Parabolas

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Intro

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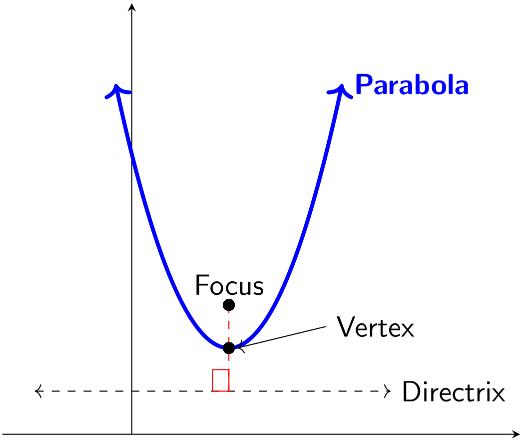
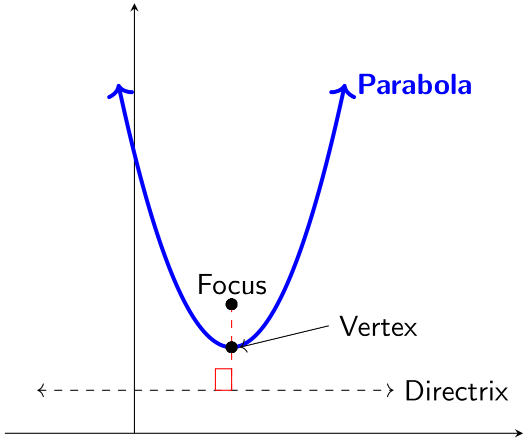
If we look at the graph of the quadratic function $f(x) = ax^2 + bx + c$, we obtain what is known as a *parabola*.

A parabola is the set of all points in the plane that are the same distance from the focus and the directrix line.


Parabola
Focus
Vertex
Directrix



The focal length is the distance between the focus and vertex (or directrix and vertex) and is $|p|$.

Equations

	Opens Up or Down	
	$(x-h)^2 = 4p(y-k)$	
Vertex	(h, k)	
Focus Point	$(h, k+p)$	
Directrix	$y = k - p$	

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Vertex	(h, k)	(h, k)
Focus Point	$(h, k+p)$	$(h+p, k)$
Directrix	$y = k - p$	$x = h - p$

Note: Sometimes the equations are written as

$$y = \frac{1}{4p}(x-h)^2 + k \text{ and } x = \frac{1}{4p}(y-k)^2 + h$$

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Finding Vertex Without Technology

For $y = ax^2 + bx + c$

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1. x-coordinate: $-\frac{b}{2a}$

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Example 1

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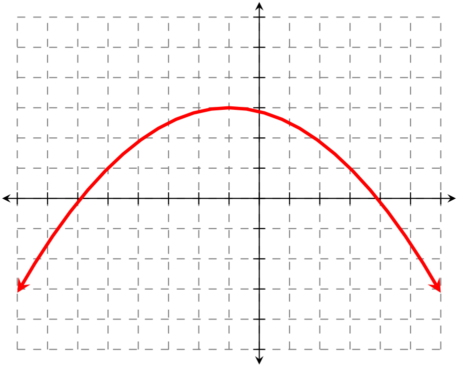

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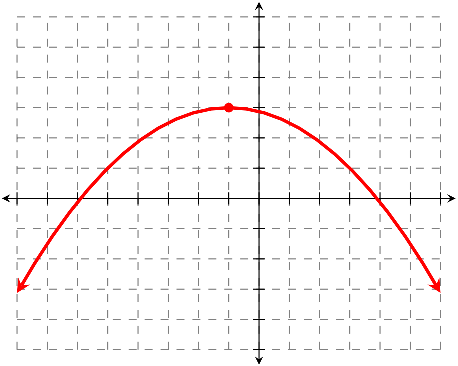

