt{2} + i\sqrt{2}$

$$\begin{aligned} z &= 2\,\mathrm{cis}\,(45^\circ) \\ &= 2^{1/3}\left(\mathrm{cis}\left(\frac{45^\circ + 360k}{3}\right)\right) \\ &= \sqrt[3]{2}\,\mathrm{cis}\,(15^\circ + 120k) \end{aligned}$$

$\sqrt[3]{2}\,\mathrm{cis}\,15^\circ, \sqrt[3]{2}\,\mathrm{cis}\,135^\circ, \sqrt[3]{2}\,\mathrm{cis}\,255^\circ$

Example 5d

(d) the five fifth roots of $z = 1$

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$$z = 1 \operatorname{cis} 0$$

Example 5d

(d) the five fifth roots of $z = 1$

$$
\begin{aligned}
z &= 1\,\mathrm{cis}\,0 \\
&= \mathrm{cis}\left(\frac{0 + 360k}{5}\right)
\end{aligned}
$$

Example 5d

(d) the five fifth roots of $z = 1$

$$\begin{aligned} z &= 1 \operatorname{cis} 0 \\ &= \operatorname{cis}\left(\frac{0 + 360k}{5}\right) \\ &= \operatorname{cis}(0 + 72k) \end{aligned}$$

Example 5d

(d) the five fifth roots of $z = 1$

$$\begin{aligned} z &= 1 \operatorname{cis} 0 \\ &= \operatorname{cis}\left(\frac{0 + 360k}{5}\right) \\ &= \operatorname{cis}(0 + 72k) \end{aligned}$$

$1, \operatorname{cis} 72^\circ, \operatorname{cis} 144^\circ, \operatorname{cis} 216^\circ, \operatorname{cis} 288^\circ$