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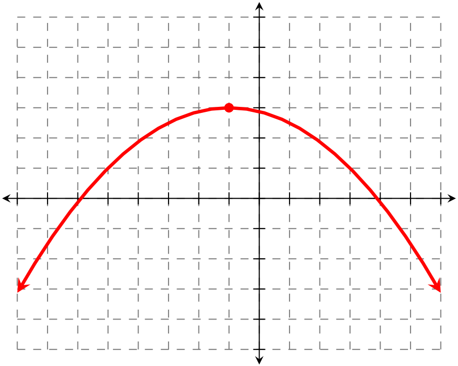

Vertex: $(-1, 3)$

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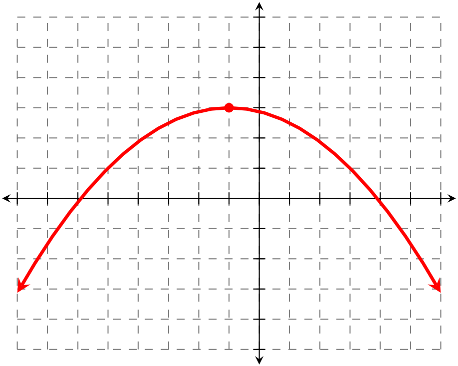

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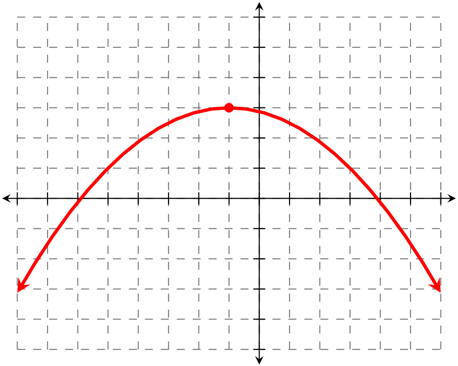

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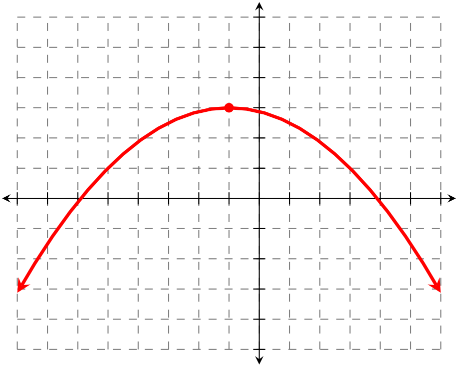

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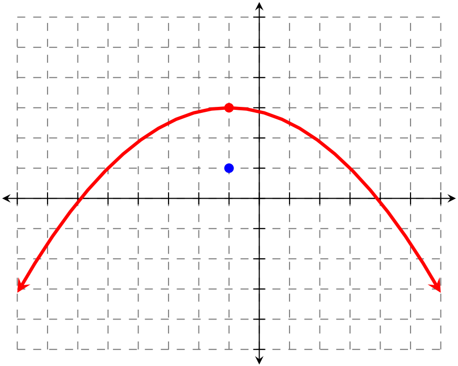

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Focus: $(-1, 3-2)$

Focus: $(-1, 1)$

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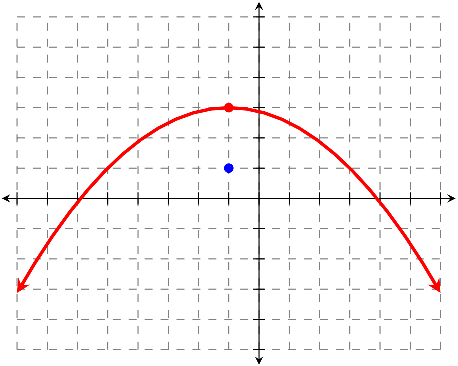

Vertex: $(-1, 3)$

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Focus: $(-1, 3-2)$

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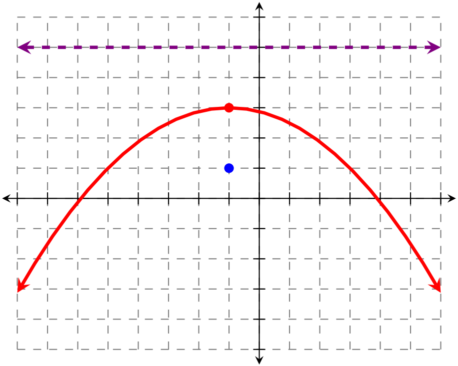

Vertex: $(-1, 3)$

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Focus: $(-1, 3-2)$

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Directrix: $y = 3+2$

Directrix: $y = 5$

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Find the vertex, focus, and directrix for $(y-2)^2 = 12(x+1)$.

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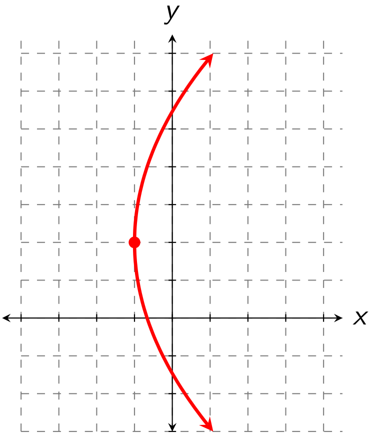

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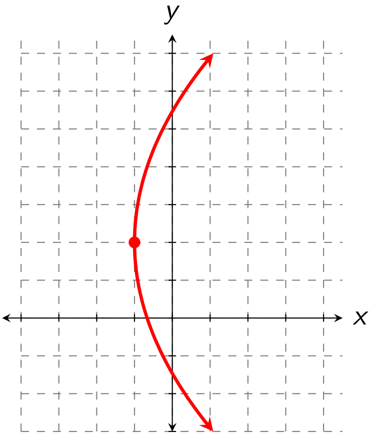

Vertex: $(-2, 1)$

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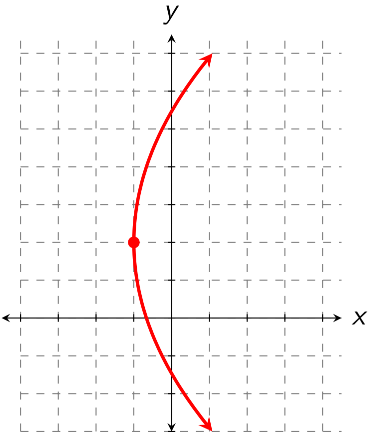

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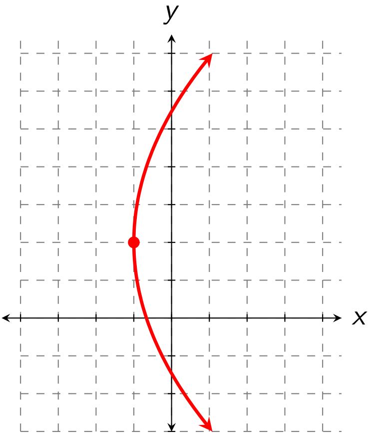

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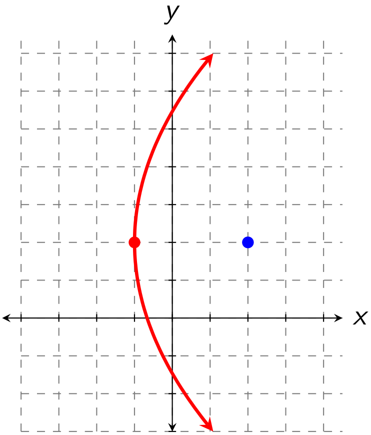

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Focus: $(-1+3, 2) \rightarrow (2, 2)$

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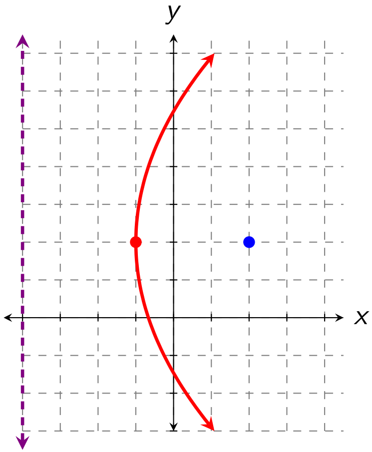

Vertex: $(-2, 1)$

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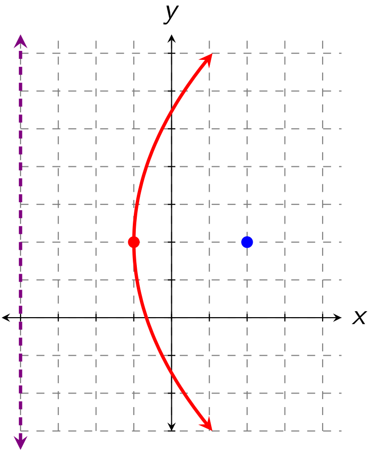

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$x = -4$

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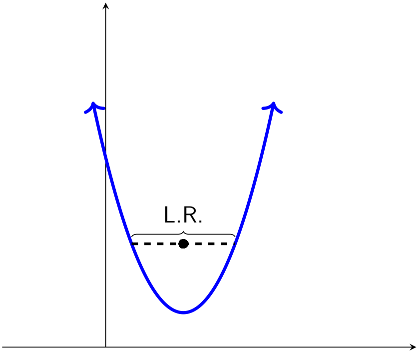
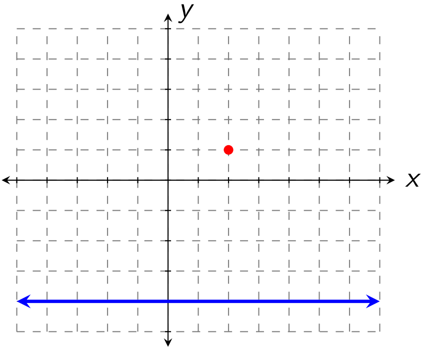


Example 3

Find the standard form of the parabola with focus $(2, 1)$ and directrix $y = -4$.



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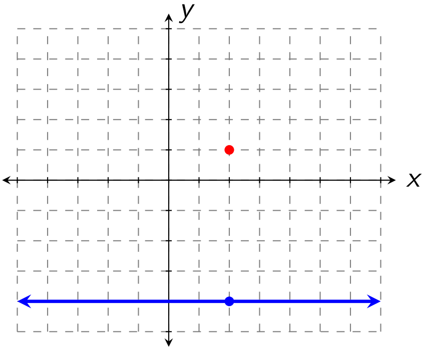

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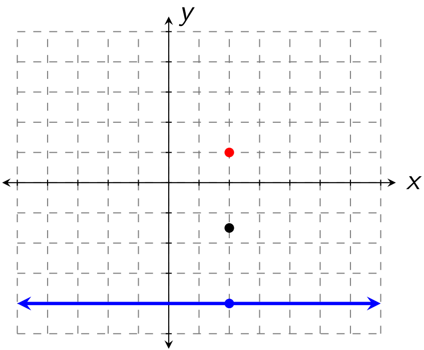

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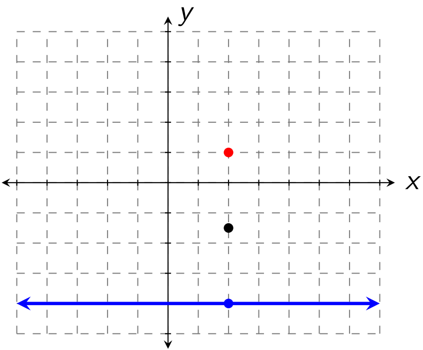

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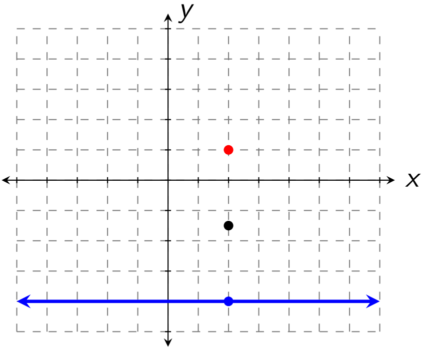

Vertex: $(2, -1.5)$

$p = 2.5$

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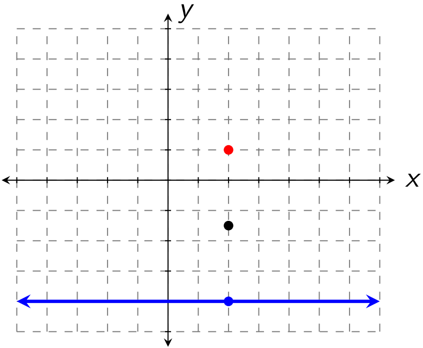

Vertex: $(2, -1.5)$

$p = 2.5$

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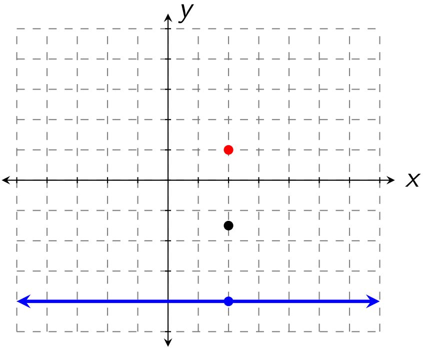

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$(x-2)^2 = 10(y+1.5)$

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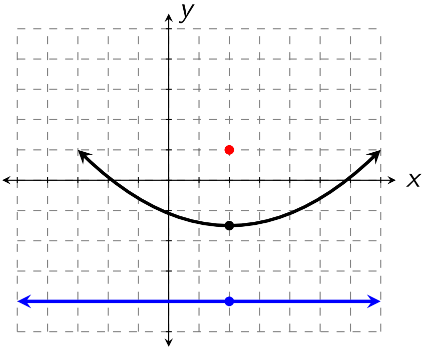

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Converting Equations

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2. Use the relationship that $4p = \frac{1}{a}$.

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Find the vertex, focus, and directrix of the following.

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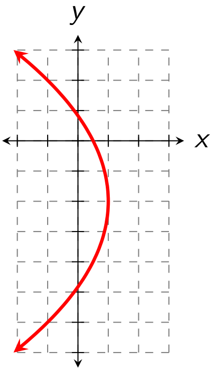

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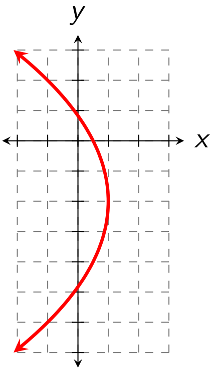

Vertex: $(1, -2)$

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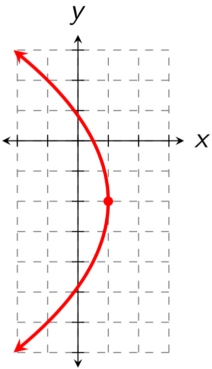

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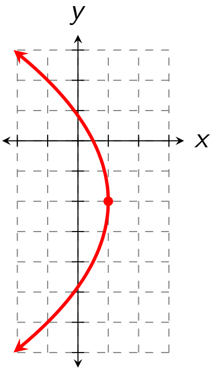

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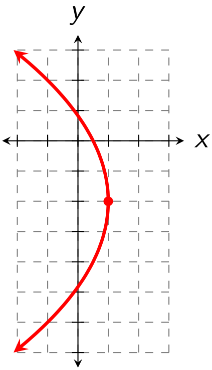

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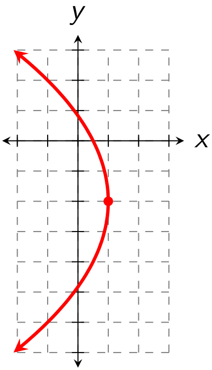

Vertex: $(1, -2)$

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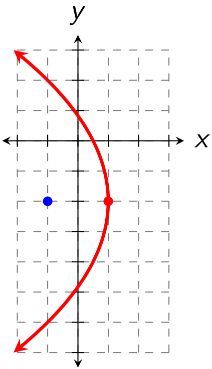

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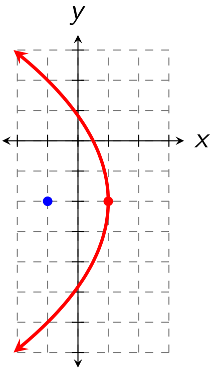

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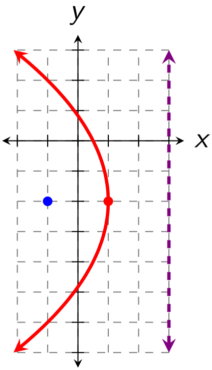

Vertex: $(1, -2)$

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Focus: $(1 - 2, -2)$

Focus: $(-1, -2)$

Directrix: $x = 1 + 2$

Directrix: $x = 3$

Example 4b

(b) $x^2 - 2x + 5y = 1$

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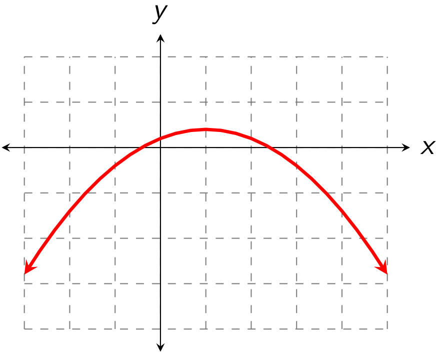
(b) $x^2 - 2x + 5y = 1$



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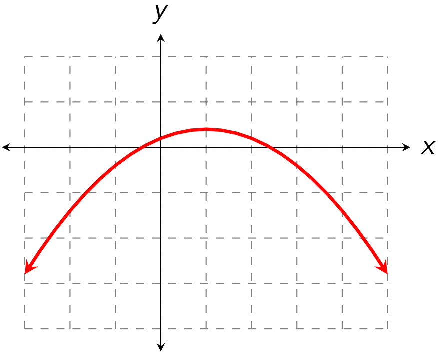
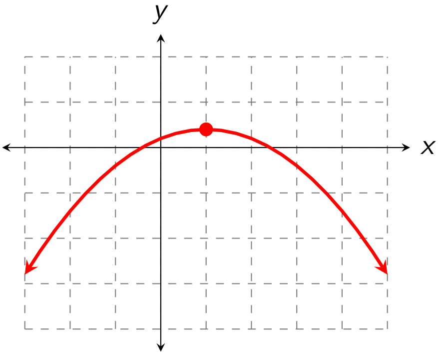


Vertex: $\left(1, \frac{2}{5}\right)$

Example 4b

(b) $x^2 - 2x + 5y = 1$



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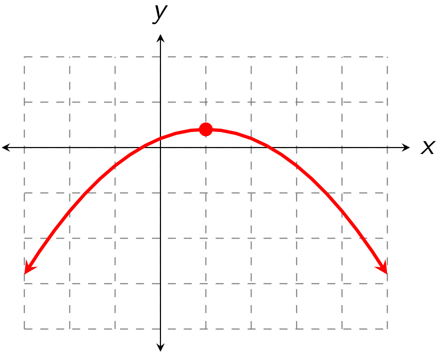

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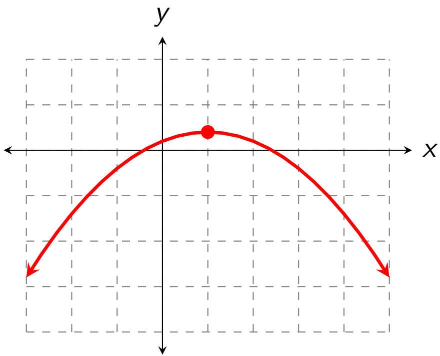

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$p = 5/4$

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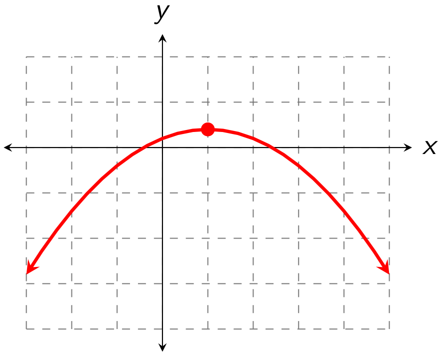


Vertex: $\left(1, \frac{2}{5}\right)$

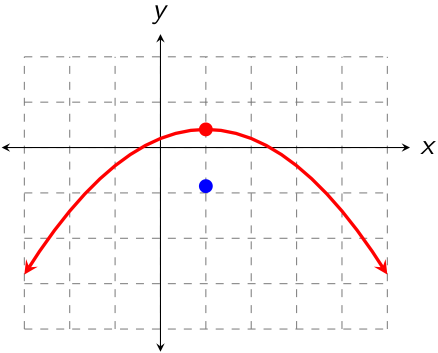
$4p = |5|$

$p = 5/4$

Focus: $\left(1, \frac{2}{5} - \frac{5}{4}\right)$

Example 4b

(b) $x^2 - 2x + 5y = 1$



Vertex: $\left(1, \frac{2}{5}\right)$

$4p = |5|$

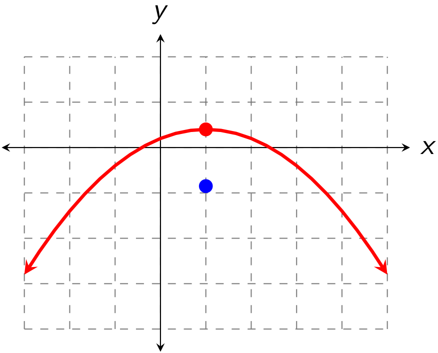
$p = 5/4$

Focus: $\left(1, \frac{2}{5} - \frac{5}{4}\right)$

Focus: $\left(1, -\frac{17}{20}\right)$

Example 4b

(b) $\quad x^2 - 2x + 5y = 1$



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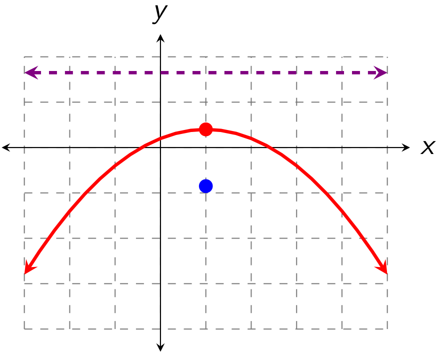
Directrix: $y = \frac{2}{5} + \frac{5}{4}$

Example 4b

(b) $x^2 - 2x + 5y = 1$



Vertex: $\left(1, \frac{2}{5}\right)$

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Directrix: $y = \frac{33}{20}$

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If we rotate a parabola around its axis of symmetry, we obtain a 3-D model of a parabola called a paraboloid, or a paraboloid of revolution.

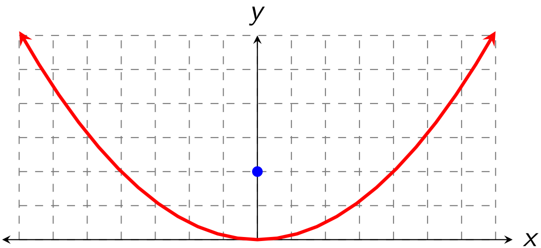
The nature of paraboloids allows signals to be sent to or from the focus in a directed manner.

Example 5

A satellite dish is to be constructed in the shape of a paraboloid of revolution. If the receiver placed at the focus is located 2 ft above the vertex of the dish, and the dish is to be 12 feet wide, how deep will the dish be?

Example 5

If we place the vertex at the origin and open the graph upward,



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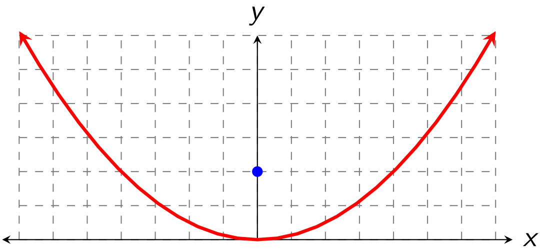

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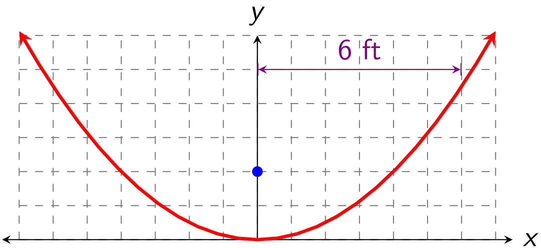

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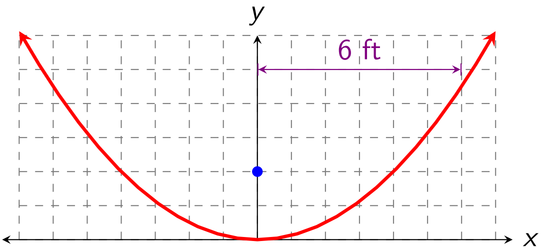

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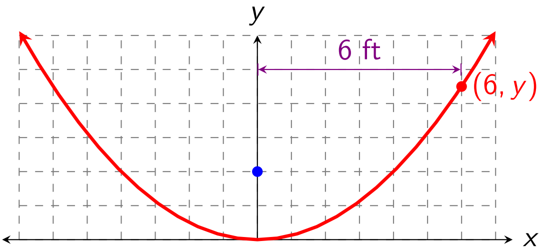

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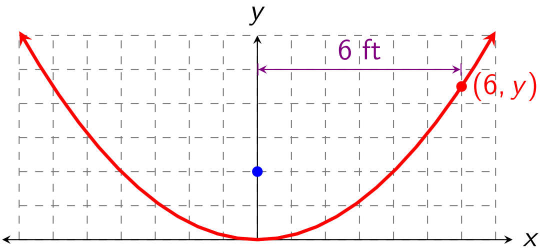

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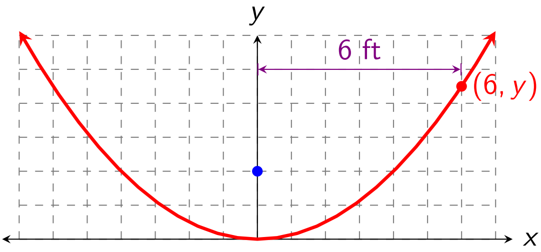

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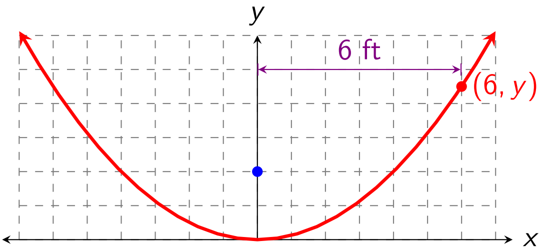

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$$y = \frac{36}{8}$$

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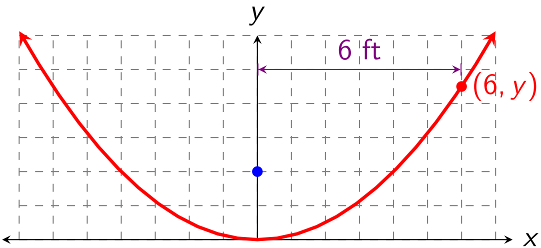

$$x^2 = 8y$$

$$6^2 = 8y$$

$$36 = 8y$$

$$y = \frac{36}{8}$$

$$y = \frac{9}{2}$$